Student loans and the allocation of graduate jobs*

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Abstract

Higher education is not just a costly signal of native talent, but also a means of raising a person's ability to carry out certain kinds of job (and at least a certain educational achievement is required to get one). Graduate jobs differentiated by quality are allocated to graduates differentiated by parental wealth and native talent through a tournament. Non-graduates jobs pay a fixed wage to those who are excluded from the tournament. If credit is rationed, poor school leavers will either buy the same amount of higher education, and end up doing graduate jobs of the same quality, as less talented but richer ones, or go straight into the non-graduate labour market. As talent is revealed ex post, the more talented poor doing the same graduate job as less talented rich will get a productivity bonus, but this will not be sufficient to give them the same salary as if they were in better jobs. We also show that student loans improve job matching, and bring educational investments closer to efficiency. If the size of the loan is not large enough, however, some poor school leavers will still be liquidity-constrained and thus buy the same amount of higher education as less talented but richer ones (on the other hand, a loan size large enough to prevent that could be socially optimal only if social preferences were very egalitarian indeed).

Key words: higher education, matching tournaments, credit rationing, partially and fully separating equilibria, productivity bonus.

JEL: C78, D82, I22, J24

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1 Introduction

The present paper addresses one of the central issues of a market economy, namely how to reconcile the parents' legitimate desire to help their own children make their way in life by buying them the best education they can afford, with society's equity and efficiency requirement that education and ultimately jobs should be assigned independently of parental wealth. Such independence would be guaranteed if credit and insurance markets were perfect, because talented young persons could then buy education on credit. In reality, however, moral hazard and adverse selection problems make it difficult for children to borrow from the market. Without corrective policy, the children of insufficiently rich parents may consequently receive less education than would be efficient, and those of sufficiently rich ones possible more. We address this issue in the context of an economy where non-graduate jobs are assigned by a competitive market, and graduate jobs by a matching tournament. The latter is a contest where heterogeneous participants compete for one or more prizes. In a matching tournament, there are two categories of participants (men and women, employers and employees, schools and students), and each member of each category seeks to form the match most advantageous to itself with a member of the other. The "prizes" to be won are thus matches. An early example of a matching tournament is provided by Becker (1973), where the participants are young men and women bent on marriage. Exploiting a result in Koopmans and Beckmann (1957), Becker shows that the most attractive man will marry the most desirable woman, the second most attractive man will marry the second most desirable woman, etc. ("positive assortative matching"). Gale and Shapley (1962) shows that an efficient matching can be achieved by a ritualized "courting" routine.[1] There are obvious parallels between these routines, and the exchanges of CVs and job offers that occur between graduates and potential employers.[2]

The early matching literature abstracts from informational problems. Since Spence (1973), however, such problems have gradually gained centre-stage. In a context where signals are wasteful, Hoppe et al. (2009) show that the allocation may be inefficient because the costs of signaling may counterbalance the gains from assortative (as against random) matching.[3] The assignment of workers differentiated by innate talent and educational investment to jobs differentiated by quality is studied in Hopkins (2012) under the implicit assumption that individual educational investment is not constrained by individual wealth. Here, education raises productivity. Educational achievement, job quality, and the productivity of the match between a worker and a job, are taken to be common knowledge, but talent is private information. As talent reduces the cost of education, educational achievement is a signal of talent. The tournament ranks

[1] Cigno (1991, Ch. 1) shows that the efficient matching may not be unique, and that the one brought about by a courting routine where the initiative rests with the men will be different from one where the initiative is taken by the women.

[2] For evidence that graduate jobs are allocated in this way, see Bratti et al. (2004) and Castagnetti and Rosti (2009).

[3] Bhaskar and Hopkins (2013) also find that there may exist inefficient equilibria.

workers on the basis of their educational achievement, and matches them with jobs in such a way that the candidate with the highest educational achievement gets the highest quality job, the one with the second-highest educational achievement gets the second-highest quality job, and so on. In such a separating equilibrium, educational achievement reveals talent, and the matching pattern is efficient. But investment is not, because the more talented students overinvest in education to separate themselves from the less talented ones. Credit rationing is introduced by Fernandez and Galí (1999), who compare the performance of tournaments with that of conventional markets in the allocation of potential students differentiated by talent and wealth to schools differentiated by quality. Assuming that the number of school places is infinitely expandable, those authors find that, if at least some of the candidates are effectively credit constrained, tournaments dominate conventional markets in terms of matching efficiency, and possibly also in terms of aggregate consumption.

Like Fernandez and Galí (1999) we recognize that the credit market is imperfect, and that potential students may differ in their ability to pay for their studies as well as in their own native talent, but like Hopkins (2012) we are ultimately interested in how graduates are matched with graduate jobs. Given credit market imperfection, wealth differentiation may exclude a number of talented children of poor families from higher education and thus from graduate jobs. In the presence of asymmetric information, it may also result in talented students from poor families reaching the same level of education, and getting graduate jobs of the same quality, as less talented students from rich families. Could that be an equilibrium? Not if all graduates doing same-quality jobs were paid the same, because the more talented and consequently more productive graduates would be lured away from their present jobs by the offer of a higher pay in other jobs of the same quality. Consistently with evidence that pay is adjusted as productivity is observed,[4] we will then assume that graduate job offers specify a fixed wage, and a fixed bonus if the realized productivity is higher than a certain level.[5] Using this framework, we examine the effects of student loans – a policy widely adopted especially in the English-speaking world – on educational investment, graduate job allocation, efficiency and welfare. We show that the government can bring individual educational investments closer to their efficient levels, and improve the matching between graduates and graduate jobs, by borrowing wholesale on the international money market and lending to students.[6] As we raise the maximum that the government is willing to lend to

[4] Analizing panel data from the US National Longitudinal Surveys (NLS), Altonji and Pierret (2001) find that, as the employer acquires more information about an employee's productivity, the pay becomes more dependent on the latter and less dependent on paper qualifications. Similar evidence is found by Posch (2015) using data from the German Socio-Economic Panel.

[5] This kind of contract is typical of the private sector. In the public administration, where discretionary payments may not be allowed, the promise of a bonus is often replaced by the prospect of rapid upgrading.

[6] In a context where graduate jobs are allocated by conventional markets, and equity is an issue, Cigno and Luporini (2009) show that the policy in question is dominated by a scholarship scheme financed by a graduate tax.

each student, the poor gradually blend-in with the rich. In particular, the more talented among the former replace the less talented among the latter. Unless student loans are so generous that nobody's investment decisions are credit constrained, however, the matching will remain inefficient. Individual investment will be inefficient in any case. Student loan provision does not bring about a Pareto-improvement. Some will gain and some will loose as a result of the policy. Indeed, if the size of individual loans is already close to that required for an efficient matching, raising it to that very high level will make almost all graduate workers worse-off.

2 Assumptions

Our agents are school leavers. There is a continuum of them differentiated by parental wealth (talent to pay for higher education), y, and innate talent, z. For simplicity, we assume that y takes only two values, $y \in \{\underline{y}, \overline{y}\}$ where $\underline{y} < \overline{y}$. Talent is distributed over "poor" ($y = \underline{y}$) and "rich" ($y = \overline{y}$) agents with the same distribution function $G(z)$ and density function $g(z)$, such that $g(z|\underline{y}) = g(z|\overline{y})$ $\forall z \in [0, \overline{z}]$. A fraction α of the agents is rich, and $(1-\alpha)$ poor.[7] An agent can go into the labour market straight after leaving school, or after a period in higher education. There is a continuum of graduate jobs differentiated by quality, $s \in [0, \overline{s}]$, with distribution function $H(s)$. We can think of s as an indicator of technological sophistication or management quality. Graduate jobs are assigned by a matching tournament. Those who are not qualified, or choose not to participate in the tournament will take a non-graduate job, and earn a fixed wage w_0.[8] We assume that there are enough jobs, but not enough graduate ones, to occupy all the agents. For simplicity, we assume that there are as many graduate jobs as there are rich agents, but this does not necessarily imply that all the rich will get a graduate job (because some of these jobs may go to the poor).

Let x denote the highest educational level achieved by an agent who attended university. We may think of x as either a degree level (e.g., BA, MA, Ph.D.) or a degree mark. The cost of achieving x for an agent with innate talent z is denoted by $c(z, x)$. The utility of an agent with parental wealth y and talent z is given by

$$U = y + w - c(z, x), \quad (1)$$

where w is the agent's wage, comprehensive of the basic wage and the productivity bonus, obviously no higher than π and no lower than w_0.

The output of the match between a graduate with talent z and educational achievement x, and a job of quality s, is denoted by $\pi(z, s, x)$. The function

[7] More formally, the Lebesgue measure of the rich is a proper fraction α of the total agent population, which we normalize to unity.

[8] In principle, non-graduate productivity, and thus, in equilibrium, non-graduate wage rates may depend on individual talent, like their graduate counterparts. Arguably, however, the kind of talent that is relevant in the non-graduate market is different from the one that is relevant in the graduate one. As our focus is on the latter, we take the non-graduate wage rate to be the same for everybody.

$\pi(.)$ is defined for $x \geq x_0$, where $x_0 > 0$ is the minimum level of education (say, a BA with a low graduation mark) necessary to perform a graduate job. Without loss of generality, we set x_0 equal to the efficient level of x for an agent with talent $z = 0$ employed in a graduate job of quality $s = 0$,

$$x_0 = \arg\max \pi(0, 0, x) - c(0, x).$$

In this framework, z has a dual role. First, it reduces the cost of achieving any given x. Second, it directly increases the output of a graduate engaged in a graduate job. We assume that x, s and π are common knowledge, but z and y are private information. The assumption about z is standard. The one about y can be justified by noting that not even the government is fully informed about individual asset holdings (and that is why a general wealth tax, as distinct from specific taxes on easy-to-ascertain assets such as real estate, is difficult to implement). All the more realistic it then is to suppose that employers would find it difficult to ascertain how rich a candidate's parents really are.[9]

Students cannot borrow from the market for the reasons mentioned in the Introduction. For simplicity, we set the interest rate equal to zero (but nothing of substance would change if we set it positive, so long as it were lower than the return to educational investment for at least the more talented agents). In sections 4 and 5, we will assume that $\underline{y}$ is lower than the cost of achieving x_0 for the most talented student, $c(\overline{z}, x_0)$, and set it equal to zero. The case where $\underline{y}$ is large enough for at least some of the poor, the more talented among them, to invest in education at the level required to participate in the tournament even without government help will be examined in Section 6. Graduate job offers specify a guaranteed basic wage, and a bonus conditional on productivity reaching at least a certain level. The equilibrium process is modelled as a two-stage game. At the first (non-cooperative) stage, the agents choose whether and how much to invest in education. At the second (cooperative) stage, graduate jobs are allocated by a matching tournament, and the product of each match is shared between the parties in such a way that the matching scheme will be stable (i.e., such that nobody has an incentive to depart from it).

In addition to assuming that the cost of education is increasing in x and decreasing in z ($c_x < 0$, $c_z < 0$), and that the product of the match is increasing in s, x and z ($\pi_s > 0$, $\pi_x > 0$, $\pi_z > 0$), we impose a number of restrictions associated with the construction and stability of the matching equilibrium. Further restrictions are imposed to simplify the algebra where this does not affect the results. The marginal cost of x constant ($c_{xx} = 0$).[10] No talent is required to acquire zero education $c(0, .) = 0$. Talent reduces the marginal cost of achieving any positive level of education ($c_{zx} < 0$). The marginal cost-reducing effect of talent is non-increasing ($c_{zz} \leq 0$). The output of the match is concave in x ($\pi_{xx} < 0$). The marginal productivity of z is constant ($\pi_{zz} = 0$).[11] Job quality

[9] In a graduate, accent and demeanour reflect education more than parental wealth.

[10] If c_{xx} were positive, we would need a further assumption (see Proof of Proposition 6).

[11] We will later show that nothing of substance changes if this productivity is decreasing ($\pi_{zz} < 0$).

raises the marginal productivity of the worker's talent ($\pi_{zs} > 0$), but not that of the worker's education ($\pi_{sx} = 0$).[12] Talent does not affect the marginal productivity of education ($\pi_{zx} = 0$).[13] Giving a little x to a zero-z, zero-x worker employed in a zero-s job raises the output of the match by more than the cost, $\pi_x(0,0,0) > c_x(0,0)$. In other words, educational investment is never wasteful.

3 First best

Before examining the possible equilibria of the two-stage game, we characterize the First Best (FB) allocation where not only x, s and π, but also y and z, are common knowledge. This allocation maximizes the Social Surplus,

$$SS = \int_z \int_s \left[\pi(z,s,x) - c(z,x)\right] ds dz,$$

subject to the resource constraint,

$$\int_z \left[\alpha \overline{y} - c(z,x)\right] g(z) dz \geq 0.$$

We will assume that the latter is not binding. In other words, there are enough initial resources to finance the efficient level of education.[14]

Koopmans and Beckmann (1957) demonstrate that this maximization yields assortative matching in (z,s).[15] Given that there are fewer graduate jobs than agents, there will be a threshold value of z, $\widetilde{z} > 0$, defined by

$$G(\widetilde{z}) = 1 - \alpha, \tag{2}$$

such that all agents with $z \geq \widetilde{z}$ will attend university independently of their y. This subpopulation of agents is distributed with distribution function $\frac{G(z)-(1-\alpha)}{\alpha}$, and density function $\frac{g(z)}{\alpha}$. Positive assortative matching then means that a worker of talent $z_i \in [\widetilde{z}, \overline{z}]$ is matched with a job of quality $s_i \in [0, \overline{s}]$, such that

$$\frac{G(z_i) - (1-\alpha)}{\alpha} = \phi\left(\frac{G(z_i) - (1-\alpha)}{\alpha}\right) = H(s_i), \tag{3}$$

where ϕ: $[0,1] \rightarrow [0,1]$ is the matching function.[16] This defines the function

[12] This is required to ensure integrability of the wage function (see below).

[13] Stability of the matching equilibrium requires $\pi_{zx} \geq 0$, but we set this cross-derivative equal to zero to simplify the algebra.

[14] Alternatively and equivalently, we could have assumed that the government can augment these resources by borrowing against the Social Surplus.

[15] In our notation, this requires complementarity of s and z in the profit function, and this is why we assume $\pi_{zs} > 0$.

[16] This function is measure-preserving and one-to-one on $\phi([0,1])$. See Hopkins (2012) for details.

$$s_{FB}(z) = H^{-1}\left(\frac{G(z) - (1-\alpha)}{\alpha}\right),$$

which associates a job of quality s to an agent of quality z.

The FB level of university education for a school leaver with talent $z \geq \widetilde{z}$ matched with a job of quality $s_{FB}(z)$, is

$$x_{FB}(z) = \arg\max\left[\pi\left(z, s_{FB}(z), x\right) - c(z, x)\right], \tag{4}$$

and it will thus satisfy the first-order condition

$$\pi_x\left(z, s_{FB}(z), x\right) - c_x(z, x) = 0. \tag{5}$$

Given that $\widetilde{z}$ is positive, it follows from the assumptions on $c(z, x)$ and $\pi(z, s, x)$ that $x_{FB}(z) > x_0 \; \forall \; z \geq \widetilde{z}$. For future reference, we define $\widetilde{x}$ as the FB level of education for the least talented agents employed in graduate jobs, so that $\widetilde{x} \equiv x_{FB}(\widetilde{z})$. Given that, in FB, the distribution of the surplus is independent of resource allocation, we say nothing on the matter. Our interest here is just to characterize an efficient allocation. The graph of $x_{FB}(z)$ is shown in Fig. 1.[17]

4 Laissez faire

We now examine equilibrium in the absence of policy. As the present section is a development of Hopkins (2012),[18] who in turn draws on Mailath (1987), we will limit ourselves to adapting their results to our framework, and pointing out the differences. In the remaining sections, we will venture into unexplored territory. A difference is that, in our framework, the tournament concerns only graduate jobs. There is an infinitely elastic demand for non-graduate labour at the wage w_0. Another difference is that, in the absence of policy, the poor are excluded from graduate jobs because $c(\overline{z}, x_0)$ is positive and greater than $\underline{y}$, rather than zero as in Hopkins. By contrast, the rich can choose not to invest in higher education, because they can find employment in the non-graduate labour market. The game has a two-stage structure. At stage 1, agents choose whether and to what an extent to invest in higher education. At stage 2, employers make graduate job offers based on educational levels. Graduates and graduate jobs are matched in such a way that the equilibrium is stable. The resulting job assignment is positively assorted in x and s. In this equilibrium (but not in other that we will consider), x reveals z, because the participants are all rich, and their choice of x is consequently not distorted by a liquidity constraint.

[17] This curve is drawn strictly concave for illustrative purposes, but we will see later that nothing of substance changes if that is not the case.

[18] Hopkins considers both the transferable utility case, where wages are bargained between employers and employees, and the nontransferable utility one, where wages are sticky (Clark 2006 establishes conditions for the existence of a unique stable matching in this case). As the second of these assumptions seems more appropriate for non-graduate wages than for graduate ones, we have assumed transferable utility for the graduate labour market, and non-transferable utility for non-graduate one.

In Laissez Faire (LF), an agent with parental wealth y and innate talent z chooses $x \geq 0$ so as to maximize his utility (1)subject to the liquidity constraint

$$c(z, x) \leq y. \tag{6}$$

An agent choosing $x = 0$, has utility $y + w_0$.

Implicitly assuming that (6) is always slack, Hopkins (2012) demonstrates that a separating incomplete-information equilibrium exists.[19] In our framework, (6) is binding for all the poor, who will consequently invest nothing and be excluded from the tournament. By contrast, all the rich (including the less talented ones) will invest in higher education and participate in the tournament, because the lowest graduate wage is equal to the non-graduate wage plus the cost for the least talented agent of acquiring the minimum level of education required to participate in the tournament, $c(x_0, 0)$. Consequently, the number of rich graduates who get a graduate job is larger in LF than in FB.[20] In order to characterize the LF separating equilibrium, we start by assuming that rich agents adopt a symmetric, differentiable and strictly increasing investment strategy $x_{LF}(z)$, with $x'_{LF}(z) > 0$, and that graduates and graduate jobs are positively assorted. Further down the section we will show that this is actually the case.

Let $F(x)$ be the distribution function of x induced by the distribution of z, $G(z)$, and the investment strategy $x_{LF}(z)$. The rank position, $F(x(z_i))$, of an agent i with talent $z_i \in [0, \overline{z}]$, achieving the education level $x_{LF}(z_i)$, will then be equal to this agent's rank $G(z_i)$ in the distribution of talent. Positive assortative matching, whereby an agent buying $x_i = x_{LF}(z_i)$ is matched with a job of quality $s_i \in [0, \overline{s}]$, is such that

$$F(x_i) = G(z_i) = \phi(G(z_i)) = H(s_i). \tag{7}$$

This condition differs from (3) because z_i is now private information, and the matching is consequently based on x_i, but it still yields a relationship between job quality and agent's talent,

$$s_{LF}(z) = H^{-1}(G(z)).$$

In contrast with FB, some graduate jobs down the quality scale will now be filled by (rich) graduates of talent $z < \widetilde{z}$.

The stage-2 stability conditions determine the wage schedule. For the equilibrium to be stable, the sum of the profit of a worker of talent z and education x matched with a firm of quality $s(z)$, and of the wage of a worker of talent $z+\varepsilon$ and education x matched with a firm of quality $s(z+\varepsilon)$, must be no lower, for ε arbitrarily small, than the output that the first firm would produce if it were matched with the second worker,

$$\pi(z, s(z), x) - w(z, s(z), x) + w(z+\varepsilon, s(z+\varepsilon), x) \geq \pi(z+\varepsilon, s(z), x). \tag{8}$$

[19] There is also a pooling equilibrium in which wages reflect the average productivity.

[20] More formally, the support of the ability distribution of graduate workers is wider in LF than in FB, where it includes only agents with $z \geq \widetilde{z}$.

Moreover, the x chosen by an agent with talent z must satisfy

$$w\left(z,s\left(z\right),x+\varepsilon\right)+\pi\left(z,s\left(z\right),x\right)-w\left(z,s\left(z\right),x\right)\geq\pi\left(z,s\left(z\right),x+\varepsilon\right). \quad (9)$$

Taking the limit of (8) and (9) for ε going to zero, gives us the stability conditions

$$w_z\left(z,s\left(z\right),x\right)+w_s\left(z,s\left(z\right),x\right)s'(z)=\pi_z\left(z,s\left(z\right),x\right) \quad (10)$$

and

$$w_x\left(z,s\left(z\right),x\right)=\pi_x\left(z,s\left(z\right),x\right). \quad (11)$$

As we move up the education scale, the worker's productivity increases for three reasons. First, because the educational level increases. Second because those who have a higher educational level have also greater native talent. Third, because the better qualified workers are in better quality jobs. The stability conditions say that the workers appropriate the first two of these productivity increases. The employers appropriate the third one. Applying Proposition 2 of Hopkins (2012) to the present context, it can be shown that the only stable matching is the positive assortative one, and that the wage schedule derived from the stability conditions (10) − (11)[21] is then

$$w_{LF}\left(z,s_{LF}\left(z\right),x\right)=\int_0^z\pi_z\left(r,s_{LF}\left(r\right),x_0\right)dr+\int_{x_0}^x\pi_x\left(z,s_{LF}\left(z\right),t\right)dt+\underline{w}_{LF}, \quad (12)$$

where $\underline{w}_{LF}$ is the lowest graduate wage, yet to be determined.

Let us now move to stage 1. Exploiting the incentive-compatibility condition that, in a separating equilibrium, it must be unprofitable for an agent of ability z to choose the education level x appropriate for an agent of ability $z'\neq z$, and using (10) − (11), Hopkins (2012) shows that

$$x'_{LF}(z)=\frac{\pi_z\left(z,s_{LF}\left(z\right),x\right)}{c_x\left(z,x\right)-\pi_x\left(z,s\left(z\right),x\right)} \quad (13)$$

is positive. Integrating this differential equation we get the investment function,[22]

$$x_{LF}(z)=\int_{\widetilde{z}}^{z}x'_{LF}(z)dz+\ x_0. \quad (14)$$

[21] In stating those conditions, we took z and thus $s(z)$ to be observable. Having assumed $\pi_{sx}=0$, however, the implied wage schedule does not depend on the functional form of $x_{LF}(.)$, and will thus be the same if z is not observable as we actually assume.

[22] The differential equation has a unique solution. To establish the boundary condition, recall that the efficient level of education for an agent with talent $z=0$, employed in a graduate job of quality $s=0$, is equal to x_0. Workers with talent $z=0$ assigned to jobs of quality $s\left(0\right)=0$ have nothing to signal and will thus choose $x(0)=x_0$, so that $c_x\left(x,0\right)=\pi_x\left(0,s\left(0\right),x\right)$. Therefore, $x'_{LF}(z)$ is not defined for $z=0$. By contrast, participating agents with $z>0$ want to signal their talent and will thus invest more than would be efficient given the matching. For these agents, the choice of x will be such, that $c_x\left(z,x\right)$ is greater than $\pi_x\left(z,s\left(z\right),x\right)$. Therefore, $x'_{LF}(z)$ is positive for $z>0$.

Together with the assortative matching scheme (7) and wage schedule (12), this function characterizes a symmetric equilibrium of the matching tournament.[23]

In Hopkins (2012), where all jobs are assigned by tournament, and $x_0 = 0$, the minimum wage is set arbitrarily. Here, by contrast, the minimum graduate wage must be large enough to cover at least the minimum cost of entering the graduate job tournament for the least talented agent,

$$\underline{w}_{LF} \geq w_0 + c(0, x_0).$$

Competition among agents will ensure that this constraint is satisfied as an equation, and thus that a school leaver of talent zero is indifferent between investing in education at the level required to get the lowest paid graduate job, or going into the non-graduate labour market straight from school and earning w_0. Therefore, the wage of a graduate of talent $z > 0$ and education $x > x_0$ is equal to the non-graduate wage rate, plus the cost of achieving the minimum education required to hold a graduate job for an agent of talent $z = 0$, plus the sum of the productivity increments that occur as talent raises from 0 to z, plus the productivity of x. As it reflects only the contribution of z and x, not that of s, $w_{LF}(z, s_{LF}(z), x)$ does not exhaust $\pi(z, s_{LF}(z), x)$. That is why an employer is not indifferent between offering a job of quality $s_{LF}(z)$ to a graduate with talent z, or to a less talented one.

The LF equilibrium is inefficient for two reasons. First, because all graduate jobs other than those of the maximum quality, $\overline{s}$, are occupied by graduates with lower talent than in FB. This reflects the fact that the agents excluded from the tournament are the poorest, rather than the least talented. This source of inefficiency is absent in Hopkins (2012), where all agents are effectively rich. Second, because x is inefficiently high (i.e., higher than in FB) for all $z > 0$. This reflects the fact that, as graduate workers are ranked according to their educational achievement, all rich agents other than the marginal ones (those who are indifferent between going to university or straight into the labour market) have an incentive to invest more in order to make a better match. Consequently, the graph of $x_{LF}(z)$, shown in Fig. 1, starts at $z = 0$ rather than $z = \widetilde{z}$, and lies everywhere above that of $x_{FB}(z)$.[24]

We can further demonstrate that all rich agents other than those with the highest talent make a better match than in FB.

Proposition 1. *For all rich agents with $z \in [\widetilde{z}, \overline{z})$, $s_{LF}(z) > s_{FB}(z)$.*

Proof. See Appendix.

[23] This statement is equivalent to Proposition 3 of Hopkins (2012).

[24] Using Mailath (1987), it can be shown that $x_{LF}(z)$ is strictly concave as drawn. We cannot say the same about $x_{FB}(z)$ because we do not have assumptions on the third derivatives of $c(z, x)$ and $\pi(z, s, x)$, but we have drawn it concave all the same because it makes no difference to the argument.

5 Student loans

We now bring the government into the picture. As y is private information,[25] wealth redistribution is out of the government's reach. Assuming that the government, unlike individual agents, can borrow wholesale from the international money market, we examine instead the possibility that the government lends to university students at stage 1 of the game, and recovers the credit at stage 2.[26] We call b the maximum that the government is willing to lend to a student. The value of b could be dictated by ability-to-repay or public-debt-management considerations, but we will show that there may be equity considerations as well. Having set the interest rate equal to zero, the utility function remains (1), because loan and loan repayment cancel out, but the liquidity constraint is now

$$c(z,x) \leq y + b. \tag{15}$$

Whether an equilibrium exists, and what its properties are if it does, depends on the value of b. Before going into the analytical detail, we summarize here the main results (see the taxonomy in Table 1).

$0 \leq b \leq b_1$	LF separating equilibrium
$b_1 < b < b_2$	No separating equilibrium
$b_2 = b < b_3$	Partially separating equilibrium: all agents with $z \geq \widetilde{z}$ participate in the tournament, poor agents with $\widetilde{z} < z < \overline{z}$ are liquidity constrained
$b_2 < b < b_3$	Partially separating equilibrium: all agents with $z \geq \widetilde{z}$ participate in the tournament, poor agents with $\widetilde{z} < z < \underline{z}(b) < \overline{z}$ are liquidity constrained
$b \geq b_3$	Separating equilibrium: all agents with $z \geq \widetilde{z}$ participate in the tournament, nobody is liquidity constrained.

Table 1. Taxonomy

There are three critical values of b, namely b_1, b_2 and b_3, with $0 \leq b_1 < b_2 < b_3$. At all levels of b up to b_1, the separating equilibrium is LF. For b higher than b_1 but still lower than b_2, there is no equilibrium. The minimum b that would allow at least some of the poor to participate in the tournament is b_2. There is thus no point in lending students less than that amount. At the opposite end, there is a loan size b_3 so high that it makes the liquidity constraint (15) slack

[25] We have assumed that parental wealth is imperfectly observable by the government, and thus by private employers. We will argue below that the poor, who in a certain type of equilibrium would have an interest in disclosing their wealth status, cannot credibly do so because the rich could falsely pretend to be poor too.

[26] For a discussion of the enforceability issue, see Cigno and Luporini (2009).

for all poor agents talented enough ($z \geq \widetilde{z}$) to qualify for a university education in FB.

Setting b equal to b_3 yields a Separating Equilibrium (SE) where graduate jobs are assigned as in FB, but all students invest more than in FB ("overinvest"). In this equilibrium, the poor not talented enough to go to university and get a graduate job in FB are as well-off as in LF, because they would excluded from the tournament anyway. Almost all the poor talented enough for that are better-off in SE, where they participate in the tournament, than in LF, where they do not (the only exception are those *just* talented enough, who are indifferent between going to university or straight into the non-graduate labour market). The rich are all worse-off, because they are either excluded from the tournament, or get worse graduate jobs than in LF.

For each value of b greater than b_2 but lower than b_3, there is a Partially Separating Equilibrium (PSE) where graduate jobs of the same quality are occupied by graduates of different talent. In other words, agents with two different z pool in their choice of x. In this range of b, there is a threshold level of talent $\underline{z}(b)$, decreasing in b, such that poor students with $\widetilde{z} \leq z < \underline{z}(b)$ are liquidity-constrained, but poor students with $z \geq \underline{z}(b)$ are not (obviously, $\underline{z}(b_3) = \widetilde{z}$). In other words, the latter may choose to borrow less than b. In PSE, all graduates (rich or poor) with $z \geq \underline{z}(b)$ occupy the same jobs as they would in FB, and thus in SE. In the talent range immediately below that, $\widetilde{z} < z < \underline{z}(b)$, the poor are liquidity-constrained, but the rich are not. Consequently, the former invest less and get lower quality jobs than the latter. There is then a range of values of b where rich graduates occupy jobs of the same quality as poor graduates more talented than themselves. Ex post, the more talented and consequently more productive poor will get a bonus, and thus earn more than the less talented rich doing the same kind of job, but not as much as the equally talented rich, who will be in better jobs. That is because the bonus reflects the difference in the productivity of two graduates with different z, but same x and s.

The latter raises two questions. The first one is, given that x reveals z if y is known, what is there to stop poor students with $\widetilde{z} < z < \underline{z}(b)$ disclosing their wealth status by providing documentary evidence that they have taken out a loan? The answer is yes, they could, but it would do them no good, because rich students in that same talent range would pretend to be poor by taking out a loan too. The second question concerns the advantage for poor agents in the said talent range to borrow the maximum offered by the government (we have already noted that the agents in the talent range above the one of interest may not). Given that their talent will be recognized ex post and rewarded with a bonus, could it be in their interest to borrow and invest *less* than b? The answer is no, because graduate jobs are assigned on the basis of educational qualifications, and the productivity of a job match depends not only on the worker's talent, but also on the worker's educational achievement, and on the quality of the job. If a poor agent borrows and invests less than b, he will then get a worse job, and receive a smaller bonus, than he otherwise would.

A further question concerns the government's choice of b. Setting $b = b_2$ is certainly better than setting it at any lower level. If b rises above b_2, the matching improves, and becomes the same as in FB for $b = b_3$. Individual investments also get closer to their FB levels, but never get there, not even for $b = b_3$. Ability-to-repay and public-debt-management considerations permitting, should the government raise b as far as b_3? As we will show in detail below, the move from the PSE associated with any $b_2 \leq b < b_3$ to the SE associated with $b = b_3$ reduces overinvestment for some (all the rich, and the more talented among the poor) and increases investment for the rest (the poor that underinvest in PSE). Therefore, the move enhances efficiency, but it is not necessarily a Pareto-improvement. The policy makes the rich, and the poor who would not be constrained even if b remained lower than b_3, worse-off. Some of the poor who become unconstrained thanks to the increase in b will be better-off. If we start from a level of b close to b_3, however, there will be very few of these. In other words, raising b to the very high level required to make the matching efficient would make almost all graduate workers less happy.

5.1 SE: Separating equilibrium ($b = b_3$)

If b is equal to b_3, all agents with talent $z \geq \widetilde{z}$ are unconstrained. Each of these agents will invest in education and participate in the matching tournament. Given the policy, the range of participating agents will then be the same as in FB. More precisely, the support of the ability distribution of participating agents is $[\widetilde{z}, \overline{z}]$ as in FB, and thus narrower than in LF. The distribution function is $\frac{G(z)-(1-\alpha)}{\alpha}$ as in FB.

We again start by assuming that the job allocation is positively assorted in s and z, and then argue that this is the only stable matching. For agent i, x_i must satisfy the equilibrium condition

$$F(x_i) = \frac{G(z_i) - (1-\alpha)}{\alpha} = \phi\left(\frac{G(z_i) - (1-\alpha)}{\alpha}\right) = H(s_i).$$

Even though the matching is now based on x, an agent with talent z will then get a job of quality

$$s_{SE}(z) = H^{-1}\left(\frac{G(z) - (1-\alpha)}{\alpha}\right),$$

where

$$s_{SE}(z) = s_{FB}(z) < s_{LF}(z)$$

for all $z \in [\widetilde{z}, \overline{z})$. All rich agents other than the most talented ones (those with $z = \overline{z}$, who will get the best jobs, $s = \overline{s}$, as in LF) now get lower quality jobs than in LF. The rich with $z < \widetilde{z}$ are now excluded from the tournament, and will get non-graduate jobs.

Proposition 2. *The job matching is the same in SE as in FB.*

Corollary 1. *The rich with $z < \overline{z}$ get worse jobs, and the poor with $z \geq \widetilde{z}$ get better ones, in SE than in LF.*

We should now show that the only stable allocation is positively assortative, and that it implies a particular wage schedule. As the argument is the same as the one we used for LF, however, we will use stability conditions analogous to $(10)-(11)$ derive the unconstrained Separating Equilibrium (SE) wage schedule

$$w_{SE}\left(z, s_{SE}\left(z\right), x\right) = \int_{\widetilde{z}}^{z} \pi_z\left(r, s_{SE}\left(r\right), x_0\right) dr + \int_{\widetilde{x}}^{x} \pi_x\left(z, s_{SE}\left(z\right), t\right) dt + \underline{w}_{SE}. \tag{16}$$

At $z = \widetilde{z}$, $w = \underline{w}_{SE}$, where

$$\underline{w}_{SE} = w_0 + c\left(\widetilde{z}, \widetilde{x}\right).$$

Agents endowed with so little talent that they are indifferent between investing in a university education and going straight into the non-graduate labour market have no interest in acquiring more than the efficient amount of x, $\widetilde{x}$, because they have nothing to signal. More talented agents, by contrast, have an interest in signalling that their z is higher than $\widetilde{z}$. They will thus adopt an investment strategy different from the FB one. At the margin, the educational investment carried out by an agent with talent z will satisfy the condition[27]

$$x'_{SE}(z) = \frac{\pi_z\left(z, s_{SE}\left(z\right), x\right)}{c_x\left(z, x\right) - \pi_x\left(z, s_{SE}\left(z\right), x\right)}. \tag{17}$$

Integrating this equation from $\widetilde{x}$ upwards, we find the SE investment function, $x_{SE}(z)$. Figure 1 compares the graph of this function with those of $x_{LF}(z)$ and $x_{FB}(z)$.

Proposition 3. *The $x_{SE}(z)$ curve lies above the $x_{FB}(z)$ curve everywhere except at $z = \widetilde{z}$, where $x_{SE}(\widetilde{z}) = x_{FB}(\widetilde{z})$, and below the $x_{LF}(z)$ curve except possibly at $z = \overline{z}$.*

Proof. *See Appendix.*

Corollary 2. *SE is less inefficient than LF.*

Proof. The corollary follows immediately from the fact that (i) in SE the matching is the same as in FB and consequently all the jobs (except those at $\overline{s}$) are filled by more talented workers in SE than in LF, and that (ii) there is less overinvestment in SE than in LF.

Proposition 4. *The rich are worse-off, and the poor no worse-off, in SE than in LF.*

Proof. *See Appendix.*

[27] This is derived from stability conditions analogous to $(10)-(11)$, and satisfies the incentive-compatibility condition that it must be unprofitable for an agent of ability z to choose the education level x appropriate for an agent of ability $z' \neq z$.

Let us see what that means. We know from Proposition 2 that the policy improves job matching with respect to LF, because it results in the same job allocation as FB. Where educational investment is concerned, the $x_{SE}(z)$ curve starts from a higher z than the $x_{LF}(z)$ curve. Poor agents of talent $z < \widetilde{z}$ invest nothing in SE as in LF. But all rich agents invest less, and poor agents of talent $z \geq \widetilde{z}$ invest more, in SE than in LF. The finding that the rich would invest more without the policy may come as a surprise because, in LF, a rich agent of any given talent faces no competition from poor agents of the same or greater talent. The intuition is that overinvestment is driven by the desire to separate from less talented agents, and that there are fewer of the latter in SE than in LF. Given that those (the rich) who, without the policy, would have overinvested now invest less, and that those (the poor of sufficiently high talent) who would have underinvested now invest more, the policy brings individual investments closer to their FB levels.

This has welfare implications. With the policy, the poor not talented enough to receive a university education in FB will still be excluded from the tournament, and they will have the same utility as in LF. The rich in that same talent range also will be excluded from the tournament, but their utility will be lower than in LF (where, remember, they would have participated in the tournament). The poor talented enough to participate in the tournament will be better-off with than without the policy. The rich who would have participated in the tournament anyway will get worse jobs but, on the other hand, they will overinvest less (and their costs will consequently be lower) with than without the policy. As the first effect predominates, the policy makes *all* the rich worse-off. By contrast, the policy makes the poor of talent $z \geq \widetilde{z}$ definitely better-off (if these agents invest in education, it must in fact be the case that the benefit is greater than the cost), and has no effect on those of lower talent.

5.2 PSE: Partially separating equilibrium ($b < b_3$)

We have compared the equilibrium without policy intervention and the one with b equal to b_3, where the government is willing to lend large enough to make all poor agents unconstrained. Let us now suppose that b is set lower than b_3, so that at least some of the poor are liquidity-constrained. We will show that this policy gives rise to a Partially Separating Equilibrium (PSE) where, at least over a certain range of x, there are two values of z for the same value of x. In this equilibrium, agents with different z pool over x. This is the case where the equilibrium does not reveal the agent's talent, and the bonus may then be paid ex post when productivity is observed. Further down in this subsection we will show (Propositions 6 and 7) that at least some of the agents endowed with a middling amount of talent will buy different amounts of x, higher if the agent is rich than if the agent is poor.

An adapted version of the approach we followed in the LF analysis can now be applied separately to constrained and unconstrained agents. As in the LF and SE, we will start by assuming positive assortative matching with respect to x, find necessary stability conditions, and then demonstrate that the equilibrium is in fact positively assorted. In contrast with LF and SE, where there was only one investment function, and consequently only one matching function, there are now two investment functions, $x_U(.)$ for the unconstrained (all the rich and possibly some of the poor) and $x_C(.)$ for the constrained poor. Consequently, there are also two matching functions, $s_U(.)$ for the unconstrained and $s_C(.)$ for the constrained, and matching will occur within each category rather than across the board. The resulting pattern will be inefficient.

Assuming that jobs of any given quality s are assigned at random among graduates with the same education x, those among these graduates who are liquidity-constrained will have a higher z and thus produce a larger π than those who are not so constrained. Two wage schedules, $w_U(.)$ for those who are not constrained and $w_C(.)$ for those who are, will then emerge. For any given s and x, the value of the latter will be higher than that of the former, and the difference between the two will constitute the productivity bonus. This did not happen in the cases examined earlier, because the ranking in terms of x was the same as the ranking in terms of z. In what follows, we will assume that a PSE exists and establish some of its necessary characteristics. The existence issue will be examined in subsequent sub-subsections. Denoting by $\widetilde{b}$ the value of b that allows poor students of talent $\widetilde{z}$ to buy the efficient amount of education $\widetilde{x}$, we can demonstrate the following.[28]

Proposition 5. *If a PSE exists for $\widetilde{b} \leq b < b_3$, it will be such that the least (most) talented rich agents participating in the tournament have the same talent, $z = \widetilde{z}$ ($z = \overline{z}$), and choose the same educational level, as the least (most) talented of the participating poor. The least talented participating agents choose the efficient education level $\widetilde{x}$.*

Proof. See Appendix.

Corollary 3. *For a PSE to exist, b must be no lower than $\widetilde{b}$.*

In order to have a PSE for b lower than b_3, it is thus necessary that at least the participating poor with the lowest and highest talent (respectively $z = \widetilde{z}$ and $z = \overline{z}$) are not liquidity-constrained in their choice of x, and thus that b is not too low. The intuition is that the maximum a student can borrow from the government is large enough to relax the liquidity constraint of a poor of talent $z = \widetilde{z}$ because this student invests little, and of a student of higher talent $z = \overline{z}$ because it costs this student little to invest in education. In what follows, we characterize first the case where b is such that only the participating poor at the two extremes of the relevant talent range are liquidity-constrained, and

[28] We assume that, in this equilibrium, the beliefs of employers and employees reflect true talent endowments. Out-of-equilibrium beliefs will satisfy the Divinity Criterion (Banks and Sobel, 1987). Without this refinement, there might in fact exist other student-loans, partially-separating equilibria where some of the rich with $z < \widetilde{z}$ go to university, while some of the poor with $z > \widetilde{z}$ go straight into the labour market. There will also exist a pooling equilibrium where all agents with $z \geq \widetilde{z}$ choose $\widetilde{x}$, and employers hold to their priors.

then the one where b is such that at least some of the poor inside that range are unconstrained too. If b is not large enough for those at the lower end of the said range ($z = \widetilde{z}$) not to be liquidity constrained, there is no PSE. If b is so low that not even those at the top end ($z = \overline{z}$) can afford to invest x_0, the equilibrium is LF. Let $b_1 = c(\overline{z}, x_0)$. For $b_1 \leq b < b_2$ there is no equilibrium. We finally show what happens as b is raised above b_2 to such a high level, b_3, that none of the agents talented enough to go to university in FB is liquidity constrained.

5.2.1 All poor agents with $\widetilde{z} < z < \overline{z}$ are liquidity constrained

Let b be such, that all the poor other than those with $z = \widetilde{z}$ and $z = \overline{z}$ are liquidity-constrained (remember that none of the rich ever is). The existence and uniqueness of such a level of b will be the subject of Proposition 6 below.[29] Let $\widetilde{b} \leq b_2 < b_3$ denote the value of b that has this property. For $b = b_2$, a poor with talent $\overline{z}$ will achieve the same level of x, call it $\overline{x}$, as an equally talented rich, while a poor with talent $\widetilde{z}$ will choose the same efficient amount of education $\widetilde{x}$ as an equally talented rich. For $\widetilde{z} < z < \overline{z}$, there will be two levels of x, lower for unconstrained than for constrained agents. We start by assuming the existence of two matching functions, $s_C(.|b_2)$ for the constrained,[30] and $s_U(.|b_2)$ for the unconstrained, both increasing in z, such that $s_C(z|b_2) < s_U(z|b_2)$, and then derive the form of the wage schedules that ensure the stability of such matchings. We will then prove the existence of an equilibrium where the matching functions are $s_C(.|b_2)$ and $s_U(.|b_2)$.

The PSE equilibrium associated with $b = b_2$ must satisfy stability conditions analogous to (10) and (11). The only difference is in that there is now a pair of these conditions for the unconstrained, and another for the constrained. The pair applicable to the former determines the wage schedule

$$w_U\left(z, s_U\left(z|b_2\right), x\right) = \int_{\widetilde{z}}^{z} \pi_z\left(r, s_U\left(r|b_2\right), x\right) dr + \int_{\widetilde{x}}^{x} \pi_x\left(z, s_U\left(z|b_2\right), t\right) dt + \underline{w}_{PSE}. \tag{18}$$

The pair applicable to the latter determines

$$w_C\left(z, s_C\left(z|b_2\right), x\right) = \int_{\widetilde{z}}^{z} \pi_z\left(r, s_C\left(r|b_2\right), x\right) dr + \int_{\widetilde{x}}^{x} \pi_x\left(z, s_C\left(z|b_2\right), t\right) dt + \underline{w}_{PSE}. \tag{19}$$

In each case, $\underline{w}_{PSE} = \underline{w}_{SE} = w_0 + c(\widetilde{x}, \widetilde{z})$, where $\widetilde{x} > x_0$ is the efficient education level common to all agents with talent $\widetilde{z}$.

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[29] We will assume that the relevant parameters are such that the cost of the FB educational investment, $c(z, x_{FB}(z))$, (i) is increasing in z, $\frac{dc}{dz} = c_z + c_x x'_{FB}(z) > 0$;
(ii) is such that $c(z, x_{FB}(\widetilde{z})) < c(z, x_{FB}(\overline{z}))$. If these conditions are not satisfied, an equilibrium where only the poor with $z = \widetilde{z}$ and with $z = \overline{z}$ are unconstrained does not necessarily exist, but there always exists a PSE equilibrium where the poor with $z = \widetilde{z}$ and some of the poor with z close to $\overline{z}$ are unconstrained as in the PSE examined in the next subsection.

[30] Strictly speaking, this function applies not only to the constrained, but also to the poor of talent $\widetilde{z}$ or $\overline{z}$, who are actually unconstrained.

Take two agents, one constrained and the other unconstrained, both with the same (s, x), such that $s_C(z'|b_2) = s_U(z|b_2)$ for $z', z \in (\widetilde{z}, \overline{z})$. The constrained agent will have higher talent, i.e., $z' > z$. Given that

$$\int_{\widetilde{x}}^{x} \pi_x(z, s_C(z|b_2), t)\, dt = \int_{\widetilde{x}}^{x} \pi_x(z, s_U(z|b_2), t)\, dt$$

for the assumption that $\pi_{zx} = \pi_{sx} = 0$, the difference between their wages is

$$\begin{aligned} & w_C(z', s_C(z'|b_2), x) - w_U(z, s_U(z|b_2), x) \qquad (20) \\ = & \int_{\widetilde{z}}^{z'} \pi_z(r, s_C(r|b_2), x)\, dr - \int_{\widetilde{z}}^{z} \pi_z(r, s_U(r|b_2), x)\, dr. \end{aligned}$$

As the first of the two integrals on the RHS of this equation is calculated over the same interval of s as, but over a wider interval of z than, the second integral, and having assumed that $\pi_{zz} = 0$,[31] the difference between the two integrals will be positive. An employer wishing to hire a worker with education x for a job of quality s will then have to offer a fixed salary equal to $w_U(z, s_U(z|b_2), x)$, and a bonus, equal to $w_C(z', s_C(z'|b_2), x) - w_U(z, s_U(z|b_2), x)$, conditional on the productivity of the match being higher than $\pi(z, s_U(z|b_2), x)$. If the employer did not do that, the worker would in fact be offered such a bonus by another employer for a job of the same quality.

Let $x_U(z|b_2)$ and $x_C(z|b_2)$ denote the investment functions (yet to be determined) of, respectively, the unconstrained and the constrained. Assume that $x_U(.|b_2)$ and $x_C(.|b_2)$ are increasing functions. The PSE matching condition is then

$$F(x_i) = \alpha F_U(x_i) + (1-\alpha) F_C(x_i) = H(s_i), \tag{21}$$

where $F_U(x)$ is the distribution of x induced by $x_U(z|b_2)$, and $F_C(x)$ the one induced by $x_C(z|b_2)$.

All agents of talent $\widetilde{z}$ now invest $\widetilde{x}$ as in FB (and thus in SE) because they have nothing to signal. Above that talent level, however, investment behaviour depends on whether the agent is constrained or unconstrained. If a PSE equilibrium exists, it satisfies the incentive-compatibility condition that it must be unprofitable for an unconstrained agent with talent z to choose the x appropriate for an unconstrained one with talent $z' \neq z$. Given conditions analogous to (10) − (11), the investment strategy of the unconstrained will then satisfy

$$x'_U(z|b_2) = \frac{\pi_z(z, s_U(z|b_2), x)}{c_x(x, z) - \pi_x(z, s_U(z|b_2), x)}, \tag{22}$$

so that, if a solution to (22) exists with initial condition $x_U(\widetilde{z}|b_2) = \widetilde{x}$,

$$x_U(z|b_2) = \int_{\widetilde{z}}^{z} x'_U(z|b_2) dz + \widetilde{x}. \tag{23}$$

[31] But the argument goes through also if π_{zz} is negative provided that the marginal product of z does not fall "too fast".

We do not have an incentive-compatibility condition for those who are constrained, because these agents borrow all that the government is willing to lend them, and their choice of x is thus determined by the equation

$$c(x,z) = b. \tag{24}$$

By the implicit function theorem, therefore,

$$x_C'(z|b) = -\frac{c_z}{c_x}, \tag{25}$$

and the investment function of the constrained poor is

$$x_C(z|b_2) = \int_{\widetilde{z}}^{z} x_C'(z)dz + \widetilde{x}. \tag{26}$$

We now turn to the issue of the existence of a PSE where all poor agents with $\widetilde{z} < z < \overline{z}$ are liquidity constrained, and establish the characteristics of this equilibrium. Recall that a poor of talent $\overline{z}$ achieves the same level of x as an equally talented rich,

$$x_U(\overline{z}|b_2) = x_C(\overline{z}|b_2).$$

Note that poor agents are unconstrained if their z is equal to $\widetilde{z}$, but will be constrained if z is even only slightly larger than $\widetilde{z}$ because, in view of (22), $x_U'(z|b_2)$ tends to infinity in a neighborhood of $\widetilde{z}$,[32] and the level of x chosen by unconstrained agents thus increases very rapidly as z rises above $\widetilde{z}$.

Proposition 6. *There is a value of b, $\widetilde{b} \le b_2 < b_3$, such that there exists a PSE where rich and poor agents with talent $\overline{z}$ choose the same level of education $\overline{x}$, while rich and poor agents of talent $\widetilde{z}$ choose the FB level of education $\widetilde{x}$. For $\widetilde{z} < z$, the rich choose a higher x than in FB. For $\widetilde{z} < z < \overline{z}$, the rich choose a higher x than the poor.*

Proof. *See Appendix.*

Corollary 4. *In the PSE associated with $b = b_2$, unconstrained (constrained) agents with $\widetilde{z} < z < \overline{z}$ are matched with higher (lower) quality jobs than in FB and SE. Agents with $z = \widetilde{z}$ and $z = \overline{z}$ are matched with the same jobs as in FB and SE. The rich (poor) are matched with worse (better) jobs than in LF.*

Proof. *See Appendix.*

Corollary 5. *The $x_U(z|b_2)$ curve lies (i) above the $x_{FB}(z)$ curve everywhere except at $z = \widetilde{z}$; (ii)* above the *$x_C(z|b_2)$ curve everywhere except at $z = \widetilde{z}$ and $z = \overline{z}$, where the two coincide, (iii) below the $x_{LF}(z)$ curve everywhere except possibly at $z = \overline{z}$.*

Proof. *See Appendix.*

The relative position of the investment curves is shown in Fig.2.

[32] See Mailath (1987, p 1355).

5.2.2 Not all poor agents with $\widetilde{z} < z < \overline{z}$ are liquidity constrained

What happens if the government sets b higher than b_2 (but still lower than b_3)? The demonstration that an equilibrium exists also in this case is analogous to the one we gave for the case where b is equal to b_2. At this level of b, the poor are unconstrained not only at $z = \widetilde{z}$ and $z = \overline{z}$, but also for a number of values of z between those extremes. Let $\underline{z}(b)$ denote the lowest z for which this is true at the given b. The poor of talent $z \geq \underline{z}(b)$ will then choose the same x as, and those of talent $\widetilde{z} < z < \underline{z}(b)$ a lower x than, the rich of the same talent. Therefore, there are again two investment functions, one for those who are not liquidity-constrained (which now include more of the poor, those with $z \geq \underline{z}(b)$, than in the previous case, as well as the rich with $z \geq \widetilde{z}$) and another for those who are so constrained (the poor with $\widetilde{z} < z < \underline{z}(b)$). For $z = \underline{z}(b)$, the two functions have the same value. For $\widetilde{z} < z < \underline{z}(b)$, the educational level achieved by those who are not liquidity-constrained (i.e., in this case, just the rich) is higher than that achieved by those who are.

Now let $\underline{x}(b)$ denote the amount of education achieved by agents of talent $\underline{z}(b)$. For $x < \underline{x}(b)$, jobs of the same quality s will again be assigned at random to agents with the same educational achievement (some of these agents will be more talented than others, but the employers do not know who). There will then be two matching functions, $s_U(.)$ for those who are not liquidity-constrained (the rich with $z \geq \widetilde{z}$, and the poor with $z > \underline{z}(b)$), and $s_C(.)$ for those who are (the poor with $\widetilde{z} < z < \underline{z}(b)$). The wage schedule, derived from stability conditions analogous to (8) and (9), is

$$w_U\left(z, s_U\left(z|b\right), x\right) = \int_{\widetilde{z}}^{z} \pi_z\left(t, s_U\left(z|b\right), \widetilde{x}\right) dt + \int_{\widetilde{x}}^{x} \pi_x\left(z, s_U\left(z|b\right), t\right) dt + \underline{w}_{PSE},$$

for those who are not liquidity-constrained, and

$$w_C\left(z, s_C\left(z|b\right), x\right) = \int_{\widetilde{z}}^{z} \pi_z\left(t, s_C\left(z|b\right), \widetilde{x}\right) dt + \int_{\widetilde{x}}^{x} \pi_x\left(z, s_C\left(z|b\right), t\right) dt + \underline{w}_{PSE}, \;\; \widetilde{z} < z < \underline{z}(b),$$

for those who are. The latter includes a bonus calculated as in (20).[33]

The functions that match jobs with agents, $s_U(z|b)$ and $s_C(z|b)$, are derived in Proposition 6 below, together with the equilibrium strategies, $x_U(z|b)$, and $x_C(z|b)$. Up to $\underline{x}(b)$, the matching condition is

$$F(x_i) = \alpha F_U(x_i) + (1-\alpha) F_C(x_i) = H(s_i).$$

Above $\underline{x}(b)$, it becomes

$$F(x_i) = \frac{G(z_i) - (1-\alpha)}{\alpha} = H(s_i),$$

[33] But, of course, the size of this bonus will be different.

and we have then the same job allocation as in FB. The educational investments of those who are not liquidity-constrained for $\widetilde{z} < z < \underline{z}(b)$ (i.e., the rich) are still governed by (23) as in the case where b is equal to b_2. With b greater than b_2, however, the s associated with each z is lower than in the previous case, because the poor can now buy more x. In other words, some bright poor agents will displace some of the dimmer rich. The amount invested by an unconstrained agent of talent $z > \underline{z}(b)$ is

$$x_U(z|b) = \int_{\underline{z}(b).}^{z} x'_U(z|b)dz + \underline{x}(b), \tag{27}$$

where $x'_U(z|b)$ has the same form as (22). The investment strategy of the liquidity-constrained poor will still satisfy (25), and will thus be given by

$$x_C\,(z|b) = \int_{\widetilde{z}}^{z} x'_C(z)dz + \widetilde{x}.$$

Proposition 7. For $b_2 < b < b_3$, *there exists a PSE where the poor are unconstrained not only for* $z = \widetilde{z}$ *and* $z = \overline{z}$, *but also for* $\overline{z} > z \geq \underline{z}(b)$. *These agents choose the same amount of* x, *higher than in FB, as the rich of the same talent. The poor with* $\widetilde{z} < z < \underline{z}(b)$ *choose less* x *than the rich of the same talent. Rich and poor agents of talent* $\widetilde{z}$ *choose the FB level of education* $\widetilde{x}$.

Proof. *See Appendix.*

Corollary 6. *In the PSE associated with* $b_2 < b < b_3$, *the unconstrained agents with* $\underline{z}(b) \leq z \leq \overline{z}$ *are matched with jobs of the same quality as in FB and SE. The same applies to agents with* $z = \widetilde{z}$. *In the* $\widetilde{z} < z < \underline{z}(b)$ *range, (i) the rich are matched with higher quality jobs than in FB and SE, but with lower quality jobs than in the PSE associated with* $b = b_2$, *(ii) the poor are matched with lower quality jobs than in FB and SE, but with higher quality ones than in the PSE associated with* $b = b_2$.

Proof. *See Appendix.*

Corollary 7. *In the PSE associated with* $b_2 < b < b_3$, $x_U(\widetilde{z}|b) = x_U(\widetilde{z}|b_2) = x_{SE}(\widetilde{z})$. *For* $z > \widetilde{z}$, *the* $x_U(z|b)$ *curve lies (i) everywhere below the* $x_U(z|b_2)$ *curve except possibly at* $z = \overline{z}$ *where it could be that* $x_U(\overline{z}|b) = x_U(\overline{z}|b_2)$, *and (ii) above the* $x_{SE}(z)$ *curve except possibly at very high levels of* z *where the two curves could coincide. Up to a certain* $z < \overline{z}$, *the* $x_C(z|b_2)$ *curve lies below the* $x_{SE}(z)$ *curve. Above that* z, *the* $x_C(z|b_2)$ *curve lies above the* $x_{SE}(z)$ *curve. As* b *rises, the* $\underline{z}(b)$ *point moves to the left, and the* $x_U(z|b)$ *curve gets closer to the* $x_{SE}(z)$ *curve. For* $b = b_3$, *the* $x_C(z|b)$ *vanishes, and* $x_U(z|b)$ *coincides with* $x_{SE}(z)$.

Proof. *See Appendix.*

One of the possible configurations envisaged in Corollary 7 is shown in Fig.3.

5.3 How b affects efficiency and distribution

Using the foregoing propositions, we can demonstrate the following.

Proposition 8. *The rich (with the possible exception of those with $z = \overline{z}$ in the special case where $x_U(\overline{z}|b_2) = x_{LF}(\overline{z})$) are worse-off in the PSE associated with b_2, than in LF. The poor with $z \geq \widetilde{z}$ are better-off in that PSE than in LF.*

Proof. *See Appendix.*

Let us see what this all means. We know from Corollary 4 that almost all the rich get better jobs, and almost all the poor get worse ones, in the PSE associated with $b = b_2$ than in FB. The only exceptions are the agents, rich or poor, whose talent is either $z = \widetilde{z}$ or $z = \overline{z}$, who will get jobs of the same quality in this PSE as in FB. Compared with LF, however, all the rich other than those with $z = \overline{z}$ get worse jobs, and all the poor with $z \geq \widetilde{z}$ get better ones in this PSE. The rich with $z < \widetilde{z}$ (who would not receive a university education in FB because they are not talented enough, but would receive one in LF) are now in non-graduate jobs. The poor in that talent range are in non-graduate jobs anyway. We also know, from Corollary 5, that all the rich other than those with $z = \widetilde{z}$ overinvest, but they overinvest less than in LF (see Fig. 2). We cannot say whether the constrained poor underinvest or overinvest, but we can say, and this is all that matters, that they invest less than the rich with the same z. Regarding utility, Proposition 8 says that offering to lend b_2 makes almost all the rich worse-off. Those with $z < \widetilde{z}$ because they are excluded from the tournament. Those with $z \geq \widetilde{z}$ because they end up in lower quality graduate jobs, and this demotion is not compensated by the reduction in overinvestment. In particular, the policy puts the rich with $z = \widetilde{z}$ in the lowest-quality graduate jobs, where the pay is so low that, even though they invest the efficient amount, these rich agents are indifferent between going to university or straight into the non-graduate labour market. The only possible exception are the rich with $z = \overline{z}$, who could be as well-off with as without the policy in the very special case where their investment level is the same as in LF. What about the poor? Those with $z < \widetilde{z}$ will still be excluded from the tournament, and will thus have the same utility as in LF. Those with $z = \widetilde{z}$ are as well-off as without the policy, because they participate in the tournament but have the same utility level as the non-graduates. Those with $z > \widetilde{z}$ are better-off with the policy. Therefore, none of the poor is worse-off with the policy, but the less talented among them are no better-off.

What happens if the policy is more generous than the one we have just examined? The following proposition holds true for b higher than b_2, but still lower than b_3 (see Fig. 3).

Proposition 9. *All agents in the* $\widetilde{z} < z < \underline{z}(b)$ *range are better-off in the PSE associated with* $b_2 < b < b_3$ *than in the one associated with* $b = b_2$ *if they are poor, worse-off if they are rich. Compared with SE, however, all the agents in that talent range are worse-off if they are poor, better-off if they are rich. In the* $z \geq \underline{z}(b)$ *range, almost all participating agents are worse-off in the PSE associated with* $b_2 < b < b_3$*, than in the one associated with* $b = b_2$*, and better off than in SE. The possible exception are the agents (rich or poor) in the upper part of the* $\widetilde{z} < z < \underline{z}(b)$ *range, in the case where the* $x_U(z|b)$ *and* $x_{SE}(z)$ *curves coincide.*

Proof. *See Appendix.*

This proposition has an obvious corollary.

Corollary 8. *In the* $\widetilde{z} < z < \underline{z}(b)$ *range, the utility of the poor (rich) is increasing (decreasing) in b. In the* $\underline{z}(b) \leq z \leq \overline{z}$ *range, the utility of all participating agents is decreasing in b, except possibly at levels of z so high that the* $x_U(z|b)$ *and* $x_{SE}(z)$ *curves may coincide.*

What it says is that raising the size of the loan above the minimum necessary to bring the talented poor into the tournament reduces the number of liquidity-constrained agents (to zero if the loan is raised as far as b_3). Where education is concerned, we move from the situation illustrated by the $x_U(z|b_2)$ and $x_C(z|b_2)$ curves of Fig. 2 to the one illustrated by the $x_U(z|b)$ and $x_C(z|b)$ curves of Fig. 3, and eventually to the one illustrated by the $x_{SE}(z)$ curve appearing in both figures. This policy reduces educational overinvestment on the part of the rich. The effect on the investment decisions of the poor is not so clear-cut. Those of them who are and remain liquidity-constrained invest more as the size of the loan increases. But, the moment the loan gets big enough to make those among them who have a certain talent unconstrained, a further increase in the size of the loan would reduce the investment of those whose talent is higher than that. This apparently paradoxical result comes from the fact that those with the latter are already unconstrained and will thus respond to the policy as if they were rich. But why do the rich invest less as the loan gets bigger? We have seen that there is a middling talent range where the poor are liquidity–constrained and thus invest less than the equally talented rich. The rise in the size of the loan induces the rich in that talent range to invest less because it reduces the quality of the jobs that will be assigned to them in equilibrium. The rich in the higher talent range also invest less, but for a different reason. The quality of their jobs will in fact remain the same, but the investment needed to differentiate themselves from those who are less talented than themselves will become smaller (because those who are less talented than themselves invest less).

Looking at the overall picture, can we then say that raising the size of the loan (but not to the level that would release *all* the participating poor from their liquidity constraints) enhances efficiency? The matching improves. Individual educational investments get closer to their FB level for the participating rich, and for the participating poor who are not or cease to be liquidity-constrained, but not necessarily for the participating poor who remain liquidity-constrained despite the policy.[34] Therefore, we cannot be sure that the policy enhances efficiency. As b increases, the poor who are still liquidity constrained could in fact overinvest. What about consumption or utility? The participating rich loose. The participating poor who would have been unconstrained even without the increase in b loose too (they would prefer the size of the loan to remain just large enough to make *them* unconstrained, because the larger loan brings in more talented competitors). Those who either remain constrained even with the policy, or become unconstrained thanks to it, gain. Remember that these are poor agents at the lower end of the talent range (i.e., with z is close to $\widetilde{z}$). Where consumption and utility are concerned, therefore, the effect of getting a better or worse quality job predominates over the effect of increasing or decreasing educational investment. But the policy does not generate a Pareto-improvement because some gain and some loose.

Does it pay to raise the size of the loan to such a high level that none of the participating poor is liquidity-constrained? As b gets closer to b_3, the number of individual investments that could be brought closer to their FB levels falls. Therefore, the efficiency-enhancing effect of the policy tends to come more and more from the improvement in the matching, but the number of agents whose matching can be improved gets smaller and smaller. In Fig. 3, the agents in question are the poor with talent higher than $\widetilde{z}$ but lower than $\underline{z}(b)$. As $\underline{z}(b)$ gets closer to $\widetilde{z}$ (because b gets closer to b_3), the number of agents who can be positively affected by the policy tends to zero. Therefore, $b \geq b_2$ is better than $b = 0$, but social preferences would have to be very egalitarian indeed for it to be optimal to set b equal or even very close to b_3.

6 If the poor are not so poor

So far, we have assumed that $\underline{y}$ is lower than $c(\overline{z}, x_0)$, and set it equal to zero. In general, however, y could vary across agents between zero and $\overline{y}$. To keep things simple, we will continue to assume that $y \in \{\underline{y}, \overline{y}\}$, $\underline{y} < \overline{y}$, and examine the possible equilibria associated with values of $\underline{y}$ within that range We can do that very easily if we re-interpret b as $\underline{y}$, because we can then exploit the results obtained in the last section. The utility function and the liquidity constraint remain, respectively, (1) and (6).

[34] In Fig.3, the FB investment curve is drawn concave, and intersects the investment curve of the liquidity-constrained poor. Up to a certain level of talent, the effect of the policy is to bring investment closer to its FB level. Above that level, the policy induces overinvestment. As pointed out in Section 3, however, we cannot be sure that the FB investment curve looks the way we have pictured it. Were it to lie everywhere below the investment curve of the liquidity-constrained poor, the policy would cause overinvestment on the part of all those who remain liquidity-constrained. As b gets closer to b_3, however, the PSE equlibrium looks more and more like SE, and the number of constrained agents gets smaller and smaller.

For $\underline{y}$ lower than b_1, we get the LF equilibrium examined in Section 4, in which case only the rich go to university and get graduate jobs. For $\underline{y}$ no lower than b_2, but lower than b_3 (and assuming $b_3 < \overline{y}$), we get a different LF equilibrium, this time one that looks like the PSE examined in Subsection 5.2, where some or all the poor with talent no lower than $\widetilde{z}$ go to university and get graduate jobs. In this kind of equilibrium, agents of different talent but same education occupy jobs of the same quality, and those among them who have higher talent get a bonus. If $\underline{y}$ is equal to b_3, the LF equilibrium looks like the SE examined in Subsection 5.1, where agents and jobs are matched as in FB. Everything we said we respect to the efficiency properties of the equilibria associated with different loan sizes applies also to the equilibria associated with different values of $\underline{y}$.

7 Conclusion

In our framework, agents (school leavers) are differentiated by native talent and parental wealth. Graduate jobs are differentiated by (technological, managerial) quality. Talent reduces the cost of achieving any given educational level (degree level, graduation mark), and directly increases the output of the match between a graduate with any given educational level and a job of any given quality. School leavers cannot borrow from the credit market. Educational achievement, job quality, and the output of each job match are common knowledge, but talent and parental wealth are private information. Graduate jobs are assigned by a matching tournament to graduates that have achieved at least a certain educational level (degree level, graduation mark). Those who do not get a graduate job find employment in the non-graduate labour market. Taken together, these assumptions yield the result that, without government intervention, a number of school leavers may be excluded from graduate jobs, not because they are insufficiently talented, but because their parents are insufficiently rich. Furthermore, all those who are rich or talented enough not to be liquidity-constrained in their educational decisions overinvest in education.

The government can change this by borrowing wholesale on the international money market, and lending to individual students. If the size of individual loans is below a certain minimum, however, either the policy is ineffective (the equilibrium is the same as in Laissez Faire) or there is no equilibrium. Above that minimum, student loans improve the matching. In particular, the policy excludes the rich not talented enough to go to university in First Best (who would go to university and occupy graduate jobs in Laissez Faire). If the loans are sufficiently large, the policy yields a Separating Equilibrium where the most talented agents get the highest quality jobs, the second-most talented agents get the second-highest quality jobs, and so on as in First Best, but all except the marginal agents (those who are indifferent between investing in education and going into the non-graduate labour market) overinvest. At in-between loan sizes, the policy yields a Partially Separating Equilibrium where all the participating rich and the more talented among the participating poor overinvest, but the less talented among the participating poor underinvest. In this equilibrium, there is a quality-of-job range where jobs of the same quality are occupied by graduates with the same education, but different talent and consequently different productivity – higher if the graduate comes from a poor family, lower if he comes from a rich one. Where that is the case, the more talented graduates get a bonus reflecting the difference between their productivity and that of the less talented ones doing the same kind of job. However, this bonus is not sufficient to fully compensate poor graduates for the fact that they are in worse jobs than if they were rich and could thus invest more.

The policy in question does not yield a Pareto-improvement. Raising the size of the loan to the minimum required to generate a separating equilibrium different from that of Laissez Faire makes all the rich worse-off, and the more talented poor (those for whom it is efficient to go to university) better-off. The less talented poor are not affected. Raising it further (but not far enough to yield a Separating Equilibrium) makes the participating poor not talented enough to be released from their liquidity constraints better-off, but all the other participants, including not only the rich, but also the more talented among the poor, worse-off. Indeed, the number of agents who would gain from a further increase in the size of the loan gets smaller, and the number of those would loose gets larger,[35] as the size of the loan gets closer to the level that would make all the poor unconstrained, and thus achieve the FB matching. Therefore, it can never be optimal to make the loan as large as that, and social preferences would have to be very egalitarian indeed (Rawlsian) for it to be optimal to get very close.

8 Appendix

8.1 Proof of Proposition 1

The support of the talent distribution is narrower in FB than in LF because, unlike the latter, the former includes only agents with $z \geq \widetilde{z}$. Therefore, as the talent distribution is the same for rich and poor,

$$\frac{G(z) - (1-\alpha)}{\alpha} < G(z) \;\forall\; z \in [\widetilde{z}, \overline{z}).$$

Given assortative matching in both FB and LF, and given that $H^{-1}(.)$ is monotonically increasing, it then follows that

$$s_{FB}(z) = H^{-1}\left(\frac{G(z) - (1-\alpha)}{\alpha}\right) < H^{-1}(G(z)) = s_{LF}(z).$$

[35] The new equilibrium would be such, that these agents would loose even more if they borrowed less than the maximum the government is willing to lend them.

8.2 Proof of Proposition 3

From the construction of the SE equilibrium, we know that, for $z < \widetilde{z}$, $x_{LF}(z) > x_{SE}(z) = x_{FB}(z) = 0$. We also know that $x_{SE}(\widetilde{z}) = x_{FB}(\widetilde{z}) = \widetilde{x}$. We must demonstrate that (i) $x_{LF}(\widetilde{z}) > \widetilde{x}$, (ii) $x_{LF}(z) > x_{SE}(z)$ for $z \in (\widetilde{z}, \overline{z})$, and (iii) $x_{SE}(z) > x_{FB}(z)$ for $z \in (\widetilde{z}, \overline{z}]$.

Concerning (i), notice that, in SE, graduates of talent $\widetilde{z}$ are matched with jobs of quality $s = 0$, while in LF they are matched with jobs of quality $s_{LF}(\widetilde{z}) > 0$. Then, $x_{SE}(\widetilde{z}) < x_{LF}(\widetilde{z})$ because $x_{SE}(\widetilde{z}) = \widetilde{x}$ is found by maximizing $\pi(\widetilde{z}, x, 0) - c(\widetilde{z}, x)$, while $x_{LF}(\widetilde{z})$ is calculated by integrating (13) from x_0 and is consequently higher than $argmax\left(\pi\left(\widetilde{z}, s_{LF}(\widetilde{z}), x\right) - c(\widetilde{z}, x)\right)$, which is in turn higher than $x_{SE}(\widetilde{z})$ for the assumption that $\pi_{sz} > 0$. Hence, at $z = \widetilde{z}$, the $x_{SE}(z)$ curve lies below the $x_{LF}(z)$ curve.

To demonstrate (ii), take any z in the $(\widetilde{z}, \overline{z})$ interval. As the slope of the $x_{SE}(z)$ curve is given by (17), and that of the $x_{LF}(z)$ curve by(13), the two curves cannot cross. To prove this, notice that the numerator of (17) is lower than the numerator of (13) because $\pi_{sz} > 0$ and $\pi_{zx} = 0$ by assumption, and $s_{LF}(z) > s_{SE}(z)$ for any $z \in (\widetilde{z}, \overline{z})$. Notice also that the denominators of (17) and (13) are increasing in x. Consequently, if there existed values of z such that $x_{SE}(z) > x_{LF}(z)$, the slope of the $x_{SE}(z)$ curve would be lower than that of the $x_{LF}(z)$ curve. Given that $x_{SE}(\widetilde{z}) < x_{LF}(\widetilde{z})$, for the two curves to cross at $z' \in (\widetilde{z}, \overline{z})$ it would then have to be true that $x_{SE}(z)$ is steeper than $x_{LF}(z)$ in some interval belonging to $(\widetilde{z}, z')$. But, for any $z = z' + \delta$ with δ arbitrarily small, the slope of $x_{SE}(z)$ should then be lower than that of $x_{LF}(z)$, thus contradicting $x_{SE}(z) > x_{LF}(z)$. Neither can the two curves coincide from point $z' \in (\widetilde{z}, \overline{z})$ upwards. Given that $\pi_{sx} = 0$, this would in fact imply that (17) and (13) have the same denominator. The numerators should then be the same, too. But this is impossible because $\pi_{sz} > 0$ implies that, for any given x, the numerator of (17) is lower than the numerator of (13). This, however, does not rule out $x_{SE}(\overline{z}) = x_{LF}(\overline{z})$.

The demonstration of (iii) is in Proposition 3 of Hopkins (2012), who in turn refers to Propositions 1 and 3 in Mailath (1987). That demonstration concerns what we call LF, but it can be easily checked that it applies also to our SE.

8.3 Proof of Proposition 4

The poor with $z < \widetilde{z}$ reach the same level of utility in SE as in LF because they are excluded from the tournament and go straight into the non-graduate labour market in both equilibria. The poor with $z = \widetilde{z}$ reach the same level of utility as in LF, because, even if they now participate in the tournament, they are kept indifferent between acquiring a university education or go straight into the non-graduate labour market. Consequently all the poor with $z \leq \widetilde{z}$ reach the non-graduate level of utility, $y + w_0$. The poor with $z > \widetilde{z}$, instead, reach a higher level of utility in SE where they get graduate education and graduate jobs than in LF where they get non-graduate jobs, otherwise they would not participate in the tournament.

The rich with $z < \widetilde{z}$ do not go to university in SE. Consequently they are worse-off than in LF, where they would go to university and participate in the tournament. In order to evaluate the utility of the rich with $z \geq \widetilde{z}$, keep in mind that this is equal to $U(\widetilde{z}) + \int_{\widetilde{z}}^{z} U'(z)dz$. We know that $U_{LF}(\widetilde{z}) > U_{SE}(\widetilde{z})$ because, in SE, a rich agent with $z = \widetilde{z}$ is indifferent between going to university or straight into the non-graduate labour market, while he reaches a higher level of utility in LF. In order to evaluate $\int_{\widetilde{z}}^{z} U'(z)dz$, recall that the utility function is $U(z) = y + w - c(z, x)$. Consequently, $U'(z) = -c_z(z, x) > 0$ by the envelope theorem, because the solution must satisfy conditions analogous to (10) and (11). We also know from Proposition 3 that $x_{SE}(z) < x_{LF}(z)$ for $z \in (\widetilde{z}, \overline{z})$. Given that $c_z(z, x) < 0$, $c_{zx}(z, x) < 0$ by assumption, it then is $U'_{SE}(z) = -c_z(z, x_{SE}(z)) < -c(z, x_{LF}(z)) = U'_{LF}(z)$. Hence, the rich have lower utility in SE than in LF, with the possible exception of those with $z = \overline{z}$ in the special case where $x_U(\overline{z}|b_2) = x_{LF}(\overline{z})$.

8.4 Proof of Proposition 5

Assuming that x_U and x_C are strictly increasing in z (the demonstration in the proofs of propositions 6 and 7 below), we can prove Proposition 5 in five steps.

Step 1. For values of z such that the poor are (are not) liquidity constrained, the amount of x chosen by a rich of talent z is higher than (the same as) the amount bought by a poor of the same talent. This in turn implies that the minimum talent level for which an agent chooses a strictly positive level of x cannot be higher for the rich than for the poor.

Step 2. There cannot exist a PSE where some rich (hence, unconstrained) agents of talent $z \leq \overline{z}$ choose a higher level of x than the constrained poor of talent $z = \overline{z}$. If such an equilibrium existed, there would in fact be a level of z, z^m, and a corresponding level of x, x^m, such that the rich of talent $z \geq z^m$ for whom it is optimal to choose $x \geq x^m$ separate themselves from the poor by acquiring more x than the poor of talent $z = \overline{z}$. That, however, cannot be an equilibrium because the employer hiring a graduate of education level x^m would be better-off hiring a worker of education level $x^m - \delta$ with δ arbitrarily small. By so doing, he would in fact have a positive probability of hiring a poor of talent $\overline{z} > z^m$. Therefore, if an equilibrium exists, all agents of talent $z = \overline{z}$ have the same amount of education independently of their wealth.

Step 3. There cannot exist a PSE where the lowest x chosen by the rich, x^n, is higher than the the lowest x chosen by the poor, x^q. Given that $x_C(z)$ is strictly increasing, and recalling Step 1, if x^n is higher than x^q, the talent of a poor choosing x^n would be strictly higher than that of a rich choosing the same level of x. Then, for an argument analogous to the one used in Step 2, a firm hiring a graduate of education level x^n would be better-off if it hired a worker of education level $x^n - \delta$ because, by so doing, it would hire a poor with higher

z than the expected level of z of the agents choosing x^n.

Step 4. There cannot exist a PSE where x^n, is lower than x^q. Suppose that such an equilibrium exists. Let z_P^q be the talent of the poor, and z_R^q that of the rich, buying x^q in this equilibrium. We know from Step 1 that $z_R^q \leq z_P^q$, and that there will thus be rich agents of talent lower than z_R^q buying positive amounts of x. Consider a level of x, $x' < x^q$. If it is profitable for a rich agent of talent $z' < z_R^q$ to choose x', it will be even more profitable to choose that amount of x for a poor of talent z'', $z' < z'' < z_P^q$, such that $c(x^q - \delta, z'') = b$ with δ arbitrarily small (so that z'' can afford x'). Hence, the equilibrium in question cannot exist.

Steps 1 to 4 tell us that, if an equilibrium exists for $b < b_3$, it will be such that the least able rich participating in the tournament choose the same amount of education as the least able of the participating poor, and that all agents of talent $\overline{z}$ choose the same amount of education independently of their wealth.

Step 5. There cannot exist an equilibrium satisfying the Divinity Criterion (Banks and Sobel, 1987) such that the common lowest level of x, say $\widehat{x}$, is chosen by rich and poor with different talent levels. If such an equilibrium existed, we know from Step 1 that $\widehat{x}$ would be chosen by the rich of talent z' and the poor of talent z'', $z'' > z'$. Then, $\underline{w}_{PSE}$ would have to satisfy $\underline{w}_{PSE} = w_0 + c\left(z', \widehat{x}\right)$, where $c\left(z', \widehat{x}\right) > c\left(z'', \widehat{x}\right)$. Since $c_{xz} < 0$, however, there is a level of x, $\widehat{x} - \delta$, such that (i) $c(z'' - \varepsilon, \widehat{x} - \delta) = b$, (ii) $\underline{w}_{PSE} - c(z' - \varepsilon, \widehat{x} - \delta) < w_0$, and (iii) $\underline{w}_{PSE} - c(z'' - \varepsilon, \widehat{x} - \delta) > w_0$ for ε arbitrarily small. If the Divinity Criterion is to be satisfied, firms observing $x = \widehat{x} - \delta$ cannot attribute a positive belief either to $z \leq z' - \varepsilon$ or to $z \geq z'' - \varepsilon$, because a poor agent of talent $z'' - \varepsilon$ is of the type that can profit most from the choice of $\widehat{x} - \delta$ as his is the only type that could strictly improve upon his equilibrium utility by accepting any of the contracts offered by the employers. But, if the belief $z'' - \varepsilon$ is attached to $\widehat{x} - \delta$, then poor agents of talent $z'' - \varepsilon$ would have an incentive to actually deviate from the equilibrium a d choose $\widehat{x} - \delta$. In fact, the offer of $\underline{w}_{PSE}$ by a firm of quality $s = 0$ dominates their equilibrium payoff because of (iii), while such offer is clearly profitable for a firm of quality $s = 0$ holding the belief $z'' - \varepsilon$. Hence, the equilibrium considered does not satisfy the Divinity Criterion.

Therefore, if an equilibrium exists for $b < b_3$, it will be such that the least talented rich participating in the tournament will not only choose the same level of x, but also have the same talent z, as the least talented of the participating poor. Given that the measure of graduate jobs is the same in PSE as in FB and SE, the common minimum talent level will then be $\widetilde{z}$. Since the agents with $z = \widetilde{z}$ have nothing to signal, they will choose their efficient level of education $\widetilde{x}$.

8.5 Proof of Proposition 6

The proof is structured as follows. First, exploiting the necessary characteristics of a PSE (in particular, conditions (22) and (24), we show that it is possible to construct two matching functions, $s_C(z|b)$ for the poor and $s_U(z|b)$ for the rich, such that there is assortative matching over z within wealth categories, and over x for the population as a whole. Even if the poor with $z = \widetilde{z}$ and those with $z = \overline{z}$ are unconstrained, we can still write $s_C(\widetilde{z}|b)$ and $s_C(\overline{z}|b)$ because we know that, if there is assortative matching, the former will be equal to 0, and the latter to $\overline{s}$. Second, having already noted that (24) has a unique solution by the implicit function theorem, we prove that (22) also has a unique solution. Third, we show that there exists a value of $b > \widetilde{b}$ such that the equilibrium exists.

We know from Proposition 5 that, if a PSE exists, the support of participating agents will be $[\widetilde{z}, \overline{z}]$, and the equilibrium will be such that all agents of talent $\widetilde{z}$ will choose $\widetilde{x}$, and all agents of talent $\overline{z}$ will choose the same level of x, $\overline{x}$, independently of their wealth.

Consider the (constrained) poor in the $\widetilde{z} < z < \overline{z}$ talent range. Recall that we have called $x_C(z|b)$ the function that solves (24). We know from the implicit function theorem that such a function exists. Given our assumptions on $c(z,x)$, $x_C(z|b)$ is continuous, convex and strictly increasing in z, with derivative $x'_C(z|b) = -\frac{c_z}{c_x}$.[36] Clearly, $x_C(z|b)$ and $x'_C(z|b)$ are increasing in b.

Consider the rich. Recall that

$$x'_U(z|b) = \frac{\pi_z\left(z, s_U\left(z|b\right), x\right)}{c_x\left(z,x\right) - \pi_x\left(z, s_U\left(z|b\right), x\right)} > 0. \tag{28}$$

Consequently, if this differential equation has a solution,

$$x_U(z|b) = \int_{\widetilde{z}}^{z} x'_U(t|b)dt + \widetilde{x}, \tag{29}$$

this is monotonically increasing in z.

Therefore, $x_i^{-1}(x|b)$, $i = U, C$, is defined, and it is decreasing in b. The distribution of x conditional on b then satisfies

$$F(x|b) = \alpha F_R(x|b) + (1-\alpha)F_P(x|b) = \alpha G\left[x_U^{-1}(x|b)\right] + (1-\alpha)G\left[x_C^{-1}(x|b)\right] \tag{30}$$

over $[\widetilde{x}, \overline{x}_U]$, where $\overline{x}_U$ denotes the level of education chosen by a rich of talent $\overline{z}$.

Assortative matching implies

$$F\left(x_i|b\right) = \phi\left(\alpha G\left[x_U^{-1}(x_i|b)\right] + (1-\alpha)G\left[x_C^{-1}(x_i|b)\right]\right) = H\left(s_i\right), \tag{31}$$

so that an agent buying x_i is matched with a job of quality s_i. We know that agents of talent $\widetilde{z}$ choosing x, will be matched with jobs of quality $s = 0$. We must derive the $s_j\left(z\right)$ functions, $j = U, C$, for values of z above $\widetilde{z}$.

First, we want to demonstrate that there is only one $s_U\left(z\right)$ function simultaneously satisfying (29) and (31), and that $s'_U\left(z\right) > 0$ (positive assortative matching). Take a talent level $z \geq \widetilde{z}$, such that the rich of that talent participate in the tournament. Suppose that (29) and (31) are simultaneously satisfied

[36] If we relax the assumptions on $c(z,x)$ by allowing c_{xx} to be positive, convexity requires $\frac{c_{xz}}{c_z} > \frac{c_{xx}}{2c_x}$.

at this level of z. Consider a talent level $z+\varepsilon$ with $\varepsilon > 0$ arbitrarily small. As $G\left[x_C^{-1}(x_i|b)\right]$ is given, there is only one value of $s_U(z+\varepsilon)$ that simultaneously satisfies (29) and (31). Given that these two conditions are satisfied by $z = \widetilde{z}$ where $s_U(\widetilde{z}|b) = 0$, and given also that $x_U(z|b)$ is continuous, if we follow this procedure starting from $z = \widetilde{z}$, we identify the unique function $s_U(z|b)$ that satisfy the conditions in question for all $z \geq \widetilde{z}$. Hence, $s_U(z|b)$ is uniquely defined by (29) and (31), and will be such that $s'_U(z) > 0$.

Second, noting that the $x_C(z|b)$ function we have just derived does not depend on $s_C(z)$, the $s_C(z|b)$ function can be derived from (31). By construction, this function will be such that $s'_C(z|b) > 0$.

So far, we have assumed that (29) has a unique solution. To prove it, we show that Theorem 1 and 2 in Mailath (1987) apply to the present case. For that to be true, the function $V(z, \widehat{z}, x) \equiv w(\widehat{z}, s(\widehat{z}), x) - c(z, x)$ must satisfy Mailath's regularity conditions (1) $V(z, \widehat{z}, x)$ is C^2 on the set of possible messages $\widehat{z}$, (2) $V_2 > 0$, (3) $V_{13} > 0$, (4) $V_3 = 0$ has a unique solution, and (5) (boundedness) $V_{33}(z, s(z), x) < 0$. Condition (1) is obviously satisfied. Condition (2) also is satisfied because, as we have shown, $V_2 = w_z + w_s s' = \pi_z$, and π_z is positive by assumption. Condition (3) is satisfied because $V_{13} = -c_{zx}$, and c_{zx} is negative by assumption. Condition (4) is satisfied because $V_3 = \pi_x - c_x$, and π_{xx} is negative and c_{xx} zero by assumption. Condition (5), finally, is satisfied because $V_{33}(z, s(z), x) = \pi_{xx}$. Additionally, the investment function $x_U(z|b)$ must be incentive-compatible, and satisfy the initial condition $x_U(\widetilde{z}|b) = \widetilde{x}$. In our case, (28) does satisfy the incentive-compatibility condition for the unconstrained, and the initial condition is the one indicated. As argued in Hopkins (2012), Mailath's Theorem 1 implies that $x_U(z|b) > x_{FB}(z)$ for all $z > \widetilde{z}$. Therefore, as stated in the proposition, all agents other than those with the minimum talent level required to participate in the tournament overinvest.

Having demonstrated the existence of a unique $x_C(z|b)$ and a unique $x_U(z|b)$ curve, we now want to show that the two are located as in Figure 2, and thus that the PSE associated with $b = b_2$ exists. We know that $x_C(\widetilde{z}|b) = x_U(\widetilde{z}|b)$. In view of (28) and of the initial condition $x_U(\widetilde{z}|b) = \widetilde{x}$, we also know that $x'_U(z|b)$ tends to infinity in a neighborhood of $\widetilde{x}$ (Mailath, 1987, p.1355). Moreover, Mailath's conditions imply concavity of $x_U(z|.)$. Consequently, given that $x_C(z|.)$ is strictly convex with bounded derivative at $x_C(\widetilde{z}|b) = x_U(\widetilde{z}|b) = \widetilde{x}$, the $x_U(z|b)$ curve will lie above the $x_C(z|b)$ curve in the neighborhood of $\widetilde{z}$

To prove that a level of $b_2 > \widetilde{b}$ sustaining this PSE exists, suppose first that $b = \widetilde{b}$. Given that $c(z, x_{FB}(z)) > c(\widetilde{z}, x_{FB}(z))$ by assumption, it must then be true that

$$\overline{x}_U \equiv x_U(\overline{z}|\widetilde{b}) > x_C(\overline{z}|\widetilde{b}),$$

where $x_C(\overline{z}|\widetilde{b})$ solves $c(\overline{z}, x) = \widetilde{b}$. Were it true that $\overline{x}_U \leq x_C(\overline{z}|\widetilde{b})$, $c(\overline{z}, x_C(\overline{z}|\widetilde{b}))$ would in fact be higher than $\widetilde{b}$, because $\overline{x}_U > x_{FB}(\overline{z})$, and the liquidity constraint would thus be violated. As the equilibrium requires $\overline{x}_U \equiv x_U(\overline{z}|b) = x_C(\overline{z}|b)$, there is then no equilibrium for $b = \widetilde{b}$. However, we can construct the $x_U(z|\widetilde{b})$ and $x_C(z|\widetilde{b})$ curves following the procedure outlined above. Knowing that the former lies above the latter in a neighborhood of $\widetilde{z}$, that $x_U(\overline{z}|\widetilde{b}) >$

$x_C(\overline{z}|\widetilde{b})$, and that $x_U(z|b)$ is concave and $x_C(z|b)$ strictly convex, the $x_U(z|\widetilde{b})$ curve will then lie above the $x_C(z|\widetilde{b})$ curve at all $z > \widetilde{z}$.

Raising b relaxes the liquidity constraints of the poor. Consequently, $x_C(z|.)$ will rise and, given (31), $x_U(z|.)$ will fall because the rich other than those of talent $z = \widetilde{z}$ will be hired for lower quality jobs than they otherwise would (i.e., $s_C(z)$ will rise and $s_U(z)$ will fall). Given that $x_C(z|b)$ is convex and $x_U(z|b)$ concave, as b rises, the two curves will come closer together and eventually coincide at the point where $z = \overline{z}$. At that level of b, call it b_2 the poor of talent $z = \overline{z}$ are not liquidity constrained, and the two investment curves are located as in Figure 2. Therefore, a PSE exists for $b = b_2$.

8.6 Proof of Corollary 4

Consider that the support of the talent as well as the mass of the agents participating in the tournament is the same as in FB and SE. We know from Proposition 3 that agents with $z = \widetilde{z}$ *and* $z = \overline{z}$ choose the same level of education ($\widetilde{x}$ and $\overline{x}$, respectively) independently of their being rich or poor. Consequently they will be matched with the same jobs (the worst and the best respectively) as in FB and SE. Moreover, we know from Proposition 6 that $s_U(z|b_2) > s_C(z|b_2)$ for all $z : \widetilde{z} < z < \overline{z}$. Consequently it must be the case that $s_U(z|b_2) > s_{FB}(z) = s_{SE}(z) > s_C(z|b_2)$ for all $z : \widetilde{z} < z < \overline{z}$. That the poor have better jobs in PSE than in LF immediately follows from the fact that in LF they are excluded from the graduate job market. That the rich have worse jobs than in LF follows from the fact that part of the graduate jobs of each given quality are now allocated to poor agents while in LF all graduate jobs are given to the rich.

8.7 Proof of Corollary 5

From the construction of the PSE, we know that, for $z < \widetilde{z}$, $x_{LF}(z) > x_U(z) = x_{FB}(z) = 0$. From Proposition 5, we also know that $x_U(\widetilde{z}|b_2) = x_C(\widetilde{z}|b_2) = x_{FB}(\widetilde{z}) = \widetilde{x}$ and that $x_U(\overline{z}|b_2) = x_C(\overline{z}|b_2)$. Part (ii) of the corollary, $x_U(z) > x_C(z)$ for $z \in (\widetilde{z}, \overline{z})$, follows from the definition of constrained agent. The proof of parts (i) and (iii) parallel those of parts (iii) and (ii) of Proposition 3 if we put $x_U(z)$ in the place of $x_{SE}(z)$.

8.8 Proof of Proposition 7

For an argument analogous to the one used in the last paragraph of the proof of Proposition 5, if we let b rise above b_2, the $x_C(z|b)$ curve will shift upward, and the $x_U(z|b)$ curve downward. The two curves will then cross at the value of z, lower than $\overline{z}$, denoted by $\underline{z}(b)$. This means that poor agents of talent $\underline{z}(b) \leq z < \overline{z}$ cease to be liquidity constrained, and buy the same amount of x as the rich of the same talent. But, as the $x_U(z|b)$ curve shifts downward, the educational investment of the rich does not remain the same. The rich of talent $z \geq \underline{z}(b)$ will in fact face more competition from the poor, and get lower quality jobs, than in the PSE associated with b_2. In the new equilibrium, the educational investment of (rich and poor) unconstrained agents, $x_U(z|b)$, is then lower than $x_U(z|b_2)$. For $z \geq \underline{z}(b)$, the job allocation function $s_U(z|b)$ is such that

$$G(z) = F(x) = H(s),$$

and thus the same as in FB. The associated investment function is $x_U(z|b) = \int_{\underline{z}(b)}^{z} x'_U(t)dt + \underline{x}(b)$, where $x'_U(z) = \frac{\pi_z(z,s_U(z|b),x)}{c_x(z,x)-\pi_x(z,s_U(z|b),x)}$ for rich and poor alike, and $\underline{x}(b)$ is the common value of x at the point where the curve representing the investment behaviour of constrained poor agents, $x_C(z|b)$, crosses the one representing the investment behaviour of unconstrained (rich or poor) agents, $x_U(z|b)$. Proofs analogous to those developed for LF and SE then apply to agents of talent $\underline{z}(b) \leq z < \overline{z}$.

For $\widetilde{z} < z < \underline{z}(b)$, there are two job allocation functions, $s_C(z|b)$ for the poor and $s_U(z|b)$ for the rich, and two investment functions, $x_C(z|b)$ and $x_U(z|b)$, constructed following the same procedure as in the proof of Proposition 6.

At $z = \widetilde{z}$ both rich and poor agents choose $\widetilde{x}$ as shown in Proposition 3.

8.9 Proof of Corollary 6

The first part of the corollary follows immediately from the proof of Proposition 7. The proof of the second part is along the same lines as the proof of Corollary 4.

8.10 Proof of Corollary 7

We know from Proposition 3 that $x_U(\widetilde{z}|b) = x_U(\widetilde{z}|b_2) = x_{SE}(\widetilde{z}) = \widetilde{x}$. Part (i) of the corollary follows immediately from the proof of Proposition 7. To prove part (ii), we recall that in the $\widetilde{z} < z < \underline{z}(b)$ range, $s_{LF}(z) > s_U(z|b) > s_{SE}(z)$ because of Corollary 6 and Corollary 4. We can then replicate the argument used in the proof of Proposition 3 to prove that $x_{LF}(z) > x_{SE}(z)$, with $x_U(z)$ in the place of $x_{LF}(z)$.

To show that up to a certain $z < \overline{z}$, the $x_C(z|b_2)$ curve lies below the $x_{SE}(z)$ curve, consider that at $\widetilde{z}$ the slope of the $x_{SE}(z)$ curve tends to infinity and is then higher than that of the $x_C(z|b)$. However it is $x_C(\underline{z}(b)|b) = x_U(\underline{z}(b)|b) > x_{SE}(z)$ at $z = \underline{z}(b)$. Given that both the $x_C(z|b)$ and the $x_{SE}(z)$ curves are continuous it must then be the cases that the $x_C(z|b)$ curve cuts the $x_{SE}(z)$ curve from below.

As b rises, the $\underline{z}(b)$ point moves to the left because less poor agents will be liquidity-constrained. This implies that the unconstrained agents with $z < \underline{z}(b)$ will now obtain worse jobs than those they obtained at lower levels of b. It can then be proved that the slope of the $x_U(z|b)$ curve will be lower because if it were not so there would arise a contradiction. In fact, the numerator of (22) is now lower than it is at a higher value of b because each agent obtains a worse job, and $\pi_{zs} > 0$. In order for the denominator to be lower enough to compensate the reduction of the numerator, we should have an increase in x at each level of z, but a higher level of x cannot result from lower values of (22) in (27).

8.11 Proof of Proposition 8

Where the rich are concerned, the proof is the same as that of Proposition 4, with $x_U(z|b_2)$ in place of $x_{SE}(z)$ and $U_{PSE}(z)$ in place of $U_{SE}(z)$. The poor with $z \geq \widetilde{z}$ are obviously better-off in PSE than in LF because, in the latter, they are excluded from university and consequently they can only reach the non-graduate utility level w_0.

8.12 Proof of Proposition 9

Recall that the utility function is $U(z) = w - c(z, x)$, and that $U_{PSE}(\widetilde{z}|b) = U_{PSE}(\widetilde{z}|b_2) = U_{SE}(\widetilde{z}|b)$. Where the rich in the $\widetilde{z} < z < \underline{z}(b)$ range are concerned, the proof is the same as that of Proposition 4 and Proposition 8. The utility of the rich with $z \geq \widetilde{z}$ is equal to $U(\widetilde{z}) + \int_{\widetilde{z}}^{z} U'(z)dz$, where $U'(z) = -c_z(z, x) > 0$ by the envelope theorem, because the solution must satisfy conditions analogous to (10) and (11). We know from Corollary 7 that $x_{SE}(z) < x_U(z|b) < x_U(z|b_2)$ for $z \in (\widetilde{z}, \underline{z}(b))$. Given that $c_{zx}(z, x) < 0$ by assumption, it then is$U'_{SE}(z) = -c_z(z, x_{SE}(z)) < -c_z(z, x_U(z|b)) = U'_{PSE}(z|b)$ and $U'_{PSE}(z|b) = -c_z(z, x_U(z|b)) < -c(z, x_U(z|b_2)) = U'_{PSE}(z|b_2)$. Consequently the rich with $\widetilde{z} < z < \underline{z}(b)$ have lower utility in SE than in the PSE associated with $b > b_2$, and lower utility in the latter than in the equilibrium associated with $b = b_2$. The utility of the poor with $\widetilde{z} < z < \underline{z}(b)$ in PSE is given by $U(\widetilde{z}) + \int_{\widetilde{z}}^{z} U'(z)dz$, where $U'(z) = \pi_z(z, s_C(z), x) + \pi_x(z, s_C(z), x)x'_C(z|b) > 0$ by the envelope theorem, because the $x_C(z|b)$ must satisfy (25). We know from Corollary 6 that the poor are matched with better jobs in the PSE associated with $b > b_2$ than in that associated with $b = b_2$. We also know, from Corollary 7, that $x_C(z|b) > x_C(z|b_2)$ for $z \in (\widetilde{z}, \underline{z}(b))$. Given that, by assumption, $\pi_{zs}(z, s, x) > 0$ and $\frac{c_{zx}(z,x)}{c_z(z,x)} > -\frac{\pi_{xx}(z,s(z),x)}{\pi_x(z,s(z),x)}$, it then follows that $U'_{PSE}(z|b) = \pi_z(z, s_C(z),\ x_C(z|b)) + \pi_x(z, s_C(z)$ and $x_C(z|b))x'_C(z|b) > \pi_z(z, s_C(z), x_C(z|b_2)) + \pi_x(z, s_C(z), x_C(z|b_2))x'_C(z|b_2) = U'_{PSE}(z|b_2)$. Consequently, the poor with $\widetilde{z} < z < \underline{z}(b)$ have higher utility in the PSE associated with $b > b_2$ than in that associated with $b = b_2$. But the same poor have lower utility in the PSE associated with the higher b than in SE because they have better jobs and overinvest less, in the latter than in the former.

Having established that $U_{SE}(z) < U_{PSE}(\underline{z}(b)|b) < U_{PSE}(\underline{z}(b)|b_2)$, the proof concerning rich and poor agents in the $z \geq \underline{z}(b)$ range parallels that given with regard to the rich in the $\widetilde{z} < z < \underline{z}(b)$ range.

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Fig 1. Student Loans Separating equilibrium ($b \geq b_3$) vs. Laissez Faire and First Best

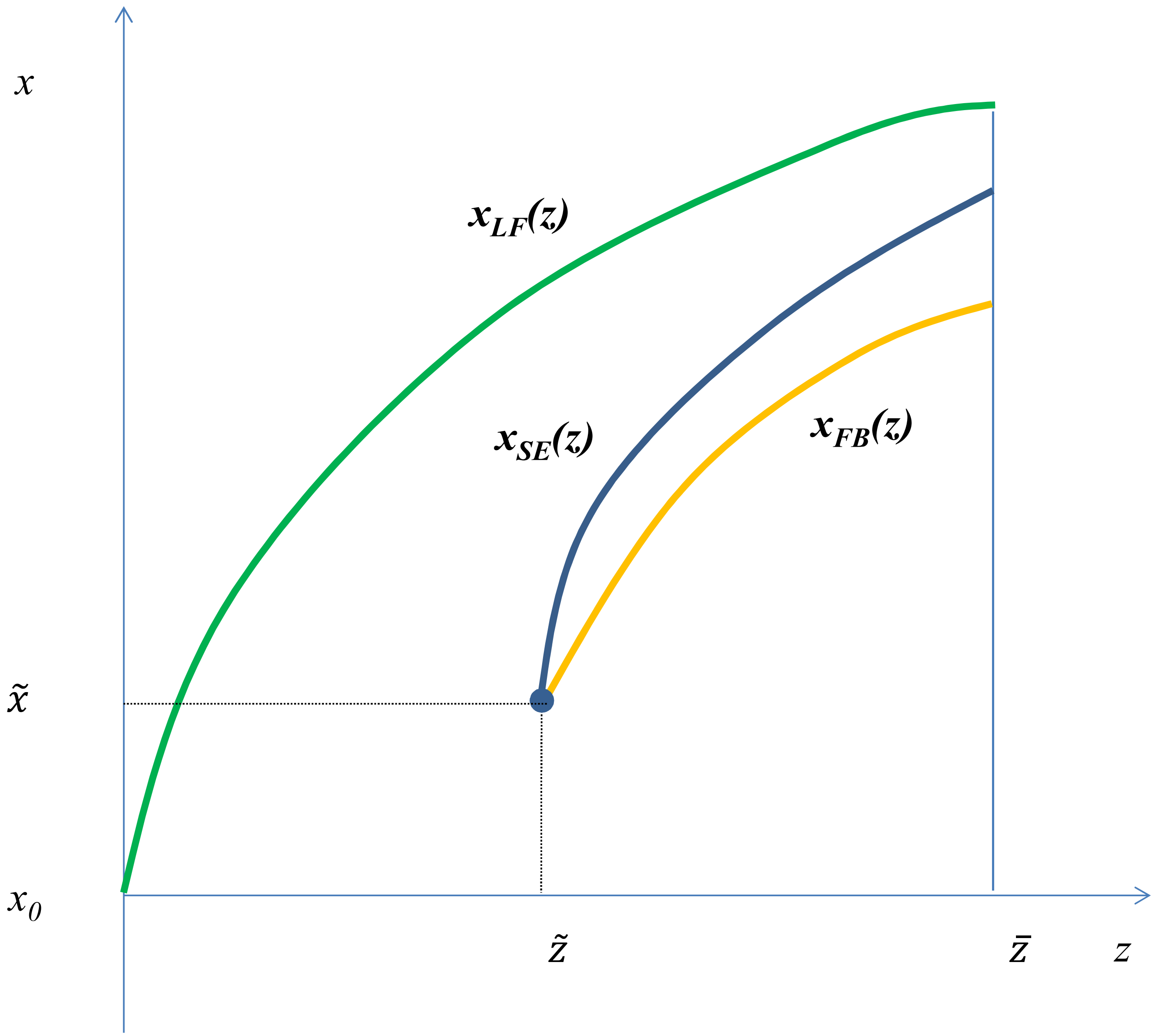
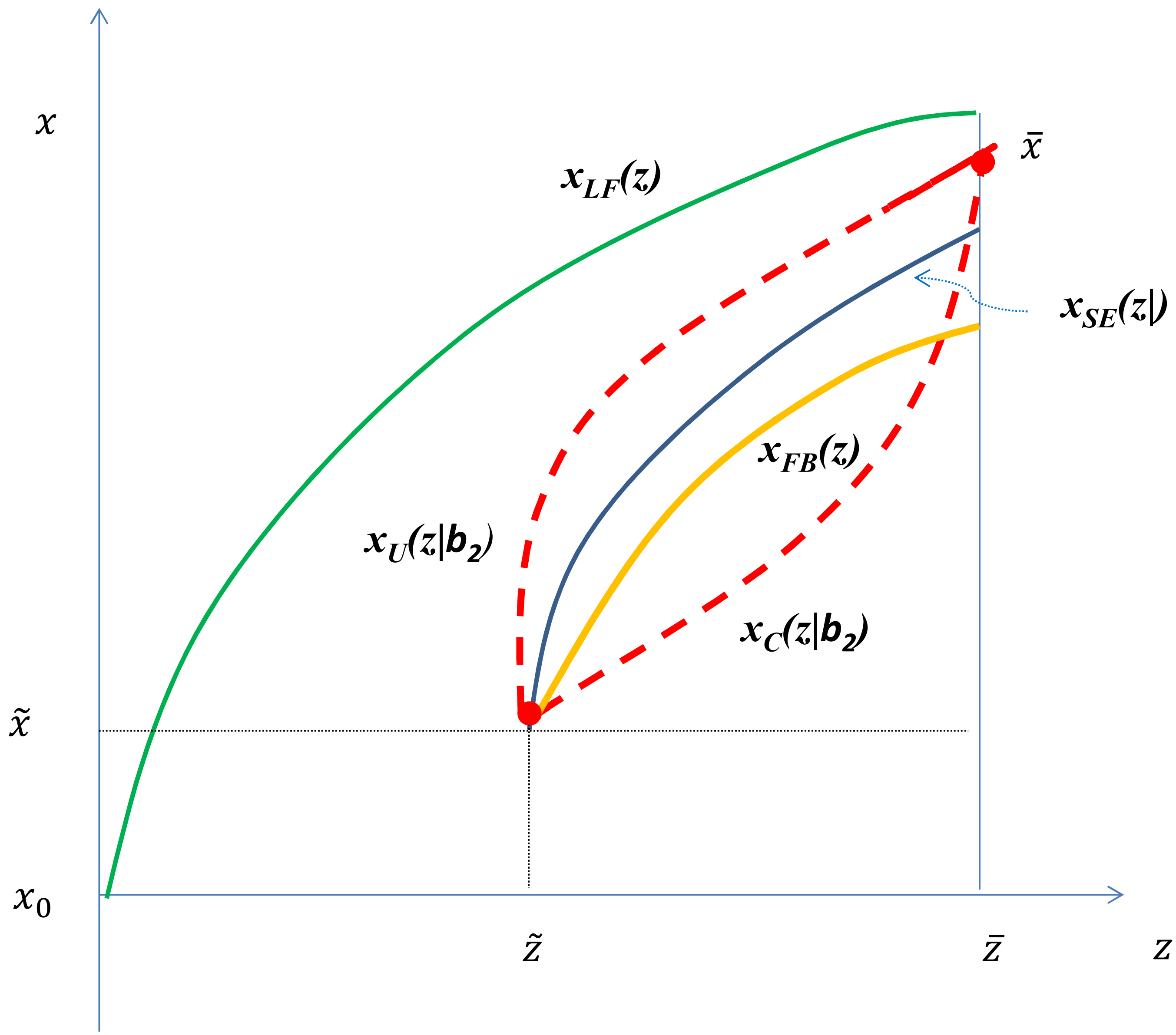


Fig. 2 Partially separating equilibrium with *b=b₂*



Fig, 3 Partially separating equilibrium with $b_3 > b > b_2$