Tax Transparency and Social Welfare: The Role of Government Commitment*

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March 2022

Abstract

Although transparency has long been held as the key principle of taxation, recent behavioral public finance theory has shown that it may *reduce* social welfare as inattention can alleviate behavioral distortions. This paper extends this analysis by modeling inattention as a noise in the tax rate signal received by Bayesian citizens. In equilibrium, we find that transparency will *improve* social welfare by ensuring the government's ability to commit to a fairly low tax rate that is socially optimal. Moreover, this model yields new sufficient statistics formulas that inform whether a policy effort to ensure tax transparency is socially worthwhile.

*I am indebted to Stephen Morris and Iván Werning for the discussions that have inspired this paper. I am grateful to Abhijit Banerjee, Amy Finkelstein, Jim Poterba, Juuso Toikka, Muhamet Yildiz, and speakers in the 2019 Sloan NOMIS Summer School on the Cognitive Foundations of Economic Behavior and 2019 briq Summer School in Behavioral Economics for their teaching, and Ph.D. students of 14.471 Public Economics I, 14.770 Political Economy, and 14.160 Behavioral Economics at Massachusetts Institute of Technology in 2015, 2016, and 2017, for many discussions in my TA recitation sections. Ashley Craig, Ray Kluender, Hideto Koizumi, Takeshi Murooka, Daniel Reck, Michael Stepner, Neil Thakral, Dan Waldinger, Matt Weinzierl, Michael Wong, anonymous referees, and especially Atsushi Yamagishi, have provided many comments that have substantially improved this paper. I was heartened by the encourgements from Jérémy Boccanfuso, Antoine Ferey, and Dmitry Taubinsky, and benefited from the seminar participants at Waseda University and International Institute of Public Finance Annual Congress 2021 and 114th Annual Conference on Taxation at National Tax Association in 2021. Philip MacLellan has provided editorial support. All errors are mine.

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> "The tax which each individual is bound to pay ... ought to be clear and plain to the contributor, and to every other person."
>
> – Adam Smith (1776)

> "If all taxes were direct, taxation would be much more perceived than at present; and there would be a security which now there is not, for economy in the public expenditure."
>
> – John Stuart Mill (1848)

1 Introduction

There has been a critical disagreement between policy practices and optimal tax theory regarding the welfare implications of tax transparency. Since *The Wealth of Nations* (Smith, 1776) and *Principles of Political Economy* (Mill, 1848), clarity has been regarded as a key principle for a good system of taxation. Today, tax authorities implement various programs to inform citizens of tax liabilities, and simplicity has been at the center of controversial tax debates, such as whether to introduce universal basic income instead of complex income transfers (See, e.g. Slemrod and Bakija, 2017.) However, prominent models in behavioral public finance have found that transparency may actually *reduce* welfare in the benchmark with a small income effect (Chetty et al., 2009 and subsequent models surveyed in Bernheim and Taubinsky, 2018[1]). The reason for this surprising result is clear: when inattention dampens the behavioral distortion of taxes, the government can raise more revenue with a smaller excess burden on its citizens. One historical example is the salience-reducing U.S. Federal income tax withholding system introduced in 1943 to finance the urgent wartime expenditure (Wagstaff, 1965).

This paper extends this taxation model by using a Bayesian model of inattention, and shows that tax transparency *improves* long-run social welfare for reasons consistent with the traditional policy discussions. This approach was first taken by the most recent and independent contribution of Boccanfuso and Ferey (2021),[2] and

[1] A number of papers (Mullainathan et al., 2012; Goldin, 2015; Reck, 2016; Taubinsky and Rees-Jones, 2018; Farhi and Gabaix, 2020; Cui et al., 2020; Kroft et al., 2020) also arrived at this insight under a variety of settings.

[2] Their first draft was made available online in 2019.

is supported by evidence that even when the taxpayers are initially uncertain about their taxes due to non-salient or complex schedules, they eventually learn with more experience and information (Finkelstein, 2009; Aghion et al., 2017; Caldwell et al., 2021). To analyze this learning effect, we consider Bayesian consumers that face uncertainty regarding their tax rates and receive signals whose noisiness reflects the extent of their inattention (Sims, 2003). Focusing on the equilibrium in which they update their prior beliefs (Harsanyi, 1967; Kreps and Wilson, 1982), we find that the inattention must *reduce* social welfare. Based on closed-form solutions of equilibrium tax rates and welfare, we then derive new sufficient statistics formulas that inform the controversial debate over the efficiency of costly policies to ensure the tax salience.

We begin by rederiving the result that transparency may reduce welfare by facilitating behavioral responses in the short-run, notwithstanding the existence of countervailing effects. In the model, Bayesian consumers rely not only on noisy signals but also on their prior beliefs to infer their tax rates. Consequently, their aggregate response to the change in the tax rate will be lower when the signals are noisier because they rely less on them. We consider an impulse response of consumption decisions to the introduction of a new tax rate from an initial state of zero tax. In the short-run, when their prior beliefs have not been updated, this inattention reduces the behavioral distortions. In addition to this effect, there are also idiosyncratic misperceptions that result in ex-post suboptimal consumption decisions. Nonetheless, so long as the overall noise is small, the former effect will dominate, and inattention will improve short-run welfare.

In the long-run, however, we show that transparency will improve social welfare. An important yet implicit assumption in taxation models is that the government sets the tax rate *before* the citizens make consumption decisions. If the tax rate is set *after* the citizens' choice, then even a benevolent government will choose a very high tax because there would be zero behavioral distortion. In equilibrium, however, rational citizens will then expect the high tax rate and consume less, resulting in the unintended consequence of large excess burden. That is, government needs an ability to *commit* to a low tax rate. Now if the tax rate signal is completely uninformative, then it is as if the government chooses the tax rate *after* the citizens' choice even if they physically choose the tax *before*. When the signal is noisy, in this way, the government will have limited ability to commit to a low tax rate.[3] Further, this long-

[3]The U.S. tax withholding system has remained even after the war, and the income tax is

run commitment effect,[4] named "taxation bias" by Boccanfuso and Ferey (2021), may be *discontinuously* large even when the noise is slight, and thus, can be important even when citizens strive to pay attention.

This long-run analysis yields new sufficient statistics formulas that inform the controversial debate over the costly programs to improve tax transparency[5]. In the U.S., the widely used estimate of compliance cost to ensure tax transparency is roughly 10 percent of its revenue (Slemrod, 1996). This model provides formulas that quantify whether this program is worthwhile, based only on the attention parameters, the policy implementation costs, and the marginal cost of public funds. While there are proposals to allow non-salient taxes by reducing this cost (Finkelstein, 2009), given reasonable range of parameter values in the literature, this formula suggests the costly effort for tax transparency is worthwhile.

Overall, this model shows that imperfect attention results in the issues of government commitment that had previously been the focus of tax debates but which had been absent in recent models reviewed in Bernheim and Taubinsky (2018). Traditionally, Smith (1776), Mill (1848) and subsequent research (e.g. Buchanan and Wagner, 1977) suggest that taxes will be *excessively* large unless they are transparent to the citizens.[6] However, with an exception of the most recent and independent work of Boccanfuso and Ferey (2021), the recent behavioral public finance models have suggested that inattention will lead to *optimally* high taxes. In this way, this paper shows that reduced-form vs. Bayesian approaches to inattention lead to vastly different equilibrium outcomes,[7] which is reminiscent of the sharp contrast between taste-based (Becker, 1957) vs. statistical (Arrow, 1973) discrimination. When the

now known to be high. Milton Friedman has attributed this increase to the lack of transparency (Friedman and Friedman, 1998), criticizing the tax level to be too high.

[4]The importance of this commitment effect was discovered independently by Boccanfuso and Ferey (2021). The first draft of my work here was, unfortunately, completed and distributed before I was informed of their important contribution by Dmitry Taubinsky. It is, however, a pleasure to acknowledge the constructive discussions I then had with Jérémy Boccanfuso and Antoine Ferey.

[5]This tax transparency program is closely related to the "bias alterlation policies" recently analyzed in Moore and Slemrod (2021). By specifying the source of behavioral bias, this paper extends their analysis by deriving a formula whose parameters can be mapped to empirical estimates.

[6]There are taxation models that embed commitment concerns *through political economy constraints of electoral processes* (e.g. Acemoglu et al. 2008; Farhi et al. 2012; Scheuer and Wolitzky 2016 among others). In contrast, this paper shows that the commitment concerns emerge under inattention *without explicitly modeling future elections*.

[7]This equilibrium analysis is related to Spiegler (2015) that shows that the explicit modeling of consumer bias can alter the welfare implications. While Spiegler (2015) focuses on the effect of market competition, this analysis focuses on the role of prior belief.

Bayesian equilibrium analysis is applied, the traditional argument of government commitment re-emerges as the principal concern.

This analysis complements the independent contribution of Boccanfuso and Ferey (2021) by identifying a transparent and tractable setting and thereby deriving new results. Specifically, their main model is quite general, involving endogenous attention, a constant bias in tax perception, and possibly non-linear tax rates, and they solve the model by directly maximizing the welfare. Under this general model, like many other optimal tax models (See e.g. Salanié, 2011), the tax rate formula is an implicit solution (Section 5), and the welfare assessment is based on numerical simulation (Section 6). In contrast, this paper harnesses the analytically attractive property that optimal taxation problems are locally a deadweight loss triangle minimization problem (Dupuit, 1844; Ramsey, 1927; Harberger, 1964), and can therefore be formulated as a *quadratic* problem. As optimal tax and Bayesian learning models tend to be mathematically involved, optimizing over the implied quadratic problem with a normal noise is an effective approach that yields transparent results such as (i) sufficient statistics formula for the tax transparency programs based on closed-form solution of equilibrium welfare and (ii) deadweight loss minimization figure in Figure 1 (III) that highlights the contrast with the seminal model of Chetty et al. (2009).

The remainder of this paper is organized as follows: Section 2 presents the model overview, the set-up, and derives its equilibrium implications for belief updating and behavioral responses; Section 3 analyzes the model's short-run and long-run implications; and Section 4 discusses the key assumptions of the model and concludes.

2 Model

This Section first provides an overview of the key argument, then introduces the taxation model with Bayesian inattention, and presents its interpretation.

2.1 Basic Model Overview

We begin with an overview of the main argument. As illustrated in Figure 1, its essential logic can be summarized by deadweight loss (DWL). In the traditional case when citizens are fully attentive, their consumption decision[8] will coincide with their

[8] Here, we consider a market of consumption demand to be consistent with the prominent models in behavioral public finance (e.g. Chetty et al., 2009). However, we can also interpret this model as

underlying valuations of the goods (Panel I). The social planner then sets the optimal tax, τ^*, which is fairly low because many attentive citizens with low valuations would otherwise cut their consumption, resulting in a large deadweight loss.

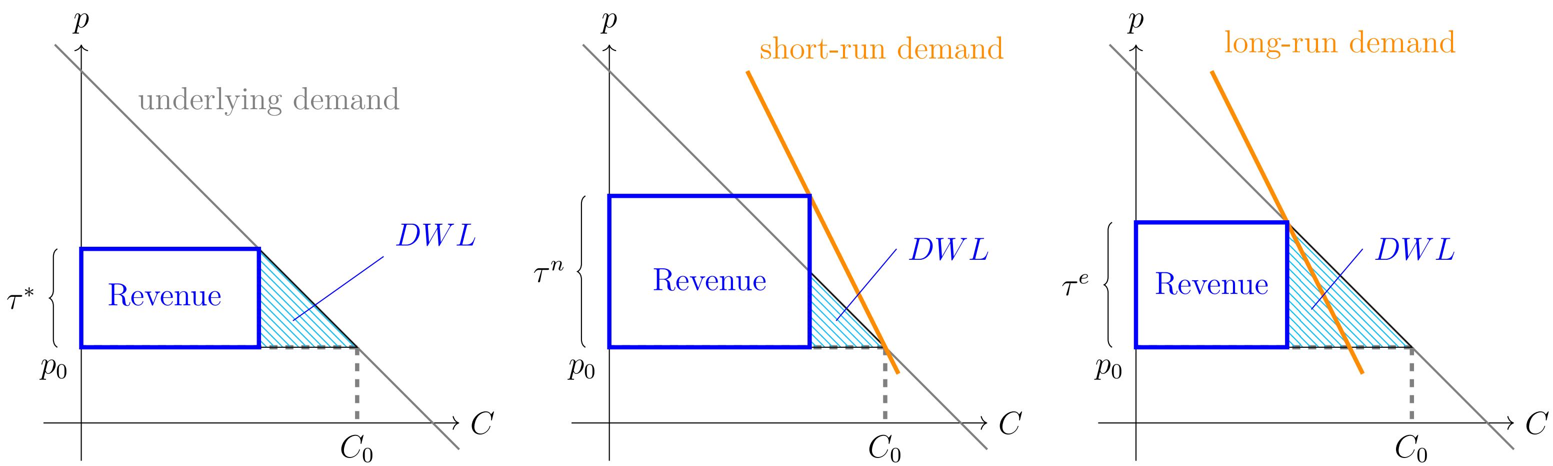
We consider an impulse response of consumption decision to introduction of a positive tax from the initial state of zero tax. In the short-run, we arrive at the findings from behavioral public finance that inattention improves welfare (Panel II, which replicates Chetty et al., 2009). Here, the only consequence of inattention is a reduction in citizens' behavioral responses, represented by the less elastic demand curve. Social welfare increases because the social planner can raise the tax rate, τ^n, to increase revenue while achieving a smaller deadweight loss.

In the long-run, however, we find that inattention lowers welfare because the Bayesian citizens' prior beliefs adjust (Panel III). After paying the taxes many times, they will eventually update their prior expectation to the equilibrium level, τ^e. That is, in equilibrium, their demand curve must coincide with the underlying demand *at the average tax level*, rather than at the initial zero tax as was assumed in the short-run. Crucially, the social planner will in equilibrium *take citizens' expectations as given*, and will increase the tax rate above the full attention benchmark, $\tau^e > \tau^*$, since demand is less elastic. But since the benchmark tax rate τ^* was socially optimal by construction, the resulting welfare will be sub-optimal. In summary, the low responsiveness to taxes due to consumer inattention compromises the government's ability to commit to the relatively low socially optimal tax rate.

Henceforth, we will build the Bayesian model with additional structures that make the analysis significantly more transparent and tractable. First, there will be a continuum of citizens who make binary decisions instead of one representative agent with a continuous decision. Second, the social planner determines the tax rate slightly imperfectly (in a sense defined precisely later) rather than with perfect control. Third, the underlying demand of citizens is assumed to be linear for the entire range of prices, and not just for some neighborhood of the equilibrium. Even though these auxiliary assumptions may appear foreign in the context of optimal taxation, they are based on standard assumptions in information economics to solve otherwise intractable Bayesian models. As illustrated in Figure 1, the essential argument henceforth will nonetheless be based on the familiar deadweight loss minimization.

a linear income taxation on labor supply by relabeling the variables. See Section 1.2.

Figure 1: Taxes and Welfare under Full and Imperfect Attention of Bayesian Citizens



(I) full attention and optimal tax (τ^*)

(II) inattention and non-equilibrium tax (τ^n)

(III) inattention and equilibrium tax (τ^e)

Notes: Figure 1 shows that, contrary to the short-run effects, inattention *reduces* long-run welfare by compromising the government's commitment to set the relatively low optimal tax. Panel (I) – (III) illustrate the government revenue (i.e. rectangles) and deadweight loss (i.e. triangles labeled as DWL) resulting from the taxes in each setting. Here, p denotes price, C denotes total consumption, and $\{p_0, C_0\}$ denotes the zero-tax equilibrium of a market of a taxed good. Panel (I) shows that the standard optimal tax (τ^*) is fairly low when the citizens are fully attentive. Panel (II) replicates the Figure 4 of Chetty et al. (2009) by depicting the short-run demand curve that is less elastic (or, steeper inverse demand curve in the Figure 1). In an impulse response of introducing a positive tax from the initial state of zero tax, inattentive citizens will be less responsive to taxes in the short-run. In this setting, the non-equilibrium tax rate (τ^n) is higher, and given the larger revenue and smaller deadweight loss, the implied welfare is also higher. Panel (III) summarizes the main argument of this paper. In the long-run, the demand curve will intersect the underlying demand *at the equilibrium tax rate* (τ^e) because the Bayesian consumers will update their prior belief. Since the demand curve is nonetheless less elastic due to their inattention, the social planner will set a tax rate that is higher than the full attention benchmark ($\tau^e > \tau^*$). But since the full attention benchmark (τ^*) was optimal by construction, the welfare under inattention must be sub-optimal.

2.2 Bayesian Model Set-up

Let us now formally introduce the static model of linear taxation. There is a continuum of Bayesian citizens with valuations $v \sim F(v)$ for a good who decide whether to consume it, $c \in \{0, 1\}$. The benevolent social planner chooses the level of the tax rate, $\tau \in \mathbb{R}$, to raise the government revenue.

The citizens consume the good if their own valuation v exceeds the expected post-tax price. In particular, normalizing the pre-tax price p_0 to be 1, the citizen with a type v chooses c to maximize the expected value of the utility,

$$u(c, v|\tau) \equiv c[v - (1 + \tau)]. \tag{1}$$

Each citizen imperfectly observes the tax rate through an idiosyncratic noisy signal,

$$\hat{\tau} = \tau + \varepsilon, \tag{2}$$

where $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is an independently distributed normal noise. We write the aggregate consumption, given a posterior perceived tax, $\tilde{\tau}$, as $C(\tilde{\tau}) = 1 - F(1 + \tilde{\tau})$.

The social planner chooses the *intended* tax, $\tau_0 \in \mathbb{R}$. Then, the *actual* tax, τ, is determined by

$$\tau = \tau_0 + \nu, \tag{3}$$

where $\nu \sim \mathcal{N}(0, \sigma_\nu^2)$ denotes some disturbances that perturb the tax rate.[9] This idiosyncratic uncertainty may reflect various determinants that the planner cannot fully control, such as legislative bargaining processes and elections. Faced with the uncertainties over the actual tax τ and the perceived tax $\tilde{\tau}$, the planner chooses the intended tax τ_0 to maximize the expected social welfare

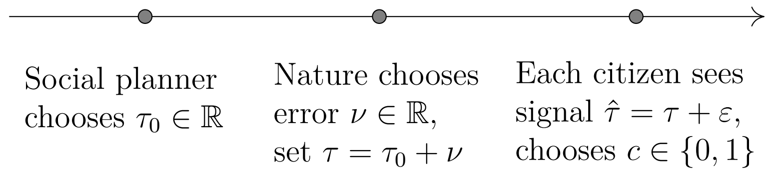
$$\mathbb{E}w \equiv \mathbb{E}\left[\lambda\tau\overline{C}(\tau) - DWL \middle| \tau_0\right], \tag{4}$$

where $\lambda > 0$ is the weight on the revenue[10]. $\overline{C}(\tau) \equiv \mathbb{E}[C(\tilde{\tau})|\tau]$ denotes the aggregate

[9] Formally, this approach is the same as the trembling hand of Selten (1975), which models the possibility of small mistakes by the decision-makers. This approach has also been adopted by recent models of inattention (Boccanfuso and Ferey, 2021; Matějka and Tabellini, 2020). Recent survey evidence also shows that the policy uncertainty is large and has substantial welfare consequences in the context of Social Security benefits (Luttmer and Samwick, 2018).

[10] Note that $\lambda = MCPF - 1$, the marginal cost of public fund *minus 1*, to avoid double counting the government revenues included in the deadweight loss as transfers.

Figure 2: Time line



Notes: Figure 2 describes the time line of this model.

consumption given actual tax τ. Here, the expected deadweight loss is

$$\mathbb{E}\left[DWL|\tau_0\right] \equiv \int \left\{ \underbrace{u\left(c_{v,\tilde{\tau}=0}, v|\, \tau = 0\right)}_{\text{zero-tax welfare}} - \mathbb{E}\left[\underbrace{u\left(c_{v,\tilde{\tau}}, v|\, \tau\right)}_{\text{welfare}} + \underbrace{\tau c_{v,\tilde{\tau}}}_{\text{revenue}} \middle| \tau_0 \right] \right\} dF\left(v\right). \quad (5)$$

This is the welfare loss resulting from taxes that does not contribute to government revenue. $c_{v,\tilde{\tau}}$ denotes the individual consumption of type v given the perceived tax $\tilde{\tau}$. While this expression may appear non-standard due to uncertainty and heterogeneity, we will henceforth show that it reflects the area of standard deadweight loss triangles in Figure 1.

The time line is as follows (Figure 2): first, the social planner chooses the intended tax rate τ_0; second, Nature chooses the perturbation parameter $\nu \sim \mathcal{N}\left(0, \sigma_\nu^2\right)$, and thereby sets the actual tax rate, $\tau = \tau_0 + \nu$; third, each citizen observes the noisy signal $\hat{\tau} = \tau + \varepsilon$, $\varepsilon \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$, updates their perceived tax rate $\tilde{\tau}$, and decides whether to consume, c, by maximizing their own expected utility; and finally, the citizens' payoffs are realized and the revenue is collected.

Interpretation as consumption tax and income tax. We can interpret this model both in terms of a consumption tax and an income tax. When c represents a commodity, τ denotes a consumption VAT tax. When the VAT tax is noted on the price tag, for example, it is very salient so that σ_ε^2 is very small (Chetty et al., 2009). When c is interpreted as the return to labor, $c = 1$ denotes labor supply, and $-\tau$ denotes an income tax or social insurance premium. When their opacity and complexity confuse the workers' understanding of the tax, the parameter σ_ε^2 is very large (Skinner, 1988; Abeler and Jäger, 2015; Caldwell et al., 2021). These tax rates are determined as a result of political negotiation and election, whose outcomes can be uncertain for the benevolent policy makers when proposing the tax level τ_0.

2.3 Equilibrium

2.3.1 "Static" Equilibrium Concept and its "Dynamic" Interpretation

This paper will focus on the Perfect Bayesian Nash Equilibrium, the standard equilibrium concept to analyze sequential-move games of incomplete information. Let the citizens' strategy be denoted by $s_c : \mathbb{R} \times \mathbb{R} \mapsto [0, 1]$, a mapping from the space of values v and signals $\hat{\tau}$ onto the probability distribution over consumption c. Let the planner's strategy be denoted by $s_p : \Delta(\mathbb{R})$, a distribution over the tax rate τ. Denote the citizens' prior belief over the government's strategy be denoted by $\mu \equiv \Delta(\Delta(\tau))$.

Definition 1 **Equilibrium** *An* equilibrium *is a tuple of strategies and beliefs* $\{s_c, s_p, \mu\}$ *such that (i) citizens strategies maximize the objective (1) in expectation given the strategies of the planner; (ii) planner strategy maximizes (4) given strategies of citizens; (iii) beliefs are consistent with the Bayes' rule and off-equilibrium strategies also maximize the objective (1).*

The key new condition, relative to the existing models, is (iii) the prior belief must be consistent with the Bayes' rule (Harsanyi, 1967; Kreps and Wilson, 1982). In the short-run immediately after the new tax rate is introduced, citizens may not be familiar with the new tax rate and thus, this condition (iii) is unlikely to be satisfied. However in the long-run when citizens have paid their taxes many times, their beliefs may well-approximate the true distribution so that this condition (iii) is reasonable. Therefore, henceforth, we will refer to the analysis that assumes the mean prior tax rate to be the initial value of 0 as the *short-run* analysis; in turn, we will refer to the analysis that imposes the (iii) prior consistency as the *long-run* analysis.[11]

[11] "Static" equilibrium can be naturally viewed as the steady state of the dynamic adjustment process of myopic players (see, e.g. Fudenberg and Levine (2009) for a survey) For example, in the context of statistical discrimination, the equilibria with and without discrimination is also analyzed with such dynamic interpretation, even when the model is static (e.g. Coate and Loury, 1993).

2.3.2 Inattention as Reduction in Aggregate Behavioral Response

We show that the average expectation over tax is the sum of the actual tax and the prior weighted by an attention parameter, and thus, the aggregate behavioral response to tax is attenuated by that parameter.

Lemma 1. Bayesian Updating with Attention Parameter. *Suppose the social planner sets some intended tax rate whose expectation is $\overline{\tau}_0$. The average perceived tax rate among citizens conditional on realized tax rate τ is*

$$\mathbb{E}\left[\tilde{\tau}|\tau\right] = m\tau + (1-m)\,\overline{\tau}_0, \tag{6}$$

where the attention parameter, m, is defined as

$$m \equiv \frac{\sigma_\nu^2}{\sigma_\nu^2 + \sigma_\varepsilon^2}. \tag{7}$$

Proof. Suppose the planner sets a distribution of intended tax rate, $G(\tau_0)$. Given each τ_0, the Bayesian citizens who receive the normal signal $\hat{\tau}$ form the perceived tax rate,

$$\tilde{\tau} = m\hat{\tau} + (1-m)\,\tau_0,$$

since the disturbances ν is also normally distributed[12]. Thus, if citizens are uncertain about τ_0,

$$\mathbb{E}\left[\tilde{\tau}|\hat{\tau}\right] = m\hat{\tau} + (1-m)\int \tau_0 dG = m\hat{\tau} + (1-m)\,\overline{\tau}_0,$$

by the Law of Iterated Expectations. Further, by linearity and mean zero additive noise,

$$\mathbb{E}\left[\mathbb{E}\left[\tilde{\tau}|\hat{\tau}\right]|\,\tau\right] = m\mathbb{E}\left[\hat{\tau}|\tau\right] + (1-m)\,\overline{\tau}_0 = m\tau + (1-m)\,\overline{\tau}_0.$$

□

Let us now introduce an assumption that makes the analysis significantly more transparent and tractable[13].

[12] Note that the behavioral response could be higher under inattention among some citizens. This implication is consistent with the evidence in Morrison and Taubinsky (2021).

[13] This assumption essentially extends the linear demand curve for negative prices and quantities, as depicted in Figure 1. While this assumption implies that the value distribution is improper, it is commonly applied in the model with Bayesian updating given normal distributions, such as

Assumption A1. Linear Demand Curve. *Given some* $\eta \in (0,1)$, $F(v) = \eta v$ *for all* $v \in \mathbb{R}$.

Lemma 2. Reduction in Aggregate Behavioral Response in the Unique Equilibrium. *Suppose Assumption A1. Then, the equilibrium will be unique. Further, inattention attenuates the effect of raising the tax rate* τ *on the aggregate consumption,* $\overline{C}(\tau)$, *by the attention parameter* m:

$$\frac{\partial}{\partial \tau}\overline{C}(\tau) = -m\eta. \tag{8}$$

Proof. Suppose the social planner chooses some distribution of intended tax rate whose expectation is $\overline{\tau}_0$. By Lemma 1, and by additivity and linearity of aggregate consumption across the entire values of v in Assumption A1,

$$\begin{aligned}
\mathbb{E}\left[C(\tilde{\tau})\,|\tau\right] &= \mathbb{E}\left[1-\eta(1+\tilde{\tau})\,|\tau\right] \\
&= 1-\eta\left\{1+\mathbb{E}\left[\tilde{\tau}|\tau\right]\right\} \\
&= C_0-\eta\left[m\tau+(1-m)\,\overline{\tau}_0\right]
\end{aligned}$$

for any τ and $\overline{\tau}_0$, where $C_0 \equiv C(0) = 1-\eta$ denotes the zero-tax level of consumption. Since $\overline{C}(\tau)$ is linear in the actual tax rate τ, $\mathbb{E}\left[\overline{C}(\tau)\,|\tau_0\right]$ is also linear in the intended tax rate τ_0. Thus, the social planner's maximization problem is quadratic in τ_0, and hence globally concave. Thus, the social planner chooses a pure strategy of a tax rate τ_0, and hence the equilibrium is unique. □

Lemmas 1 and 2 are consistent with the two key assumptions imposed in the canonical models of inattention over taxes, both incorporated in the general model of Farhi and Gabaix (2020). Lemma 1 is consistent with the model in Finkelstein (2009), which assumes that the perceived tax rate is a linear combination of the actual tax and its expectation. Lemma 2 is consistent with the models in Chetty et al. (2009), which formulates the attenuation of behavioral responses by the attention

"beauty contest models" in macroeconomics (Morris and Shin, 2002). Morris and Shin (2001), and more recently, Ambrus and Kolb (2021) contains a thorough discussion that shows validity of this approach. In the context of Ramsey taxation, this assumption is equivalent to assuming quadratic utility of a representative agent over his aggregate consumption. Without this assumption, one has to work with the truncated normal distributions, which will not allow for decomposition of various effects. While this assumption may not hold literally, the result is approximately valid since normal distribution is thin-tailed. This approximation argument has also been made in the related works that also use normal distributions (Maggi, 1999; Boccanfuso and Ferey, 2021).

parameter[14]. As forcefully argued previously (Mullainathan et al., 2012; Chetty et al., 2009), the inattention due to noisy signals in Bayes' rule also generates a reduction in behavioral responses similar to other forms of adjustment frictions. While these canonical models often assumed these results exogenously, this model has derived them endogenously based on the Bayes' rule in a unique equilibrium.

3 Analyses

Section 2 has laid out the Bayesian model of taxation with inattention. Here, we show that this model suggests that the inattention reduces social welfare in equilibrium, as illustrated in the 2.1 Basic Model Overview. Further, this analysis leads to a new sufficient statistics formula to assess the optimality of tax transparency programs. Note that, henceforth, the detailed proofs will be relegated to the Appendix A1 and A2.

3.1 Benchmark Analysis with Full Attention ($m = 1$)

Let us begin with the benchmark analysis with a full attention that implies the optimal tax formula with an inverse elasticity rule.

Proposition 1. Optimal Tax Rate. *Suppose Assumption A1. Suppose the tax is observed without noise, $\sigma_\varepsilon^2 = 0$. Then, the optimal intended tax rate, τ_0^*, is*

$$\tau_0^* = \frac{C_0}{\eta}\frac{\lambda}{2\lambda+1}. \tag{9}$$

The optimal expected welfare, $\mathbb{E}w^*$, is

$$\mathbb{E}w^* = \frac{C_0^2}{2\eta}\frac{\lambda^2}{2\lambda+1} - \eta\left[\lambda + \frac{1}{2}\right]\sigma_\nu^2 \tag{10}$$

Sketch of Proof. As illustrated by Figure 1 (I), the aggregate consumption is consistent with the underlying demand curve because the perceived tax rate equals

[14] The relative weight on the tax signal is called an *attention parameter*, following the terminology of the behavioral public finance literature. Its notation, m, follows Farhi and Gabaix (2020), and corresponds to θ in Chetty et al. (2009) and $\delta_1(\theta)$ in Finkelstein (2009). This approach dates back at least to Peek and Wilcox (1984).

the actual tax rate, $\tilde{\tau} = \tau$. That is, $m = 1$, and by Lemma 2,

$$\overline{C}(\tau) = C_0 - \eta\tau.$$

The deadweight loss conditional on the actual tax τ is thus the triangle,

$$DWL(\tau) = \eta\frac{\tau^2}{2}.$$

The social planner chooses the intended tax rate τ_0, and the first order condition implies the tax rate (9). Since there is an idiosyncratic deviation of the actual tax τ from the intended tax τ_0, there will be an additional welfare loss, $-\eta\left[\lambda + 1/2\right]\sigma_\nu^2$. Nonetheless, as shown in Appendix Proposition 1, this variance cost is additively separable due to the Assumption A1, and thus, the optimal intended tax rate (9) is still the standard inverse elasticity formula. □

Proposition 1 shows that the set-up with full attention replicates the classical result of the inverse elasticity rule (Ramsey, 1927).[15] Further, the model yields a closed-form expression for the attained welfare because the demand function is linear by Assumption A1. Henceforth, we will use these results as the benchmark to assess the implications of Bayesian inattention.

3.2 Short-run Analysis with Inattention ($m < 1, \overline{\tau}_0 = 0$)

Let us next consider the effect of inattention in the short-run as an impulse response to increase in tax rates when the consumers' prior belief equals the initial tax rate of zero. The results are broadly consistent with the existing models of taxation with inattention: inattention increases the tax rate, and improves the welfare so long as the overall misperception is small.

Proposition 2. Non-equilibrium Tax Rate. *Suppose Assumption A1. Sup-*

[15]The initial consumption term, C_0, may appear unfamiliar in the optimal tax formula. However, this formula is exactly identical to the standard inverse elasticity rule. Dividing both sides by $1+\tau_0$, we find

$$\frac{\tau_0}{1+\tau_0} = \frac{1}{1+\tau_0}\frac{C_0}{\eta}\frac{\lambda}{2\lambda+1} = \frac{1}{e}\frac{\lambda}{2\lambda+1},$$

since

$$\frac{1}{1+\tau_0}\frac{C_0}{\eta} = \frac{C_0}{1+\tau_0}\frac{\partial(1+\tau_0)}{\partial C_0} = 1/\left(\frac{\partial \log C_0}{\partial \log(1+\tau_0)}\right) = 1/e,$$

where e denotes the elasticity.

pose the tax is observed with noise, $\sigma_\varepsilon^2 > 0$. Fix the prior tax rate to be $\overline{\tau}_0 = 0$. Then, the non-equilibrium intended tax rate, τ_0^n, is

$$\tau_0^n = \frac{C_0}{m\eta}\frac{\lambda}{2\lambda + m}. \tag{11}$$

The non-equilibrium expected welfare, $\mathbb{E}w^n$, is

$$\mathbb{E}w^n = \frac{C_0^2}{2m\eta}\frac{\lambda^2}{2\lambda + m} - \eta\left[\lambda + \frac{1}{2}\right]\sigma_\nu^2 - m\eta\frac{\sigma_\varepsilon^2}{2}. \tag{12}$$

Thus, the tax rate level is higher than the full attention optimum, $\tau_0^n > \tau_0^*$. Moreover, whenever the uncertainty is sufficiently low, i.e. $\sigma_\varepsilon^2 < \overline{\sigma}_\varepsilon^2$ for some $\overline{\sigma}_\varepsilon > 0$, the attained welfare level is also higher than the full attention setting, $\mathbb{E}w^n > \mathbb{E}w^*$.

Sketch of Proof. As illustrated by the slope of short-run demand curve in Figure 1 (II), inattention reduces the behavioral response to the tax rate. By Lemma 2, the aggregate consumption is

$$\overline{C}(\tau) = C_0 - m\eta\tau,$$

where $m < 1$.

The deadweight loss given the actual tax τ consists of the smaller triangle, as well as the errors due to idiosyncratic misperception:

$$DWL(\tau) = \eta\frac{(m\tau)^2}{2} + \eta\frac{m^2\sigma_\varepsilon^2}{2}. \tag{13}$$

While there are idiosyncratic errors in both actual tax and its perception, as shown in Appendix Proposition 1, they are additively separable by the Assumption A1. Thus, the social planner chooses the intended tax rate (11) by the first order condition, and the welfare expression (12) is derived by substitution.

The non-equilibrium tax rate is higher than the optimal tax rate because, for any $m \in [0, 1]$ and $\lambda > 0$,

$$\frac{1}{2\lambda m + m^2} > \frac{1}{2\lambda + 1}.$$

However, so long as the uncertainty in perceived tax rate, σ_ε^2, is sufficiently small, the non-equilibrium welfare is higher than the full attention benchmark. □

Proposition 2 confirms that, when the priors have not been updated, the Bayesian model replicates the results of models with reduced-form inattention (Chetty et al., 2009). Specifically, the social planner chooses the tax rate higher than the full attention benchmark because a higher revenue can be collected with a smaller deadweight loss. As note by Chetty et al. (2009), there will also be a welfare loss from consumers who misperceive the tax to be too low, but it does not increase the deadweight loss since their consumption results in the government revenue. As noted by Farhi and Gabaix (2020), the formula (11) also shows that this effect is in orders of m^2, suggesting that the tax rate increases "relatively fast" when inattention increases slightly from the benchmark of full attention.

However, the analysis also shows that the welfare implications of inattention become more nuanced due to the noise in their tax perceptions[16]. That is, citizens' perceived tax not only deviate systematically but also differ idiosyncratically from the actual tax. Even though this effect does not alter the aggregate consumption and cannot be visualized in Figure 1 (II), it still reduces the total consumer welfare. Nonetheless, so long as this uncertainty, σ_ε, is small, this effect will also be negligible. Thus, in the "neighborhood" of the full attention model, inattention still improves social welfare in the short-run.

3.3 Long-run Analysis with Inattention ($m < 1, \overline{\tau}_0 = \tau_0^e$)

When the tax rate is raised, the inattentive Bayesian consumers will not notice in the short-run. However, in the long-run, they will learn that the tax rate is high, even without observing precise signals. We will now show that this equilibrium effect alters the results found in the short-run analysis.

Proposition 3. Equilibrium Tax Rate. *Suppose Assumption A1. Suppose the tax is observed with noise, $\sigma_\varepsilon^2 > 0$. Then, in the unique equilibrium, the equilibrium intended tax rate, τ_0^e, is*

$$\tau_0^e = \frac{C_0}{\eta} \frac{\lambda}{\lambda (1 + m) + m}. \tag{14}$$

The equilibrium expected welfare, $\mathbb{E}w^e$, is

$$\mathbb{E}w^e = \frac{C_0^2}{2\eta} \frac{\lambda^2}{2\lambda + 1} \Lambda (\lambda, m) - \eta \left[\lambda + \frac{1}{2}\right] \sigma_\nu^2 - m\eta \frac{\sigma_\varepsilon^2}{2}, \tag{15}$$

[16] This effect is related to but differs from the attention variation effects in Taubinsky and Rees-Jones (2018) since here the attention level m is fixed but the citizens perceptions vary.

where the value of commtiment, $\Lambda(\lambda, m)$, is defined as

$$\Lambda(\lambda, m) \equiv 1 - \left[\frac{(\lambda+1)(1-m)}{\lambda(1+m)+m}\right]^2. \tag{16}$$

Thus, the increase in tax rate due to the inattention is mitigated, $\tau_0^e \in (\tau_0^*, \tau_0^n)$, and the resulting welfare is lower than under the optimal tax rate, $\mathbb{E}w^e < \mathbb{E}w^*$.

Sketch of Proof. As illustrated by Figure 1 (III) and Figure 3, the long-run demand curve is not only less elastic with respect to the actual tax rate τ, but also is shifted in parallel to intersect with the underlying demand curve at the prior tax rate $\overline{\tau}_0$. That is, by Lemma 1, the aggregate consumption is

$$\overline{C}(\tau) = C_0 - \eta\left[m\tau + (1-m)\overline{\tau}_0\right]. \tag{17}$$

Given the actual tax rate τ, the deadweight loss becomes

$$DWL(\tau) = \frac{\eta}{2}\left[m\tau + (1-m)\overline{\tau}_0\right]^2 + \eta\frac{m^2\sigma_\varepsilon^2}{2}. \tag{18}$$

The key is that the social planner takes the citizens expectation $\overline{\tau}_0$ as given, and chooses the intended tax rate τ_0 to solve

$$\max_{\tau_0} \mathbb{E}\left[\lambda\tau\overline{C}(\tau) - DWL(\tau)\,|\tau_0\right]. \tag{19}$$

And by the Appendix Proposition 1, the first order condition of (19) with respect to τ_0 implies

$$\lambda\left\{C_0 - \eta\left[2m\tau_0 + (1-m)\overline{\tau}_0\right]\right\} = m\eta\left[m\tau_0 + (1-m)\overline{\tau}_0\right]. \tag{20}$$

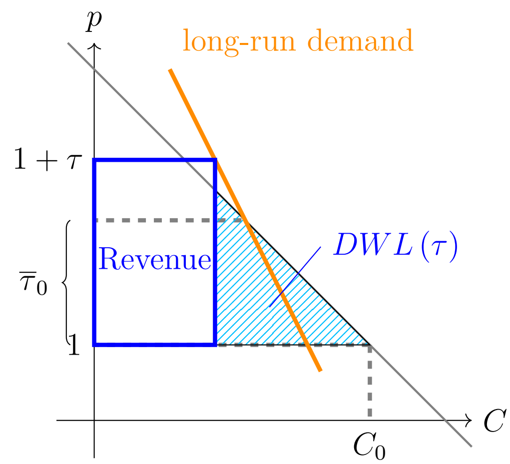
Nonetheless, in equilibrium, the prior must coincide with the intended tax rate by the Bayes' rule:

$$\overline{\tau}_0 = \tau_0. \tag{21}$$

We substitute this consistency condition (21) into (20) to derive the equilibrium tax formula (14). The expression (15) for the resulting welfare is obtained by substitution.

Note that $\tau_0^o < \tau_0^e < \tau_0^n$ because, by $m \in (0,1)$,

Figure 3: Revenue and Deadweight Loss under Inattention in the Long-run



Notes: Figure 3 illustrates the optimal tax problem under inattention in the long-run. The long-run demand curve with inattention (the thick orange solid line) is less elastic than the underlying demand curve (the thin black straight line) because the consumers are inattentive to taxes. Nonetheless, they interact at the expected tax rate $\overline{\tau}_0$ because the citizens' prior belief will adjust in the long-run. Crucially, the social planner takes the citizens' prior as given when considering the welfare effects of tax rate τ.

$$\frac{1}{2\lambda+1} < \frac{1}{\lambda(1+m)+m} < \frac{1}{2\lambda m + m^2}.$$

The attained welfare becomes unambiguously lower than the full attention benchmark both because the value of commitment is less than 1 and because idiosyncratic misperceptions further reduce the welfare. □

Proposition 3 shows that the equilibrium tax rate (14) will be higher than the optimal rate (9) but not as high as the non-equilibrium tax rate (11).[17] Here, the increase in tax rate will be moderated because the aggregate consumption, and consequently

[17] It is instructive to interpret the limit cases of full attention ($m = 1$) and no attention ($m = 0$). When $m = 1$, the optimal tax formula (9) is recovered for both non-equilibrium (11) and equilibrium (14) tax formula, . For the equilibrium tax rate, as the attention decreases to $m = 0$ in equilibrium, the aggregate price $1 + \tau^e$ equals $1/\eta$, the intercept of the demand curve. That is, when there is no attention at all, the government will raise tax maximally so that the average aggregate consumption will be 0.

the marginal revenue arising from taxation, will be low due to the updated prior belief. Unlike the short-run analysis, the effect of inattention is no longer quadratic, but instead linear, in the attention parameter. This result is quantitatively notable. Concretely, a comparative static analysis shows, starting from the benchmark of full attention, the effect of inattention on the implied tax rates will be halved compared to the short-run analysis: for any λ and η,

$$\left.\frac{\partial \tau_0^e}{\partial m}\right|_{m=1} = \frac{1}{2} \left.\frac{\partial \tau_0^n}{\partial m}\right|_{m=1}. \tag{22}$$

The derivation is contained in the Appendix A3. This upward bias is named "taxation bias" by Boccanfuso and Ferey (2021), and its magnitude implied in this model is consistent with their taxation bias formula as shown in the Appendix A4.

Furthermore, the formula (15) shows that the inattention *reduces* the resulting equilibrium welfare. That is, the equilibrium tax rate is not optimally but excessively high because inattention compromises the government's ability to commit to the optimal tax rate. In any optimal tax models, if the government is allowed to choose the tax rate *after* the citizens' consumption decision, then it will choose a very high tax rate since there is zero behavioral distortion. But anticipating such high taxes, the citizens will not consume, and the government's enhanced ability to set tax rates after the consumption perversely reduces welfare. Because inattention essentially leads the consumers to make decisions without precise observation of tax rates, its effect is analogous to allowing the government to change taxes afterwards. That is, even though inattention may reduce the behavioral responses in the short-run, government will have to incur its cost in the long-run as the consumers' priors adjust. Together with the welfare reduction in idiosyncratic errors, the analysis shows that the long-run welfare must be reduced by citizens' inattention; conversely, tax transparency will unambiguously improve the social welfare in this model.

3.4 Implications for Sufficient Statistics Formulas

The long-run analysis of taxation under inattention leads to new empirical implications for the policies of tax transparency and tax rates.

3.4.1 Efficiency of Costly Tax Transparency Policy

We can use the closed-form welfare expressions to derive sufficient statistics formulas that inform whether a costly policy to increase tax transparency is welfare-improving. In the U.S., approximately 10 percent of the tax revenue is lost as a compliance cost (Slemrod, 1996), and this is partly motivated to ensure the transparency of the tax system.[18] While faced with the concerns of sacrificing transparency, there have been controversial proposals to reduce this cost (Finkelstein, 2009). Does the model support the argument to eliminate the costly effort for tax transparency?

Proposition 4.1. Sufficient Statistics Formula for Tax Transparency Policy. *Suppose Assumption A1. Suppose a tax transparency program increases the citizens' attention from m_0 to m_1, $m_0 < m_1$, by reducing σ_ε^2 at some fixed cost $\kappa \leq \lambda$.*[19] *The tax transparency policy improves the equilibrium welfare for any parameters $\{\eta, C_0, \sigma_\nu^2\}$ if*

$$m_0 < \frac{1}{2(1+\lambda)} \tag{23}$$

and

$$m_1 > \frac{1}{2(1+\lambda-\kappa)}. \tag{24}$$

Proof. By Propositions 1 and 3, a sufficient condition for the tax transparency program to improve welfare is

$$\frac{C_0^2}{2\eta}\frac{(\lambda-\kappa)^2}{2(\lambda-\kappa)+1}\Lambda(\lambda-\kappa, m_1) > \frac{C_0^2}{2\eta}\frac{\lambda^2}{2\lambda+1}\Lambda(\lambda, m_0)$$

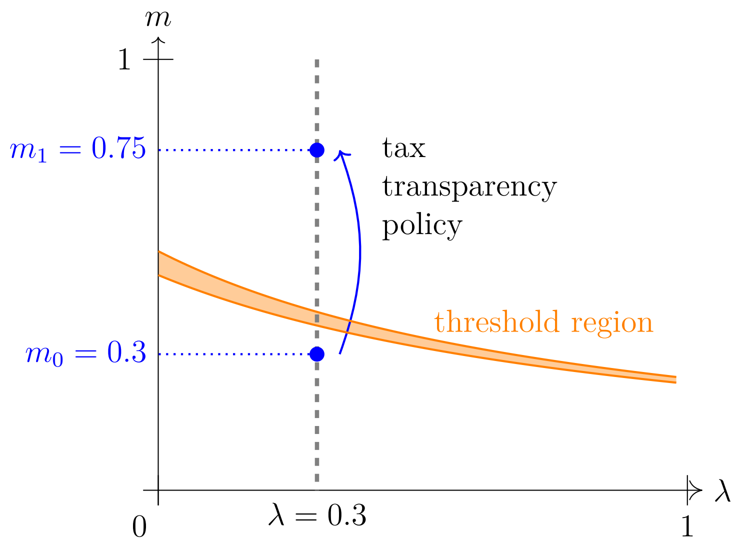
since the welfare loss due to idiosyncratic errors is also reduced. We can focus on the value of commitment, $\Lambda(\lambda, m)$, to observe that this condition is always satisfied whenever

$$\Lambda(\lambda, m_0) < 0 \Rightarrow m_0 < \frac{1}{2(1+\lambda)},$$
$$\Lambda(\lambda-\kappa, m_1) > 0 \Rightarrow m_1 > \frac{1}{2(1+\lambda-\kappa)}.$$

[18] De Neve et al. (2021) has recently shown that the simplicity and transparency can substantially reduce the compliance issues.

[19] Note that, if $\kappa > \lambda$, then the tax will always lower welfare, and thus, the analysis of Proposition 1 and 3 will not apply.

Figure 4: Optimality Condition for Tax Transparency Policy



Notes: Figure 4 summarizes the key implications of the sufficient statistics formulas. This depicts the threshold condition for the attention parameters, m, that is needed to ensure the social optimality of the program given λ. The thick orange region represents the sufficient statistics formulas. The policy is socially worthwhile for any parameter values if the original attention level, m_0, is below this region, and the improved attention level, m_1, goes above it. Given a typical estimate of $\lambda = 0.3$ (Ballard et al., 1985; Poterba, 1995; Finkelstein and McKnight, 2008), the improvement from attention level under non-transparent policy, $m_0 = 0.3$ (Chetty et al., 2009; Taubinsky and Rees-Jones, 2018), to the higher level under a transparent policy, $m_1 = 0.75$ (Rees-Jones and Taubinsky, 2019; Boccanfuso and Ferey, 2021), is shown to be socially worthwhile.

□

In words, incurring the costs to improve the tax transparency is socially worthwhile if the attention without the program is very poor, and if the attention with the program is sufficiently improved. This condition is illustrated in Figure 4. Note that these conditions are a conservative *sufficient* condition, not a necessary condition. Nonetheless, they provide an attractively simple condition to examine.[20]

[20] The necessary and sufficient condition is

$$\frac{2\lambda + 1}{2(\lambda - \kappa) + 1}\left(1 - \frac{\kappa}{\lambda}\right)^2 > \frac{\Lambda(\lambda, m_0)}{\Lambda(\lambda - \kappa, m_1)},$$

where $\Lambda()$ is defined in (16). Here, I consider the simpler sufficient condition for the clarity of the exposition.

Back-of-the-envelope calculation: We can quantitatively examine the efficiency of the tax transparency effort for the U.S. income taxes by applying the estimates from the literature. Here, we choose $\lambda = 0.3$ as the commonly used estimate in the literature (Ballard et al., 1985; Poterba, 1995; Finkelstein and McKnight, 2008) and set $\kappa = 0.1$ following Slemrod (1996)[21], which satisfies $\lambda \geq \kappa$.[22] Recently, Boccanfuso and Ferey (2021) has argued $m_1 = 0.75$ as a reasonable estimate of the attention to income taxes, incorporating the estimates in Rees-Jones and Taubinsky (2019). While we do not know how inattentive citizens will be in the counterfactual situation without the transparency effort, many studies suggest that the inattention can be very severe, including $m = 0.35$ in the grocery store experiment of Chetty et al. (2009) and $m = 0.25$ in the online shopping experiment of Taubinsky and Rees-Jones (2018). Given these influential studies, setting $m_0 = 0.3$ in the absence of the policy is reasonable. But since these typical estimates satisfy the sufficient statistics formulas (23) and (24), this model supports the argument that the costly efforts to ensure tax transparency are worthwhile.

3.4.2 Discontinuously Large Role of Small Noise

Theories of costly attention (See Gabaix, 2019; Maćkowiak et al., 2020 for reviews) suggest that citizens will pay more attention when the welfare consequences are high. In contrast with the effect of idiosyncratic errors that become small (Section 3.2), here we show that this long-run commitment effect can be large even when these uncertainties become negligible.

Proposition 4.2 Discontinuity under Small Noise. *Suppose Assumption A1. Fix any $m \in [0,1]$, and set $\sigma_\nu^2 = [2\lambda m + m^2]\,\sigma_\varepsilon^2$. Then, the resulting equilibrium intended tax rate, $\tau_0^e(\sigma_\varepsilon^2)$, will have a discontinuity at $\sigma_\varepsilon^2 = 0$:*

- *when $\sigma_\varepsilon^2 = 0$,*

$$\tau_0^e\left(\sigma_\varepsilon^2 = 0\right) = \tau_0^* \tag{25}$$

- *when $\sigma_\varepsilon^2 \to 0$,*

$$\lim_{\sigma_\varepsilon^2 \to 0} \tau_0^e\left(\sigma_\varepsilon^2\right) > \tau_0^*. \tag{26}$$

[21] A more recent estimate from of the Taxpayer Advocate (2010) also suggests a similar estimate of $\kappa = 0.11$.

[22] The original paper of Ballard et al. (1985) suggests the value of λ between 0.17 and 0.56. As Slemrod (1996) was careful to note, the wide range of plausible values of κ may be wide. Nonetheless, since the smallest suggested value of λ is 0.17, the condition $\kappa \leq \lambda$ is plausible.

Proof. The equilibrium tax formula (14) shows that the mean tax rate will depend on the attention parameter, m, which reflects the *relative* precision of signal, but not its *absolute* precision.[23] Thus, for every τ_0^e and its corresponding m, there exist sequence of $\{\sigma_\varepsilon^2, \sigma_\nu^2\}$ such that the overall uncertainty, $\sigma_\varepsilon^2 + \sigma_\nu^2$, converges to 0 while maintaining m constant. Thus, even if σ_ε^2 is arbitrarily small, the implied equilibrium tax rate will be higher so long as σ_ν^2 is also proportionately small. □

Standard tax models assume full attention of consumers ($\sigma_\varepsilon^2 = 0$) and full control of government over tax rates ($\sigma_\nu^2 = 0$). This modeling choice is made because people are believed to be fairly attentive and the hypothetical planner makes little mistakes, at least for important items of consumption as experimentally shown in Morrison and Taubinsky (2021). In many models, implications are approximately valid whenever their assumptions also approximately hold. This analysis shows, however, that such approximate validity of optimal taxation models is *not* guaranteed. So long as the policy-making process also becomes proportionately more careful and less variable for important items, then the commitment effect will persist. In this sense, the standard taxation models are a "knife-edge" case: there exists a perturbation of the full attention and full control benchmark that changes their implications for taxes and welfare discontinuously. This *fragility* may be important in considering the robustness of the optimal tax analyses (e.g. Lockwood et al., 2020), and in understanding when and why the actual tax chosen deviates from the socially optimal tax rates.

[23] Besides the proof based on the relative precision of signals, we can also interpret this result based on higher-order beliefs (See e.g. Morris and Shin, 2002). To see this, let us consider citizens that receive the signal $\hat{\tau}$ to form an expectation $\mathbb{E}[\tau|\hat{\tau}]$ over the tax rate. Citizens also know that the planner sets the intended tax rate based on the government's expectation of citizen's expectation,

$$\mathbb{E}\left[\mathbb{E}\left[\tau|\hat{\tau}\right]|\tau_0\right].$$

However, this higher order belief will always be closer to the prior tax rate. That is, even when the tax rate is higher than the expectation, that increase will be only partially perceived by the citizens on average; conversely, even when the planner sets the tax rate level to be lower, that decrease will be only noisily perceived in expectation. In the model, citizens also know the government objective to increase the tax rate when the behavioral responses are mitigated. Thus, this model implies that the citizens will expect a higher tax rate under inattention. This result is inspired by game theory's insight that commitment is fragile under slightly imperfect observability (Bagwell, 1995). The new element is the insight from global games (Carlsson and van Damme, 1993; Morris and Shin, 1998; Weinstein and Yildiz, 2007) that the equilibrium implication will depend crucially on the relative precision, but not on the absolute precision, of signals. This approach stands in contrast with the other subsequent papers (van Damme and Hurkens, 1997; Maggi, 1999) that considered a limit of small noise while keeping the environmental uncertainties constant.

4 Discussions and Conclusion

This Section discusses the key assumptions of the model, and concludes.

4.1 Discussions

The model has derived its result under many assumptions, such as (1) Bayesian learning, (2) full updating, (3) myopic objective in dynamic interpretation, (4) constant degree of attention in the population, (5) specific distributional assumptions, (6) constant degree of risk aversion in the population, and (7) government's social welfare maximization objective. Let us discuss if the main results hold when these assumptions are altered.

1. **Evidence on Bayesian updating:** *Are there evidence that the citizens update their belief over the tax rates?*

   Yes. Evidence from multiple studies suggests that even the inattentive citizens learn about their taxes over time.[24] Recently, Caldwell et al. (2021) has elicited the mean, confidence level, and probablistic beliefs over tax refunds among low-income households in Boston. They provide direct evidence that, while the tax filers face substantial uncertainties, they update their beliefs with new information in a way consistent with Bayesian updating[25]. In addition, Aghion et al. (2017) finds that, among self-employed workers in France, the tax complexity has large welfare costs, and while the citizens do not immediately understand the tax regimes, they learn over time to adjust to the optimal choice.

2. **Partial updating:** *Will tax transparency still be beneficial when the citizens update their belief only partially?*

   While this paper has focused on the cases with no and full updating, the citizens may update their belief only partially. Such bias in tax perception is

[24]Some papers suggest that the citizens perceive the tax rates accurately (Fujii and Hawley, 1988), at least on average (Gideon, 2017; Caldwell et al., 2021), while others suggest that they may be confused about the tax schedules (Feldman et al., 2016), or have mistaken beliefs about them (Ballard and Gupta, 2018). See, for example, Fochmann et al. (2010) for a comprehensive survey.

[25]Relatedly, Finkelstein (2009) surveys the belief of the drivers who pay the tolls by cash or electronically, and finds that those who are inattentive to the tolls because they are pay electronically tend to expect higher tolls, consistent with the idea that people adjust their priors even though they are inattentive.

documented in Chetty et al. (2009), which suggests that the consumers are informed of tax rates but may not remember them at the time of purchasing decisions. The partial updating can be modeled by including a bias parameter, $\zeta \in [0, 1]$, in setting the prior belief:

$$\overline{\tau}_0 = \zeta \tau_0.$$

As reported in Appendix Proposition 2, there will be a mixed effect of inattention: while the perception bias $\zeta < 1$ lowers behavioral distortion, the imperfect observability $m < 1$ compromises the government commitment. The Appendix A5 shows that the transparency will still improve the welfare so long as the former effect is small (or, $\zeta \geq \overline{\zeta}$ for some $\overline{\zeta}$).

3. **Time horizon of government objective:** *If the government is far sighted and maximizes the long-run welfare, would the commitment issue be resolved?*

   Yes. The time horizon of government objective can be modeled by whether the behavioral response is measured in either short-term or long-term. As citizens receive more information, the signal precision $(1/\sigma_\varepsilon^2)$ increases over time. Thus, in the short-term, the attention parameter and thus the behavioral response are lower while in the long-term, they are both higher.

4. **Heterogeneity in attention parameters:** *Are the model's implications robust to population heterogeneity in the attention parameter?*

   Yes. Both the implied equilibrium tax rate and the welfare in Proposition 3 depends only on the average attention parameter. This result on tax rate (14) is consistent with the empirical research that focuses on the average elasticities (See, e.g. Saez, 2010). Further, the welfare consequence of inattention is also invariant to the heterogeneity so long as the government places the same weight on the errors made by different populations as shown in Appendix A6. Thus, even though the formula (15) shows the welfare becomes lower when the signal is noisier[26], it is invariant to the heterogeneity in the degree of noisiness. This is notable for the sufficient statistics analyses because a number of recent papers (Hoopes et al., 2015; Aghion et al., 2017; Taubinsky and Rees-Jones, 2018) highlights the importance of the attention heterogeneity.

[26] This is consistent with the result derived in Taubinsky and Rees-Jones (2018).

5. **Distributional assumptions:** *If the assumptions of uniform distribution and normal distribution were replaced by some other distributions, could the results hold?*

   The quadratic objective, which results from the uniform distribution, and normal type and noise distributions, are a pair of assumptions that is commonly used (e.g. Morris and Shin, 2002; Sims, 2003) to derive closed-form solutions. Since the optimal tax problem yields closed-form solutions when modeled by the quadratic objective (Dupuit, 1844; Ramsey, 1927), the uniform and normal distributions are adopted in this paper. But as shown by the general model of Boccanfuso and Ferey (2021), these distributional assumptions are not the driving force of the main results regarding commitment.

6. **Potential benefit of tax uncertainties:** *Both commitment effect and idiosyncratic errors of tax uncertainty suggest that the tax uncertainty decreases welfare. Are there any reasons that the tax uncertainty may help?*

   Yes. From the subjective perspective of the citizens, the uncertainty over tax rates implies that the realized tax rates will be random. When there is heterogeneity of risk aversion, randomization of tax rates can improve welfare through efficient screening (Atkinson and Stiglitz, 1976; Stiglitz, 1982). In the context of income taxation, if high ability citizens are more risk averse than low ability citizens, then subjective uncertainty over the tax rate given low income can prevent the high ability ones from pretending to be low abilities. More broadly, if citizens are prudent, then future income uncertainty due to random taxes can encourage them to work more. While this paper sets aside this effect by assuming that the risk aversion is constant (i.e. quadratic utility), this effect could exist in the real world.

7. **Other constraints on government objective:** *If the government faces other constraints, such as political constraints, so that it sets the tax rate lower than the social optimum, could inattention improve the social welfare to offset the pre-existing biases?*

   Yes. By the theory of second best (Lipsey and Lancaster, 1956), the inefficiency due to inattention can address other inefficiencies. Mill (1848) discusses the possibility that the "excessively" collected revenues may be used efficiently for valuable public expenditures that do not have political support.

4.2 Conclusion

While long and widely regarded as the key policy concern, tax transparency has been a difficult theme for optimal tax analyses. Based on the evidence that non-transparency alleviates behavioral responses, Chetty et al. (2009) has modeled the transparency as a "reduced-form" parameter that alters behavioral elasticities. Farhi and Gabaix (2020) has generalized this analysis and other subsequent papers using the sparcity model of inattention (Gabaix, 2014), which is more tractable than Bayesian learning. However, these models found that the transparency *reduces* social welfare in the benchmark with small income effects, the conclusion that many authors have argued against with various reasons[27].

This paper, in contrast, revisits the traditional insights since Dupuit (1844) that the welfare losses can be formulated as a quadratic problem. As the optimal taxation problem is concerned with this quadratic objective (Dupuit, 1844; Ramsey, 1927; Harberger, 1964), this paper develops a tractable Bayesian model of tax transparency with the normal distributions of states and signals. This model yields new sufficient statistics formulas to assess the social optimality of the controversial tax transparency policies. In this way, this paper extends the most recent analysis of Boccanfuso and Ferey (2021) that also shows, by the Bayesian equilibrium effects, the tax transparency *improves* social welfare. These conclusions are consistent with traditional discussions of Smith (1776) and Mill (1848), as well as policy practices to promote the transparency today.

[27] For example, Chetty et al. (2009) emphasizes the role of income effects, and Taubinsky and Rees-Jones (2018) highlights the role of individual mistakes in consumption.

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Appendix

This Appendix contains the formal proofs. A1 shows that the social planner's problem is quadratic in the intended tax rate, providing the basis for the Propositions 1-3 in the main text; A2 derives the attained welfare in the Propositions; A3 shows the comparative static of the tax rate with respect to the attention parameter; A4 derives the taxation bias formula; A5 analyzes the setting with partial updating; and A6 examines the welfare implications of heterogeneity in the attention parameter.

A1. Social Welfare Formula

In the model set-up, the deadweight was defined as the reduction in welfare due to taxes not included in the revenue. Here, we show that the expression (5) translates into the tractable quadratic formula, as used in the Propositions.

Appendix Proposition 1. *Suppose Assumption A1. Given the expected value of the prior tax rate, $\overline{\tau}_0$, the social welfare (4) is*

$$\mathbb{E}\left[w|\tau_0\right] = -\alpha\tau_0^2 + \beta\tau_0 + \gamma, \tag{27}$$

where

$$\begin{aligned}
\alpha &= m\eta\left[\lambda + \frac{m}{2}\right] \\
\beta &= \lambda C_0 - \eta\left(1-m\right)\left(\lambda+m\right)\overline{\tau}_0 \\
\gamma &= -\eta\left[\lambda + \frac{1}{2}\right]\sigma_\nu^2 - m\eta\frac{\sigma_\varepsilon^2}{2} - \eta\frac{\left(1-m\right)^2\overline{\tau}_0^2}{2}.
\end{aligned}$$

Proof. The proof consists of two steps: first, we derive the individual utilities given the consumption choices, and use the expressions to derive the components of objective; second, we combine them to derive the formula (27).

Step 1. Welfare expressions of each component: the social planner's objective consists of (i) the zero-tax welfare and (ii) the equilibrium welfare and revenue. When we derive each component, we consider definite integrals over a domain $v \in [-\overline{v}, \overline{v}]$ with a uniform density.

(i) pre-tax welfare: since the first-best efficient decision is to consume if and only if $v \geq 1$,

$$\begin{aligned}
\int_{-\overline{v}}^{\overline{v}} \max_{c} u\left(c, v | \tau=0\right) dv &= \int_{-\overline{v}}^{\overline{v}} \mathbb{1}\left(v \geq 1\right)\left(v-1\right) dv \\
&= \int_{1}^{\overline{v}} \left(v-1\right) dv \\
&= \left(\frac{v^2}{2} - v\right)\Big|_{1}^{\overline{v}} \\
&= \frac{\overline{v}^2}{2} - \overline{v} + \frac{1}{2}. \qquad (28)
\end{aligned}$$

(ii) equilibrium welfare and revenue: since the equilibrium decision is to consume if and only if $v \geq 1 + \tilde{\tau}$,

$$\int_{-\overline{v}}^{\overline{v}} \mathbb{E}\left[u\left(c_{v,\tilde{\tau}}, v|\tau\right) + \tau c_{v,\tilde{\tau}} \mid \tau\right] dv = \int_{-\overline{v}}^{\overline{v}} \mathbb{P}\left(v \geq 1 + \tilde{\tau} \mid \tau\right)\left(v-1\right) dv$$

Since $\tilde{\tau} = m\hat{\tau} + (1-m)\,\overline{\tau}_0$ by Lemma 1, the variance of the posterior distribution conditional on the actual tax τ is

$$\begin{aligned}
Var\left(\tilde{\tau}|\tau\right) &= m^2 Var\left(\hat{\tau}|\tau\right) + \left(1-m\right)^2 Var\left(\overline{\tau}_0|\tau\right) \\
&= m^2\sigma_\varepsilon^2 \qquad (29)
\end{aligned}$$

Therefore, the fraction of citizens of type v who consume (or, $c^* = 1$) is given by

$$\mathbb{P}\left(v \geq 1 + \tilde{\tau} \mid \tau\right) = \Phi\left(v|\tau\right) \equiv \Phi_0\left(\frac{v - 1 - \left[m\tau + \left(1-m\right)\overline{\tau}_0\right]}{m\sigma_\varepsilon}\right),$$

where $\Phi_0\left(\cdot\right)$ is the distribution function of the standard normal distribution. By the integration by parts,

$$\begin{aligned}
\int_{-\overline{v}}^{\overline{v}} \Phi\left(v|\tau\right)\left(v-1\right) dv &= \left[\frac{v^2}{2} - v\right]\Phi\left(v|\tau\right)\Big|_{-\overline{v}}^{\overline{v}} - \int_{-\overline{v}}^{\overline{v}} \left[\frac{v^2}{2} - v\right]\phi\left(v|\tau\right) dv \\
&= \frac{\overline{v}^2}{2}\left[\Phi\left(\overline{v}|\tau\right) - \Phi\left(-\overline{v}|\tau\right)\right] - \overline{v}\left[\Phi\left(\overline{v}|\tau\right) + \Phi\left(-\overline{v}|\tau\right)\right] \\
&\quad - \frac{1}{2}\mathbb{E}\left[v^2\Big|\,\tau, |v| \leq \overline{v}\right] + \mathbb{E}\left[v|\,\tau, |v| \leq \overline{v}\right], \qquad (30)
\end{aligned}$$

where $\phi\left(v|\tau\right)$ is the density, and $\mathbb{E}\left[\cdot\right]$ is the expectation of $\Phi\left(v|\tau\right)$ for the domain $v \in \left[-\overline{v}, \overline{v}\right]$.

Step 2. Expression for total welfare: we now use the expressions in Step 1, (28) and (30), to show that the total welfare (4) is quadratic in the intended tax rate τ_0.

(i) total deadweight loss conditional on τ: the total deadweight loss conditional on the actual tax τ is

$$DWL(\tau) = \lim_{\overline{v}\to\infty} \int_{-\overline{v}}^{\overline{v}} [\mathbb{1}(v \geq 1) - \Phi(v|\tau)](v-1) f(v)\, dv. \tag{31}$$

Since the density $f(v) = \eta$ for all v by the Assumption A1, we combine and reorganize (28) and (30) to have

$$DWL(\tau)/\eta = \lim_{\overline{v}\to\infty} \frac{\overline{v}^2}{2} [1 - \Phi(\overline{v}|\tau) + \Phi(-\overline{v}|\tau)] \tag{32}$$

$$+ \lim_{\overline{v}\to\infty} \overline{v} [\Phi(\overline{v}|\tau) + \Phi(-\overline{v}|\tau) - 1] \tag{33}$$

$$+ \lim_{\overline{v}\to\infty} \left\{ \frac{1}{2} \mathbb{E}\left[v^2|\tau, |v| \leq \overline{v}\right] - \mathbb{E}[v|\tau, |v| \leq \overline{v}] + \frac{1}{2} \right\} \tag{34}$$

Since the distribution function of normal distribution converges to 0 or 1 faster than the linear or quadratic terms, and since the normal distribution is thin-tailed, the expressions (32) and (33) equal zero by the L'Hopital's rule. Since $\mathbb{E}[v^2] = \mathbb{E}[v]^2 + Var(v)$, taking the limit as $\overline{v} \to \infty$, the expression (34) becomes

$$\begin{aligned} \frac{1}{2} + \frac{1}{2}\mathbb{E}\left[v^2|\tau\right] - \mathbb{E}[v|\tau] &= \frac{\{\mathbb{E}[v|\tau] - 1\}^2 + Var(v|\tau)}{2} \\ &= \frac{[m\tau + (1-m)\overline{\tau}_0]^2}{2} + \frac{m^2\sigma_\varepsilon^2}{2} \end{aligned}$$

by Lemma 1 and (29). Combining these observations, the deadweight loss (31) is

$$DWL(\tau) = \frac{\eta}{2} \left\{ [m\tau + (1-m)\overline{\tau}_0]^2 + m^2\sigma_\varepsilon^2 \right\}.$$

(ii) total welfare conditional on τ_0: using Lemma 1 to combine with the effects on revenues, we have

$$\begin{aligned}
\mathbb{E}\left[w|\tau_0\right] &= \lambda\mathbb{E}\left[\tau\left\{C_0 - \eta\left[m\tau + (1-m)\,\overline{\tau}_0\right]\right\}|\tau_0\right] \\
&- \frac{\eta}{2}\mathbb{E}\left[\left[m\tau + (1-m)\,\tau_0\right]^2|\tau_0\right] - \frac{\eta}{2}m^2\sigma_\varepsilon^2 \\
&= \lambda\tau_0\left\{C_0 - \eta\left[m\tau_0 + (1-m)\,\overline{\tau}_0\right]\right\} - \lambda m\eta\sigma_\nu^2 \\
&- \frac{\eta}{2}\left[m\tau_0 + (1-m)\,\overline{\tau}_0\right]^2 - \frac{\eta}{2}m^2\left(\sigma_\nu^2 + \sigma_\varepsilon^2\right)
\end{aligned}$$

since $\tau = \tau_0 + \nu$, $\mathbb{E}\nu = 0$, and $\mathbb{E}\nu^2 = \sigma_\nu^2$. Thus, re-organizing the terms, we obtain (27). □

A2. Attained Welfare

Here, we derive the expressions for attained welfare in Propositions 1, 2, and 3. From the Appendix Proposition 1, we can complete the square to obtain

$$\mathbb{E}\left[w|\tau_0\right] = \alpha\left(\tau_0 - \frac{\beta}{2\alpha}\right)^2 - \frac{\beta^2}{4\alpha} + \gamma.$$

(i) Proposition 1. Optimal Tax under Full Attention: since $\sigma_\varepsilon^2 = 0$ under full attention, we have $m = 1$. Thus,

$$\begin{aligned}
\alpha &= \eta\left[\lambda + \frac{1}{2}\right] \\
\beta &= \lambda C_0 \\
\gamma &= -\eta\left[\lambda + \frac{1}{2}\right]\sigma_\nu^2.
\end{aligned}$$

Since this welfare is optimized, we have $\mathbb{E}w^* = -\beta^2/4\alpha + \gamma$ to derive (10).

(ii) Proposition 2. Non-equilibrium Tax under Inattention: while $m < 1$ due to inattention, in the short-run, $\overline{\tau}_0 = 0$. Thus,

$$\begin{aligned}
\alpha &= m\eta\left[\lambda + \frac{m}{2}\right] \\
\beta &= \lambda C_0 \\
\gamma &= -\eta\left[\lambda + \frac{1}{2}\right]\sigma_\nu^2 - m\eta\frac{\sigma_\varepsilon^2}{2}.
\end{aligned}$$

Since this welfare is optimized, we again have $\mathbb{E}w^n = -\beta^2/4\alpha + \gamma$ to derive (12).

(iii) Proposition 3. Equilibrium Tax under Inattention: while $m < 1$ due to inattention, the prior belief will be consistent in the long run, $\overline{\tau}_0 = \tau_0^e$. Since we can substitute this into the welfare expression to obtain that the welfare formula is

$$\mathbb{E}w^e = \alpha \left(\tau_0^e - \tau_0^*\right)^2 - \frac{\beta^2}{4\alpha} + \gamma,$$

where

$$\alpha = \eta \left[\lambda + \frac{1}{2}\right]$$
$$\beta = \lambda C_0$$
$$\gamma = -\eta \left[\lambda + \frac{1}{2}\right] \sigma_\nu^2 - m\eta \frac{\sigma_\varepsilon^2}{2}.$$

That is, except the additional term of σ_ε^2, the objectives is identical to the full attention benchmark since $m\tau_0 + (1-m)\overline{\tau}_0 = \tau_0$. By the Propositions 1 and 3,

$$\tau_0^e - \tau_0^* = \frac{\lambda C_0}{\eta} \left(\frac{1}{\lambda(1+m) + m} - \frac{1}{2\lambda + 1} \right),$$

and we derive (15) by substitution.

A3. Comparative Static of Tax Rate with respect to Attention Parameter

We wish to prove the ratio of comparative static (22) in Section 3.3. Given the formulas (11) and (14), let us denote

$$M^n(m) = 2\lambda m + m^2$$
$$M^e(m) = \lambda(1+m) + m.$$

By differentiating the formulas,

$$\frac{\partial\tau_0^n}{\partial m}\mid_{m=1}=\frac{1-\eta}{\eta}\frac{-\lambda}{M^n(m)^2}\frac{\partial M^n(m)}{\partial m}\mid_{m=1}=-\frac{1-\eta}{\eta}\frac{\lambda}{(2\lambda+1)^2}(2\lambda+2)$$
$$\frac{\partial\tau_0^e}{\partial m}\mid_{m=1}=\frac{1-\eta}{\eta}\frac{-\lambda}{M^e(m)^2}\frac{\partial M^e(m)}{\partial m}\mid_{m=1}=-\frac{1-\eta}{\eta}\frac{\lambda}{(2\lambda+1)^2}(\lambda+1).$$

Thus, the ratio (22) holds.

A4. Taxation Bias Formula

Following Boccanfuso and Ferey (2021), let us quantify the taxation bias in this model. In particular, based on Propositions 1 and 3, we can write[28]

$$\tau_0^e-\tau_0^*=\frac{(1-m)(1+\lambda)}{(1+m)\lambda+m}\tau_0^*.$$

As we consider the revenue-maximizing case by $\lambda\to\infty$, this taxation bias expression becomes

$$\tau_0^e-\tau_0^*=\frac{1-m}{1+m}\tau_0^*. \quad (35)$$

This expression is consistent with the taxation bias expression obtained in Boccanfuso and Ferey (2021) (in Section 2), adopting the benchmark case of revenue-maximizing objective for simplicity,

$$\tau_0^e-\tau_0^*=\frac{(1-m)e}{1+me}\tau_0^*, \quad (36)$$

where e here denotes an elasticity. Note that, in this model, the elasticity is 1 since the quadratic utility is assumed. When $e=1$ in their formula (36), the taxation bias formula (35) of this model is derived.

[28]This result follows from

$$\tau_0^e-\tau_0^*=\frac{\lambda C_0}{\eta}\left[\frac{1}{2\lambda+1}-\frac{1}{\lambda(1+m)+m}\right]=\frac{C_0}{\eta}\frac{\lambda(\lambda+1)(1-m)}{(2\lambda+1)[\lambda(1+m)+m]}.$$

A5. Partial Updating

Besides the lack of information, the citizens may hold perception bias that discounts the taxes. Here, we generalize the model to consider such cases.

Appendix Proposition 2. Equilibrium Tax Rate with Partial Updating. *Suppose Assumption A1. Suppose the tax is observed with noise, $\sigma_\varepsilon^2 > 0$. Suppose that the equilibrium prior mean partially reflects the equilibrium tax rate,*

$$\overline{\tau}_0 = \zeta \tau_0,$$

for $\zeta \in (0, 1)$. Then, the equilibrium intended tax rate with partial updating, τ_0^p, is

$$\tau_0^p = \frac{C_0}{\eta} \frac{\lambda}{m (2\lambda + m) + (1 - m) (\lambda + m) \zeta}. \tag{37}$$

The equilibrium expected welfare, $\mathbb{E}w^p$, is

$$\mathbb{E}w^n = \frac{C_0^2}{2\eta} \frac{\lambda^2 \left[m (2\lambda + m) - (1 - m)^2 \zeta^2 \right]}{\left[m (2\lambda + m) + (1 - m) (\lambda + m) \zeta \right]^2} \tag{38}$$

$$- \eta \left[\lambda + \frac{1}{2} \right] \sigma_\nu^2 - m\eta \frac{\sigma_\varepsilon^2}{2}. \tag{39}$$

Thus, the tax rate level is higher than the full attention optimum, $\tau_0^p > \tau_0^*$. Moreover, the attained welfare level can be lower than the full attention setting if and only if ζ is sufficiently high.

Proof. To derive the tax rate, let us replace the equilibrium condition $\overline{\tau}_0 = \tau_0$ (21) in Proposition 3 by $\overline{\tau}_0 = \zeta \tau_0$ above. Then, the first order condition (20) becomes

$$\lambda \left\{ C_0 - \eta \left[2m\tau_0 + (1 - m) \zeta \tau_0 \right] \right\} = m\eta \left[m\tau_0 + (1 - m) \zeta \tau_0 \right]. \tag{40}$$

Rearranging, we obtain the tax formula (37).

Using this expression, we can express each component of welfare (27) by

$$
\begin{aligned}
-\alpha\tau_0^2 &= m\left[\lambda + \frac{m}{2}\right]\frac{C_0^2}{\eta}\frac{\lambda^2}{Z^2} \\
\beta\tau_0 &= \lambda C_0\tau_0 - \eta\left(1-m\right)\left(\lambda+m\right)\zeta\tau_0^2 \\
&= m\left(2\lambda+m\right)\frac{C_0^2}{\eta}\frac{\lambda^2}{Z^2} \\
\gamma &= -\frac{\left(1-m\right)^2\zeta^2}{2}\frac{C_0^2}{\eta}\frac{\lambda^2}{Z^2} - \eta\left[\lambda+\frac{1}{2}\right]\sigma_\nu^2 - m\eta\frac{\sigma_\varepsilon^2}{2},
\end{aligned}
$$

where $Z \equiv m\left(2\lambda+m\right) + \left(1-m\right)\left(\lambda+m\right)\zeta$. By adding these terms, we obtain (38).

Thus, the attained welfare is lower than the full attention setting in Proposition 1 if and only if

$$
\frac{C_0^2}{2\eta}\frac{\lambda^2\left[m\left(2\lambda+m\right)-\left(1-m\right)^2\zeta^2\right]}{\left[m\left(2\lambda+m\right)+\left(1-m\right)\left(\lambda+m\right)\zeta\right]^2} - m\eta\frac{\sigma_\varepsilon^2}{2} \leq \frac{C_0^2}{2\eta}\frac{\lambda^2}{2\lambda+1} \tag{41}
$$

Note that this condition is guaranteed to be satisfied for $\zeta = 1$, the full updating case, by the Proposition 3. We can rearrange this condition as

$$
\tilde{\alpha}\zeta^2 + \tilde{\beta}\zeta + \tilde{\gamma} \geq 0,
$$

where

$$
\begin{aligned}
\tilde{\alpha} &= \left(1-m\right)^2\left\{1+\left(\lambda+m\right)^2\left[\frac{1}{2\lambda+1}+m\left(\frac{\eta\sigma_\varepsilon}{\lambda C_0}\right)^2\right]\right\} \\
\tilde{\beta} &= 2\left[\frac{1}{2\lambda+1}+m\left(\frac{\eta\sigma_\varepsilon}{\lambda C_0}\right)^2\right]m\left(1-m\right)\left(2\lambda+m\right)\left(\lambda+m\right) \\
\tilde{\gamma} &= m\left(2\lambda+m\right)\left\{m\left(2\lambda+m\right)\left[\frac{1}{2\lambda+1}+m\left(\frac{\eta\sigma_\varepsilon}{\lambda C_0}\right)^2\right]-1\right\}.
\end{aligned}
$$

Thus, the above condition (41) is equivalent to $\zeta \geq \overline{\zeta}$, where

$$
\overline{\zeta} = \max\left\{\frac{-\tilde{\beta}+\sqrt{\tilde{\beta}^2-4\tilde{\alpha}\tilde{\gamma}}}{2\tilde{\alpha}}, 0\right\}.
$$

□

A6. Welfare Consequence of Heterogeneity in Attention Parameter

To examine the effect of heterogeneity on equilibrium welfare, we consider a mean-preserving spread of the attention parameter m into $m+(1-q)\,\Delta$ for a fraction q of the population, and into $m-q\Delta$ for another $1-q$ of the population. When we keep the variance of policy σ_ν^2 to be constant and vary the variance of noise σ_ε^2 to be either $\sigma_{\varepsilon,H}^2$ and $\sigma_{\varepsilon,L}^2$ to match this perturbation, we have

$$m+(1-q)\,\Delta=\frac{\sigma_\nu^2}{\sigma_{\varepsilon,L}^2+\sigma_\nu^2},\;\; m-q\Delta=\frac{\sigma_\nu^2}{\sigma_{\varepsilon,H}^2+\sigma_\nu^2}$$

by the formula of attention parameter (7). Rearranging, we consider

$$\sigma_{\varepsilon,L}^2=\sigma_\nu^2\left\{\frac{1}{m+(1-q)\,\Delta}-1\right\},\;\; \sigma_{\varepsilon,H}^2=\sigma_\nu^2\left\{\frac{1}{m-q\Delta}-1\right\}.$$

Thus, by the welfare formula (15), the welfare loss due to idiosyncratic errors is

$$\begin{aligned}
&-\frac{\eta}{2}\mathbb{E}\left[m\sigma_\varepsilon^2\right]\\
&=-\frac{\eta}{2}\left[q\left\{m+(1-q)\,\Delta\right\}\times\sigma_\nu^2\left\{\frac{1}{m+(1-q)\,\Delta}-1\right\}+(1-q)\,(m-q\Delta)\times\sigma_\nu^2\left\{\frac{1}{m-q\Delta}-1\right\}\right]\\
&=-\frac{\eta}{2}\left[q\sigma_\nu^2\left\{1-\left\{m+(1-q)\,\Delta\right\}\right\}+(1-q)\,\sigma_\nu^2\left\{1-(m-q\Delta)\right\}\right]\\
&=-\frac{\eta}{2}\sigma_\nu^2\,(1-m)\\
&=-\frac{\eta}{2}\frac{\sigma_\varepsilon^2\sigma_\nu^2}{\sigma_\varepsilon^2+\sigma_\nu^2}\\
&=-\frac{\eta}{2}m\sigma_\varepsilon^2.
\end{aligned}$$

Therefore, the welfare is invariant to the heterogeneity in inattention. In the model of this paper, whether the idiosyncratic errors are concentrated on some sub-population or dispersed across the population is not consequential.