# Agglomeration in Purely Neoclassical and Symmetric Economies*

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**Abstract**

This article demonstrates the emergence of agglomeration unaccompanied by any conventional explanatory factors such as scale economies, externalities or comparative advantages. Agglomeration forms out of inter-regional migration prompted by households seeking the type of consumers that complements their endowments. We construct a general equilibrium model with four commodities, four types of heterogeneous households, and linear production over two regions. Spatial sorting leads to an uneven distribution of people in equilibrium. This is driven by consumers' inclination to co-locate with a certain type of households who are endowed with the commodity they like to consume as is or to be used as an input to produce the commodity they like to consume. Our findings are robust against additional assumptions, including inter-regional trades and portability of endowments.

**Keywords**: Agglomeration, general equilibrium, spacial sorting
**JEL classification**: R13

"Therefore, it follows that if assumptions a1-a4 are upheld, there exists either a trivial solution, or no (price taking) competitive equilibrium. In short, the spatial impossibility theorem says that the smooth market mechanism alone cannot generate spatial agglomeration of activities." (Fujita [Fuj86], pp. 113-114).

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# 1 Introduction

Here we examine the circumstances underlying equilibrium population agglomeration in the context of a completely standard economy, namely without externalities or imperfect competition, but with ordinary utility functions and constant returns to scale production. Whatever equilibria there are will clearly be Pareto efficient. And symmetric equilibria will be present. In such a situation, what force can possibly cause population to agglomerate, and importantly, can this force complement or substitute for the agglomerative forces more commonly used in the literature, such as the New Economic Geography or externalities?

As we shall explain, it is a bit puzzling and surprising that agglomeration can be generated in such a simple neoclassical model, starting with a completely symmetric situation. In fact, transportation cost can be zero or positive; the results are identical. In equilibrium, the regions or locations are autarkic, but the population distributions can be asymmetric. In the end, it is complementarity of types of consumers through their endowments that causes agglomeration. Next, we detail the strategy for our analysis.

Our focus is on a very specific example for tractability and expository reasons. We adopt and then adapt the example of Kehoe [Keh85]. This classical example is aspatial, so it is best to imagine it to have only one region. There are four commodities and four consumers with different Cobb-Douglas utilities, but two different producers with constant returns to scale technologies. Constant returns to scale simplifies matters, since equilibrium profits must be zero. Thus, there is no need to worry about profit distribution and the zero profit conditions yield restrictions on equilibrium prices, useful for computational purposes. The key properties of this example are that it is quite simple **but features 3 equilibria**. Heterogeneous income effects play a big role both in Kehoe's example and in our work.

Next, we adapt Kehoe's model to the spatial context. There will be 2 identical regions or locations. There will be measure 1 of each of the four types of consumer. The same production technologies are available in each region. There are now 8 commodities, 4 in each region. Consumers can move between regions at no cost, as is standard in the literature.

We consider three versions of the model with differing portability of endowments. In the first version, endowments move with the consumers. An example of a portable endowment is labor. In the second, endowments are not portable but income derived from endowments moves with the consumers. Notice that land is an example of an endowment that is not portable. In the third, both types of endowments above are present.

Our model and results are perfectly consistent with the spatial impossibility theorem as stated by Fujita and Thisse ([FT13], p. 39), even though we have a continuum of agents:

**The Spatial Impossibility Theorem**. Assume a two-region economy with a finite number of consumers and firms. If space is homogeneous, transport is costly, and preferences are locally nonsatiated, there is no competitive equilibrium involving transportation.

The remainder of the paper proceeds as follows: In the following section, we lay out the model. We present three versions of it: Endowments are portable in section 3, not portable in section 4, and partially portable in section 5. We then discuss the possibility of inter-regional trade in section 6. Section 7 examines the role the number of commodities plays in generating agglomeration. Section 8 concludes.

## 2 The Model

We build our model on the production economy analyzed by Kehoe [Keh85]. His model features a single region with four commodities $i=1,\cdots,4$, four consumers $j=1,\cdots,4$, and linear technology. We add one more region to it and examine if agglomeration takes place in the absence of scale economies.

There is a unit mass of each of four types of consumers, who take up residence in either region $a$ or $b$. Their relocation incurs no cost. We denote the population distribution by $\lambda=\begin{bmatrix}\lambda_1 & \lambda_2 & \lambda_3 & \lambda_4\end{bmatrix}$, where $\lambda_j\in[0,\ 1]$ is a fraction of type-$j$ consumers who live in region $a$. In what follows we use a superscript to denote a row and commodity $i$, and a subscript to denote a column and consumer type $j$ and/or region $a$ or $b$.

A consumer of type $j$ maximizes $u_j\left(x_j\right)=\prod_{i=1}^{4}\left(x_j^i\right)^{\alpha_j^i}$ subject to $\pi\cdot x_j\leq\pi\cdot w_j$, where $x_j=\begin{bmatrix}x_j^1 & x_j^2 & x_j^3 & x_j^4\end{bmatrix}^\top$ is his consumption bundle, $w_j=\begin{bmatrix}w_j^1 & w_j^2 & w_j^3 & w_j^4\end{bmatrix}^\top$ is his endowment, and $\pi=\begin{bmatrix}\pi^1 & \pi^2 & \pi^3 & \pi^4\end{bmatrix}^\top$ is a price vector. **As we will show below, the equilibrium price vector will be the same in both regions**. Expenditure share $\alpha$ and endowment $w$ are specified as

$$\alpha=\begin{bmatrix}.52 & .86 & .5 & .06\\ .4 & .1 & .2 & .25\\ .04 & .02 & .2975 & .0025\\ .04 & .02 & .0025 & .6875\end{bmatrix}\quad\text{and}\quad w=\begin{bmatrix}50 & 0 & 0 & 0\\ 0 & 50 & 0 & 0\\ 0 & 0 & 400 & 0\\ 0 & 0 & 0 & 400\end{bmatrix}. \tag{1}$$

For instance, type-4 consumer's expenditure share of commodity 1 is .06, and he is endowed with zero units of it.

Technology is linear and specified by technological process

$$A=\begin{bmatrix}-1 & 0 & 0 & 0 & 6 & -1\\ 0 & -1 & 0 & 0 & -1 & 3\\ 0 & 0 & -1 & 0 & -4 & -1\\ 0 & 0 & 0 & -1 & -1 & -1\end{bmatrix}. \tag{2}$$

The supply is $Ay$, where $y$ is a $6\times 1$ non-negative vector indicating how much of each column of $A$ is deployed in production. Since all commodities are goods, the first four entries of $y$ are zero in equilibrium, i.e., the first four firms do not engage in production.

Inter-regional trade does not occur in equilibrium, no matter transport cost (see section 6 to follow). That is because equilibrium prices equate across regions. This is similar to the Factor Price Equalization Theorem. Here, utility levels play the role of product prices, and goods prices play the role of factor prices.

A region is in **intra-regional equilibrium** when each consumer maximizes his utility level subject to his budget and all eight commodity markets clear. Namely, excess demand $(x_a - w)\lambda^\top - Ay_a = \mathbb{0}$ in region $a$; similarly, $(x_b - w)(\mathbb{1} - \lambda)^\top - Ay_b = \mathbb{0}$ in region $b$.[1] Furthermore, two regions are in **inter-regional equilibrium** if 1) every region is in intra-regional equilibrium, and 2) utility levels are the same in both regions type by type.[2] Whereas the first requirement guarantees that the gains from trade are exhausted region by region, the second requirement guarantees that the utility gains from relocation are exhausted across regions.

A firm earns zero profit in equilibrium because of constant returns to scale. Thus, the intra-regional equilibrium price vector must be orthogonal to the column space of $A$. In addition, Walras' law enables the normalization of prices, $\sum \pi^i = 1$. Combined, these imply that the intra-regional equilibrium price vector $\pi$ must be of the form $\begin{bmatrix} \pi^1 & \frac{1}{4} & \frac{7\pi^1 - 1}{3} & \frac{-10\pi^1}{3} + \frac{13}{12} \end{bmatrix}^\top$, $\pi^1 \in \left(\frac{1}{7}, \frac{13}{14}\right)$ in intra-regional equilibrium. Let $\Pi^\perp$ be a set of all such price vectors. Note that four units of commodity 2 function as a numéraire in our economy. Also note that $\pi^2$, $\pi^3$ and $\pi^4$ are a linear and thus monotone function of $\pi^1$ over $\Pi^\perp$. Moreover, non-numéraire commodity prices $\pi^3$ and $\pi^4$ are strictly monotone in $\pi^1$, rendering them interchangeable when evaluating the monotonicity of a function. In what follows we say a function is monotone over $\Pi^\perp$ to mean that within the restricted domain $\Pi^\perp (\subset \mathbb{R}^4_{++})$ a function is monotone in terms of a non-numéraire price $\pi^1$, $\pi^3$ or $\pi^4$.

We classify the economy into three categories by portability of endowments. Section 3 presents an economy where the endowment must move with its owner, and in section 4 it cannot do so. Section 5 presents a hybrid between the two. In all three cases, endowments or outputs cannot be shipped. The only time they move is when they accompany their owner in section 3 and part of section 5. We discuss the case where they can be transported in a separate section 6 to follow.

To better distinguish section 6 from sections 3 to 5, we use the term "portable" to indicate whether a consumer must or cannot take his endowments with him when he relocates,[3] and "tradable" to indicate whether outputs can be shipped out to another

[1] A number in script font denotes a column or row vector (whichever is appropriate) consisting of repeated entries of a same number, e.g., $\mathbb{0} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^\top$ and $\mathbb{1} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$ in the preceding equations.

[2] Except when $\lambda_j = 0$ or $1$.

[3] In our context, portability implies that a consumer not simply **can** but **must** move with his endowment and **put it to use where he resides**. This eliminates inter-regional commute as labor cannot be employed

| term | portable | tradable | mobile | transferable |
|---|---|---|---|---|
| applicable to | endowment | output | people | income |
| section 3 | • | | • | n/a* |
| section 4 | | | • | • |
| section 5 | partially | | • | • |
| section 6 | either | • | • | n/a* |
| section 7 | either | either | • | • |

**Table 1.** Types of cross-border movements. *Transferability makes difference only when portability is absent. In sections 3 and 6, all types take their endowments with them so that income is **always** generated where its recipient is. There is no reason to transfer income whether it is transferable or not.

region. Income from endowments is "transferable" if it can be cashed in in a different region than where it was generated. We reserve the term "mobile" to refer to worker's geographic mobility.

We assume throughout the paper that workers are perfectly mobile and incomes are transferable at no cost. Perfect mobility enables us to characterize equilibrium as a state where utility levels equate across regions by virtue of internal migration. It also rationalizes portability: We are not able to discuss portability unless workers are mobile in the first place. Transferability makes no difference when endowments are portable. However, a lack of it would add inessential constraints to the analysis when endowments are not portable. In addition, we assume that endowment cannot cross borders unaccompanied by its owner. If it can, it will open the way for an illogical possibility where endowment can be shipped but output cannot. Namely, tradability is defined exclusively in terms of output; endowment can never be shipped by itself. Table 1 summarizes the terminologies used in this paper.

For example, labor is portable and land is not portable. Neither one of them is tradable. Their portability or tradability notwithstanding, their owners are perfectly mobile and income generated from them are freely transferable.

# 3 Portable Endowments

## 3.1 Equilibria

We begin with the economy where consumers must take their endowment with them and use it where they move to.

The value functions or utility levels of type 3 and 4 are strictly monotone over $\Pi^{\perp}$ (cf.

outside where a worker lives.

figure 1. We picked $\pi^1$ for illustrative purposes). In inter-regional equilibrium, consumer $j$ achieves the same utility level regardless of his residency as a result of free mobility. If a price differs between the regions the utility level will not equate among type 3, nor among type 4. Therefore, in inter-regional equilibrium, $\pi$ must be identical in both regions. Furthermore, since consumers face the same prices wherever they live and the individual endowments are independent from $\lambda$, the **individual** consumption levels are the same in both regions. The next proposition summarizes this observation:

Proposition 3.1 Inter-Regional Equilibrium

*Suppose that at least one type of consumer has a strictly monotone value function over $\Pi^{\perp}$. If an inter-regional equilibrium exists, $\pi_a = \pi_b$ and $x_a = x_b$.*

*Proof.* Suppose that a type-$j$ consumer has a strictly monotone value function over $\Pi^{\perp}$, and that $\pi_a \neq \pi_b$. Then his utility level changes depending on where he is: $u_j\left[x_j\left(\pi_a,\ w_j\right)\right] \neq u_j\left[x_j\left(\pi_b,\ w_j\right)\right]$, and thus $\pi_a$ and $\pi_b$ do not make an inter-regional equilibrium price vector. Therefore, if an inter-regional equilibrium exists, $\pi_a = \pi_b$. Accordingly, $x\left(\pi_a,\ w\right) = x\left(\pi_b,\ w\right)$. $\square$

Regardless, **region-wide** consumption $x\lambda^{\top}$ and $x(\mathbb{1}-\lambda)^{\top}$ will differ from each other because the population will not necessarily split evenly between the two regions.

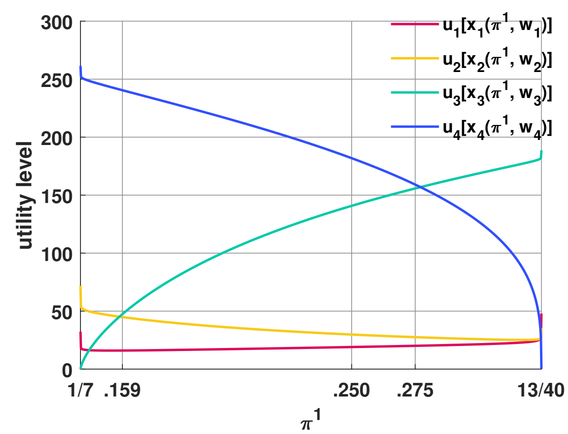



**Figure 1.** Value functions

There are many inter-regional equilibria (see appendix A.1 for details). We present below three of them for example. The equilibrium specification includes the inter-regional equilibrium price vector $\pi$ $(= \pi_a = \pi_b)$, population distribution $\lambda$, total population in each region $\lambda\mathbb{1}$ and $(\mathbb{1}-\lambda)\mathbb{1}$ $(= 4-\lambda\mathbb{1})$, individual demand $x$ $(= x_a = x_b)$, utility level $u$ $(= u_a = u_b)$, and activity levels $y_a$ and $y_b$. We begin with the equilibria in Kehoe [Keh85] mirrored across the two regions. Whereas this is not part of our inter-regional equilibria, we place Kehoe's value of $y$ side by side with corresponding $y_a$ and $y_b$ in each equilibrium. The subsequent section will draw a comparison between them. What is crucial is the value of $\lambda$, representing agglomeration. Note that computational algorithms do not find unstable equilibria. The following are therefore stable.

Equilibrium #1

$$\begin{aligned}
\pi &= \begin{bmatrix} 0.159 & 0.250 & 0.0387 & 0.552 \end{bmatrix}^\top \\
\lambda &= \begin{bmatrix} 0.705 & 0.493 & 0.822 & 0.871 \end{bmatrix} \\
\begin{bmatrix} \lambda\mathbb{1} & 4-\lambda\mathbb{1} \end{bmatrix} &= \begin{bmatrix} 2.89 & 1.11 \end{bmatrix} \\
x &= \begin{bmatrix} 26 & 67.43 & 48.49 & 83.09 \\ 12.75 & 5 & 12.37 & 220.77 \\ 8.25 & 6.47 & 119 & 14.28 \\ 0.58 & 0.45 & 0.07 & 27 \end{bmatrix} \\
u &= \begin{bmatrix} 16.0 & 44.9 & 47.4 & 240.5 \end{bmatrix} \\
y_a &= \begin{bmatrix} 0 & 0 & 0 & 0 & 33.8 & 74.3 \end{bmatrix}^\top \\
y_b &= \begin{bmatrix} 0 & 0 & 0 & 0 & 8.9 & 6.9 \end{bmatrix}^\top \\
y &= \begin{bmatrix} 0 & 0 & 0 & 0 & 42.7 & 81.2 \end{bmatrix}^\top (= y_a + y_b)
\end{aligned}$$

Equilibrium #2

$$\begin{aligned}
\pi &= \begin{bmatrix} 0.250 & 0.250 & 0.250 & 0.250 \end{bmatrix}^\top \\
\lambda &= \begin{bmatrix} 0.839 & 0.884 & 0.252 & 0.227 \end{bmatrix} \\
\begin{bmatrix} \lambda\mathbb{1} & 4-\lambda\mathbb{1} \end{bmatrix} &= \begin{bmatrix} 2.20 & 1.80 \end{bmatrix} \\
x &= \begin{bmatrix} 26 & 43 & 200 & 24 \\ 20 & 5 & 80 & 100 \\ 2 & 1 & 119 & 1 \\ 2 & 1 & 1 & 275 \end{bmatrix} \\
u &= \begin{bmatrix} 19.1 & 29.8 & 140.8 & 181.9 \end{bmatrix} \\
y_a &= \begin{bmatrix} 0 & 0 & 0 & 0 & 14.2 & 11.3 \end{bmatrix}^\top \\
y_b &= \begin{bmatrix} 0 & 0 & 0 & 0 & 37.8 & 57.7 \end{bmatrix}^\top \\
y &= \begin{bmatrix} 0 & 0 & 0 & 0 & 52.0 & 69.0 \end{bmatrix}^\top (= y_a + y_b)
\end{aligned}$$

Equilibrium #3

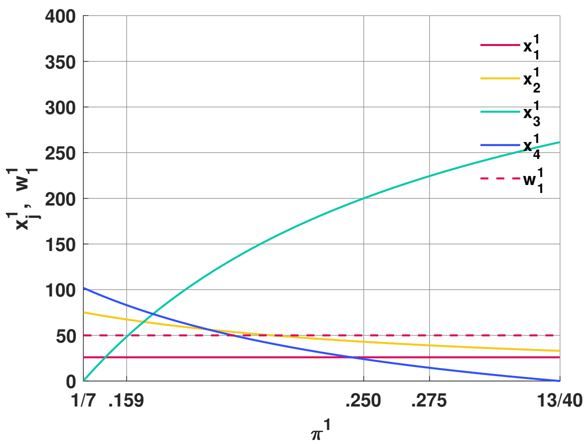
$$
\begin{aligned}
\pi &= \begin{bmatrix} 0.275 & 0.250 & 0.309 & 0.166 \end{bmatrix}^\top \\
\lambda &= \begin{bmatrix} 0.267 & 0.924 & 0.161 & 0.105 \end{bmatrix} \\
\begin{bmatrix} \lambda\mathbb{1} & 4-\lambda\mathbb{1} \end{bmatrix} &= \begin{bmatrix} 1.46 & 2.54 \end{bmatrix} \\
x &= \begin{bmatrix} 26 & 39.07 & 224.36 & 14.50 \\ 22.01 & 5 & 98.77 & 66.49 \\ 1.78 & 0.81 & 119 & 0.54 \\ 3.31 & 1.50 & 1.86 & 27 \end{bmatrix} \\
u &= \begin{bmatrix} 20.1 & 27.6 & 155.8 & 159.1 \end{bmatrix} \\
y_a &= \begin{bmatrix} 0 & 0 & 0 & 0 & 42.7 & 46.3 \end{bmatrix}^\top \\
y_b &= \begin{bmatrix} 0 & 0 & 0 & 0 & 10.5 & 18.8 \end{bmatrix}^\top \\
y &= \begin{bmatrix} 0 & 0 & 0 & 0 & 53.2 & 65.1 \end{bmatrix}^\top (= y_a + y_b)
\end{aligned}
$$

Not only the size but also the composition of types differ between the regions in equilibrium. For instance, at $\pi_a^1 = \pi_b^1 = .250$, type 1 and 2 sort into one region and type 3 and 4 into the other. Thus, each region consists of a different mix of types. We explain the reason behind the spatial sorting in appendix A.1.

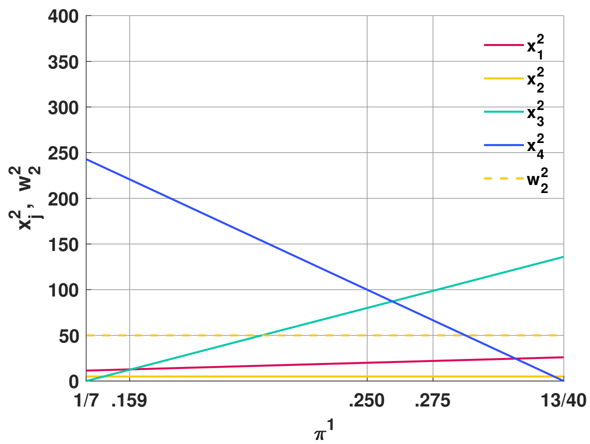
It is only the economy-wide **demand** that is influenced by $\lambda$. Production simply scales up or down as needed in an attempt to fill the demand both large and small regardless of the price. The demand is **non-linear** over $\Pi^\perp$. In contrast, the supply **does not depend on** $\pi$ in the sense that so long as $\pi \in \Pi^\perp$ the firms always earn zero profit whatever $y_a$ and $y_b$ they choose. If $Ay_a$ and $Ay_b$ happen to square with $(x-w)\lambda^\top$ and $(x-w)(\mathbb{1}-\lambda)^\top$, then that is an inter-regional equilibrium; or else there is no inter-regional equilibrium at the $\pi \in \Pi^\perp$ and $\lambda$ under consideration.

### 3.2 Comparison between Single- and Two-Region Economies

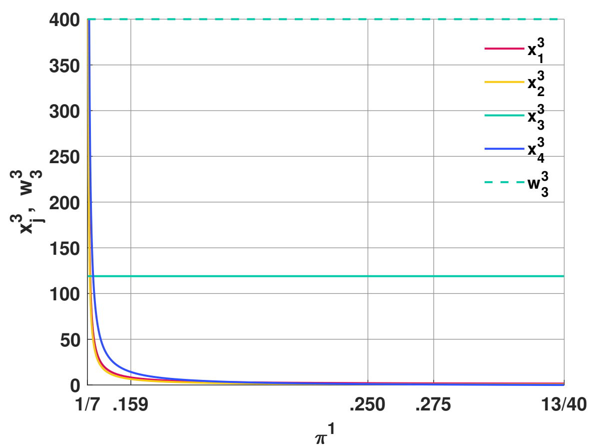
All the inter-regional equilibria we found in section 3.1 are closely related to the three equilibria in Kehoe [Keh85] in several ways. Let us call our two-region economy $E^{2R}$ and Kehoe's single-region economy $E^{1R}$. In $E^{1R}$ the aggregate net demand for commodity $i$ is a simple sum of the individual net demand, $(x^i - w^i)\mathbb{1}$. It appears as a vertical sum of each type's demand (quantity is on the vertical axis), less the endowment in figure 2. By contrast, in $E^{2R}$ the economy-wide net demand for commodity $i$ becomes a weighted sum of the individual net demand, $(x^i - w^i)\lambda^\top$ and $(x^i - w^i)(\mathbb{1}-\lambda)^\top$. It appears as a vertical sum of each type's demand, less the endowment in figure 2 with an uneven weight of $\lambda$ in region $a$ and $\mathbb{1}-\lambda$ in region $b$. $E^{1R}$ can be thought of as a special case of $E^{2R}$ where $\lambda = \mathbb{0}$ or $\mathbb{1}$, with the requirement $u_a(\cdot) = u_b(\cdot)$ removed.



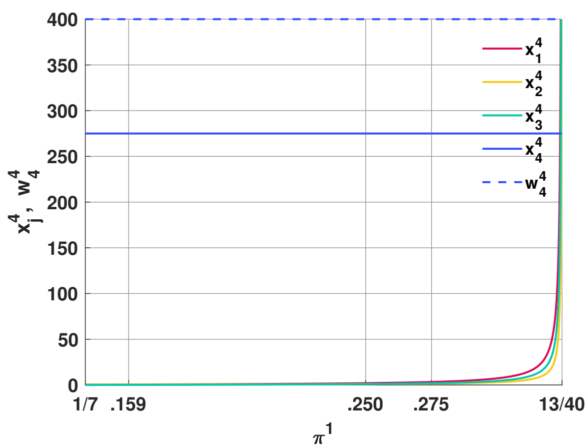

(a) Commodity 1




(b) Commodity 2




(c) Commodity 3




(d) Commodity 4

**Figure 2.** Individual demand for each commodity by type in solid lines and endowment in broken lines.

With this added degree of freedom, one may be tempted to speculate that $E^{2R}$ takes a different equilibrium price than $E^{1R}$, and that there are more than three equilibrium prices possible. For instance, whereas the aggregate net demand for commodity 1 is monotone increasing in $\pi^1$, with the right mix of $\lambda$ the economy-wide net demand for commodity 1 may no longer be increasing or monotone. However, this turns out not to be the case:

Proposition 3.2 Equilibrium Prices in Single- and Two-Region Economies

*Suppose that at least one type of consumer has a strictly monotone value function over $\Pi^{\perp}$. The set of inter-regional equilibrium price vectors $\Pi^{2R}$ in $E^{2R}$ is a subset of its counterpart $\Pi^{1R} \times \Pi^{1R}$ in $E^{1R}$.*

*Proof.* Suppose there exists a pair of price vectors $(\pi_a, \pi_b) \in \Pi^{2R}$ but $\pi_a \notin \Pi^{1R}$. Recall from proposition 3.1 that $\pi_a = \pi_b$ in inter-regional equilibrium. Since $\pi_a (= \pi_b)$ clears all four markets in each region of $E^{2R}$, there exists such $y_a,\ y_b \geq \mathbb{0}$ that

$$\begin{aligned} [x(\pi_a) - w]\,\lambda^\top &= Ay_a, \quad \text{and} \\ [x(\pi_a) - w]\,(\mathbb{1} - \lambda)^\top &= Ay_b. \end{aligned} \tag{3}$$

Aggregate them to obtain the countrywide market clearance in $E^{2R}$:

$$[x(\pi_a) - w]\,\mathbb{1} = A(y_a + y_b). \tag{4}$$

On the other hand, since $\pi_a \notin \Pi^{1R}$, there is no $y \geq \mathbb{0}$ such that

$$[x(\pi_a) - w]\,\mathbb{1} = Ay \tag{5}$$

in $E^{1R}$. Since the left-hand sides of (4) and (5) are identical, $A(y_a + y_b) = Ay$. Then

$$y_a + y_b = y. \tag{6}$$

Whereas $y_a + y_b$ exists, $y$ does not, contradicting each other. Therefore, if $\pi_a$ clears the regional markets in $E^{2R}$, it also clears the markets in $E^{1R}$. Hence $\pi_a \in \Pi^{1R}$ and thus $\Pi^{2R} \subseteq \Pi^{1R} \times \Pi^{1R}$. □

Depending on the weight $\lambda$, one of the regions can and does feature an **intra**-regional equilibrium whose price vector falls outside $\Pi^{1R}$ thanks to the added degree of freedom mentioned above. However, these equilibria will not make an **inter**-regional equilibrium because the inter-regional equilibrium price vector has to be a member of $\Pi^{1R}$ as proved above. We present one such example in figure 3.[4] Observe that region $a$ features one intra-regional equilibrium price vector in $\Pi^{1R}$ and two intra-regional equilibrium price vectors outside $\Pi^{1R}$. The latter two will not make the list for inter-regional equilibria because there is no corresponding intra-regional price vectors found in region $b$. Thus, as we discussed earlier in this section, the expansion of a set of $\lambda$ from $\{\mathbb{0},\ \mathbb{1}\}$ of $E^{1R}$ to $[0,\ 1]^4$ of $E^{2R}$ does unleash lots of price vectors outside $\Pi^{1R}$, but these are only intra-regional equilibrium price vectors. In inter-regional equilibrium, the price vector still has to be selected from $\Pi^{1R}$. See appendix A.2 for more on net demand.

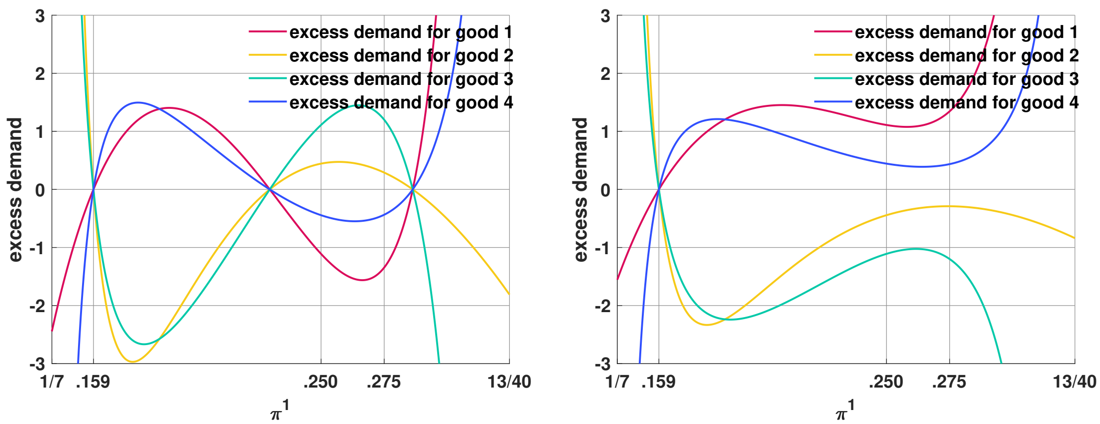
Note also that the zero-profit condition further implies $\Pi^{2R} \subseteq \Pi^{1R} \times \Pi^{1R} \subseteq \Pi^{\perp} \times \Pi^{\perp}$.

We derive two equivalencies from proposition 3.2:

Corollary 3.1 Supply and Demand in Single- and Two-Region Economies

*Suppose that at least one type's value function is strictly monotone over $\Pi^{\perp}$. The countrywide net demand in $E^{2R}$ is identical to its corresponding aggregate net demand in $E^{1R}$ in equilibrium. Furthermore, the countrywide activity level $y_a + y_b$ in $E^{2R}$ is equal to its corresponding $y$ in $E^{1R}$ in equilibrium.*

[4] As mentioned earlier in this section, $y_a$ and $y_b$ are not a function of $\pi \in \Pi^{\perp}$. We compute excess demand $(x - w)\lambda^\top - Ay_a$ with the midpoint between $y_a$ that clears the even-numbered markets and $y_a$ that clears the odd-numbered markets in figure 3 for illustrative purposes. These $y_a$'s are equal to each other only in intra-regional equilibrium, turning excess demand zero. The same goes for region $b$ as well.




(a) Excess demand in region $a$.

(b) Excess demand in region $b$.

**Figure 3.** Regional excess demand when $\lambda = \begin{bmatrix} .972 & .126 & .569 & .641 \end{bmatrix}$. A region is in intra-regional equilibrium when all four excess demands are zero. Two regions are furthermore in intra-regional equilibrium when they share the same intra-regional equilibrium price vector. In this case, region $a$ has three intra-regional equilibria and region $b$ has one; $E^{2R}$ as a whole has one inter-regional equilibrium.

*Proof.* Aggregate region $a$'s net demand on the left-hand side of (3) with region $b$'s to obtain $x(\pi_a)\lambda^\top + x(\pi_b)(\mathbb{1} - \lambda^\top) - w$. Since at least one type has a strictly monotone value function, proposition 3.1 implies $\pi_a = \pi_b$. Then the countrywide net demand becomes $x(\pi_a) - w$, which is the net demand in $E^{1R}$ when the price vector is $\pi_a$.

Furthermore, since the net aggregate demand is the same in both economies, the aggregate supply $A(y_a + y_b)$ in $E^{2R}$ is the same as $Ay$ in $E^{1R}$. □

*Remark.* This explains why $y_a + y_b = y$ in all three equilibria we listed in section 3.1. Indeed in any inter-regional equilibrium, $y_b = y - y_a$. Whereas there are many inter-regional equilibrium $y_a$ and $y_b$ depending on $\lambda$, $y$ remains the same because there is only one $y$ each for three price vectors in $\Pi^{1R}$ in $E^{1R}$. Put differently, there is a wide range of equilibrium $y_a$ and $y_b$ because the possible range of $\lambda$ is $[0,\ 1]^4$ in $E^{2R}$, but the sum of $y_a$ and $y_b$ has to come to one of only three $y$'s because the possible range of $\lambda$ in $E^{1R}$ is $\{\mathbb{0},\ \mathbb{1}\}(\subset [0,\ 1]^4)$.

In short, the demand has to keep to $u_a(\cdot) = u_b(\cdot)$, and the supply has to keep to $y_a + y_b = y$ in inter-regional equilibrium.

### 3.3 Scalable Equilibria and Spatial Sorting

Whereas a sample of inter-regional equilibria listed in section 3.1 involves an uneven presence of each type in a region, heterogeneous preferences do not necessarily lead to regional sorting. This section will establish that $\lambda$ of the form $c\mathbb{1}\left(= \begin{bmatrix} c & c & c & c \end{bmatrix}\right)$ with

$c \in [0,\ 1]$ constitutes an inter-regional equilibrium. In particular, $c = .5$ indicates that spatial sorting is not a requisite of inter-regional equilibria.

Evidently, such distributions constitute an **intra**-regional equilibrium because linear technology allows firms to rescale their production by a factor of $c$ and $1-c$ in respective regions to meet the smaller, but proportionally down scaled net aggregate demand from $E^{1R}$. In contrast, it is not all too obvious whether they further constitute an **inter**-regional equilibrium. We verify that each region will achieve the same utility level regardless of the value of $c$ selected.

Consider any equilibrium in $E^{1R}$. Material balance implies $\left(x^{1R} - w\right)\mathbb{1} = Ay^{1R}$. Multiply both sides by $c$ to obtain $\left(x^{1R} - w\right)(c\mathbb{1}) = A(cy^{1R})$. Given the equilibrium price in $E^{1R}$, the optimal bundle $x_a$ in $E^{2R}$ coincides with $x^{1R}$ in $E^{1R}$ because the individual demand is independent of $\lambda$, or in this case, $c$. In addition let $y_a = cy^{1R}$. Then the equation can be rewritten as $(x_a - w)(c\mathbb{1}) = Ay_a$, which is none other than the material balance in region $a$ itself. Therefore, region $a$ reaches an intra-regional equilibrium, as does region $b$. Furthermore, there is no inter-regional migration of consumers of any type. Since individual demand is independent of $c$, each type achieves the same utility level in either region under the $\pi^{1R}$ selected. On the supply end, $y_a + y_b = cy + (1-c)y = y$, in keeping with corollary 3.1. Therefore, any $c$ constitutes an inter-regional equilibrium, with any equilibrium price inherited from $E^{1R}$. Put differently, equilibria in $E^{1R}$ are scalable: any equilibrium in $E^{1R}$ can be implemented as an inter-regional equilibrium in $E^{2R}$ with an arbitrary $c \in [0,\ 1]$. In this case, both regions are simply a miniature copy of $E^{1R}$ with the identical composition of types. Consequently, spatial sorting can but does not have to take place in inter-regional equilibrium.

This observation draws on two features of the model. On the one hand, supply is linear. If $Ay^{1R}$ is in the production set, so are $Ay_a = cAy^{1R}$ and $Ay_b = (1-c)Ay^{1R}$. On the other hand, $\pi_a$ and $\pi_b$ aside, there is no channel through which individual demand matrices $x_a$ and $x_b$ respond to $c$. Thus, net aggregate demands in $E^{2R}$ are simply a scalar multiple of $(x^{1R} - w)\mathbb{1}$ in $E^{1R}$. Since the equilibrium price in $E^{1R}$ clears markets in $E^{2R}$ as shown above, the resultant allocation constitutes an inter-regional equilibrium. All combined, an equilibrium in $E^{1R}$ can be scaled down by an arbitrary factor without changing the level of utility, which in turn guarantees an existence of (infinitely many) corresponding inter-regional equilibria in $E^{2R}$.

# 4 Non-Portable Endowments

## 4.1 Inter-Regional Equilibria

This section considers the economy where endowments are not portable. We call the preceding economy with portable endowments $E^P$ and the economy with non-portable endowments currently under consideration $E^{NP}$. The previous section established that inter-regional equilibria exist in $E^P$. A lack of portability increases the complexity of the system in $E^{NP}$: Two regions are now interconnected not only through free mobility of consumers but also through combined incomes earned in respective regions. Added complexity notwithstanding, inter-regional equilibria exist in $E^{NP}$ as well.

Let us denote the exogenous distribution of endowments by $\mu \in [0,\ 1]^4$, where $\mu^i$ denotes a fraction of $w_j^i$ located in region $a$ for any $j$. Whereas the endowment that **belongs to** the residents in region $a$ is $\begin{bmatrix} \lambda_1 w_1 & \cdots & \lambda_4 w_4 \end{bmatrix}$, the endowment physically **located** in region $a$ is $\begin{bmatrix} \mu^1 w^1 \\ \vdots \\ \mu^4 w^4 \end{bmatrix}$. The former only determines the part of personal income earned in region $a$ (with the remainder earned in region $b$). It varies with (endogenous) $\lambda$. The latter is what residents consume or employ for production in region $a$. It does not vary because $\mu$ is exogenous and there is no importing or exporting of goods between two regions. Previously, $E^P$ is a version of an economy where $\mu$ is endogenous and set equal to $\lambda^\top$ so that the two matrices are identical.[5] In contrast, $E^{NP}$ is a version where $\mu$ is exogenous and $\lambda^\top$ may differ from $\mu$.

A type-$j$ consumer in $E^{NP}$ earns $\pi_a \cdot (\mu \circ w_j) + \pi_b \cdot ([\mathbb{1} - \mu] \circ w_j)$ [6] **regardless of his region of residence** in comparison to $E^P$ in section 3, where he instead earns either $\pi_a \cdot w_j$ or $\pi_b \cdot w_j$ **depending on where he lives**. Endowments themselves cannot cross borders. However, income generated from them are transferable, i.e., it travels free by way of banking so that his combined income is a location-free value.

The case $\mu = \mathbb{0}$ reduces to the economy with only one region. In order to neutralize the effect of such spatial inhomogeneity, we assume $\mu = .\mathbb{5}$.

[5]Since all off-diagonal entries of $w$ in (1) are zero, so long as $\mu = \lambda^\top$, the two matrices are identical (otherwise both $\mu$ and $\lambda$ have to be of the form $c\mathbb{1}$).

[6]We denote an entry-wise product by $\mu \circ w_j := \begin{bmatrix} \mu^1 w_j^1 & \cdots & \mu^4 w_j^4 \end{bmatrix}$.

There are many equilibria in $E^{NP}$. We present one of them below:[7]

$$
\begin{aligned}
\pi &= \begin{bmatrix} 0.159 & 0.250 & 0.0387 & 0.552 \end{bmatrix}^\top \\
\lambda &= \begin{bmatrix} 0.336 & 0.283 & 0.603 & 0.511 \end{bmatrix} \\
\begin{bmatrix} \lambda \mathbb{1} & 4 - \lambda \mathbb{1} \end{bmatrix} &= \begin{bmatrix} 1.73 & 2.27 \end{bmatrix} \\
\mu &= \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}^\top \\
x &= \begin{bmatrix} 26 & 67.43 & 48.49 & 83.09 \\ 12.75 & 5 & 12.37 & 220.77 \\ 8.25 & 6.47 & 119 & 14.28 \\ 0.58 & 0.45 & 0.07 & 275 \end{bmatrix} \\
u &= \begin{bmatrix} 16.0 & 44.9 & 47.4 & 240.5 \end{bmatrix} \\
y_a &= \begin{bmatrix} 0 & 0 & 0 & 0 & 19.1 & 40.0 \end{bmatrix}^\top \\
y_b &= \begin{bmatrix} 0 & 0 & 0 & 0 & 23.6 & 41.2 \end{bmatrix}^\top \\
y &= \begin{bmatrix} 0 & 0 & 0 & 0 & 42.7 & 81.2 \end{bmatrix}^\top (= y_a + y_b)
\end{aligned}
$$

There are more residents in region $b$ than in $a$ and thus agglomeration takes place in this equilibrium. Appendix A.3 lists other equilibria found, the majority of which feature agglomeration as above. We did not find any equilibrium whose prices differ by region or from the three equilibrium prices in $E^{1R}$.

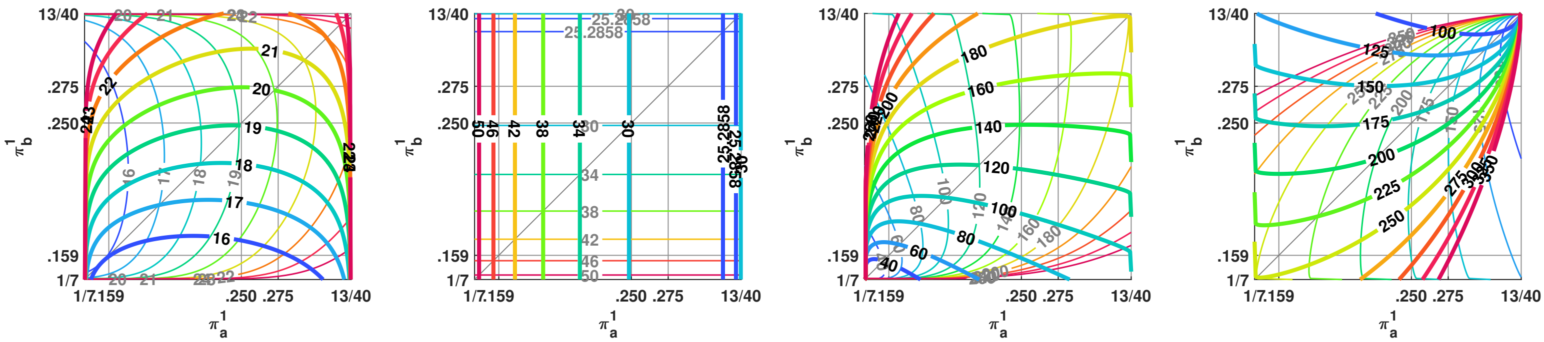
To better understand the equilibrium in $E^{NP}$, let us examine its value functions in some depth. Utility levels equate across regions by type in inter-regional equilibrium in $E^{NP}$ as in $E^P$. In $E^P$, indirect utility functions are plotted over $\pi_a^1$ alone in figure 1. In contrast, they need to be plotted over $\left(\pi_a^1,\ \pi_b^1\right)$ in $E^{NP}$: Demand in region $a$ depends not only on $\pi_a$ but also on $\pi_b$ by way of income collected from region $b$. Define indirect utility function by $v_{j,\ a}(\pi_a,\ \pi_b,\ w) := u_{j,\ a}\left(x_{j,\ a}(\pi_a,\ \pi_b,\ w)\right)$.[8] Figure 4 [9] represents the level sets of $v_{j,\ a}(\pi_a,\ \pi_b,\ w)$ and $v_{j,\ b}(\pi_b,\ \pi_a,\ w)$. Note that if we slice the value function along the 45° line, the cut surface is identical to figure 1: Along the 45° line, $\pi_a^1 = \pi_b^1$ so that type $j$'s income $\pi_a \cdot \left(\mu \circ w_j\right) + \pi_b \cdot \left[(\mathbb{1} - \mu) \circ w_j\right]$ reduces to $\pi_a \cdot w_j$ irrespective of the value of $\mu$. Thus $v_{j,\ a}(\pi_a,\ \pi_b,\ w) = v_{j,\ a}(\pi_a,\ w)$ in effect, which is indeed what figure 1 represents.[10]

[7] Whereas it is not part of the inter-regional equilibrium, we nevertheless include weight $y$, an $E^{1R}$ counterpart to $y_a$ and $y_b$, as we did in section 3. Proposition 4.1 explains why $y_a + y_b$ comes to $y$.

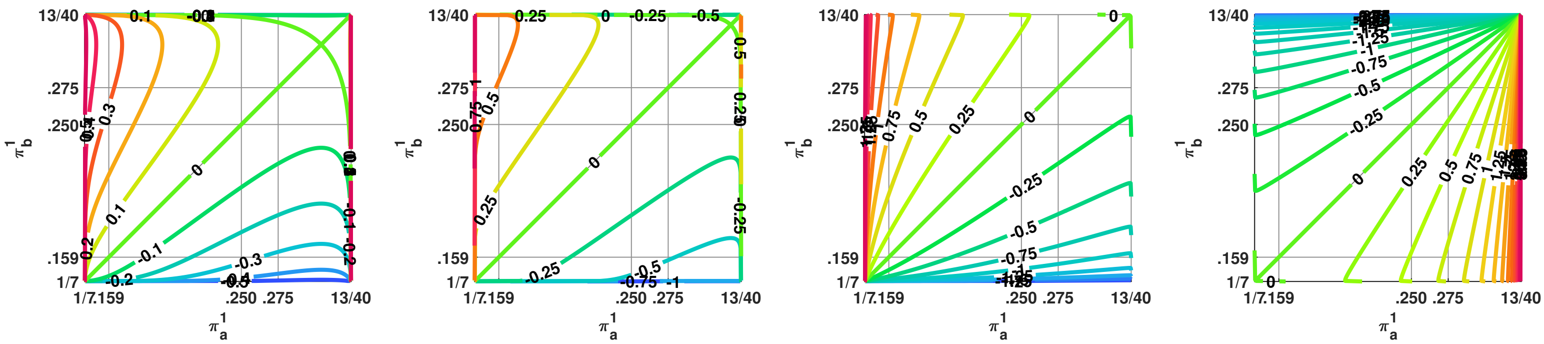
[8] We list the home price first, followed by the cross-regional price regardless of the region in question so as to maintain the symmetry. As such, the indirect utility function in region $b$ is written as $v_{j,\ b}(\pi_b,\ \pi_a, w)$ rather than $v_{j,\ b}(\pi_a,\ \pi_b,\ w)$.

[9] We restrict the domain of price vectors to $\Pi^\perp$ because orthogonality, being the condition for production rather than consumption, still applies to $E^{NP}$.

[10] Except that the horizontal axis is stretched by $\sqrt{2}$.



(a) $v_{j,\ a}(\cdot)$ and $v_{j,\ b}(\cdot)$.



(b) $\log v_{j,\ a}(\cdot) - \log v_{j,\ b}(\cdot)$.

**Figure 4.** An $E^{NP}$ counterpart to figure 1 in $E^{P}$. From left to right: type 1, 2, 3 and 4. Level sets of value functions $v_{j,\ a}(\pi_a,\ \pi_b,\ w)$ are represented by bold lines with dark numbers indicating utility levels, and $v_{j,\ b}(\pi_a,\ \pi_b,\ w)$ by thin lines with light numbers in figure 4(a). Warmer colors correspond to higher utility levels in either region. Note that $\frac{\partial v_{1,\ a}(\pi_a,\ \pi_b,\ w)}{\partial \pi_b^1} > 0$, $\frac{\partial v_{2,\ a}(\cdot)}{\partial \pi_b^1} = 0$, $\frac{\partial v_{3,\ a}(\cdot)}{\partial \pi_b^1} > 0$, and $\frac{\partial v_{4,\ a}(\cdot)}{\partial \pi_b^1} < 0$ because the cross-regional price produces income effects only. Consequently, the value functions of type 1, 3 and 4 are strictly monotone over the cross-regional price for any given home price, as can be seen in the first, third and fourth plots in figure 4(a); and that of type 2 takes the same value regardless of the cross-regional price as in the second plot. Figure 4(b) plots the corresponding utility differential between two regions that appears in figure 4(a). Inter-regional migration stops where the gap is zero (a contour line in green) for all types (see figure 10).

If $(\pi_a,\ \pi_b)$ constitutes an inter-regional equilibrium, $v_a(\pi_a,\ \pi_b,\ w) = v_b(\pi_b,\ \pi_a,\ w)$. On a graph, this translates to contour lines of the same utility level crossing at $(\pi_a,\ \pi_b)$.

Note that $v_{j,\ a}\big(\pi,\ \pi',\ w\big) = v_{j,\ b}\big(\pi,\ \pi',\ w\big)$ (whether the economy is in inter-regional equilibrium or not). The equality does not hold in general because the individual income of type $j$ differs between the two sides of the equality: $\mu\pi \cdot w_j + (1-\mu)\pi' \cdot w_j$ in region $a$ on the left, where $(\pi_a,\ \pi_b) = \big(\pi,\ \pi'\big)$; and $\mu\pi' \cdot w_j + (1-\mu)\pi \cdot w_j$ in region $b$ on the right, where $(\pi_b,\ \pi_a) = \big(\pi,\ \pi'\big)$. However, since we set $\mu = .5$, these values will be the same on both sides. Consequently, demand is $x_{j,\ a}\big(\pi,\ \pi',\ w\big) = x_{j,\ b}\big(\pi,\ \pi',\ w\big)$ and thus $v_{j,\ a}\big(\pi,\ \pi',\ w\big) = v_{j,\ b}\big(\pi,\ \pi',\ w\big)$.

This translates to the level set of $v_{j,\ b}(\pi_b,\ \pi_a,\ w)$ being axially symmetric to that of

| | regional price differences | regional income differences |
|---|---|---|
| $E^P$ in general | possible | possible |
| $E^P$ with $v_j(\cdot)$ strictly monotone over $\pi_a$ or $\pi_b$ | no | no |
| $E^{NP}$ in general | possible | no |
| $E^{NP}$ with $v_j(\cdot)$ strictly monotone over $\pi_a$ and $\pi_b$ | no | no |

**Table 2.** Possible inter-regional equilibrium outcomes in $E^P$ and $E^{NP}$.

$v_{j,\,a}(\pi_a,\ \pi_b,\ w)$ about the 45° line in figure 4.[11]

This further indicates that $\pi_a = \pi_b$ is a sufficient condition for inter-regional equilibria. Since $v_{j,\,a}\big(\pi,\ \pi',\ w\big) = v_{j,\,b}\big(\pi,\ \pi',\ w\big)$, these functions take the same value if $\pi = \pi_a = \pi_b$, and $\pi' = \pi_b = \pi_a$, in which case $\pi_a = \pi_b$. In figure 4, region $a$'s contour line of a given utility level crosses region $b$'s contour line of the same utility level on the 45° line. Thus, utility differentials are zero along the line. See figure 4(b).

$E^{NP}$'s relation to $E^{1R}$ resembles that of $E^P$. We present the following proposition that parallels proposition 3.2 of $E^P$.

Proposition 4.1 Equilibrium Prices in $E^{1R}$ and $E^{NP}$

*Let $\Pi^{NP}$ be a set of equilibrium prices in $E^{NP}$. If $\pi_a = \pi_b$, $\Pi^{NP} \subseteq \Pi^{1R} \times \Pi^{1R}$.*

*Proof.* See appendix A.5. □

There is a small difference between proposition 3.2 and 4.1. Whereas proposition 4.1 is conditional on $\pi_a = \pi_b$, proposition 3.2 capitalizes on proposition 3.1 instead. We discuss an $E^{NP}$-equivalent of proposition 3.1 below.

## 4.2 Comparison between $E^P$ and $E^{NP}$

$E^{NP}$ differs from $E^P$ on two grounds: composition of income and competition for endowment. The former acts on the prices and the latter bears on the degree of agglomerative force.

### 4.2.1 Flexibility of Income and Symmetric Prices

The first essential difference is whether individual incomes can regionally vary or not (cf. table 2). In $E^P$, $\pi_a \cdot w_j$ does not have to match up with $\pi_b \cdot w_j$. In our example, however, they do because price vectors coincides with each other due to strictly monotone value functions (cf. proposition 3.1). By contrast, in $E^{NP}$ strict monotonicity over home price

[11]Note that $v_{j,\,a}(\pi_a,\ \pi_b,\ w)$ is plotted with the home price $\pi_a^1$ (first input) on the horizontal axis and the cross-regional price $\pi_b^1$ (second input) on the vertical axis; on the contrary, $v_{j,\,b}(\pi_b,\ \pi_a,\ w)$ is plotted in reverse with the cross-regional price $\pi_a^1$ (second input) on the horizontal axis and the home price $\pi_b^1$ (first input) on the vertical axis, resulting in a diagonally flipped image of $v_{j,\,a}(\cdot)$ on a graph.

alone is not sufficient to observe $\pi_a = \pi_b$ as value functions are defined over $\pi_a$ **and** $\pi_b$. Thus, we would need to assume in addition that at least one of the value functions are strictly monotone over cross-regional price for any given home price as well. This may seem to indicate that $E^{NP}$ is more likely to support inter-regional price differentials than $E^P$.

Quite the contrary, it becomes even harder to find asymmetric equilibrium prices in $E^{NP}$. The stricter restrictions on value functions are merely a sufficient condition for symmetric prices. Indeed, our functions do not meet this condition (cf. figure 4) and yet we did not find any inter-regional equilibrium featuring regional differences in price. This is due to the additional constraint that income imposes in $E^{NP}$. In $E^{NP}$ income must be universal: $\pi_a \cdot (\mu \circ w_j) + \pi_b \cdot ([\mathbb{1} - \mu] \circ w_j)$, i.e., in choosing a region of residence, a consumer needs to take into account regional price differences (if exist) but his location choice has no consequences on his income level. If $\pi_a \neq \pi_b$ in inter-regional equilibrium, then regional utility variation needs to be cleared only through prices but not through incomes. This leaves only a small degree of freedom to realize $\pi_a \neq \pi_b$ in equilibrium.

Agglomeration takes place even in the absence of any regional price variations. Consumers move to a region where what they like to consume in large quantities is inexpensive. This may shrink regional price gaps as they capitalize on any such gaps to exhaust any spatial arbitrage opportunities left. However, prices may still differ by region in inter-regional equilibrium so long as utility levels equate type by type. Whereas each type may have the same utility level outside $\pi_a = \pi_b$ (cf. figure 4(b)), it becomes increasingly difficult to find $\pi_a \neq \pi_b$ where **all** types' utility levels equate between regions (cf. figure 10). Consequently, inter-regional equilibria are more likely to be of the form $\pi_a = \pi_b$. See appendix A.4 for details.

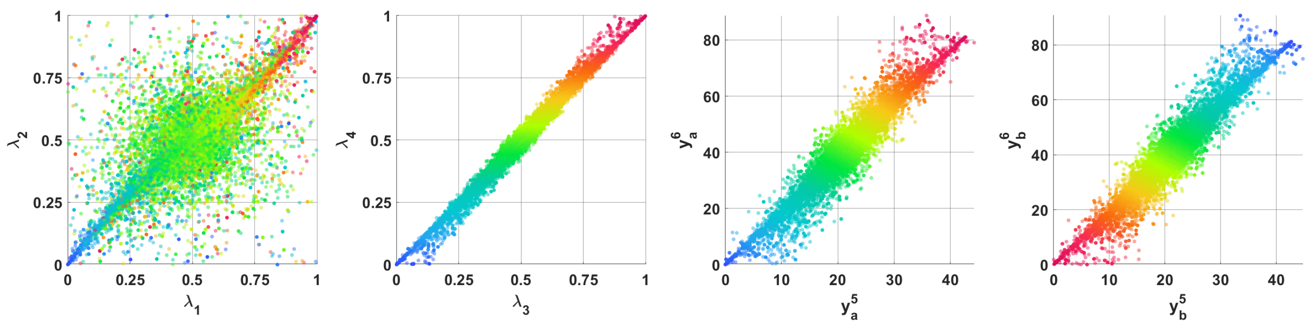
In view of this, let us suppose $\pi_a = \pi_b$. Comparing (3) to (8), $Ay_a^{NP} = Ay_a^P + w\lambda^\top - \mu \circ w\mathbb{1}$. An inter-regional equilibrium in $E^{NP}$ may or may not have a corresponding inter-regional equilibrium in $E^P$. It does as long as the firms in region $a$ are capable of producing the difference $w\lambda^\top - \mu \circ w\mathbb{1}$, and similarly, the ones in region $b$ are capable of producing the difference $w(\mathbb{1} - \lambda^\top) - (\mathbb{1} - \mu) \circ w\mathbb{1}$ to be made up for, while keeping to $y_a^{NP} + y_b^{NP} = y^{1R}$ as a whole (cf. *Remark* in appendix A.5).

We can trivially and retroactively set $\mu$ equal to one of $\lambda^\top$'s found in section 3.1 to ensure an inter-regional equilibrium even when $w$ is not portable. With that, we assume that $w$ is portable for simplicity and for the ease of notation in section 6 to follow. We do so with the understanding that there are at least as many equilibria in $E^{NP}$ as in $E^P$ by simply setting $\mu = \lambda^\top$.

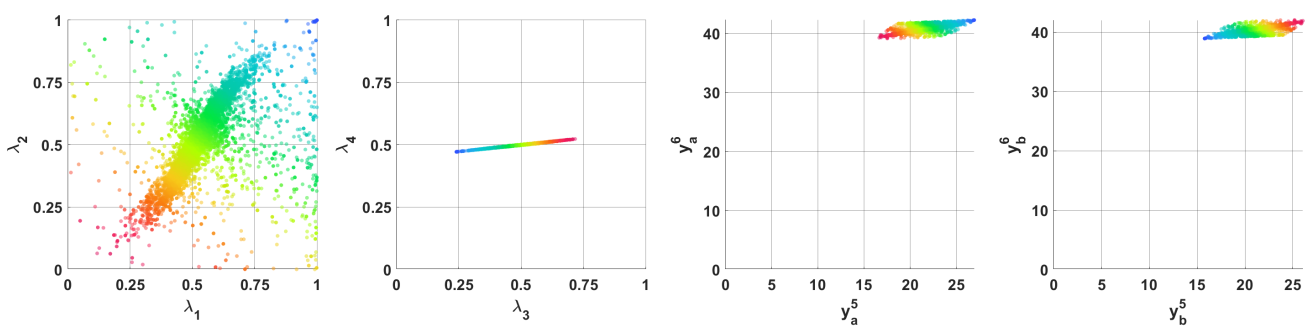
#### 4.2.2 Portability of Endowments and Strength of Agglomerative Force

The second major difference is the way demands are met. In $E^P$, demands are filled through two variable channels: production level $Ay_a$ and $Ay_b$, and available endowments with the help of inter-regional migration. Any change in demands is accommodated by adjusting one or both of the channels. $E^{NP}$ cannot rely on the second channel for adjustment. Labor mobility is guaranteed (cf. table 1) but internal migration does not change endowments available in this context.

This difference leaves a significant mark on the way agglomeration is formed. In general, the agglomerative force is more potent in $E^P$ than in $E^{NP}$ because of portability. We use figure 5 to compare two economies at $\pi_a^1 = \pi_b^1 = .159$ for example. Along with




(a) $E^P$.




(b) $E^{NP}$.

**Figure 5.** Both at $\pi_a^1 = \pi_b^1 = .159$. Each dot represents an inter-regional equilibrium, colored by the value of $\lambda_4$ to show correspondence among four figures.

$\lambda$, accompanying activity level $y_a$ and $y_b$ are presented to visualize how production and endowment complement each other in $E^P$ and not so in $E^{NP}$. Whereas they both feature agglomeration in most cases, the gap in population between two regions is typically more pronounced in $E^P$ than in $E^{NP}$.

Whereas the range that $\lambda_1$ and $\lambda_2$ take does not differ much between figures 5(a) and 5(b), that of $\lambda_3$ and $\lambda_4$ are markedly different. In addition, the composition of types in each region makes a contrasting difference as well.

In figure 5(a), type 3 and 4 tend to sort into the same region. Type 3 has a relatively high expenditure share of commodity 1 at $\alpha_3^1 = .5$. Their demand can be met with $w_1^1$. This explains a positive correlation between $\lambda_3$ (as a consumer of commodity 1) and $\lambda_1$

(as an owner of endowment 1). However, their tie does not need to be strong because $x_3^1$ can be met through production as well. Indeed, where $\lambda_3$ is high, deployment of the fifth column of $A$ picks up. In figure 5(a), $y_a^5$ is high when $\lambda_3$ is high. And vice versa, in a region scarcely populated with type 3, barely any production takes place via $y^5$.

The same argument goes for type 4 and commodity 2. Whereas $\alpha_4^2 = .25$ is not high, $x_4^2$ is sizable at $\pi^1 = .159$ (cf. figure 2(d)). This results in a heavy use of the sixth column of $A$ in the presence of high $\lambda_4$, as indicated by a high value of $y_a^6$ in figure 5(a). The weight $y$ changes liberally to cater to a differing presence of type 3 and 4.

During the co-production of commodity 1 and 2, the fifth column needs to employ endowment 4 as it cannot be produced, and the sixth column needs endowment 3 for the same reason. Thus, type 3 needs type 4 to be in the same region in the same proportion for the production of commodity 1, and type 4 in turn needs type 3 to be in the same region for the production of commodity 2. This creates mutually reinforced sorting forces that result in co-habitation of the two. This does not immediately lead to agglomeration because they only need to be in the same region in roughly **equal** proportions, including non-agglomerative mix of $\begin{bmatrix}\lambda_3 & \lambda_4\end{bmatrix} = .\mathbb{5}$. However, this sorting can support and does generate **extreme** agglomeration $\begin{bmatrix}\lambda_3 & \lambda_4\end{bmatrix} \approx \mathbb{0}$ and $\mathbb{1}$ as recorded in figure 5(a).

All in all, $E^P$ can support a wide range of size distributions, including sweeping agglomeration of the form $\lambda \approx \mathbb{0}$ or $\mathbb{1}$. This is due to added flexibility in the production process made possible by portability. Portability is particularly helpful in the procurement of non-producible inputs. In conjunction with free mobility, production stays responsive to demands regardless of the type composition. This is further aided by the second channel mentioned above: endowments directly fill any unmet demand left after production.

In contrast, agglomerative forces in $E^{NP}$ are weaker. The range of $\lambda_4$ is severely restricted, as does $\lambda_3$ to a lessor extent. Unlike $E^P$, region $a$ cannot increase or decrease the production of commodity 1 or 2 because supply of non-producible commodity 3 and 4 is fixed at $\mu^3 = \mu^4 = .5$. Thus, $\lambda_3$ and $\lambda_4$ are unlikely to reach near 0 or 1 because they would have to split the rigid supply of commodity 1 and 2 with many consumers.

Since $\lambda_4$ cannot break out of the 50-50 split anyway, there is no need to vary $y^6$ in figure 5(b). The lack of portability significantly reduces the range of quantities available both for consumption and input use. This in turn restrains the mobility of consumers. The resultant distribution still features agglomeration, but it is much less pronounced than in $E^P$.

In both cases, $\lambda_1$ and $\lambda_2$ is free of endogenously imposed mobility restraints above, and can be found anywhere between 0 and 1. This is because they do not show much affinity towards non-producible goods ($\alpha_1^3 = \alpha_1^4 = .04$ and $\alpha_2^3 = \alpha_2^4 = .02$). Since they do not need much of these to keep their utility levels even across regions, they have little reason to stay with the source of supply, type 3 and 4. Consequently, $\lambda_1$ and $\lambda_2$ easily take a value far from .5 in $E^{NP}$.

However, $\lambda_1$ and $\lambda_2$ do tend to take a similar value. This is because $\alpha_2^1 = .86$. Individual demand $x_2^1$ can be supplied through the fifth column of $A$, which requires the use of non-producible commodity 3 and 4, which are furthermore non portable in $E^{NP}$. Nonetheless, $x_2^1$ can be directly filled by endowment 1 without orchestrating onerous production process above, resulting in co-presence of type 1 (as an owner) and 2 (as a consumer). Endowment 1 is not portable in figure 5(b), which makes values near 0 or 1 less frequent than in figure 5(a). However, the range of $\begin{bmatrix}\lambda_1 & \lambda_2\end{bmatrix}$ is still broader than $\begin{bmatrix}\lambda_3 & \lambda_4\end{bmatrix}$.

In summary, price vectors tend to take the same value across regions in $E^{NP}$ because income cannot vary by region to accommodate asymmetric prices. Against a backdrop of symmetric prices, agglomerative forces are attenuated because non-portability severely restrains the range of supply. As a result, fraction $\lambda_j$ of type 3 and 4, who require a large quantity of non-producible commodities, cannot mark a significant departure from endowment location $\mu^3$ and $\mu^4$.

## 5 Mixed Portability of Endowments

In preceding sections 3 and 4 endowments are either all portable or all non portable. In this section, we examine a more realistic setup where some are portable and some are not.

Commodities have two attributes: 1) whether they can be produced, and 2) whether endowments are portable. Given technological process $A$ in (2), the last two commodities can only be an input. In this section, we render endowment 1 and 3 portable and the remainder non portable to include all possible combinations of attributes as shown in table 3.[12] Consider, for example, that commodity 3 is labor and commodity 4 is land.

In relation to regional income differences presented in table 2, this economy is a 50-50 cross between $E^P$ and $E^{NP}$. The individual incomes of type 1 and 3 depend on the region they choose to live in (either $\pi_a \cdot w_j$ or $\pi_b \cdot w_j$). Type 2 and 4 respectively earn the same income $\pi_a \cdot (\mu \circ w_j) + \pi_b \cdot ([\mathbb{1} - \mu] \circ w_j)$ regardless of their choice.

| | | producibility: yes | producibility: no |
|---|---|---|---|
| portability | yes (as in $E^P$) | 1 | 3 (e.g., labor) |
| | no (as in $E^{NP}$) | 2 | 4 (e.g., land) |

**Table 3.** Four goods sorted according to two attributes in our setup.

As with sections 3 and 4, inter-regional equilibria exist. We present one of them below. It is computed with $\mu = \begin{bmatrix}\lambda_1 & .5 & \lambda_3 & .5\end{bmatrix}^\top$ to represent the setup in table 3.[13]

[12] The existence of inter-regional equilibrium is not contingent on the way portable endowments are selected. Appendix A.6 represents equilibria in other combinations.

[13] Note that $\lambda$ does not have any direct impact on individual income. Type 1 and 3's income does not involve any $\lambda$ to begin with. Type 2 and 4's income depends on $\mu^2$ and $\mu^4$ respectively, but these are set

$$
\begin{aligned}
\pi &= \begin{bmatrix} 0.159 & 0.250 & 0.0387 & 0.552 \end{bmatrix}^\top \\
\lambda &= \begin{bmatrix} 0.821 & 0.577 & 0.512 & 0.496 \end{bmatrix} \\
\begin{bmatrix} \lambda\mathbb{1} & 4-\lambda\mathbb{1} \end{bmatrix} &= \begin{bmatrix} 2.41 & 1.59 \end{bmatrix} \\
\mu &= \begin{bmatrix} 0.821 & 0.500 & 0.512 & 0.500 \end{bmatrix}^\top \\
x &= \begin{bmatrix} 26 & 67.43 & 48.49 & 83.09 \\ 12.75 & 5 & 12.37 & 220.77 \\ 8.25 & 6.47 & 119 & 14.28 \\ 0.58 & 0.45 & 0.07 & 27 \end{bmatrix} \\
u &= \begin{bmatrix} 19.1 & 29.8 & 140.8 & 181.9 \end{bmatrix} \\
y_a &= \begin{bmatrix} 0 & 0 & 0 & 0 & 21.2 & 41.8 \end{bmatrix}^\top \\
y_b &= \begin{bmatrix} 0 & 0 & 0 & 0 & 21.5 & 39.4 \end{bmatrix}^\top \\
y_a + y_b &= \begin{bmatrix} 0 & 0 & 0 & 0 & 42.7 & 81.2 \end{bmatrix}^\top (= y^{1R})
\end{aligned}
$$

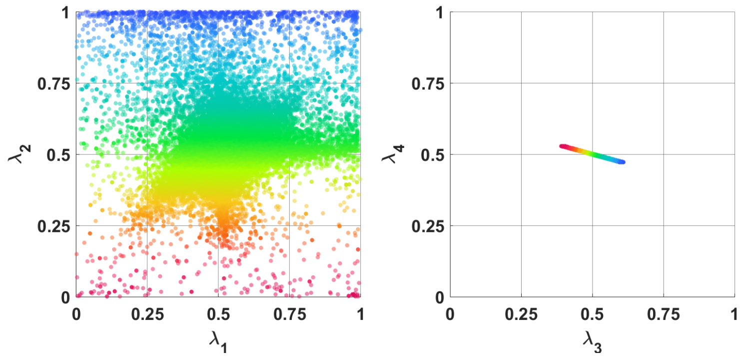
See figure 6 for a collection of equilibria found. In this economy $\pi_a = \pi_b$ in inter-regional equilibrium because proposition 3.1 applies in this case. This is because of type 3. Since $w^3$ is portable, type 3's income is unaffected by cross-regional price. Accordingly, their value function is strictly monotone increasing over home price for any given cross-regional price (cf. figure 1).[14] Therefore, their utility levels will not equate across regions unless $\pi_a = \pi_b$.

The proportion of type 4 tends to coincide with $\mu^4$. Indeed if we replace $\mu = \begin{bmatrix} \lambda_1 & .5 & \lambda_3 & .5 \end{bmatrix}^\top$ with $\begin{bmatrix} \lambda_1 & .75 & \lambda_3 & .75 \end{bmatrix}^\top$, $\lambda_4$ rises accordingly (see figure 6(d) for $\pi^1 = .250$). Since commodity 4 cannot be produced, the only way to supply them is through endowments. Type 4 consumes commodity 4 far more than other types at any one of the three equilibrium prices (cf. figure 2). Consequently, they need to reside where commodity 4 is located. For example, when $\mu^4 = .5$, $\lambda_4$ is unlikely to be found near 1. If $\lambda_4 \approx 1$, type 4 in region $a$ is severely worse off because they will have to split $\mu^4 w^4 \mathbb{1} = 200$ units of endowment 4 located in the region with as many as $\lambda_4 \approx 1$ consumers, as opposed to in region $b$, where they split the same amount with only $1 - \lambda_4 \approx 0$ consumers. The outflow of type 4 from region $a$ into $b$ is expected to level the welfare gap between two regions. Thus, the lack of portability leaves type 4 no choice but to align themselves to the similar proportion as (exogenous) endowment allocations.

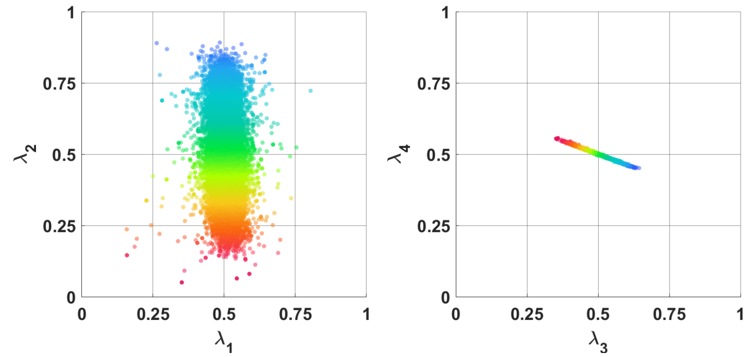
The same argument does not apply to type 2 despite the fact that their endowments are predetermined as type 4. This is in part because their expenditure share on commodity 2

equal to .5 rather than $\lambda_2$ or $\lambda_4$.

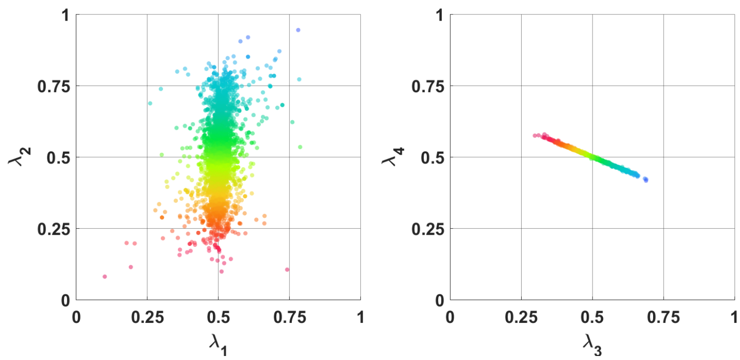
[14]Unlike section 3, type 4 does not fit this description. However, proposition 3.1 applies so long as at least **one** of four types has a strictly monotone value function.



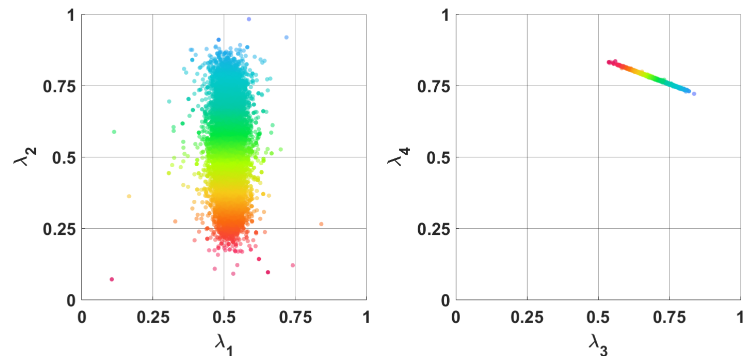

(a) $\pi^1_a = \pi^1_b = .159$.




(b) $\pi^1_a = \pi^1_b = .250$.




(c) $\pi^1_a = \pi^1_b = .275$.




(d) $\pi^1_a = \pi^1_b = .250$ and $\mu = \begin{bmatrix} \lambda_1 & .75 & \lambda_3 & .75 \end{bmatrix}^\top$.

**Figure 6.** Inter-regional equilibria sorted according to the three equilibrium prices found. The figure on left plots vector $\begin{bmatrix} \lambda_1 & \lambda_2 \end{bmatrix}$ and the figure on the right plots remaining entries $\begin{bmatrix} \lambda_3 & \lambda_4 \end{bmatrix}$ for each price. Each equilibrium is colored according to its $\lambda_4$ value in order to show correspondence in respective pairs of figures. Endowment 2 and 4 are evenly allocated except in figure 6(d).

is not high ($\alpha^2_2 = .1$) and also because commodity 2 can be produced (cf. table 3) whereas commodity 4 cannot be produced. Producibility frees type 2 from mobility constraints above that type 4 experiences. Even if $\lambda_2$ takes a value close to 1, type 2 is not necessarily worse off in region $a$ than in region $b$. It is true that they will have to split $\mu^2 w^2 \mathbb{1} = 25$ with more consumers in region $a$. However, this is not a severe disadvantage because commodity 2 can simply be produced instead of relying exclusively on endowment 2 for its supply. As such, $\lambda_2$ ranges wider than $\lambda_4$ in figure 6. In addition, changing $\mu^2$ does not have much impact on $\lambda_2$ as can be seen in figure 6(d), because production activities redress any imbalance in endowment allocations.

It is rather type 1 who tend to stay where endowment 2 is. Their expenditure share is relatively high at $\alpha^2_1 = .4$. In addition, their consumption of it increases with $\pi^1$ (cf. figure 2(b)). They tend to be where commodity 2 is easily supplied from endowment ($\mu^2$) or easily produced using the non-portable and non-producible inputs ($\mu^4$). Consequently, $\lambda_1 \approx \mu^2 = \mu^4$ as can be seen in figures 6(b) and 6(c).

Outside of these, type 3 and 4 tend to repel each other. Type 3 has inclination towards commodity 1 and type 4 towards commodity 2. If both type 3 and 4 co-locate, type 3 would use commodity 2 to produce commodity 1, and type 4 would do the opposite. They would then be in direct competition with each other for resources, one for consumption and the other for input.

In contrast, type 2 and 3 tend to co-locate. During the production of commodity 1 that type 2 and 3 prefer to consume, the firm employs a high volume of commodity 3. Type 2 is then better off co-habitating with type 3 for ease of procurement.

Type 1 and 2 do not consume any particular commodity out of proportion. Furthermore, even if their demand for commodity 1 and 2 was disproportionately larger as what commodity 3 and 4 are for type 3 and 4, their mobility would not be affected by this because commodity 1 and 2 can be supplied through production as much as through endowments. Accordingly, $\begin{bmatrix}\lambda_1 & \lambda_2\end{bmatrix}$ does not exhibit as distinct a pattern as $\begin{bmatrix}\lambda_3 & \lambda_4\end{bmatrix}$ does.

# 6 Transportation Cost and Inter-Regional Trade

Agglomeration takes place whether inter-regional trade is allowed or not. Put differently, agglomeration in our model is **not** due to comparative advantages, scale economies or heterogeneous technologies that would underlie inter-regional trade because our model does not feature any of them.

Let $t \geq 1$ be the units of commodity required to be shipped from one region to receive one unit of it in the other region. In section 3, we assumed that $t \to \infty$ so that there is no point in engaging in inter-regional trades. Trading beyond the regional boundaries is effectively equivalent to deploying the first four columns of $A$, i.e., disposal of commodities for free. Let us now consider two other cases: $t > 1$, and $t = 1$. This section focuses on $E^P$ for the reason outlined in section 4.2.1. The same argument applies to $E^{NP}$ in section 4 or partially portable economy in section 5.

When $t > 1$, the inter-regional equilibria remain the same as above. No one engages in inter-regional trades in this case either. Such trades only incur transport costs with no gain in return. Technology is not heterogeneous by region to warrant comparative advantages, nor does it exhibit increasing returns to scale to warrant exclusive production in a particular region. As seen above, regional prices are symmetric in our case. Consequently, imported goods are always priced higher than locally produced goods and thus no one buys them.

When $t = 1$, $E^{2R}$ reduces to $E^{1R}$ in effect, with a token presence of $\lambda$. The location of production or consumption is of no consequence as commodities can flow freely. As such, any $\lambda$ constitutes an inter-regional equilibrium.

Although we did not find any such equilibria, suppose that there is a price-asymmetric equilibrium. If in addition $\pi_b^i = t\pi_a^i$, then there could be commodity flow from region $a$ to $b$. The firm producing commodity $i$ becomes indifferent between selling it in region $a$ or export it to region $b$. However, as discussed in sections 3 and 4, $\pi_a \neq \pi_b$ is not likely in inter-regional equilibrium, and even less so when there is an additional equality, $\pi_b^i = t\pi_a^i$ to meet.

# 7 Economy with Two Types

We have demonstrated that agglomeration occurs in a four-type, four-commodity setting. Let us denote the number of commodities by $I$ and the number of types by $J$. This section examines the role $I$ and $J$ play in forming agglomeration. For agglomeration to develop out of spatial sorting, we need at least $I \geq 3$ and $J \geq 2$.

Consider a downscaled economy[15] with $I = 2$, $J = 2$ and $\operatorname{rank}(A) = 1$. In particular, we isolate type 1 and 2, and commodity 1 and 2 from the preceding economy, and replace $A$ with a $2 \times 1$ vector $\hat{A}$ with one positive and one negative entry. As in section 2, the firm earns zero profit so that $\pi_a^\top \hat{A} = \pi_b^\top \hat{A} = 0$.[16] Consequently, $\pi_a = \pi_b$. Since $\pi$ does not differ by region, neither does $x - w =: z$, a $2 \times 2$ individual net demand matrix, nor does $u_j(x_j)$ for any $j$. The argument so far does not involve any $\lambda$. As such, inter-regional utility equalization does not impose any restrictions on $\lambda$.

Material balance implies $z\lambda^\top = \hat{A}y_a$ and $z(\mathbb{1} - \lambda^\top) = \hat{A}y_b$ in respective regions. Combined,

$$\begin{bmatrix} z & -\hat{A} & \mathbb{0} \\ z & \mathbb{0} & \hat{A} \end{bmatrix} \begin{bmatrix} \lambda^\top \\ Y \end{bmatrix} = \begin{bmatrix} \mathbb{0} \\ z\mathbb{1} \end{bmatrix}, \tag{7}$$

where $Y := [y_a \quad y_b]^\top$ (note that $z$ is independent from $\lambda$). If the first matrix was invertible, distribution $\lambda$ would be unique. However, it is not full rank. Thus, there are infinitely many solutions to (7). Intuitively, constant returns to scale enable firms to counter any distribution $\lambda$ by simply rescaling their production level to meet the regional demand $z\lambda^\top$ and $z(1 - \lambda^\top)$ with no footprints on the price.

The individual budget constraint $\pi \cdot z = \mathbb{0}$ does not involve $\lambda$ either. Consumers do not receive any dividends from firms because they do not make any profit. Therefore, the budget constraints do not impose any restrictions on $\lambda$ either.

All in all, this economy supports any $\lambda \in [0,\ 1]^2$ in equilibrium. Agglomeration takes place but it is rather coincidental compared to section 2. Linear production plays two contrasting roles in this. On the one hand, it adds rigidity to the economy: it single-handedly dictates what the equilibrium price is. It does not require any involvement of

[15]Due to the symmetric prices, portability makes no difference, i.e., the subsequent argument applies to any of the three economies examined in sections 3 to 5.

[16]Consumers are free to enter into a direct transaction among them at a rate of exchange different from such $\pi$. However, one of the parties involved in such exchange will be better off trading with the firm at $\pi$ anyway.

demand (and by extension, $\lambda$) because $\hat{A}$ alone determines the unique direction of the price vector. On the other hand, it adds flexibility to the economy: because it is linear, firms can easily scale up or down their production to meet the regional demand that varies with $\lambda$, without affecting the price, which itself is of linear technology's own making as explained above.

The original example with $I = J = 4$ compares to the current example with $I = J = 2$ as follows: In section 2, zero-profit condition only narrows the candidate prices down to **infinitely many** vectors in $\Pi^{\perp}$. Here, in contrast, such $\pi$ is **unique** (up to a scalar multiple). As described above, once $\hat{A}$ is given, $\pi(= \begin{bmatrix} \frac{1}{7} & \frac{6}{7} \end{bmatrix}^{\top})$ is uniquely determined, as does $z$. Moreover, since $\hat{A}$ does not differ by region, neither does $z$.

Inter-regional trade will not take place if $t > 1$ because there is no price differential to justify costly transport of commodities. If $t = 1$, inter-regional trade may take place but it does not have any bearing on $\lambda$, which is randomly determined outside of market transactions, be it intra- or inter-regional in nature.

By contrast, section 2 cannot be written as a linear system because $A$ alone cannot narrow $\pi$ down to a single vector and thus $z$ involves $\pi$ in it rather than being treated as a constant as in (7). This in turn helps reduce the degree of freedom to pin down $\lambda$. Distribution $\begin{bmatrix} \lambda\mathbb{1} & 4 - \lambda\mathbb{1} \end{bmatrix}$ thus obtained features agglomeration[17] as a result of intentional spatial sorting rather than at random in the 2×2 economy above. To induce conscious agglomeration, it is essential to equip the economy with at least $I \geq 3$ commodities to have more than one price vector in $\Pi^{\perp}$.

## 8 Conclusions

Agglomeration is conventionally thought of as a production-driven phenomenon. Scale economies favor a concentration of inputs within a close proximity.

Barring scale economies, can there still be agglomeration? To examine whether agglomeration can be driven by consumption rather than production, we worked on a general equilibrium model with constant returns to scale proposed by Kehoe [Keh85]. We established that agglomeration does not necessitate the presence of scale economies. Heterogeneity among consumers creates asymmetry in population distribution, in particular through complementarities in endowments.

Our model does not entail any externalities. The said complementarities are externalities of pecuniary nature. Mossay and Picard [MP11] also derive agglomeration from consumers' end. They model return from social interactions that attenuates with distance. Both endowments and preferences are homogenous so that agglomeration is due to externalities from social contacts. If we incorporate their setup in our model, social interactions may enhance spatial sorting, rendering agglomeration even more clear-cut.

[17] Except few instances referred to in section 3.3.

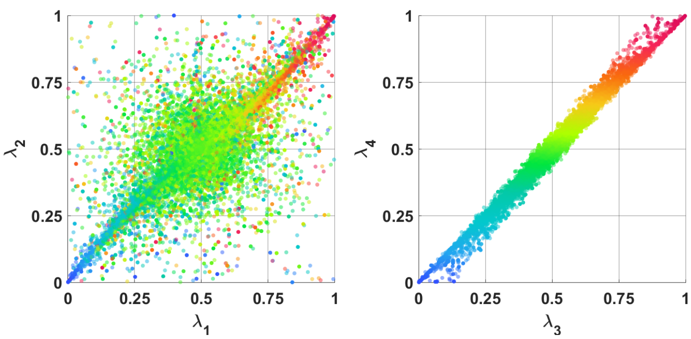
We are not intent on overriding the existing knowledge about production-oriented agglomeration. Rather, we cast light on the role consumption plays in generating agglomeration, which combined should illustrate a more realistic mechanism behind agglomeration.

As did Kehoe [Keh85], we worked on a specific class of preferences in the interest of tractability. We defer to future research for reproduction of our results in a general setting.

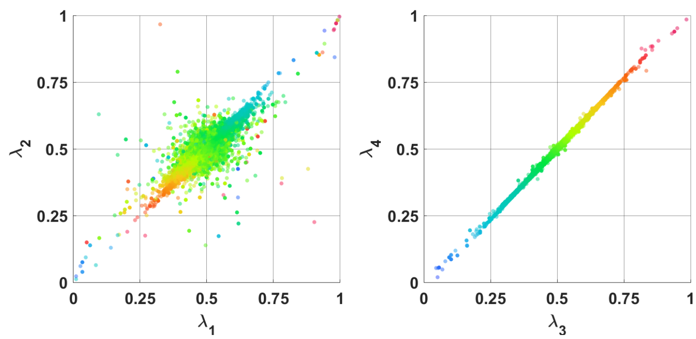
# A Appendix

## A.1 List of Inter-Regional Equilibria in $E^P$

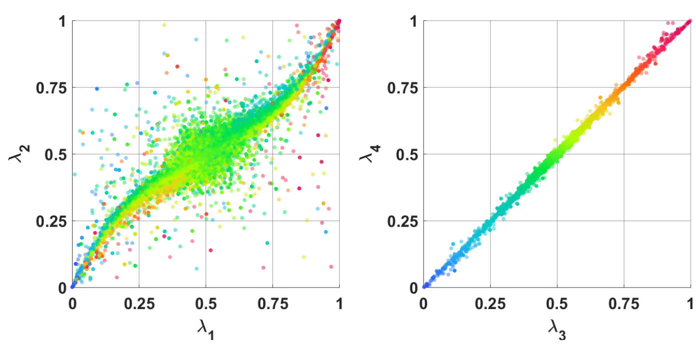
An inter-regional equilibrium takes one of three prices: $\pi^1 = .159,\ .250$ or $.275 \in \Pi^{1R}$. Figure 7 sorts the equilibria according to these prices and plots population distribution $\lambda$, and activity level $y_a$ and $y_b$ under each price.




(a) $\pi^1 = .159$.




(b) $\pi^1 = .250$.




(c) $\pi^1 = .275$.

**Figure 7.** Inter-regional equilibria in $E^P$. Each equilibrium is colored according to the value of $\lambda_4$ to show correspondence between a pair of plots at each price.

In general, a similar proportion of type 3 and 4 sort into the same region. This is due in part to a large excess supply of commodity 3 that type-3 consumers have, and similarly to a large excess supply of commodity 4 that type-4 consumers have, which heavily drags down the excess demand for these commodities. Without co-presence of two types, either commodity 3 or 4 will have a large excess in supply. They both can be used as an input to produce commodity 1 or 2.[18] However, commodity 3 **cannot** be used to produce commodity 4, and neither can commodity 4 be used to produce commodity 3. Thus, a region can even out the supply of commodity 1, 2 and 3; or 1, 2 and 4; but not all four of them at once through production activities. Since preferences are convex, imbalance between commodity 3 and 4 (and by extension, type 3 and 4) does not last and will be rectified through inter-regional migration. Therefore, each region tends to host a roughly equal portion of type 3 and 4. Indeed, $y^5$ and $y^6$ are positively correlated in order to produce **both** commodity 1 and 2 with the aim of providing a full range of commodities in proportion.

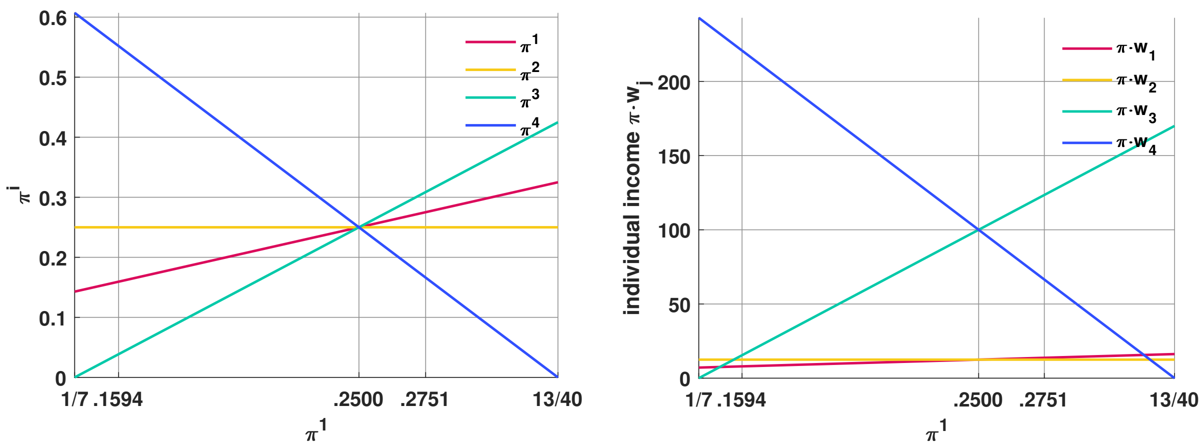
As for type 1 and 2, in general, they tend to co-locate in a similar proportion. Their relation to type 3 and 4 is sensitive to the prices. When $\pi^1 = .159$, they co-locate with type 3 and 4. When $\pi^1 = .250$, they avoid type 3 and 4. When $\pi^1 = .275$, there is no distinct pattern of relationship. By and large, their pattern of distribution is not as definitive as type 3 and 4 as above. This is in part because type 1 and 2 are endowed with commodities that **can** be produced. Their lack of presence can easily be made up for by other types through production of commodity 1 and 2. In addition, their excess demand $x_1^1 - w_1^1$ and $x_2^2 - w_2^2$ are smaller in magnitude than $x_3^3 - w_3^3$ and $x_4^4 - w_4^4$ (cf. figure 2). Thus their presence, or the lack thereof, does not have as significant a bearing as type 3 and 4. This renders their distribution more fluid and sensitive to the price than type 3 and 4's.

## A.2 Net Demand

To further comprehend the construction of net demand in figure 2 let us consider consumer 3 in detail for example. He is endowed with $w_3^3 = 400$ units of commodity 3, whose price is increasing in $\pi^1$ (cf. figure 8(a)). As such, his income increases with $\pi^1$ (cf. figure 8(b)). His net demand for commodity 1, $x_3^1 - w_3^1$ is traced by the green line in figure 2(a). His income effect on commodity 1 exceeds the substitution effect. His net demand for commodity 1 **grows** with $\pi^1$ as a result. On the contrary, his net demand for commodity 4, as appearing in figure 2(d), **diminishes** with $\pi^4$ (which itself is decreasing in $\pi^1$). In this case, his income effect on commodity 4 falls behind the substitution effect. Indeed his expenditure share of commodity 4 is only $\alpha_3^4 = .0025$ and thus the effect of income growth is easily trumped by realignment of his consumption towards commodity 1 and 2. On the other hand, his net demand for commodity 3, as depicted by the flat green line in figure 2(c), remains **invariable**. His only source of income is commodity 3 and thus the effect of a change in $\pi^3$ on his demand for commodity 3 is exactly offset by the

[18] Note that activity level $y_a$ and $y_b$ do pick up where type-3 and -4 consumers are in figure 7.

associated change in his income. The same argument goes for commodity 1 for consumer 1, commodity 2 for consumer 2, and commodity 4 for consumer 4. As such, the diagonal entries of demand matrix $x$ always take the same value regardless of the price as seen in section 3.1 (the same goes for sections 3 to 5). $E^{1R}$ features more than one equilibrium in part because of the conflicting gradients in figures 2(a) and 2(b).




(a) A set of $\pi$ that is strictly positive and orthogonal to the column space of $A$. Color corresponds to **commodity**.

(b) Income by type. Color corresponds to **type**.

**Figure 8.**

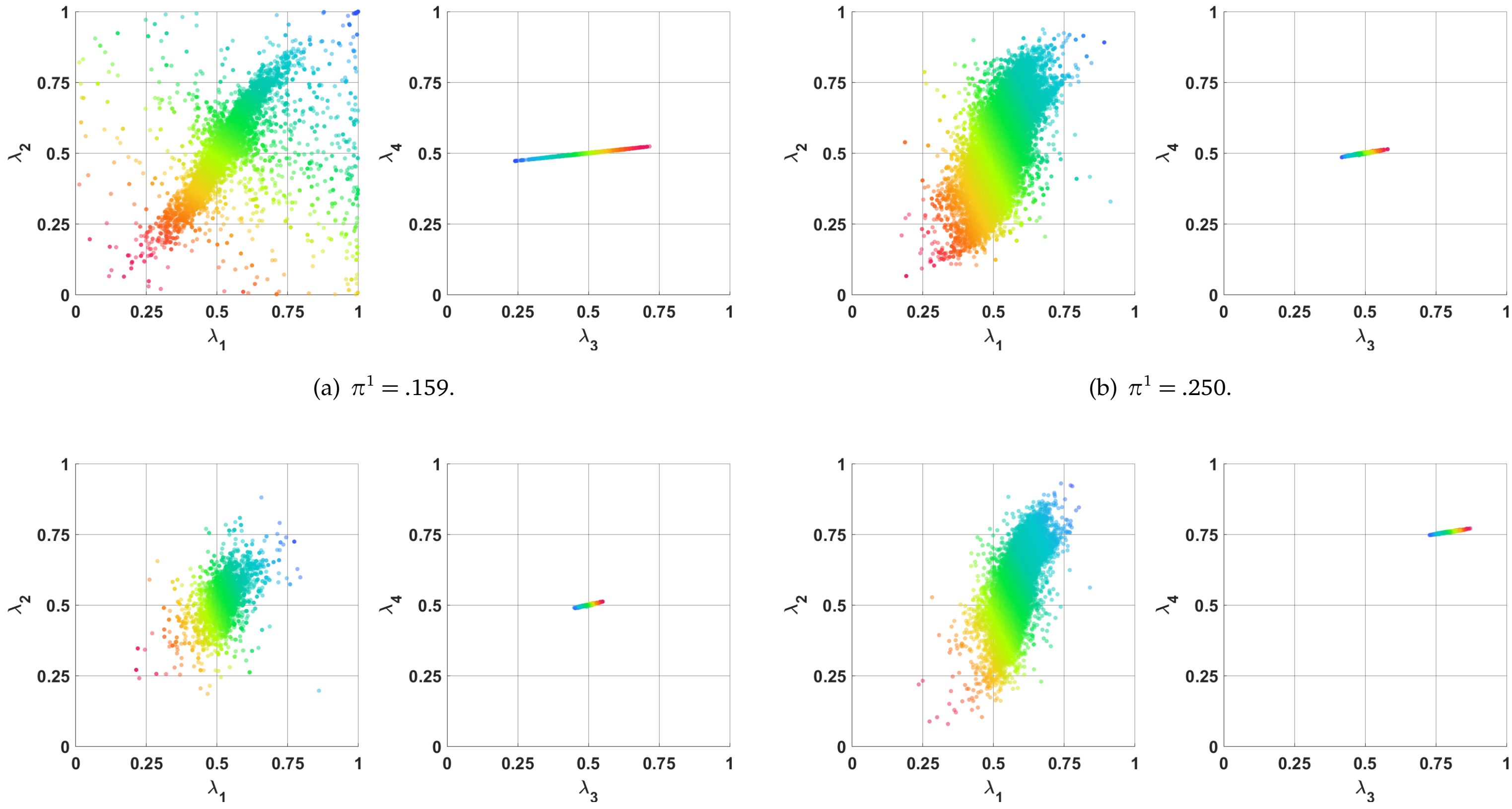
## A.3 List of Inter-Regional Equilibria in $E^{NP}$

Figure 9 lists inter-regional equilibria found in $E^{NP}$. In this economy, type 4 strongly aligns itself with where endowments are located. When $\mu = .\bar{5}$, $\lambda_4$ falls within the tight range around .5. If $\mu$ increases, it follows suit as in figure 9(d). Type 4 has a high expenditure share on commodity 4 at $\alpha_4^4 = .6875$. Since commodity 4 cannot be produced from other commodities, they need to be where endowment 4 is.

The same applies to type 3 as commodity 3 cannot be produced. However, $\alpha_3^3 = .2975$ is not as high as $\alpha_4^4$. Thus, $\lambda_3$ admits a wider range of values than $\lambda_4$.

Type 1 and 2 tend to co-locate in one region and type 3 and 4 tend to settle in the other region. Both type 1 and 2 have an inclination towards commodity 1. The fifth column of $A$ will be engaged to meet their demand. In so doing, the firm will eat into endowment 3 and 4, which type 1 and 2 do not care much for.

Type 3 and 4 have an inclination towards commodity 3 and 4. If they co-reside with type 1 and 2, they would have to vie with the firm to secure endowment 3 and 4 for their consumption. Since $w$ is not portable, an increased presence of type 3 and 4 does not equate with an increased supply of endowment 3 and 4. Thus, type 3 and 4 are better off sorting into a different region than where type 1 and 2 occupy to avoid putting themselves in direct competition with the firm.




(a) $\pi^1 = .159$.

(b) $\pi^1 = .250$.

(c) $\pi^1 = .275$.

(d) $\pi^1 = .250$ and $\mu = \begin{bmatrix} .75 & .75 & .75 & .75 \end{bmatrix}$.

**Figure 9.** Endowments are evenly allocated except in figure 9(d).

## A.4 Inter-Regional Difference in Utility Level

Proposition 4.1 is conditional on $\pi_a = \pi_b$. Let us consider if $\pi_a$ can differ from $\pi_b$ in inter-regional equilibrium.

Off the 45° line, type 1 and 2 realize $v_{j,\,a}(\cdot) = v_{j,\,b}(\cdot)$ near the northeastern corner in figure 4(b). Type 3 realizes $v_{3,\,a}(\cdot) = v_{3,\,b}(\cdot)$ near $\left(\pi_a^1,\ \pi_b^1\right) = (13/40,\ 13/40)$ on the top right corner in figure 4(b), and type 4 near $\left(\pi_a^1,\ \pi_b^1\right) = (1/7,\ 1/7)$ on the bottom left corner in figure 4(b). This is due to the sharp bend that appears in figure 4, which in turn is from the steep ascent near $\pi^1 = 1/7$ and $13/40$ observed in figure 1. However, these points will not constitute an inter-regional equilibrium because **all** types need to be indifferent between two regions. Figure 10 presents the value of $\sum_j |\log v_{j,\,a}(\cdot) - \log v_{j,\,b}(\cdot)|$. Thus, under the current set of parameters we adopt, $\pi_a = \pi_b$ in inter-regional equilibrium.

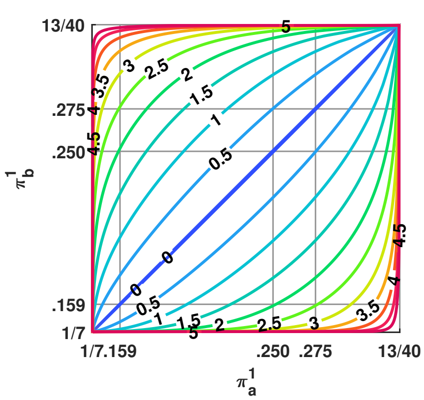



**Figure 10.** Sum of the utility differential $\sum_j |\log v_{j,\,a}(\cdot) - \log v_{j,\,b}(\cdot)|$. All types are indifferent between two regions when this value takes zero.

## A.5 Proof of Proposition 4.1

The proof of proposition 4.1 is similar to that of proposition 3.2.

*Proof.* Suppose there exists a pair of price vectors $(\pi_a, \pi_b) \in \Pi^{NP}$ but $\pi_a \notin \Pi^{1R}$. The markets in region $a$ clear when

$$x(\pi_a, \pi_b)\lambda^\top - \mu \circ w\mathbb{1} = Ay_a. \tag{8}$$

Similarly in region $b$, $x(\pi_b, \pi_a)(\mathbb{1} - \lambda^\top) - (\mathbb{1} - \mu) \circ w\mathbb{1} = Ay_b$. Since $\pi_a = \pi_b$ by assumption, both $x(\pi_a, \pi_b)$ and $x(\pi_b, \pi_a)$ reduce to $x(\pi_a)$. Aggregate them to obtain the country-wide market clearance:

$$[x(\pi_a) - w]\mathbb{1} = A(y_a + y_b). \tag{9}$$

On the other hand, $\pi_a$ is not a market-clearing price vector in $E^{1R}$ so that there is no $y^{1R} \geq \mathbb{0}$ such that

$$[x(\pi_a) - w]\mathbb{1} = Ay^{1R}. \tag{10}$$

In conjunction with (9), (10) imply $y_a + y_b = y^{1R}$. Since $(\pi_a, \pi_a) \in \Pi^{NP}$, both $y_a \geq \mathbb{0}$ and $y_b \geq \mathbb{0}$ exist, running counter to $y^{1R}$ being nonexistent. Therefore, $\Pi^{NP} \subseteq \Pi^{1R} \times \Pi^{1R}$. □

*Remark.* As in proposition 3.2, (9) and (10) imply that $y_a + y_b$ in $E^{NP}$ comes to the corresponding $y^{1R}$ under the same price vector in $E^{1R}$.

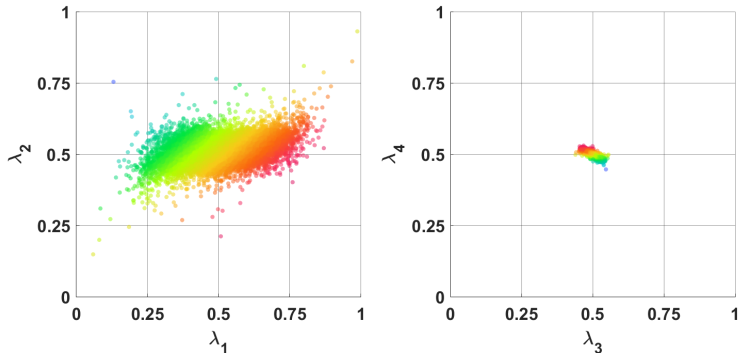
## A.6 Inter-Regional Equilibria in Various Portability Settings

The existence of inter-regional equilibira does not depend on a particular choice of portable endowments in section 5. Figure 11 lists inter-regional equilibria in other configurations as represented in table 4. We present the case for $\pi_a^1 = \pi_b^1 = .250$ only for comparison purposes. Inter-regional equilibria exist at any of the three prices in $\Pi^{1R}$. Figure 11(a) shows that equilibria exist if we render endowment 2 and 4 portable instead of 1 and 3 as in figure 6 in section 5. Figure 11(b) renders only producible commodities portable; figure 11(c) does the opposite. The remaining figures render only one rather than two endowments portable, producible commodity in figure 11(d) and non-producible commodity in figure 11(e). Inter-regional equilibria are robust and we found them in all the configurations tested above.

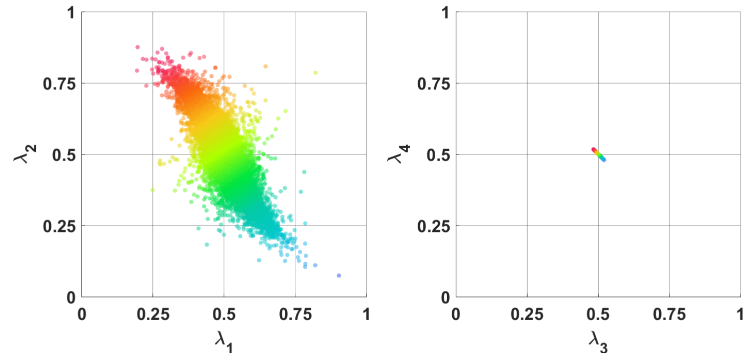
| commodity | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| producibility | • | • | | |
| figure 6 | • | | • | |
| figure 11(a) | | • | | • |
| figure 11(b) | • | • | | |
| figure 11(c) | | | • | • |
| figure 11(d) | • | | | |
| figure 11(e) | | | • | |

**Table 4.** Portability of endowments.

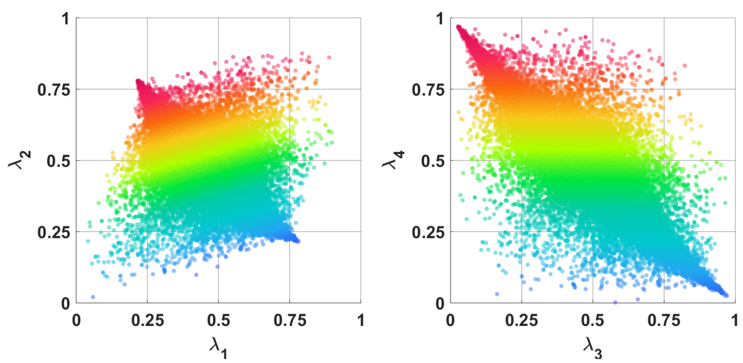
In figure 11(a), $\lambda_1$ widens its range and $\lambda_2$ clusters around .5, reversing the tendency observed in figure 6(b). It is now type 2 who tend to stay where endowment 1 and 3 are for ease of consumption of commodity 1.



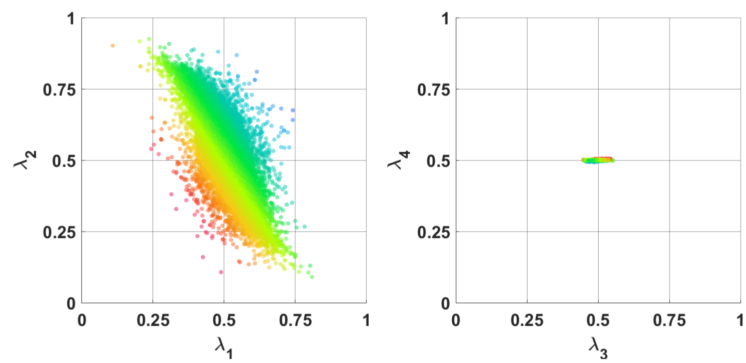

(a) Endowment 2 and 4 are portable.



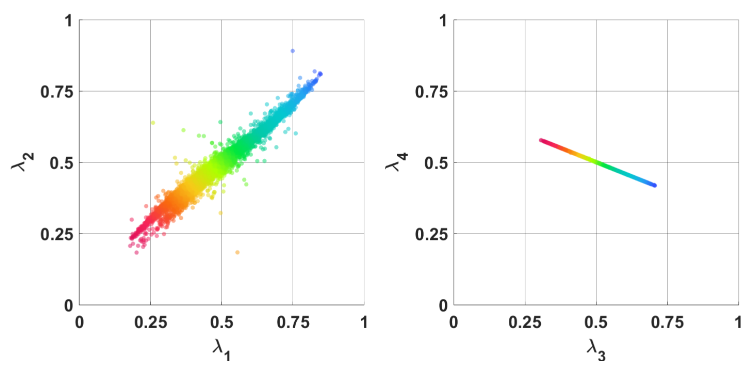

(b) Endowment 1 and 2 are portable.




(c) Endowment 3 and 4 are portable




(d) Endowment 1 is portable.




(e) Endowment 3 is portable.

**Figure 11.** All equilibria feature $\pi_a = \pi_b = .250$.

In figure 11(b), the range of $\lambda_3$ and $\lambda_4$ is even tighter than in figure 6(b). When a producible commodity is non portable, the economy could still support some variance in $\lambda_3$ and $\lambda_4$ by way of producing unmet demands. It is now impossible to do so because any commodity in short supply cannot be produced at all.

The opposite applies to figure 11(c). A lack of portability tends to restrain the distribution of types, but this tendency is lessened when the commodity in question is producible. Comparison between figures 11(d) and 11(e) attests to this. Since only one (rather than two) endowment is portable, the range of distribution is restrained in figures 11(d) and 11(e) than in figure 11(c). However, within figures 11(d) and 11(e), figure 11(e) supports a wider range of distributions of $\lambda_3$ and $\lambda_4$ because non-producible commodity is portable in this case.

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