Waiting for Affordable Housing*

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Abstract

We develop a new dynamic equilibrium model of housing markets for low- and moderate-income households, which is consistent with the key supply restrictions and search frictions that arise in rental markets for public and affordable housing. We estimate the model using data collected by the New York Housing Vacancy Survey in 2011. We find that having access to public (rent stabilized) housing increases household welfare by up to \$60,000 (\$50,000). The estimated average time on the waitlist for public housing in Manhattan is 19 years. The search frictions in the rent stabilized market imply an average search time of four years.

Keywords: Affordable Housing, Public Housing Communities, Excess Demand, Rationing, Search Frictions, Queuing, Welfare Analysis, Stationary Equilibrium.

JEL classification: C33, C83, D45, D58, H72, R31.

1 Introduction

As many urban and metropolitan areas have shifted toward a knowledge-based economy, most large cities in the U.S. and other developed countries have continued to attract highly skilled and educated households. As a consequence real estate prices and rents have continued to soar in many metropolitan areas for the past two decades. This leaves few options for low- and moderate-income households who struggle to pay the increasingly expensive costs of living in these cities. Nevertheless, low- and moderate-income households play a key role in the provision of many local goods and services in the urban economy. Their presence is particularly essential with extreme-skill complementarity in the production function of large cities (Eeckhout, Pinheiro, and Schmidheiny, 2014).

Despite the importance of providing public and affordable housing for low- and moderate income households in many large, affluent cities, there are few compelling dynamic models that allow us to study housing choices of low- and moderate-income households. The main purpose of this paper is to fill this gap. We develop and estimate a new dynamic equilibrium model that is consistent with the observed market search frictions, the existence of long queues for public housing, and the need to search for a long time to obtain access to other types of affordable housing.

A compelling model of affordable housing should capture the existence of three different types of rental markets: public, regulated, and unregulated markets. It must also capture the dynamic incentives faced by households, such as income dynamics, long waiting list for public housing, and long search time for regulated housing. We model the unregulated private housing rental market as frictionless. Households can purchase any quantity or quality of housing given the prevailing market price. In addition, low- and moderate income households may have access to publicly provided housing and other types of subsidized housing such as rent-stabilized or rent-controlled housing. Low-income households are eligible for public housing assistance in the U.S. if their income is below a threshold that depends on household composition and region. The rent charged for public housing is a fixed percentage of household income. Hence, there is no price mechanism to ensure that public housing markets clear. Since demand for public housing vastly exceeds the available supply in most affluent U.S. cities, there are long waitlists to get into public housing communities.[1]

If households are forward looking, they understand that they will only receive an offer to move into public housing when they reach the top of the waitlist. Moreover, most housing authorities do not evict households after they have lost their eligibility for housing aid. Consequently, public housing provides a consumption subsidy and also partial insurance against negative income shocks. We show that these incentives of current U.S. housing policies give rise to a large degree of mismatch of low- and moderate-income households in housing markets.

In addition, low- and moderate-income households may have access to other forms of subsidized housing. Many affluent cities also use rent control or rent stabilization laws to provide affordable housing. For example, over one million households live in rent regulated housing units in New York City. The rental price for stabilized housing is typically significantly lower than the equivalent market price in the unregulated market. Since the demand for rent-stabilized units typically exceeds the supply, there are significant frictions in the rent-stabilized market. In contrast to public housing, where rationing is achieved by placing households on waitlists, the key friction here is that finding a rent-stabilized apartment involves significant search efforts and luck.[2] We capture these market frictions by endogenizing the probability that a household who is actively searching for rent stabilized housing will receive an offer to move into a stabilized unit. The length of the waitlist for public housing and the probability of finding a rent-stabilized unit are, therefore, all endogenously determined in equilibrium of our model.[3]

[1]Wait lists are also common in housing assistance programs in other countries.

[2]Our paper is also related to search and matching models that have been applied to study housing markets. See, for example, Wheaton (1990), Krainer (2001), or Albrecht, Anderson, Smith, and Vroman (2007). Most of the papers in this literature focus on the markets for owner occupied housing which are distinctly different from affordable rental markets that are the focus of this paper.

[3]Our modeling approach is thus consistent with Glaeser and Luttmer (2003) who show that rent controls lead to misallocations in housing markets. Potential misallocations arise in our model due to search frictions

We define and characterize a stationary equilibrium with rationing.[4] The equilibrium has the property that those with the highest priority score for public housing receive an offer to move into public housing. In addition, a fraction of the households with the second highest priority score also get an offer. The discreteness of the priority score effectively partitions the demand for public housing into a finite number of cohorts. To smooth out the flow of households into public housing and equate the inflow with the voluntary outflow of households, the housing authority must randomize among households with the second highest priority score. Households in the second highest priority group that lose the lottery then move into the highest priority group and will obtain an offer in the next period.

In equilibrium low-income households prefer to live in public housing due to the large rent subsidy, which implies a large increase in numeraire consumption, and the relatively high quality of these housing units. Rent stabilized housing appeals to a large range of low- and moderate-income households due to the significant rental price discount relative to the unregulated market. Higher income households prefer to rent in the unregulated market. Due to the existence of rationing in public housing and search frictions in rent stabilized housing a fraction of low- and moderate-income households must also rent in the unregulated market in equilibrium. Due to the no-eviction policy of the housing authority, a significant fraction of ineligible moderate-income households live in public housing. Our model can support long waitlists, and it is also consistent with the observed mismatch in public and rent-stabilized housing markets.

The parameters of our model can be identified based on the observed moments in the data. Our proof of identification is constructive and can be used to define a method of moments estimator. This estimator matches the sorting of households by income and family type among housing options and the average time spent in different housing markets. The estimator also matches the average rental payment for each housing type.

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and the existence of public housing.

[4]Geyer and Sieg (2013) consider a static model of public housing with myopic households and provide a detailed discussion of the related literature.

Our empirical analysis focuses on New York City (NYC). While many cities in the U.S. and abroad face the challenge to provide an adequate supply of affordable housing, NYC has been at the center of the debate over affordable housing.[5] Studying the housing markets for low- and moderate-income households in NYC is promising for a variety of compelling reasons. First, NYC has the largest stock of rental apartments of all cities in the U.S. and is generally perceived to be one of the most expensive rental market in the world. Second, New York City also has the largest stock of public housing units of all cities in the U.S. Finally, NYC has ever declared a housing emergency every year since 1973 and has adopted strict rent stabilization programs over an extended period of time.[6] NYC, therefore, serves as a laboratory to explore the effectiveness and impact of a variety of different affordable housing policies.

Our empirical analysis is based on the 2011 sample of the New York City Housing and Vacancy Survey (NYCHVS). This survey provides comprehensive data about household and housing characteristics. In particular, we observe household income and family status, the time that the household has spent in the housing unit, as well as a large number of structural characteristics of the housing unit that the household occupies. In addition, it allows us to classify households as living in public housing, rent-stabilized housing, or unregulated housing. We implement our estimator focusing on Manhattan, since waitlists in NYC are operated at the borough level.[7]

The data show that approximately 10 percent of our sample of low- and moderate-income households lived in public housing communities in 2011. 58 percent of households lived in rent stabilized units and the remaining 32 percent rented in the unregulated housing market. At the time of the survey, households spent, on average, 16 years in public housing, 9.5 years in

[5] As a candidate, current mayor of NYC, Bill de Blasio, successfully ran on a platform that promised significant increases the provision of affordable housing. Once in office, he proposed and city council recently adopted a 10-year plan to build 200,000 affordable housing units in the NYC area through various rezoning laws.

[6] An early analysis of the benefits and costs of public housing in New York City is given by Olsen and Barton (1983).

[7] As a robustness check we have also estimated our model using all five boroughs in NYC.

regulated housing and only 4 years in unregulated housing. Not surprisingly, households in public housing are much poorer than households in rent stabilized and unregulated housing.

We estimate the structural parameters of the model. We find that our model fits the sorting of households by income among the three housing options. Our model captures differences in rental prices as well as time spent in the housing units. We find that rental prices for stabilized housing are approximately 50 percent of the prices in the unregulated market in Manhattan. This significant discount explains the popularity of rent-stabilized units. We find that households that have access to rent stabilized housing are willing to pay up to $50,000 for this benefit. The probability of finding a rent-stabilized unit is approximately 25 percent per year. Public housing communities are also attractive options for low- and moderate-income households. We find that households that have access to public housing are willing to pay up to $60,000 for this benefit. As expected, this creates an excess demand for public housing. We find that the average wait time for public housing in Manhattan is 18 years.

It is important to note we cannot conclude from this analysis that affordable housing policies are desirable because a comprehensive welfare analysis also needs to take the costs of providing affordable housing into consideration. Nevertheless, we have shown that the benefits associated with these policies can be substantial.

In addition, there seems to be some scope for improving current housing policies. One notable feature of existing public housing policies is that housing authorities rarely ask households to leave public housing once their income exceeds the eligibility threshold. Approximately 17 percent of households living in public housing in Manhattan have income that exceeds the eligibility threshold, which is 80 percent of the median income. For households that remain in public housing despite their ability to leave, housing aid is de facto an open-ended entitlement program.

As a counterfactual experiment we consider the impact of a policy that strictly limits access to housing aid for households making less than the median income. We find that such a policy has some interesting distributional effects. The main losers of this policy are households that are currently living in public housing. Their welfare is significantly reduced for two reasons. First, households that lose eligibility need to move out of public housing and rent in the unregulated or rent stabilized markets. Second, currently eligible households may lose eligibility in the future if they experience a sequence of positive income shocks. The main beneficiaries of this policy are low-income households that are currently on the waitlist. Enforcing eligibility criteria creates more openings and thus significantly reduces the expected time spent on the waitlist. Hence needy households are more likely to obtain access to public housing. Welfare also increases for ineligible households outside of public housing, since public housing becomes more readily available and provides limited insurance against a sequence of negative income shocks.

Our work is related to a substantial literature that analyzes policies aimed at providing affordable housing for low-income households. As pointed out by Olsen (2003) one justification for housing subsidies to low-income households is that most taxpayers want to help households in poverty, but feel that, at least, some low-income households undervalue housing. In recent years, proponents of housing subsidies have frequently argued that the primary housing problem of low-income households is an excessive rent-income ratio rather that inadequate housing. Rent stabilization programs directly try to address this concern. However, it is well understood that rent stabilization creates other sources of inefficiencies. Glaeser and Luttmer (2003) find that 21 percent of New York apartment renters live in units with more or fewer rooms than they would if they rented in the unregulated market in 1990.

There may also be large inefficiencies in the assignment of public housing. Thakral (2015) considers a dynamic model of matching and introduces a multiple waitlist procedure. His analysis suggests that there are large potential welfare gains associated with this allocation mechanism. In contrast, we focus on the lack of enforcement of eligibility criteria as another potential source of inefficiency. Low-income housing programs can be justified due to the potential negative externalities to public health and safety that result from low-cost, high-density housing neighborhoods for poor families. Children tend to suffer more than adults from poor neighborhoods with inadequate housing and shelter. These children grow up with less education and lower earning power. They are more likely to have drug addictions, psychological trauma and disease, or become incarcerated.[8]

There is some mixed evidence suggesting negative spill-over effects of low income housing programs, such as higher crime rates and lower educational achievement, discussed in Currie and Yelowitz (2000). In contrast, Jacob (2004) finds that there are very few positive effects associated with moving out of housing projects in Chicago using a variety of different outcome measures. Kling, Liebman, and Katz (2007) find that moving to lower poverty neighborhoods improves physical and mental health but produces mixed outcomes for children's behavior and has little impact on employment outcomes.[9] Chetty, Hendren, and Katz (2016) find large effects on earnings for children that left public housing when they were younger than 13. If public housing is inferior to unregulated housing, an argument can be made for subsidizing the supply of privately provided affordable housing. As detailed in Erickson and Rosenthal (2011), the Low Income Housing Tax Credit program was created in 1986 as part of the Tax Reform Act of 1986 as an alternative to public housing. They find, however, that this program has failed to result in new construction that serves the population served by public housing, largely due to crowd-out effects. Moreover, considerable evidence suggests that rent control and stabilization programs are very crude policy tools that often create misallocation in housing markets.[10]

The rest of the paper is organized as follows. Section 2 discusses affordable housing policies in NYC and our data. Section 3 provides a new dynamic model of affordable housing markets. Section 4 discusses identification and estimation of the parameters of our model. Section 5 presents our empirical findings. Section 6 reports the findings from our policy analysis. Section 7 offers our conclusions.

[8]For detailed discussion of the literature see, among others, Cutler, Glaeser, and Vigdor (1999) and Oreopoulos (2003).

[9]See also Galiani, Murphy, and Pantano (2015) who estimate preferences over neighborhoods and amenities for low income households using data from the MTO experiment.

[10]Earlier work on rent control include Olsen (1972) and Suen (1989). Gyourko (1989) provide an empirical analysis on NYC markets.

2 Data

Providing adequate housing and shelter for low- and moderate-income households has been a policy goal of most federal, state, and city administrations in the United States since the passage of the Public Housing Act of 1937. There are clear and documented benefits for low-income families which are given the opportunity to live in high-quality housing communities. Beyond the obvious improved housing quality, other benefits include increased safety, improved property management, better access to neighborhood amenities, improved public health, and improved mental health from a reduction in stress.[11] Of course, public housing communities have a complicated and sometimes notorious history in the United States. The potential gains have not always materialized for reasons such as poor design of communities, neglect, corruption, and mismanagement. Despite its problems, public housing remains a highly demanded commodity, especially in the most expensive metropolitan areas in the U.S.

Our empirical analysis focuses on housing markets in New York City. The New York City Housing Authority (NYCHA) provides public housing and administers Section 8 housing vouchers for low- and moderate-income residents throughout the five boroughs of New York City. Households whose incomes do not exceed 80% (50%) of median income are eligible for the public housing program (voucher program). In addition, income limits are functions of family size. For example, in 2011 the income limit for a single person household was $45,850 ($28,500) while it was $65,450 ($40,900) for a family of four.

Applications for public housing are assigned a priority code based upon information that includes employment status, income, family size, and quality of previous residence provided. Households are then placed on the housing authority's preliminary waiting list for an eligibility interview. Households are required to update or renew their applications every two years if they have not been scheduled for an interview. Upon passing the interview and background

[11] Whether or not low-income families have benefited economically or educationally is contested. Similarly, there is mixed evidence of the benefits of moving low-income households to more mixed-income neighborhoods. We discuss some of the evidence in detail below.

checks, applicants are then placed on a (borough wide) waiting list.

More than 403,000 New Yorkers reside in NYCHA's 177,666 public housing apartments across the city's five boroughs. Another 235,000 residents receive subsidized rental assistance in private homes through the NYCHA-administered Section 8 program. The NYCHA reported that 270,201 families were on the waiting list for conventional public housing and 121,356 families on the waiting list for Section 8. Little is known about the annual flows of waitlisted individuals into public housing. The NYT reported on July 23, 2013 that "the queue moves slowly. The apartments are so coveted that few leave them. Only 5,400 to 5,800 open up annually." As of December 10, 2009 NYCHA stopped processing any new Section 8 applications due to the long waiting list. As consequence, there is almost no mobility in and out of Section 8 housing markets. We, therefore, treat Section 8 housing as a completely separate market and focus on public housing in this paper.

Housing markets have also been heavily regulated in NYC since the 1930's. The stock of rent-regulated units includes a relatively small number of rent controlled units - approximately 38,000 - but a much larger number of rent-stabilized units. Rent control primarily affects old units. As of 2011, over one million units were rent-stabilized representing roughly 47 percent of the rental housing stock in NYC. [12]

Rent stabilization generally applies to buildings of six or more units built between February 1, 1947 and December 31, 1973, and to those units that have exited from the rent-control program. Approximately 8 percent of the city's stabilized units and nearly all stabilized units in buildings constructed after 1974 were voluntarily subjected to rent stabilization by their owners in exchange for tax incentives from the city. Under the 421-a program, developers currently have to set aside 20 percent of new apartments for poor and working-class tenants

[12] The New York Times (NYT) reported on July 23, 2015 that the de Blasio administration "had lined up financing for more than 20,000 affordable apartments - about 8,500 to be newly built and 11,800 preserved - through deals with landlords to lock in low rents for decades. That is an aggressive pace. Not since 1989, when a decade-long program begun under Mayor Edward Koch was transforming rubbly mountains of blight into miles of solid apartment blocks, has the city achieved so much in a single year."

to receive tax abatements lasting 35 years.[13]

Involuntarily stabilized units, representing 92 percent of the stabilized stock, are regulated based on a "housing emergency" declared by the city in 1974 and renewed every three years since. Under New York States Rent Stabilization Law, the city may declare a housing emergency whenever the city's rental vacancy rate drops below five percent. This law was most recently renewed in June 2015 and affects units with a maximum rent of $2,700. Rent stabilization sets maximum rates for annual rent increases. It also entitles tenants to have their leases renewed. The rent guidelines board meets every year to determine how much the landlord can set future rents on the lease.

The empirical analysis is based on the New York City Housing Vacancy Survey (NYCHVS) in 2011. The main advantage of this data set is that it matches households with units (i.e., it contains detailed information about both household characteristics and housing characteristics).

Table 1: Descriptive Statistics

housing type	market share	rent	number of years	income	female head	kids	working family
Public	0.10	—	16.18	32,930	0.73	0.92	0.70
Regulated	0.58	1317	9.49	54,739	0.53	0.38	0.83
Unregulated	0.33	2640	3.85	71,045	0.54	0.17	0.87

Source: New York City Housing Vacancy Survey 2011

A household is defined as working if the labor income share is higher than 50 percent of total income.

Regulated units include rent-stabilized units, HUD-regulated units, and Michell-Lama rental units.

We focus on affordable housing for low- and moderate-income households which imposes three sample restrictions. First, we drop households whose average incomes exceed 200% of median income level. This sample restriction is motivated by the fact that high-income New Yorkers are likely to own a condominium or house and, therefore, face a different choice set

[13]The de Blasio administration has been pushing to increase that fraction to 35 percent.

than low- and moderate-income households face.[14] Second, we drop all low-income households that receive vouchers since that market has been closed for at least 6 years. Finally, we drop all households not living in Manhattan since waitlists are operated at the borough level rather than city-wide. These restrictions reduce our sample size to 1,557.

Tables 1 provides some descriptive statistics of the Manhattan housing market for 2011. Table 1 shows that a large fraction of the rental units in Manhattan are under rent stabilization. The fraction was 58 percent in 2011. At the same time, the average rent was \$2,640 in the unregulated market and \$1,317 in the regulated market.

Households tend to stay for long periods in their apartments. On average in 2011, households had occupied their apartments 16.18 years for public housing and 9.49 years for rent stabilized housing. The turnover is much higher in the unregulated housing market. Not surprising, households in public housing are much poorer than household in rent stabilized and unregulated housing. Families in public housing tend towards single parent households, the majority headed by a female. Public housing families have more children, on average, than households in rent-stabilized or unregulated housing.

3 A Dynamic Model of Affordable Housing Markets

We consider a local housing market with three housing options: public housing (p), rent-regulated housing (r), and housing provided by the unregulated market (m). The exogenous housing supply in public and rent regulated housing are given by k_p and k_r. The assumption of fixed supply of public and rent stabilized housing is appropriate for NYC. There has been limited recent construction of new housing communities in NYC.[15] We can, therefore, treat supply as price inelastic and fixed in the short run.

Time is discrete, $t = 0, ..., \infty$. Households are infinitely lived and forward looking. House-

[14] None of the key findings of this paper qualitatively or quantitatively depend on these choices.

[15] If anything, the supply of rent stabilized housing has declined in the past decades.

holds have a common discount factor β and maximize expected lifetime utility. In the baseline model, households only differ by income, denoted by y, which evolves according to a stochastic law of motion that can be described by a stationary Markov process with transition density $f(y'|y)$. Below we extend our model to allow for additional sources of household heterogeneity.

Household flow utility is defined over housing quality, h, and a numeraire good, b. Consider a household that rents in the unregulated market. Housing services can be purchased at price p_m.[16] Flow utility is, therefore, given by:

$$\begin{aligned} u_m(y) &= \max_{h,b} \; U(b,h) \\ & \quad s.t. \;\; p_m \, h + b = y \end{aligned} \tag{1}$$

Note that we are imposing the realistic assumption that low and moderate–income households do not save and cannot borrow against uncertain future income. They are liquidity constrained and spend their income on housing and consumption goods in each period.

There are R discrete different levels of housing quality in the stabilized market. The flow utility associated with a rent regulated unit of quality h_r and price $p_r < p_m$ is given by:

$$u_r(y) = U(y - p_r h_r, h_r) \quad r = 1, .., R \tag{2}$$

The next assumption captures the search frictions in that market.

Assumption 1

a) Each period, there is a positive probability q_r that a household receives an offer to move into a rent regulated unit of quality h_r.

b) Each household receives, at most one, offer per period.

The probabilities of receiving an offer to move into a stabilized housing unit are endogenous and depends on the supply and the voluntary outflow from regulated housing as discussed below in detail.

[16] We implicitly assume that unregulated housing supply is perfectly elastic at price p_m. This assumption can be easily relaxed to endogenize the price of housing in the unregulated market by allowing for an upward sloping supply function.

To simplify the notation we set $R = 1$ for the remainder of this section. But all results can be easily generalized to account for heterogeneity in the quality and supply of rent stabilized units.[17] In our quantitative analysis below, we estimate a model with such heterogeneity ($R > 1$).

Public housing provides a constant level of housing consumption, h_p, and taxes individual at constant rate τ. Per period utility in public housing is, therefore, given by:

$$u_p(y) = U((1 - \tau)y, h_p) \tag{3}$$

The local housing authority that administers the public housing program manages a waitlist. The priority score of a household is a monotonic function of the time spent on the waitlist. More formally, let w denote the time that a household has been on the wait list. Let $p(w)$ denote the probability that a household that has been on the waitlist for w periods receives an offer to move into public housing. The next assumption captures the behavior of the housing authority.

Assumption 2

a) The housing authority makes take it or leave it offers,i.e if a household rejects an offer, it will go to the end of the waitlist ($w = 0$).

b) The outflow of public housing is voluntary (i.e., the housing authority does not evict households from public housing).

c) Eligibility is determined by an income cut-off, denoted by $\bar{y}$ and is checked every time period. Loss of eligibility means that the household is removed from the waitlist ($w = 0$).

These assumptions are uncontroversial and reflect common practice of housing authorities in NYC and other U.S. metropolitan areas. Note that the distribution of priority scores is endogenous and determined in equilibrium as we discuss below.

The timing of decisions is as follows:

[17] An appendix that contains a detailed derivation of all key equations is available upon request from the authors.

1. Each household gets a realization of income which determines the income distributions at the beginning of the period.

2. Some households get an offer to move into public housing generated with probability $p(w)$.

3. Some households get an offer to move into rent-regulated housing generated with probability q_r.

4. Households decide to move and obtain the flow utility that depends on their decisions.

5. Wait times are updated.

Note that utility is realized after households have relocated.

The two state variables in this model are wait time, w, and income, y. Define the conditional value functions associated with the three choices:

$$\begin{aligned} v_p(y) &= u_p(y) + \beta \int V_p(y') \, f(y'|y) \, dy' \\ v_m(y,w) &= u_m(y) + \beta \int V_m(y',w') \, f(y',w'|y,w) \, dy'dw' \\ v_r(y,w) &= u_r(y) + \beta \int V_r(y',w') \, f(y',w'|y,w) \, dy'dw' \end{aligned} \tag{4}$$

We can derive recursive expressions for the unconditional value functions. The value function of a household with characteristics (w, y) that rents in the regulated market is given by:

$$\begin{aligned} V_r(y,w) &= p(w) \, 1\{y \leq \bar{y}\} \, \max\{v_p(y), v_m(y,0), v_r(y,0)\} \\ &+ (1-p(w)) \, 1\{y \leq \bar{y}\} \, \max\{v_m(y,w+1), v_r(y,w+1)\} \\ &+ 1\{y > \bar{y}\} \, \max\{v_m(y,0), v_r(y,0)\} \end{aligned} \tag{5}$$

The value function of a household with characteristics (w, y) that rents in the unregulated market is then given by:

$$
\begin{aligned}
V_m(y,w) &= q_r\, V_r(y,w) \\
&+ (1-q_r)\, p(w)\, 1\{y \leq \bar{y}\}\ \max\{v_m(y,0), v_p(y)\} \\
&+ (1-q_r)\, (1-p(w))\, 1\{y \leq \bar{y}\}\ v_m(y,w+1) \\
&+ (1-q_r)\, 1\{y > \bar{y}\}\ v_m(y,0)
\end{aligned} \tag{6}
$$

Finally, the value function of a household living in public housing satisfies:

$$
\begin{aligned}
V_p(y) &= (1-q_r) \max\{v_p(y),\ v_m(y,0)\} \\
&+ q_r\ \max\{v_p(y),\ v_m(y,0), v_r(y,0)\}
\end{aligned} \tag{7}
$$

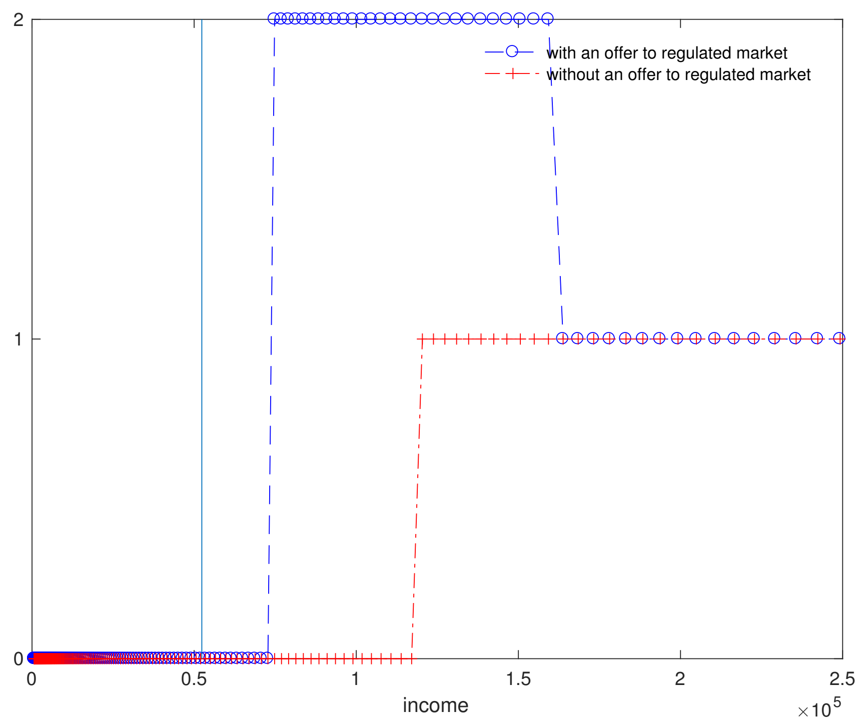
These value functions determine the optimal decision rules for each household.[18]

To illustrate the optimal decision rules we consider a simple estimated version of model with only one type of stabilized housing. Figure 1 plots the policy function for a household in public housing. The blue vertical line indicates the income eligibility threshold for public housing. Optimal decision rules can be characterized by thresholds. The blue line indicates the decision rule of a household that received an offer to move into regulated housing while the red line is a household without an offer. Low-income households prefer to live in public housing, moderate-income households prefer rent-regulated housing while higher income households prefer renting in the unregulated market.

Figure 2 shows the decision rule for a household, who is currently in the regulated market, has been on the waitlist for 5 periods, and does not receive an offer to move into public housing. Here we find that low- and high-income households prefer the unregulated markets while moderate-income households prefer units in the rent-regulated market. The non-monotonicity of the decision rule is partially due to the fact that the quality of housing in the regulated market is relatively high exceeding the quality in public housing in this example.

[18] Note that our analysis abstracts from moving costs which are likely to be low in rental markets. It is straightforward to extend the model to allow for both monetary and psychic mobility costs.

Figure 1: Policy Function (0=public, 1=unregulated, 2=regulated)



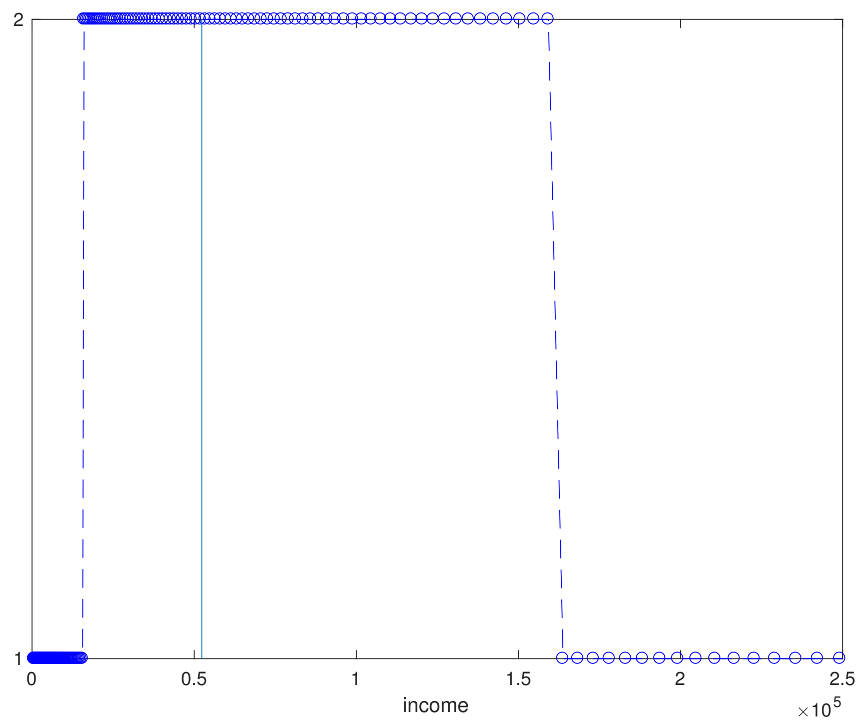
Let $g_m(w)$ ($g_r(w)$) denote the marginal distribution of wait times for households in unregulated (rent regulated) housing in stationary equilibrium. Let $g_p(y)$ denote the density of income of households that are inside public housing at the beginning of each period (before households have moved). Similarly let $g_m(y|w)$ ($g_r(y|w)$) denote the stationary density of income conditional on wait time for households in the unregulated (regulated) market.

The voluntary flow of households out of public housing is given by:

$$\begin{aligned} OF_p &= k_p\,(1-q_r)\int 1\{v_m(y,0) > v_p(y)\}\; g_p(y)\; dy \\ &+ k_p\; q_r \int 1\{v_m(y,0) \geq \max[v_p(y), v_r(y,0)]\}\; g_p(y)\; dy \\ &+ k_p\; q_r \int 1\{v_r(y,0) \geq \max[v_p(y), v_m(y,0)]\}\; g_p(y)\; dy \end{aligned} \tag{8}$$

Note that the first two terms is the outflow to the unregulated market and the third term

Figure 2: Policy Function (1=unregulated, 2=regulated, $w = 5$)



captures the outflow to the rent regulated market. The flow into public housing is given by:

$$\begin{aligned} IF_p &= k_m \sum_{j=0}^{\infty} p(w_j)\; g_m(w_j)\; IF_{mp}(w_j) \\ &+ k_r \sum_{j=0}^{\infty} p(w_j)\; g_r(w_j)\; IF_{rp}(w_j) \end{aligned} \tag{9}$$

where the inflow from the unregulated market conditional on wait time is:

$$\begin{aligned} IF_{mp}(w_j) &= (1 - q_r) \int_{y \leq \bar{y}} 1\{v_p(y) \geq v_m(y, 0)\}\; g_m(y|w_j)\; dy \qquad (10) \\ &+ q_r \int_{y \leq \bar{y}} 1\{v_p(y) \geq \max[v_m(y, 0), v_r(y, 0)]\}\;\; g_m(y|w_j)\; dy \end{aligned}$$

and the inflow from the rent regulated market is given by:

$$IF_{rp}(w_j) = \int_{y \leq \bar{y}} 1\{v_p(y) \geq \max[v_m(y, 0), v_r(y, 0)]\}\; g_r(y|w_j)\; dy \tag{11}$$

Similarly, the voluntary flow of households out of rent regulated housing is given by:

$$\begin{aligned}
OF_r &= k_r \sum_{j=0}^{\infty} p(w_j)\, g_r(w_j) \int_{y \leq \bar{y}} 1\{v_r(y,0) \leq \max[v_p(y), v_m(y,0)]\}\, g_r(y|w_j)\, dy \\
&+ k_r \sum_{j=0}^{\infty} (1-p(w_j))\, g_r(w_j) \int_{y \leq \bar{y}} 1\{v_m(y,w_j+1) \geq v_r(y,w_j+1)\}\, g_r(y|w_j)\, dy \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{y > \bar{y}} 1\{v_m(y,0) \geq v_r(y,0)\}\, g_r(y|w_j)\, dy
\end{aligned} \tag{12}$$

Note that the first term is the outflow of those households that have an offer to move into public housing. The second term is the outflow of households eligible for public housing who do not have an offer to move into public housing. The last term is the outflow of households above the eligibility threshold to unregulated housing. The flow into rent regulated housing is given by:

$$\begin{aligned}
IF_r &= k_p\, q_r \int 1\{v_r(y,0) \geq \max[v_m(y,0), v_p(y)]\}\, g_p(y)\, dy \\
&+ k_m \sum_{j=0}^{\infty} p(w_j)\, q_r \int_{y \leq \bar{y}} 1\{v_r(y,0) \geq \max[v_m(y,0), v_p(y)]\}\; g_m(y|w_j)\, dy \\
&+ k_m \sum_{j=0}^{\infty} (1-p(w_j))\, q_r \int_{y \leq \bar{y}} 1\{v_r(y,w_j+1) \geq v_m(y,w_j+1)\}\; g_m(y|w_j)\, dy \\
&+ k_m \sum_{j=0}^{\infty} q_r \int_{y > \bar{y}} 1\{v_r(y,0) \geq v_m(y,0)\}\; g_m(y|w_j)\, dy
\end{aligned} \tag{13}$$

In a stationary equilibrium, the inflow has to be equal to the outflow of households for public and rent regulated housing.[19]

Definition 1 *A stationary equilibrium for this model consists of the following: a) offer probabilities $p(w)$ and q_r, b) distributions $g_p(y)$, $g_m(w), g_r(w)$, $g_m(y|w)$, and $g_r(y|w)$, and c) value functions $V_p(y)$, $V_m(y,w)$ and $V_r(y,w)$, such that:*

1. *Households behave optimally and value functions satisfy the equations above.*

[19]The vacancy rate in NYC has been around 2 percent during the time period of interest. Hence we ignore vacancies.

2. *The housing authority behaves according the administrative rules described above.*

3. *The densities are is consistent with the laws of motion and optimal household behavior.*

4. *$p(w)$ satisfies the market clearing condition for public housing:*

$$OF_p \quad = \quad IF_p \tag{14}$$

5. *q_r satisfies the market clearing condition for rent regulated housing:*

$$OF_r \quad = \quad IF_r \tag{15}$$

Finally note that we can endogenize the price of housing in the unregulated market by assuming that that there is an upward sloping housing supply function $H_m^s(p_m)$ and by requiring that the demand for unregulated housing given by

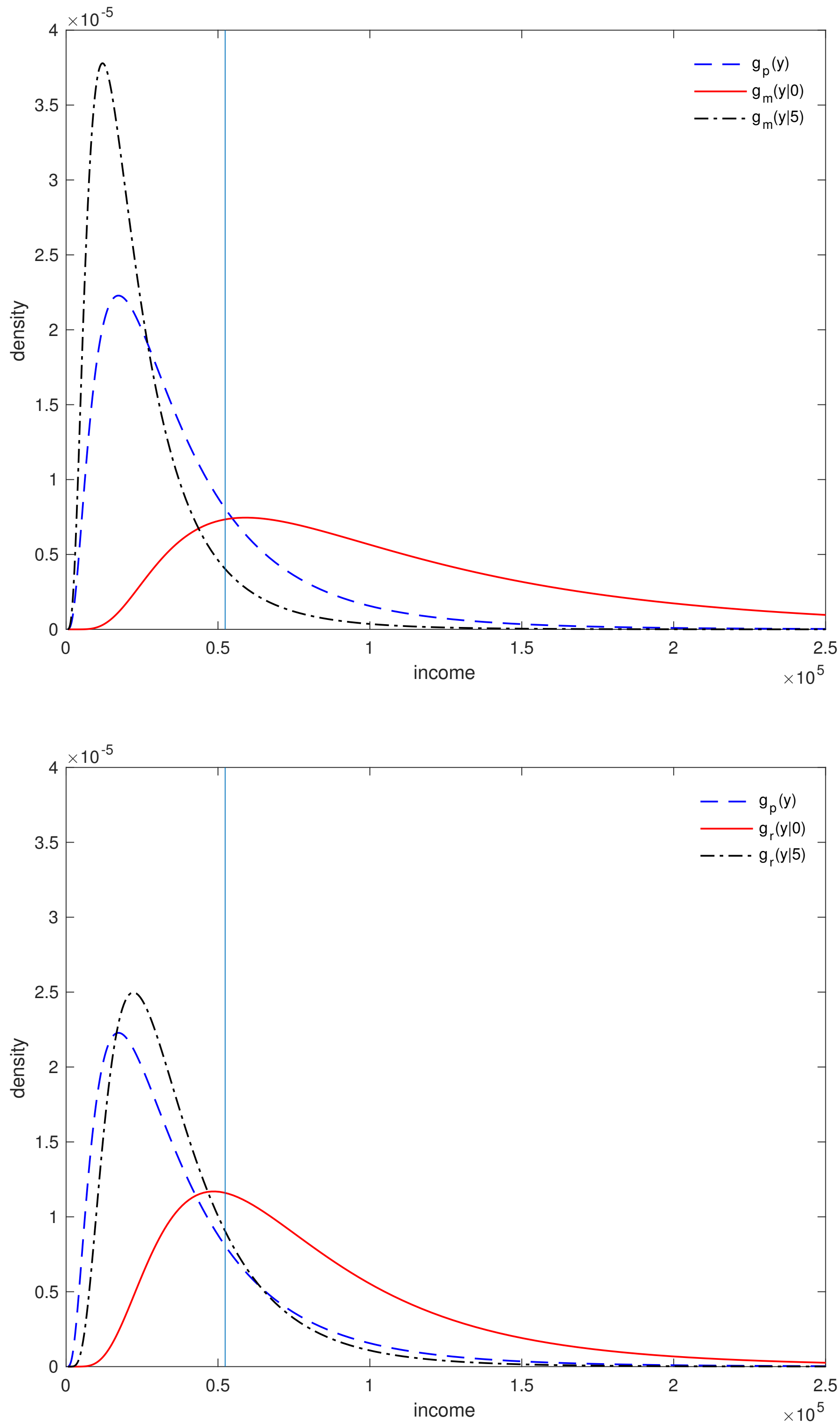
$$H_m^d = (1 - k_p - k_r) \sum_j g(w_j) \int h(p_m, y) g_m(y|w_j) \, dy \tag{16}$$

is equal to the supply.

Figure 3 illustrates the stationary equilibrium densities of income for a specification of our estimated model with one household type and only one type of rent stabilized housing.

The top panel compares the income distribution of households in the unregulated market with those that are in public housing. Not surprising we find that households with a priority score of zero, who are ineligible for public housing, have much higher income than those who live in public housing. More surprising is the result that households with a priority score of five years have lower income, on average. This due to the fact that households with high priority must have had income below the eligibility threshold for a number of consecutive periods to remain eligible, while this criteria does not apply for households in public housing. The lower panel compares the income distribution of households in the rent stabilized market with the distribution of households in public housing. Again we find similar qualitative patterns. Households that live in stabilized housing with high priority scores look similar to households in public housing.

Figure 3: Stationary Distributions



Next we characterize the properties of equilibria with rationing. The main analytical result is summarized by the following proposition.

Proposition 1 *Any stationary equilibrium with sufficiently strong excess demand for public housing has the property that there exists a value* $\bar{w} < \infty$ *such that:* $p(\bar{w}+1) = 1$, $0 \leq p(\bar{w}) < 1$, *and* $p(\bar{w} - j) = 0$ *for all* $j \geq 1$.

Note that $p(\bar{w} + 1) = 1$ implies that there are no households with priority score greater than $\bar{w} + 1$, i.e. $g(\bar{w} + 1 + j) = 0$, for $j \geq 1$.

Proof:

We use a proof by contradiction. Suppose not, then

$$p(\bar{w} + 1) < 1 \tag{17}$$

and next period there exists some households with priority score $\bar{w} + 2$, hence $g(\bar{w} + 2) > 0$ which violates the stationarity definition and the definition of $\bar{w}$.

Suppose that $p(\bar{w} - j) > 0$ and $p(\bar{w}) \leq 1$. This case violates the assumption that offers to households with lower priority ranks can only be made if all households with higher ranks receive offers.

Suppose that $p(\bar{w} - j) > 0$, $p(\bar{w}) = 1$ and $p(\bar{w} + 1) = 1$, then there will be no household in the next period which priority score $\bar{w} + 1$ which violates the stationarity assumption and that and that $g(\bar{w} + 1) > 0$.

Q.E.D.

The equilibrium has the property that everybody in the highest priority group obtains an offer to move into public housing. In addition, a fraction of the households with the second highest priority also gets an offer. The remaining households with the second highest priority score who do not get an offer this period, will obtain an offer in the next period. The intuition for this result is the following. The waitlist partitions the potential demand into $\bar{w} + 2$ cohorts. By adjusting $p(\bar{\omega})$, we can smooth out the fraction of individuals that obtains an offer. Note that $p(w_j)$ is not uniquely defined for $w_j > \bar{w} + 1$. Since the housing authority makers take-it-or-leave-it offers, there will be no households with wait times larger than $\bar{w}+1$. Without loss of generality, we can, therefore, set $p(\bar{w}+j) = 1$ for all $j > 1$.

Given this equilibrium offer function, the inflow into public housing has two components and is equal to:

$$\begin{aligned} IF_p &= p(\bar{w})\,[k_m\ g_m(\bar{w})\ IF_{mp}(\bar{w})\ +\ k_r\ g_r(\bar{w})\ IF_{rp}(\bar{w})\ ] \\ &+\ [\ k_m g_m(\bar{w}+1))\ IF_{mp}(\bar{w}+1)\ +\ k_r\ g_r(\bar{w}+1)\ IF_{rp}(\bar{w}+1)\ ] \end{aligned} \tag{18}$$

To finish the characterization of the equilibrium, we need to provide the laws of motion for the equilibrium densities. Equations (21) - (29) in Appendix A provide the details.

Finally, we would like to point out that households differ across many attributes besides income such as family size, race, ethnicity or gender of the household head. We extend our model to capture these differences using discrete household types, which also allows us to include differences in preferences over public housing and differences in access to rent stabilized units.[20] We discuss in Appendix B how to extend our model to allow for heterogeneity among households. We also estimate versions of the model with heterogeneity that account for separate wait lists for households with different characteristics.

4 Identification and Estimation

Since equilibria can only be computed numerically, we need to introduce a parametrization of the model and discuss identification and estimation. We can normalize the price of housing in the unregulated market to be equal to one since the units of housing services are arbitrary. To identify and estimate the price discount in the rent regulated market, we assume that market rents can be decomposed into a price and a quality index. We assume that the quality index is the same for units in the unregulated and the regulated markets, but the prices are not.

[20] This approach could also be used to capture the fact that some households may not consider public housing a desirable housing choice due to its stigma as suggested by Moffitt (1983).

We can, therefore use the techniques discussed in Sieg, Smith, Banzhaf, and Walsh (2002) to identify and estimate the price discount in the regulated market. We can also classify rent stabilized units into different types based on the quality levels predicted by the regression model. That approach allows us to discretize the underlying distribution of quality of rent regulated housing units.

We assume that the logarithm of income for each household follows an AR(1) process:

$$\ln(y_{it}) \;=\; \mu \;+\; \rho\, \ln(y_{it-1}) \;+\; \epsilon_{it}^{y} \tag{19}$$

The mean and the variance of income is identified of the observed income distributions in the data. The autocorrelation parameter is identified of the persistence of housing choices measured by time spent in each housing type.[21]

We assume that the flow utility functions can be approximated by a Cobb-Douglas utility function, and hence we have:

$$\begin{aligned} u_p(y, h_p) &= [(1-\tau)y]^{(1-\alpha)}\; h_p^{\alpha} \\ u_m(y) &= \alpha^{\alpha}\,(1-\alpha)^{1-\alpha}\; y\; p_m^{-\alpha} \\ u_r(y, h_r) &= [y - p_r h_r]^{(1-\alpha)}\; h_r^{\alpha} \end{aligned} \tag{20}$$

Recall that α is the housing share parameter, which is identified from the observed joint distribution of housing and income. Public housing quality, h_p, is identify from the observed demand for public housing. The quality parameters h_r for $r = 1, .., R$ are identified based on our classification algorithm discussed above and the observed market rents for each type of unit type conditional on observed characteristics. The model also predicts that the time spent in the unit is an increasing function in housing quality.

All parameters of the income process and household preferences depend on household type in the extended model. As noted before, we assume that household type is observed by the

[21] Alternatively, we could use moments from a panel data set such as the SIPP to identify the autocorrelation parameter.

econometrician. Hence, the identification argument extends to that model since all relevant moments are observed conditional on type.

Next consider the tax rate in public housing, denoted by τ. This parameter is determined by the administration of public housing programs. It is a state policy that renters in public housing pay roughly 30 percent of their income in rent.[22]

The arguments for identification are constructive and suggest that we can estimate the parameters of our model using a methods of moments estimator. We use the following moments in estimation: the fraction of each housing type, the average time spent in unit by housing type, the average income by housing type, the variance of income by type, the autocorrelation of income by type, and the housing expenditure shares by housing type. Asymptotic standard errors can be consistently estimated using the standard formula for a parametric method of moments estimator provided, for example, in Newey and McFadden (1994).

5 Empirical Results

We estimate the relative price of rent stabilized housing as discussed in the previous section. We find that rent stabilized apartments are offered at a 51 percent discount in Manhattan.[23] This explains why rent stabilized units are extremely popular in Manhattan.

Table 2 reports estimated parameters and standard errors for a variety of models. First, we estimate the baseline model (Column I). We then add heterogeneity in regulated housing types (Column II) , heterogeneity in preferences by household type (Column III), and finally explore a model with heterogeneity in household types and multiple waitlists (Column IV). Overall, we find that all parameter estimates are reasonable and estimated with good precision.

First consider the baseline model in Column I. The parameter α captures the housing expenditure share for households that rent in the unregulated market. Low- and moderate-

[22] Also note that k_m, k_p and k_r are observed in the data.

[23] See Appendix B for details of how to measure the price discount for stabilized housing.

Table 2: Estimated Parameters

	I	II	III		IV	
	Baseline	1 Type	2 Type - 1 Queue		2 Type - 2 Queue	
	all	all	female	male	female	male
α	0.45 (0.01)	0.46 (0.01)	0.50 (0.02)	0.43 (0.01)	0.47 (0.01)	0.44 (0.01)
μ_y	10.62 (0.03)	10.64 (0.02)	10.59 (0.03)	10.69 (0.03)	10.56 (0.03)	10.70 (0.06)
σ	0.54 (0.02)	0.53 (0.02)	0.50 (0.01)	0.58 (0.03)	0.49 (0.01)	0.59 (0.06)
ρ	0.76 (0.02)	0.76 (0.03)	0.77 (0.03)	0.72 (0.04)	0.80 (0.02)	0.69 (0.02)
h_p	26,552 (515)	25,902 (866)	25,985 (670)		24,189 (2296)	29,841 (1278)
h_1	32,240 (673)	26,795 (604)	27,110 (620)		26,527 (618)	
h_2		37,980 (1087)	37,605 (918)		37,072 (440)	

Standard errors are in parenthesis.

income households in Manhattan spend approximately 45 percent of their income on housing if they rent in the unregulated market. Allowing for heterogeneity among households in Columns III and IV shows that female headed households have slightly larger housing share parameters than male headed households.

The parameters of the income process also depend on the observed household type. Comparing the estimates in Column I and II with those in Columns III and IV, we find that male headed households tend to have higher, more volatile, and less persistent incomes than female headed households. The autocorrelation coefficient ranges between 0.69 and 0.80 suggesting that income shocks are fairly persistent.

Housing quality is measured as equivalent expenditures in the unregulated market. Column I shows that an average public housing unit in Manhattan provides the same quality as a unit that rents for approximately $26,000 dollars in the unregulated market. The average quality of rent stabilized housing in the baseline model is approximately $32,000. Allowing for heterogeneity in rent stabilized housing in Columns II-IV indicates that the quality for a low (high) quality rent stabilized apartment is approximately $27,000 ($38,000). Low quality stabilized units are, therefore, similar to public housing units while high quality units are significantly nicer than units in public housing.

Turning our attention to the model with multiple wait lists in Column IV, we find that the main empirical results are qualitatively and quantitatively similar to the model with one wait list (Columns I-III). The main difference is that male headed households tend to value public housing higher than female headed households.

Table 3: Properties of Equilibrium

		Baseline	1 Type	2 Type - 1 Queue	2 Type - 2 Queue	
wait	$\bar{w}$	18	17	17	19	18
times	$p(\bar{w})$	0.82	0.53	0.96	0.75	0.72
search	q_1	0.25	0.13	0.14	0.14	
frictions	q_2		0.10	0.11	0.11	

Table 3 summarizes some properties that correspond to the equilibria that are implied by the parameter estimates. In equilibrium, our model generates wait times of approximately 18 years. The probability of finding a rent stabilized unit is approximately 25 percent, 11 percent for high quality units and 14 percent for low quality units. Male headed households tend to have slightly shorter wait times then female headed households, but the predicted difference is only one year or approximately 5 percent of the wait time.

Tables 4 reports a variety of goodness of fit statistics. We report the key statistics for three models. Overall, we find that our models fit the key moments used in estimation well.

Table 4: Model Fit

	housing	percent		years		income		market rent	
				Baseline					
	Public	9.90	9.90	16.18	16.37	32930	33914	——	——
	Regulated	57.20	57.20	9.49	9.20	54739	55615	1317	1309
	Market	32.90	32.90	3.85	4.22	71045	70262	2640	2642
				2 Type - 1 Queue					
	Public	6.55	6.55	15.39	16.75	28796	33732	——	——
female	Regulated1	12.55	13.15	10.03	8.90	45516	43625	1048	1101
	Regulated2	14.90	14.79	10.41	10.41	55184	59342	1484	1527
	Market	16.00	15.51	3.70	4.19	69970	65844	2555	2729
	Public	2.95	2.95	18.34	13.41	44298	36075	——	——
male	Regulated1	16.55	15.95	8.37	8.54	53550	50321	1093	1101
	Regulated2	13.45	13.56	8.99	8.59	66288	66296	1695	1527
	Market	17.05	17.55	4.04	4.22	72300	74908	2743	2673
				2 Type - 2 Queue					
	Public	6.55	6.55	15.39	16.02	28796	30942	——	——
female	Regulated1	12.55	13.20	10.03	8.67	45516	44186	1048	1077
	Regulated2	14.90	14.38	10.41	10.29	55184	59558	1484	1506
	Market	16.00	15.87	3.70	4.21	69970	67341	2555	2654
	Public	2.95	2.95	18.34	18.99	44298	42304	——	——
male	Regulated1	16.55	15.90	8.37	7.90	53550	48874	1093	1077
	Regulated2	13.45	13.97	8.99	8.71	66288	64866	1695	1506
	Market	17.05	17.18	4.04	3.95	72300	74827	2743	2743

6 Policy Analysis

Our estimates imply that public and regulated housing units are attractive options for low- and moderate-income households in Manhattan. To gain some additional insights, we compute the monetary benefits associated with having access to public housing. Our benefits measures are compensating variations. Note that a one-time income increase is persistent in our model since income follows an AR(1) process. We, therefore, compute the present value of the income stream that is associated with a one-time compensating increase in income.[24] Figure 4 plots the net present values of income that make households indifferent between having access to public housing and not.

Figure 4: Public Housing versus Unregulated Housing

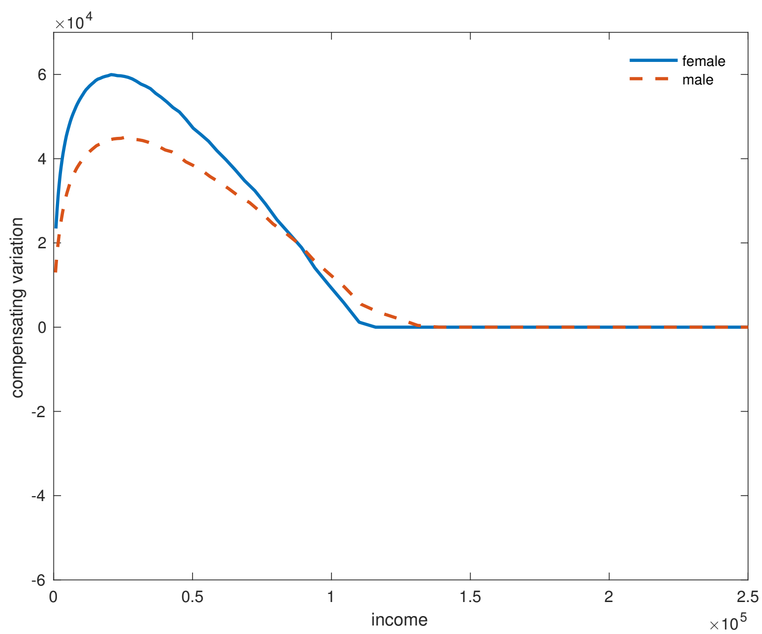


Figure 4 shows that there are substantial benefits for low- and moderate-income households. These gains are up to $60,000. These benefits primarily arise from two sources. First, public housing provides higher quality housing than housing purchased in equilibrium in the unregulated markets. These gains decrease with income. Second, households in public housing

[24]Alternatively we computed percentage increases in consumption that make households indifferent.

have significantly higher levels of non-housing consumption than households that rent in the unregulated market. This second effect arises from the fact that households in public housing only pay 30 percent of their income in rent while households that rent in the unregulated market pay between 45 and 50 percent of their income in rent.

Table 5: Public Housing and Unregulated Housing

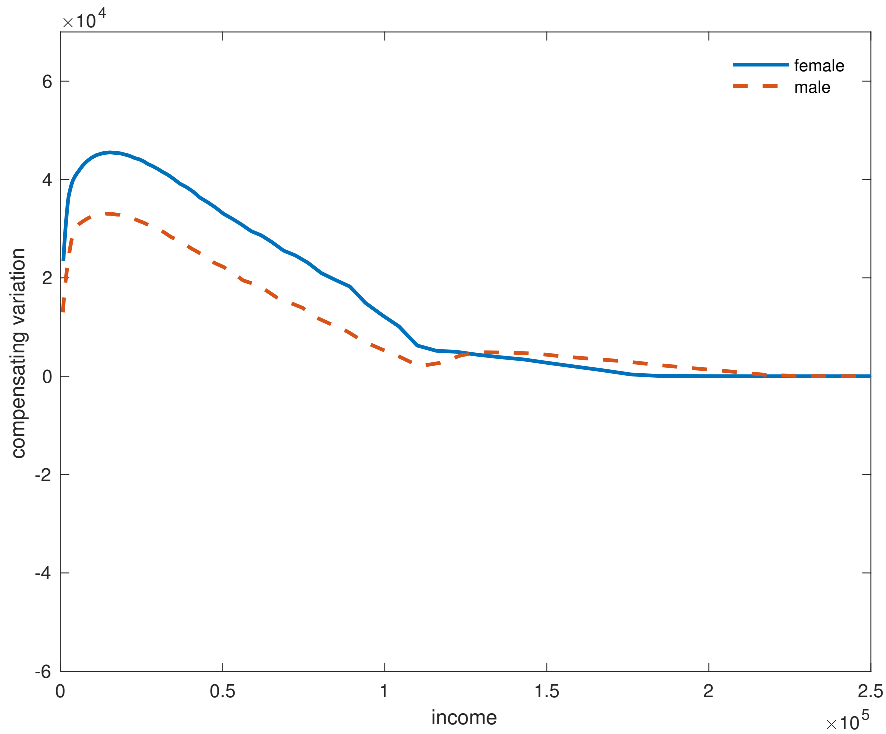
	Housing		Non-housing	
income	public	unregulated	public	unregulated
10,000	26,552	4,500	7,000	5,500
30,000	26,552	13,500	21,000	16,500
50,000	26,552	22,500	35,000	27,500
70,000	26,552	31,500	49,000	38,500

Table 5 illustrates these two effects for households with different income levels. Low-income household obtain very large housing subsidies, but only small consumption subsidies. Moderate-income households with incomes ranging between $30,000 and $50,000 obtain significant housing and consumptions subsidies. Households with $70,000 income experience a mismatch because public housing quality is too low relative to their preferred housing quality purchased in the unregulated market.

Figure 5 plots the net present values of income that make households indifferent between public housing and rent stabilized housing. The top panel focuses on low quality units while the lower panel focuses on high quality units. We find that low-income households tend to prefer public housing while moderate income households prefer high quality rent regulated apartments. High quality rent stabilized housing provides much higher housing services than public housing and hence is a better match for moderate-income households.

Finally we compare rent stabilized housing with unregulated housing. Figure 6 plots the net present values of income that make households indifferent between rent stabilized housing and unregulated housing. The top panel focuses on low quality units while the lower panel focuses on high quality units. Overall, we find an inverted-u shaped relationship. Moderate

Figure 5: Public Housing versus Rent Stabilized Housing



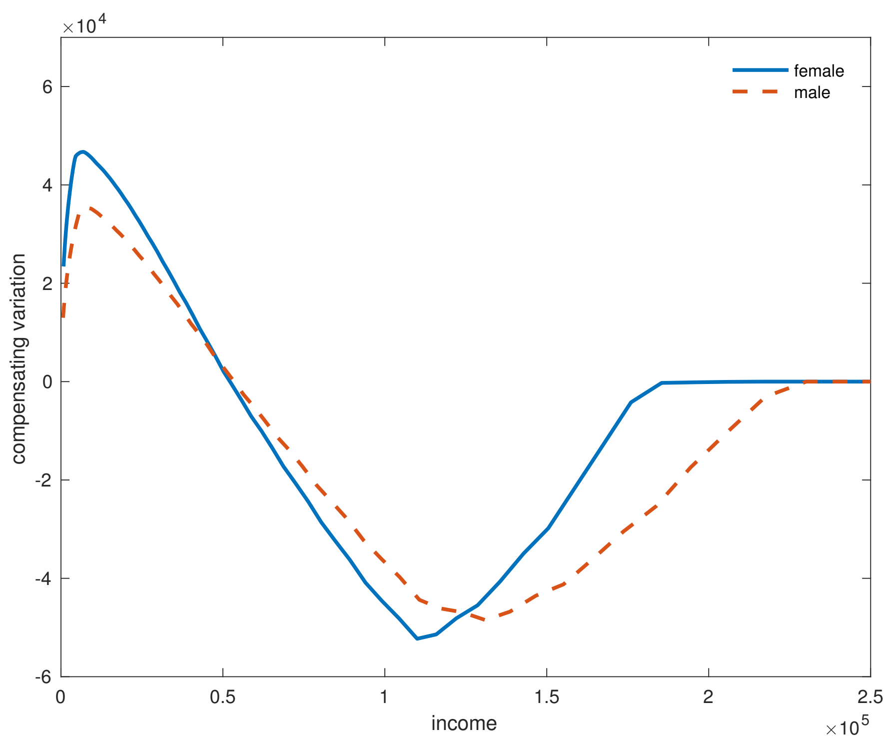
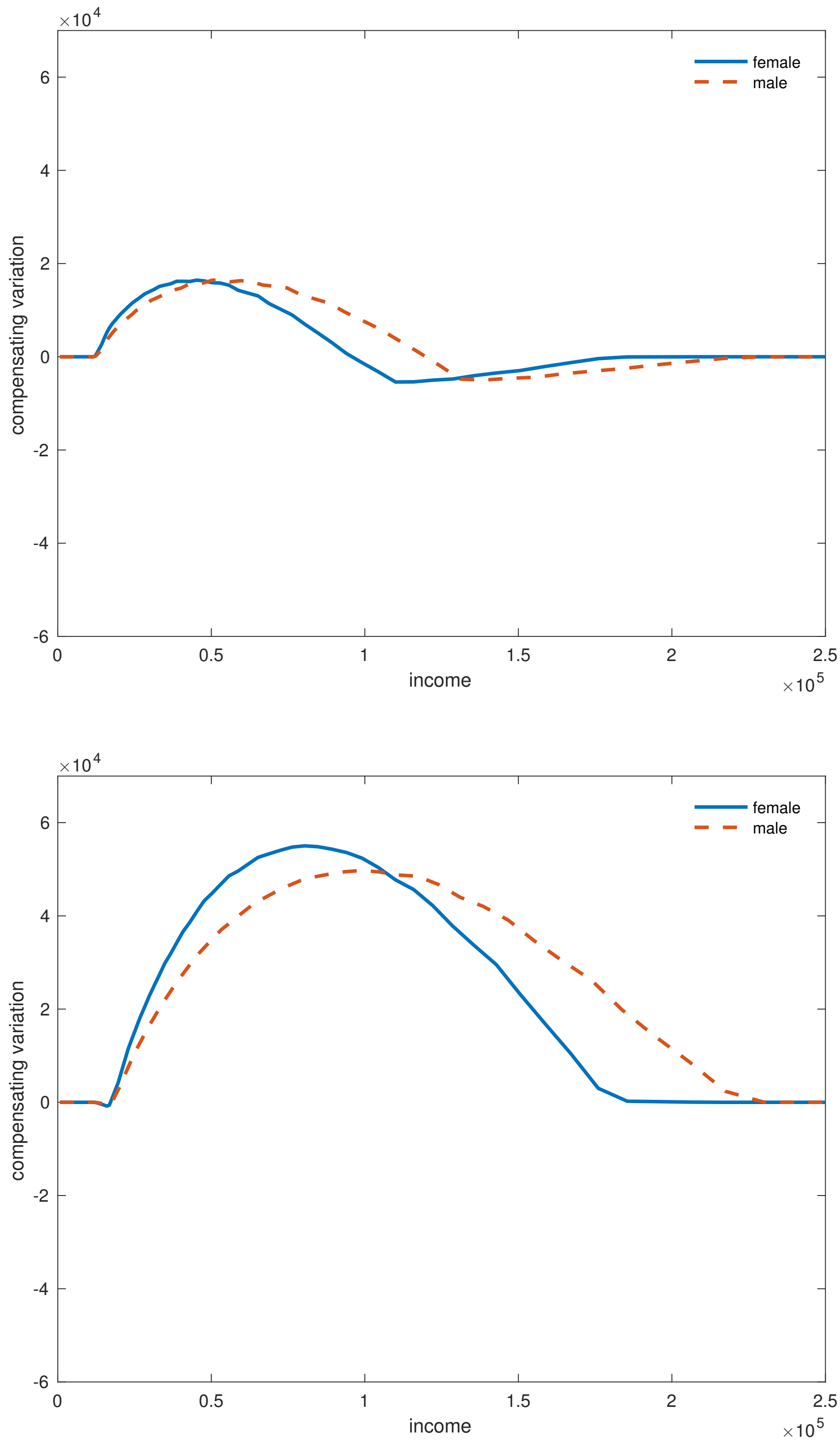


Figure 6: Rent Stabilized Housing versus Unregulated Housing



income households benefit the most from rent stabilized housing. The welfare gains are up to $20,000 for low quality units and up to $50,000 for high quality units.

We, therefore, conclude that there are substantial benefits associated with having access to public or rent stabilized housing. It should be clear that we cannot conclude that these policies are welfare improving since a full and comprehensive welfare analysis also needs to take the cost associated with the affordable housing policies into consideration.

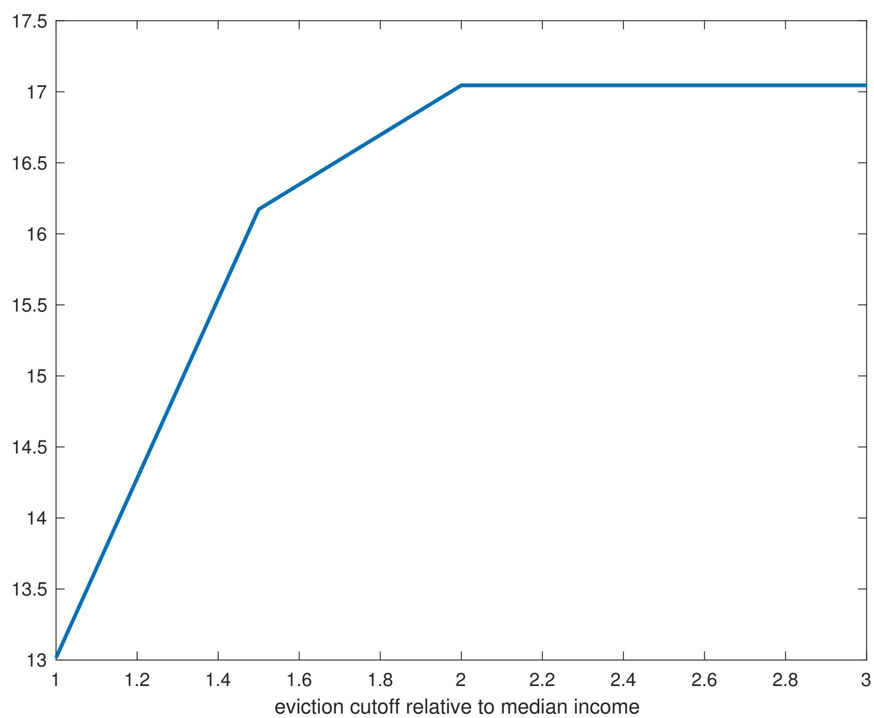
In addition, there seems to be some scope for improving existing housing policies. One surprising feature of existing public housing policies is that housing authorities rarely ask households to leave public housing even if household income significantly exceeds the eligibility threshold. Exit from public housing is purely voluntary. Current housing policies, therefore, imply that many eligible households do not have access to public housing, while some ineligible households receive large subsidies. This suggests that housing aid is not effectively targeted towards the most needy households. Instead, housing aid can be an open-ended entitlement for the lucky few that were initially deserving but whose circumstances have since changed.

Table 6: Public Housing Mismatch

	Fraction of Ineligible Households		Percent Income over Threshold	
	Data	Model	Data	Model
baseline	0.17	0.18	0.36	0.38
one type	0.17	0.19	0.36	0.38
1-queue-female	0.12	0.18	0.32	0.34
1-queue-male	0.30	0.21	0.41	0.42
2-queue-female	0.12	0.13	0.32	0.34
2-queue-male	0.30	0.29	0.41	0.51

Tables 6 shows that our model fits the overall fraction of ineligible households in public housing well. We also match the average difference between the income threshold and realized incomes. Our model seems well suited to study the welfare implications of public housing mismatch. A natural question is whether it is possible to design a different housing policy that provides a more fair distribution of benefits by targeting aid to the most needy in the population. One way to accomplish this goal is by strictly enforcing the current set of eligibility criteria. Recent reforms of the welfare system have tried to stress the importance of helping individuals become self-sufficient by imposing strict limits on the eligibility of welfare benefits.[25] Housing aid is the only major welfare program in the U.S. that does not strictly enforce its eligibility criteria.[26] It is also useful to conduct this counterfactual analysis since it provides some interesting insights into the equilibrium properties of our dynamic model.

Figure 7: Wait Times

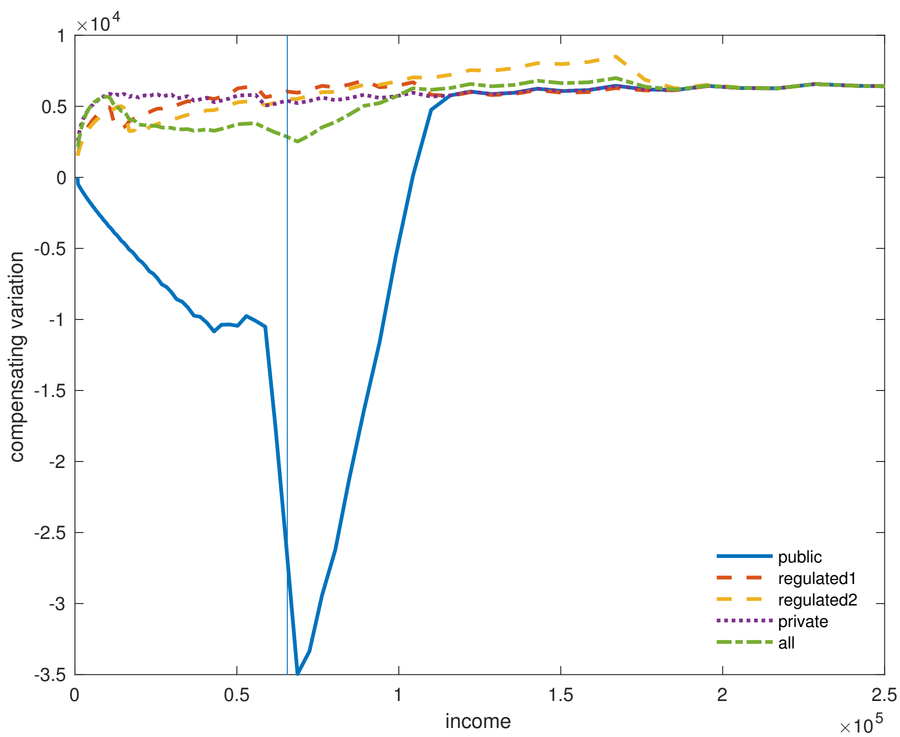


[25]Self-sufficiency was Johnson's original goal in launching his War on Poverty. The stated objective was to remove the causes, not just the consequences of poverty.

[26]Imposing time limits and requiring work requirements has been another attempt to increase self-sufficiency among welfare recipients. Enforcing eligibility criteria may cause some practical transitional problems, especially in a high rent city such as NYC. A natural concern would be to minimize potential displacement costs for young children.

We evaluate this policy using our estimated two-type model with one wait list. Figures 7 plots the impact of this policy on wait times as a function of the eviction cut-off. We find that the more restrictive the enforcement policy, the shorter the average wait times. Consider the case in which the eviction cut-off is set at the median income. Our model suggests that average wait times would be reduced by 25 percent without any additions to the public housing stock. From this, we can conclude that a more stringent enforcement policy would alleviate some of the bottlenecks in the supply of public housing without relying on costly new investments.

Figure 8: Compensating Variations: Enforcing Eligibility Criteria



We also find that such a policy has some interesting distributional welfare effects. Figure 8 plots compensating variations associated with this policy as function of income and housing status for a female headed household using median income as the eviction threshold.[27].

Naturally, the main losers of the policy of enforcing eligibility criteria are households that

[27]The findings are similar for male headed households and are available form the authors.

are currently living in public housing. Their welfare is significantly reduced for two reasons. First, households that lose eligibility are evicted from public housing and must rent in the unregulated market. Second, currently eligible households will face possible eviction in the future if they experience a sequence of positive income shocks. The main beneficiaries of this policies are poor and eligible household that are currently on the waitlist. Evicting higher income households from public housing creates more vacancies and thus reduces the expected time spent on the waitlist. Hence, these households are more likely to move into public housing. Welfare also increases for ineligible households outside of public housing, since public housing becomes more readily available and provides limited insurance against negative income shocks.

Figure 9: Benefits by Household Type

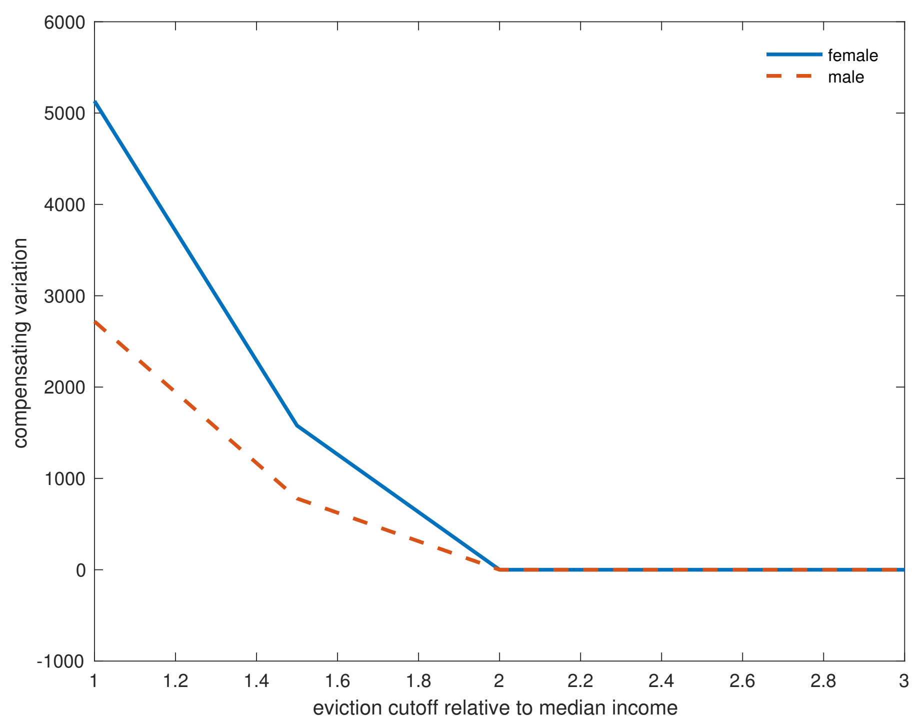


Figure 9 illustrates the changes in welfare as a function of the eviction threshold. We find that male headed households tend to gain less compared to female headed households under the more stringent enforcement policies. This largely follows from the fact that male headed households tend to have higher levels of income.

In practice it may be difficult to enforce eligibility criteria. In addition, one would need to take family status into consideration since it may undesirable to displace young children. As a consequence, it may be easier to phase out the subsidy and charge households market rents if they lose eligibility. However, the analysis above suggests that there is some scope for improving existing housing policies by targeting them to the most deserving subpopulation.

7 Conclusions

We have developed a new dynamic model that captures search frictions and queuing in the market for rent stabilized housing for low- and moderate-income households. We have characterized the stationary equilibrium with rationing that arises in the model, and have shown how to identify and estimate the structural parameters of the model. Our application focus is on the housing markets of Manhattan in 2011. Overall, our model fits the observed sorting of households well. We have characterized the distribution of welfare that arises in our model and shown that access to public (rent stabilized) housing can increase welfare by as much as $60,000 ($50,000). Of course, we cannot conclude from this analysis that these policies are desirable because a full welfare analysis also needs to take the costs of providing these goods into consideration. Nevertheless, we have shown that the benefits associated with the policies can be substantial.

Despite these benefits, there seems to be some scope for improving current housing policies. We have considered the impact of a public housing policy that strictly enforces eligibility criteria. We find that such a policy benefits eligible low-income households that are currently on the waitlist and hurts households that are currently in public housing. Strict enforcement of eligibility criteria would significantly lowers the average wait time for public housing and provide more aid to the neediest families in the population, even absent costly new investments in public housing stock.

Our research provides ample scope for future work. Current policies may provide disincentives to work. Our data suggest that 70 percent of households that currently life in public housing work, at least, part time defined as making at least $10,000 in labor income in 2011. In contrast, 83 percent of all households in rent stabilized housing and 87 percent of households that rent in the unregulated market are in the work force. As we discuss in detail in Appendix D of the paper, we do not find any evidence in the data that households manipulate their incomes to stay below the eligibility threshold for public housing. Nevertheless, some policy makers currently contemplate whether or not to impose work requirements for housing aid recipients.[28] A interesting, but non-trivial extension of our model would endogenize labor market participation to study the impact of public housing policies on work incentives and the distribution of welfare.

Finally, our model does not allow households to enter or leave local housing market in response to market conditions. It is not difficult to extend the model to allow for such an outside option. We did not pursue this option in this paper since the NYCHVS is a repeated cross-sectional sample. Hence, it is not possible to track households over time. Moreover, it is difficult to characterize mobility in and out of the NY metropolitan area based on these data. Nevertheless, one could conduct a comparative static equilibrium analysis over longer time horizons, which may provide some insights into important topics such as the impact of supply side changes affecting the availability of rent stabilized housing in Manhattan.

[28]The WSJ reported on May 7, 2013 that the Obama administration was introducing a pilot program to study these changes in the 2014 budget.

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A Law of Motions for the Income Distributions

The equilibrium rationing rule then implies the following law of motion for the stationary income distributions:

$$
\begin{aligned}
g_p(y) \;=\;& k_p\,(1-q_r) \int 1\{v_p(x) \geq v_m(x,0)\}\; f(y|x)\; g_p(x)\; dx \\
+\;& k_p\, q_r \int 1\{v_p(x) \geq \max[v_m(x,0), v_r(x,0)]\}\; f(y|x)\; g_p(x)\; dx \\
+\;& k_m \sum_{j=0}^{\infty} g_m(w_j)\; p(w_j)\; (1-q_r) \int_{x \leq \bar{y}} 1\{v_p(x) \geq v_m(y,0))\}\; f(y|x)\; g_m(x|w_j)\; dx \\
+\;& k_m \sum_{j=0}^{\infty} g_m(w_j)\; p(w_j)\; q_r \int_{x \leq \bar{y}} 1\{v_p(x) \geq \max[v_m(x,0), v_r(x,0)]\}\; f(y|x)\; g_m(x|w_j) dx \\
+\;& k_r \sum_{j=0}^{\infty} g_r(w_j)\; p(w_j) \int_{x \leq \bar{y}} 1\{v_p(x) \geq \max[v_m(x,0), v_r(x,0)]\}\; f(y|x)\; g_r(x|w_j)\; dx
\end{aligned}
\tag{21}
$$

and

$$
\begin{aligned}
g_m(y|0) \;=\;& k_p\,(1-q_r) \int 1\{v_m(x,0) \geq v_p(x)\}\; f(y|x)\; g_p(x)\; dx \\
+\;& k_p\, q_r \int 1\{v_m(x,0) \geq \max[v_p(x), v_r(x,0)]\}\; f(y|x)\; g_p(x)\; dx \\
+\;& k_m \sum_{j=0}^{\infty} g_m(w_j)\; p(w_j)\; (1-q_r) \int_{x \leq \bar{y}} 1\{v_m(x,0) \geq v_p(x)\}\; f(y|x)\; g_m(x|w_j)\; dx \\
+\;& k_m \sum_{j=0}^{\infty} g_m(w_j)\; p(w_j)\; q_r \int_{x \leq \bar{y}} 1\{v_m(x,0) \geq \max[v_p(x), v_r(x,0)]\}\; f(y|x)\; g_m(x|w_j)\; dx \\
+\;& k_r \sum_{j=0}^{\infty} g_r(w_j)\; p(w_j) \int_{x \leq \bar{y}} 1\{v_m(x,0) \geq \max[v_p(x), v_r(x,0)]\}\; f(y|x)\; g_r(x|w_j)\; dx \\
+\;& k_m \sum_{j=0}^{\infty} g_m(w_j)\; (1-q_r) \int_{x > \bar{y}} f(y|x)\; g_m(x|w_j)\; dx \\
+\;& k_m \sum_{j=0}^{\infty} g_m(w_j)\; q_r \int_{x > \bar{y}} 1\{v_m(x,0) \geq v_r(x,0)\}\; f(y|x)\; g_m(x|w_j)\; dx \\
+\;& k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > \bar{y}} 1\{v_m(x,0) \geq v_r(x,0)\}\; f(y|x)\; g_r(x|w_j)\; dx
\end{aligned}
\tag{22}
$$

and

$$
\begin{aligned}
k_m\ g_m(0) &= k_p\ (1-q_r) \int 1\{v_m(x,0) \geq v_p(x)\}\ g_p(x)\ dx \\
&+ k_p\ q_r \int 1\{v_m(x,0) \geq \max[v_p(x), v_r(x,0)]\}\ g_p(x)\ dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j)\ p(w_j)\ (1-q_r) \int_{x\leq\bar{y}} 1\{v_m(x,0) \geq v_p(x)\}\ \ g_m(x|w_j)\ dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j)\ p(w_j)\ q_r \int_{x\leq\bar{y}} 1\{v_m(x,0) \geq \max[v_p(x), v_r(x,0)]\}\ g_m(x|w_j)\ dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j)\ p(w_j) \int_{x\leq\bar{y}} 1\{v_m(x,0) \geq \max[v_p(x), v_r(x,0)]\}\ g_r(x|w_j)\ dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j)\ (1-q_r) \int_{x>\bar{y}} g_m(x|w_j)\ dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j)\ q_r \int_{x>\bar{y}} 1\{v_m(x,0) \geq v_r(x,0)\}\ g_m(x|w_j)\ dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x>\bar{y}} 1\{v_m(x,0) \geq v_r(x,0)\}\ g_r(x|w_j)\ dx
\end{aligned} \tag{23}
$$

Moreover,

$$
\begin{aligned}
g_r(y|0) &= k_p\ q_r \int 1\{v_r(x,0) \geq \max[v_p(x), v_m(x,0)]\}\ f(y|x)\ g_p(x)\ dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j)\ p(w_j)\ q_r \int_{x\leq\bar{y}} 1\{v_r(x,0) \geq \max[v_p(x), v_m(x,0)]\}\ f(y|x)\ g_m(x|w_j)\ dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j)\ p(w_j) \int_{x\leq\bar{y}} 1\{v_r(x,0) \geq \max[v_p(x), v_m(x,0)]\}\ f(y|x)\ g_r(x|w_j)\ dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j)\ q_r \int_{x>\bar{y}} 1\{v_r(x,0) \geq v_m(x,0)\}\ f(y|x)\ g_m(x|w_j)\ dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x>\bar{y}} 1\{v_r(x,0) \geq v_m(x,0)\}\ f(y|x)\ g_r(x|w_j)\ dx
\end{aligned} \tag{24}
$$

and

$$
\begin{aligned}
k_r\ g_r(0) &= k_p\ q_r \int 1\{v_r(x,0) \geq \max[v_p(x), v_m(x,0)]\}\ g_p(x)\ dx \qquad (25)\\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j)\ p(w_j)\ q_r \int_{x\leq\bar{y}} 1\{v_r(x,0) \geq \max[v_p(x), v_m(x,0)]\}\ g_m(x|w_j)\ dx\\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j)\ p(w_j) \int_{x\leq\bar{y}} 1\{v_r(x,0) \geq \max[v_p(x), v_m(x,0)]\}\ g_r(x|w_j)\ dx\\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j)\ q_r \int_{x>\bar{y}} 1\{v_r(x,0) \geq v_m(x,0)\}\ g_m(x|w_j)\ dx\\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x>\bar{y}} 1\{v_r(x,0) \geq v_m(x,0)\}\ g_r(x|w_j)\ dx
\end{aligned}
$$

$$
\begin{aligned}
g_m(y|w_j) &= k_m\ g_m(w_j-1)\ (1-q_r) \int_{x\leq\bar{y}} f(y|x)\ g_m(x|w_j-1)\ dx \qquad (26)\\
&+ k_m\ g_m(w_j-1)\ q_r \int_{x\leq\bar{y}} 1\{v_m(x,w_j) \geq v_r(x,w_j)\}\ f(y|x)\ g_m(x|w_j-1)\ dx\\
&+ k_r\ g_r(w_j-1) \int_{x\leq\bar{y}} 1\{v_m(y,w_j) \geq v_r(y,w_j)\}\ f(y|x)\ g_r(x|w_j-1)\ dx
\end{aligned}
$$

and

$$
\begin{aligned}
g_m(w_j) &= g_m(w_j-1)\ (1-p(w_j-1))\ (1-q_r) \int_{x\leq\bar{y}} g_m(x|w_j-1)\ dx \qquad (27)\\
&+ g_m(w_j-1)\ (1-p(w_j-1))\ q_r \int_{x\leq\bar{y}} 1\{v_m(x,w_j) \geq v_r(x,w_j)\}\ g_m(x|w_j-1)\ dx\\
&+ \frac{k_r}{k_m}\ g_r(w_j-1)\ (1-p(w_j-1)) \int_{x\leq\bar{y}} 1\{v_m(x,w_j) \geq v_r(x,w_j)\}\ g_r(x|w_j-1)\ dx
\end{aligned}
$$

$$
\begin{aligned}
g_r(y|w_j) &= k_r\ g_r(w_j-1) \int_{x\leq\bar{y}} 1\{v_r(x,w_j) \geq v_m(x,w_j)\}\ f(y|x)\ g_r(x|w_j-1)\ dx \qquad (28)\\
&+ k_m\ g_m(w_j-1)\ q_r \int_{x\leq\bar{y}} 1\{v_r(x,w_j) \geq v_m(x,w_j)\}\ f(y|x)\ g_m(x|w_j-1)\ dx
\end{aligned}
$$

and

$$g_r(w_j) = g_r(w_j - 1)\,(1 - p(w_j - 1)) \int_{x \leq \bar{y}} 1\{v_r(x, w_j) \geq v_m(x, w_j)\}\, g_r(x|w_j - 1)\, dx \quad (29)$$
$$+ \frac{k_m}{k_r}\, g_m(w_j - 1)\,(1 - p(w_j - 1))\, q_r \int_{x \leq \bar{y}} 1\{v_r(x, w_j) \geq v_m(x, w_j)\}\, g_m(x|w_j - 1)\, dx$$

B Extending the Model to Allow for Multiple Household Types

We can allow for different discrete types allowing for differences in family structure. Assume that there are I types of households. Household types are defined by family structure (number of kids, number of adults etc.) Each household has a fixed share denoted by s_i, where $\sum_{i=1}^{I} s_i = 1$.[29] We make the following simplifying assumption which can be easily relaxed.

Assumption 3 *The housing authority operates one waitlist for all types and all types compete for the same housing units in the unregulated and regulated markets.*

Let (k_{ip}, k_{ir}, k_{im}) denote the relevant type specific market shares. Let $g_{im}(w)$ ($g_{ir}(w)$) denote the marginal distribution of wait times for households of type i in unregulated (rent regulated) housing in stationary equilibrium. Let $g_{ip}(y)$ denote the density of income of households of type i that are inside public housing at the beginning of each period. Similarly let $g_{im}(y|w)$ ($g_{ir}(y|w)$) denote the stationary density of income conditional on wait time for households in the unregulated (regulated) market.

[29] This approach is in the spirit of Heckman and Singer (1984) although we will treat the household type as observed.

The voluntary flow of type i households out of public housing is given by:

$$\begin{aligned} OF_{ip} &= k_{ip}\,(1-q_r)\int 1\{v_{im}(y,0) > v_{ip}(y)\}\; g_{ip}(y)\; dy \\ &+ k_{ip}\; q_r \int 1\{v_{im}(y,0) \geq \max[v_{ip}(y), v_{ir}(y,0)]\}\; g_{ip}(y)\; dy \\ &+ k_{ip}\; q_r \int 1\{v_{ir}(y,0) \geq \max[v_{ip}(y), v_{im}(y,0)]\}\; g_{ip}(y)\; dy \end{aligned} \tag{30}$$

Note that the first two terms is the outflow to the unregulated market and the third term captures the outflow to the rent regulated market.

The flow into public housing of type i households is given by:

$$\begin{aligned} IF_{ip} &= p(\bar{w})\,[k_{im}\; g_{im}(\bar{w})\; IF_{imp}(\bar{w}) \;+\; k_{ir}\; g_{ir}(\bar{w})\; IF_{irp}(\bar{w})\;] \\ &+ \left[\; k_{im} g_{im}(\bar{w}+1))\; IF^{i}_{imp}(\bar{w}+1) \;+\; k_{ir}\; g_{ir}(\bar{w}+1)\; IF_{irp}(\bar{w}+1)\;\right] \end{aligned} \tag{31}$$

where the inflow from the unregulated market conditional on wait time is:

$$\begin{aligned} IF_{imp}(w) &= (1-q_r)\int_{y\leq\bar{y}} 1\{v_{ip}(y) \geq v_{im}(y,0)\}\; g_{im}(y|w)\; dy \\ &+ q_r \int_{y\leq\bar{y}} 1\{v_{ip}(y) \geq \max[v_{im}(y,0), v_{ir}(y,0)]\}\;\; g_{im}(y|w)\; dy \end{aligned} \tag{32}$$

and the inflow from the rent regulated market is given by:

$$IF_{irp}(w) = \int_{y\leq\bar{y}} 1\{v_{ip}(y) \geq \max[v_{im}(y,0), v_{ir}(y,0)]\}\; g_{ir}(y|w)\; dy \tag{33}$$

Equilibrium in public housing requires that for each housing type i, we have

$$IF_p = \sum_{i=1}^{I} IF_{ip} \;=\; \sum_{i=1}^{I} OF_{ip} = OF_p \tag{34}$$

Next consider the market for regulated housing. The voluntary flow of type i households out of rent regulated housing is given by:

$$\begin{aligned} OF_{ir} &= k_{ir} \sum_{j=0}^{\infty} p(w_j)\, g_{ir}(w_j) \int_{y \le \bar{y}} 1\{v_{ip}(y) \ge \max[v_{im}(y,0), v_{ir}(y,0)]\}\, g_{ir}(y|w_j)\, dy \\ &+ k_{ir} \sum_{j=0}^{\infty} p(w_j)\, g_{ir}(w_j) \int_{y \le \bar{y}} 1\{v_{im}(y,0) \ge \max[v_{ip}(y), v_{ir}(y,0)]\}\, g_{ir}(y|w_j)\, dy \\ &+ k_{ir} \sum_{j=0}^{\infty} (1 - p(w_j))\, g_{ir}(w_j) \int_{y \le \bar{y}} 1\{v_{im}(y, w_j + 1) \ge \max[v_{ir}(y, w_j + 1)]\}\, g_{ir}(y|w_j)\, dy \\ &+ k_{ir} \sum_{j=0}^{\infty} g_{ir}(w_j) \int_{y > \bar{y}} 1\{v_{im}(y,0) \ge \max[v_{ir}(y,0)]\}\, g_{ir}(y|w_j)\, dy \end{aligned} \tag{35}$$

Note that the first term is the outflow to public housing. The second term is the outflow to unregulated housing if you have an offer to move into public housing. The last two terms are the outflow to unregulated housing if you do not have an offer to move into public housing.

The flow into rent regulated housing is given by:

$$IF_{ir} = k_{im} \sum_{j=0}^{\infty} g_{im}(w_j)\, IF_{imr}(w_j) + k_{ip}\, IF_{ipr} \tag{36}$$

where the inflow from the unregulated market conditional on wait time is:

$$\begin{aligned} IF_{imr}(w_j) &= q_r\, p(w_j) \int_{y \le \bar{y}} 1\{v_{ir}(y,0) \ge \max[v_{im}(y,0), v_{ip}(y)]\}\ \ g_{im}(y|w_j)\, dy \\ &+ q_r\, (1 - p(w_j)) \int_{y \le \bar{y}} 1\{v_{ir}(y, w_j + 1) \ge v_{im}(y, w_j + 1)\}\ \ g_{im}(y|w_j)\, dy \\ &+ q_r \int_{y > \bar{y}} 1\{v_{ir}(y,0) \ge v_{im}(y,0)\}\ \ g_{im}(y|w_j)\, dy \end{aligned} \tag{37}$$

and the flow from public housing market to rent regulated housing is given by:

$$IF_{ipr} = q_r \int 1\{v_{ir}(y,0) \ge \max[v_{im}(y,0), v_{ip}(y)]\}\, g_{ip}(y)\, dy \tag{38}$$

Equilibrium requires that the aggregate outflow equal the aggregate inflow

$$IF_r = \sum_{i=1}^{I} IF_{ir} = \sum_{i=1}^{I} OF_{ir} = OF_r \tag{39}$$

As before, we can define a stationary equilibria with rationing as follows:

Definition 2 *A stationary equilibrium with rationing for the extended model consists of the following: a) market shares* (k_{ip}, k_{ir}, k_{im}) $i = 1, .., I$, *b) offer probability* $p(w)$ *and* q_r, *c) distributions* $g_{ip}(y)$, $g_{im}(w), g_{ir}(w)$, $g_{im}(y|w)$, *and* $g_{ir}(y|w)$, *and d) value functions* $V_{ip}(y)$, $V_{im}(y, w)$ *and* $V_{ir}(y, w)$, *such that:*

1. *Households behave optimally and value functions satisfy the equations above.*

2. *The housing authority behaves according the administrative rules described above.*

3. *The densities are is consistent with the laws of motion and optimal household behavior.*

4. $p(w)$ *satisfies the market clearing condition for public housing:*

$$OF_p = IF_p \tag{40}$$

5. q_r *satisfies the market clearing condition for rent regulated housing:*

$$OF_r = IF_r \tag{41}$$

6. *The following identities hold for the market shares:*

$$\begin{aligned} \sum_{i=1}^{I} k_{ir} &= k_r \\ \sum_{i=1}^{I} k_{im} &= k_m \\ k_{ip} + k_{ir} + k_{im} &= s_i \qquad i = 1, ..., I \end{aligned} \tag{42}$$

It is fairly straightforward to extend the law of motions for the equilibrium densities.[30]

C Measuring the Discount in Rent Stabilized Housing

To measure the relative price between unregulated and regulated housing, we estimate a hedonic regression using data on housing units in both market. As discussed in Section 4 we

[30] An appendix is available upon request from the authors that provides the relevant equations.

assume that the quantity index that relates structural and neighborhood characteristics to housing service flows is constant among the two markets. We can, therefore, use these regressions to measure price differences between regulated and unregulated housing markets.

Table 7: Log Rent Regression

regulated	-0.513***
# of bed rooms	0.124***
# of other rooms	-0.00249
complete kitchen	0.370**
complete plumbing	0.622**
Constant	7.188***
Observations	1416

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The regression also includes dummy variables that indicate whether the building has an elevator, the building age, the building size, a dummy for the fuel type, a dummy for condo/coop, a dummy for bad walls, a unit floor control and household characteristic controls, a swell as sub-borough controls. Table 7 summarizes our findings. We find that rent regulated units are, on average, 51 percent cheaper in Manhattan compared to the market rated units.

D Testing for Income Bunching

We have also have detailed data on income categories. We define a household as working if the labor income share is higher than 50 percent of total income. That fraction of working households is higher in unregulated and regulated housing than in public housing. Nevertheless, more than 70 percent of all households in public housing receive a significant fraction of their income from labor income in 2011. Note that individuals have some incentives to keep income below the eligibility threshold to retain their priority on the waitlist. We conducted some empirical analysis that did not reveal any evidence of income bunching below the threshold. Figure 10 plots the income distribution relative to the eligibility threshold for public housing.

Figure 10: Income Relative to Eligibility Threshold

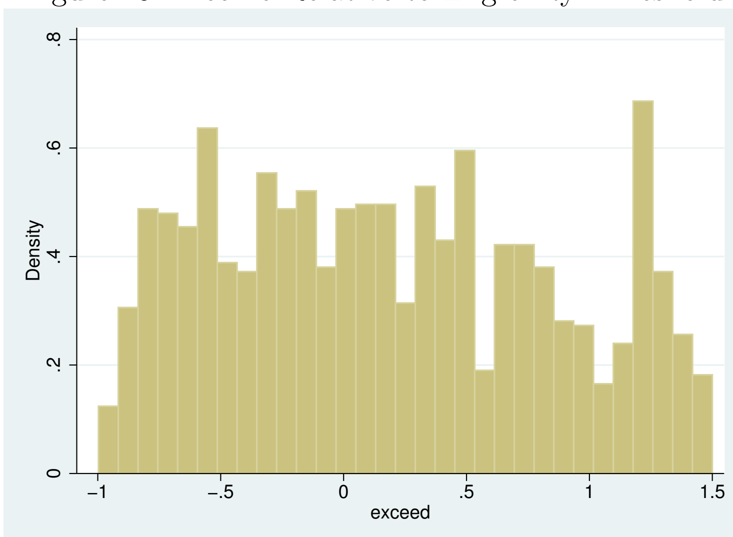

Figure 10 does not provide any evidence that suggests households strategically manipulate their income or if they try to, they are unsuccessful. Given the length of the wait times in New York City this result may not be that surprising. We, therefore, treat income as exogenous in our model below and discuss potential extensions in the conclusions.