Endogenous Public Information and Welfare in Market Games*

Xavier Vives

IESE Business School

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Abstract

This paper performs a welfare analysis of markets with private information in which agents condition on prices in the rational expectations tradition. Price-contingent strategies introduce two externalities in the use of private information: a pecuniary externality and a learning externality. With decreasing marginal utility the pecuniary externality induces agents to put too much weight on private information and in the normal case, where the allocation role of the price prevails over its informational role, overwhelms the learning externality which impinges in the opposite way. The price may be very informative but at the cost of an excessive dispersion of the actions of agents. The welfare loss at the market solution may be increasing in the precision of private information. The analysis provides insights into optimal business cycle policy and a rationale for a Tobin-like tax for financial transactions.

Keywords: learning externality, asymmetric information, pecuniary externality, team solution, rational expectations, cursed equilibrium, Tobin tax, business cycle policy.

JEL Codes: D82, D83, G14, H23.

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1. Introduction

We show that when agents can condition on prices the presumption that they will put too little weight on private information need not hold. Agents may put too much weight on private information and prices may contain "too much" information from a second best perspective for reasons other than the well-known Hirshleifer (1971) effect of destruction of insurance opportunities.[1] This typically happens, in fact, in a scenario with decreasing marginal utility. The results provide a rationalization of pro-cyclical business cycle policy and a Tobin-style tax on financial transactions in the tradition of Greenwald and Stiglitz (1986).

In many markets agents compete in demand and/or supply schedules and therefore condition on prices. This is very common in financial markets, asset auctions, and some goods markets such as wholesale electricity. Prices are main providers of endogenous public information. In financial markets, prices are noisy statistics that arise from the decisions of traders. In goods markets, prices aggregate information on the preferences of consumers and the quality of the products.

The received literature on information externalities points at agents typically relying too much on public information. The reason is that agents do not take into account that their reaction to private information affects the informativeness of public statistics and general welfare. In other words, agents do not internalize an information externality. Pure information externalities will make agents insufficiently responsive to their private information (Vives 1993, 1997; Amador and Weill 2012) and, in the limit to disregard it (Banerjee 1992, Bikhchandani et al. 1992). For example, Morris and Shin (2005) point to the paradox that a central bank by publishing aggregate statistics makes those less reliable by inducing agents in the economy to rely less on their private signals.

In order to speak meaningfully of excessive or insufficient weight to private information, i.e. to perform a welfare analysis in a world with asymmetric information, we require a benchmark against which to test market equilibria. An appropriate benchmark for measuring inefficiency at the market equilibrium is the team solution

1 See the general analysis of the value of public information in Schlee (2001).

in which agents internalize collective welfare but must still rely on private information when making their own decisions (Radner 1979; Vives 1988; Angeletos and Pavan 2007). This is in the spirit of Hayek (1945), where the private signals of agents cannot be communicated to a center. The team-efficient solution internalizes the payoff and information externalities associated with the actions of agents in the market. Collective welfare may refer to the surplus of all market participants, active or passive, or may be restricted to the internal welfare of the active agents.

The under-reliance on private information result extends to some classes of economies with endogenous public information. Indeed, consider an economy in which equilibria are team-efficient when public information is exogenous as, for example, a Cournot market with a continuum of firms and private information (Vives 1988). Then increasing public information has to be good marginally, and under regularity conditions the result is global. This implies that more weight to private information is needed in relation to the market (Angeletos and Pavan 2009). We show that this logic breaks down in a market game where agents condition on the price, say firms competing in supply functions, because then there is a pecuniary externality related to the use of private information, even if public information were to be exogenous, which makes the market inefficient. This pecuniary externality may counteract the learning from the price externality and lead agents to put too much weight on private information.

We consider a tractable linear-quadratic-Gaussian model. The context is a market game, where external effects go through the price. There is uncertainty about a common valuation parameter (say cost shock) about which agents have private information, and the price is potentially noisy (say because of a demand shock). We use a model with a rational expectations flavor but in the context of a well-specified game where a continuum of agents compete in schedules. We focus our attention on linear Bayesian equilibria. The model is flexible and admits several interpretations in terms of firms competing in a homogenous product market, monopolistic competition, and trading in a financial market. (We will follow the first interpretation when developing the model and results.)

Let us discuss the results in some more detail. For concreteness, consider a homogenous product market with random demand and a continuum of ex ante identical firms competing in supply schedules with increasing marginal costs with uncertain intercept. Each firm receives a private signal as well as a public signal about the marginal cost intercept.

Conditioning on prices introduces two information-related externalities. The first one, termed pecuniary, arises even if firms do not take into account the information content of prices, i.e., they are naïve as in a fully cursed equilibrium (Eyster and Rabin 2005), and even if there is no noise in demand. There is a pecuniary externality in the use of private information at the (naïve) competitive equilibrium because firms use price-contingent strategies but they do not take into account how their response to private information affects the price. With decreasing marginal utility (and downward sloping demand) this externality leads firms to put too much weight on their private signals. This externality and its effects are novel. The second externality is the by now well understood learning externality which leads firms to underweight private information because firms do not anticipate the influence of their actions on the information content of the price.

The driving force of the pecuniary externality can be understood in three steps as follows. Note first that under asymmetric information when costs are high the price-cost margin tends to be low. This is so since the error term in the cost signal of a firm is positively correlated with the margin (say that the cost shock of the firm is positive, then this firm tends to produce less and marginal cost tends to be low but the price is not affected -since error terms wash out in the aggregate- and therefore the margin tends to be high). Since in equilibrium the signal itself is uncorrelated with the margin, this implies that the price-cost margin co-varies negatively with the cost level. Second, suppose that demand is downward sloping and that firms are naïve in not taking into account the information content of the price. Let us then increase the response to private information from the market equilibrium level. For concreteness suppose also that costs are high, then a firm when reacting more to private information raises the market price (since a higher reaction to bad news reduces production and the price is decreasing in aggregate supply) but this is not taken into account by the firm. This implies that firms tend to supply more (since they compete in supply functions and supply increases with the price) but this is bad for profits since when costs are high the price-cost margin tends to be low. Finally, note this is bad also for total surplus since the price-cost margin provides the right welfare pointer in our model with a continuum of firms. In conclusion, welfare can be improved by decreasing the response to private information. The result is reversed when demand is upward sloping and increasing output increases the market price. Those results do not depend on the level of noise in demand and, indeed, they hold with no noise in demand. The driving force of the learning externality is well known and leads to under-reliance on private information.

Consider now sophisticated firms taking into account the information content of the price. In this case we do need to have noise in demand to avoid the price being fully revealing. In this context, firms correct the slope of their strategy according to what they learn from the price. With downward sloping demand the price's informational and allocational roles conflict and firms face adverse selection. In this case a high price is bad news (high cost) and the equilibrium schedule is steeper than with full information. In fact, in equilibrium schedules may slope downwards when the informational role of prices dominates their allocational role.[2] This will occur when there is little noise in the price. With upward sloping demand there is no conflict: a high price is good news, and the equilibrium schedule is flatter than with full information.

With downward sloping demand depending on the strength of the learning externality we may overcome or not the overweighting result due to the pecuniary externality. The point where both externalities cancel each other is when firms use vertical supply schedules. In the normal case where supply is upward sloping, which happens when noise in demand is high, the allocational effect of the price prevails and the learning externality is weak. In this case the pecuniary externality effect wins over the learning externality and the weight to private information is too large. When the supply function is downward sloping, which happens when noise in demand is low, the informational component of the price prevails and the learning externality is strong. In

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2 See Wilson (1979) for a model in which adverse selection makes demand schedules upward sloping.

this case the learning externality wins over the pecuniary externality and the weight to private information is too small. With upward sloping demand then both externalities go in the same direction and there is always underweighting of private information.

In the economy considered the full information equilibrium is efficient since it is competitive. In this equilibrium all firms produce the same amount since they all have full information on costs, which are symmetric. With private information there is both aggregate and productive inefficiency. Aggregate inefficiency refers to a distorted total output and productive inefficiency refers to a distorted distribution of a given total output. The team-efficient solution in an economy with asymmetric information optimally trades off the tension between the two sources of welfare loss, aggregate and productive inefficiency. The somewhat surprising possibility that prices are "too informative" may arise since even though to have more informative prices is good for aggregate efficiency this comes in a second best world at the cost of increasing dispersion and productive inefficiency. Indeed, to have more informative prices firms have to respond more to their private signals and this magnifies the noise they contain.

More precise information, be it public or private, reduces the welfare loss at the team-efficient solution. The reason is that the direct impact of the increased precision is to decrease the welfare loss and this is the whole effect since at the team-efficient solution the responses to private and public information are already (socially) optimized (this is as in Angeletos and Pavan 2009). In contrast, at the market solution an increase in, say, the precision of private information will increase the response of a firm to its private signal and this will tend to increase the welfare loss when the market calls already for a too large response to private information. If this indirect effect is strong enough the welfare loss may be increasing with the precision of private information. In principle the same effect could happen with the precision of public information but we can show that the indirect effect of changes in both the exogenous public precision of information and the precision of the noise in the endogenous public signal are always dominated by the direct effect. The result is that the welfare loss at the market solution is always decreasing with the precisions of public information.

The team-efficient solution can be implemented with tax-subsidy schemes; in particular, with a quadratic transaction tax. This may rationalize a Tobin-like tax in the context of a financial market whenever the allocational role of the price prevails over its informational role.[3] In this case the transaction tax makes informed traders to internalize the pecuniary externality in the use of private information. The end result is a price which contains less information and it may even result in a deeper market. Similarly, the model may rationalize the imposition of adjustment costs in the labor force to moderate the excessive responsiveness of firms to their private information on productivity in detriment of general welfare.

The results can be extended to the internal team-efficient benchmark (where only the collective welfare of the players is taken into account, for example, ignoring passive consumers). Then the full information market does not achieve an efficient outcome. In this case also, endogenous public information may overturn conclusions reached using exogenous information models (e.g., Angeletos and Pavan 2007) when the informational role of the price is in conflict and dominates its allocational role.

The paper follows the tradition of the literature on the welfare analysis of private information economies (Palfrey 1985, Vives 1988, Angeletos and Pavan 2007, 2009), extending the analysis to endogenous public information when the public signal is the price. To do so it builds on the models of strategic competition on schedules such as Kyle (1985) and Vives (2011) but in a continuum of agents framework.[4] We contribute to the recent surge of interest in the welfare analysis of economies with private information and in particular on the role of public information in such economies (see, e.g., Morris and Shin 2002; Angeletos and Pavan 2007; Amador and Weill 2010). Our results qualify the usual intuition of informational externality models (Vives 1997, Amador and Weill 2010, 2012) in a market game model. It is

3 A tax on short-term speculation was proposed by Keynes (1936) and advocated by Tobin (1978) later on with the celebrated phrase "to throw sand in the wheels of the excessively efficient international money markets". The Tobin tax has been advocated by, among others, Stiglitz (1989) and Summers and Summers (1989) who argue for its benefits even if it reduces market liquidity. Taxes of this sort have been in place in several countries (such as the UK and Sweden) and more recently, after the financial crisis, the Financial Transactions Tax (FTT) is on the agenda: 11 European countries have committed to introduce it, with delayed implementation to January 1st, 2016, and France has already moved to introduce a version of a FTT in August 2012.

4 Vives (2014) also uses a continuum of agents framework to study the Grossman-Stiglitz paradox in a related model where agents have private valuation utility parameters.

worth noting that pecuniary externalities are associated to inefficiency in competitive but incomplete markets and/or in the presence of private information since then the conditions of the first fundamental welfare theorem are not fulfilled. Competitive equilibria are not constrained efficient in those circumstances (Greenwald and Stiglitz 1986).[5] In our paper (as in Laffont 1985) competitive noisy rational expectations equilibria (REE), in which traders take into account information from prices, are not constrained efficient.[6] In our quasilinear utility model there is no room for the Hirshleifer (1971) effect according to which REE may destroy insurance opportunities by revealing too much information. We provide therefore an instance of REE which may reveal too much information on a fundamental (from a second best perspective) which is independent of the Hirshleifer effect.

Recent literature has examined the circumstances under which more public information actually reduces welfare (as in Burguet and Vives 2000; Morris and Shin 2002; Angeletos and Pavan 2007; Amador and Weill 2010, 2011). In Burguet and Vives (2000) a higher (exogenous) public precision may discourage private information acquisition and lead to a higher welfare loss in a purely informational externality model. In Morris and Shin (2002) the result is driven by a socially excessive incentive to coordinate by agents. Angeletos and Pavan (2007) qualify this result and relate it to the payoff externalities present in a more general model. In Amador and Weill (2010) a public release of information reduces the informational efficiency of prices and this effect may dominate the direct information provision effect. Their model is purely driven by information externalities in the presence of strategic complementarities in terms of responses to private information.[7] In our model, which is based on competition on schedules, more public information is not damaging welfare but more private precision may be.

5 For example, pecuniary externalities in markets with financial frictions (borrowing or collateral constraints) can explain market failure (see, e.g., Caballero and Krishnamurthy 2001 and Jeanne and Korinek 2010).

6 If the signals of agents can be communicated to a center, as in Laffont (1985) then questions arise concerning the incentives to reveal information and how welfare allocations may be modified. This issue is analyzed in a related model by Messner and Vives (2006).

7 In Amador and Weill (2010) there is no direct complementarity or substitutability in actions. However, complementarity or substitutability arises indirectly because workers learn from prices, and the informativeness of prices is affected by the actions of agents.

The plan of the paper is as follows. Section 2 presents the model with the leading interpretation of firms competing in a homogenous product market. Section 3 introduces the welfare benchmark. Section 4 characterizes equilibrium and welfare with exogenous public information. Section 5 considers endogenous public information with firms learning from prices. Section 6 deals with demand function competition and the role of optimal taxes. Section 7 studies the internal team-efficient benchmark. Concluding remarks are provided in Section 8 and proofs are gathered in the Appendix. Supplementary material, including a detailed comparative static analysis of the equilibrium and an analysis of the Cournot case with endogenous public information is provided in the online appendix.

2. The market game

Consider a continuum of firms indexed within the interval $[0,1]$ (endowed with the Lebesgue measure), x_i is the output of firm i, produced at cost $C(x_i) = \theta x_i + (\lambda/2)x_i^2$ where θ is random and $\lambda > 0$. Firms face an inverse demand for an homogenous product $p = \alpha + u - \beta\tilde{x}$, where u is a normally distributed demand shock, $u \sim N(0,\sigma_u^2)$ with $\sigma_u^2 \geq 0$. $\alpha > 0$, $\beta \in \mathbb{R}$, and $\tilde{x} = \int_0^1 x_i \, di$ is the aggregate output. The demand can be derived from a quadratic utility function from a representative consumer with quasilinear preferences (see the Appendix for a microfoundation of the model). In the normal case of decreasing marginal utility we have that $\beta > 0$ and demand is downward sloping. If $\beta < 0$, demand is upward sloping. The latter situation may arise, for example, in the case of a good which is addictive.

The parameter θ has prior Gaussian distribution with mean $\bar{\theta}$ and variance $\sigma_\theta^2 > 0$ ($\theta \sim N(\bar{\theta},\sigma_\theta^2)$ and, to ease notation, set $\bar{\theta} = 0$). Firm i receives a private signal $s_i = \theta + \varepsilon_i$ with $\varepsilon_i \sim N(0,\sigma_\varepsilon^2)$, $\text{cov}[\varepsilon_i, \varepsilon_j] = 0$, $j \neq i$, and a public signal $z = \kappa\theta + \omega$ where $\kappa \geq 0$ and $\omega \sim N(0,\sigma_\omega^2)$.[8] The error term ω will be independent of u if the public signal is exogenous and $\omega = u$ if the public signal is endogenous (linked to the

[8] As an example, the cost parameter θ could be a unit ex post pollution damage that is assessed on firm *i*, say an electricity generator in a distributed generation system, and for which the firm has a private estimate s_i before submitting its supply function.

market price). In the first case the random variables $\{\theta, \varepsilon_i, u, \omega\}$ are mutually independent, and in the second $\{\theta, \varepsilon_i, u\}$ are. In any case the information set of firm i is therefore $I_i = \{s_i, z\}$.[9] Firms use supply functions as strategies.

Given a random variable y we denote by $\tau_y \equiv 1/\sigma_y^2$ its precision. We follow the convention that error terms cancel in the aggregate: $\int_0^1 \varepsilon_i \, di = 0$ almost surely (a.s.).[10] Then the aggregation of all individual signals will reveal the underlying uncertainty: $\int_0^1 s_i \, di = \theta + \int_0^1 \varepsilon_i = \theta$.[11]

The timing of events is as follows. At $t = 0$, the random variables θ and u are drawn but not observed. At $t = 1$, consumers and producers form demand and supply plans. A consumer maximizes utility knowing the realization of u. Each firm submits a supply schedule $X_i(I_i;\cdot)$ contingent on his information set $I_i = \{s_i, z\}$ with $x_i = X_i(I_i; p)$ where p is the price. The strategy of a firm is a map from the signal vector space to the space of schedules. At $t = 2$ the market clears, the price is formed by finding a p that solves $p = \alpha + u - \beta\left(\int_0^1 X_j(I_j; p)\, dj\right)$. Finally, consumption occurs and payoffs are collected.

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9 Normality of random variables means that prices and quantities can be negative with positive probability. The probability of this event can be controlled, if necessary, by an appropriate choice of means and variances. Furthermore, for this analysis the key property of Gaussian distributions is that conditional expectations are linear. Other prior-likelihood conjugate pairs (e.g., beta-binomial and gamma-Poisson) share this linearity property and can display bounded supports.

10 Equality of random variables has to be understood to hold almost surely. We will not insist on this in the paper.

11 Suppose that $(q_i)_{i\in[0,1]}$ is a process of independent random variables with means $\mathbb{E}[q_i]$ and uniformly bounded variances $\mathrm{var}[q_i]$. Then we let $\int_0^1 q_i \, di = \int_0^1 \mathbb{E}[q_i]\, di$ (a.s.). This convention will be used while taking as given the usual linearity property of integrals. (Equality of random variables is assumed to hold almost surely always.) In short, we assume that the strong law of large numbers (SLLN) holds for a continuum of independent random variables with uniformly bounded variances.

Let us assume that there is a unique price $\hat{p}\left(\left(X_j\left(I_j;\cdot\right)\right)_{j\in[0,1]}\right)$ for any realization of the signals.[12] Then, for a given profile $\left(X_j\left(I_j;\cdot\right)\right)_{j\in[0,1]}$ of firms' schedules and realization of the signals, the profits for firm i are given by

$$\pi_i = (p-\theta)x_i - \frac{\lambda}{2}x_i^2,$$

where $x_i = X_i\left(I_i; p\right)$, and $p = \hat{p}\left(\left(X_j\left(I_j;\cdot\right)\right)_{j\in[0,1]}\right)$. This defines a Bayesian game in schedules. If the public signal z in $I_i = \{s_i, z\}$ is exogenous then the firms are "naïve" and do not take into account the information content of the price. If the public signal is endogenous (the price) then firms are sophisticated and the formulation has a rational expectations flavor but in the context of a well-specified schedule game. In this second case we will restrict our attention to linear Bayesian equilibria of the schedule game.[13]

It is worth to remark that in the market game with sophisticated firms both payoff and learning externalities go through the market price p, which has both an allocational and an informational role. When $\beta = 0$, the price is independent of $\tilde{x}$ and there are neither payoff nor informational externalities among players.

The model admits other interpretations in terms of demand function competition (see Section 6) or monopolistic competition (Section 7).[14]

We will study Bayesian equilibria of the supply function game in two versions. In the first firms will be "naïve" in the sense of not taking into account the information content of the price and in the second they will. In the first therefore the public signal z will be exogenous while in the second it will reflect the information content of the

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12 We assign zero payoffs to the players if there is no p that solves the fixed point problem. If there are multiple solutions, then the one that maximizes volume is chosen.

13 Normality of random variables means that prices and quantities can be negative with positive probability. The probability of this event can be controlled, if necessary, by an appropriate choice of means and variances. Furthermore, for this analysis the key property of Gaussian distributions is that conditional expectations are linear. Other prior-likelihood conjugate pairs (e.g., beta-binomial and gamma-Poisson) share this linearity property and can display bounded supports.

14 See also Chapter 3 in Vives (2008) for an overview of the connection between supply function competition and rational expectations models, as well as examples.

price. In the exogenous information case noise in demand plays no special role and we can have $\sigma_u^2 = 0$ while when there is learning from the price we need $\sigma_u^2 > 0$ to avoid prices being fully revealing of θ. Before that in the next section we define the appropriate welfare benchmark for the use of information in our economy.

3. The welfare benchmark

Consider an allocation assigning output x_i to firm i and with average output $\tilde{x}$. We will use utilitarian welfare criteria which in our quasilinear world is equivalent with total surplus:

$$\mathrm{TS} \equiv \left(\alpha + u - \beta\frac{\tilde{x}}{2}\right)\tilde{x} - \int_0^1 \left(\theta x_i + \frac{\lambda}{2}x_i^2\right) di.$$

We assume that $2\beta + \lambda > 0$, which guarantees that profits are strictly concave in output at symmetric solutions ($\partial^2\pi/(\partial x)^2 = -(2\beta + \lambda) < 0$). A fortiori, we have that $\beta + \lambda > 0$ and it follows that TS is also strictly concave for symmetric solutions.

It is immediate that the first-best (full information) allocation has all firms producing the same amount, $x^o = (\lambda + \beta)^{-1}(\alpha + u - \theta)$. Denote by TS^o total surplus at the full information first best. The first-best allocation is attained by the competitive market when there is full information (i.e., firms receive perfect signals about θ).

The market equilibria we consider under asymmetric information will not attain in general the first best. The reason is that suppliers produce under uncertainty and rely on imperfect idiosyncratic estimation of the common cost component; hence they end up producing different amounts even though costs are identical and strictly convex. However, since firms are competitive they will produce in expected value the right amount at the equilibrium: $\mathbb{E}[\tilde{x}] = \mathbb{E}[x^o] = \alpha(\lambda + \beta)^{-1}$.

Using the expression for first best output x^o we have that for a symmetric allocation (in the sense that $\mathbb{E}\left[\int_0^1 (x_i - \tilde{x})^2 di\right] = \mathbb{E}\left[(x_i - \tilde{x})^2\right]$), $\mathbb{E}[\mathrm{TS}] = \mathbb{E}\left[\mathrm{TS}^o\right] - \mathrm{WL}$ where

$$\text{WL} = \left((\beta+\lambda)\mathbb{E}\left[\left(\tilde{x}-x^{o}\right)^{2}\right]+\lambda\mathbb{E}\left[\left(x_{i}-\tilde{x}\right)^{2}\right]\right)\Big/2 .$$

The expression for WL follows since using the fact that $x^{o}(\lambda+\beta)=\alpha+u-\theta$, we obtain $\text{TS}^{o}=(\lambda+\beta)\left(x^{o}\right)^{2}\big/2$ and $\text{TS}^{o}-\text{TS}=\left((\beta+\lambda)\left(\tilde{x}-x^{o}\right)^{2}+\lambda\int_{0}^{1}\left(x_{i}-\tilde{x}\right)^{2}di\right)\Big/2$.

The first term in the expected welfare loss WL corresponds to aggregate inefficiency (how distorted is the average quantity $\tilde{x}$ while producing in a cost-minimizing way), which is proportional to $\mathbb{E}\left[\left(\tilde{x}-x^{o}\right)^{2}\right]$, and the second term to productive inefficiency (how distorted is the distribution of production of a given average quantity $\tilde{x}$), which is proportional to the dispersion of outputs $\mathbb{E}\left[\left(x_{i}-\tilde{x}\right)^{2}\right]$. Let p^{o} be the full information first best price. Note that $\mathbb{E}\left[\left(p-p^{o}\right)^{2}\right]=\beta^{2}\mathbb{E}\left[\left(\tilde{x}-x^{o}\right)^{2}\right]$.

The welfare benchmark we use is the team solution maximizing expected total surplus subject to employing linear decentralized strategies (as in Vives 1988; Angeletos and Pavan 2007). This team-efficient solution internalizes the payoff and information externalities of the actions of agents, and it is restricted to using the same type of strategies (decentralized and linear) that the market employs. The question then is whether the team solution can improve upon the market allocation.

It is worth noting that in the economy considered if firms were not to condition on prices, i.e. if each firm would set use quantities as strategies, conditioning only on its available signals as in a Cournot market, instead of using a supply function, then the market solution would be team-efficient (Vives 1988, see Section 4.3). That is, in the Cournot economy, the private information equilibrium is team-efficient for given public information. We will see that this is not the case in our market game with price-contingent strategies even when firms disregard the information content of the price (i.e. with exogenous public information) because of a pecuniary externality in the use of private information.

4. Equilibrium and welfare with exogenous public information

In this section we consider that firms are naïve in the sense that they do not realize the informational value of the price (i.e., there is no learning from the price). This situation corresponds to the case of fully cursed equilibrium of Eyster and Rabin (2005). Indeed, in a fully cursed equilibrium each player assumes no connection between the actions and types of other players. In this case the price is perceived to have no information about the parameter θ and the information set of firm i is $I_i = \{s_i, z\}$ where $z = \kappa\theta + \omega$ and u and ω are independent.

4.1 Equilibrium

At the market equilibrium , firm i solves

$$\max_{x_i} \left(p - \mathbb{E}\left[\theta \middle| s_i, z\right] - \frac{\lambda}{2} x_i \right) x_i .$$

The solution is both unique (given strict concavity of profits) and symmetric across firms (since the cost function and signal structure are symmetric across firms):

$$X\left(s_i, z; p\right) = \lambda^{-1}\left(p - \mathbb{E}\left[\theta \middle| s_i, z\right]\right).$$

Let $\hat{z} = \mathbb{E}[\theta \mid z]$. We have that $\mathbb{E}\left[\theta \mid s_i, z\right] = \gamma s_i + \left(1-\gamma\right)\hat{z}$ where the weight to private information $\gamma = \tau_\varepsilon\left(\tau_\varepsilon + \tau\right)^{-1}$ is the Bayesian weight, with $\tau \equiv \left(\operatorname{var}\left[\theta \middle| z\right]\right)^{-1} = \tau_\theta + \tau_\omega \kappa^2$.[15] Averaging the demand of firms we obtain $\tilde{x} = \lambda^{-1}\left[p - \left(\gamma\theta + \left(1-\gamma\right)\hat{z}\right)\right]$, and from market clearing, $p = \alpha + u - \beta\tilde{x}$, it is immediate that the equilibrium is unique, symmetric and linear.

4.2 Team solution

At the team-efficient solution with exogenous public information expected total surplus $\mathbb{E}\left[\mathrm{TS}\right]$ is maximized subject to firms using decentralized linear (affine) production strategies contingent on their information set $I_i = \{s_i, z\}$ and on the price

[15] Indeed, γ minimizes the mean square error of predicting θ with the private and public signals ($I_i = \{s_i, z\}$): $\min_\gamma \frac{1}{2\lambda}\left(\frac{\left(1-\gamma\right)^2}{\tau} + \frac{\gamma^2}{\tau_\varepsilon} \right)$.

(but with no learning from it; i.e., perceiving no covariance between θ and the price, $\mathbb{E}[\theta p]=0$). That is, the team solution solves the program:

$$\max_{a,\hat{b},c,\hat{c}} \mathbb{E}[\mathrm{TS}] \qquad (\mathbf{T}_{exo})$$

subject to $x_i=\hat{b}-as_i-cz+\hat{c}p$, $p=\alpha+u-\beta\tilde{x}$, $\mathbb{E}[\theta p]=0$, $\tilde{x}=\hat{b}-a\theta-cz+\hat{c}p$, and $z=k\theta+w$.

It can be shown (see Claim 1 in the Appendix) then that team strategy is of the same form as in the market solution, $X(s_i,z;p)=\lambda^{-1}\left(p-\left(\gamma s_i+(1-\gamma)\hat{z}\right)\right)$, but where now the weight to private information γ may differ from the Bayesian weight. At a candidate team strategy public information is optimally used for a given weight to private information, i.e. the weights to private and public information ($\hat{z}=\mathbb{E}[\theta\,|\,z]$) have to add up to one. We will see that the team solution can improve upon the market even when restricting strategies to those with the same form as in the market solution.

It follows then that the welfare loss at any candidate team solution will depend only on the response to private information γ, or equivalently on the response to private information in the *strategy* of a firm: $a=\lambda^{-1}\gamma$. We have then that $\tilde{x}=\left(\alpha+u-\left(\lambda a\theta+(1-\lambda a)\hat{z}\right)\right)/(\beta+\lambda)$ and, using the expression for x^o, we find that $\tilde{x}-x^o=(1-\lambda a)(\theta-\hat{z})/(\beta+\lambda)$. Since $\tau=\left(\mathrm{var}\left[\theta|z\right]\right)^{-1}$ we obtain $\mathbb{E}\left[\left(\tilde{x}-x^o\right)^2\right]=(1-\lambda a)^2/\left(\tau(\beta+\lambda)^2\right)$. Similarly we obtain $x_i-\tilde{x}=-\lambda^{-1}\gamma(s_i-\theta)=-a\varepsilon_i$ and, conclude that $\mathbb{E}\left[\left(x_i-\tilde{x}\right)^2\right]=a^2\sigma_\varepsilon^2$. It follows from the expression of the welfare loss in Section 3 that

$$\mathrm{WL}(a;\tau)=\frac{1}{2}\left(\frac{(1-\lambda a)^2}{\tau(\beta+\lambda)}+\frac{\lambda a^2}{\tau_\varepsilon}\right),$$

where $\tau=\tau_\theta+\tau_\omega\kappa^2$ which is easily seen strictly convex in a. Changing a has opposite effects on both sources of the welfare loss since aggregate inefficiency decreases with a, as the average quantity gets close to the full information allocation, but productive inefficiency increases with a as dispersion increases.

The team solution for given exogenous public precision τ, denoted $a^*_{exo}(\tau)$, minimizes WL and optimally trades off the sources of inefficiency among decentralized strategies. It is worth noting that WL is independent of σ_u^2 and therefore $a^*_{exo}(\tau)$ will also be independent of σ_u^2. We have that

$$a^{\mathrm{T}}_{\mathrm{exo}}(\tau) = \frac{\tau_\varepsilon}{\lambda(\tau_\varepsilon + \tau) + \beta\tau}.$$

Denote by $a^*_{\mathrm{exo}}(\tau)$ the market solution, then $a^*_{\mathrm{exo}}(\tau) = \lambda^{-1}\tau_\varepsilon(\tau_\varepsilon + \tau)^{-1}$. It follows that $a^{\mathrm{T}}_{\mathrm{exo}}(\tau) \le a^*_{\mathrm{exo}}(\tau)$ if and only if $\beta \ge 0$. The following proposition states the result.

<u>Proposition 1</u>. *Let* $\infty > \tau_\varepsilon > 0$ and $\sigma_u^2 \ge 0$, *suppose that firms receive a public signal of precision* τ *and ignore the information content of the price. Then, at the unique equilibrium, firms respond more (less) to private information* $a^*_{\mathrm{exo}}(\tau)$ *than at the team optimal solution* $a^{\mathrm{T}}_{\mathrm{exo}}(\tau)$ *when* $\beta > 0$ ($\beta < 0$), *i.e.* $\mathrm{sgn}\{a^*_{\mathrm{exo}}(\tau) - a^{\mathrm{T}}_{\mathrm{exo}}(\tau)\} = \mathrm{sgn}\{\beta\}$.

Noise in demand plays no role in the result (i.e. the proposition holds for $\sigma_u^2 = 0$) but asymmetric information is crucial. Indeed, if $\tau_\varepsilon = 0$, then $a^*_{\mathrm{exo}}(\tau) = a^{\mathrm{T}}_{\mathrm{exo}}(\tau) = 0$ and if $\tau_\varepsilon \to \infty$, then both tend to λ^{-1}. When information is symmetric (there is no private information, $\tau_\varepsilon = 0$, or information is perfect, $\tau_\varepsilon = \infty$) the market is efficient since it is competitive and pecuniary externalities are internalized. However, with asymmetric information, the team solution depends indeed on β, with the term $\beta\tau$ reflecting a pecuniary externality at the market solution. There is a pecuniary externality in the use of private information at the (naïve) competitive equilibrium because firms use price-contingent strategies but they do not take into account how their response to private information affects the price and therefore the average quantity. The result is that the market $\tilde{x}$ overreacts to θ, for a given public signal, when $\beta > 0$ since then $a^*_{\mathrm{exo}} > a^{\mathrm{T}}_{\mathrm{exo}}$. The opposite happens when $\beta < 0$. Indeed, we have that $\partial\tilde{x}/\partial\theta = -\lambda a/(\beta + \lambda)$.

In order to understand the pecuniary externality in the use of private information generated from conditioning on the price when $\beta \neq 0$ let us consider the effect of a change in the response to private information a at the market solution $a^*_{\text{exo}}(\tau)$. From the candidate strategy for firm i, $x_i = \lambda^{-1}\left(p - \left(\lambda a s_i + (1-\lambda a)\hat{z}\right)\right)$, we have that

$$\frac{\partial x_i}{\partial a} = \left.\frac{\partial x_i}{\partial a}\right|_p + \frac{\partial x_i}{\partial p}\frac{\partial p}{\partial a}$$

and

$$\frac{\partial \mathbb{E}[\text{TS}]}{\partial a} = \mathbb{E}\left[(p - \text{MC}(x_i)) \left.\frac{\partial x_i}{\partial a}\right|_p \right] + \mathbb{E}\left[(p - \text{MC}(x_i)) \frac{\partial x_i}{\partial p}\frac{\partial p}{\partial a} \right],$$

where the first term is what the market equates to zero, since firms take as given the price, and the second corresponds to the pecuniary externality in the use of private information since firms do not take into account that they influence the price when they change their response to private information.

At the market solution, $\mathbb{E}\left[\left(p - \text{MC}(x_i)\right) \left.\frac{\partial x_i}{\partial a}\right|_p \right] = \mathbb{E}\left[\left(p - \text{MC}(x_i)\right)\left(-(s_i - \hat{z})\right)\right] = 0$ since $\partial x_i / \partial a|_p = -(s_i - \hat{z})$. Note that if signals were perfect ($\sigma_\varepsilon^2 = 0$ and $s_i = \theta$, a.s.) we would have $\mathbb{E}\left[\left(p - \text{MC}(x_i)\right)(\theta - \hat{z})\right] = 0$. However, with noisy signals ($\sigma_\varepsilon^2 > 0$) the margin $\left(p - \text{MC}(x_i)\right)$ covaries negatively with the difference between costs and their public expectation $(\theta - \hat{z})$, i.e., $\mathbb{E}\left[\left(p - \text{MC}(x_i)\right)(\theta - \hat{z})\right] < 0$. That is, in equilibrium, a high cost realization tends to go together with a lower margin. The reason is that at the market solution

$$\mathbb{E}\left[\left(p - \text{MC}(x_i)\right)(s_i - \hat{z})\right] = \mathbb{E}\left[\left(p - \text{MC}(x_i)\right)(\theta - \hat{z})\right] + \mathbb{E}\left[\left(p - \text{MC}(x_i)\right)\varepsilon_i\right] = 0 \ ,$$

and $\mathbb{E}\left[\left(p - \text{MC}(x_i)\right)\varepsilon_i\right] = -\mathbb{E}\left[\text{MC}(x_i)\varepsilon_i\right] = -\mathbb{E}\left[(\theta + \lambda x_i)\varepsilon_i\right] = \lambda a^*_{\text{exo}}\sigma_\varepsilon^2 > 0$. For example, when $\varepsilon_i > 0$ firm i tends to produce less and marginal cost tends to be low but the price is not affected (since error terms wash out in the aggregate) and therefore the margin tends to be high.

Since from market clearing $p = \left((\alpha+u)\lambda + \beta\left(\gamma\theta + (1-\gamma)\hat{z}\right) \right)(\beta+\lambda)^{-1}$, $\partial p/\partial a = \beta\lambda(\beta+\lambda)^{-1}(\theta-\hat{z})$, and $\partial x_i/\partial p = \lambda^{-1}$, we obtain the effect of the pecuniary externality for given public information at the market solution:

$$\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right) \frac{\partial x_i}{\partial p} \frac{\partial p}{\partial a} \right] = \beta\lambda(\beta+\lambda)^{-1} \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)(\theta - \hat{z}) \right].$$

As a consequence, the sign of the externality depends on whether β is positive or negative. Consider the normal case with $\beta > 0$ and, say, costs are high ($\theta - \hat{z} > 0$) then an increase in a raises p ($\partial p/\partial a = \beta\lambda(\beta+\lambda)^{-1}(\theta-\hat{z}) > 0$) since a higher reaction to bad news decreases production and the price is decreasing in aggregate supply. However, the price increase will lead to a higher output x_i since the supply function is upward sloping but this is inefficient (both for profits and total surplus) since when costs are high the price-cost margin tends to be low. This is bad for expected profits and for welfare since the margin provides the right signal to produce in our competitive economy. The result is that the market puts too much weight on private information and the aggregate quantity overreacts to θ when $\beta > 0$.

Consistently with the result in Proposition 1 it follows that at the market solution for given τ the sign of the pecuniary externality depends on the sign of β,

$$\operatorname{sgn}\left\{ \partial \mathbb{E}[\mathrm{TS}]/\partial a \right\} = \operatorname{sgn}\left\{ \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)\left(\frac{\partial x_i}{\partial p} \frac{\partial p}{\partial a} \right) \right] \right\} = \operatorname{sgn}\{-\beta\}.[16]$$

4.3 Comparison with Cournot competition

In this section we show that if firms were to compete à la Cournot by setting quantities contingent only on their information then the market solution would be team-efficient. In this case a strategy for firm i is a mapping from signals $I_i = \{s_i, z\}$ into outputs: $X_i(s_i, z)$. This is the model considered in Vives (1988) (and Angeletos and Pavan, Section 6.1, 2007) from where it follows that, under the same distributional assumptions as in Section 2, there is a unique Bayesian Cournot equilibrium and it is symmetric and linear:

16 And, in fact, the result of Proposition 1 follows since $\mathbb{E}[\mathrm{TS}]$ is strictly concave in a.

$$X(s_i, z) = (\beta + \lambda)^{-1}(\alpha - (\gamma s_i + (1-\gamma)\hat{z}))$$

where $\gamma = (\beta + \lambda)a$ and $a = \dfrac{\tau_\varepsilon}{\lambda(\tau + \tau_\varepsilon) + \beta\tau_\varepsilon}$.

The profits of firm i are $\pi_i = (\alpha + u - \beta\tilde{x})x_i - C(x_i)$. Note that in the game in outputs, we have that $\partial^2\pi / \partial x_i \partial\tilde{x} = -\beta$, and strategies are strategic substitutes when demand is downward sloping ($\beta > 0$) and strategic complements when it is upward sloping ($\beta < 0$). The equilibrium follows immediately from the optimization problem of firm i, $\max_{x_i} \mathbb{E}[\pi_i | s_i, z]$. Since $p = \alpha + u - \beta\tilde{x}$, the associated FOC (which are also sufficient) are $\mathbb{E}[p - \mathrm{MC}(x_i) | s_i, z] = 0$, where the difference from our market game is that firms do *not* condition on the price. It follows that the market solution is team efficient since the same FOC hold also for the maximization of $\mathbb{E}[\mathrm{TS}]$ subject to decentralized production strategies. Indeed, under our assumptions, it is easily seen that the solution is symmetric and with the same FOC as the market

$$\mathbb{E}\left[\frac{\partial \mathrm{TS}}{\partial x_i} | s_i, z\right] = \mathbb{E}[p - MC(x_i) | s_i, z] = 0.$$

In the terminology of Angeletos and Pavan (2007), the economy in which agents use non-price contingent strategies displays exactly the right degree of coordination or complementarity.

The difference between the Cournot and the supply function mechanism can be seen easily noting that the candidate team strategies with Cournot strategies are of the same form as the market $X(s_i, z) = (\beta + \lambda)^{-1}(\alpha - ((\beta + \lambda)as_i + (1 - (\beta + \lambda)a)\hat{z}))$ but again with potentially a different response a to private information. At the team optimum we have that:

$$\frac{\partial \mathbb{E}[\mathrm{TS}]}{\partial a} = \mathbb{E}\left[(p - \mathrm{MC}(x_i))\frac{\partial x_i}{\partial a}\right] = \mathbb{E}[(p - \mathrm{MC}(x_i))(-(s_i - \hat{z}))] = 0.$$

This is exactly the same FOC than at the market solution where firms maximize expected profits. In contrast, with supply function competition firms condition on the price and do not take into account the pecuniary externality term $\mathbb{E}\left[\left(p-\mathrm{MC}\left(x_i\right)\right)\frac{\partial x_i}{\partial p}\frac{\partial p}{\partial a}\right]$ analyzed in the previous section.

5. Equilibrium and welfare with endogenous public information

We consider now the case where firms are sophisticated and do take into account the information content of prices.

5.1 Equilibrium

Suppose that firms receive no public signal except for the price. That is, the information set available to firm i is $\{s_i, p\}$. We are interested in a linear (Bayesian) equilibrium—equilibrium, for short—of the schedule game for which the public statistic functional is of type $P(\theta, u)$. Since the payoffs and the information structure are symmetric and since payoffs are strictly concave, there is no loss of generality in restricting our attention to symmetric equilibria. Indeed, the solution to the problem of firm i,

$$\max_{x_i} \mathbb{E}\left[\left(p-\theta-\frac{\lambda}{2}x_i\right)x_i \middle| s_i, p\right],$$

is both unique (given strict concavity of profits) and symmetric across firms (since the cost function and signal structure are symmetric across firms):

$$X\left(s_i, p\right)=\lambda^{-1}\left(p-\mathbb{E}\left[\theta \middle| s_i, p\right]\right),$$

where $p=P(\theta, u)$. A strategy for firm i may be written as

$$x_i=\hat{b}+\hat{c}p-as_i,$$

in which case the aggregate action is given by

$$\tilde{x}=\int_0^1 x_i\, di=\hat{b}+\hat{c}p-a\theta.$$

It then follows from $p=\alpha+u-\beta\tilde{x}$ that, provided $\hat{c}\neq-\beta^{-1}$,

$$p=P(\theta, u)=\left(1+\beta\hat{c}\right)^{-1}\left(\alpha-\beta\hat{b}+z\right);$$

here the random variable $z=\kappa\theta+\omega$ has $\kappa=\beta a$ and $\omega=u$ and z is informationally equivalent to the price p. Because u is random, z (and the price) will typically generate a noisy signal of the unknown parameter θ. Let τ denote the precision of the price p or of z in the estimation of θ, $\tau \equiv \left(\operatorname{var}\left[\theta | z\right]\right)^{-1}$. From the properties of Gaussian random variables it is immediate that $\tau = \tau_\theta + \tau_u \beta^2 a^2$.

The information available to firm i is $\{s_i, p\}$ or, equivalently, $I_i = \{s_i, z\}$ where $z = \beta a \theta + u$. We can write the strategy of the firm as $X\left(I_i ; p\right) = \lambda^{-1}\left(p - \mathbb{E}\left[\theta | I_i\right]\right)$ but the price p is linearly related to z (and, indeed, $\mathbb{E}\left[\theta | s_i, p\right] = \mathbb{E}\left[\theta | s_i, z\right]$); therefore, with some abuse of notation, we can posit strategies of the form

$$X\left(s_i, z\right) = b - a s_i + cz$$

and obtain that $p = \alpha - \beta b + \left(1 - \beta c\right) z$. If $1 + \beta \hat{c} > 0$ then $1 - \beta c > 0$ (since $\hat{c} = \left(c^{-1} - \beta\right)^{-1}$ and $1 + \beta \hat{c} = \left(1 - \beta c\right)^{-1}$) and so p and z will move together. The strategy of player i is then given by

$$X\left(s_i, z\right) = \lambda^{-1}\left(\alpha - \beta b + \left(1 - \beta c\right) z - \mathbb{E}\left[\theta | s_i, z\right]\right).$$

We can solve for the linear equilibrium in the usual way: identifying coefficients with the candidate linear strategy $x_i = b - a s_i + cz$ by calculating $\mathbb{E}\left[\theta | s_i, z\right]$ and using the supply function of a firm. In equilibrium, firms take public information z, with precision $\tau \equiv \left(\operatorname{var}\left[\theta | z\right]\right)^{-1}$, as given and use it to form probabilistic beliefs about the underlying uncertain parameter θ. As in Section 4.1 the market chooses the weight to private information in $\mathbb{E}\left[\theta | s_i, z\right]$, $\gamma = \tau_\varepsilon \left(\tau_\varepsilon + \tau\right)^{-1}$, in a Bayesian way. Revised beliefs and optimization determine thus the coefficients a ($= \lambda^{-1}\gamma$) and c for private and public information, respectively. In equilibrium, the informativeness of public information z depends on the sensitivity of strategies to private information $a : \tau = \tau_\theta + \tau_u \beta^2 a^2$. Firms behave as information takers and so, from the perspective of an individual firm, public information in the price is exogenous. This fact is at the root of a learning externality: firms fail to account for the impact of their own actions on public information (the price) and hence on other firms. As we have seen in Section 4.2 a second, pecuniary externality in the use of information arises even if firms with private signals do not to learn from prices but still use price-contingent strategies (for example, in the present case it arises even if the price is extremely noisy, $\tau_u = 0$). We will deal with the combined effect of them in the welfare analysis section.

The following proposition characterizes the equilibrium.

Proposition 2. *Let* $\tau_\varepsilon \geq 0$ *and* $\tau_u \geq 0$. *There is a unique (and symmetric) equilibrium*

$$X\left(s_i, p\right) = \lambda^{-1}\left(p - \mathbb{E}\left[\theta \middle| s_i, p\right]\right) = \hat{b} - as_i + \hat{c}p,$$

where a *is the unique (real) solution of the equation* $a = \tau_\varepsilon \lambda^{-1}\left(\tau_\varepsilon + \tau\right)^{-1}$ *with* $\tau = \tau_\theta + \tau_u \beta^2 a^2$, $\hat{c} = \left(\left(\beta + \lambda\right)\left(1 - \beta\lambda\tau_u a^2 \tau_\varepsilon^{-1}\right)^{-1} - \beta\right)^{-1}$, *and* $\hat{b} = \alpha\left(1 - \lambda\hat{c}\right)/\left(\beta + \lambda\right)$.

Corollary: *Let* $\tau_\varepsilon > 0$ *and* $\tau_u > 0$. *In equilibrium,*

- $a \in \left(0, \tau_\varepsilon \lambda^{-1}\left(\tau_\theta + \tau_\varepsilon\right)^{-1}\right)$ *decreases with* τ_u, τ_θ, $|\beta|$ *and* λ, *and increases with* τ_ε;
- $\operatorname{sgn}\left\{\partial\hat{c}/\partial\tau_u\right\} = \operatorname{sgn}\left\{-\partial\hat{c}/\partial\tau_\theta\right\} = \operatorname{sgn}\left\{-\beta\right\}$, *and market depth* $\left(\partial P/\partial u\right)^{-1} = 1 + \beta\hat{c} > 0$ *is decreasing in* τ_u *and increasing in* τ_θ; *and*
- *price informativeness* τ *is increasing in* $|\beta|$, τ_u, τ_θ *and* τ_ε, *and decreasing in* λ.

Remark 1. We have examined linear equilibria of the schedule game for which the price functional is of type $P\left(\theta, u\right)$. In fact, these are the linear equilibria in strategies with bounded means and with uniformly (across players) bounded variances. (See Claim A in the Online Appendix.)

Remark 2. We can show that the equilibrium in the continuum economy is the limit of equilibria in replica economies that approach the limit economy. Take the market with a finite number of firms n and inverse demand $p_n = \alpha + u - \beta\tilde{x}_n$, where $\tilde{x}_n$ is the average output per firm, and with the same informational assumptions. In this case, given the results in Section 5.2 of Vives (2011), the supply function equilibrium of the finite n-replica market converges to the equilibrium in Proposition 2.

The price serves a dual role as index of scarcity and conveyor of information. Indeed, a high price has the direct effect of increasing a firm's competitive supply, but it also conveys news about costs—namely, that costs are high (low) if $\beta > 0$ ($\beta < 0$). Consider as a benchmark the full information case with perfectly informative signals ($\tau_\varepsilon = \infty$). This is a full information competitive equilibrium and we have $c = (\beta + \lambda)^{-1}$, $a = \hat{c} = \lambda^{-1}$, and $X(\theta, p) = \lambda^{-1}(p - \theta)$. In this case, agents have nothing to learn from the price. If signals become noisy ($\tau_\varepsilon < \infty$) then $a < \lambda^{-1}$ and $\hat{c} < \lambda^{-1}$ for $\beta > 0$, with supply functions becoming steeper (lower $\hat{c}$) as agents protect themselves from adverse selection. The opposite happens ($\hat{c} > \lambda^{-1}$ and flatter supply functions) when $\beta < 0$, since then a high price is good news (entailing lower costs). [17] There is then "favorable" selection.

There are several other cases in which $\hat{c} = \lambda^{-1}$ and there is no learning from the price: (i) When signals are uninformative about the common parameter θ $(\tau_\varepsilon = 0)$ or when there is no uncertainty ($\tau_\theta = \infty$ and $\theta = \overline{\theta}$ (a.s.)), the price has no information to convey; a = 0 and $X(s_i, p) = \lambda^{-1}(p - \overline{\theta})$; (ii) When the public statistic is extremely noisy ($\tau_u = 0$) or when $\beta = 0$ (in which case there is no payoff externality, either), then public information is pure noise, $a = \lambda^{-1}\tau_\varepsilon(\tau_\theta + \tau_\varepsilon)^{-1}$, with $X(s_i, p) = \lambda^{-1}(p - \mathbb{E}[\theta|s_i])$.

As τ_u tends to ∞, the precision of prices τ also tends to ∞, the weight given to private information a tends to 0, and the equilibrium collapses (with market depth $1 + \beta\hat{c} \to 0$). Indeed, the equilibrium becomes fully revealing and is not implementable. The informational component of the price increases with τ_u and decreases with τ_θ (since firms are endowed with better prior information with a larger τ_θ). With $\beta > 0$, as τ_u increases from 0, $\hat{c}$ decreases from λ^{-1} (and the slope of supply increases) because of the price's increased informational component (a high price indicates higher costs). As τ_u increases more, $\hat{c}$ becomes zero at some point

[17] This follows because we assume that $2\beta + \lambda > 0$ and therefore $\lambda > -\beta$.

and then turns negative; as τ_u tends to ∞, $\hat{c}$ tends to $-\beta^{-1}$.[18] At the point where the allocational and informational effects balance, agents place zero weight ($\hat{c}=0$) on the price. In this case the model reduces to a quantity-setting model à la Cournot (however, not reacting to the price is optimal). If τ_θ increases then the informational component of the price diminishes and we have a more elastic supply (higher $\hat{c}$).

When $\beta<0$ then a high price conveys the good news that average quantity tends to be high and that costs therefore tend to be low. In this case, increasing τ_u, which reinforces the informational component of the price, increases $\hat{c}$—the opposite of what happens when τ_θ increases. It follows that in either case ($\beta>0$ or $\beta<0$) market depth $\left(\partial P/\partial u\right)^{-1}=1+\beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ. (See the Online Appendix for a complete statement of the comparative statics properties of the equilibrium.)

5.2 The team solution

Now at the team-efficient solution, expected total surplus $\mathbb{E}[\mathrm{TS}]$ is maximized under the constraint that firms use decentralized linear production strategies contingent on endogenous public information (price p or the equivalent variable z). That is,

$$\max_{a,b,c}\mathbb{E}[\mathrm{TS}] \qquad (\mathbf{T})$$

subject to $\quad x_i=b-as_i+cz$, $\tilde{x}=b-a\theta+cz$, and $z=u+\beta a\theta$.

Note that in this problem the variable z comes from market clearing and incorporates the conditioning on the price. It is easily seen (see Claim 2 in the Appendix) that the form of the optimal team strategy is

$$x_i=\lambda^{-1}\left(p-\left(\lambda as_i+(1-\lambda a)\mathbb{E}[\theta\mid z]\right)\right).$$

As in Section 4.1 this yields a strictly convex WL as a function of a:

$$\mathrm{WL}\left(a;\tau(a)\right)=\frac{1}{2}\left(\frac{(1-\lambda a)^2}{\tau(a)(\beta+\lambda)}+\frac{\lambda a^2}{\tau_\varepsilon}\right) \text{ where } \tau(a)=\tau_\theta+\tau_u\beta^2a^2.$$

[18] Downward sloping supply bids have been allowed in some wholesale electricity markets (e.g. in the Nord Pool before 2007).

Now aggregate inefficiency decreases with increases in a also because price informativeness $\tau(a)$ increases and the average quantity gets close to the full information allocation, and again productive inefficiency increases with a as dispersion increases. A higher response to private information induces a more informative price (higher τ) and reduces allocative inefficiency but increases productive inefficiency. The team solution, denoted a^{T}, minimizes WL and optimally trades off the sources of inefficiency among decentralized strategies taking into account that τ depends on a. This generates a learning externality that we characterize next.

5.2.1 Endogenous public information: The learning externality

When firms do take into account the information content of the price there is a learning externality and an added reason for the market solution to be inefficient. We know from the received literature that the learning externality will tend to make agents put too little weight on private information (Vives 1997, Amador and Weill 2012). The reason is that an agent when responding to its private information does not take into account the improved informativeness of public statistics. This effect will also be present in our case but with the price as public statistic. Indeed, when the informational value of the price is accounted for, public information is endogenous and the response to private information a affects the precision of the price $\tau = \tau_\theta + \tau_u \beta^2 a^2$.

As stated before, $\mathrm{WL}(a;\tau(a))$ is a strictly convex function of a and the following FOC characterizes the team solution a^{T}

$$\frac{d\mathrm{WL}}{da} = \frac{\partial \mathrm{WL}}{\partial a} + \underbrace{\frac{\partial \mathrm{WL}}{\partial \tau}}_{-}\underbrace{\frac{\partial \tau}{\partial a}}_{+} = 0$$

The solution is unique, and $\lambda^{-1} > a^T > 0$ provided $\infty > \tau_\varepsilon > 0$.

The first term $\partial \mathrm{WL}/\partial a$ corresponds to the direct effect of changing a for a fixed τ and the second corresponds to the indirect effect through the public precision τ. This second term is the effect of the learning externality and it is negative since

$\partial \mathrm{WL}/\partial\tau < 0$ and $\partial\tau/\partial a > 0$. This implies that for any given τ we want to increase a from the optimal level with exogenous public information. Indeed, we have that $\partial \mathrm{WL}\left(a^{\mathrm{T}}_{\mathrm{exo}}(\tau);\tau\right)/\partial a = 0$ and therefore, $\mathrm{dWL}\left(a^{\mathrm{T}}_{\mathrm{exo}}(\tau);\tau\right)/da < 0$: when $a = a^{\mathrm{T}}_{\mathrm{exo}}$, increasing a induces a first order gain making $\tilde{x}$ closer to x^o and reducing aggregate inefficiency while there is no first order loss in the trade-off between aggregate and productive inefficiency. This confirms the idea that the learning externality biases the market solution towards putting too little weight on private information. The following lemma states the result.

Lemma 1. *Let $\tau_u > 0$. At the team solution with exogenous public precision τ by increasing a the welfare loss is reduced; i.e.* $\mathrm{dWL}\left(a^{\mathrm{T}}_{\mathrm{exo}}(\tau);\tau\right)/da < 0$.

5.2.2 The combined effect of the externalities

We examine now the combined effect of the two (pecuniary and learning) externalities in the use of information characterized in Proposition 1 and Lemma 1. We know that the learning externality always leads agents to underweight private information and that the pecuniary externality leads to overweight or to underweight depending on whether demand is downward or upward sloping. From this it follows that with upward sloping demand we would have always underweighting of private information. However, in the normal case of downward sloping demand depending on the strength of the learning externality we may overcome or not the overweighting result due to the pecuniary externality.

From the FOC $d\mathrm{WL}\left(a;\tau(a)\right)/da = 0$ with

$$\frac{\partial \mathrm{WL}}{\partial a} = \frac{\lambda a}{\tau_\varepsilon} - \frac{\lambda(1-\lambda a)}{(\beta+\lambda)\tau} \text{ and } \frac{\partial \mathrm{WL}}{\partial \tau}\frac{\partial \tau}{\partial a} = -\frac{(1-\lambda a)^2\beta^2 a\tau_u}{(\beta+\lambda)\tau^2},$$

we obtain that a^T fulfills

$$a = \frac{\tau_\varepsilon}{\lambda(\tau(a)+\tau_\varepsilon)+\beta\tau(a)-\Delta(a)}$$

where $\beta\tau(a)$ corresponds to the pecuniary externality and $\Delta(a) = \frac{(1-\lambda a)^2\beta^2\tau_u\tau_\varepsilon}{\lambda\tau(a)} \geq 0$ to the learning externality.

At the market solution, denoted by *, in the normal case ($\beta > 0$) the pecuniary and learning externalities cancel each other exactly when $\beta\tau = \Delta$, in which case $a^* = a^{\mathrm{T}}$. This happens when $c^* = 0$. We have that $\beta\tau - \Delta > 0$ when $c^* > 0$ and $\beta\tau - \Delta < 0$ when $c^* < 0$ (see Claim 3 in the Appendix). This suggests that $a^* < a^{\mathrm{T}}$ when $c^* < 0$ and $a^* > a^{\mathrm{T}}$ when $c^* > 0$. The first case happens when τ_u is large, the supply function is downward sloping because the informational component of the price prevails, and the learning externality wins over the pecuniary externality.[19] The second case happens when τ_u is low, the supply function is upward sloping because the allocational effect of the price prevails, and the learning externality is overpowered by the pecuniary externality. With $\beta < 0$ the pecuniary and learning externalities reinforce each other and $a^* < a^{\mathrm{T}}$.

When $\beta > 0$ and firms do not respond to the price ($c^* = 0$), the model is equivalent to a quantity-setting model with private information. Indeed, the strategy used by a firm reduces to a Cournot strategy because, in the given parameter constellation, the allocation weight to the price in the supply function $X\left(s_i, p\right) = \lambda^{-1}\left(p - \mathbb{E}\left[\theta \middle| s_i, p\right]\right)$, equal to 1, exactly matches its informational weight (the weight to the price in $\mathbb{E}\left[\theta \middle| s_i, p\right]$).

The result is given in the following proposition.

<u>Proposition 3</u>. *Let* $\infty > \tau_\varepsilon > 0$. *Then the team problem has a unique solution with* $\lambda^{-1} > a^{\mathrm{T}} > 0$ *and* $\operatorname{sgn}\left\{a^* - a^{\mathrm{T}}\right\} = \operatorname{sgn}\left\{\beta c^*\right\}$.

From the expression for WL we obtain directly that $\left.\dfrac{d\mathrm{WL}}{da}\right|_{a=a^*} = \lambda a^* \sigma_\varepsilon^2 \beta c^*$ and WL is strictly convex with one minimum. The result follows since $a^* > 0$ when $\tau_\varepsilon > 0$.

[19] Recall that c^* is decreasing in τ_u.

An alternative argument which isolates the effect of the externalities associated to the use of private information because agents use price-contingent strategies, as explained in Section 4.2, is as follows. The strategy for firm i is of the form $x_i = \lambda^{-1}\left(p - \left(\lambda a s_i + (1-\lambda a)\,\hat{z}\right)\right)$, where $\hat{z} = \mathbb{E}[\theta \mid z]$, $z = \beta a\theta + u$. We have that

$$\frac{\partial x_i}{\partial a} = \left.\frac{\partial x_i}{\partial a}\right|_{p,\hat{z}} + \frac{\partial x_i}{\partial p}\left.\frac{\partial p}{\partial a}\right|_{\hat{z}} + \frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a},$$

where the first term corresponds to market behavior, the second to the pecuniary externality with exogenous public information, and the last term to the learning externality:

$$\frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a} = \left(\frac{\partial x_i}{\partial p}\frac{\partial p}{\partial \hat{z}} + \frac{\partial x_i}{\partial \hat{z}}\right)\frac{\partial \hat{z}}{\partial a}.$$

From Section 4.2 we obtain the effect of the pecuniary externality for given public information at the market solution a^*:

$$\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)\frac{\partial x_i}{\partial p}\left.\frac{\partial p}{\partial a}\right|_{\hat{z}\ \mathrm{ct.}}\right] = -\beta\lambda(\beta+\lambda)^{-1}a^*\sigma_\varepsilon^2.$$

Furthermore, the learning externality has the expected sign

$$\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)\left(\frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a}\right)\right] = \frac{1-\lambda a^*}{\beta+\lambda}\frac{\beta^2\tau_u a^*}{\tau^*}\left(\lambda a^*\sigma_\varepsilon^2\right) > 0$$

and adding up both externality effects delivers the desired result $\mathrm{sgn}\{\partial\mathbb{E}[\mathrm{TS}]/\partial a\} = \mathrm{sgn}\{-\beta c^*\}$ (using the fact that $c^* = (\beta+\lambda)^{-1}\left(1 - \beta\tau_u a^*\left(\tau_\varepsilon + \tau^*\right)^{-1}\right)$ from the proof of Proposition 2).

If $\beta = 0$ then there is neither a learning nor a pecuniary externality, and the team and market solutions coincide. For $\beta \neq 0$, $\tau_\varepsilon > 0$, and $\tau_u > 0$, the solutions coincide only if $c^* = 0$. When signals are uninformative ($\tau_\varepsilon = 0$) or perfect ($\tau_\varepsilon = \infty$) there is no private information, there is no learning externality and the pecuniary externality is internalized at the competitive equilibrium. As a result the team and the market solution coincide (with $a = 0$ when $\tau_\varepsilon = 0$). When the price contains no information ($\tau_u = 0$) there is no learning externality, $c = 1/(\beta+\lambda)$ both for the team and the market solutions, and only the pecuniary externality remains with the result that $\operatorname{sgn}\{a^* - a^{\mathrm{T}}\} = \operatorname{sgn}\{\beta\}$ (as in Proposition 1).

In conclusion, in the usual case with downwards sloping demand, $\beta > 0$, and upward sloping supply functions, $c^* > 0$, there is too much dispersion and productive inefficiency. With downward sloping supply functions, $c^* < 0$, firms give insufficient weight to private information there is too much aggregate inefficiency. This is the case also when $\beta < 0$.

Remark (Cournot): If our firms would compete à la Cournot as in Section 4.3 but receive an endogenous noisy quantity signal (say the average quantity plus noise) then at the market solution they would rely too little on their private information because of the learning externality.[20]

Two corollaries follow immediately from Proposition 3. The first is on market quality and the second on the implementation of the team solution with tax-subsidy schemes.

Corollary 3.1 (market quality). *At the market solution:*

- *In relation to the team optimum, when $\beta c^* > 0$ price informativeness τ and dispersion $\mathbb{E}\left[\left(x_i - \tilde{x}\right)^2\right]$ are too high, and aggregate inefficiency $\mathbb{E}\left[\left(\tilde{x} - x^o\right)^2\right]$ too low. The opposite is true when $\beta c^* < 0$.*
- *In relation to the first best (where $\mathbb{E}\left[\left(\tilde{x} - x^o\right)^2\right] = \mathbb{E}\left[\left(x_i - \tilde{x}\right)^2\right] = 0$), price informativeness is too low, and aggregate inefficiency and dispersion are too high.*

It is worth noting that *market depth* $1 + \beta\hat{c}$ is always too low with respect to the first best and it may be too low also with respect to the team solution when $\beta c^* > 0$.[21]

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[20] See the Online Appendix for a precise statement and proof of this result.

[21] The result for the first best follows immediately given that at the first best $\hat{c} = \lambda^{-1}$ and from the market equilibrium expression for $\hat{c}$.

Corollary 3.2 (implementation of team solution). *The team solution* a^{T} *can be implemented with a quadratic tax* $(\delta/2)x_i^2$ on firms where $\delta = \dfrac{\beta\tau(a^{\mathrm{T}}) - \Delta(a^{\mathrm{T}})}{\tau(a^{\mathrm{T}}) + \tau_\varepsilon}$.

Corollary 3.2 follows since at the team solution

$$a = \frac{\tau_\varepsilon}{\lambda(\tau(a)+\tau_\varepsilon)+\beta\tau(a)-\Delta(a)} = \frac{\tau_\varepsilon/(\tau(a)+\tau_\varepsilon)}{\lambda+\delta},$$

which is the expression for the market responsiveness to private information when the quadratic cost parameter is $\lambda+\delta$. The implementation of the team solution requires, as expected, a tax with $\delta > 0$ when $a^* > a^{\mathrm{T}}$ (in which case $\beta\tau(a^{\mathrm{T}}) - \Delta(a^{\mathrm{T}}) > 0$) and a subsidy with $\delta < 0$ when $a^* < a^{\mathrm{T}}$ (in which case $\beta\tau(a^{\mathrm{T}}) - \Delta(a^{\mathrm{T}}) < 0$). The tax or subsidy may be returned/charged in expectation to the firms and therefore it can satisfy budget balance in expected terms. The imposition of an optimal tax ($\delta > 0$) reduces price informativeness (and may increase market depth when $\beta c^* > 0$).[22]

5.3 Can more information hurt?

The question arises as of how the welfare loss WL at the market solution depends on precisions of private and public information as well as on the noise in demand and costs. To elucidate these questions let us consider the model with *both* an exogenous public signal (adding precision $\kappa^2\tau_\omega$) and the price as endogenous public signal. The effect of the exogenous public signal is the same as adding $\kappa^2\tau_\omega$ to the prior precision τ_θ since WL depends only on the total public precision τ and on private precision τ_ε. Therefore the comparative statics of τ_θ and τ_ω will be identical.

We know that WL is a strictly convex function of a attaining a minimum at the team-efficient solution a^{T}. It is immediate then that $\mathrm{WL}(a^{\mathrm{T}})$ is decreasing in τ_ε, τ_u and τ_θ. This is so since WL is decreasing in τ_ε, τ_u and τ_θ for a given a and $d\mathrm{WL}(a^{\mathrm{T}})/da = 0$. Things are potentially different at the market solution a^* since

22 See Angeletos and Pavan (2009), Lorenzoni (2010), and Angeletos and La'O (2012) for examples of tax-subsidy schemes to implement team-efficient solutions. Different from them our analysis is based on competition on schedules.

then $d\mathrm{WL}(a^*)/da$ is positive or negative depending on whether $a^* > a^{\mathrm{T}}$ or $a^* < a^{\mathrm{T}}$. Since a^* is decreasing in τ_u and τ_θ, and increasing in τ_ε (see the Corollary to Proposition 2) we have thus that $\mathrm{WL}(a^*)$ is decreasing in τ_u and τ_θ when $a^* > a^{\mathrm{T}}$ and in τ_ε when $a^* < a^{\mathrm{T}}$.[23] It would be possible in principle that increasing precisions τ_u and τ_θ increases the welfare loss when $a^* < a^{\mathrm{T}}$ when the direct effect of the increase of τ_u or τ_θ is dominated by the indirect effect via the induced decrease in a^* (and similarly for an increase in τ_ε when $a^* > a^{\mathrm{T}}$). We can check, however, that $\mathrm{WL}(a^*)$ is always decreasing in τ_θ and τ_u. This need not be the case when changing τ_ε. In any case, as the information precisions τ_θ, τ_u and τ_ε tend to infinity $\mathrm{WL}(a^*)$ tends to 0.[24] The following proposition summarizes the results.

<u>Proposition 4</u>. *The welfare loss at the team-efficient solution is decreasing in* $\tau_\varepsilon, \tau_\omega, \tau_u$ *and* τ_θ. *The welfare loss at the market solution is also decreasing in* τ_ω, τ_θ *and* τ_u, *and it may be decreasing or increasing in* τ_ε *(it will be increasing for* $\beta > \lambda$ *and* $\tau_\varepsilon/\tau_\theta$ *small enough). As any of the precisions* τ_θ, τ_u, τ_ω *and* τ_ε *tend to infinity welfare losses tend to zero.*

More precise public (τ_ω) or private (τ_ε) information reduces the welfare loss at the team-efficient solution. This is in accordance with the results in Angeletos and Pavan (2007, 2009) where more information can not hurt when it is used efficiently. The welfare loss at the market solution is also always decreasing with the precision of public information. However, the welfare loss at the market solution may be increasing with the precision of private information when the market calls already for a too large response to private information. The reason is that an increase in the

23 We have that a^T is increasing in τ_ε and decreasing in τ_θ (since $\partial^2\mathrm{WL}/\partial a\,\partial\tau_\varepsilon < 0$, $\partial^2\mathrm{WL}/\partial a\,\partial\tau_\theta > 0$, and WL is strictly convex in a) ; and with simulations we obtain that a^T is hump-shaped or decreasing in τ_u.

24 This follows since as $\tau_\varepsilon \to \infty$, $a^* \to \lambda^{-1}$; and as τ_θ or $\tau_u \to \infty$, $a^* \to 0$ and $\tau \to \infty$.

precision of private information will increase the response of an agent to his private signal and this indirect effect may dominate.

The welfare result of the market solution is in contrast with received results in the literature where more *public* information may be damaging to welfare (Burguet and Vives 2000; Morris and Shin 2002; Angeletos and Pavan 2007; Amador and Weill 2010, 2011). In those papers more public information discourages the use and/or acquisition of private information. In the present paper this also happens but the direct effect of public information provision prevails.

A possible extension of the model would study the private incentives to acquire information (as in Vives 1988; Burguet and Vives 2000; Hellwig and Veldkamp 2009; Myatt and Wallace 2012; Llosa and Venkateswaran 2013; and Colombo et al. 2014).

5.4 Application: Business cycle policy

Consider a standard "island" economy business cycle model with CRR utilities and CES aggregators augmented with incomplete information (see Angeletos and Lao 2013).[25] A reduced form of the model has players being the islands in the economy (with representative household and firm), the actions are productions (which can be strategic substitutes or strategic complements), and types the local information sets consisting of exogenous and endogenous private and public signals (the endogenous public signal being a noisy aggregate quantity, say a macro forecast). The equilibrium is in log-linear strategies (of the Bayesian-Cournot type) and it is unique under certain parametric conditions. The team welfare function is analogous to ours but in constant elasticity form and the optimum is found in the class of log-linear decentralized strategies. The model so far is akin to the Cournot version of our model (see Section 4.3), albeit quite a bit more complex. The result is that if prices are flexible and there are no endogenous signals the equilibrium is team efficient (as in our Cournot economy).[26] With endogenous signals there is an informational externality and the equilibrium is not team efficient. In this case the optimal policy is countercyclical in the sense that it induces agents to put more weight on their private information to

25 And Colombo et al. (2014) for models of the same family.

26 This result will not hold in general in price-setting economies with complementarities (see, e.g., Hellwig 2005 and Lorenzoni 2010).

internalize the informational externality. This is, indeed, the result we obtain in our Cournot version of the model with an endogenous public signal (see the Online Appendix). However, we may ask where does the aggregate public signal comes from. If the signal is an average price across islands, something very plausible, then the results of the present paper apply: a) Even if prices are flexible and agents are naïve and do not learn from prices the equilibrium is not team efficient; b) In the normal case with strategic substitutes competition, agents put too much weight on their private information and optimal policy should be pro-cyclical. We see, therefore, how providing a plausible interpretation for the public signal as a price index we overturn optimal policy.

6. Demand schedule competition and optimal taxes

In this section we reinterpret the model in terms of competition in demand schedules. Let a buyer of a homogenous good with unknown ex post value θ face an inverse supply $p = \alpha + u + \beta\tilde{y}$, where $\tilde{y} = \int_0^1 y_i \, di$ and y_i is the demand of buyer i. The suppliers face a cost of supply of $\left(\alpha + u + \beta\frac{\tilde{y}}{2}\right)\tilde{y}$. The marginal cost of supply is increasing (decreasing) in the amount supplied when $\beta > 0$ ($\beta < 0$). The case $\beta < 0$ may correspond, for example, to a situation where there is learning by doing in the supply (see the Appendix for a microfoundation of the model). The buyer's net benefit is given by $\pi_i = (\theta - p)y_i - (\lambda/2)y_i^2$, where λy_i^2 is a transaction or opportunity cost (or an adjustment for risk aversion). The timing of events runs parallel to one in the basic model in Section 2. The model fits this setup if we let $y_i \equiv -x_i$. We illustrate the results for a financial market (another illustration would have firms hiring labor of unknown productivity).

Traders in a financial market.[27]

Informed speculators have information on the liquidation value θ of a risky asset and face quadratic transaction costs (alternatively, the parameter λ proxies for risk aversion). Liquidity suppliers trade according to the elastic aggregate demand $(\alpha + u - p)/\beta$, where u is random. In the normal case, when $\beta > 0$, liquidity

[27] A variation of this example can be used to model Treasury or liquidity auctions.

suppliers buy (sell) when the price is low (high); when $\beta < 0$, liquidity suppliers buy (sell) when the price is high (low). In this case we could interpret liquidity suppliers are program traders following a portfolio insurance strategy.[28]

Our results apply. In the normal case with $\beta > 0$ and downward-sloping demand schedules for informed traders, those overreact to their private signals. This will happen when the volume of liquidity trading is high (i.e. when τ_u is low). In this case a Tobin-style tax on privately informed speculators is warranted. If the tax is set at the optimal level (see Corollary 3.2) it will reduce the responsiveness to private information of speculators and implement the team optimum. The tax may increase market depth. It is worth noting again that the tax is not optimal because of a Hirshleifer effect of prices being "too" informative and destroying insurance opportunities (see, e.g., Dow and Rahi 2000).[29] The tax corrects a pecuniary externality which arises because informed traders condition on prices when trading.

To levy a tax only on privately informed speculators may not be feasible. In fact, a common criticism to the Tobin tax is that it cannot distinguish between speculators and liquidity suppliers. It is easy to see, however, that an appropriate tax on all traders will also work. This is so since the responsiveness to information of informed speculators decreases not only with their transaction cost λ but also with $|\beta|$ (Corollary to Proposition 2). Consider thus a quadratic tax δ on both informed, and liquidity traders. Then the inverse supply is given by $p = \alpha + u + (\beta + \delta)\tilde{y}$ and at the market solution $a^*(\delta)$,

$$a = \frac{\tau_\varepsilon}{(\lambda + \delta)\left(\tau_\varepsilon + \tau_\theta + (\beta + \delta)^2 \tau_u a^2\right)}.$$

Consider the case with $\beta > 0$ and downward-sloping demand schedules for informed traders ($c^* > 0$). We know then that $a^* > a^{\mathrm{T}} > 0$ when $\delta = 0$ (Proposition 3). It is

[28] See Gennotte and Leland (1990). Hendershott and Seasholes (2009) find that program trading accounts for almost 14% of the average daily market volume at the NYSE in 1999-2005 and that program traders lose money on average.

[29] Dow and Rahi (2000) consider quadratic transaction taxes in models with risk averse informed traders and find conditions under which a transaction tax can be Pareto improving even if the tax revenue is wasted. Subrahmanyam (1998) also considers quadratic transaction taxes and finds that the tax reduces market liquidity.

immediate that $a^*(\delta)$ decreases with δ, and ranges from a^* to 0 as δ goes from 0 to ∞. Therefore, there is a $\delta > 0$ for which $a^*(\delta) = a^T$. This is the common transaction tax that implements the team solution. It is worth noting that this δ is strictly lower than the transaction tax targeted only to speculators (as given in Corollary 3.2).

In the parameter region where demand schedules for the informed are upward sloping, those traders underreact to their private information. The same applies in the case $\beta < 0$. In those cases a transaction subsidy would be optimal.

7. Internal welfare benchmark

In this section we explore a different welfare benchmark where only the welfare of the producers (firms) is taken into account. This is a collusive benchmark where the welfare of consumers is disregarded. We term it the internal team solution and consider directly the case where firms do take into account the information content of prices.

At the internal team–efficient solution, expected average profit $\mathbb{E}[\tilde{\pi}]$,where $\tilde{\pi} = \int_0^1 \pi_i \, di$, is maximized under the constraint that firms use decentralized linear strategies. Since the solution is symmetric we have that $\mathbb{E}[\tilde{\pi}] = \mathbb{E}[\pi_i]$. This is the cooperative solution from the firms' perspective. That is,

$$\max_{a,b,c} \mathbb{E}[\pi_i]$$

$$\text{subject to} \quad x_i = b - as_i + cz, \;\; \tilde{x} = b - a\theta + cz, \text{ and } \; z = u + \beta a\theta .$$

Note again that the team strategy conditions on the variable $z = u + \beta a\theta$ which is informationally equivalent to the price. It should be clear that the market solution, not even with complete information, will attain the full information cooperative outcome (denoted M for monopoly, for which $x^{\mathrm{M}} = (\lambda + 2\beta)^{-1}(\alpha + u - \theta)$) where joint profits are maximized under full information. This is so since the market solution does not internalize the competition (payoff) externality and therefore if $\beta \neq 0$ it will produce an expected output $\mathbb{E}\left[\tilde{x}^*\right]=\alpha\left(\beta+\lambda\right)^{-1}$ which is too high (low) with $\beta>0$ ($\beta<0$) in relation to the optimal $\mathbb{E}\left[x^{\mathrm{M}}\right]=\alpha\left(2\beta+\lambda\right)^{-1}$. Furthermore, the market solution does not internalize the externalities in the use of information arising from price-contingent strategies. At the *internal team* (IT) benchmark, joint profits are maximized and information-related externalities internalized with decentralized strategies.[30] The question is whether the market solution allocates the correct weights (from the firms' collective welfare viewpoint) to private and public information. We show that the answer to this question is qualitatively similar to the one derived when analyzing the *total surplus* team benchmark but in this case with a larger bias towards the market displaying too much weight on private information.

As before, it can be seen that the internal team-efficient solution minimizes, over the restricted strategies, the expected loss Ω with respect to the full information cooperative outcome x^{M}, and that

$$\Omega=\left(\left(2\beta+\lambda\right)\mathbb{E}\left[\left(\tilde{x}-x^{\mathrm{M}}\right)^2\right]+\lambda\mathbb{E}\left[\left(x_i-\tilde{x}\right)^2\right]\right)\Big/2.$$

The first term in the sum corresponds to aggregate inefficiency in the average quantity, which is proportional to $\mathbb{E}\left[\left(\tilde{x}-x^{\mathrm{M}}\right)^2\right]$, and the second term to productive inefficiency, which is proportional to $\mathbb{E}\left[\left(x_i-\tilde{x}\right)^2\right]$.

It can be checked that the form of the internal optimal team strategy is $x_i=\left(\lambda+\beta\right)^{-1}\left(p-\left(\gamma s_i+\left(1-\gamma\right)\mathbb{E}\left[\theta\mid z\right]\right)\right)$ where $\gamma=\left(\lambda+\beta\right)a$ (while at the market solution we have that $\gamma=\lambda a$). The loss at any candidate internal team solution (which internalizes the competition payoff externality and for which $\mathbb{E}\left[\tilde{x}\right]=\alpha\left(2\beta+\lambda\right)^{-1}$) will depend only on the response to private information a since at this candidate solution we have $\mathbb{E}\left[\left(\tilde{x}-x^{\mathrm{M}}\right)^2\right]=\left(1-\left(\lambda+\beta\right)a\right)^2\Big/\left(\tau\left(2\beta+\lambda\right)^2\right)$ and $\mathbb{E}\left[\left(x_i-\tilde{x}\right)^2\right]=a^2/\tau_{\varepsilon}$. This yields a strictly convex Ω as a function of a. As before,

[30] Indeed, when $\beta=0$ there are no externalities (payoff or informational) and the internal team and market solutions coincide.

changing a has opposite effects on both sources of the loss. Now the internal team solution optimally trades off the sources of the loss with respect to the responsiveness to private information among decentralized strategies which internalize the competition payoff externality.

In this case at the market solution there is as before a combined, pecuniary and learning, price-contingent strategy externality (PE+LE) in the use of private information, and also a competition payoff (CE) externality through the impact of aggregate output on price in the use of information, since even with full information the market solution is not efficient (i.e. cooperative). The impact of the externalities on the response to private information can be assessed similarly as before. The market takes the public statistic z or p as given while the internal team solution takes into account all externalities:

$$\frac{\partial \mathbb{E}[\pi_i]}{\partial a} = \underbrace{\mathbb{E}\left[(p - MC(x_i))\left(\frac{\partial x_i}{\partial a} \right)_z \right]}_{\text{Market}} + \underbrace{\mathbb{E}\left[(p - MC(x_i))\left(\frac{\partial x_i}{\partial z}\frac{\partial z}{\partial a} \right) \right]}_{\text{PE+LE}} + \underbrace{\mathbb{E}\left[x_i\left(\frac{\partial p}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial a} \right) \right]}_{\text{CE}} .$$

The market term is null at the market solution and the sum of the PE+LE and CE terms at the market solution can be evaluated as follows:

$$\left.\frac{\partial \mathbb{E}[\pi_i]}{\partial a}\right|_{a=a^*} = -\beta a^*\left(c^*\lambda\sigma_\varepsilon^2 + \left(c^*\beta - 1 \right)^2 \sigma_\theta^2 \right).$$

It is worth noting that while, as before, $\operatorname{sgn}\{\text{PE+LE}\} = \operatorname{sgn}\{-\beta c^*\}$ we have that $\operatorname{sgn}\{\text{CE}\} = \operatorname{sgn}\{-\beta\}$ since $\left(c^*\beta - 1 \right)^2 \sigma_\theta^2 > 0$, and therefore the CE term will call for a lower (higher) response to private information with downward (upward) sloping demand than the market solution. If $\beta > 0$ a high price indicates high costs. If, say, costs are high ($\theta - \bar{\theta} > 0$) then an increase in a will increase p ($\frac{\partial p}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial a} = -\beta(c\beta - 1)(\theta - \bar{\theta}) > 0$ since at the market solution $c\beta < 1$) while x_i will tend to be low (since at the market solution $\mathbb{E}\left[\left(\theta-\bar{\theta}\right)x_i\right]=a\sigma_\theta^2\left(c\beta-1\right)<0$). This means that if $\beta>0$, CE < 0 and a must be reduced. Similarly, we have that CE > 0 if $\beta<0$. The results on the payoff externality CE are in line with the results obtained by Angeletos and Pavan (Section 6.5, 2007) with non price-contingent strategies. We will see how the effect of the PE+LE term may overturn this result when $c<0$.

The next proposition characterizes the response to private information.

Proposition 5. *Let* $\infty>\tau_\varepsilon>0$. *Then the internal team problem has a unique solution with* $\left(\lambda+\beta\right)^{-1}>a^{\text{IT}}>0$, *and* $\text{sgn}\left\{a^*-a^{\text{IT}}\right\}=\text{sgn}\left\{\beta\left(c^*\lambda\sigma_\varepsilon^2+\left(c^*\beta-1\right)^2\sigma_\theta^2\right)\right\}$.

If $c^*\geq 0$ then $\text{sgn}\left\{a^*-a^{\text{IT}}\right\}=\text{sgn}\left\{\beta\right\}$. Therefore, as before, under $\beta<0$, there is too little response to private information, $a^*<a^{\text{IT}}$. Indeed, the characterization yields the same qualitative result as in the previous section if $c^*>0$: too much or too little response to private information depending on the sign of β. In this case, however, if agents use Cournot strategies (i.e., if $c^*=0$) then the market is not internal team–efficient. This should not be surprising when one considers that, when $c^*=0$, the combined externality for the use of price-contingent strategies is nil, yet the competition payoff externality is not internalized, as firms set a quantity that is too large (small) when $\beta>0$ ($\beta<0$). If $\beta>0$ and $c^*<0$, then $c^*\lambda\sigma_\varepsilon^2+\left(c^*\beta-1\right)^2\sigma_\theta^2>0$ for c^* close to zero or sufficiently negative (τ_u large). Only for intermediate values of c^* we have $c^*\lambda\sigma_\varepsilon^2+\left(c^*\beta-1\right)^2\sigma_\theta^2<0$ and $a^{\text{IT}}>a^*$. With $\beta>0$ the market will bias the solution more towards putting too high a weight on private information since we may have $c^*\lambda\sigma_\varepsilon^2+\left(c^*\beta-1\right)^2\sigma_\theta^2>0$ even if $c^*<0$.

Remark 4. The weights to private information in the internal team and market solutions are, respectively, $\gamma^{\text{IT}}=\left(\lambda+\beta\right)a^{\text{IT}}$ and $\gamma^*=\lambda a^*$. It is easy to see that for τ_u small enough (and $\tau_\theta>0$) we have that $\gamma^{\text{IT}}>\gamma^*$. The same result applies when

$\beta > 0$ and $c^{*}\lambda\sigma_{\varepsilon}^{2} + \left(c^{*}\beta - 1\right)^{2}\sigma_{\theta}^{2} < 0$ in which case $a^{\mathrm{IT}} > a^{*}$ and therefore $\lambda\gamma^{\mathrm{IT}} > \left(\lambda + \beta\right)\gamma^{*} > \lambda\gamma^{*}$.

Application. *Monopolistic competition*. The model applies also to a monopolistically competitive market with quantity-setting firms; in this case, either $\beta > 0$ (goods are substitutes) or $\beta < 0$ (goods are complements). Firm i faces the inverse demand for its product, $p_i = \alpha + u - \beta\tilde{x} - \left(\lambda/2\right)x_i$, and has costs θx_i. Each firm uses a supply function that is contingent on its own price: $X\left(s_i, p_i\right)$ for firm i. It follows then that observing the price p_i is informationally equivalent (for firm i) to observing $p \equiv \alpha + u - \beta\tilde{x}$. Under monopolistic competition, the total surplus function (consistent with the differentiated demand system) is slightly different:

$$\mathrm{TS} = \left(\alpha + u - \theta\right)\tilde{x} - \left(\beta\tilde{x}^{2} + \left(\lambda/2\right)\int_{0}^{1} x_i^{2}\, di\right)\Big/2 .$$

Here the market is not efficient under complete information because price is not equal to marginal cost. Each firm has some residual market power. The welfare results of Section 5 do not apply but those of the present section apply when firms collude. It is interesting to note then that, if agents cannot use price-contingent strategies (as in the cases of Cournot or Bertrand competition), Angeletos and Pavan (Section 6.5, 2007) argue that with strategic substitutability ($\beta > 0$ in our case) we would have always excessive response to private information in contrast with the case with supply functions as strategies, where either excessive or insufficient response to private information is possible.

8. Concluding remarks.

We find that price-contingent strategies, on top of the usual learning externality, introduce a pecuniary externality in the use of private information which induces agents to overweight private information in the normal case of decreasing marginal utility. This externality dominates the usual learning-from-prices externality when the allocational role of prices prevails over their informational role. The inefficiency of the market solution opens the door to the possibility that more precise public or private information will lead to an increased welfare loss. This is the case when the market already calls for a too large response to private information, then more precise private information exacerbates the problem (but not more precise public information).

The practical implication of the result is that in market games, where agents condition on prices, the presumption that agents rely too much on public information and too little in private information will not hold. Efficiency can be restored with an optimal tax, which in the case of financial markets is a quadratic Tobin-like tax, and which induces traders to internalize the externalities they generate. The results have also implications for business cycle policy when firms have private information on productivity. They may rationalize the use of pro-cyclical policy to moderate the response of firms to their private information. The policy implications have to be understood as illustrations of the results in the context of the very stylized model presented.

The results extend to an economy which is not efficient with full information. Then the potential bias towards putting too much weight on private information is increased. It follows that received results on the optimal relative weights to be placed on private and public information (when the latter is exogenous) may be overturned when the informational role of the price conflicts with its allocational role and the former is important enough.

Appendix

Microfoundation of the model

We provide here a foundation for the demand and supply model of Section 2. Consider a continuum of households integrated by a consumer and a producer of a homogeneous good indexed within the interval $[0,1]$ (endowed with the Lebesgue measure). Household i has a quasilinear utility function $v_i = v(q_i, u) + m_i$ where q_i is the consumption of the good and m_i of the numeraire and $v(q_i, u) = \left(\alpha + u - (\beta/2) q_i\right) q_i$ with $\alpha > 0$ and $\beta \in \mathbb{R}$. Marginal utility $\partial v / \partial q_i = \alpha + u - \beta q_i$ decreases with consumption if $\beta > 0$ and increases with it if $\beta < 0$. His budget constraint is $pq_i + m_i = \pi_i + M$ where p is the price of the good,

M the endowment of the numeraire, and π_i are the profits of the producer (firm) of the household.[31] The output of the firm i, x_i, is produced at cost $C(x_i,\theta)=\theta x_i+(\lambda/2)x_i^2$ yielding a payoff $\pi_i=(p-\theta)x_i-\frac{\lambda}{2}x_i^2$. The distributions of the random variables and information structure is as stated in Section 2.

The timing of events is as follows. At $t=1$, consumers and producers form demand and supply plans conditional on their information. A consumer maximizes utility subject to his budget knowing the realization of u. Given quasilinear utility, this is equivalent (assuming that the consumer is allowed to borrow if the budget is insufficient) to

$$\max_{q_i}\{v(q_i,u)-pq_i\},$$

yielding an inverse demand for the good $p=\alpha+u-\beta q_i$.[32] Each firm submits a supply schedule $X_i(I_i;\cdot)$ contingent on his information set $I_i=\{s_i,z\}$ with $x_i=X_i(I_i;p)$ where p is the price. At $t=2$ the market clears, the price is formed by finding a p that solves $p=\alpha+u-\beta\left(\int_0^1 X_j(I_j;p)\,dj\right)$. Note that all consumers will consume the same and therefore from market clearing we have that $q_i=\tilde{x}$ for all i and we have an inverse demand as in the usual partial equilibrium market

$$p=\alpha+u-\beta\tilde{x}.$$

Finally, consumption occurs and payoffs are collected. Household i will have ex post utility $v_i=v(\tilde{x},u)-p\tilde{x}+\pi_i+M=(\beta/2)\tilde{x}^2+\pi_i+M$ and therefore producer i will choose its supply function to maximize profits. Note that an individual producer cannot influence average output $\tilde{x}$. We use a utilitarian welfare criteria which in our quasilinear world coincides with total surplus (TS):

[31] This is the demand model considered in Vives (1988) and the formulation is borrowed from Angeletos and Pavan, Section 6.1 (2007).

[32] Note that the consumer need not know the realized profits of the producer.

$$\int_0^1 v_i \, di = v(\tilde{x},u) - \int_0^1 C(x_i,\theta) di + M =$$
$$\left(\alpha + u - \beta \frac{\tilde{x}}{2}\right)\tilde{x} - \int_0^1 \left(\theta x_i + \frac{\lambda}{2} x_i^2\right) di + M.$$

Demand schedule competition. Similarly, we can provide a foundation for the reinterpretation the model in terms of competition in demand schedules. Consider now a continuum of households integrated by a supplier and a purchaser/buyer of a homogeneous good. The cost of supplying ℓ_i is $\varsigma(\ell_i,u) = \left(\alpha + u + (\beta/2)\ell_i\right)\ell_i$. The marginal cost of supply $\partial\varsigma/\partial\ell_i = \alpha + u + \beta\ell_i$ is increasing (decreasing) in ℓ_i when $\beta > 0$ ($\beta < 0$). The case $\beta < 0$ may correspond, for example, to a situation where there is learning by doing in the supply. Household i has a quasilinear utility function $v_i = m_i - \varsigma(\ell_i,u)$ and his budget is $m_i - p\ell_i = \pi_i + M$. Let y_i be the quantity demanded by buyer i . The buyer's net benefit is given by $\pi_i = (\theta - p) y_i - (\lambda/2) y_i^2$, where λy_i^2 is a transaction or opportunity cost. The timing of events runs parallel to the one in the basic model.

Equilibrium and welfare characterization results: proofs

Claim 1: The strategy at the team solution with exogenous public information is of the form $X(s_i, z; p) = \lambda^{-1}\left(p - \left(\gamma s_i + (1-\gamma)\hat{z}\right)\right)$ where $\hat{z} \equiv \mathbb{E}[\theta \mid z]$.

Proof: The team solution with exogenous public information solves program ($\mathbf{T}_{exo}$). It can be checked that $\partial^2\mathbb{E}[\mathrm{TS}]/\partial^2\hat{b} < 0$ whenever $\beta + \lambda > 0$, $\partial^2\mathbb{E}[\mathrm{TS}]/\partial^2 c < 0$, and $\partial^2\mathbb{E}[\mathrm{TS}]/\partial^2\hat{c} < 0$ whenever $\mathrm{var}[p] > 0$. Given that $\partial x_i/\partial\hat{b} = 1$, $\partial x_i/\partial c = -z$, and $\partial x_i/\partial\hat{c} = p$ we can optimize $\mathbb{E}[\mathrm{TS}]$ with respect to b , c , $\hat{c}$ to obtain

$$\begin{aligned}
\frac{\partial\mathbb{E}[\mathrm{TS}]}{\partial\hat{b}} &= \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)\right] = 0, \\
\frac{\partial\mathbb{E}[\mathrm{TS}]}{\partial c} &= \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)z\right] = 0, \\
\frac{\partial\mathbb{E}[\mathrm{TS}]}{\partial\hat{c}} &= \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)p\right] = 0,
\end{aligned}$$

where $\mathrm{MC}(x_i) = \theta + \lambda x_i$. The constraint $\mathbb{E}\left[p - \mathrm{MC}(x_i)\right] = 0$ can be seen equivalent to $(1 - \lambda \hat{c})\mathbb{E}[p] = \lambda \hat{b}$ (using the fact that $\mathbb{E}[\theta] = \mathbb{E}[s_i] = \mathbb{E}[z] = 0$), and $\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)p\right] = 0$ equivalent to $(1 - \lambda \hat{c})\mathbb{E}[p^2] = \lambda \hat{b}\mathbb{E}[p]$ (using the assumption that $\mathbb{E}[\theta p] = 0$ and therefore $\mathbb{E}[zp] = 0$). The equations $(1 - \lambda \hat{c})\mathbb{E}[p] = \lambda \hat{b}$ and $(1 - \lambda \hat{c})\mathbb{E}[p^2] = \lambda \hat{b}\mathbb{E}[p]$ can hold with $\mathbb{E}[p] > 0$ and $\mathrm{var}[p] > 0$ if and only if $1 - \lambda \hat{c} = \lambda \hat{b} = 0$. (In equilibrium we necessarily have $\mathbb{E}[p] > 0$, provided $\alpha > 0$, and $\mathrm{var}[p] > 0$ provided $\sigma_\theta^2 > 0$.) Therefore we conclude that $\hat{c} = \lambda^{-1}$ and $\hat{b} = 0$. Furthermore, $\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)z\right] = 0$ can be seen equivalent to $c = \dfrac{(1 - a\lambda)k\tau_w}{\lambda\left(\tau_\theta + \tau_w k^2\right)}$ (using the fact that $\mathbb{E}[zp] = 0$). Note that $cz = \lambda^{-1}(1 - \lambda a)\hat{z}$ since $\hat{z} \equiv \mathbb{E}[\theta \mid z] = \dfrac{k\tau_w}{\tau_\theta + \tau_w k^2} z$. It follows therefore that we can write the team strategy as $x_i = -as_i - \lambda^{-1}(1 - \lambda a)\hat{z} + \lambda^{-1}p = \lambda^{-1}\left(p - (\gamma s_i + (1 - \gamma)\hat{z})\right)$ where $\gamma = \lambda a$. ♦

<u>Proof of Proposition 2</u>: From the posited strategy $X(s_i, z) = b - as_i + cz$, where $z = u + \beta a\theta$ and $1 - \beta c \neq 0$, we obtain that $p = \alpha - \beta b + (1 - \beta c)z$. From the first-order condition for player i we have

$$X(s_i, z) = \lambda^{-1}\left(\alpha - \beta b + (1 - \beta c)z - \mathbb{E}\left[\theta \middle| s_i, z\right]\right).$$

Here $\mathbb{E}[\theta \mid s_i, z] = \gamma s_i + (1 - \gamma)\mathbb{E}[\theta \mid z]$ with $\gamma = \tau_\varepsilon(\tau_\varepsilon + \tau)^{-1}$, $\mathbb{E}[\theta \mid z] = \beta \tau_u a \tau^{-1} z$ (recall that we have normalized $\bar{\theta} = 0$), and $\tau = \tau_\theta + \beta^2 a^2 \tau_u$ from the projection theorem for Gaussian random variables. Note that $\mathbb{E}[\theta \mid s_i, z] = \gamma s_i + hz$ where $h = \beta a \tau_u (\tau_\varepsilon + \tau)^{-1}$. Identifying coefficients with $X(s_i, z) = b - as_i + cz$, we can immediately obtain

$$a = \frac{\gamma}{\lambda} = \frac{\tau_\varepsilon}{\lambda(\tau_\varepsilon + \tau)}, \quad c = \frac{1 - h}{\beta + \lambda} = \frac{1}{\beta + \lambda} - \frac{\beta a \tau_u}{(\beta + \lambda)(\tau_\varepsilon + \tau)}, \quad \text{and} \quad b = \frac{\alpha}{\beta + \lambda}.$$

It follows that the equilibrium parameter a is determined as the unique (real), of the following cubic equations, that is positive and lies in the interval $a \in \left(0, \tau_\varepsilon \lambda^{-1}\left(\tau_\theta + \tau_\varepsilon\right)^{-1}\right)$:

$$a = \frac{\tau_\varepsilon}{\lambda\left(\tau_\varepsilon + \tau_\theta + \beta^2 a^2 \tau_u\right)} \quad \text{or} \quad \beta^2 \tau_u a^3 + \left(\tau_\varepsilon + \tau_\theta\right) a - \lambda^{-1} \tau_\varepsilon = 0$$

and

$$c = \frac{1}{(\beta + \lambda)} - \frac{\beta \lambda \tau_u a^2}{(\beta + \lambda)\tau_\varepsilon}.$$

It is immediate from the preceding equality for c that $c < (\beta + \lambda)^{-1}$ (since $a \geq 0$) and that $1 - \beta c > 0$ (since $\beta + \lambda > 0$); therefore,

$$\beta c = \frac{\beta}{\beta + \lambda} - \frac{\beta^2 a \tau_u}{(\beta + \lambda)(\tau_\varepsilon + \tau)} < 1.$$

It follows that

$$X(s_i, p) = \hat{b} - a s_i + \hat{c} p,$$

where $\hat{b} = b(1 - \lambda \hat{c})$, $b = \alpha/(\beta + \lambda)$, and $\hat{c} = c/(1 - \beta c) = \left(c^{-1} - \beta\right)^{-1}$ with $1 + \beta \hat{c} > 0$.♦

<u>Claim 2</u>: The strategy at the team solution is of the form $X(s_i, z; p) = \lambda^{-1}\left(p - \left(\gamma s_i + (1 - \gamma)\hat{z}\right)\right)$ where $\hat{z} \equiv \mathbb{E}[\theta \mid z]$.

<u>Proof</u>: The team solution solves program (**T**). It can be checked that $\partial^2 \mathbb{E}[\mathrm{TS}]/\partial^2 b < 0$ and $\partial^2 \mathbb{E}[\mathrm{TS}]/\partial^2 c < 0$ whenever $\beta + \lambda > 0$. Given that $\partial x_i/\partial b = 1$, and $\partial x_i / \partial c = z$, we can optimize with respect to b and c to obtain

$$\frac{\partial \mathbb{E}[\mathrm{TS}]}{\partial b} = \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)\right] = 0,$$
$$\frac{\partial \mathbb{E}[\mathrm{TS}]}{\partial c} = \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right) z\right] = 0,$$

where $p = \alpha + u - \beta \tilde{x}$ and $\mathrm{MC}(x_i) = \theta + \lambda x_i$. The constraint $\mathbb{E}\left[p - \mathrm{MC}(x_i)\right] = 0$ can be seen equivalent to $b = \alpha/(\beta + \lambda)$, and $\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right) z\right] = 0$ to $c = c(a) \equiv \frac{1}{\beta + \lambda} - \frac{\beta a \tau_u (1 - \lambda a)}{\tau(\beta + \lambda)}$. Those constraints are also fulfilled by the market solution since the first-order condition (FOC) for player i is $\mathbb{E}\left[p-\mathrm{MC}\left(x_i\right)\middle|s_i,z\right]=0$, from which it follows, according to the properties of Gaussian distributions, that $\mathbb{E}\left[p-\mathrm{MC}\left(x_i\right)\right]=0$, and $\mathbb{E}\left[\left(p-\mathrm{MC}\left(x_i\right)\right)z\right]=0$ (as well as $\mathbb{E}\left[\left(p-\mathrm{MC}\left(x_i\right)\right)s_i\right]=0$, which is equivalent to $c=\frac{a(\lambda(\tau_\varepsilon+\tau_\theta)+\beta\tau_\varepsilon)-\tau_\varepsilon}{a\beta\tau_\varepsilon(\lambda+\beta)}$). Using the expressions for $b = \alpha/\left(\beta+\lambda\right)$ and $c=\frac{1}{\beta+\lambda}-\frac{\beta a\tau_u\left(1-\lambda a\right)}{\tau\left(\beta+\lambda\right)}$, using the fact that $\mathbb{E}\left[\theta\mid z\right]=\beta\tau_u a\tau^{-1}z$ and $\tilde{x}=b-a\theta+cz$, we find that $x_i=\lambda^{-1}\left(p-\left(\gamma s_i+(1-\gamma)\mathbb{E}\left[\theta\mid z\right]\right)\right)$ where $\gamma=\lambda a$. ♦

<u>Claim 3</u>: *When* $\beta>0$, *at the market solution* $\operatorname{sgn}\left\{\beta\tau-\Delta\right\}=\operatorname{sgn}\left\{c^*\right\}$.

<u>Proof</u>: When at the market solution we have that $c^*=0$ then $\beta\tau=\Delta$. This is so since we can check that $\Delta=\frac{\tau\tau_\varepsilon(1-(\beta+\lambda)c)^2}{\lambda\tau_u a^2}$ and therefore $\beta\tau=\Delta$ is equivalent to $\beta=\frac{\tau_\varepsilon}{\lambda\tau_u a^2}$ when $c=0$. The result follows since at the market equilibrium $c=\left(\beta+\lambda\right)^{-1}\left(1-\beta\lambda\tau_u a^2\tau_\varepsilon^{-1}\right)$ (from Proposition 1) and therefore $1=\beta\lambda\tau_u a^2\tau_\varepsilon^{-1}$ when $c^*=0$. At the market solution, when $c^*>0$ we have that $\beta\tau-\Delta>0$ and when $c^*<0$ we have that $\beta\tau-\Delta<0$. This is immediate since at the market solution $\frac{\tau_\varepsilon(1-(\beta+\lambda)c)^2}{\lambda\tau_u a^2}=\beta\left(1-(\beta+\lambda)c\right)$. ♦

<u>Proof of Proposition 4</u>. The welfare loss at the team-efficient solution is given by $\mathrm{WL}\left(a^{\mathrm{T}}\right)$, which is decreasing in $\tau_\varepsilon,\tau_\omega,\tau_u$ and τ_θ since WL is decreasing in those parameters for a given a and $d\mathrm{WL}\left(a^{\mathrm{T}}\right)/da=0$. With respect to the market solution we have that

$$\frac{d\mathrm{WL}}{d\tau_\theta}\left(a^*\right)=\frac{\partial\mathrm{WL}}{\partial a}\frac{\partial a^*}{\partial\tau_\theta}+\frac{\partial\mathrm{WL}}{\partial\tau_\theta},$$

where $\frac{\partial a^*}{\partial\tau_\theta}=-\frac{a}{\tau_\theta+\tau_\varepsilon+3a^2\beta^2\tau_u}$ and a^* solves $\beta^2\tau_u a^3+\left(\tau_\varepsilon+\tau_\theta\right)a-\lambda^{-1}\tau_\varepsilon=0$.

Given that

$$\mathrm{WL} = \frac{1}{2}\left(\frac{(1-\lambda a)^2}{(\tau_\theta + \tau_u \beta^2 a^2)(\beta+\lambda)} + \frac{\lambda a^2}{\tau_\varepsilon} \right),$$

it is possible to show that

$$\frac{d\mathrm{WL}}{d\tau_\theta}(a^*) < 0 \text{ if and only if } \frac{\tau_\theta + \tau_u \beta^2 a^2}{\tau_\varepsilon} > -\frac{2\beta+\lambda}{\lambda},$$

which is always true since $2\beta+\lambda>0$. Exactly the same condition holds for $d\mathrm{WL}(a^*)/d\tau_u < 0$. Furthermore, we can show that $d\mathrm{WL}(a^*)/d\tau_\varepsilon \le 0$ if and only if $\beta - \lambda \le \frac{\tau_u \beta^2 a^*}{\tau_\theta}\left(a^* (\beta+\lambda) + 2 \right) + (\beta+\lambda)\frac{\tau_\varepsilon}{\tau_\theta}$. It follows that WL will be increasing in τ_ε for $\beta > \lambda$ and $\tau_\varepsilon/\tau_\theta$ small enough (since a^* is increasing in $\tau_\varepsilon/\tau_\theta$). ♦

Proof of Proposition 5: It proceeds in a parallel way to the proof of Proposition 3. Again, it can be checked first that $\partial^2 \mathbb{E}[\pi_i]/\partial^2 b < 0$ and $\partial^2 \mathbb{E}[\pi_i]/\partial^2 c < 0$ whenever $2\beta+\lambda>0$. Given that $\pi_i = p x_i - \mathrm{C}(x_i)$, $p = \alpha + u - \beta\tilde{x}$, $\partial x_i/\partial b = 1$, and $\partial x_i/\partial c = \partial\tilde{x}/\partial c = z$ and $\partial p/\partial\tilde{x} = -\beta$ we can optimize with respect to b and c to obtain

$$\frac{\partial \mathbb{E}[\pi_i]}{\partial b} = \mathbb{E}\left[(p - \mathrm{MC}(x_i)) - \beta x_i \right] = 0,$$
$$\frac{\partial \mathbb{E}[\pi_i]}{\partial c} = \mathbb{E}\left[(p - \mathrm{MC}(x_i)) z - \beta x_i z \right] = 0.$$

where $\mathrm{MC}(x_i) = \theta + \lambda x_i$. The constraint $\mathbb{E}\left[(p - \mathrm{MC}(x_i)) - \beta x_i \right] = 0$ is equivalent to $b = \alpha/(2\beta+\lambda)$; we can also check that $\mathbb{E}\left[(p - \mathrm{MC}(x_i)) z - \beta x_i z \right] = 0$ is equivalent to $c = c^{\mathrm{IT}}(a)$, where

$$c^{\mathrm{IT}}(a) = \frac{1}{2\beta+\lambda} - \frac{\beta a \tau_u (1-(\lambda+\beta)a)}{\tau(2\beta+\lambda)} \quad \text{and} \quad \tau = \tau_\theta + \beta^2 \tau_u a^2.$$

Note that due to the competition payoff externality ($\partial p/\partial\tilde{x} = -\beta$) the expressions for b and for c are different than in the market solution. It follows that the form of the internal team optimal strategy is $x_i = (\lambda+\beta)^{-1}\left(p - (\gamma s_i + (1-\gamma)\mathbb{E}[\theta \mid z]) \right)$ where $\gamma = (\lambda+\beta)a$. We have that $\tilde{x} = (\lambda+\beta)^{-1}\left(p - (\gamma\theta + (1-\gamma)\mathbb{E}[\theta \mid z]) \right)$ and that

$\tilde{x}-x^{\mathrm{M}}=(1-\gamma)(\theta-\mathbb{E}[\theta \mid z])/(2\beta+\lambda)$ and, since $\tau=(\operatorname{var}[\theta|z])^{-1}$ we obtain $\mathbb{E}\left[\left(\tilde{x}-x^{\mathrm{M}}\right)^2\right]=(1-(\lambda+\beta)a)^2/\left(\tau(2\beta+\lambda)^2\right)$. We have that $\mathbb{E}\left[(x_i-\tilde{x})^2\right]=a^2/\tau_\varepsilon$.

Let $\Omega=\mathbb{E}[\pi_i^{\mathrm{M}}]-\mathbb{E}[\pi_i]$. Similarly as before we can obtain that $\Omega=\left((2\beta+\lambda)\mathbb{E}\left[\left(\tilde{x}-x^{\mathrm{M}}\right)^2\right]+\lambda\mathbb{E}\left[(x_i-\tilde{x})^2\right]\right)/2$. It follows that

$$\Omega(a)=\frac{1}{2}\left(\frac{(1-(\lambda+\beta)a)^2}{\left(\tau_\theta+\tau_u\beta^2a^2\right)(2\beta+\lambda)}+\frac{\lambda a^2}{\tau_\varepsilon}\right),$$

which is easily seen strictly convex in a and with a unique solution $(\lambda+\beta)^{-1}>a^{\mathrm{IT}}>0$. (Note that $(\lambda+\beta)^{-1}<a$ is dominated by $a=(\lambda+\beta)^{-1}$ and that $a<0$ is dominated by $-a>0$. Furthermore, it is immediate that $\Omega'(0)<0$ and therefore $a>0$ at the solution.)

The impact of a on $\mathbb{E}[\pi_i]$ is easily characterized (noting that $\partial\mathbb{E}[\pi_i]/\partial c=0$ and therefore disregarding the indirect impact of a on $\mathbb{E}[\pi_i]$ via a change in c):

$$\begin{aligned}\frac{\partial\mathbb{E}[\pi_i]}{\partial a}&=\underbrace{\mathbb{E}\left[(p-MC(x_i))\left(\frac{\partial x_i}{\partial a}\right)_{z\ ct.}\right]}_{\text{Market}}+\\&\underbrace{\mathbb{E}\left[(p-MC(x_i))\left(\frac{\partial x_i}{\partial z}\frac{\partial z}{\partial a}\right)\right]}_{\text{PE+LE}}+\\&\underbrace{\mathbb{E}\left[x_i\left(\frac{\partial p}{\partial\tilde{x}}\frac{\partial\tilde{x}}{\partial a}\right)\right]}_{\text{CE}}\\&=\mathbb{E}\left[(p-\mathrm{MC}(x_i))(-s_i+c\beta\theta)-\beta(c\beta-1)\theta x_i\right]\end{aligned}$$

given that $(\partial x_i/\partial a)_{z\text{ ct.}}=-s_i$, $\partial x_i/\partial z=c$, $\partial z/\partial a=\beta\theta$, $\partial p/\partial\tilde{x}=-\beta$ and $\partial\tilde{x}/\partial a=(c\beta-1)\theta$. Evaluating $\partial\mathbb{E}[\pi_i]/\partial a$ at the market solution, where $\mathbb{E}\left[(p-\mathrm{MC}(x_i))s_i\right]=0$, we obtain

$$\frac{\partial\mathbb{E}[\pi_i]}{\partial a}=\beta\mathbb{E}\left[c(p-\mathrm{MC}(x_i))\theta-(c\beta-1)\theta x_i\right].$$

We know that $\mathbb{E}\left[\left(p-\mathrm{MC}\left(x_i\right)\right)\theta\right]=-\lambda a\sigma_\varepsilon^2<0$ and, recalling that $\bar{\theta}=0$, it is easily checked that $\mathbb{E}\left[\theta x_i\right]=a\sigma_\theta^2\left(c\beta-1\right)$. At the equilibrium we have therefore[33]

$$\frac{\partial\mathbb{E}\left[\pi_i\right]}{\partial a}=-\beta a^*\left(c^*\lambda\sigma_\varepsilon^2+\left(c^*\beta-1\right)^2\sigma_\theta^2\right).$$

Since $\mathbb{E}\left[\pi_i\right]$ is single-peaked for $a>0$ and has a unique maximum at $a^{\mathrm{IT}}>0$ and $a^*>0$, it follows that $\operatorname{sgn}\left\{a^{\mathrm{IT}}-a^*\right\}=\operatorname{sgn}\left\{\left.\frac{\partial\mathbb{E}\left[\pi_i\right]}{\partial a}\right|_{a=a^*}\right\}=$

$\operatorname{sgn}\left\{-\beta\left(c^*\lambda\sigma_\varepsilon^2+\left(c^*\beta-1\right)^2\sigma_\theta^2\right)\right\}$. ♦

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[33] Note also that at the equilibrium $c\beta-1<0$.

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Online Appendix to

Endogenous Public Information and Welfare in Market Games

1. Suplement to the equilibrium characterization of Section 5.1

Claim A. *Linear equilibria in strategies with bounded means and with uniformly (across players) bounded variances yield linear equilibria of the schedule game for which the public statistic function is of type* $P(\theta,\ u)$.

Proof: If for player i we posit the strategy

$$x_i = \hat{b}_i + \hat{c}_i p - a_i s_i$$

then the aggregate action is given by

$$\tilde{x} = \int_0^1 x_i \, di = \hat{b} + \hat{c}p - a\theta - \int_0^1 a_i \, \varepsilon_i \, di = \hat{b} + \hat{c}p - a\theta \, ,$$

where $\hat{b} = \int_0^1 \hat{b}_i \, di$, $\hat{c} = \int_0^1 \hat{c}_i \, di$, and $a = \int_0^1 a_i \, di$ (assuming that all terms are well-defined). Observe that, according to our convention on the average error terms of the signals, $\int_0^1 a_i \varepsilon_i \, di = 0$ a.s. provided that $\mathrm{var}\left[a_i \, \varepsilon_i \right]$ is uniformly bounded across agents (since $\mathrm{var}\left[\varepsilon_i \right] = \sigma_\varepsilon^2$, it is enough that a_i be uniformly bounded). In equilibrium, this will be the case. Therefore, if we restrict attention to candidate linear equilibria with parameters a_i uniformly bounded in i and with well-defined average parameters $\hat{b}$ and $\hat{c}$, then $\tilde{x} = \hat{b} + \hat{c}p - a\theta$ and the public statistic function is of the type $P(\theta,\ u)$. ♦

2. Comparative statics of the equilibrium

This section studies the comparative statics properties of the equilibrium and how the weights and the responses to public and private information vary with underlying parameters. The following proposition presents a first set of results. The effects of changes in the degree of complementarity are dealt with afterwards.

Proposition A1. *Let* $\tau_\varepsilon > 0$ *and* $\tau_u > 0$. *In equilibrium, the following statements hold.*

(i) *Responsiveness to private information* a *decreases from* $\lambda^{-1}\tau_\varepsilon(\tau_\theta+\tau_\varepsilon)^{-1}$ *to* 0 *as* τ_u *ranges from* 0 *to* ∞, *decreases with* τ_θ, $|\beta|$ *and* λ, *and increases with* τ_ε.

(ii) *Responsiveness to the public statistic* $\hat{c}$ *goes from* λ^{-1} *to* $-\beta^{-1}$ *as* τ_u *ranges from* 0 *to* ∞. Furthermore, $\operatorname{sgn}\{\partial\hat{c}/\partial\tau_u\}=\operatorname{sgn}\{-\partial\hat{c}/\partial\tau_\theta\}=\operatorname{sgn}\{-\beta\}$ *and* $\operatorname{sgn}\{\partial\hat{c}/\partial\tau_\varepsilon\}=\operatorname{sgn}\{\beta(\beta^2\tau_u\tau_\varepsilon^2+4\lambda^2\tau_\theta^2(\tau_\varepsilon-\tau_\theta))\}$. *Market depth* $1+\beta\hat{c}$ *is decreasing in* τ_u *and increasing in* τ_θ.

(iii) *Price informativeness* τ *is increasing in* $|\beta|$, τ_u , τ_θ *and* τ_ε, *and decreasing in* λ.

(iv) *Dispersion* $\mathbb{E}\left[(x_i-\tilde{x})^2\right]$ *decreases with* τ_u , τ_θ , $|\beta|$ *and* λ.

Proof: (i) From the equation determining the responsiveness to private information a, $\beta^2\tau_u a^3+(\tau_\varepsilon+\tau_\theta)a-\lambda^{-1}\tau_\varepsilon=0$, it is immediate that a decreases with τ_u, τ_θ, β^2 and λ, that a increases with τ_ε. Note that $\operatorname{sgn}\{\partial a/\partial\beta\}=\operatorname{sgn}\{-\beta\}$. As τ_u ranges from 0 to ∞, a decreases from $\lambda^{-1}\tau_\varepsilon(\tau_\theta+\tau_\varepsilon)^{-1}$ to 0.

(ii) As τ_u ranges from 0 to ∞, the responsiveness to public information c goes from $(\beta+\lambda)^{-1}$ to $-\infty$ (resp. $+\infty$) if $\beta>0$ (resp. $\beta<0$). The result follows since, in equilibrium,

$$c=\frac{1}{\beta+\lambda}-\frac{\beta\lambda\tau_u a^2}{(\beta+\lambda)\tau_\varepsilon}=\frac{1}{\beta+\lambda}-\frac{1}{(\beta+\lambda)\beta}\left(\frac{1}{a}-\lambda\left(1+\frac{\tau_\theta}{\tau_\varepsilon}\right)\right)$$

and $a\to 0$ as $\tau_u\to\infty$. It follows that $\operatorname{sgn}\{\partial c/\partial\tau_u\}=\operatorname{sgn}\{-\beta\}$ because $\partial a/\partial\tau_u<0$. Similarly, from the first part of the expression for c we have $\operatorname{sgn}\{\partial c/\partial\tau_\theta\}=\operatorname{sgn}\{\beta\}$ since $\partial a/\partial\tau_\theta<0$. Furthermore, with some work it is possible to show that, in equilibrium,

$$\frac{\partial c}{\partial\tau_\varepsilon}=(\beta+\lambda)^{-1}\lambda\beta\tau_u\tau_\varepsilon^{-1}a\left(2\frac{a\lambda-1}{\lambda(\tau_\theta+\tau_\varepsilon+3a^2\beta^2\tau_u)}+a\tau_\varepsilon^{-1}\right)\quad\text{and}$$

$$\operatorname{sgn}\left\{2\frac{a\lambda-1}{\lambda\left(\tau_\theta+\tau_\varepsilon+3a^2\beta^2\tau_u\right)}+a\tau_\varepsilon^{-1}\right\}=\operatorname{sgn}\left\{a\lambda\tau_\theta-2\tau_\varepsilon+3a\lambda\tau_\varepsilon+3a^3\beta^2\lambda\tau_u\right\}$$
$$=\operatorname{sgn}\left\{-2a\lambda\tau_\theta+\tau_\varepsilon\right\}$$
$$=\operatorname{sgn}\left\{\beta^2\tau_u\tau_\varepsilon^2+4\lambda^2\tau_\theta^2\left(\tau_\varepsilon-\tau_\theta\right)\right\}.$$

Hence we conclude that $\operatorname{sgn}\left\{\partial c/\partial\tau_\varepsilon\right\}=\operatorname{sgn}\left\{\beta\left(\beta^2\tau_u\tau_\varepsilon^2+4\lambda^2\tau_\theta^2\left(\tau_\varepsilon-\tau_\theta\right)\right)\right\}$. Since $\hat{c}=\left(c^{-1}-\beta\right)^{-1}$, it follows that $\hat{c}$ goes from λ^{-1} to $-\beta^{-1}$ as τ_u ranges from 0 to ∞,[1] $\operatorname{sgn}\left\{\partial\hat{c}/\partial\tau_u\right\}=\operatorname{sgn}\left\{-\partial\hat{c}/\partial\tau_\theta\right\}=\operatorname{sgn}\left\{-\beta\right\}$, and $\operatorname{sgn}\left\{\partial\hat{c}/\partial\tau_\varepsilon\right\}=\operatorname{sgn}\left\{\partial c/\partial\tau_\varepsilon\right\}$. It is then immediate that $1+\beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ.

(iii) Price informativeness $\tau=\tau_\theta+\beta^2a^2\tau_u$ is increasing in τ_ε (since a increases with τ_ε) and also in τ_u (since $a=\lambda^{-1}\tau_\varepsilon\left(\tau_\varepsilon+\tau\right)^{-1}$ and a decreases with τ_u). Using the expression for $\partial a/\partial\tau_\theta$ we have that

$$\frac{\partial\tau}{\partial\tau_\theta}=1+2\beta^2\tau_u a\frac{\partial a}{\partial\tau_\theta}=1-\frac{2\beta^2a^2\tau_u}{\tau_\theta+\tau_\varepsilon+3a^2\beta^2\tau_u}=\frac{\tau_\theta+\tau_\varepsilon+a^2\beta^2\tau_u}{\tau_\theta+\tau_\varepsilon+3a^2\beta^2\tau_u}>0.$$

Furthermore,

$$\frac{\partial\tau}{\partial\beta}=\tau_u\left(2\beta a^2+2\beta^2a\frac{\partial a}{\partial\beta}\right)=2\beta a\tau_u\left(a-\frac{2a^4\tau_u\lambda\beta^2}{1+2a^3\tau_u\lambda\beta^2}\right)=\frac{2\beta a^2\tau_u}{1+2a^3\tau_u\lambda\beta^2},$$

and therefore $\operatorname{sgn}\left\{\partial\tau/\partial\beta\right\}=\operatorname{sgn}\left\{\beta\right\}$.

(iv) From $x_i=\lambda^{-1}\left[p-\mathbb{E}\left[\theta|s_i,z\right]\right]$ and $\mathbb{E}\left[\theta\,|\,s_i,z\right]=\gamma s_i+\left(1-\gamma\right)\mathbb{E}\left[\theta\,|\,z\right]$ we obtain $x_i-\tilde{x}=\lambda^{-1}\gamma\left(s_i-\theta\right)=\lambda^{-1}\gamma\varepsilon_i$ and, noting that $\gamma=\lambda a$ we conclude that $\mathbb{E}\left[\left(x_i-\tilde{x}\right)^2\right]=a^2\sigma_\varepsilon^2$. The results then follow from the comparative statics results for a in (i). ♦

How the equilibrium weights to private and public information vary with the deep parameters of the model help to explain the results. We have that $\mathbb{E}\left[\theta\,|\,s_i,z\right]=\gamma s_i+hz$ where $h=\beta a\tau_u\left(\tau_\varepsilon+\tau\right)^{-1}$. Identify the informational component

[1] Note that if $\beta<0$ and $\beta+\lambda>0$ then $\lambda^{-1}<-\beta^{-1}$.

of the price with the weight $|h|$ on public information z, with $\text{sgn}\{h\} = \text{sgn}\{\beta\}$. When $\beta > 0$ there is adverse selection (a high price is bad news about costs) and $h > 0$ while when $\beta < 0$, $h < 0$ and there is favorable selection (a high price is good news). We have that $\text{sgn}\{\partial|h|/\partial\beta\} = \text{sgn}\{\beta\}$. As β is decreased from $\beta > 0$ adverse selection is lessened, and when $\beta < 0$ we have favorable selection with $h < 0$ and $\partial|h|/\partial\beta < 0$. The result is that an increase in $|\beta|$ increases the public precision[2] τ and decreases the response to private information. We have also that increasing the precision of the prior decreases the informational component of the price, $\partial|h|/\partial\tau_\theta < 0$, while that increasing the precision of the noise in the price increases it, $\partial|h|/\partial\tau_u > 0$ (see Claim B). The effect of τ_ε is ambiguous.

Claim B. $\mathbb{E}\left[\theta \mid s_i, z\right] = \gamma s_i + hz$ *with* $h = \lambda\beta\tau_\varepsilon^{-1}\tau_u a^2$, $\partial|h|/\partial\tau_\theta < 0$, $\partial|h|/\partial\tau_u > 0$ *and* $\text{sgn}\{\partial|h|/\partial\beta\} = \text{sgn}\{\beta\}$.

Proof: From $h = \beta a\tau_u\left(\tau_\varepsilon + \tau\right)^{-1}$ in the proof of Proposition 1 it is immediate that $h = \lambda\beta\tau_\varepsilon^{-1}\tau_u a^2$. We have that $\partial|h|/\partial\tau_\theta < 0$ since $\partial a/\partial\tau_\theta < 0$; $\partial|h|/\partial\tau_u > 0$ since $\partial\tau/\partial\tau_u > 0$ and therefore $\partial\left(\tau_u a^2\right)/\partial\tau_u > 0$. Finally, we have that in equilibrium

$$\frac{\partial c}{\partial\beta} = -\frac{1}{(\lambda+\beta)\tau_\varepsilon}\left(\frac{\tau_\varepsilon + \lambda^2\tau_u a^2}{\lambda+\beta} + \frac{4a^5\lambda^3\tau_u^2\beta^2}{1+2a^3\tau_u\lambda\beta^2}\right) < 0,$$

and from $c = (1-h)(\beta+\lambda)^{-1}$ we can obtain $\partial h/\partial\beta > 0$, and therefore, $\text{sgn}\{\partial|h|/\partial\beta\} = \text{sgn}\{\beta\}$. ♦

To interpret the results consider first the case $\beta > 0$. As τ_u increases from 0, $\hat{c}$ decreases from λ^{-1} (and the slope of supply increases) because of the price's increased informational component $h > 0$. Agents are more cautious when seeing a high price because it may mean higher costs. As τ_u increases more, $\hat{c}$ becomes zero

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2 An increase in $|\beta|$ has a direct positive effect on τ and an indirect negative effect via the induced change in a. The direct effect prevails. Note that changing β modifies not only the public statistic p but also the degree of complementarity in the payoff.

at some point and then turns negative; as τ_u tends to ∞, $\hat{c}$ tends to $-\beta^{-1}$.At the point where the scarcity and informational effects balance, agents place zero weight ($\hat{c}=0$) on the price. If τ_θ increases then the informational component of the price diminishes since the agents are now endowed with better prior information, and induces a higher $\hat{c}$ (and a more elastic supply). An increase in the precision of private information τ_ε always increases responsiveness to the private signal but has an ambiguous effect on the slope of supply. The parameter $\hat{c}$ is U-shaped with respect to τ_ε. Observe that $\hat{c}=\lambda^{-1}$ not only when $\tau_\varepsilon=\infty$ but also when $\tau_\varepsilon=0$ and that $\hat{c}<\lambda^{-1}$ for $\tau_\varepsilon\in(0,\infty)$. If τ_ε is high, then a further increase in τ_ε (less noise in the signals) lowers adverse selection (and h) and increases $\hat{c}$. If τ_ε is low then the price is relatively uninformative, and an increase in τ_ε increases adverse selection (and h) while lowering $\hat{c}$.

If $\beta<0$ then a high price conveys goods news in terms of both scarcity effects and informational effects, so supply is always upward sloping in this case. Indeed, when $\beta<0$ we have $\hat{c}>\lambda^{-1}$. A high price conveys the good news that average quantity tends to be high and that costs therefore tend to be low ($h<0$). In this case, increasing τ_u, which reinforces the informational component of the price, increases $\hat{c}$—the opposite of what happens when τ_θ increases. An increase in the precision of private information τ_ε increases responsiveness to the private signal but, as before, has an ambiguous effect on the slope of supply. Now the parameter $\hat{c}$ is hump-shaped with respect to τ_ε because $\hat{c}>\lambda^{-1}$ for $\tau_\varepsilon\in(0,\infty)$ and $\hat{c}=\lambda^{-1}$ in the extremes of the interval $(0,\infty)$.

In either case ($\beta>0$ or $\beta<0$) market depth $(\partial P/\partial u)^{-1}=1+\beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ.

Table 1 summarizes the comparative statics results on the equilibrium strategy.

Table 1: Comparative Statics on the Equilibrium Strategy

sgn	$\partial\tau_u$	$\partial\tau_\theta$	$\partial\tau_\varepsilon$
∂a	$-$	$-$	$+$
$\partial\hat{c}$	$-\beta$	β	$\beta\left(\beta^2\tau_u\tau_\varepsilon^2+4\lambda^2\tau_\theta\left(\tau_\varepsilon-\tau_\theta\right)\right)$

3. Efficiency in the Cournot market with endogenous public signals

In this section we assume that firms compete in quantities. We have shown in Section 4.3 that with exogenous public information the market solution is efficient. Suppose now that public signal z comes from an endogenous noisy quantity signal, $q=\tilde{x}+\eta$ where $\eta \sim N\left(0,\tau_\eta^{-1}\right)$ is independent of the other random variables in the model. Then positing that firms use a linear strategy $x_i=b-as_i-\hat{c}q$ it is easily seen that $q=\left(1+\hat{c}\right)^{-1}\left(b-z\right)$ where $z=a\theta-\eta$. Letting $\hat{z}=\mathbb{E}[\theta\,|\,z]$, the strategy $X\left(s_i,z\right)$ has the same form as before but now $\tau=\tau_\theta+\tau_\eta a^2$ is endogenous.

We may conjecture that the endogenous quantity signal will lead firms to put too little weight on their private information due to the presence of an information externality . We confirm that this is indeed the case. It can be checked that candidate team strategies are of the same form as the market but with potentially a different response a to private information. We have that:

$$\frac{\partial\mathbb{E}[\mathrm{TS}]}{\partial a}=\mathbb{E}\left[\left(p-\mathrm{MC}(x_i)\right)\frac{\partial x_i}{\partial a}\bigg|_{\hat{z}\text{ ct.}}\right]+\mathbb{E}\left[\left(p-\mathrm{MC}(x_i)\right)\left(\frac{\partial x_i}{\partial\hat{z}}\frac{\partial\hat{z}}{\partial a}\right)\right].$$

At the (Cournot) market solution $\mathbb{E}\left[\left(p-\mathrm{MC}\left(x_i\right)\right)\frac{\partial x_i}{\partial a}\bigg|_{\hat{z}\text{ ct.}}\right]=0$ since firms take z as given and the learning externality term is positive, $\mathbb{E}\left[\left(p-\mathrm{MC}\left(x_i\right)\right)\left(\frac{\partial x_i}{\partial\hat{z}}\frac{\partial\hat{z}}{\partial a}\right)\right]>0$, and therefore $\partial\mathbb{E}[\mathrm{TS}]/\partial a>0$. This indicates that a has to be increased from the market level and, since $\mathbb{E}[\mathrm{TS}]$ is strictly concave in a, we conclude as expected that the information externality leads to a too small response to private information. We confirm in Lemma A1 that this is indeed the case.

Lemma A1 (*Cournot*): *Consider the Cournot model of Section 4.3 with the information set of firms augmented with a noisy quantity signal. Let* $\tau_\varepsilon > 0$*, then the market solution has a smaller response to private information than the team solution.*

Proof: It can be checked that candidate team strategies are of the same form as the market $X(s_i, z) = \alpha(\beta+\lambda)^{-1} - \left(as_i + \left((\beta+\lambda)^{-1} - a\right)\hat{z}\right)$ but with potentially a different response a to private information. We have that:

$$\frac{\partial \mathbb{E}[\mathrm{TS}]}{\partial a} = \mathbb{E}\left[(p - \mathrm{MC}(x_i))\left.\frac{\partial x_i}{\partial a}\right|_{\hat{z}}\right] + \mathbb{E}\left[(p - \mathrm{MC}(x_i))\left(\frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a}\right)\right]$$
$$= \mathbb{E}\left[(p - \mathrm{MC}(x_i))\left(-(s_i - \hat{z}) - \left((\beta+\lambda)^{-1} - a\right)\left(\tau_\eta a\tau^{-1}\theta + z\frac{\partial\left(\tau_\eta a\tau^{-1}\right)}{\partial a}\right)\right)\right]$$

since $\hat{z} = \mathbb{E}[\theta \mid z] = \tau_\eta a\tau^{-1}z$, $z = a\theta - \eta$. At the market solution $\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)\left.\frac{\partial x_i}{\partial a}\right|_{\hat{z}}\right] = \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)(s_i - \hat{z})\right] = 0$ since firms take z as given, $\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)\hat{z}\right] = \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)z\right] = 0$. We have that at the market solution $0 < a < (\beta+\lambda)^{-1}$ since $\tau_\varepsilon > 0$, and

$$\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)(s_i - \hat{z})\right] = \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)(\theta - \hat{z})\right] + \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)\varepsilon_i\right] = 0.$$

Therefore,

$$\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)(\theta - \hat{z})\right] = \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)\theta\right] = -\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)\varepsilon_i\right]$$
$$= \mathbb{E}\left[\mathrm{MC}(x_i)\varepsilon_i\right] = \mathbb{E}\left[(\theta + \lambda x_i)\varepsilon_i\right] = -\lambda a\sigma_\varepsilon^2 < 0$$

since ε_i is independent of θ. We conclude that $\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)\theta\right] = -\lambda a\sigma_\varepsilon^2$ and therefore $\partial\mathbb{E}[\mathrm{TS}]/\partial a = \lambda\tau_\varepsilon^{-1}\left((\beta+\lambda)^{-1} - a\right)\tau_\eta a^2\tau^{-1} > 0$. Furthermore, it can be checked that $\mathbb{E}[\mathrm{TS}]$ is strictly concave in a and we can conclude that the team solution calls for a larger response to private information than the market. ♦