

Network-motivated Lending Decisions*

Yoshiaki Ogura,^{a†} Ryo Okui,^{b‡} and Yukiko Umeno Saito^{c§}

^a*School of Political Science and Economics, Waseda University,
1-6-1 Nishi Waseda, Shinjuku-ku, Tokyo 169-8050, Japan*

^b*Department of Econometrics & OR, Vrije Universiteit Amsterdam,
De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands,
and Kyoto Institute of Economic Research, Kyoto University,
Yoshida Honmachi, Sakyo-ku, Kyoto 606-8501, Japan*

^c*Research Institute of Economy, Trade and Industry,
1-3-1, Kasumigaseki, Chiyoda-ku, Tokyo 100-8901, Japan*

April 2016

*This paper is a result of the research of the Study Group on Dynamics of Corporate Finance and Corporate Behavior at the Research Institute of Economy, Trade, and Industry (RIETI) in Tokyo, Japan. We are grateful for insightful comments by Kosuke Aoki, Salvatore Capasso, Kyota Eguchi, Shin'ichi Fukuda, Masaharu Hanasaki, Takeo Hoshi, Kaoru Hosono, Masami Imai, Hideshi Ito, Keiichiro Kobayashi, Teruyoshi Kobayashi, Yoshinori Kon, Hideaki Miyajima, Daisuke Miyakawa, Hisashi Nakamura, Makoto Nirei, Arito Ono, Hiroo Sasaki, Etsuro Shioji, Yasunobu Tomoda, Hirofumi Uchida, Greg Udell, Ken'ichi Udeda, Iichiro Uesugi, Yukihiro Yasuda, Alberto Zazzaro, and other participants in the workshops at Chuo University, the Development Bank of Japan, the Financial Service Agency Institute, Hitotsubashi University, Hokkaido University, Kobe University, Osaka University, RIETI, Waseda University, Yokohama National University, and University of British Columbia, and those in the sessions at the 5th Regional Finance Conference at Chuo University, Tokyo, Japan, the Western Economic Association International 10th Biennial Pacific Rim Conference at Keio University, Tokyo, Japan, and the 2015 Japanese Economic Association Fall Meeting at Sophia University, Tokyo, Japan. We gratefully acknowledge financial support, KAKENHI grants 23243050 and 26590051.

[†]E-mail address: yogura@waseda.jp.

[‡]E-mail address: okui.ryo.3@gmail.com, r.okui@vu.nl, okui@kier.kyoto-u.ac.jp.

[§]E-mail address: saito-yukiko@rieti.go.jp.

Abstract

We theoretically and empirically demonstrate that monopolistic or collusive banks will keep lending to a loss-making firm at an interest rate lower than the prime rate if the firm is located in an influential position in an inter-firm supply network. An influential firm generates a positive externality, and its exit damages the sales in the supply network. To internalize this externality, the profit-maximizing banks undertake such forbearance lending for an influential firm when the cost to support the loss-making influential firm can be recouped by imposing high interest on less influential firms. Our empirical study with a unique dataset containing information about inter-firm transactions and main banks provides evidence for such network-motivated lending decisions. This effect is more clearly observed at less credit-worthy firms whose main bank is a regional bank. Notably, we observe that such banks are often dominant lenders in the local loan market, and most of their clientele do not have direct access to the stock and bond market.

Keyword: supply network, influence coefficient, centrality, forbearance, bail-out, zombie.
JEL Classification: C55, D57, G21, G32, L13, L14

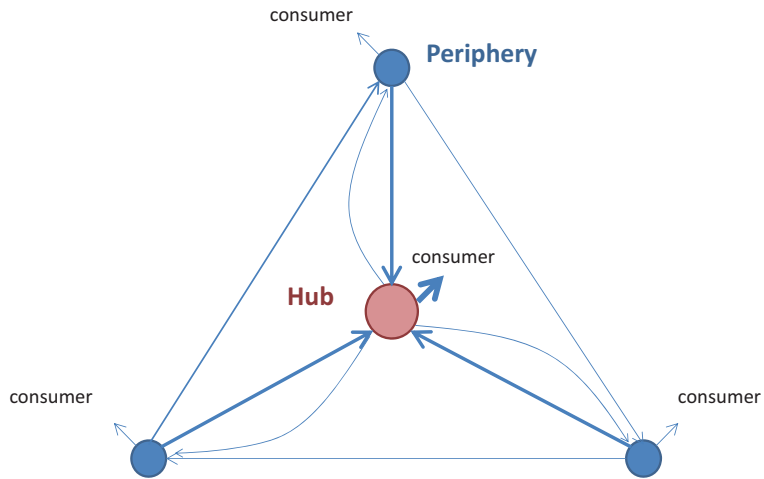
1 Introduction

In economic downturns, we frequently observe governments and financial institutions rescuing “too big to fail” and “too connected to fail” companies. For example, the U.S. government bailed out the country’s Big Three automakers by providing some 8 billion USD in the midst of the 2008 global financial crisis, despite heated controversy over this action. A news article reports, “The Big Three directly employ almost 250,000, [...], not counting the vast network of suppliers and dealers whose businesses are intertwined. In all, administration officials estimate that the failure of the U.S. auto makers would cost the economy more than one million jobs”.¹ It is also well recognized in the academic literature that Japanese major banks engaged in extensive *forbearance* or *zombie* lending in the 1990s (e.g., [Sekine et al., 2003](#); [Peek and Rosengren, 2005](#); [Caballero et al., 2008](#)). Forbearance or zombie lending refers to the behavior of a bank that keeps extending additional loans at a lower interest rate to under-performing or non-performing firms to which the bank has existing exposure.

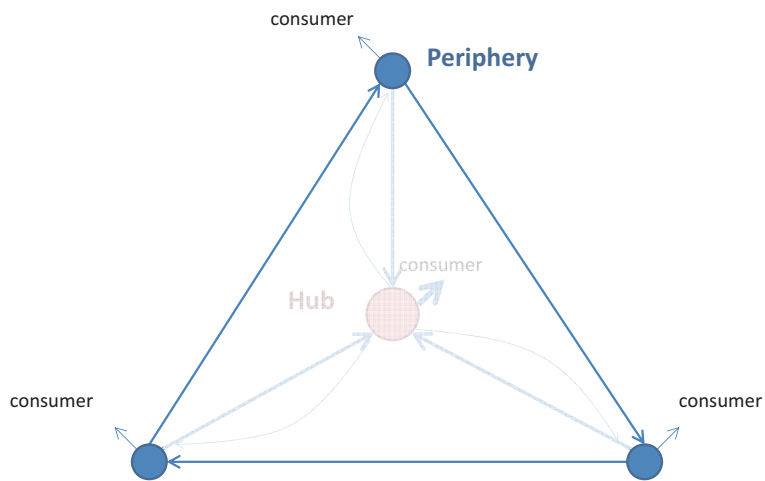
In this paper, we theoretically and empirically demonstrate the possibility that profit-maximizing banks undertake forbearance lending. We argue that some recipient firms are influential in a supply network in the sense that its existence creates positive externalities for the total sales and profit of the network. Even when that firm is loss-making, forbearance may enhance the total profit of the network when it internalizes the externalities caused by the existence of this firm. A profit-maximizing bank that is a dominant financier in a region or an industrial group may have an incentive to undertake forbearance in order to internalize this externality. In particular, the bank can increase its profit by supporting a loss-making influential firm and recouping the cost by imposing higher interests on the other firms that depend on sales to the influential firm. We conduct an empirical study that uses a unique dataset of inter-firm transactions matched with the main bank information of each firm. The empirical results are consistent with our theory.

¹“Detroit Gets Access To Bailout Funds,” Dec. 13, 2008, *Wall Street Journal*.

Figure 1: Supply network as a directed and weighted network
(Note) Each node is a hub or peripheral firm. The direction of each arrow represents the direction of product sales and the thickness indicates the amounts of sales.



(a) Keeping the hub open



(b) Closing the hub

To illustrate the economic problem that we try to address in this study, let us consider the following example, illustrated in Figure 1. There are two types of companies. One type of company is a large company that heavily depends on intermediate inputs from other companies. We call it a hub company (panel (a) of Figure 1). The other type is small companies that are not as dependent on intermediate goods but are major providers of intermediate inputs; we call them periphery firms. The direction of the arrows in the figure indicates the direction of the flow of products, and the thicknesses indicate the amount of sales. The negative impact of the closure of the hub company propagates throughout the network by reducing sales. This can even trigger a chain of closures of peripheral companies, as is shown in panel (b) of the figure. If the total profit of this network is positive despite that the hub firm is making losses, it can be efficient to keep supporting the hub firm afloat. A bank is willing to do so if it is a dominant financier for this network, and that it expects that it can increase its profit by supporting the loss-making hub company and recouping the cost by imposing higher interest on the peripheral companies.

To formalize and generalize the insights obtained above, we construct a theoretical model of a supply network. We formulate the inter-firm supply network as an incomplete directed and weighted network of sales among oligopolistic firms offering differentiated products that are intermediate inputs as well as final products. Under the assumption that each firm has to depend on external financing to pay its fixed costs, we identify the conditions under which a monopolistic bank or collusive banks will strategically extend a loan to a loss-making but influential company at a rate lower than the prime rate.

In our inter-firm supply network, each firm has both positive and negative externalities. The positive externality comes from the fact that the production of a firm induces demand for intermediate goods from suppliers, which propagates through the supply network. The extent of this externality is measured by the influence coefficient, which is well-known in industry-level input-output analysis and has been extended to firm-level analysis by [Acemoglu et al. \(2012\)](#). The negative externality is the business-stealing effect ([Mankiw and Whinston, 1986](#)) or the congestion effect ([Caballero et al., 2008](#)). The model shows that a monopolistic bank or collusive banks will engage in forbearance lending when the former outweighs the latter.

Our empirical study shows evidence for the network-motivated lending decisions. We com-

pute the influence coefficients of firms from a unique dataset, the *TSR Corporate Relationship Database*, which provides information about inter-firm transactions in Japan. We then examine the relationship between influence coefficients and interest payments.

TSR Corporate Relationship Database contains records of inter-firm transactions among about 650,000 firms in Japan. By using the data set above, we write the directed adjacency matrix of firms for each network. Our theoretical model indicates a spatial autoregressive model of sales with the weighting matrix being approximated by the adjacency matrix. We estimate the resulting spatial autoregressive model and compute the influence coefficients by using the supply networks consisting of firms that share the same main bank under the assumption that a main bank observes the supply network among its borrowers.

We then match the estimated influence coefficients with corporate financial data and examine the effects of the influence of a firm on its paid interest rates. We find that the interest rates paid by influential firms are lower than those of the other firms in a distress, and significantly so in both the statistical and economic senses for those whose main bank is a regional bank. This result holds after controlling for relevant observable characteristics of each company and unobservable fixed effect of each bank. We note that, in Japan, regional banks are often a dominant lender in a more concentrated local loan market, and most of their borrowers are bank-dependent companies that are not publicly traded in the stock exchange. Thus, regional banks are likely to observe a entire part of the supply network within a regional market and able to recoup the cost for a forbearance from peripheral bank-dependent firms. These empirical results are consistent with our theory of network-motivated lending decisions.

This paper contributes to the literature in various ways. First, and most importantly, this paper points out the importance of firms' influence in a supply network for the capital costs. In particular, we show that forbearance can occur as a result of internalizing externalities that run through the supply network. Second, our empirical study that uses both inter-firm transaction data, including small businesses, and the main bank information is new in the corporate finance literature. This dataset enables us to observe the relationship between inter-firm supply networks and bank lending decisions, which has never been directly examined in the literature. This paper illustrates the usefulness of this type of data in the context of corporate finance.

This paper also highlights some policy implications. We theoretically clarify the conditions

under which forbearance lending is profitable for a bank. Our analysis additionally suggests that information about the influence coefficients of companies is useful for a bank to quickly judge whether forbearance is profitable.

Related literature

Our study proposes a novel viewpoint for academic discussion about the mechanism by which a bank will choose to engage in forbearance or zombie lending. Existing theories on this subject have focused on the one-to-one bank–firm relationship (e.g., [Dewatripont and Maskin, 1995](#); [Berglöf and Roland, 1997](#)) or the gambling for resurrection (e.g., [Bruche and Llobet, 2014](#)). The novelty of our explanation is that it focuses on the fact that a bank lends to thousands of firms interconnected through a supply network and can take into account the network effect.

Many researchers investigated the zombie lending in Japan in the 1990s and early 2000s. For example, [Sekine et al. \(2003\)](#), [Peek and Rosengren \(2005\)](#) and [Caballero et al. \(2008\)](#) find it in listed large companies, whereas [Sakai et al. \(2010\)](#) and [Hamao et al. \(2012\)](#) find that it was less likely for non-listed small companies.² Our theory and empirical finding are consistent with these existing studies, given that listed companies are more likely to be a hub company within a network. These exiting empirical studies mainly consider the existence of zombie lending and the consequences of such lending, while we consider the mechanism that leads to zombie lending or forbearance. Since our dataset does not cover the periods that these researches targeted, our empirical results do not necessarily explain the events in the 1990s and early 2000s in Japan. Nonetheless, our theory is useful in understanding the motivation behind zombie lending generally. It is said that zombie lending in the lost decade in Japan occurred because the government induced the banks to extend loans to loss-making large companies.³ Our theory indicates that such a controversial policy may be of interest for governments because large firms are likely to have high influence in supply networks, at least, in the short run.

Analytical models of the supply network have already been proposed in the macroeconomics

²Several researches also report a similar result in Japanese articles; for example [Fukuda et al. \(2007\)](#), [Hosono \(2008\)](#), and [Ogawa \(2008\)](#).

³For example, about a loss-making giant retailer in Japan, “Daiei (pronounced die-ay) has been on artificial support for several years: despite sales that have fallen 25 percent since 1999, it has managed to borrow trillions more yen from its banks.... Daiei’s major banks have been encouraged by politicians and government officials to keep it going at all cost. [... omitted ...] [The] Minister of Economy, Trade and Industry announced earlier this year that Daiei, which employs 96,000 people, was simply too big to be allowed to fail.” (“They’re Alive! They’re Alive! Not!; Japan Hesitates to Put an End to its ‘Zombie’ Businesses,” October 25, 2002, *New York Times*).

literature. However, studies within macroeconomics tend to focus on the impact of sector-specific shocks on aggregate variables. The list of related macroeconomic studies includes [Long and Plosser \(1983\)](#), and [Horvath \(2000\)](#) about the simulation of aggregate variables, [Battiston et al. \(2007\)](#) about firm distributions, [Acemoglu et al. \(2012\)](#) about the possibility of an idiosyncratic shock to be transformed into an aggregate shock, and [Bigio and La'O \(2013\)](#) about the aggregate impact of collateral constraints.⁴ The present paper does not examine the effect of supply networks on aggregate macroeconomic variables; instead, it focuses on the behavior of a bank facing a supply network. Another important difference from the macroeconomic strand in the literature is that research in macroeconomics typically use the industry-level input-output table to empirically examine the network effect. We use a dataset of inter-firm transactions instead.

Analytical models to formulate the chain of inter-firm trade credit are also abundant, but the focus of such models is on aggregate volatility ([Giesecke and Weber, 2006](#)) and on the possibility of contagion through the chain of trade credit ([Kiyotaki and Moore, 1997](#); [Boissay, 2006](#)). Among empirical studies on this point, several studies find significant effects of a liquidity shock, a bankruptcy, and other negative shocks of a firm, as reflected in stock returns, the yield spreads of bonds, and the supply of trade credits of its direct suppliers, customers, or competitors ([Hertzel et al., 2008](#); [Boissay and Gropp, 2013](#); [Chen et al., 2013](#); [Calvalho et al., 2014](#)). However, none of these studies looks at the influence of a shock on the response of a bank. Moreover, we consider the higher-order influence of a shock as measured by the influence coefficient while existing studies, in contrast, examine only first-order impacts.

Network models have been popular in the banking literature. However, to the best of our knowledge, our paper is the first to examine the role of supply network on the lending decision of banks. The existing studies in the banking literature focus mainly on inter-bank networks and concern how a shock to a single bank is propagated to the entire network of banks (see [Acemoglu et al., 2015](#); [Allen and Gale, 2000](#); [Billio et al., 2012](#); [Castiglionesi and Navarro, 2011](#); [Eisenberg and Noe, 2001](#); [Elliot et al., 2014](#); [Gai et al., 2011](#); [Nier et al., 2007](#), among others), and a possible autonomous bailout to avoid such a contagion (see [Leitner, 2005](#); [Rogers and Veraart, 2013](#)). While the present paper also belongs to the literature of banking, we consider networks of non-financial firms, not networks of banks.

⁴[Dupor \(1999\)](#), in contrast, argues that the effect of the input-output structure on the aggregates might be minor.

Our paper is also related to development finance. For example, [Morck \(2009\)](#) discusses the possibility that business groups and financial transactions within such groups, which are often found in developing economies, contribute to economic development by complementing the underdeveloped financial sector. Business groups can be interpreted as a network of group firms and group financial institutions. The role of business groups as a financial institution is empirically examined in [Gopalan et al. \(2007\)](#), and [Almeida et al. \(2015\)](#). Our analysis, based on inter-firm supply networks, is readily applicable to this context.

Organization of the paper

The remaining part of this paper is organized as follows. We introduce a model of a supply network and derive the equilibrium in [Section 2](#). Our main theoretical results are in [Section 3](#). There, we show the possibility and the welfare implications of network-motivated forbearance. We specify the hypotheses to be tested in our empirical study in [Section 4](#). [Section 5](#) explains the dataset used in our empirical analysis. The estimation of the influence coefficient is explained in [Section 6](#). [Section 7](#) presents the results of our empirical analysis of the relationship between interest rate and influence coefficient. A robustness check of our empirical study is described in [Section 8](#). [Section 9](#) discusses the welfare implications of our results. [Section 10](#) is the conclusion and comments on possible future research topics.

2 Theoretical analysis

In this section, we present a theoretical model of a supply network with the financial sector to show a network-motivated forbearance by a profit-maximizing bank.

2.1 Setup

Our model is a version of a one-period oligopoly model with a banking sector. We assume that production technology requires intermediate goods and that this creates the supply network. We also assume that firms require fixed costs to keep operating. The role of the bank is to finance the fixed costs.

The basic setup of the model is as follows. There are H households, indexed by h ($= 1, \dots, H$); n firms, indexed by i ($= 1, \dots, n$); and a monopolistic bank. There are $(n + 1)$ goods in the economy. These goods are differentiated. Good 0 is a pure intermediate good (i.e., not

a consumption good). It is supplied from outside of the supply network in the model. Good i , for $0 < i \leq n$, is produced by firm i . The aggregate quantity and the nominal price of good i are denoted by x_i and p_i , respectively. The nominal price of good 0, p_0 , is exogenously given. We assume that the banking sector is a monopoly for the time being to elucidate the logic of our model in the simplest manner. An economy with multiple banks is discussed later. More details of each economic agent are described below.

We assume the decision timing and the settlement scheme are as follows.

Time 1. The bank decides on which firms or an outside opportunity it will invest in. Those firms whose fixed costs are not financed by the bank exit from the economy. Which firms keep operating becomes common knowledge.

Time 2. Firms decide the production level and their demand for input. At the same time, households decide the product demand. Each firm sets the price of its product in response to the demand function. All payments are settled by trade credit.

Time 3. Firm profits and outside investment outcomes are realized. All trade credits are cleared. The bank captures all the profits of the firms and obtains the return of the outside investment opportunity.

Let e_i ($i = 1, 2, \dots, n$) equal one if firm i operates and zero otherwise. Good j is not supplied if $e_j = 0$ for some j . We assume $e_0 = 1$ always, that is, the input supply from the outside of the network always exists.

2.1.1 Households

Households are utility maximizing, with the CES utility function of household h given by

$$U_h = \left(\sum_{j=1}^n c_{hj}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1, \quad (1)$$

where c_{hj} is the consumption of good j by household h , and θ is the elasticity of substitution.

We assume that households are symmetric so that the value of θ does not vary across households.

The budget constraint of household h is

$$\sum_{j=1}^n c_{hj} p_j \leq R_h, \quad (2)$$

where R_h is the nominal income of household h , which is exogenously given. Our model is a partial equilibrium model at this point. The households also faces the availability constraint:

$$(1 - e_j)c_{hj} = 0. \quad (3)$$

This constraint implies that the demand has to be zero if the good is not supplied whereas it can be any non-negative value otherwise.

2.1.2 Firms

Firms are profit maximizing and have the CES production function

$$x_i = \left(\sum_{j=0}^n w_{ij}^{\frac{1}{\theta}} x_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1, \quad (4)$$

where θ is the elasticity of input substitution, w_{ij} is the “technological importance” of the input j for the production of firm i , and x_{ij} is the quantity of the input quantity from firm j into the production of firm i . We assume that the elasticity of input substitution θ is equal to θ in the utility function to simplify the analysis.

$\{\{w_{ij}\}_{i=1}^n\}_{j=0}^n$ is the most important determinant of the supply network. We assume that the supply network is rigid in the sense that w_{ij} does not change, even if a supplier of an intermediate product ends up closing.⁵ This assumption also means that there is no free-entry of new firms into the market. We assume that $\{\{w_{ij}\}_{i=1}^n\}_{j=0}^n$ satisfies $0 \leq w_{ij} \leq 1, \forall i, j; w_{ii} = 0$; and $0 < \sum_{j=0}^n w_{ij} \leq 1$.⁶ Note that since some firms may fail to operate, the supply network the firms actually face is $\{\{e_j w_{ij}\}_{i=1}^n\}_{j=0}^n$.

Each firm has to expend an exogenously given fixed cost F_i ($i = 1, 2, \dots, n$) to operate. The value of F_i is fixed in the real term but varies by firm. Each firm has to rely on external financing for this expense.⁷ Those firms whose fixed costs are financed always operate even

⁵This assumption is more plausible in industries where the designs of input products or the contents of services are highly customized, information- or skill-intensive, and specific to each user. Automobiles, construction, and some types of retailers/wholesalers dealing in custom-made items are of this type.

⁶The value of $\sum_{j=0}^n w_{ij}$ represents the productivity level. Note also that the condition $\sum_{j=0}^n w_{ij} \leq 1$ guarantees the existence of the equilibrium price vector (12) as well as the existence of an equilibrium.

⁷This assumption is employed to make the analysis tractable and is a simplified version of the situation in which external finance is necessary. A literal interpretation of this assumption may be the upfront payment for obtaining or renting fixed assets. An alternative example may be from the case of the bail-out of the Big Three car companies. One of their most important problems was the cost of pensions owed to their retired employees. This pension cost must be financed in order for them to operate. However, it is not directly related to their output-level. For example, a news article reports that the Big Three car makers “saddled themselves with a cost structure in flush times that has proved unsustainable as their market share has eroded. They have made great strides of late in shedding legacy pension and health-care costs, but they took decades to do so.” (“The Next Bailout: Detroit,” August 22, 2008, *Wall Street Journal*).

when the operation results in a loss; presumably, this is because of the perfect monitoring by the bank or some private benefit for firm owners. Payments for fixed costs exit the economy.

2.1.3 Financial market

We assume that the financial market is a monopoly. Each household is endowed with a numéraire κ/H . They deposit this at the monopolistic bank because it is the only agent that permits investment in firms and other investment opportunities. The bank chooses which firms will be financed so as to maximize the total return from these investments. The bank captures the entire profits (and the losses as well) of firms that are financed by the bank. The bank also has an outside opportunity that can yield the real risk-free rate of return $\rho (> 0)$, which we call the prime rate. We assume that $\kappa \geq \sum_{i=1}^n F_i$, that is, the total deposit or loanable funds of the bank exceeds the total demand for funds.

2.2 Equilibrium

We derive the market equilibrium backwardly. That is, we first derive the equilibrium outcome in the input and product markets and then derive the optimal investment decision by the bank.

2.2.1 Final demand for each product

The final demand function for each product is determined by the utility maximization of the households after observing the list of operating firms $\{e_i\}_{i=0}^n$. By solving the maximization problem of the utility in (1) with respect to c_{hi} ($i = 1, 2, \dots, n$) under the budget constraint (2) and the availability constraint (3), we obtain the demand function of household h :

$$c_{hi} = \frac{e_i R_h}{p_c} \cdot \left(\frac{p_c}{p_i} \right)^\theta, \text{ where } p_c \equiv \left(\sum_{j=1}^n e_j p_j^{1-\theta} \right)^{\frac{1}{1-\theta}}. \quad (5)$$

Note that p_c is understood as the consumer price index (CPI).

2.2.2 Intermediate demand for each product

The demand function for each product as an intermediate good is derived by solving the cost minimization problem of firms given their production levels. The problem for firm i is

$$\min_{\{x_j\}_{j=0}^n} \sum_{j=0}^n p_j x_{ij}, \quad \text{s.t., } x_i = \left(\sum_{j=0}^n w_{ij}^{\frac{1}{\theta}} x_{ij}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \text{ and } (1 - e_j)x_{ij} = 0,$$

where the second constraint is the availability constraint. The usual cost minimization gives the demand of firm i for good j :

$$x_{ij} = \left(\frac{p^i}{p_j}\right)^\theta e_j w_{ji} x_i, \text{ where } p^i \equiv \left(\sum_{j=0}^n e_j w_{ij} p_j^{1-\theta}\right)^{\frac{1}{1-\theta}}, \quad (6)$$

for $j = 0, 1, \dots, n$. We note that p^i is the average cost of producing good i because

$$\sum_{j=0}^n p_j x_{ij} = p^i x_i. \quad (7)$$

Similarly to CPI, p^i is understood as the input price index (IPI).

2.2.3 Profit maximization by each firm

Each firm sets its price so as to maximize its profit since each firm produces a differentiated product. We assume that each firm ignores the impact of its pricing strategy on the price indexes (CPI, IPI): thus, firm i maximizes profit under the assumption that $\partial p_c / (\partial p_i) = 0$ and $\partial p^j / (\partial p_i) = 0$.

The profit maximization problem for firm i is

$$\max_{p_i} (p_i - p^i) x_i, \quad (8)$$

under the demand function for firm i ,

$$x_i = \sum_{h=1}^H c_{hi} + \sum_{j=1}^n x_{ji} = \frac{\sum_{h=1}^H R_h}{p_c} \cdot \left(\frac{p_c}{p_i}\right)^\theta + \sum_{j=1}^n \left(\frac{p^j}{p_i}\right)^\theta e_j w_{ji} x_j. \quad (9)$$

The first term and the second term in the demand function are the final demand (5) and the intermediate demand (6), respectively.

Solving the profit maximization problem, we obtain the price level and the profit of firm i . The first-order condition provides the price level:

$$p_i = \frac{\theta}{\theta - 1} p^i, \quad (10)$$

for each i ($= 1, 2, \dots, n$). Without loss of generality, we assume that (10) holds for firm i with $e_i = 0$. The coefficient $\theta / (\theta - 1)$ is the mark-up. The (nominal) profit of firm i is

$$\pi_i = \frac{p_i x_i}{\theta}. \quad (11)$$

2.2.4 Equilibrium outcomes of the product market

We now describe the equilibrium level of price, sales, and household income in the product market. These equilibrium outcomes are uniquely determined.

The equilibrium price level is obtained by solving the system of simultaneous equations characterized by (10) (raising both sides to the power $1 - \theta$) and the definition of IPI. Note that the system is linear on $p_i^{1-\theta}$. Therefore, a simple matrix calculation gives

$$\mathbf{p}_\theta = \left\{ \mathbf{I} - \left(\frac{\theta - 1}{\theta} \right)^{\theta-1} \mathbf{W}\mathbf{E} \right\}^{-1} \mathbf{w}_0 p_0^{1-\theta}, \quad (12)$$

where $\mathbf{p}_\theta \equiv (p_1^{1-\theta}, p_2^{1-\theta}, \dots, p_n^{1-\theta})'$, \mathbf{I}_n is the $n \times n$ identity matrix, $\mathbf{w}_0 \equiv (w_{10}, w_{20}, \dots, w_{n0})'$, \mathbf{W} is the $n \times n$ matrix whose (i, j) element is equal to w_{ij} .⁸, and \mathbf{E} is a diagonal matrix whose i th diagonal element is e_i . The price level is thus uniquely determined.

Next, we derive the sales in the equilibrium. The consumption level is determined by (5). Let $c_i = \sum_{h=1}^H c_{hi}$ be the total consumption of good i . Multiplying both sides of (9) by $p_i e_i$ gives the vector of total sales:

$$\mathbf{s} = \mathbf{f} + \mathbf{Q}\mathbf{s}, \quad (13)$$

where \mathbf{s} (total sales) $\equiv (e_1 p_1 x_1, e_2 p_2 x_2, \dots, e_n p_n x_n)'$; \mathbf{Q} is the $n \times n$ matrix whose (i, j) element is $q_{ij} \equiv e_i w_{ji} p_i^{1-\theta} p_j^\theta / p_j$; and \mathbf{f} (sales to consumers) $\equiv (e_1 p_1 c_1, e_2 p_2 c_2, \dots, e_n p_n c_n)'$. By the assumptions on w_{ij} and the definition of p^i , the matrix $\mathbf{I} - \mathbf{Q}$ is invertible.⁹ Therefore, the sales vector is uniquely determined and is written as

$$\mathbf{s} = (\mathbf{I} - \mathbf{Q})^{-1} \mathbf{f} = \sum_{k=0}^{\infty} \mathbf{Q}^k \mathbf{f}. \quad (14)$$

We note that the sales vector \mathbf{s} is conceptually similar to the Bonacich centrality (Bonacich, 1987).

⁸Note that the matrix inverse in (12) is well-defined because $((\theta - 1)/\theta)^{\theta-1} < 1$ and the largest eigenvalue of \mathbf{W} is less than 1 by the assumption that $0 < \sum_{j=0}^n w_{ij} \leq 1$.

⁹Let $\|\cdot\|_1$ denote the norm by Bowker (1947), defined as $\|\mathbf{Q}\|_1 \equiv \max_j \sum_{i=1}^n \|Q_{ij}\|$. Let λ be an eigenvalue of \mathbf{Q} . It is known that $\|\lambda\| \leq \|\mathbf{Q}\|$. $\|\mathbf{Q}\|_1 < 1$ holds because

$$\begin{aligned} \sum_{i=1}^n q_{ij} &= e_i (p_j^{1-\theta} - w_{j0} p_0^{1-\theta}) \frac{p_j^\theta}{p_j} \\ &= \frac{e_i p_j}{p_j} \left(1 - \frac{w_{j0} p_0^{1-\theta}}{\sum_{i=0}^n w_{ji} p_i^{1-\theta}} \right). \end{aligned}$$

The first term, which is the inverse of the mark-up rate, is smaller than one by (10). The second term is also smaller than 1. It follows that $\|\lambda\| \leq \|\mathbf{Q}\|_1 < 1$.

Thus, the equilibrium in the product market is uniquely determined according to the unique price vector (12). The price level determines the sales by (5), and (14).

2.2.5 Financial market

Lastly, we discuss the equilibrium of the financial market. The profit maximization problem of the monopolistic bank is to determine which firms to be financed. Since the profit of firm i in real terms is $x_i p_i / (\theta p_c)$ and the opportunity cost of financing the fixed costs of firm i is $(1 + \rho)F_i$, the bank's profit maximization problem is

$$\max_{\{e_i\}_{i=1}^n} \sum_{i=1}^n e_i \left\{ \frac{p_i x_i}{\theta p_c} - (1 + \rho)F_i \right\}. \quad (15)$$

Let $\{e_i^*\}_{i=1}^n$ be the solution to this problem. We note that since the set of values that $\{e_i\}_{i=1}^n$ can take is finite, the bank's problem has a solution, although the solution is not guaranteed to be unique.

3 Network-motivated Forbearance for an Influential Firm

Given the above equilibrium, we now discuss the main result of this paper: the possibility that a profit maximizing bank may rationally undertake forbearance. We also show that firms with a strong influence on the aggregate profit through the supply network are likely to be the target of forbearance. We first define the influence coefficients of firms.

3.1 Influence vector

The concept of influence coefficient is useful in analyzing rational forbearance. The influence vector \mathbf{v} is defined as

$$\mathbf{v}' \equiv \mathbf{1}'(\mathbf{I} - \mathbf{Q})^{-1} = \mathbf{1}' \sum_{k=0}^{\infty} \mathbf{Q}^k, \quad (16)$$

where $\mathbf{1}$ is the $n \times 1$ vector of ones. The i th element of \mathbf{v} , denoted by v_i , is called the *influence coefficient* of firm i . The vector \mathbf{v} is a modified version of *the influence vector* (i.e., the vector of the influence coefficients) proposed by [Acemoglu et al. \(2012\)](#).

The influence coefficient represents the influence of firm i on the aggregate profit. This observation comes from the fact that, by (11), the aggregate profit is θ^{-1} times

$$\mathbf{1}'\mathbf{s} = \mathbf{1}'(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{f} = \mathbf{v}'\mathbf{f}. \quad (17)$$

In words, the influence coefficient v_i indicates the magnitude of the influence on the total network sales of the change in the sales of firm i to households, that is,

$$v_i = \frac{\Delta \text{Total sales of the network}}{\Delta \text{Sales of firm } i \text{ to households}}.$$

Higher values of v_i indicate that a negative shock to the final sales of firm i is more damaging to the aggregate profit than a negative shock to a firm j with $v_j < v_i$. It is important to recognize that the influence coefficient takes into account not only the first-order impact (i.e., the impact to the adjacent neighbors) but also all the higher-order impacts. Note that the matrix $\mathbf{I} - \mathbf{Q}$ is conceptually the same as the Leontief matrix in the input–output analysis and the logic of input–output analysis applies here. Specifically, the first-order impact of an increase in the sales of firm i to households on the total sales of firm j is determined by the factor of the (j, i) element of \mathbf{Q} ; the second-order impact is determined by the factor of the (j, i) element of \mathbf{Q}^2 ; and so forth.

3.2 Rational forbearance

We now discuss the possibility that the profit maximizing monopolistic bank may undertake forbearance for a loss-making but influential firm. Our argument is based on externalities working through the supply network. By the same logic as in Leontief input–output analysis, the level of sales to households of a firm with a higher influence coefficient has a greater impact on the levels of sales of the other firms and, thereby, on the total profit of the network. The monopolistic bank can take this positive externality into account to maximize the total profit. The concept of the influence vector \mathbf{v} is useful as a way to characterize the conditions in which this positive externality is large.

We first define forbearance in the context of our model.

Definition 1 (Forbearance) *We say a bank undertakes **forbearance** if it extends a loan to firm z despite that firm’s real economic profit being negative; namely,*

$$e_z = 1 \text{ and } \frac{x_z p_z}{\theta p_c} - (1 + \rho)F_z < 0. \quad (18)$$

We assume that the real profit is deflated by CPI p_c since households are the owner of firms and banks.

We then discuss the condition under which forbearance occurs. Let $\{\tilde{e}_i\}_{i=1}^n$ be the solution to (15) with the additional constraint $e_z = 0$. We indicate the equilibrium outcomes of x_i , p_i , and p_c under $\{e_i^*\}_{i=1}^n$ by the superscript $*$ and those under $\{\tilde{e}_i\}_{i=1}^n$ by a tilde. The condition for firm z to be financed, that is, the condition for forbearance to be provided to firm z , is

$$\sum_{i=1}^n e_i^* \left(\frac{x_i^* p_i^*}{\theta p_c^*} - (1 + \rho) F_i \right) \geq \sum_{i=1}^n \tilde{e}_i \left(\frac{\tilde{x}_i \tilde{p}_i}{\theta \tilde{p}_c} - (1 + \rho) F_i \right).$$

Using the influence vector, the condition can be written as

$$\sum_{i=1}^n e_i^* \left(\frac{v_i^* p_i^* c_i^*}{\theta p_c^*} - (1 + \rho) F_i \right) \geq \sum_{i=1}^n \tilde{e}_i \left(\frac{\tilde{v}_i \tilde{p}_i \tilde{c}_i}{\theta \tilde{p}_c} - (1 + \rho) F_i \right), \quad (19)$$

by (17). Rearranging this inequality so as to separate the terms of firm z and those of other firms, we obtain

$$\underbrace{\frac{v_z^* p_z^* c_z^* - p_z^* x_z^*}{\theta p_c^*}}_{\text{influence-coefficient effect}} \quad (20)$$

influence-coefficient effect

$$+ \underbrace{\sum_{i \neq z} \left[e_i^* \left(\frac{v_i^* p_i^* c_i^*}{\theta p_c^*} - (1 + \rho) F_i \right) - \tilde{e}_i \left(\frac{\tilde{v}_i \tilde{p}_i \tilde{c}_i}{\theta \tilde{p}_c} - (1 + \rho) F_i \right) \right]}_{\text{business-stealing effect/influence-enhancing effect by firm } z} \quad (21)$$

business-stealing effect/influence-enhancing effect by firm z

$$> \underbrace{- \left(\frac{p_z^* x_z^*}{\theta p_c^*} - (1 + \rho) F_z \right)}_{\text{direct cost to support firm } z} > 0. \quad (22)$$

direct cost to support firm z

The left-hand side of the inequality captures the externalities to the system-wide profit from keeping firm z open. The right-hand side (22) is the direct cost to keep firm z open.

The term in (20) is the positive externality of firm z to the other firms. This term is always positive so long as at least two firms including firm z and another firm that supplies firm z operate at the optimum for the bank. The higher the influence coefficient v_z^* of firm z at the optimum is, the larger the propagation effect to the sales and profits of the other firms, and so the more beneficial it is for the bank to support firm z .

The term in (21) is the effect of the existence of firm z on the influence of the other firms. A part of this effect is referred to as the business-stealing effect (Mankiw and Whinston, 1986) or the congestion effect (Caballero et al., 2008). If the business-stealing effect of firm z is dominant, this part is negative. In contrast, this term could be positive if the existence of firm z increases the influence coefficient of the other firms by improving their connectivity. A more precise explanation is given in Appendix 2.

If the bank undertakes network-motivated forbearance, then the bank clearly requires an interest rate less than the prime rate ρ , and perhaps even a negative rate. The interest rate for firm z is

$$\frac{x_z^* p_z^*}{\theta p_c^* F_z} - 1. \quad (23)$$

Under the condition (18), this interest rate is less than ρ . This point supports the empirical strategy of using the information about whether a firm borrows at a rate below the prime rate as an indicator for the existence of forbearance lending, following Caballero et al. (2008). Under the conditions of (20), (21), and (22), the cost of extending a loan to a loss-making firm at a rate less than the prime rate is covered by the higher interest payments from loans to other firms profiting by selling their products to the loss-making firm.

Proposition 1 (Network-motivated Forbearance) *The monopolistic bank can maximize its profit by undertaking forbearance for firm z when the inequality consisting of (20), (21), and (22) holds. The interest rate of a loan to firm z is below the prime rate, and that of at least a loan to the other firm is over the prime rate.*

The first term in (20) suggests that firm z is more likely to be the target of forbearance if it is more “influential” than the others; that is, when v_z^* is larger than v_i^* ($i \neq z$). This also implies that the larger the level of sales to households by firm z (i.e., $p_z^* c_z^*$), the more likely the firm is to be the target of forbearance. This conjecture is not obvious under the complicated interdependence of the two terms (20) and (21) through the equilibrium price (12). The next proposition shows that this conjecture is true if the bank ignores the impact of the forbearance to the product prices and the products are less substitutable. The proof is given in Appendix 1.

Proposition 2 *For a given set of operating firms, $S = \{i | e_i = 1, i \neq z\}$, the monopolistic bank is more likely to provide a loan to firm z with a larger influence coefficient v_z if θ is sufficiently close to one, and the bank assumes that the impacts of its loan supply decision to the equilibrium product prices are negligible.*

3.3 Closure of a profitable firm

The bank may also take a strategy opposite to forbearance and decline to supply funds to a firm that is individually profitable but less influential. The explicit condition for this case to emerge

is the opposite side of the condition (20), (21), and (22). That is, it occurs when

$$\begin{aligned} & \frac{v_z^* p_z^* c_z^* - p_z^* x_z^*}{\theta p_c^*} + \sum_{i \neq z} \left[e_i^* \left(\frac{v_i^* p_i^* c_i^*}{\theta p_c^*} - (1 + \rho) F_i \right) - \tilde{e}_i \left(\frac{\tilde{v}_i \tilde{p}_i \tilde{c}_i}{\theta \tilde{p}_c} - (1 + \rho) F_i \right) \right] \\ & < - \left(\frac{p_z^* x_z^*}{\theta p_c^*} - (1 + \rho) F_z \right) \leq 0. \end{aligned} \quad (24)$$

This condition holds when the business stealing effect dominates the positive externality. In other words, if the profit of firm z mainly comes from stealing the sales of the other firms, then the bank will refuse to finance the fixed costs of firm z to maximize its profit even though firm z is individually profitable. As before, because social welfare and bank profit coincide, it is welfare-improving to close a profitable firm when (24) holds.

Proposition 3 (Closure of a profitable firm) *The monopolistic bank can maximize its profit by refusing financing and closing firm z if the inequality (24) holds.*

3.4 Numerical example

To illustrate the above propositions, we present a numerical example of 4 firms that supply inputs with each other in a network similar to Figure 1. We assume

$$\mathbf{W} = \begin{pmatrix} 0 & .3 & .3 & .3 \\ .1 & 0 & .1 & 0 \\ .1 & 0 & 0 & .1 \\ .1 & .1 & 0 & 0 \end{pmatrix} \quad \mathbf{w}_0 = \begin{pmatrix} .1 \\ .8 \\ .8 \\ .8 \end{pmatrix} \quad \mathbf{F} = \begin{pmatrix} 5.2 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$\theta = 4$, $\rho = 0.1$, $p_0 = 1$, and $R = 100$. Firm 1 depends on the inputs from the other firms more than the others, and its fixed cost is much larger than the others. The other firms are symmetric. The influence coefficient of Firm 1 is 1.6069 and those of others are 1.0588. The monopolistic bank can achieve the maximum profit of 31.90 by financing the fixed costs of all these firms and making them operate despite that the loans to Firm 1 incurs a loss of 0.0513. The required spread that have to be obtained from the other firms to support the loss making firm 1 is 1.71%. The bank can collect this additional premium from the other firms since each of them can earn a profit of 10.65. This case is an example for Proposition 1.

Firm 1 will not be supported if the influence coefficient is lower. If we replace \mathbf{W} and \mathbf{w}_0 as follows,

$$\mathbf{W} = \begin{pmatrix} 0 & .1 & .1 & .1 \\ .1 & 0 & .1 & 0 \\ .1 & 0 & 0 & .1 \\ .1 & .1 & 0 & 0 \end{pmatrix} \quad \mathbf{w}_0 = \begin{pmatrix} .7 \\ .8 \\ .8 \\ .8 \end{pmatrix},$$

we obtain lower influence coefficient of Firm 1, 1.1088, since firm 1 becomes much less dependent on the inputs from the other firms. Those of the other firms are now 1.0664. The best portfolio for the monopolistic bank is to lend to all firms except Firm 1 despite that Firm 1 can earn a positive profit of 4.1028 if it operates. This case is an example for Proposition 3.

3.5 Decentralized financial market

In this section, we examine the case in which the financial market is decentralized. So far, we have considered the extreme case of a monopolistic bank to keep the exposition simpler. However, the financial market in the reality is more decentralized. We show the possibility that forbearance lending can happen in the decentralized financial market in a form of tacit collusion or coalition. We also conjecture that so-called relationship banking, which yields monopolistic power to the relational lender, generates the possibility for lenders to recoup the costs of forbearance lending.

3.5.1 Tacit coalition

When the financial market is oligopolistic, banks may be able to collude and engage in forbearance that is profitable and improves welfare.

We consider an arrangement where each of competing banks holds a share of the optimal portfolio that the monopolistic bank would have. If one of them declines to extend a loan to an influential but unprofitable firm z , this can lead to the closure of firm z and a resulting cascade of firm closures. If each bank profit is larger in the former case than in the latter, then each bank has an incentive to keep the coalition intact.

More precisely, suppose that bank b purchases the share s_{bi} of firm i by giving the numéraire of $s_{bi}F_i$, where $\sum_b s_{bi} = 1$, $0 < s_{bi} \leq 1$, $\forall b, i$ (i.e., no short-selling), and $\sum_i s_{bi}F_i \leq \kappa_b$ where κ_b is the amount of numéraires deposited at bank b (liquidity constraint of banks). The product market structure is the same as the case for monopolistic financial market. After a production cycle, the bank obtains $s_{bi}\pi_i$, where π_i is defined as in (11). We denote bank b 's share of the optimal portfolio of the monopolistic bank by s_b^* .

We examine the Nash equilibrium of the game played by the banks. We consider the case where bank k deviates from this optimal portfolio s_k^* while the other banks hold the optimal portfolio s_b^* ($b \neq k$), and examine the conditions under which this is unprofitable for bank k .

This deviated portfolio of bank k , denoted by \hat{s}_{ki} , is

$$\hat{s}_{ki} \equiv \operatorname{argmax}_{s_{ki}} \sum_i e_i(s_{ki}, s_b^*) \left(\frac{p_i(s_{ki}, s_b^*) x_i(s_{ki}, s_b^*)}{\theta p_c(s_{ki}, s_b^*)} - (1 + \rho) F_i \right) s_{ki}.$$

If the deviation by bank k results in the closure of firm z , then the necessary and sufficient condition for bank k not to deviate from the optimal portfolio s_k^* is

$$\left[\sum_i e_i^* \left(\frac{x_i^* p_i^*}{\theta p_c^*} - (1 + \rho) F_i \right) \right] s_k^* \geq \sum_i \hat{e}_i \left(\frac{\hat{x}_i \hat{p}_i}{\theta \hat{p}_c} - (1 + \rho) F_i \right) \hat{s}_{ki},$$

where the superscript $*$ indicates the values at the optimum of the monopolistic bank as defined before, and the hat indicates the values at the deviated portfolio where bank k chooses $\{\hat{s}_{ki}\}_i$ and bank $b(\neq k)$ chooses s_b^* . We can rewrite this condition by using the influence coefficient,

$$\begin{aligned} s_k^* \left(\frac{v_z^* p_z^* c_z^*}{\theta p_c^*} - \frac{p_z^* x_z^*}{\theta p_c^*} \right) + \sum_{i \neq z} \left\{ e_i^* s_k^* \cdot \frac{v_i^* p_i^* c_i^*}{\theta p_c^*} - \hat{e}_i \hat{s}_{ki} \cdot \frac{\hat{v}_i \hat{p}_i \hat{c}_i}{\theta \hat{p}_c} \right\} - (1 + \rho) \sum_{i \neq z} (e_i^* s_k^* - \hat{e}_i \hat{s}_{ki}) F_i \\ \geq -s_k^* \left(\frac{p_z^* x_z^*}{\theta p_c^*} - (1 + \rho) F_z \right) \geq 0, \quad (25) \end{aligned}$$

where \hat{v}_i is the influence vector under the deviated portfolio. From this inequality, we can conclude again that firm z is more likely to be collusively supported when v_z^* is sufficiently higher than v_i^* ($i \neq z$). This result is qualitatively the same as what we obtain in the case of a monopolistic financial market.

3.5.2 Relational lending

The other possible situation in which rational forbearance happens in an oligopolistic financial market is related to so-called relational lending. Suppose that a bank has a monopolistic power over some firms. In this case, the bank may have an incentive to undertake forbearance for a firm that influences the profits of those firms that are under relational lending with the bank.

A bank has a monopolistic power over some firms, for example, when that bank has a long-term relationship with those firms and has some informational advantages. It is widely recognized that a bank can earn a quasi-rent by maintaining lending relationships, by achieving the information advantage over rival banks, or by providing firm-specific value-adding services by making use of this information advantage (Sharpe, 1990; Rajan, 1992; Boot and Thakor, 2000). Many empirical studies provide evidence supportive of this possibility in the financing of small businesses (Degryse and van Cayseele, 2000; Ioannidou and Ongena, 2010), which are presumably peripheral in the supply network described here.

There is a possibility that the bank engages in network-motivated forbearance for a firm that has a strong influence on the profits of firms under relationship banking with the bank. Suppose that forbearance increases the profits of those small firms that are under relational banking with the bank. Since the bank has monopoly power over those firms, it can extract the benefits of forbearance from those small firms. If the excess return of the bank from relational lending to small businesses is large enough and the bank covers a large enough part of the financing of the supply network that is connected to an influential firm, then the bank has an ability to recoup the loss from forbearance toward an influential firm. In this way, forbearance is possible even when banks do not collude.

4 Hypothesis setting for the empirical study

Our model analysis provides us with several hypotheses that can be statistically tested on the relationship between the supply network and bank’s lending decision. In this section, we present the hypotheses that are tested empirically in our study.

Propositions 1 and 2 indicate that a firm with a high influence coefficient is likely to obtain forbearance. In our model, forbearance is interpreted as lending to a distressed firm at an interest rate lower than the prime rate.¹⁰ We test the following hypothesis.

Hypothesis 1 *The influence coefficient has a negative impact on the interest rate paid by a less credit-worthy firm.*

This is our main hypothesis. To test Hypothesis 1, we estimate the following interest rate equation:

$$rate_i = b_0 + b_1 \cdot \ln(v_i) + b_2 \cdot score_i + b_3 \cdot \ln(v_i) \times score_i + \mathbf{b}_4' \mathbf{X}_i + \nu_m + \epsilon_i, \quad (26)$$

¹⁰ We focus on interest rate in our analysis because other measures of bank’s support may not be suitable for our analysis. For example, the effect of the influence coefficient on loan amount is ambiguous. Banks’ support through loan amounts could take two forms. One is to keep lending at a lower rate, and the other is to discharge a firm from its obligation on overdue debts. Indeed, about 5.9 trillion JPY are reserved for possible loan losses out of 389 trillion JPY (1.5%) loans in total, and, additionally, about 398.5 billion JPY of loans (0.1%) were written off in 2006 in Japan (the author calculated these figures from the non-consolidated financial statements of major banks—including Mitsubishi UFJ, Sumitomo Mitsui, Mizuho, Risona, and Saitama Risona—and regional banks—including those who are members of the second association of regional banks. These lists are available from the website of the Japanese Bankers’ Association). If an influential firm is more likely to be supported by a bank, then the change in loans outstanding should have a positive correlation with the influence coefficient in the former case, while it is negative in the latter case. Thus, the test using changes in loan amounts is expected to produce ambiguous results. In contrast, it is expected that the actual interest payment for loans have a negative correlation with the influence coefficient in either of the above cases.

where $rate_i$ is the interest rate for a loan to firm i , $ln(v_i)$ is the natural logarithm of the influence coefficient of firm i , $score_i$ is the credit score used to measure the credit-worthiness of firm i (with higher scores meaning more credit-worthy), \mathbf{X}_i is the column vector of control variables for firm i , ν_m is the fixed effect of each main bank, ϵ_i is the error term, and bs are coefficients to be estimated. Hypothesis 1 predicts that b_1 is negative, and b_3 is positive.

We also examine whether regional banks are more likely to engage in network-motivated forbearance.

Hypothesis 2 *The effect of the influence coefficient is larger for firms whose main bank is a regional bank.*

This hypothesis is established by the observation that regional banks in Japan are often a single dominant lender in a regional lending market and that they widely engage in relationship banking.¹¹ Forbearance is more likely when a bank is dominant in a lending market, such that it lends to both hub companies and peripheral companies in the market. The branch networks of major banks are mostly located in metropolitan areas, overlapping with each other, and their sizes are similar.¹² Therefore, none of them is likely to be a single dominant lender in a metropolitan market. In contrast, regional banks are often a single dominant lender in a regional lending market.¹³ For example, a single regional bank holds a market share of more than 40% in the lending market of 18 prefectures and a market share of more than 30% in 34 prefectures out of 47 prefectures in Japan, as of March 2006.¹⁴ Moreover, it is known that regional banks in Japan are more active in relationship banking than the major banks are (see, e.g., [Uchida et al., 2008, 2012](#); [Nemoto et al., 2011](#)). Indeed, the ratio of companies listed on the stock exchange, which supposedly depend on arm's length financing and are less prone to be exploited by a main bank, is much higher for major banks.¹⁵ The analysis in Section 3.5.2 indicates that banks engaging in relationship banking are more likely to undertake network-motivated forbearance.

¹¹It is also noteworthy that most of Japanese regional banks are listed on the stock exchange. Majority shareholders of any Japanese regional bank are neither the government, a municipality nor a non-financial local company.

¹²Outstanding loans and bills discounted as of March 2006 at the largest three major banks—Mitsubishi UFJ Bank, Mizuho Bank (including Mizuho Corporate Bank), and Sumitomo Mitsui Bank—are 70 trillion JPY, 62 trillion JPY, and 52 trillion JPY, respectively.

¹³In the case of the U.S., [Slovin et al. \(1999\)](#) argue that regional banking markets are not contestable.

¹⁴The market share information is from page 11 in *Gekkan Kin'yu Janaru, Zokan Go, Kin'yu Mappu 2007 Nen Ban* (Monthly Finance Journal, Extra Edition, Financial Map 2007 Edition), December 2006, Kin'yu Janaru Sha.

¹⁵The average ratio of listed companies in the largest three major banks (Mitsubishi UFJ, Sumitomo Mitsui, and Mizuho) is 22%, whereas that of regional banks is merely 3% in our dataset.

Thus, the interest reduction effect is expected to be more prominent among regional banks than among major banks. For Hypothesis 2, we split the sample according to the type of main bank.

Note that our model analysis suggests other hypotheses that are not tested in this paper. For example, Proposition 2 suggests that a firm with a smaller influence coefficient and with a larger business-stealing effect may be closed even when it is profitable. Another possible hypothesis is that forbearance is more likely in sectors where it is hard for firms to switch suppliers or clients because of relation-specific factors. This hypothesis is motivated by the observation that our theoretical results depend on the assumption that the supply network is rigid. We do not consider these hypotheses in our empirical analysis; instead, we leave them for future research.

5 Data

This section explains the datasets used in our empirical study. We use several different datasets on Japanese firms. The TSR Corporate Relationship Database provides information about inter-firm transactions, and it is used here to estimate the influence coefficients of firms. Detailed data on the characteristics of firms are obtained through the TSR financial Information Database and TSR Company Information Database.

5.1 TSR Relationship Database

We use the Corporate Relationship Database (*TSR Kigyō Sōkan File*) provided by the Tokyo Shoko Research (TSR) as of 2005 (hereafter, we call this dataset the *TSR Relationship Data*) to estimate the influence coefficient of each firm. This database contains the names and IDs of important corporate clients and suppliers up to the largest 24 for each company, including those in the process of a bankruptcy.¹⁶ The dataset also contains the basic items in the financial statement from the prior three years, such as sales and profits, and other characteristics including the credit score provided by TSR, the number of employees, the names and IDs of the largest 10 lending banks, the head office address, and industry classification.

We use the sample comprising firms listed in the TSR Relationship Data for which the number of employees and positive sales in (Year, Year -1) are recorded. After dropping the observations whose latest accounting year is before August 2004, we obtain 651,913 observations.¹⁷

¹⁶We keep bankrupt companies if their sales are reported since, in that case, they are still operating with the aim of revival.

¹⁷We also conduct the whole analysis after dropping observations whose latest accounting year is before June

We compute the value of the influence coefficients for 306,354 firms about which information on main banks is available.

5.2 TSR Financial Information Database

We match the estimated influence coefficients with the financial data and the data on firms' characteristics to construct the dataset for our empirical study that tests the hypotheses about network-motivated lending. The financial data are collected from the TSR Financial Information Database (*TSR Zaimu Joho File*). The credit score provided by TSR and other corporate information are collected from the TSR Company Information Database as of 2006 (*TSR Kigyō Joho File*). We obtained detailed information about the balance sheet and the income statement in the accounting period of 12 months ending in any month in 2006 from 16,369 companies, which are chosen by random sampling stratified independently on employment-size class, capital-size class, and industry category. We drop those without interest rate measures, which we define below (3,488 firms dropped). We drop an observation if the number of borrowers of a bank in the database to estimate the influence coefficient of the observation is less than 10, to avoid the extra noise due to the measurement error in the influence coefficient (485 firms). We dropped firms whose main bank is a government-owned bank or an agricultural/fishery cooperatives, which do not maximize their profit (2,291 firms). We also dropped outliers in the top 1% with respect to the interest rate measure (102 firms), and those reporting negative or zero interest rates (336 firms). We also dropped those with infeasible ratios of tangible asset to total assets (51 firms). We further dropped outliers in terms of variables of which maximum or minimum is too far away from the 99 percentile or the 1 percentile¹⁸ (437 firms). After dropping those lacking in the information required for our regression analysis (2,065 firms), we obtain 7,114 sample firms for our regression analysis.

2004. We confirm that the results are hardly changed. Similarly, dropping observations whose latest accounting year is before September 2004 does not alter the results.

¹⁸We find that PROFITABLE, CURRENT, EBITDA_G, and SALE_G, which are defined in Table 3, contain outliers, such that the maximum value is more than ten times of the 99 percentile. We also find that PROFITABLE contains outliers on the left tail of the distribution, such that the absolute value of the minimum is more than ten times of the absolute value of the 99 percentile. We dropped those outliers in the top 1 percent with respect to all these variables, and those in the bottom 1 percent with respect to PROFITABLE.

6 Estimating the influence coefficient

In this section, we discuss how the influence coefficients are estimated from the TSR Relationship data. The dataset contains information about which firms are connected but does not provide the price levels of the products nor the magnitude of the trade between firms. The influence vector appearing in our theoretical model cannot, therefore, be computed directly. We first estimate a spatial autoregressive model of the supply network. We then compute the influence coefficient of each firm by using the estimates from the spatial autoregressive model.

6.1 Spatial autoregressive model

We first estimate the spatial autoregressive model of the supply network indicated by (13). To eliminate the firm fixed effects, the differenced version of (13) is considered:

$$\Delta \mathbf{s} = \mathbf{Q} \Delta \mathbf{s} + \Delta \mathbf{f}, \quad (27)$$

where Δ indicates the difference between the value in the latest accounting year and that in the previous accounting year. Note that while $\Delta \mathbf{s}$ (the difference in sales) can be observed in the data, \mathbf{Q} and $\Delta \mathbf{f}$ cannot.

We consider the following approximation of \mathbf{Q} and $\Delta \mathbf{f}$. We approximate \mathbf{Q} by $\beta \mathbf{G}$, where β is a parameter to be estimated and \mathbf{G} is the adjacency matrix for the supply network. We assume that firm i purchases a product from firm j if firm i identifies firm j as an important supplier or firm j identifies firm i as an important client.¹⁹ The (i, j) th element of the adjacency matrix \mathbf{G} is $g_{ij} = 1$ when firm i sells its product to firm j and $g_{ij} = 0$ otherwise. Note that when $g_{ij} = 0$, it is also the case that q_{ij} (the (i, j) th element of \mathbf{Q}) is 0 since $q_{ij} = p_i^{1-\theta} (w_{ji} p^j)^\theta / p_j$. We examine alternative approximations of \mathbf{Q} in Section 8. We assume that $\Delta \mathbf{f}$ is written as $\Delta \mathbf{f} = \gamma_I' \mathbf{Ind} + \gamma_P' \mathbf{Pre} + \mathbf{u}$, where γ_I and γ_P are unknown coefficients, \mathbf{Ind} is the matrix of industry dummies, \mathbf{Pre} is the matrix of prefecture dummies, and \mathbf{u} is the vector of an unobserved error term.²⁰

Under these assumptions, we have the following spatial autoregressive model that is esti-

¹⁹ Note that while the dataset contains the information about the 24 most important corporate clients and suppliers, the observed degree of a firm, that is, the number of suppliers for a firm, can be larger than 24 by adopting our assumptions.

²⁰ The industries are categorized according to the two-digit Japan Standard Industry Classification (96 sectors). The number of prefectures in Japan is 47.

Table 1: Estimation results of the spatial autoregressive model

	Est. coef.	s.e.	
β	0.00197	0.0000494	***
Industry factor	yes		
Prefecture factor	yes		
Adj. R-squared	0.1451		
# of observations	652,280		

(note) Industry factor is controlled by the 95 sector dummies, which indicate one of the 2-digit “chu-bunrui” classifications of the Japanese standard industry classification, revised in 2002. Prefecture factor is controlled by 46 prefecture dummies (Hokkaido is the base class).

mated by ordinary least squares (OLS):

$$\Delta \mathbf{s} = \beta \mathbf{G} \Delta \mathbf{s} + \gamma_I' \mathbf{Ind} + \gamma_P' \mathbf{Pre} + \epsilon, \quad (28)$$

where ϵ is the sum of \mathbf{u} and the approximation error of $\beta \mathbf{G}$. We estimate this model by using the sample of 652,280 observations from TSR Relationship Data. The estimation result is summarized in Table 1. The estimated β is positive and statistically significant, although the value is small. We comment on our use of OLS to estimate the spatial auto-regressive models. It is known in the literature on spatial models that the OLS estimator is not guaranteed to be consistent (Kelejian and Prucha, 1998). The reason for this is that the regressor $\mathbf{G}\mathbf{s}$ and the error term ϵ are likely to be correlated, by construction. However, in our case, the correlation is likely to be small. Appendix A.1 provides a more detailed discussion. Note that Lee (2002) also examines the conditions in which the OLS estimator is consistent. However, that argument cannot be applied here and our discussion is different.

As a robustness check, we also estimate the parameters by use of the instrumental variables estimator, as suggested by Kelejian and Prucha (1998). For example, when β_1 is estimated, $\mathbf{G}\mathbf{s}$ is instrumented by $\mathbf{G} \times \mathbf{Ind}$ and $\mathbf{G} \times \mathbf{Pre}$. The resulting estimates are very imprecise, possibly because of the weakness of the instruments. Moreover, the resulting influence vector is very highly correlated with the one obtained by using the OLS estimate. Therefore, we decided to rely on OLS to estimate β values.

6.2 Influence coefficients

We now compute the influence coefficient of each firm. We measure the influence of a firm on the other firms that share the main bank. We construct the supply network for each bank and

compute the influence coefficient of each firm in that supply network.

We primarily consider the influence of a firm on the profits of other firms in the same supply network, where each supply network consists of firms that share the same main bank. Let $\mathbf{G}^{(b)}$ be the adjacency matrix of the supply network that consists of firms whose main bank is bank b . Let $\mathbf{v}^{(b)}$ be the vector of influence coefficients in the supply network for bank b . The influence vector for the supply network determined by b is estimated by

$$\mathbf{v}^{(b)'} = \mathbf{1}' \sum_{k=0}^{100} (\hat{\beta} \mathbf{G}^{(b)})^k, \quad (29)$$

where $\mathbf{1}$ is the vector of ones whose dimension is the same as $\mathbf{v}^{(b)}$ and $\hat{\beta}$ is the estimated coefficient in (28). Note that the influence vector is defined in (16), and it is $\mathbf{v}' = \mathbf{1}' \sum_{k=0}^{\infty} \mathbf{Q}^k$. Our influence vector estimate is obtained by approximating the infinite series $\sum_{k=0}^{\infty} \mathbf{Q}^k$ by $\sum_{k=0}^{100} \mathbf{Q}^k$ and then replacing \mathbf{Q} with $\hat{\beta} \mathbf{G}^{(b)}$.²¹

We argue that this definition of supply network and influence is appropriate for our analysis on forbearance. A bank does not have any incentive to undertake forbearance if the benefits of the forbearance are received by firms with which the bank does not have any relationship. An alternative argument may be that each bank may observe the supply network only among firms for which the bank is the main bank. There are as many supply networks as the number of banks that act as a main bank since we construct a network for each main bank. We assume that no firm has multiple main banks, so that supply networks are mutually exclusive, to simplify the analysis.²²

It is important to note that to estimate β we use the model of an economy-wide supply network while influence coefficients are computed for each bank-specific supply network, which is a sub-network of the economy-wide network. In other words, β is estimated with the adjacency matrix \mathbf{G} defined in the economy-wide supply network, but $\mathbf{v}^{(b)}$ is computed with $\mathbf{G}^{(b)}$. There are two main reasons that we estimate the parameters by using the economy-wide supply network instead of the bank-level supply networks. First, for some banks, only a small number of firms choose it as their main bank, and therefore it is difficult to allow the parameters to be bank-specific. The other and more important reason is that since the product market covers the entire economy, not restricted to within firms who share the main bank, and \mathbf{Q} is an outcome of the

²¹We have verified the accuracy of the approximation of the infinite series by trying various truncation points.

²²In the data, we observe which bank has had the most transactions for each firm and we define that bank as the main bank of that firm.

Table 2: Descriptive statistics of the influence coefficient

# of obs.	mean	sd	min	p10	med	p75	p90	p95	p99	max
306,354	1.003	0.011	1.000	1.000	1.002	1.004	1.006	1.010	1.025	2.837

equilibrium in the product market, it would be more appropriate to use the information about the economy-wide supply network to estimate the parameter that determines \mathbf{Q} .

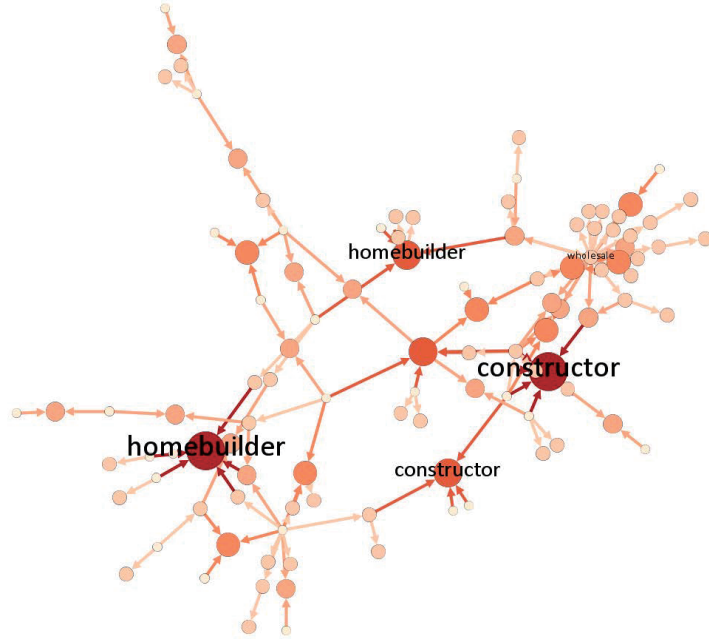
6.3 Descriptive statistics of the influence coefficient

Table 2 shows the descriptive statistics of the estimated influence coefficients of the supply networks of firms with a common main bank v , which is estimated by (29). The influence coefficients range from 1 to about 3. This means that a one unit increase in sales to households by a firm generates at most three units of increase in total sales, including intermediate inputs, within the common-main-bank network. The distribution of the influence coefficient is highly skewed to the left. More than half of firms have a factor equal to 1. Even at the 99th percentile, the factor is very close to 1. Most of the variation is concentrate in the top one percent. This is plausible given the fact that large companies, which are more likely to be a major procurer from multiple suppliers, account for less than 1% of the total number of companies in Japan.²³

Figure 2 is an example of the supply networks of firms with a common main bank. The size of each node indicates the level of the influence coefficient v . The largest ones are a construction company and a homebuilder ($v = 1.0119$). They purchase a wide variety of materials and services for constructions. In contrast, a construction-material wholesale company (upper right in the figure) has a relatively small influence coefficient ($v = 1.002$) despite its high out-degree. This is because the wholesale company is not an important corporate client in the network but is, rather, an important supplier for the supply network. Our theory predicts that the interest cost for the construction company and a homebuilder is lower than that for less influential firms.

²³*Establishment and Enterprise Census* (Ministry of Internal Affairs and Communications) in 2004 reports the number of small and medium-sized enterprises (SMEs) is 1,508,194, excluding sole proprietorships, in the non-agricultural sectors. The number of large enterprises is 11,793 in the non-agricultural sectors. The latter accounts for only 0.8% of the total number of enterprises. An enterprise is classified as an SME if its number of full-time employees is 300 persons or less, or its capital is 300 million JPY (about 3 million USD) or less. These thresholds are 100 persons and 100 million JPY for the wholesale sector, 50 persons and 50 million JPY for the retail and restaurant sectors, and 100 persons and 50 million JPY for the service sector.

Figure 2: Example of a bank-specific supply network



7 Interest rate regression

In this section, we present the results of our empirical study on the relationship between influence coefficients and interest rates of loans. This section provides our main empirical findings. We first explain the definitions and the characteristics of the variables used in the empirical study. We then examine the relationship between influence coefficients and interest rates.

7.1 Variables

We first explain the characteristics of the variables used in the regression analysis. The precise definitions of variables for our regression analysis are listed in Table 3.

7.1.1 Key variables

The key variable in our regression analysis are interest rate (which is the dependent variable), influence coefficient, and credit score.

We define the dependent variable $rate_i$ by using the items in the financial statement of each

Table 3: Definition of variables

Variable	Definition
rate	Interest costs (t) / $\{0.5(\text{total asset } (t-1) + \text{total asset } (t))\} \times 100$ (%).
$\ln(v)$	Natural logarithm of the influence coefficient v .
score	Normalized credit score from Tokyo Shoko Research, (original score) / 50 - 1.
DISTRESS	A dummy variable equal to 1 if score < 0, or zero otherwise.
INSOLVENT	A dummy variable equal to 1 if asset < liability, or zero otherwise.
LN(INT_COV)	Natural logarithm of $(x+1)$, where $x := \text{EBITDA} / (1 + \text{interest payment})$ if $\text{EBITDA} \geq 0$, or 0 otherwise, and $\text{EBITDA} = \text{operating profit} + \text{depreciation}$.
LEVERAGE	Total assets / capital.
TANGIBLE	Tangible fixed assets / total assets.
CURRENT	Short-term assets / short-term liability.
PROFITABLE	EBITDA / sales.
EBITDA_G	$\{\text{EBITDA}(t) - \text{EBITDA}(t-1)\} / \text{total assets}(t-1)$.
SALES_G	$\{\text{sales}(t) / \text{sales}(t-1) - 1\} \times 100\%$.
LN(SALES)	Natural logarithm of sales.
LN(LOAN)	Natural logarithm of the average of outstanding loans in the current and previous accounting year end (mil. JPY).
LN(FIRM AGE)	Natural logarithm of firm age in years.
LN(EMPLOY)	Natural logarithm of the number of employees plus one.
LISTED	1 if the firm is listed on a stock market, 0 otherwise.
BOND_RATIO	Bonds outstanding / (bonds outstanding + loans outstanding). Set equal to 0 if the firm does not issue bonds and loans.
#LENDING_BKS	Number of lending banks. The maximum is 10 banks.
HI	Herfindahl index of the number of branches of banks (excluding government-owned ones) and Shinkin banks (larger cooperative banks) in the telephone area-code area where the head office of the firm is located. The index is calculated after excluding Shinkin banks if the capital of a firm is 900 million JPY or more.

firm, following the existing literature (e.g., [Caballero et al., 2008](#)).

$$rate_i \equiv \frac{\text{firm } i\text{'s interest expense in the accounting period ending}}{\text{average of firm } i\text{'s outstanding loans in current and previous years}}$$

The key independent variable is the influence coefficient \hat{v} . It is computed by the procedure documented in Section 6. We find that the using the natural logarithm yields a better fit for the model. Thus, we use $\ln(v)$ in our regression analysis.

The most important control variable is *score*. We use a normalized version of the credit score provided by TSR. TSR credit score (*Hyo Ten*) ranges from 0 (high default risk) to 100 (no default risk): a higher score indicates a higher credit-worthiness. The score is calculated on the basis of both quantitative and qualitative information including manager's competence (20 points), growth potential (25 points), stability (45 points), and disclosure/reputation (10

points). TSR instructs that a score below 50 indicates an “alerting” situation.²⁴ We normalize it so that it equals 0 when the original score is at 50. That is, we define *score* by $(\text{original score})/50 - 1$.

We also use an indicator variable, DISTRESS, which equals one when *score* is negative. The scatter plot of *score* and *rate* indicates that they are negatively correlated when *score* is positive, whereas they are positively correlated when *score* is negative. This is probably because of the default of interest payments or the resulting rescheduling for repayments.²⁵ The variable DISTRESS is used to control for this effect. As another indicator of a distressed firm, we also use an indicator variable, INSOLVENT, which equals one if the liability of a firm is greater than its total asset, or zero otherwise.

7.1.2 Control variables

In addition, we consider three categories of control variables. The first set of control variables is related to the characteristics of each firm. The second set characterizes access to the financial market and the degree of competition in the financial market. We also consider the sectoral and regional dummies.

The first set of control variables on the characteristics of each firm minimizes the possibility that the influence coefficient acts as a proxy for other firm characteristics, that is, it controls for the omitted variable bias. For example, the number of suppliers to a credit-worthy firm could be larger than those less credit-worthy thanks to the lower probability of default. If so, the influence coefficient may reflect the credit-worthiness of the firm. We minimize this possibility by using these control variables. Following the existing empirical literature (e.g., [Petersen and Rajan, 1994](#); [Peek and Rosengren, 2005](#); [Bharath et al., 2011](#)), we include the following set of control variables related to firm characteristics: LN(INT_COV) (natural logarithm of the interest coverage ratio), LEVERAGE (ratio of total debts over total assets), TANGIBLE (ratio of collateralizable assets over total assets), CURRENT (current ratio), PROFITABLE (EBITDA / sales), EBITDA_G (growth of EBITDA / total assets), SALES_G (growth rate of

²⁴TSR instructs that a score between 30 and 49 indicates “Ichiou Keikai” (alerting provisionally), and that between 0-29 indicates “Keikai” (alerting).

²⁵ The non-performing loan ratio, i.e., the ratio of risk-management loans over total loans, is 1.9% for major banks, 4.6% for regional banks including the members of the second association of regional banks, and 7.8% for Shinkin banks (cooperative banks) in March 2006. The aggregate non-performing loan ratio is 4.1%. These numbers are calculated from the financial statement of each bank.

sales), LN(SALES) (natural logarithm of sales), LN(LOAN) (natural logarithm of loans), and LN(AGE) (natural logarithm of firm age). Many existing empirical studies include a dummy variable to indicate whether a firm is a corporation, but we do not use this variable since we have only one firm that is not a corporation in our sample.

The second set characterizes access to the financial market and the degree of competition there. These factors affect the influence coefficient as well as interest rates. A firm with direct access to the financial market tends to be more credit-worthy and larger, and so its main bank is more likely to be a large bank. The supply network among borrowers of a large bank connects a larger number of firms, and so the influence coefficient can theoretically be larger. Likewise, the market structure affects the size of the network and the influence coefficient of each firm in it. We avoid the possibility that the influence coefficient acts as a proxy for these factors by introducing the second set of control variables. The list of variables includes LISTED (a dummy variable to indicate whether a firm is listed on the stock exchange), BOND_RATIO (ratio of bond financing to total debts), #LENDING_BKS (the number of lending banks), and HI (Herfindahl index of bank branches in the area where the head office of the firm is located).

To control for the unobservable characteristics of each main bank, industrial sector and region, we include the fixed effects of each main bank, industrial sector, and region. The number of main banks included in the full sample is 210. Among them, nine banks are major banks, 98 are regional banks, and 103 are cooperative banks.²⁶ Major banks mainly operate in the metropolitan areas in Japan. 38% of firms in our sample use a major bank as their main bank. Regional banks operate on a prefecture-wide basis, including adjacent prefectures. In our sample, 54% of firms use a regional bank as their main bank. Regional banks often maintain the market largest share of the lending market in the prefecture where they are located. The last category, cooperative banks, are smaller than regional banks in terms of asset size and operating area. About 8% of our sample firms use cooperative banks as their main bank.

The industrial sectors include the following 8 sectors: manufacturing (31.1% in the sample), construction (21.6%), real estate (2.4%), retail (9.7%), wholesale (19.0%), communications

²⁶Major banks include Mizuho, Mizuho Corporate, Mitsubishi UFJ, Sumitomo Mitsui, Risona, Saitama Risona, Shinsei, Aozora, Mitsubishi UFJ Trust, Chuo-Mitsui Trust, Sumitomo Trust, and Mizuho Trust. We lack in the observations for Shinsei, Aozora, and Mizuho Trust. Regional banks include those banks belonging to the Regional Banks Association of Japan, or the Second Association of Regional Banks. Cooperative banks includes Shinkin banks and Shinkumi banks. Some are dropped from our dataset due to the lack of the number of observations.

(2.3%), logistics (2.9%), and others (11.0%). The regions include the following 10 regions: Hokkaido (6.5%), Tohoku (10.7%), Kanto (27.5%), Koshinetsu (8.1%), Tokai (9.8%), Hokuriku (4.7%), Kansai (13.3%), Chugoku (7.3%), Shikoku (4.3%), Kyushu (6.9%), and Okinawa (0.8%).

7.1.3 Descriptive statistics

Table 4, Panel (a), shows the descriptive statistics of variables in the full sample. Next, we discuss the basic characteristics of the firms in our sample, the degree of accessibility to the financial market and the characteristics of the financial market.

The dependent variable *rate* ranges from 0.001% to about 11.4%. We do not repeat the discussion on the influence coefficient; instead, see Section 6.3. The mean of the dummy variable DISTRESS indicates that 14.8% of our sample firms are in the situation of distress or near to distress by our definition. The mean and median of LN(SALES) are both about 8, which means that the median or mean of sales of our sample firms is about 3 billion JPY (30 million USD at a rate of 100 JPY = 1 USD). The dataset also includes quite large firms in terms of sales, 31.8 billion JPY at the 90th percentile, and a maximum of 11.1 trillion JPY. The mean and median ages of firms in our data are about 36 and 41 years old, respectively.

The descriptive statistics indicate the importance of bank lending. The mean of the dummy variable LISTED indicates that about 12% of our sample firms are listed at a stock exchange. Given the fact that a company that is not listed in the stock exchange rarely issues a public bond, we can reasonably expect that bank lending is still the most important external financing source for the remaining 88% of firms. More than 95% of firms obtain loans from multiple banks (#LENDING_BKS).

The Herfindahl index (HI) indicates that more than half of firms are located in an area where the index is less than 0.18, and the lending market is relatively competitive according to the U.S. regulatory standard; however, some firms are located in areas of highly concentrated banks, where the index equals 1.

Table 4, Panel (b), shows the descriptive statistics of variables in the sample of firms whose main bank is a regional bank. The range of *rate* and its descriptive statistics are similar to that in the full sample. The ratio of distressed firms is 17.4% (mean of DISTRESS), and is higher than that of the full sample. The mean and median of LN(SALES) is somewhat smaller

Table 4: Descriptive statistics of variables for interest rate regression

(a) Full sample

	N	mean	sd	min	p1	p10	p50	p90	p99	max
rate	7,114	2.271	1.352	0.001	0.230	0.909	2.027	3.771	7.227	11.383
$\ln(v)$	7,114	0.011	0.034	0.000	0.000	0.000	0.004	0.022	0.130	0.979
score	7,114	0.138	0.151	-1.000	-0.200	-0.040	0.120	0.340	0.500	0.840
DISTRESS	7,114	0.148	0.355	0	0	0	0	1	1	1
INSOLVENT	7,114	0.044	0.205	0	0	0	0	0	1	1
LN(INT_COV)	7,114	1.834	1.389	0.000	0.000	0.000	1.629	3.724	5.839	10.571
LEVERAGE	7,114	0.721	0.282	0.072	0.190	0.405	0.739	0.945	1.395	6.783
TANGIBLE	7,114	0.292	0.201	0.000	0.001	0.040	0.267	0.571	0.828	0.989
CURRENT	7,114	1.505	0.930	0.024	0.272	0.743	1.258	2.502	5.553	7.995
PROFITABLE	7,114	0.039	0.055	-0.158	-0.097	-0.006	0.027	0.104	0.234	0.340
EBITDA_G	7,114	0.001	0.050	-0.987	-0.140	-0.040	0.001	0.046	0.155	0.255
SALES_G	7,114	0.034	0.167	-0.871	-0.409	-0.141	0.024	0.219	0.576	0.866
LN(SALES)	7,114	7.950	1.811	2.421	4.577	5.777	7.769	10.367	12.936	16.221
LN(LOAN)	7,114	6.387	1.971	-2.198	1.940	3.929	6.330	8.842	11.552	15.864
LN(FIRM AGE)	7,114	3.585	0.566	-1.792	1.792	2.778	3.718	4.099	4.477	4.827
LN(EMPLOY)	7,114	4.180	1.524	0.000	1.386	2.303	4.025	6.248	8.242	11.094
LISTED	7,114	0.120	0.325	0	0	0	0	1	1	1
BOND_RATIO	7,114	0.060	0.153	0.000	0.000	0.000	0.000	0.224	0.787	1.000
#LENDING_BKS	7,114	4.689	2.292	1	1	2	4	8	10	10
HI	7,114	0.182	0.109	0.050	0.050	0.050	0.164	0.324	0.510	1.000

(b) Regional bank sample

	N	mean	sd	min	p1	p10	p50	p90	p99	max
rate	3,836	2.387	1.345	0.002	0.289	1.005	2.168	3.876	7.377	11.148
$\ln(v)$	3,836	0.009	0.019	0.000	0.000	0.000	0.004	0.020	0.095	0.379
score	3,836	0.121	0.146	-0.900	-0.200	-0.040	0.100	0.320	0.460	0.640
DISTRESS	3,836	0.174	0.379	0	0	0	0	1	1	1
INSOLVENT	3,836	0.051	0.220	0	0	0	0	0	1	1
LN(INT_COV)	3,836	1.587	1.254	0.000	0.000	0.000	1.402	3.299	5.251	7.914
LEVERAGE	3,836	0.737	0.289	0.093	0.197	0.419	0.757	0.954	1.463	6.452
TANGIBLE	3,836	0.324	0.203	0.000	0.001	0.065	0.303	0.604	0.858	0.989
CURRENT	3,836	1.459	0.924	0.032	0.231	0.701	1.223	2.403	5.553	7.995
PROFITABLE	3,836	0.034	0.054	-0.150	-0.100	-0.011	0.024	0.097	0.232	0.340
EBITDA_G	3,836	0.001	0.054	-0.987	-0.152	-0.044	0.000	0.049	0.164	0.255
SALES_G	3,836	0.033	0.174	-0.871	-0.428	-0.152	0.020	0.231	0.588	0.864
LN(SALES)	3,836	7.397	1.499	2.421	4.456	5.559	7.288	9.429	11.200	12.740
LN(LOAN)	3,836	5.907	1.770	-2.198	1.658	3.678	5.916	8.191	9.946	11.516
LN(FIRM AGE)	3,836	3.525	0.551	1.179	1.778	2.708	3.651	4.056	4.420	4.730
LN(EMPLOY)	3,836	3.783	1.267	0.000	1.386	2.197	3.664	5.525	6.934	8.090
LISTED	3,836	0.045	0.206	0	0	0	0	0	1	1
BOND_RATIO	3,836	0.029	0.102	0.000	0.000	0.000	0.000	0.089	0.512	1.000
#LENDING_BKS	3,836	4.281	2.222	1	1	2	4	8	10	10
HI	3,836	0.227	0.092	0.050	0.052	0.115	0.219	0.343	0.510	1.000

Table 5: Correlation matrix of variables: Full sample (N=7,114)

		1	2	3	4	5	6	7	8	9	10	11
1	rate	1.000										
2	ln(<i>v</i>)	-0.088	1.000									
3	ln(<i>v</i>) × score	-0.094	0.875	1.000								
4	score	-0.345	0.242	0.308	1.000							
5	DISTRESS	0.218	-0.056	-0.081	-0.590	1.000						
6	INSOLVENT	0.083	-0.041	-0.042	-0.341	0.413	1.000					
7	score × DISTRESS	-0.145	0.037	0.064	0.540	-0.660	-0.514	1.000				
8	LN(INT_COV)	-0.327	0.140	0.173	0.633	-0.315	-0.173	0.224	1.000			
9	LEVERAGE	0.160	-0.061	-0.091	-0.496	0.385	0.499	-0.395	-0.381	1.000		
10	TANGIBLE	-0.018	-0.053	-0.042	-0.035	0.060	0.017	-0.046	-0.057	-0.004	1.000	
11	CURRENT	-0.016	-0.054	-0.015	0.105	-0.095	-0.110	0.091	0.103	-0.364	-0.190	1.000
12	PROFITABLE	-0.099	0.051	0.080	0.346	-0.161	-0.121	0.128	0.597	-0.192	0.211	0.054
13	EBITDA_G	0.014	0.007	0.012	0.018	0.015	-0.040	-0.007	0.230	-0.004	-0.006	-0.006
14	SALES_G	0.010	-0.003	0.014	0.099	-0.067	-0.057	0.062	0.202	-0.018	-0.044	-0.024
15	LN(SALES)	-0.275	0.420	0.343	0.583	-0.313	-0.197	0.231	0.488	-0.160	-0.088	-0.167
16	LN(LOAN)	-0.279	0.354	0.270	0.325	-0.142	-0.093	0.102	0.180	0.084	0.143	-0.250
17	LN(AGE)	-0.131	0.178	0.130	0.246	-0.130	-0.100	0.115	0.127	-0.170	0.129	-0.067
18	LN(EMPLOY)	-0.291	0.394	0.322	0.548	-0.286	-0.180	0.209	0.479	-0.198	0.034	-0.107
19	LISTED	-0.172	0.304	0.253	0.330	-0.108	-0.073	0.080	0.318	-0.210	-0.050	-0.013
20	BOND_RATIO	-0.112	0.153	0.173	0.230	-0.122	-0.077	0.089	0.150	-0.121	-0.059	0.120
21	#LENDING_BKS	-0.100	0.135	0.075	0.195	-0.128	-0.117	0.115	0.109	0.023	0.039	-0.103
22	HI	0.082	0.003	-0.025	-0.088	0.086	0.044	-0.067	-0.131	0.025	0.190	-0.067

		12	13	14	15	16	17	18	19	20	21
12	PROFITABLE	1.000									
13	EBITDA_G	0.312	1.000								
14	SALES_G	0.176	0.386	1.000							
15	LN(SALES)	0.228	0.029	0.093	1.000						
16	LN(LOAN)	0.295	0.015	0.010	0.800	1.000					
17	LN(AGE)	0.068	0.003	-0.065	0.376	0.378	1.000				
18	LN(EMPLOY)	0.269	0.020	0.038	0.876	0.745	0.406	1.000			
19	LISTED	0.232	0.013	0.008	0.526	0.450	0.209	0.525	1.000		
20	BOND_RATIO	0.113	0.004	0.022	0.264	0.243	0.138	0.257	0.199	1.000	
21	#LENDING_BKS	0.163	0.013	0.033	0.477	0.558	0.241	0.440	0.245	0.150	1.000
22	HI	-0.057	-0.024	-0.037	-0.241	-0.181	-0.052	-0.162	-0.119	-0.182	-0.172

than the full sample. The ratio of listed firms is about 4.4%, and is significantly smaller than that of the full sample. The Herfindahl index (HI) is above 2, and is higher than that of the full sample. Thus, firms whose main bank is a regional bank are somewhat less creditworthy, smaller, and are located in the area where the lending market is more concentrated.

Table 5 is the matrix of correlation coefficients among variables to be used in the regression analysis. The table indicates that the influence coefficient has a negative correlation with *rate*, but at a smaller magnitude relative to the traditional indicators of financial soundness, such as *score*, LN(INT_COV), LN(SALES), and LN(AGE). The influence coefficients have positive correlations with LN(SALES). The correlation coefficient is about 0.42. This suggests that larger firms are more likely to have larger influence, but not always. LISTED, too, has a negative correlation with *rate*. This may be because only those credit-worthy enough can be approved to

Table 6: Correlation matrix of variables: Regional bank sample (N=3,847)

		1	2	3	4	5	6	7	8	9	10	11
1	rate	1.000										
2	ln(v)	-0.069	1.000									
3	ln(v) \times score	-0.124	0.747	1.000								
4	score	-0.333	0.140	0.405	1.000							
5	DISTRESS	0.194	-0.051	-0.182	-0.618	1.000						
6	INSOLVENT	0.072	-0.044	-0.098	-0.351	0.398	1.000					
7	score \times DISTRESS	-0.133	0.033	0.156	0.575	-0.666	-0.514	1.000				
8	LN(INT_COV)	-0.293	0.104	0.255	0.614	-0.309	-0.171	0.232	1.000			
9	LEVERAGE	0.117	-0.039	-0.155	-0.484	0.383	0.519	-0.400	-0.337	1.000		
10	TANGIBLE	-0.045	-0.007	0.020	0.014	0.028	-0.007	-0.021	0.003	-0.021	1.000	
11	CURRENT	0.007	-0.079	-0.023	0.115	-0.102	-0.132	0.096	0.092	-0.387	-0.189	1.000
12	PROFITABLE	-0.073	0.006	0.083	0.315	-0.150	-0.111	0.131	0.623	-0.142	0.244	0.040
13	EBITDA_G	0.027	-0.001	0.002	0.000	0.028	-0.052	-0.011	0.228	-0.003	0.000	-0.015
14	SALES_G	-0.006	-0.025	0.007	0.092	-0.054	-0.063	0.071	0.199	-0.014	-0.038	-0.023
15	LN(SALES)	-0.269	0.415	0.402	0.523	-0.298	-0.176	0.229	0.442	-0.101	0.023	-0.212
16	LN(LOAN)	-0.265	0.328	0.252	0.213	-0.089	-0.054	0.065	0.124	0.176	0.263	-0.305
17	LN(AGE)	-0.134	0.189	0.155	0.202	-0.115	-0.096	0.109	0.087	-0.147	0.155	-0.083
18	LN(EMPLOY)	-0.282	0.386	0.371	0.471	-0.260	-0.159	0.192	0.420	-0.135	0.161	-0.155
19	LISTED	-0.118	0.285	0.308	0.196	-0.062	-0.033	0.049	0.222	-0.123	0.018	-0.037
20	BOND_RATIO	-0.066	0.042	0.055	0.146	-0.104	-0.047	0.059	0.088	-0.084	-0.034	0.105
21	#LENDING_BKS	-0.128	0.221	0.190	0.196	-0.118	-0.094	0.093	0.137	0.044	0.093	-0.139
22	HI	0.000	0.162	0.122	0.013	0.002	-0.015	-0.006	-0.011	-0.050	0.090	-0.022

		12	13	14	15	16	17	18	19	20	21
12	PROFITABLE	1.000									
13	EBITDA_G	0.310	1.000								
14	SALES_G	0.170	0.373	1.000							
15	LN(SALES)	0.186	0.027	0.102	1.000						
16	LN(LOAN)	0.257	0.015	0.005	0.753	1.000					
17	LN(AGE)	0.031	0.013	-0.072	0.341	0.352	1.000				
18	LN(EMPLOY)	0.228	0.014	0.025	0.843	0.705	0.398	1.000			
19	LISTED	0.138	0.011	-0.005	0.374	0.286	0.109	0.381	1.000		
20	BOND_RATIO	0.057	-0.007	-0.002	0.149	0.117	0.088	0.155	0.081	1.000	
21	#LENDING_BKS	0.168	0.010	0.021	0.518	0.581	0.222	0.479	0.235	0.101	1.000
22	HI	-0.012	-0.037	-0.027	-0.008	0.001	0.071	0.052	0.051	-0.048	-0.049

be publicly traded in the stock exchange. The alternative explanation may be that banks may exploit firms that lack direct access to finance markets.

These tendencies are qualitatively the same in the subsample of firms whose main bank is a regional bank (Table 6).

7.2 Econometric frameworks

We estimate the equation (26) to test Hypothesis 1. The hypothesis predicts that b_1 is negative, and b_3 is positive. In addition, we estimate the model using the DISTRESS dummy or the INSOLVENT dummy in place of *score*. Hypothesis 1 predicts that the influence coefficient reduces the interest rates when a firm is in a distress, i.e., b_3 and $b_1 + b_3$ are negative.

The model is estimated by the main-bank fixed-effect model. We compute the main-bank clustered standard errors, which take into account the estimation error in the influence coeffi-

cients. Note that we do not observe the true influence coefficients but instead need to estimate them. They therefore suffer from measurement errors. Online appendix explains how to modify the standard errors to take into account the estimation error in the influence coefficients.

7.3 Baseline results: regional banks

We first estimate the model using the observations of firms whose main banks are regional banks. We focus on this subsample because regional banks are often dominant in local lending markets and this subsample would accord with our theoretical framework. The regression results are supportive of our hypotheses. They indicate that a firm with a high influence coefficient receives a lower interest rate on average. Moreover, we find that the effect is more prominent for less credit-worthy firms.

Table 7 shows the results of the baseline regression of (26). Column (1) shows that the influence coefficient and *score* are negatively correlated with interest rates, and that the interaction term between the influence coefficient and *score* has a positive and significant coefficient. Thus, the regression analysis supports Hypothesis 1.

Columns (3) in Table 7 shows that the coefficient of the interaction term of $\ln(v)$ and DISTRESS is negative and significant although the coefficient of $\ln(v)$ without interaction is not significant. The sum of these coefficient is negative and significant. This result implies that the influence coefficient reduces interest payments only for distressed firms. Thus, this result supports Hypothesis 1. We obtain a somewhat weaker but similar result when we use INSOLVENT in place of DISTRESS (column (4)).

To gauge the economic importance of the effect of the influence coefficient, we compute the marginal effect of $\ln(v)$ at each grid cell of *score* to *rate*, which is listed in Table 8 and illustrated in Figure 3 (i). The interest reduction effect of the influence coefficient is statistically significant for distressed firms (i.e., firms with a score less than 0). This effect is not significant or even reversed for financially sound firms, for which the main bank does not need to consider forbearance lending. The estimates indicate, for example, that the interest rate for a firm with an influence coefficient at the median is larger than that for a firm with an influence coefficient at the 90th percentile by about 13 basis points when their scores are -0.2. The difference is larger, at 78 basis points, when we compare with firms having an influence factor at the 99th

Table 7: Baseline regression (regional bank only, dependent variable: rate)

	(1)		(2)		(3)		(4)
	coef.		coef.		coef.		coef.
	(s.e.)		(s.e.)		(s.e.)		(s.e.)
ln(v)	-5.986	***	-3.508	**	1.546		1.274
	(1.907)		(1.618)		(1.432)		(1.407)
ln(v) × score	21.023	**	22.854	***			
	(9.660)		(8.204)				
ln(v) × DISTRESS					-6.775	***	
					(2.171)		
ln(v) × INSOLVENT							-7.294
							(4.387)
score	-2.932	***	-2.594	***	-2.284	***	-2.222
	(0.203)		(0.237)		(0.220)		(0.219)
DISTRESS			0.193	***	0.253	***	0.206
			(0.070)		(0.076)		(0.072)
INSOLVENT							-0.154
							(0.128)
score × DISTRESS			1.934	***	1.817	***	1.401
			(0.534)		(0.529)		(0.518)
LN(INT_COV)			-0.540	***	-0.539	***	-0.537
			(0.036)		(0.037)		(0.036)
LEVERAGE			0.200	*	0.205	*	0.282
			(0.117)		(0.117)		(0.125)
TANGIBLE			0.256	**	0.268	**	0.280
			(0.124)		(0.123)		(0.123)
CURRENT			-0.006		-0.007		-0.005
			(0.027)		(0.027)		(0.027)
PROFITABLE			9.618	***	9.551	***	9.518
			(0.791)		(0.790)		(0.790)
EBITDA_G			0.618		0.629		0.606
			(0.546)		(0.548)		(0.554)
SALES_G			-0.048		-0.045		-0.051
			(0.153)		(0.153)		(0.154)
LN(SALES)			0.491	***	0.484	***	0.480
			(0.054)		(0.053)		(0.053)
LN(LOAN)			-0.515	***	-0.514	***	-0.518
			(0.059)		(0.059)		(0.059)
LN(AGE)			0.007		0.004		0.005
			(0.043)		(0.044)		(0.044)
LISTED			-0.082		-0.050		-0.038
			(0.060)		(0.056)		(0.059)
BOND_RATIO			0.057		0.035		0.043
			(0.182)		(0.181)		(0.181)
#LENDING_BKS			0.032	***	0.032	***	0.031
			(0.010)		(0.010)		(0.010)
HI			-0.206		-0.193		-0.213
			(0.251)		(0.254)		(0.249)
industry factor	yes		yes		yes		yes
region factor	yes		yes		yes		yes
main-bank fixed effect	yes		yes		yes		yes
t-stat. (p-value)							
H_0 : coef of ln(v) + coef					-2.62	(0.009)	-1.44
of ln(v) × (DISTRESS							(0.151)
or INSOLVENT) = 0							
N	3,836		3,836		3,836		3,836
number of groups	101		101		101		101
R-squared (within)	0.124		0.251		0.249		0.250
(between)	0.006		0.083		0.081		0.078
adj. R-squared	0.119		0.244		0.242		0.242

(Note) Estimated by the fixed effect model of the main bank factor. Dependent variable is rate. The sample includes those whose main bank is a regional bank only. Standard errors are the main-bank cluster robust standard errors, and are adjusted to include the standard error of the estimated influence coefficients. Constant term, the coefficients of industry (manufacturing, construction, real estate, retail, wholesale, communication, logistics, and others) and regional factors (Hokkaido, Tohoku, Kanto, Koshinetsu, Tokai, Hokuriku, Kansai, Chugoku, Shikoku, Kyushu, and Okinawa) are omitted from the report. *, **, and ***, indicate that the estimated coefficient is statistically significant at a 10%, 5%, and 1% significance level, respectively.

Table 8: Marginal effects

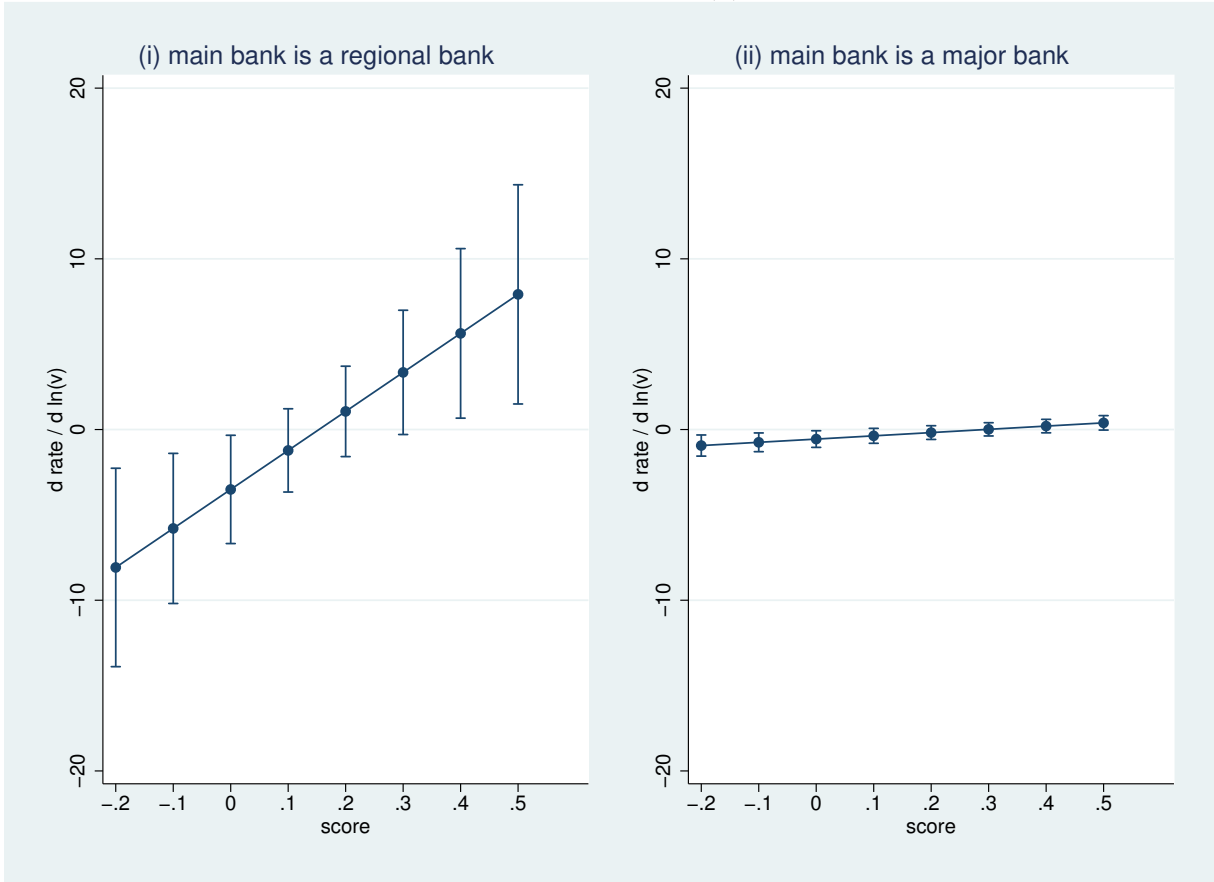
Table 7 (2)			
at score =	d rate/d ln(v)	(s.e.)	
-0.2	-8.079	2.965	***
-0.1	-5.794	2.243	**
0	-3.508	1.618	**
0.1	-1.223	1.244	
0.2	1.062	1.351	
0.3	3.348	1.857	*
0.4	5.633	2.534	**
0.5	7.918	3.276	**

(Note) Variables except score is set at sample mean. Calculated from the results (2) in Table 7. *, **, and ***, indicate that the estimated marginal effect is statistically significant at a 10%, 5%, and 1% significance level, respectively.

percentile. Thus, the effect of the influence coefficient is more economically significant for firms with a higher influence coefficient.

Among the control variables, DISTRESS has a positive and significant coefficient, and the interaction term of *score* and DISTRESS is positive and significant. This result suggests that the interest rates for distressed firms are typically higher, but that more severely distressed firms cannot afford to do anything but postpone the repayment of agreed interest (note that we define *rate* on the basis of actual interest payments by each firm). Those financially sound and secure firms with higher LN(INT_COV) pay lower interest. Those with larger amount of loans LN(LOAN) pay less interest per unit of loans. This result perhaps comes from the economy of scale in the provision of loans and entailed information production. The other indicators of creditworthiness of a firm, TANGIBLE, PROFITABLE, and LN(SALES) have a positive and significant coefficient. This result is counter-intuitive. It is likely that the result is driven by the fact that our interest rate measure is based on the interest that the firm actually paid rather than the contracted interest rate, and so the measure decreases by the delay in interest payments of less credit-worthy firms. Firms borrowing from more banks pay higher interests. This result indicates that those who are more credit-worthy and able to pay higher interests can borrow from multiple banks.

Figure 3: Marginal effect on $rate$ of $\ln(v)$, by main bank type



(Note) Vertical line segments indicate the 95% confidence interval.

7.4 Too-big-to-fail vs. too-connected-to-fail

A possible criticism against the above interpretation of the results is that the influence coefficient works as a proxy for the size of a firm. To make sure that the above results are not driven by this too-big-to-fail argument, we estimate the baseline model augmented by the interaction term of a proxy for the firm size, the natural logarithm of the number of employees, $\text{LN}(\text{EMPLOY})$, with the sample of firms whose main bank is a regional bank.

Column (1) in Table 9 is the result of the estimation by using the specification (2) of the baseline regression (Table 7) that is augmented by $\text{LN}(\text{EMPLOY})$ and its interaction with $score$. The signs of coefficients of the influence coefficient and its interaction term barely differ from Column (2) in Table 7. The coefficients of $\text{LN}(\text{EMPLOY})$ and its interaction term do not have a significant coefficient. Column (2) in Table 9 is the result by using the specification (3) of the baseline regression that is augmented by $\text{LN}(\text{EMPLOY})$. The coefficient of the influence

coefficient and its interaction term shows the result similar to the baseline regression of column (3) in Table 7. The coefficients indicate that the influence coefficient reduces the interest costs for firms in a distress. The coefficients of LN(EMPLOY) and its interaction term is significant but the signs are opposite to the prediction of the too-big-to-fail argument. Column (3) in Table 9 is the result of the augmented version of the specification (4) in Table 7. The signs and magnitudes of the estimated coefficients of the influence coefficients are similar to the baseline regression although they lose the statistical significance. The additional terms of LN(EMPLOY) do not have a significant coefficients. These results show that the result of the influence coefficient in the baseline regression is not driven by the too-big-to-fail argument.

7.5 Types of main bank

We estimate the model using the subsample of firms whose main bank is a major bank operating nation-wide or internationally, and the full sample including both groups of firms whose main bank is a regional bank and others. Comparing the results of these estimations and that in the previous section provides an empirical test of Hypothesis 2. Recall that we have surmised from our theoretical analysis that forbearance is more likely when a bank is dominant in its lending market and/or it actively engages in relationship banking. We thus expect that the interest reduction effect will be more weaker among firms with a major bank as their main bank.

Our empirical result supports this conjecture. The results are listed in Table 10. Column (1) is the result for firms whose main bank is a major bank. The marginal effect of the influence coefficient is illustrated in Figure 3 (ii). The result shows that the marginal effect of the influence coefficient is statistically significant. However, the magnitude is much smaller and is about one eighth of that of the firms whose main bank is a regional bank. Because of this, the estimation with the full sample including all groups of firms with major banks, regional banks, or cooperative banks as their main bank, shows less economically significant effect of the influence coefficient. Thus, the results support Hypothesis 2.

7.6 What features of regional banks matter?

As is noted above, many of regional banks operate as a dominant lender within a less competitive market in a rural area, and most of their borrowers are bank-dependent and privately-held companies that are not listed on the stock exchange. To examine which of these features is more

Table 9: Too-big-to-fail vs too-connected-to-fail (regional bank only, dependent variable: *rate*)

	(1) with LN(EMPLOY) × score	(2) with LN(EMPLOY) × DISTRESS	(3) LN(EMPLOY) × INSOLVENT
	coef. (s.e.)	coef. (s.e.)	coef. (s.e.)
ln(<i>v</i>)	-3.267 * (1.948)	2.017 (1.474)	1.324 (1.426)
ln(<i>v</i>) × score	21.693 ** (10.605)		
ln(<i>v</i>) × DISTRESS		-11.505 *** (2.664)	
ln(<i>v</i>) × INSOLVENT			-9.606 (6.033)
LN(EMPLOY)	-0.050 (0.040)	-0.067 * (0.037)	-0.043 (0.036)
LN(EMPLOY) × score	0.045 (0.161)		
LN(EMPLOY) × DISTRESS		0.145 *** (0.048)	
LN(EMPLOY) × INSOLVENT			-0.005 (0.043)
controls	yes	yes	yes
region factor	yes	yes	yes
industry factor	yes	yes	yes
main-bank fixed effect	yes	yes	yes
t-stat. (p-value)		-3.90 (0.000)	-1.43 (0.151)
H_0 : coef. of ln(<i>v</i>) + coef. of ln(<i>v</i>) × (DISTRESS or INSOLVENT) = 0			
t-stat. (p-value)		1.57 (0.116)	-0.80 (0.425)
H_0 : coef. of ln(EMPLOY) + coef. of ln(EMPLOY) × (DISTRESS or INSOLVENT) = 0			
R-squared (within)	0.252	0.252	0.250
(between)	0.086	0.089	0.085
adj. R-squared	0.244	0.244	0.242
N	3,836	3,828	3,828
Number of groups	101	101	101

(Note) Estimated by the fixed effect model of the main bank factor with the sample firms whose main bank is a regional bank. The dependent variable is rate. Standard errors are the main-bank cluster robust standard errors, and are adjusted to include the standard error of the estimated influence coefficients. The set of the control variables in Column (1), (2), and (3), are the same as that in the column (2), (3), and (4) in Table 5, respectively. *, **, and ***, indicate that the estimated coefficient is statistically significant at a 10%, 5%, and 1% significance level, respectively.

Table 10: Regression with the sample including major banks (dependent variable: *rate*)

	(1) major banks only			(2) full sample		
	coef.	(s.e.)		coef.	(s.e.)	
$\ln(v)$	-0.573	(0.244)	**	-1.095	(0.606)	*
$\ln(v) \times \text{score}$	1.922	(0.460)	***	3.895	(1.241)	***
controls	yes			yes		
region factor	yes			yes		
industry factor	yes			yes		
main bank fe	yes			yes		
R-squared (within)	0.335			0.268		
(between)	0.662			0.193		
adj. R-squared	0.326			0.264		
N	2,723			7,114		
Number of groups	17			294		
[Marginal Effect]	d rate/d $\ln(v)$	(s.e.)		d rate/d $\ln(v)$	(s.e.)	
at score = -0.2	-0.958	(0.312)	***	-1.874	(0.833)	**
-0.1	-0.766	(0.276)	***	-1.484	(0.718)	**
0	-0.573	(0.244)	**	-1.095	(0.606)	*
0.1	-0.381	(0.217)	*	-0.706	(0.499)	
0.2	-0.189	(0.197)		-0.316	(0.403)	
0.3	0.003	(0.186)		0.073	(0.326)	
0.4	0.195	(0.187)		0.463	(0.285)	
0.5	0.388	(0.198)	*	0.852	(0.295)	***

(Note) Estimated by the fixed effect model of the main bank factor. Standard errors are the main-bank cluster robust standard errors, and are adjusted to include the standard error of the estimated influence coefficients. Column (1) is the result of the estimation with the sample firms whose main bank is a major bank. Column (2) is the result of the estimation with the full sample. We include the major bank dummy and the regional bank dummy into the set of control variables for the estimation of (2). The other variables are the same as that in the column (2) in Table 7. In the calculation of the marginal effects, control variables except score are all set at sample mean. *, **, and ***, indicate that the estimated coefficient or the marginal effect is statistically significant at a 10 %, 5%, and 1% significance level, respectively.

relevant, we estimated the baseline model with subsamples, first, those split at the median of the Herfindahl index of bank branches (HI), and, second, those split at the bank-level sample median of the ratio of publicly-traded companies among borrowers of each bank.

The results of the subsample regression with respect to the market concentration are listed in Table 11. The estimated marginal effects show that the negative effect of the influence coefficient at a lower score is economically and statistically significant in the concentrated market only, although the statistical significance is marginal (column (2)).

The results of the subsample regression with respect to the bank dependence of borrowers are listed in Table 12. The estimated marginal effect is negative and statistically significant for firms with the score below zero in both subsamples. However, the economic significance is much

Table 11: Subsample Regression: Competitive market vs. concentrated market (dependent variable: *rate*)

	(1) competitive market			(2) concentrated market		
	coef.	(s.e.)		coef.	(s.e.)	
$\ln(v)$	-0.269	(0.337)		-2.448	(1.572)	
$\ln(v) \times \text{score}$	1.708	(0.584)	***	9.129	(3.866)	**
controls	yes			yes		
region factor	yes			yes		
industry factor	yes			yes		
main bank fe	yes			yes		
R-squared (within)	0.296			0.257		
(between)	0.205			0.144		
adj. R-squared	0.289			0.250		
N	3,467			3,647		
Number of groups	139			242		
[Marginal Effect]	d rate/d $\ln(v)$	(s.e.)		d rate/d $\ln(v)$	(s.e.)	
at score = -0.2	-0.611	(0.432)		-4.274	(2.207)	*
-0.1	-0.440	(0.383)		-3.361	(1.877)	*
0	-0.269	(0.337)		-2.448	(1.572)	
0.1	-0.099	(0.295)		-1.535	(1.312)	
0.2	0.072	(0.260)		-0.622	(1.126)	
0.3	0.243	(0.234)		0.290	(1.056)	
0.4	0.414	(0.221)	*	1.203	(1.122)	
0.5	0.585	(0.223)	***	2.116	(1.305)	

(Note) Estimated by the fixed effect model of the main bank factor. Column (1) is the estimation result with the sample firms whose HI (Herfindahl index of bank branches) are below median, 0.1642. Column (2) is that with the sample whose HI is median or above. Standard errors are the main-bank cluster robust standard errors, and are adjusted to include the standard error of the estimated influence coefficients. Control variables include those in the column (2) in Table 7. In the calculation of the marginal effects control variables except score are all set at sample mean. *, **, and ***, indicate that the estimated coefficient or the marginal effect is statistically significant at a 10%, 5%, and 1% significance level, respectively.

larger for the subsample of firms whose main bank lends more to non-listed firms (column (2)).

Thus, both characteristics of regional banks, as a dominant lender in a concentrated market, and as a major financier for bank-dependent non-listed companies, generate the motivation for regional banks to undertake a forbearance lending.

8 Robustness check

In this section, we examine the effects of alternative specifications of the influence coefficients to assure the robustness of our results.

Table 12: Subsample Regression: More listed companies vs. less (dependent variable: *rate*)

	(1) more listed			(2) less listed		
	coef.	(s.e.)		coef.	(s.e.)	
$\ln(v)$	-0.588	(0.346)	*	-3.159	(2.055)	
$\ln(v) \times \text{score}$	2.443	(0.714)	***	19.313	(5.872)	***
controls	yes			yes		
region factor	yes			yes		
industry factor	yes			yes		
main bank fe	yes			yes		
R-squared (within)	0.310			0.241		
(between)	0.438			0.187		
adj. R-squared	0.303			0.234		
N	3,483			3,631		
Number of groups	23			271		
[Marginal Effect]	d rate/d $\ln(v)$	(s.e.)		d rate/d $\ln(v)$	(s.e.)	
at score = -0.2	-1.076	(0.461)	**	-7.021	(2.997)	**
-0.1	-0.832	(0.401)	**	-5.090	(2.502)	*
0	-0.588	(0.346)	*	-3.159	(2.055)	
0.1	-0.343	(0.297)		-1.227	(1.696)	
0.2	-0.099	(0.259)		0.704	(1.490)	
0.3	0.145	(0.238)		2.635	(1.501)	*
0.4	0.389	(0.236)		4.567	(1.725)	***
0.5	0.634	(0.256)	**	6.498	(2.094)	***

(Note) Estimated by the fixed effect model of the main bank factor. Column (1) is the estimation result with the sample firms whose main bank has a ratio of companies listed on the stock exchange equal or higher than the median, 0.073. Column (2) is that with the other sample firms. Standard errors are the main-bank cluster robust standard errors, and are adjusted to include the standard error of the estimated influence coefficients. Control variables include those in the column (2) in Table 7. In the calculation of the marginal effects control variables except score are all set at sample mean. *, **, and ***, indicate that the estimated coefficient or the marginal effect is statistically significant at a 10%, 5%, and 1% significance level, respectively.

8.1 Alternative specification for influence coefficients

Since the influence coefficients are estimated and not directly observable, it is important to check whether our results are driven by the particular specification used in estimation. To do so, we consider an alternative procedure for estimating the influence coefficients. The result demonstrates that our results are robust in this regard.

The alternative methods for estimating influence coefficients are also based on the sales equation (13) but use a different specification of the matrix \mathbf{Q} . The alternative specification of \mathbf{Q} is $\mathbf{Q}_s = \beta_s \mathbf{G}\mathbf{S}$, where \mathbf{S} is the $n \times n$ diagonal matrix whose i th diagonal element is the square root of firm i 's credit score (as a percent of maximum, achieved by dividing the raw score by 100) provided by TSR as of 2005. In this way, \mathbf{Q}_s reflects to some extent the possibility of firm

Table 13: Alternative influence coefficients

Variable	N	mean	sd	min	p10	p50	p75	p90	p95	p99	max
v_s	306,354	1.003	0.011	1.000	1.000	1.002	1.003	1.005	1.008	1.022	2.700
v_w	651,913	1.010	0.054	1.000	1.000	1.004	1.010	1.019	1.028	1.071	11.490
v_{ws}	651,913	1.008	0.051	1.000	1.000	1.003	1.008	1.015	1.023	1.062	11.161
$v - v_w$	306,354	0.011	0.069	0.000	0.000	0.004	0.011	0.022	0.034	0.093	9.690

(a) Description of alternative influence coefficients

	Estimate	s.e.	
β_s	0.00337	0.0000708	***
Industry factor	yes		
Prefecture factor	yes		
Adj. R-squared	0.1457		

(b) First-stage regression to calculate influence coefficient v_s (N = 651,913)

Variable	v	v_s	v_w
v_s	0.9969	1	
v_w	0.7481	0.7525	1
v_{ws}	0.7493	0.7573	0.9981

(c) Pairwise correlation coefficients among alternative influence coefficients

(Note for panel (b)) Industry factor is controlled by the 95 sector dummies, which indicate one 2-digit *chu-bunrui* classification. Prefecture factor is controlled by 46 prefecture dummies (Hokkaido is the base class).

defaults. We estimate the following model by OLS

$$\Delta \mathbf{s} = \beta_s \mathbf{G} \mathbf{S} \Delta \mathbf{s} + \gamma_I' \mathbf{Ind} + \gamma_P' \mathbf{Pre} + \epsilon.$$

We then compute

$$\hat{\mathbf{v}}_s^{(b)'} = \mathbf{1}' \sum_{k=1}^{100} (\hat{\beta}_s \mathbf{G}^{(b)} \mathbf{S}^{(b)})^k, \quad (30)$$

where $\mathbf{S}^{(b)}$ is the submatrix of \mathbf{S} induced by including only firms in bank b 's network.

We find that the two types of influence coefficient estimates are similar and which estimate is used does not affect our main results for regression on interest rate. The first row in panel (a) of Table 13 shows the descriptive statistics of the estimated influence coefficients for the supply networks of firms with a common main bank v_s , which is estimated by (30). The underlying

Table 14: Regression with the alternative influence coefficients (regional bank only, dependent variable: *rate*)

influence coef. =	(1) $\ln(v_s)$	(2) $\ln(v_w)$	(3) $\ln(v_{ws})$
	coef.	coef.	coef.
	(s.e.)	(s.e.)	(s.e.)
influ. coef.	-4.308 ** (1.942)	-1.462 ** (0.669)	-1.816 ** (0.795)
influ. coef. \times score	26.110 *** (9.097)	8.182 *** (2.024)	9.460 *** (2.280)
controls	yes	yes	yes
industry factor	yes	yes	yes
region factor	yes	yes	yes
main-bank fixed effect	yes	yes	yes
N	3,836	4,690	4,690
Number of groups	101	107	107
R-squared (within)	0.251	0.246	0.246
(between)	0.083	0.221	0.223
adj. R-squared	0.244	0.240	0.240

(Note) Estimated by the fixed effect model of the main bank factor with the sample firms whose main bank is a regional bank. Standard errors are the main-bank cluster robust standard errors, and are adjusted to include the standard error of the estimated influence coefficients. Constant term, the coefficients of industry and regional factors are omitted from the report. The control variables are the same as that in the column (2) in Table 5. *, **, and ***, indicate that the estimated coefficient is statistically significant at a 10%, 5%, and 1% significance level, respectively.

regression results are presented in panel (b). The two versions of influence coefficients, v and v_s , are highly correlated with each other. The correlation coefficients between v and v_s is 0.9969, as seen in panel (c). The empirical results with the alternative version of the influence coefficient (column (1) in Table 14) are very similar to the baseline result in column (2), Table 7.

8.2 Economy-wide network

We also re-estimate the baseline model with the influence coefficient in the economy-wide supply network. We conduct this analysis to confirm that the effect of influence coefficients appears through the network-motivated lending decision, and not through some other mechanism that affects both the location of firms in the network and the interest rate. Our result showing that this is related to Hypothesis 2 indicates that influence coefficients defined in terms of bank-specific supply networks are more appropriate than economy-wide influence coefficients. This is because a bank is highly likely to be a dominant financier and be able to observe the influence of each firm in the supply network of its borrowers. In contrast, it is less likely that a bank will

Table 15: Marginal Effect

at score =	Table 14 (2)		
	d rate/d ln(v)	(s.e.)	
-0.2	-3.098	0.978	***
-0.1	-2.280	0.813	***
0	-1.462	0.669	**
0.1	-0.644	0.563	
0.2	0.175	0.517	
0.3	0.993	0.548	*
0.4	1.811	0.644	***
0.5	2.629	0.782	***

(Note) Variables except score is set at sample mean. Calculated from the results in (2) in Table 14. *, **, and ***, indicate that the estimated coefficient is statistically significant at a 10%, 5%, and 1% significance level, respectively.

consider the influence of a borrower on the economy-wide supply network. We thus expect to observe a weak effect of the influence coefficient when it is defined in terms of the economy-wide supply network. Consistent with this prediction, we find a smaller effect from the economy-wide influence coefficients than from the bank-specific coefficients.

The economy-wide influence coefficients are computed similarly to (29) and (30), but we use the entire matrix \mathbf{Q} . For example, v_w , which is the economy-wide version of v , is computed by

$$\hat{\mathbf{v}}'_w = \mathbf{1}' \sum_{k=1}^{100} (\hat{\beta} \mathbf{G})^k,$$

where $\hat{\mathbf{v}}_w$ is the estimated vector of v_w for all the firms. Similarly, $\hat{\mathbf{v}}_{ws}$, which is the economy-wide version of $\hat{\mathbf{v}}_s$, is computed by

$$\hat{\mathbf{v}}'_{ws} = \mathbf{1}' \sum_{k=1}^{100} (\hat{\beta}_s \mathbf{G}_s)^k.$$

Rows 2 and 3 in panel (a) of Table 13 show the descriptive statistics of the estimated influence coefficients of the economy-wide supply network, v_w and v_{sw} (panel (b)). Compared with the influence coefficients defined in terms of bank-specific networks, the economy-wide influence coefficients are larger and more variable. Their distribution is more strongly skewed. This result is natural because the number of firms in the economy-wide supply network is much larger and many firms have transactions with firms that do not share their main bank.

There is a considerable difference between the economy-wide influence coefficient and the bank-specific influence coefficient. The correlation coefficient between the alternative economy-wide influence coefficients and those defined in the bank-specific networks is about 0.75 (panel

(c)), while the correlation coefficient between alternative economy-wide influence coefficients is almost one. The last row in panel (a) is the descriptive statistics of the difference between the bank-specific influence coefficient and the economy-wide influence coefficient. The difference is larger in the upper tail.

The results in columns (2) and (3) of Table 14 demonstrate that the economy-wide influence coefficients have a much weaker effect on average interest rate, as expected. The coefficient of the influence-coefficient term is negative and statistically significant, and the interaction term between the influence coefficient and *score* is positive and statistically significant. The statistical significance is stronger in the economy-wide network due to the higher variation of the influence coefficient. However, the coefficient and the marginal effect are less than half of the estimate obtained by using bank-specific networks (Table 15). This result is consistent with our theoretical analysis.

9 Discussions

So far, we have shown the possibility of forbearance resulting from the profit-maximization of a bank. However, we have not discussed its welfare implication.

In our specific model, we can show that forbearance is welfare-improving. The social welfare in this case is the aggregate indirect utility of households minus the total cost of producing final products after netting out the intermediate inputs within the network. We can show that the aggregate indirect utility of households equals $\sum_{h=1}^H R_h/p_c$ from the households' optimization. Thus, the social welfare is equal to

$$\frac{\sum_{h=1}^H R_h}{p_c} - (1 + \rho) \sum_{i=1}^n e_i F_i - \frac{p_0 \sum_{i=1}^n e_i x_{i0}}{p_c}. \quad (31)$$

It is easy to show that this is equivalent to the total real profit of firms in the network so long as good 0, the intermediate input from the outside of the network, is used only for production

and not for consumption. The total real profit of firms within the network is²⁷

$$\frac{\sum_{i=1}^n e_i(p_i x_i - p^i x_i)}{p_c} - (1 + \rho) \sum_{i=1}^n e_i F_i = \frac{\sum_{h=1}^H R_h}{p_c} - \frac{p_0 \sum_{i=1}^n e_i x_{i0}}{p_c} - (1 + \rho) \sum_{i=1}^n e_i F_i.$$

Clearly, this is equivalent to the social welfare (31). Thus, the profit maximizing bank behavior also maximizes the social welfare in our setup.

However, in light of the special features in our model; 1. network is fixed, 2. fund demand is fixed, and 3. the moral hazard of the influence firm is ignored, we need to raise several caveats about the above welfare implications.

First, as we note, our analysis is a short-run analysis in the sense that the new link is not allowed. We assume that the technological importance of inputs w_{ij} , which is the primary determinant of the influence coefficient of each firm, is fixed. In other words, we do not allow for entries from outside of the given supply network. This assumption may be plausible in the short run. The analytical part of Caballero et al. (2008) adopts the assumption at the opposite extreme: that there are always potential entrants who are more efficient than the incumbent firms. That assumption is suitable for a long-run welfare analysis because the supply network will be flexible in the long run. Empirical studies are needed to clarify how rigid the supply network is.²⁸ Nonetheless, we conjecture that whether such efficient potential entrants exist depends on the type of industry and the economic environment at each time point. For example, if an industry requires the accumulation of relation-specific information and design to improve productivity or product quality, which is known to occur in the automotive and financial sectors, then potential entrants are not likely to be more efficient than incumbent firms. However, if an industry treats commodities that are less differentiated and do not require relation-specific investments, then potential entrants could be more efficient than incumbent firms. Thus, the

²⁷This equality holds because

$$\begin{aligned} \sum_{i=1}^n e_i(p_i x_i - p^i x_i) &= \sum_{i=1}^n e_i p_i x_i - \sum_{i=1}^n \sum_{j=0}^n e_i p_j x_{ij} = \sum_{i=1}^n e_i p_i c_i + \sum_{i=1}^n p_i \sum_{j=1}^n e_i x_{ji} - \sum_{j=0}^n p_j \sum_{i=1}^n e_i x_{ij} \\ &= \sum_{h=1}^H R_h - p_0 \sum_{i=1}^n e_i x_{i0}. \end{aligned}$$

²⁸Known empirical results on this point are mixed. For example, Fukao and Kwon (2006) and Nishimura et al. (2005) find that less efficient companies increased their market shares and were less likely to exit in the 1990s in Japan. In contrast, Sakai et al. (2010) find that those that exited were less profitable than those that survived in the 1990s, from examining extensive microdata of small business lending in Japan. Calvalho et al. (2014) finds that a firm whose suppliers or corporate customers are damaged by a great earthquake is more likely to search and find a new supplier or a new customer. However, the economic significance of such adjustment in a circumstance without such an extreme shock could be smaller.

applicability of our results on the welfare analysis will differ across economies and sectors. In any case, it is important to note that what we examine is short-run welfare, and the implications for long-run welfare could be different.

Second, while we discuss the welfare implication for an economy with rigid supply networks, there is some concern about whether an economy with rigid supply networks is efficient. It has been pointed out that economies in which specificity works as a barrier to entry, termed *sclerosis* in Caballero and Hammour (1998), is inherently inefficient. Our efficiency result for network-motivated forbearance says that forbearance could be better for the economic efficiency under the assumption that the economy had already been constructed and is full of relation-specific investments. Although it is clearly beyond the scope of the current paper, it would be interesting to examine the possibility that constructing the economy without such specificity from the beginning would be better for social welfare.

Third, in our setting, the capital cost of external finance is not related to the output level. The usual route from the monopolistic capital cost to welfare loss is through the reduction of output caused by high capital costs. However, this channel of welfare loss is shut down by making sales independent of capital costs in our setup to make the analysis tractable. If this route is taken into account, the ability of the bank to recoup the forbearance cost by imposing higher interest rates on loans to peripheral firms could be limited. This might reduce the chance of forbearance, and the welfare consequences of forbearance will be ambiguous under this scenario.

Fourth, we do not explicitly include the possible moral hazard for an influential company entailed by being considered too big to fail or too connected to fail. Such a company can expect a bailout by a bank or a government, and this assurance is likely to provide perverse incentives for managers and shareholders. This point might affect the welfare implications. In particular, influential firms might become more inefficient and thereby inflate the cost of forbearance. Even in this case, we conjecture that rational forbearance by a profit maximizing bank would improve welfare. However, this possibility makes it difficult to examine whether public bailouts improve welfare.

10 Concluding remarks

We showed that a bank acting as the dominant financier to an inter-firm supply network will be motivated to strategically support an influential producer in the network by forbearance lending, and we illustrated the conditions under which this behavior would emerge by using an oligopoly model of final and intermediate products with an incomplete supply network. Our analytical model shows that this network-motivated forbearance is an economically rational response by a bank. Our empirical study provides evidence for this motivation by statistically verifying that the interest rate of loans is lower for influential firms within the supply networks among borrowers of a bank, and this tendency is more pronounced for financially less sound firms and those whose main bank is a regional bank, even after controlling for all other observable factors.

Our findings suggest that the terms of a financial contract can be affected by the importance of the contracting firm relative to other firms within the portfolio of a bank. Thus, we need to look at the relative position of each firm within the portfolio of a bank and also at the characteristics of individual firms and banks in examining the economic efficiency of loan contracts and the financial market.

As concluding remarks, we would like to comment on some of the abundant remaining research questions and directions. First, it would be interesting to extend our analysis to the case in which the supply network is flexible. We have assumed that the supply network is rigid, which implies that we examine the short-run effect. In the short run, it is hard to find a corporate client and to create a new link in the supply network. However, these strategic behaviors of banks could harm economic efficiency in the long run because, in the long run, a firm can find new corporate clients. Thus, forbearance may deter natural selection and crowd out potentially efficient new entrants, as is shown by [Caballero et al. \(2008\)](#). Evaluating the welfare implication of forbearance lending (or public bailouts) with taking into account both of these effects remains a challenging future research subject.

Second, cross-country comparisons or comparisons between bank-dominant markets and market-oriented markets would be useful. Our model shows that a bank that is dominant in some market is more likely to undertake network-motivated lending decisions. This finding highlights a novel viewpoint for discussing the efficient structure of the financial market in conjunction with the argument in the previous paragraph.

Lastly, our empirical study illustrates the usefulness of the inter-firm transaction database for financial researchers as well as for policy makers. In this study, we estimate the influence coefficient of each firm by making use of limited, but available, information on inter-firm transactions. More precise inter-firm transaction data would improve the accuracy of influence coefficients and ease computation of them. This sort of database enables us to identify influential companies within a portfolio of a bank or within an economy, and is useful for policy makers, banks, institutional investors, and academic researchers.

References

- Acemoglu, Daron, Vasco Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2012) “The Network Origins of Aggregate Fluctuations,” *Econometrica*, Vol. 80, No. 5, pp. 1977-2016.
- Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2015) “Systemic Risk and Stability in Financial Networks,” *American Economic Review*, Vol. 105, No. 2, pp. 564-608.
- Allen, Franklin and Douglas Gale (2000) “Financial Contagion,” *Journal of Political Economy*, Vol. 108, No. 1, pp. 1-33.
- Almeida, Heitor, Chang-Soo Kim, and Hwanki Brian Kim (2015) “Internal Capital Markets in Business Group: Evidence from the Asian Financial Crisis,” *Journal of Finance*, Vol. 70, No. 6, pp. 2539-86.
- Battiston, Stefano, Domenico Delli Gatti, Mauro Gallegati, Bruce Greenwald, and Joseph E. Stiglitz (2007) “Credit Chains and Bankruptcy Propagation in Production Networks,” *Journal of Economic Dynamics & Control*, Vol. 31, pp. 2061-2084.
- Berglöf, Erik and Gérard Roland (1997) “Soft Budget Constraints and Credit Crunches in Financial Transition,” *European Economic Review*, Vol. 41, No. 3-5, pp. 807-817.
- Bharath, Sreedhar, Sandeep Dahiya, Anthony Saunders, and Anand Srinivasan (2011) “Lending Relationships and Loan Contract Terms,” *Review of Financial Studies*, Vol. 24, pp. 1141-1203.
- Bigio, Saki and Jennifer La’O (2013) “Financial Frictions in Production Networks,” July, mimeo. Columbia University.

- Billio, Monica, Mila Getmansky, Andrew W. Lo, and Lioriana Pelizzon (2012) “Econometric measures of connectedness and systemic risk in the finance and insurance sectors,” *Journal of Financial Economics*, Vol. 104, pp. 535-559.
- Boissay, Frederic (2006) “Credit Chains and the Propagation of Financial Distress,” European Central Bank, Working Paper Series No. 573.
- Boissay, Frederic and Reint Gropp (2013) “Payment Defaults and Interfirm Liquidity Provision,” *Review of Finance*, Vol. 17, pp. 1853-1894.
- Bonacich, Phillip (1987) “Power and Centrality: A Family of Measures,” *American Journal of Sociology*, Vol. 92, pp. 1170-1182.
- Boot, Arnoud W. A. and Anjan V. Thakor (2000) “Can Relationship Banking Survive Competition?” *Journal of Finance*, Vol. 55, No. 2, pp. 679-713.
- Bowker, Albert H. (1947) “On the Norm of a Matrix,,” *Annals of Mathematical Statistics*, Vol. 18, No. 2, pp. 285-288.
- Bruche, Max and Gerard Llobet (2014) “Preventing Zombie Lending,” *Review of Financial Studies*, Vol. 27, No. 3, pp. 923-956.
- Caballero, Ricardo J., Takeo Hoshi, and Anil K. Kashyap (2008) “Zombie Lending and Depressed Restructuring in Japan,” *American Economic Review*, Vol. 98, No. 5, pp. 1943-1977.
- Caballero, Ricardo J. and Mohamad L. Hammour (1998) “The Macroeconomics of Specificity,” *Journal of Political Economy*, Vol. 106, No. 4, pp. 724-767.
- Calvalho, Vasco M., Makoto Nirei, and Yukiko Umeno Saito (2014) “Supply Chain Disruptions: Evidence from the Great East Japan Earthquake,” RIETI Discussion Paper Series 14E035.
- Castiglionesi, Fabio and Noemí Navarro (2011) “Fragile Financial Networks,” mimeo.
- Chen, Tsung-Kang, Hsien-Hsing Liao, and Hui-Ju Kuo (2013) “Internal Liquidity Risk, Financial Bullwhip Effects, and Corporate Bond Yield Spreads: Supply Chain Perspectives,” *Journal of Banking and Finance*, Vol. 37, pp. 2434-2456.

- Degryse, Hans and Patrick van Cayseele (2000) "Relationship Lending within A Bank-Based System: Evidence from European Small Business Data," *Journal of Financial Intermediation*, Vol. 9, No. 1, pp. 90-109.
- Dewatripont, Mathias F. and Eric S. Maskin (1995) "Credit and Efficiency in Centralized and Decentralized Economies," *Review of Economic Studies*, Vol. 62, No. 4, pp. 541-555.
- Dupor, Bill (1999) "Aggregation and irrelevance in multi-sector models," *Journal of Monetary Economics*, Vol. 43, pp. 391-409.
- Eisenberg, Larry and Thomas H. Noe (2001) "Systemic Risk in Financial Systems," *Management Science*, Vol. 47, No. 2, pp. 236-249.
- Elliot, Matthew, Benjamin Golub, and Matthew O. Jackson (2014) "Financial Networks and Contagion," *American Economic Review*, Vol. 104, No. 10, pp. 3115-3153.
- Fukao, Kyoji and Hyeog-Ug Kwon (2006) "Why Did Japan's TFP Growth Slow Down in the Lost Decade? An Empirical Analysis Based on Firm-Level Data of Manufacturing Firms," *Japanese Economic Review*, Vol. 57, No. 2, pp. 195-228.
- Fukuda, Shin'ichi, Munehisa Kasuya, and Jochi Nakajima (2007) "Hi Jojo Kigyo Ni "Oikashi" Wa Sonzai Shitaka? (Did forbearance exist among non-listed companies?)," *Kin'yu Kenkyu (Bank of Japan)*, pp. 73-104, February.
- Gai, Prasanna, Andrew Haldane, and Sujit Kapadia (2011) "Complexity, concentration and contagion," *Journal of Monetary Economics*, Vol. 58, pp. 453-470.
- Giesecke, Kay and Stefan Weber (2006) "Credit Contagion and Aggregate Losses," *Journal of Economic Dynamics & Control*, Vol. 30, pp. 741-767.
- Gopalan, Radhakrishnan, Vikram Nanda, and Amit Seru (2007) "Affiliated firms and financial support: Evidence from Indian business groups," *Journal of Financial Economics*, Vol. 86, pp. 759-795.
- Hamao, Yasushi, Kenji Kutsuna, and Joe Peek (2012) "Nice to Be on the A-list," Federal Reserve Bank of Boston Working Papers No. 12-13.

- Hertzel, Michael G., Zhi Li, Micah S. Officer, and Kimberly J. Rodgers (2008) “Inter-Firm Linkages and the Wealth Effects of Financial Distress along the Supply Chain,” *Journal of Financial Economics*, Vol. 87, pp. 374-387.
- Horvath, Michael (2000) “Sectoral Shocks and Aggregate Fluctuations,,” *Journal of Monetary Economics*, Vol. 45, pp. 69-106.
- Hosono, Kaoru (2008) “Chusho Kigyo Muke Yushi Wa Tekisetsu Ni Kin’ri Settei Sareteiruka (Are the SME loans priced appropriately?),” in Watanabe, Tsutomu and Iichiro Uesugi eds. *Ken’sho Chusho Kigyo Kin’yu (Investigation: Small and Medium-Sized Enterprise Financing)*: Nihon Keizai Shinbun Shuppan Sha, (in Japanese).
- Ioannidou, Vasso and Steven Ongena (2010) “‘Time for a Change’: Loan Conditions and Bank Behavior When Firms Switch Banks,” *Journal of Finance*, Vol. 65, No. 5, pp. 1847-1877.
- Kelejian, Harry H. and Ingmar R. Prucha (1998) “A Generalized Spatial Two-Stage Least Squares Procedure for Estimating a Spatial Autoregressive Model with Autoregressive Disturbances,” *Journal of Real Estate Finance and Economics*, Vol. 17, No. 1, pp. 99–121.
- Kiyotaki, Nobuhiro and John Moore (1997) “Credit Chains,” mimeo. University of Minnesota and London School of Economics.
- Lee, Lung-Fei (2002) “Consistency and Efficiency of Least Squares Estimation for Mixed Regressive, Spatial Autoregressive Models,” *Econometric Theory*, Vol. 18, pp. 252–277.
- Leitner, Yaron (2005) “Financial Networks: Contagion, Commitment, and Private Sector Bailouts,” *Journal of Finance*, Vol. 60, No. 6, pp. 2925-2953.
- Long, John B. and Charles I. Plosser (1983) “Real Business Cycles,” *Journal of Political Economy*, Vol. 91, pp. 39-69.
- Mankiw, Gregory and Michael D. Whinston (1986) “Free Entry and Social Inefficiency,” *RAND Journal of Economics*, Vol. 17, pp. 48-58.
- Morck, Randall (2009) “The Riddle of the Great Pyramids,” NBER Working Paper Series 14858.
- Nemoto, Tadanobu, Yoshiaki Ogura, and Wako Watanabe (2011) “An Estimation of the Inside Bank Premium,” RIETI Discussion Paper Series 11E067.

- Nier, Erlend, Jing Yang, Tanju Yorulmazer, and Amadeo Alentorn (2007) “Network models and financial stability,” *Journal of Economic Dynamics & Control*, Vol. 31, pp. 2033-2060.
- Nishimura, Kiyohiko, Takanobu Nakajima, and Kozo Kiyota (2005) “Does the Natural Selection Mechanism Still Work in Severe Recessions? Examination of the Japanese Economy in the 1990s,” *Journal of Economic Behavior & Organization*, Vol. 58, No. 1, pp. 53-78.
- Ogawa, Kazuo (2008) “Kashi Hagashi No Eikyo Wa Shinkoku Dattanoka (Are the impact of the withdrawal of loans serious?),” in Watanabe, Tsutomu and Iichiro Uesugi eds. *Ken'sho Chusho Kigyo Kin'yu (Investigation: Small and Medium-Sized Enterprise Financing)*: Nihon Keizai Shinbun Shuppan Sha, (in Japanese).
- Peek, Joe and Eric S. Rosengren (2005) “Unnatural selection: Perverse incentives and the misallocation of credit in Japan,” *American Economic Review*, Vol. 95, pp. 1144-1166.
- Petersen, Mitchell A. and Raghuram G. Rajan (1994) “The Benefits of Lending Relationships: Evidence from Small Business Data,” *Journal of Finance*, Vol. 49, No. 1, pp. 3-37.
- Rajan, Raghuram G. (1992) “Insiders and Outsiders: The Choice between Informed and Arm's-Length Debt,” *Journal of Finance*, Vol. 47, No. 4, pp. 191-212.
- Rogers, L. C. G. and L. A. M. Veraart (2013) “Failure and Rescue in and Interbank Network,” *Management Science*, Vol. 59, No. 4, pp. 882-898.
- Sakai, Koji, Iichiro Uesugi, and Tsutomu Watanabe (2010) “Firm Age and the Evolution of Borrowing Costs: Evidence from Japanese Small Firms,” *Journal of Banking & Finance*, Vol. 34, No. 8, pp. 1970-1981.
- Sekine, Toshitaka, Keiichiro Kobayashi, and Yumi Saita (2003) “Forbearance Lending: The Case of Japanese Firms,” *Monetary and Economic Studies*, Vol. 21, No. 2, pp. 69-92.
- Sharpe, Steven A. (1990) “Asymmetric Information, Bank Lending and Implicit Contracts: A Stylized Model of Customer Relationships,” *Journal of Finance*, Vol. 45, No. 4, pp. 1069-1087.
- Slovin, Myron B., Marie E. Sushka, and John A. Polonchek (1999) “An analysis of contagion and competitive effects at commercial banks,” *Journal of Financial Economics*, Vol. 54, pp. 197-225.

Uchida, Hirofumi, Gregory F. Udell, and Wako Watanabe (2008) “Bank Size and Lending Relationships,” *Journal of the Japanese and International Economies*, Vol. 22, No. 2, pp. 242-267.

Uchida, Hirofumi, Gregory F. Udell, and Nobuyoshi Yamori (2012) “Loan Officers and Relationship Lending to SMEs,” *Journal of Financial Intermediation*, Vol. 21, No. 1, pp. 97-122.

Appendix 1: Proof of Proposition 2.

The real profit of the network before subtracting the fixed costs of firms under the given equilibrium product prices is

$$\pi(\mathbf{E}) = \frac{\mathbf{v}'\mathbf{f}}{p_c\theta} = \frac{Rp_c^{\theta-2}}{\theta} \cdot \mathbf{1}'\mathbf{P}_\theta \left[\mathbf{I} - \left(\frac{\theta-1}{\theta} \right)^\theta \mathbf{E}\mathbf{W}' \right]^{-1} \mathbf{E}\mathbf{1}, \quad (32)$$

where \mathbf{P}_θ is a $n \times n$ diagonal matrix of which i th diagonal element is $p_i^{1-\theta}$, and \mathbf{E} is a $n \times n$ diagonal matrix of which i th diagonal element is e_i . We plug the f.o.c. of firm i (10) into p^j of the definition of $q_{ij} \equiv e_i w_{ji} p_i^{1-\theta} p^j / p_j$ to get this expression.

Let \mathbf{E}_1 be the $n \times n$ diagonal matrix whose i -th diagonal element is $e_i = 1$ for $i \in S$, $e_i = 0$ for $i \in S^c \setminus \{z\}$, and $e_z = 1$. Let \mathbf{E}_0 be the $n \times n$ diagonal matrix whose i -th diagonal element is $e_i = 1$ for $i \in S$, $e_i = 0$ for $i \in S^c \setminus \{z\}$, and $e_z = 1$.

We then compute $\pi(\mathbf{E}_1) - \pi(\mathbf{E}_0)$ holding p_c and \mathbf{P}_θ fixed. Let

$$\mathbf{A}_{e_z} = \mathbf{I} - \left(\frac{\theta-1}{\theta} \right)^\theta \mathbf{E}_{e_z} \mathbf{W}', \quad e_z \in \{0, 1\}. \quad (33)$$

Then, we have

$$\begin{aligned} \pi(\mathbf{E}_1) - \pi(\mathbf{E}_0) &= \frac{Rp_c^{\theta-2}}{\theta} \cdot \mathbf{1}'\mathbf{P}_\theta (\mathbf{A}_1^{-1} \mathbf{E}_1 \mathbf{1} - \mathbf{A}_0^{-1} \mathbf{E}_0 \mathbf{1}) \\ &= \frac{Rp_c^{\theta-2}}{\theta} \cdot \mathbf{1}'\mathbf{P}_\theta \mathbf{A}_0^{-1} ((\mathbf{A}_0 - \mathbf{A}_1) \mathbf{A}_1^{-1} \mathbf{E}_1 \mathbf{1} + (\mathbf{E}_1 \mathbf{1} - \mathbf{E}_0 \mathbf{1})) \\ &= \frac{Rp_c^{\theta-2}}{\theta} \cdot \mathbf{1}'\mathbf{P}_\theta \mathbf{A}_1^{-1} (\mathbf{E}_1 \mathbf{1} - \mathbf{E}_0 \mathbf{1}) \end{aligned} \quad (34)$$

$$+ \frac{Rp_c^{\theta-2}}{\theta} \cdot \mathbf{1}'\mathbf{P}_\theta \mathbf{A}_1^{-1} (\mathbf{A}_1 - \mathbf{A}_0) \mathbf{A}_0^{-1} (\mathbf{E}_1 \mathbf{1} - \mathbf{E}_0 \mathbf{1}) \quad (35)$$

$$+ \frac{Rp_c^{\theta-2}}{\theta} \cdot \mathbf{1}'\mathbf{P}_\theta \mathbf{A}_0^{-1} (\mathbf{A}_0 - \mathbf{A}_1) \mathbf{A}_1^{-1} \mathbf{E}_1 \mathbf{1}. \quad (36)$$

We note that $\mathbf{E}_1\mathbf{1} - \mathbf{E}_0\mathbf{1}$ is the z -th unit vector, the vector whose z -th element is 1 and other elements are 0. Thus, the term (34) is

$$\frac{Rp_c^{\theta-2}}{\theta} \cdot \mathbf{1}'\mathbf{P}_\theta\mathbf{A}_1^{-1}(\mathbf{E}_1\mathbf{1} - \mathbf{E}_0\mathbf{1}) = \frac{v_z p_z c_z}{\theta p_c} \Big|_{\mathbf{E}=\mathbf{E}_1}. \quad (37)$$

Next, we consider the second term (35). Note that

$$\mathbf{A}_0 - \mathbf{A}_1 = \left(\frac{\theta-1}{\theta}\right)^\theta (\mathbf{E}_1 - \mathbf{E}_0)\mathbf{W}'. \quad (38)$$

Thus, we have

$$\begin{aligned} & \frac{Rp_c^{\theta-2}}{\theta} \cdot \mathbf{1}'\mathbf{P}_\theta\mathbf{A}_1^{-1}(\mathbf{A}_1 - \mathbf{A}_0)\mathbf{A}_0^{-1}(\mathbf{E}_1\mathbf{1} - \mathbf{E}_0\mathbf{1}) \\ &= \frac{Rp_c^{\theta-2}}{\theta} \left(\frac{\theta-1}{\theta}\right)^\theta \cdot \mathbf{1}'\mathbf{P}_\theta\mathbf{A}_1^{-1}(\mathbf{E}_1 - \mathbf{E}_0)\mathbf{W}'\mathbf{A}_0^{-1}(\mathbf{E}_1\mathbf{1} - \mathbf{E}_0\mathbf{1}) \\ &= \frac{Rp_c^{\theta-2}}{\theta} \left(\frac{\theta-1}{\theta}\right)^\theta v_{z,\mathbf{E}_1} p_z^{1-\theta} \sum_{k=1}^N w_{kz} \omega_0^{kz} \\ &= \left(\frac{\theta-1}{\theta}\right)^\theta \frac{v_z p_z c_z}{\theta p_c} \Big|_{\mathbf{E}=\mathbf{E}_1} \sum_{k=1}^N w_{kz} \omega_0^{kz}. \end{aligned} \quad (39)$$

where v_{z,\mathbf{E}_1} is v_z evaluated under $\mathbf{E} = \mathbf{E}_1$, $\omega_{e_z}^{kz}$ is the (k, z) -th element of $\mathbf{A}_{e_z}^{-1}$. Note that this is positive and tends to 0 as $\theta \rightarrow 1$. Lastly, we consider the third term. We have

$$\begin{aligned} & \frac{Rp_c^{\theta-2}}{\theta} \cdot \mathbf{1}'\mathbf{P}_\theta\mathbf{A}_0^{-1}(\mathbf{A}_0 - \mathbf{A}_1)\mathbf{A}_1^{-1}\mathbf{E}_1\mathbf{1} \\ &= \frac{Rp_c^{\theta-2}}{\theta} \left(\frac{\theta-1}{\theta}\right)^\theta \cdot \mathbf{1}'\mathbf{P}_\theta\mathbf{A}_0^{-1}(\mathbf{E}_1 - \mathbf{E}_0)\mathbf{W}'\mathbf{A}_1^{-1}\mathbf{E}_1\mathbf{1} \\ &= \frac{Rp_c^{\theta-2}}{\theta} \left(\frac{\theta-1}{\theta}\right)^\theta v_{z,\mathbf{E}_0} p_z^{1-\theta} \sum_{j \in \text{SU}\{z\}} \sum_{j \in \text{SU}\{z\}} w_{kz} \omega_1^{kj} \\ &= \left(\frac{\theta-1}{\theta}\right)^\theta \frac{v_z p_z c_z}{\theta p_c} \Big|_{\mathbf{E}=\mathbf{E}_0} \sum_{j \in \text{SU}\{z\}} \sum_{k=1}^n w_{kz} \omega_1^{kj}, \end{aligned} \quad (40)$$

where v_{z,\mathbf{E}_0} is v_z evaluated under $\mathbf{E} = \mathbf{E}_0$. Note that this is positive and tends to 0 as $\theta \rightarrow 1$.

Thus, the effect of the forbearance lending to the bank profit is

$$\pi(\mathbf{E}_1) - \pi(\mathbf{E}_0) \rightarrow \frac{v_z p_z c_z}{\theta p_c} \Big|_{\mathbf{E}=\mathbf{E}_1} \quad (41)$$

as $\theta \rightarrow 1$. This is positive and increasing in v_z under fixed product prices. \square

Appendix 2: The detail of the effect of e_z to the bank profit.

The full effect of the lending decision e_z is evaluated by the calculation of $\pi(\mathbf{E}_1) - \pi(\mathbf{E}_0)$ taking into account the effect through changes in p_c and \mathbf{P}_θ . Let p_{c,e_z} be the value of p_c under \mathbf{E}_{e_z} . Similarly, let \mathbf{P}_{θ,e_z} be the value of \mathbf{P}_θ under \mathbf{E}_{e_z} .

We observe that

$$\begin{aligned}\pi(\mathbf{E}_1) - \pi(\mathbf{E}_0) &= \frac{Rp_{c,1}^{\theta-2}}{\theta} \cdot \mathbf{1}' \mathbf{P}_{\theta,1} \mathbf{A}_1^{-1} \mathbf{E}_1 \mathbf{1} - \frac{Rp_{c,0}^{\theta-2}}{\theta} \cdot \mathbf{1}' \mathbf{P}_{\theta,0} \mathbf{A}_0^{-1} \mathbf{E}_0 \mathbf{1} \\ &= \frac{Rp_{c,1}^{\theta-2}}{\theta} \cdot \mathbf{1}' \mathbf{P}_{\theta,1} \mathbf{A}_1^{-1} (\mathbf{E}_1 \mathbf{1} - \mathbf{E}_0 \mathbf{1})\end{aligned}\quad (42)$$

$$+ \frac{Rp_{c,1}^{\theta-2}}{\theta} \cdot \mathbf{1}' \mathbf{P}_{\theta,1} (\mathbf{A}_1^{-1} - \mathbf{A}_0^{-1}) \mathbf{E}_0 \mathbf{1}\quad (43)$$

$$+ \frac{Rp_{c,1}^{\theta-2}}{\theta} \cdot \mathbf{1}' (\mathbf{P}_{\theta,1} - \mathbf{P}_{\theta,0}) \mathbf{A}_0^{-1} \mathbf{E}_0 \mathbf{1}\quad (44)$$

$$+ \frac{R}{\theta} (p_{c,1}^{\theta-2} - p_{c,0}^{\theta-2}) \cdot \mathbf{1}' \mathbf{P}_{\theta,0} \mathbf{A}_0^{-1} \mathbf{E}_0 \mathbf{1}.\quad (45)$$

As in the proof of Proposition 2, the first term (42) is

$$\frac{Rp_{c,1}^{\theta-2}}{\theta} \cdot \mathbf{1}' \mathbf{P}_{\theta,1} \mathbf{A}_1^{-1} (\mathbf{E}_1 \mathbf{1} - \mathbf{E}_0 \mathbf{1}) = \left. \frac{v_z p_z c_z}{\theta p_c} \right|_{\mathbf{E}=\mathbf{E}_1}.\quad (46)$$

This part captures the propagation effect through a given influence coefficient of z , which corresponds to the expression (20) in the main text.

The argument in the proof of Proposition 2 also implies that the second term (43) is

$$\frac{Rp_{c,1}^{\theta-2}}{\theta} \cdot \mathbf{1}' \mathbf{P}_{\theta,1} (\mathbf{A}_1^{-1} - \mathbf{A}_0^{-1}) \mathbf{E}_0 \mathbf{1} = \left(\frac{\theta - 1}{\theta} \right)^\theta \frac{v_z p_z c_z}{\theta p_c} \Big|_{\mathbf{E}=\mathbf{E}_0} \sum_{j \in SU\{z\}} \sum_{k=1}^n w_{kz} \omega_1^{kj}.\quad (47)$$

This effect comes from the e_z in q_{zj} . This effect also always increases the network profit. The dependence on the input produced within the network increases as the number of suppliers within the network increases. This results in the increase of the influence coefficient. The term (21) in the main text captures this effect and the remaining two effects inclusively.

We then consider the two additional terms. The third term (44) is

$$\begin{aligned}& \frac{Rp_{c,1}^{\theta-2}}{\theta} \cdot \mathbf{1}' (\mathbf{P}_{\theta,1} - \mathbf{P}_{\theta,0}) \mathbf{A}_0^{-1} \mathbf{E}_0 \mathbf{1} \\ &= \frac{Rp_{c,1}^{\theta-2}}{\theta} \cdot p_z^{1-\theta} \sum_{k \in S} \omega_0^{zk} = \left. \frac{p_z c_z}{\theta p_c} \right|_{\mathbf{E}=\mathbf{E}_0} \sum_{k \in S} \omega_0^{zk}.\end{aligned}\quad (48)$$

Note that this is positive. This part comes from \mathbf{P}_θ , the product prices. More firms brings more variety in inputs, which improves the productivity of every firm. This results in the reduction of the product price relative to p_0 (price of the outside good). Cheaper inputs increases the input demand of goods, and so the dependence on the inputs provided by firms within the network. This increases the influence coefficient of each firm.

Lastly, the fourth term (45) is

$$\frac{R}{\theta}(p_{c,1}^{\theta-2} - p_{c,0}^{\theta-2}) \cdot \mathbf{1}' \mathbf{P}_{\theta,0} \mathbf{A}_0^{-1} \mathbf{E}_0 \mathbf{1} = \frac{p_{c,1}^{\theta-2} - p_{c,0}^{\theta-2}}{p_{c,0}^{\theta-2}} \pi(\mathbf{E}_0). \quad (49)$$

Note that we have

$$p_{c,e_z} = (\mathbf{w}'_0 \mathbf{A}_{e_z}^{-1} \mathbf{E}_{e_z} \mathbf{1})^{\frac{1}{1-\theta}}. \quad (50)$$

Thus, to determine the sign of this fourth term, we just need to see the sign of

$$\mathbf{w}'_0 \mathbf{A}_1^{-1} \mathbf{E}_1 \mathbf{1} - \mathbf{w}'_0 \mathbf{A}_0^{-1} \mathbf{E}_0 \mathbf{1}. \quad (51)$$

Observe that

$$\begin{aligned} & \mathbf{w}'_0 \mathbf{A}_1^{-1} \mathbf{E}_1 \mathbf{1} - \mathbf{w}'_0 \mathbf{A}_0^{-1} \mathbf{E}_0 \mathbf{1} \\ &= \mathbf{w}'_0 \mathbf{A}_1^{-1} (\mathbf{E}_1 - \mathbf{E}_0) \mathbf{1} + \mathbf{w}'_0 (\mathbf{A}_1^{-1} - \mathbf{A}_0^{-1}) \mathbf{E}_0 \mathbf{1}. \end{aligned} \quad (52)$$

We have

$$\mathbf{w}'_0 \mathbf{A}_1^{-1} (\mathbf{E}_1 - \mathbf{E}_0) \mathbf{1} = \sum_{k=1}^n w_{0k} \omega_1^{kz}. \quad (53)$$

And also we have

$$\begin{aligned} & \mathbf{w}'_0 (\mathbf{A}_1^{-1} - \mathbf{A}_0^{-1}) \mathbf{E}_0 \mathbf{1} \\ &= \mathbf{w}'_0 \mathbf{A}_0^{-1} (\mathbf{A}_0 - \mathbf{A}_1) \mathbf{A}_1^{-1} \mathbf{E}_0 \mathbf{1} \\ &= \left(\frac{\theta - 1}{\theta} \right)^\theta \mathbf{w}'_0 \mathbf{A}_0^{-1} (\mathbf{E}_1 - \mathbf{E}_0) \mathbf{W}' \mathbf{A}_1^{-1} \mathbf{E}_0 \mathbf{1} \\ &= \left(\frac{\theta - 1}{\theta} \right)^\theta \left(\sum_{k=1}^n w_{0,k} \omega_0^{kz} \right) \left(\sum_{j \in S} \sum_{k=1}^n w_{kz} \omega_1^{kj} \right). \end{aligned} \quad (54)$$

Note that this is positive. Thus, if $\theta < 2$, then the fourth term (45) is positive. If $\theta > 2$, then this term is negative. This effect comes through p_c . This route has two opposite effects to the bank profit. First, it increases the real income of households R/p_c . Second, it increases the relative price of each product except z . The former increases the sales to households whereas the latter decreases that of the existing products except z . If θ is closer to one, i.e., the elasticity of substitution is lower, the latter effect is dominated by the former, and so the impact to the bank profit through this route is positive. However, if θ is greater than two, the the latter dominates the former, and the impact to the bank profit is negative.