# Spatial Price Competition under Kinked Transportation Cost

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**Abstract**

This thesis investigates a spatial Bertrand competition model in which the economy consist of two cites given by line segments with different unit transportation costs. It is shown that firms always choose locations that minimize the total costs of delivering products to consumers and the equilibrium market share depends on both relative transportation costs and relative size between two cities.

## 1 Introduction

Since the work of Hotelling (1929), a rich and diverse literature on spatial competition has emerged. Hoover (1936) analyzed spatial price discrimination by firms with fixed locations. He concluded that a firm serving a particular market point would be constrained in its local price by the marginal cost of service faced by other firms in serving that point. In situations where demand elasticity is 'not too high', the price at the market point is equal to the marginal cost of the firm with the next lowest marginal cost of serving the market point. This result was extended and reinforced later by Hurter and Lowe (1976a, b).

Hurter and Lederer (1985) developed model with two firms competing in location and price is analyzed for the case of spatial discrimination. Two games are compared. In one game, firms first choose locations and then quantity schedules; in the other, the final stage is choice of price schedules. Prices and transport costs are lower under Bertrand competition. They showed that the profits are higher under Cournot competition for low transport costs, but the reverse holds for transportation costs. They also showed that the aggregate welfare is higher in Bertrand competition case. In both games, firms locate in such a way as to minimize their own transport costs.

On the other hand, Hamilton et al. (1989) and Anderson and Neven (1991) showed that two firms agglomerate at the center of the linear market in the two-stage location-quantity game.

In these papers, the unit transportation cost is assumed to be proportional to the distance between the location of the firm and the market and common across the firms over the whole linear city. But, in reality, the transportation cost may be differ across firms or regions. A typical example is different toll fee by province in China. Or in a product variety problem, the transportation costs can be interpreted as the additional production costs of switching to other products. Technologies of some products are closely related to each other but others are not. This may suggest additional switching costs at some point on a linear market area.

This thesis extends the analysis by introducing a heterogeneous unit transportation cost across different segments of the linear city aiming at investigating how the gap of transportation costs affects the duopolistic location-price competition.

It is shown that the transportation cost gap together with the size difference between two provinces determines equilibrium of the spatial competition.

This thesis is organized as follows. Section 2 formulates the model. Section 3 presents the result of the model under basic assumptions. Section 4 endogenizes the unit transportation cost in a numerical example. Section 5 concludes the thesis. Proofs and calculations are presented in the Appendix.

# 2 Model

We assume a linear market area from 0 to 1 on a straight line as the Hotelling model did. And additionally we assume the whole market area is divided into two parts, say province A and B. Province A takes charge of the area between $[0, Y]$ , and province B takes charge of the area between $(Y, 1]$, where Y represents the border point.

On the demand side, a continuum of consumers are uniformly distributed over the market area and each of them purchases one unit of good from a firm providing a lower price. The utility level of a consumer at location $x$ is given by $u(x) = U - p(x)$ , where $U$ is a positive constant and $p(x)$ is the price at location $x \in [0, 1]$.

On the supply side, let two firms, denoted A and B, are selling a homogeneous product. Let $a$ (resp. $b$) be the distance between firm A's (resp. B's) location and regional border $Y$.There is regional restriction such that firm $i$ belonging to province $i$, can locate anywhere in their province but is not allowed to locate another province. Each of them is able to supply the whole market.

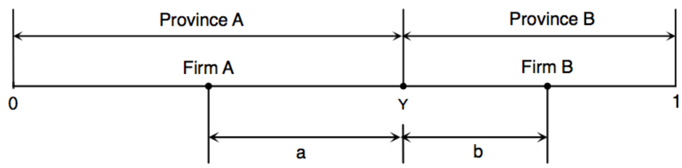


Figure 2.1: linear city structure

In order to highlight the impact of different transportation costs between provinces, we assume a shipping model such that firm $i$ bears the transportation cost and charge delivered price $p_i(x)$ to the consumer at $x$. In other words, the firm discriminates among consumers according to their location. To ship a unit of product from its location to market at $x$, firm $i$ should pay $t_i x$ amount of transportation cost, where $t_i$ is the unit transportation cost in province $i$ , which is constant in the same province, but differs in another province.

The marginal cost of production is normalized to zero for both firms without loss of generality. The total cost of firm $i$ to supply market $x$ is as follows

$$C_A(x) = \begin{cases} t_A(Y-a) - t_A x & when \quad x \in [0,\ Y-a] \\ t_A x - t_A(Y-a) & when \quad x \in (Y-a,\ Y] \\ t_B x - (t_B Y + t_A a) & otherwise \end{cases}$$

$$C_B(x) = \begin{cases} (t_B b + t_A Y) - t_A x & when \quad x \in [0,\ Y] \\ t_B(b+Y) - t_B x & when \quad x \in (Y,\ Y+b] \\ t_B x - t_B(Y+b) & otherwise \end{cases}$$

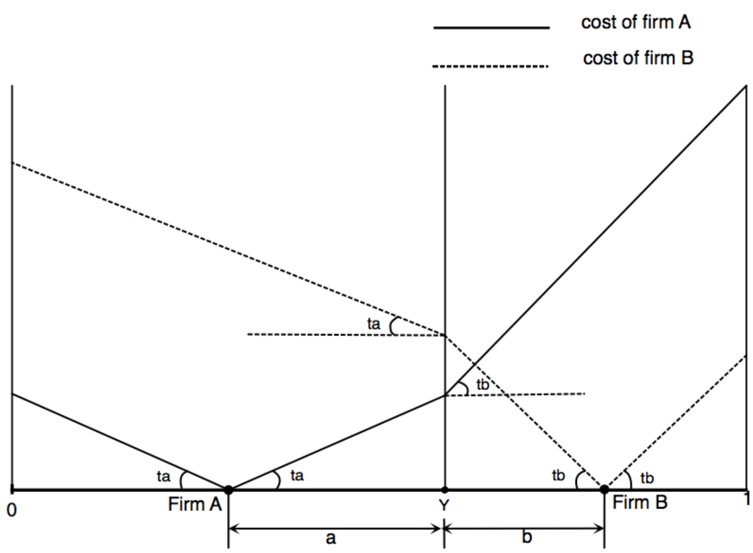


Figure 2.2: firms' delivery cost when $t_A < t_B$

Given border $Y$ and the marginal transportation costs of each province ( $t_A$ and $t_B$ ), duopolists engage in the following two-stage game. In the first stage, firms simultaneously decide how far they locate from the border in each province ($a$ and $b$ ). After observing the rival's location, in the second stage¸ each firm competes in terms of price. Each firm simultaneously chooses its price level at every location $x$ in the continuum [0,1] in oder to maximize its profit. The profit of firm $i$ at location $x$ is given by $\pi_i(x) = p_i(x) - t_i x$.

# 3 Equilibria

## 3.1 Equilibrium patterns

To find the subgame perfect Nash equilibrium, we solve the game by backward induction.

In the last stage, firms will decide their price schedules over the market area they supply. Because each firm delivers its product, a firm's price decision at a particular location has no effect on actions at other locations. We can break down the problem into subproblems at each location. First of all, at each point $x$, firms would not propose a price level lower than their marginal cost $C(x)$, otherwise, the profit at $x$ would be negative such that it is better to give up providing a product at location $x$. Then, if $C_i(x) < C_j(x)$ holds, firm $i$ can gain monopoly status through offering a price $p_i(x) \in [C_i(x), C_j(x))$. Obviously, as

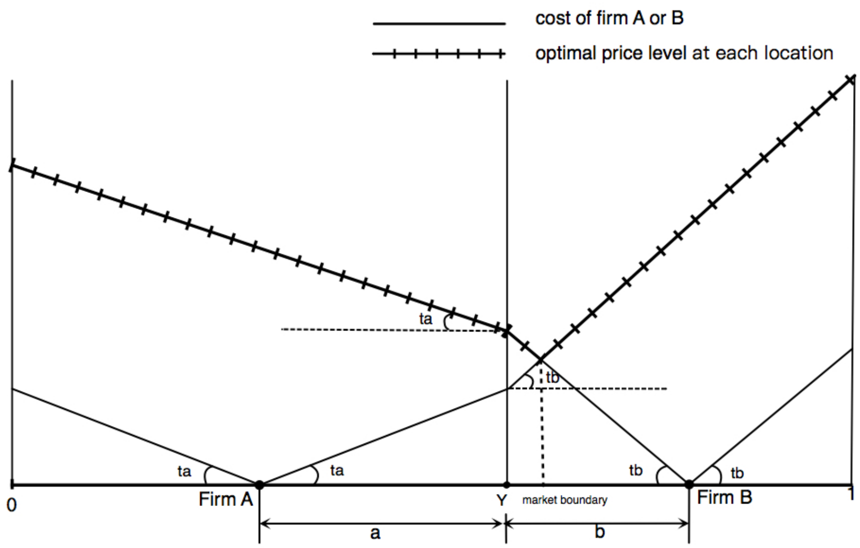


Figure 3.1: optimal price level at each market point $x$ given firms location fixed

long as they locate in different points, there always exist a market boundary $\bar{x}$, where the transportation costs of two firms are the same. That is, firm A supplies the market in the interval of $[0, \bar{x})$ with price $p_A(x) = C_B(x) - \epsilon$ , and firm B supplies the market in the interval of $(\bar{x}, 1]$ with price $p_B(x) = C_A(x) - \epsilon$, where $\epsilon$ is positive and sufficiently small.

Given the optimal price schedules, in the first stage, each firm chooses a location to maximize profits over its whole market area given the rival's location. Formally, a Nash location equilibrium is a pair $(a^*, b^*)$ such that $a^*$ maximizes $\int_0^{\bar{x}} [p_A(x) - C_A(x)]dx$, and $b^*$ maximizes $\int_{\bar{x}}^1 [p_B(x) - C_B(x)]dx$ for all $x \in [0, Y]$.

However, those objective functions are continuous but are not differentiable to firms' locations. Three possible cases of equilibria are represented as follows.

Equilibrium pattern-I: firm A and firm B split the market of province A and firm B become a monopolist in province B, i.e., $\bar{x} \in (0, Y)$.

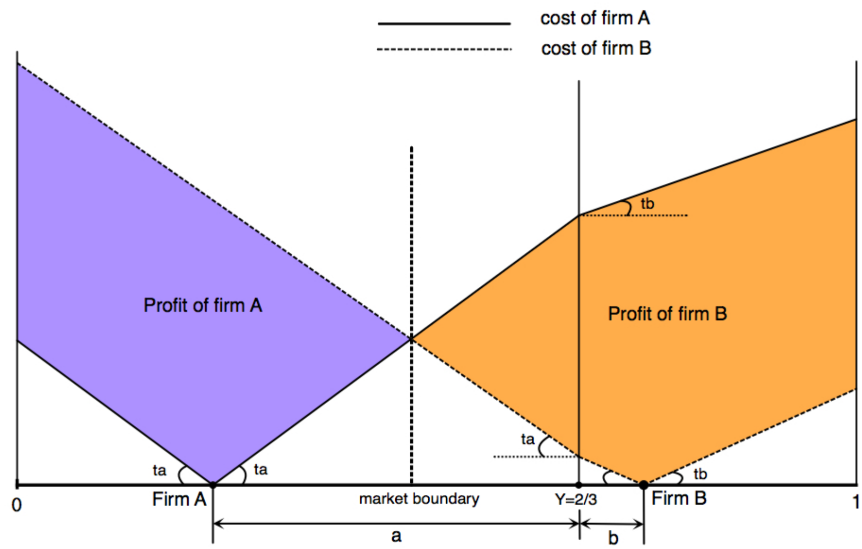


Figure 3.2: equilibrium pattern-I: firm A and firm B split the market of province A and firm B become a monopolist in province B

The profits of two firms are:

$$\pi_A = t_A[\bar{x}^2 - (Y-a)^2]$$
$$\pi_B = \tfrac{t_B}{2}[3Y\tfrac{t_A}{t_B}a + 4Y - 3Y^2 - 1 - b^2 - (1-Y-b)^2 - \tfrac{t_A}{t_B}\bar{x}a - (Y-\bar{x})b]$$

where $\bar{x}$ is the market boundary and in this case given by $\bar{x}$=Y-$\frac{1}{2}[a - \frac{t_B}{t_A}b]$.

Then, solving the profit maximization problem for each firm, the best responses are

$$a^*(b) = \frac{1}{3}\left(2Y - \frac{t_B}{t_A}b\right) \tag{3.1}$$

$$b^*(a) = \frac{2(1-Y)-a}{4-\frac{t_B}{t_A}} \tag{3.2}$$

If we solve simultaneous equations (3.1)-(3.2), the equilibrium locations are given by

$$a^* = \frac{4Y - \frac{t_B}{t_A}}{6-2\frac{t_B}{t_A}} = Y - \frac{\bar{x}}{2} \tag{3.3}$$

$$b^* = \frac{4(1-Y)-1}{6-2\frac{t_B}{t_A}} = \frac{1+\bar{x}}{2} - Y \tag{3.4}$$

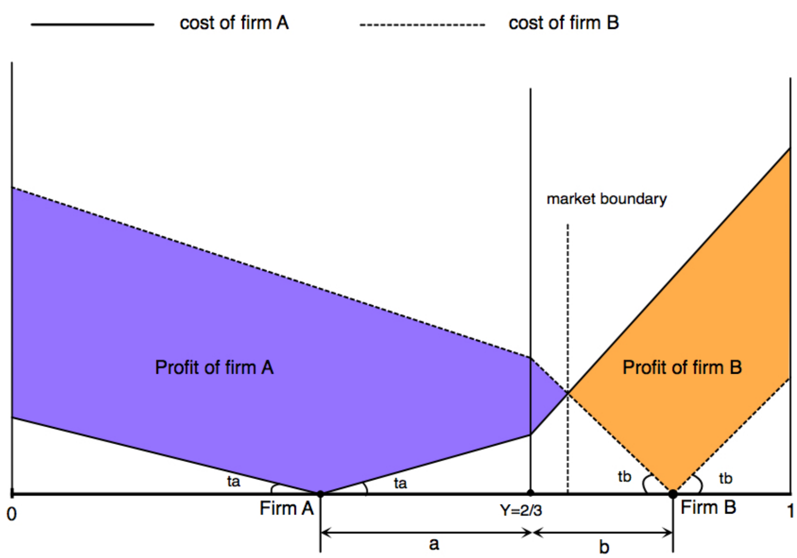


Figure 3.3: equilibrium pattern-II: firm A and firm B split the market of province B and firm A become a monopolist in province A

Equilibrium pattern-II: firm A and firm B split the market of province B and firm A become a monopolist in province A, i.e., $\bar{x} \in (Y, 1)$.

The profits are

$$\pi_A = \tfrac{1}{2}[(3t_A a + t_B b)Y - (t_A a - t_B b)\bar{x} - 2t_A a^2]$$
$$\pi_B = (t_A a + t_B b)\left[(1 - Y - b) + \tfrac{1}{4t_B}(t_A a + t_B b)\right]$$

where the market boundary $\bar{x}$ is given by $Y + \frac{1}{2}(b - \frac{t_A}{t_B}a)$.

Hence, the best response of firm A is

$$a^*(b) = \frac{2Y - b}{4 - \frac{t_A}{t_B}} \tag{3.5}$$

and that of firm B is

$$b^*(a) = \frac{1}{3}\left[2(1 - Y) - \frac{t_A}{t_B}a\right] \tag{3.6}$$

Similarly, solving equations (3.5) and (3.6), we obtain

$$a^* = \frac{4Y - 1}{6 - 2\frac{t_A}{t_B}} \tag{3.7}$$

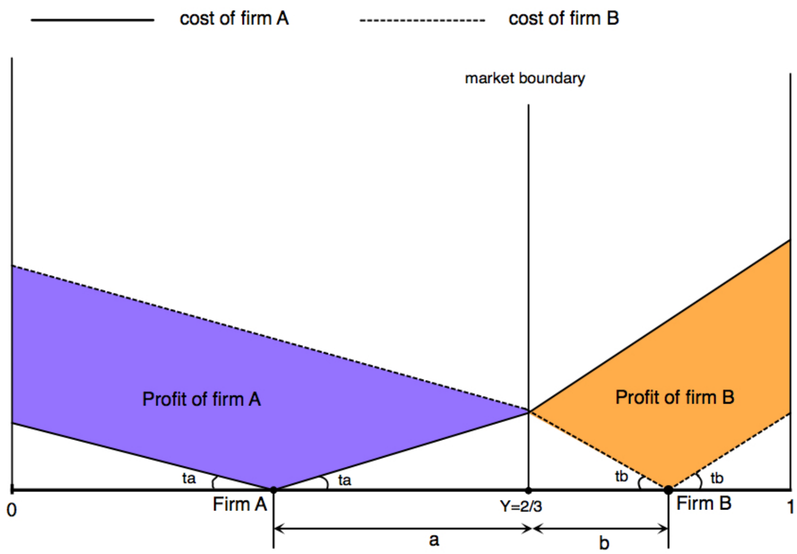


Figure 3.4: equilibrium pattern-III: each firm become a monopolist in each province

$$b^* = \frac{4(1-Y) - \frac{t_A}{t_B}}{6 - 2\frac{t_A}{t_B}} \tag{3.8}$$

Equilibrium pattern-III: each firm fully supplies the region each belongs to but not supplies the other one, i.e., $\bar{x} = Y$. We call it the separated market equilibrium. Since the objective function is continuous with respect to $a$ and $b$, this pattern can be included in either of the former two equilibria as a special case. For example, when $t_A = t_B$, and $Y = \frac{1}{2}$, the two firms compete under symmetric cost conditions so that the equilibrium locations are $a^* = b^* = \frac{1}{4}$.

Which kind of equilibrium occurs depends on the value of $Y$ and the ratio of unit transportation costs $\frac{t_A}{t_B}$. Furthermore, there are not only these three equilibria, but also varieties of corner solution equilibria in which at least one of the firms locates on point 0, $Y$ or 1 in equilibrium. However, such corner solutions happen under situations that the gap between two provinces are extremely large, namely, the cases in which the area of the larger region is 4 times larger than that of the smaller one, or the higher unit transportation cost is 3 times higher than that of the lower one. In this thesis, we only consider interior solutions by assuming $Y \in (\frac{1}{4}, \frac{3}{4})$ and $\frac{t_A}{t_B} \in (\frac{1}{3}, 3)$.

If define $t \equiv \frac{t_A}{t_B}$, it can be shown that equilibrium pattern-I occurs when

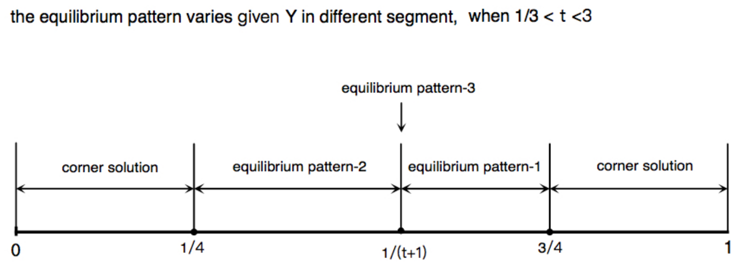


Figure 3.5: given transportation cost which equilibrium pattern occurs depends on where the border Y is

$Y \in (\frac{1}{t+1}, \frac{3}{4})$, and that equilibrium pattern-II occurs when $Y \in (\frac{1}{4}, \frac{1}{t+1})$. If $Y = \frac{1}{t+1}$, the equilibrium is pattern-III. Therefore¸ we can infer that even if there is a large province, as long as the unit transportation cost there is relatively low, the market will not be splitted. Namely, a separated market equilibrium occurs such that each firm becomes a monopolist in each province.

## 3.2 Equilibrium analysis

First of all, if we exclude corner solutions mentioned above, we can always show that firms locate on the midpoint of the market area they supply (i.e. $a^* = \frac{\bar{x}}{2}, b^* = \frac{1-\bar{x}}{2}$), which leads to a fact that the distance between the two firms will always be kept as $\frac{1}{2}$ in equilibrium. This can be verified by a straightforward calculation of adding $a^*$ and $b^*$ up. The underlying reason is that closer location leads to tougher price competition. Firms choose their locations as far apart as possible in order to relax price competition.

If we know the market boundary $\bar{x}$, we can easily compute the equilibrium locations of firms. Hence, to analyze the marginal effect of regional size on the location choice of firms ($\frac{\partial a}{\partial Y}, \frac{\partial b}{\partial Y}$), we only have to figure out how the market boundary $\bar{x}$ responds to the change in the regional size. A sensible result is, in any equilibrium, $\frac{\partial \bar{x}}{\partial Y} > 0$ holds, implying that the market share of firm A expands if the area of province A becomes larger, and of course that of firm B shrinks.

However, the profit change is not straightforward. This depends on the equilibrium market structure. The profit of firm A is monotonically increasing in $Y$ if firm A supplies market in province A only as illustrated in Figure 3.6. This is because as the area of province A enlarges, the market share of firm A becomes larger, and thus, firm A moves right in equilibrium. Then the relocation induces an increase in the transportation cost at markets on the left-hand side of firm A, which reduces firm A's profit. However, the profit increases because the following two gains dominates the profit loss. One is that the relocation

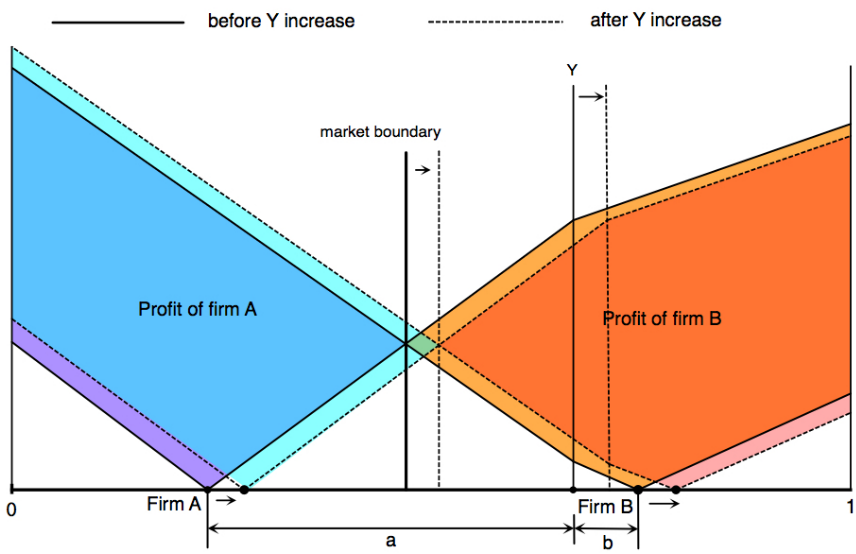


Figure 3.6: the profit of firm A is monotonically increasing in Y if firm A supplies market in province A only

causes a decrease in transportation cost at market on the right-hand side. And another is that the rival shrinks back such that it becomes more costly to supply the market area of firm A. This allows firm A to set a higher price in its market area.

If firm A supplies a part of province B in addition to the whole province A, then whether its profit increases or not depends on the value of $Y$ and the relative transportation cost between the two provinces. First of all, if border $Y$ moves right, both firm A and firm B move right in equilibrium. Similar to the former case, the relocation of firm A makes its transportation cost increase at the market on the left-hand side but decreases at the market on the right-hand side. However, the main difference is that firm B's shipping cost to the market in province A does not necessarily increase in pattern-II equilibrium as illustrated in Figure 3.7. And obviously, a decrease in firm B's shipping cost will damage firm A's profit since firm A has to lower its price schedule in each market in province A. Therefore, we cannot obtain a consistent conclusion on the effect of increasing $Y$ on firm A's profit in equilibrium pattern-II.

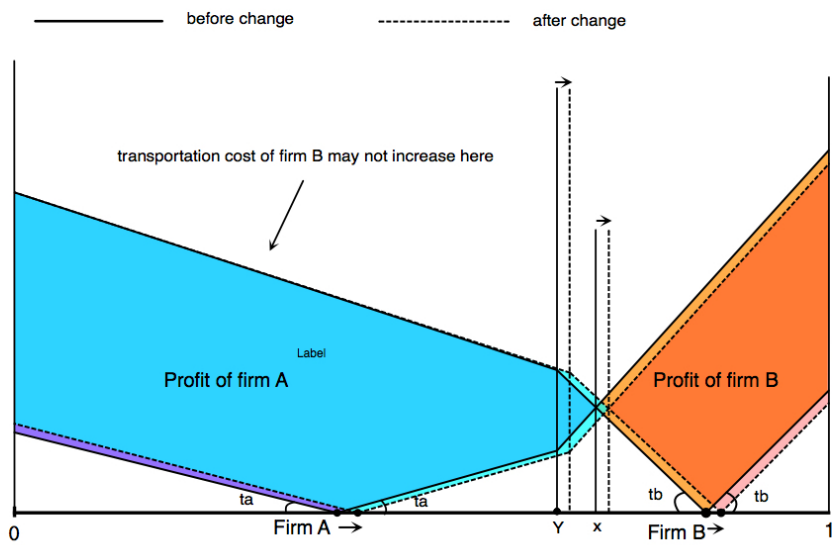


Figure 3.7: the profit of firm A may decrease in Y in equilibrium pattern-II

# 4 Endogenous transportation costs

So far, we treated the unit transportation costs in each province as given. In this section, we study how the optimal unit transportation cost is determined from a social welfare point of view. For this purpose, we introduce governments of two provinces to the game. Furthermore, we split the unit transportation cost into two components, $t_i = T + \tau_i$. The fuel cost $T$ are exogenous and fixed over the segment $[0, 1]$, and the toll fee $\tau_i$ $(i = A, B)$ is determined by the local government in province $i$. The total toll revenues in each province are attributed to each government.

Accordingly, a preceding game stage should be added to the game. In this stage, the local governments set $\tau_i$ simultaneously to maximize the social welfare of own province, which consists of three parts: the consumer's surplus, the firm's profit and the toll revenue of the government.

Here we consider a numerical case in which we set $Y$ at $\frac{2}{3}$. The social welfare of province A in equilibrium pattern-I is represented by the blue area in Figure 4.1. The first order condition for government A's maximization problem is given by

$$\frac{T + \tau_A}{T + \tau_B} = \frac{1}{2}$$

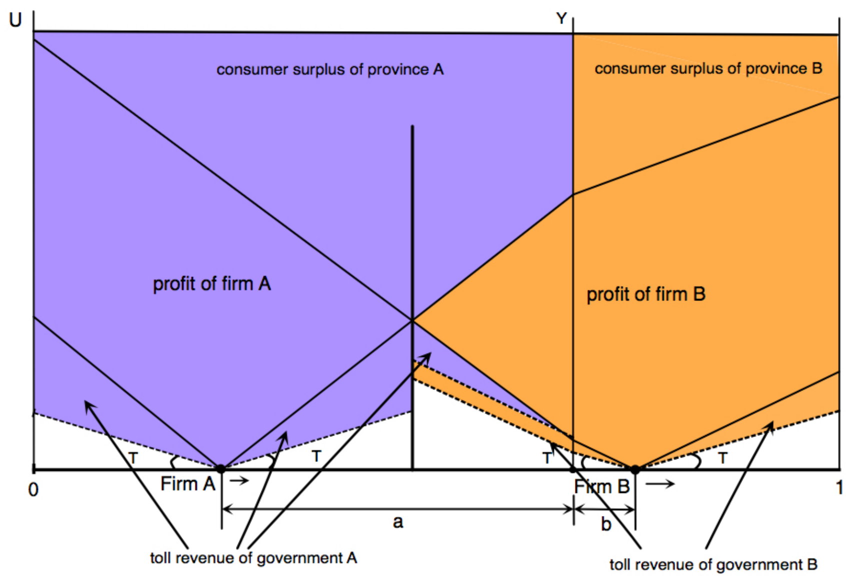


Figure 4.1: shape of social welfare in equilibrium pattern-I

That is exactly the same condition for the separated market equilibrium (pattern-III). In other words, given $\tau_B$ and $Y = \frac{2}{3}$, since the pattern-I equilibrium is more efficient for province A, the government of province A tends to set $\tau_A$ to ensure the equilibrium pattern-III instead of pattern-I.

Meanwhile, government B's best response is to set $\tau_B = 0$ for any value of $\tau_A$ in the case of the equilibrium pattern-I or III. Because in such equilibrium firm B is supplying a segment in province A, with a lower toll fee $\tau_B$ firm B can gain more market share in province A. This is to shift the market boundary $\bar{x}$ to the left.

Then, if we exclude the possibility of subsidy ($\tau_i < 0$), both of two government will set the toll fee equal to 0 in equilibrium.

# 5 Conclusions

A model of location-price competition between two firms who are confronted with kinked transportation cost has been presented. In the first stage, firms choose their locations under regional restriction. Then in the second stage, each firm sets the delivered price schedules to maximize its profit.

We showed the existence and necessary conditions of different kinds of equilibria. There are two main results.

1. Equilibria in such a model can be classified into 3 patterns based on how firms split the market area of two provinces. The necessary conditions for a paticular equilibrium to realize are determined by the relative transportation costs and relative sizes of porivences. A firm in a large province can get monopoly position, as long as the unit transportation cost of that province is lower than another province. On the contrary, markets in a province with high transportation cost are more likely to be supplied by outsiders.

2. In equilibrium locations, each firm's location minimizes the costs of serving its own market area, and the distance between the two locations is shown to be maintaned as $\frac{1}{2}$. The main reason is that in a spatial price competition model, firms do not agglomerate to avoid profit loss due to intense competition. This result is consistent with d'Aspremont, Gabszewicz, and Thisse (1979).

Finally, a limitation of the model is that firms are not able to choose which province they belong to. Thus, a possible extension of this model is to add another game stage in which firms simultaneously choose a province they locate in. Another possible extension is to reconstruct the model by assuming sequential entry of firms. Then, new patterns of equilibrium may emerge such as firms agglomerate in only one province.

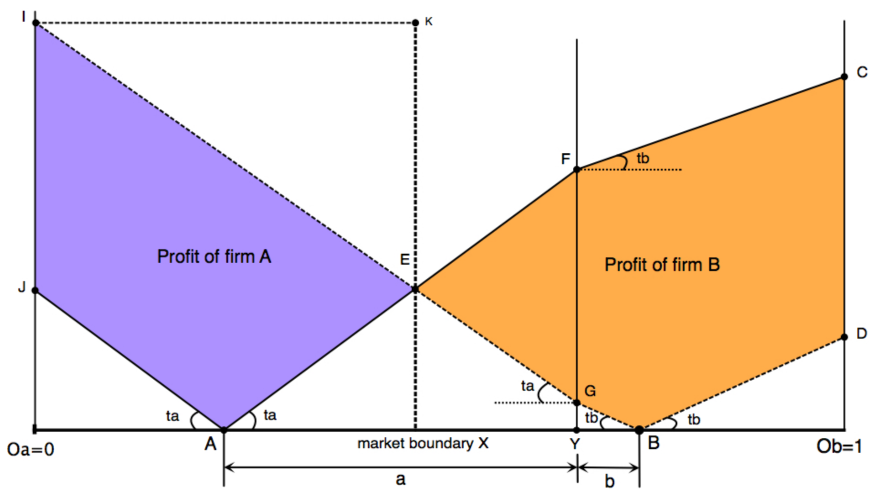


Figure 5.1:

## Appendix

### First order conditions of firms' profit maximization problem

For pattern-I, the profit of firm A can be represented by the area $IEAJ$ in Figure 5.1, which is equal to $O_aIKX - IKE - O_aJA - AEX$.

Then, we can easily compute the size of them as follows.

$$O_aIKX = [t_AX + \frac{1}{2}(t_Aa + t_Bb)]X$$
$$IKE = \frac{1}{2}X^2t_A$$
$$O_aJA = \frac{1}{2}(Y-a)^2t_A$$
$$AEX = \frac{1}{2}(a-Y+X)^2t_A$$

Since the height of point E can be expressed as $[a-(Y-X)]t_A$ or $t_Bb+(Y-X)t_A$. By equating them, we can derive the market boundary $X$ as X=Y-$\frac{1}{2}(a-\frac{t_B}{t_A}b)$.

The profit maximization problem of firm A is given by

$$\max_a \quad \pi_A = t_A[\bar{x}^2 - (Y-a)^2]$$
$$s.t. \quad \bar{x}=Y-\frac{1}{2}(a-\frac{t_B}{t_A}b)$$

The first order condition is given by

$$a = \frac{2Y - \frac{t_B}{t_A}b}{3}$$

Similarly, the profit maximization problem of firm B can be written as

$$\max_b \quad \pi_B = \frac{t_B}{2}[(3Y-\bar{x})\frac{t_A}{t_B}a-(2Y-1)^2-(1-Y-b)^2-(Y-\bar{x})b+Y^2-b^2]$$
$$s.t. \quad \bar{x}\text{=Y-}\frac{1}{2}(a-\frac{t_B}{t_A}b)$$

The first order condition is given by

$$b=\frac{2(1-Y)-a}{4-\frac{t_B}{t_A}}$$

For pattern-II, the profit maximization problem of firm A is given as

$$\max_a \quad \pi_A=\frac{1}{2}[(3t_Aa+t_Bb)Y-2t_Aa^2+(t_Bb-t_Aa)\bar{x}]$$
$$s.t. \quad \bar{x}=Y+\frac{1}{2}(b-\frac{t_A}{t_B}a)$$

The first order condition is given by

$$a=\frac{2Y-b}{4-\frac{t_A}{t_B}}$$

Similarly, the profit maximization problem of firm B is

$$\max_b \quad \pi_B=(t_Aa+t_Bb)[(1-Y-b)+\frac{1}{4t_B}(t_Aa+t_Bb)]$$
$$s.t. \quad \bar{x}=Y+\frac{1}{2}(b-\frac{t_A}{t_B}a)$$

The first order condition is given by

$$b=\frac{2(1-Y)-\frac{t_A}{t_B}a}{3}$$

**Necessary conditions for each equilibrium pattern**

Because the equilibrium locations given by equations (3.3) and (3.4), the necessary conditions for pattern-I are

$$\begin{array}{ccc} a^*>0 & \Rightarrow & Y>\frac{t_B}{4t_A} \\ b^*>0 & \Rightarrow & Y<\frac{3}{4} \\ \frac{t_A}{t_B}a^*>b^* & \Rightarrow & Y>\frac{1}{\frac{t_A}{t_B}+1} \end{array}$$

The last inequality comes form the fact that the transportation cost of firm A to supply point Y is higher than that of firm B.

If we restict $\frac{t_A}{t_B}>\frac{1}{3}$, the intersection of above conditions is $Y\in(\frac{1}{\frac{t_A}{t_B}+1},\frac{3}{4})$.

The same approach applies to equilibrium pattern-II. From the conditions, we have

$$\begin{array}{ccc} a^*>0 & \Rightarrow & Y>\frac{1}{4} \\ b^*>0 & \Rightarrow & Y<1-\frac{t_A}{4t_B} \\ \frac{t_A}{t_B}a^*<b^* & \Rightarrow & Y<\frac{1}{\frac{t_A}{t_B}+1} \end{array}$$

and the intersection will be $Y \in (\frac{1}{4}, \frac{1}{\frac{t_A}{t_B}+1})$.

Then it is easy to check that equilibrium pattern-III happens when $Y = \frac{1}{\frac{t_A}{t_B}+1}$.

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