Working out the impacts of labor market integration

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March 1, 2013

Abstract

We develop a competitive search model involving multiple regions and migration between them. Equilibrium migration patterns are analyzed and characterized, which show that a shock to a particular region, such as a productivity shock, can propagate to other regions through migration. It is also shown that equilibrium migration patterns are not efficient. We further demonstrate by calibrating our framework to Japanese regional data that it can be used to quantify the impact of regional labor market integration: integration can reduce national unemployment and dispersion of regional unemployment by more than 10 percent.

Very Preliminary Version. Please do not quote or circulate.

1 Introduction

This paper studies the possible impacts of regional labor market integration on the local and national labor markets and social welfare. As well observed in many countries, there exists considerable labor mobility within a nation, and such migration has been shown to be sensitive to local labor market conditions.[1] We then naturally expect that migration would eventually eliminate regional differences in labor market conditions such as wages and unemployment rates. However, contrary to this expectation, we have been observing persistent and large differences in labor market outcomes such as wages and unemployment rates: for instance, Lkhagvasuren

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[1]For earlier contributions on this issue, see Blanchard and Katz [4], Borjas et al [5], and Topel [23] among others. Recent contributions include Hatton and Tani [7], Kennan and Walker [9], and Rabe and Taylor [20].

[8] showed that the magnitude of cross-state unemployment differences is roughly the same size and the cyclical variation in the national unemployment rate.[2]

The sensitivity of migration to labor market conditions and the persistent regional differences in labor market outcomes imply that regional labor markets are only imperfectly integrated, which would be mainly attributed to the existence of migration costs in general. Such migration costs include those related to job turnover, which depend on institution and regulation regarding labor markets such as mutual recognition of professional degrees among different regions and occupational licenser requirements, those of moving, selling and finding houses, which depend on technology of transportation and communication, and those of adjusting to a new environment and re-constructing social networks. The aim of this paper is to qualify and quantify the effects of general migration costs on local and national labor markets.[3]

We develop a competitive search model involving multiple regions and migration costs. As modeled in Acemoglu and Shimer [1] [2] and Moen [13], firms post wages when opening their vacancies and job search is directed.[4] Search is off-the-job and only unemployed workers can move between regions. Although job searchers can search jobs (i.e., can access information on vacancies) both within and outside of their places of residence, they need to incur migration costs when landing a job to work in a region different from their places of residence.

Our analysis first uncovers the effects of migration costs on the migration patterns qualitatively. The intriguing result is that a change in migration costs exhibits spillover effects through migration responses, which can result in a counter intuitive result: a better access from a particular region to a region with a better economic condition (higher productivity) may hurt the source region. It increases job settlements from the region to the better region whereas it decreases job settlements to other regions, which may result in a higher unemployment rate in the region. Hence, an improvement in access between a particular pair of regions may widen the difference between the two regions.

Second, equilibrium of the model is shown to be inefficient: migration flow is inefficiently

[2] The same holds true for Japanese prefectures. Population census of Japan reports prefectural unemployment rates for every five years. The coefficients of variation of cross-prefecture unemployment for the years 1975, 1985, 1995, and 2005 are around 0.47, 0.35, 0.31, and 0.23, respectively. That of time series unemployment for these years is 0.39.

[3] In the international context, the degree of labor market integration also depends on the formation of political and economic unions such as the European Union. Although we base our arguments on the migration within a nation in this paper, our framework can be applied to such unions as well.

[4] See, among others, Rogerson et al [21] for recent developments in the literature of job search models that include a competitive search model.

low when the destination (resp. source) region has a relatively high (resp. low) asset value of an unemployed worker. A high asset value of an unemployed worker in the destination region implies that in-migration of job searchers to the region is socially beneficial. However, firms in the destination region ignore such benefits of migration in opening their vacancies, leading to inefficiently few job settlements and small migration. When the asset value of an unemployed worker in the source region is low, out-migration of job searchers from the region is socially beneficial. Again, firms in the destination region ignore such benefits when opening vacancies, resulting in too small migration.

Further, we demonstrate how to quantify the impacts of changes in migration costs. For this purpose, we calibrate our framework to Japanese prefectural data and then consider a counterfactual experiment where there exists no migration cost. The counterfactual analysis shows that in the benchmark case, disappearance of migration cost leads to reductions in the national unemployment rate by 0.006 points and the dispersion of regional unemployment rate measured by the coefficient of variation by 0.025 points, which correspond to 12.4 percent decrease and 14.5 percent decrease, respectively. These figures are considerably high, which indicates the significance of migration costs in shaping the labor markets.

Several previous studies investigated the role of migration and possible effects of labor market integration. Lkhagvasuren [8] extended the island model of Lucas and Prescott [10] by introducing job search frictions in each island as modeled in the Mortensen-Pissarides model.[5] In his model, a worker's productivity is subject to a shock specific to the worker-location match. As a result, a job searcher who is hit by a negative productivity shock may have an incentive to move to other islands even if her/his current location has good probability of finding jobs, which leads to a possibility of simultaneous in- and out-migration. By using this framework, he showed that regional differences in the unemployment rate can persist regardless of high labor mobility between regions, and that labor mobility is procyclical. Although our model is similar to that developed in Lkhagvasuren [8] in the sense that both exhibit labor mobility and regional unemployment differences at the same time, they are different in focus: we aim to uncover the possible role of migration costs in determining migration patterns whereas he examined the role of productivity shock.

In the literature of immigration, Ortega [17] developed a two-country job search model where workers can decide where to search jobs. The workers need to bear migration costs if they search

[5] For details on the Mortensen-Pissarides model, see, among others, Mortensen and Pissarides [15] and Pissarides [19].

jobs in a country different from their native country. Due to difference in the job separation rate, workers in the high job separation country may have an incentive to migrate to the low job separation country. Because wages are determined by Nash bargaining, firms expect that they can pay low wages to immigrants who have high search costs, which results in an incentive to increase vacancies. Thus, the incentive to migrate and that to increase vacancies can reinforce each other, resulting in Pareto-ranked multiple equilibria. In contrast, we employ a competitive search model where wages are posted and search is directed. This modeling strategy results in a unique equilibrium, which enables us to focus on the analysis of migration patterns.

The following studies shed light on the positive effects of labor market integration on human capital accumulation and specialization. Miyagiwa [11], in the context of immigration between countries, shows that if the scale economies exist in education, the migration of skilled people benefits the host region by increasing the skilled labor ratio, whereas it hurts the source region by discouraging skill formation there. In such an environment, regional integration represented by reductions in migration costs induces people in the host region to invest more in human capital whereas it discourages people in the source region from investing in it. Wildasin [24] presented a multi-region model where human capital investment increases specialization but exposes skilled workers to region specific earnings risk. He then showed that the mobility of skilled workers across regions mitigates such risk and improves efficiency. He also examined how the ways of financing investments, which include local taxes, affect the efficiency. However, these studies treat migration in a highly simple way, and they can not provide a solid basis for the analysis of changes in and efficiency properties of migration patterns in detail, which we try to do in this paper.

Our quantitative analysis is also related to recent studies on migration such as Bayer and Juessen [3], Coen-Pirani [6], and Kennan and Walker [9]. Bayer and Juessen [3] and Kennan and Walker [9] estimated partial equilibrium models of migration in which worker's migration decisions are motivated by idiosyncratic and location specific factors. Especially, Bayer and Juessen [3] has several points in common with our quantitative analysis: they obtained a migration cost estimate, which is of roughly two-thirds of an average annual household income, and then considered a counterfactual experiment in which migration costs are set to zero. It results in increases in the migration rate from 3.7% to 12.6% and increases in the average income by $1-2$ % in the baseline case. In contrast, our focus is on the general equilibrium effects of migration, which is in common with Coen-Pirani [6], and on the impacts of removing migration cost on the regional and national unemployment rates. Coen-Pirani [6] developed a general equilibrium model of migration based on Lucas and Prescott [10] to show that the model can replicate several stylized facts on migration in the United States. We depart from his study by investigating the quantitative impacts of labor market integration on the regional and national labor markets.

The remainder of the paper is organized as follows. In Section 2, we present the basic setups. Section 3 then analyzes the equilibrium migration patterns. Section 4 presents the efficiency property of equilibrium. Section 5 quantifies the effects of migration costs. Section 6 finally concludes.

2 General settings

Consider H regions (region $1, 2, ..., H$) in which there is a continuum of workers of size N. Workers are either employed or unemployed. While employed, a worker can not move between regions. An unemployed worker can move between regions by bearing migration costs t_{ij}. Moreover, she/he can search jobs outside of the region of her/his residence as well as in the region of residence but need to pay t_{ij} when landing a job.[6][7] We assume that finding a job in the current region of residence requires no migration cost, $t_{ii} = 0$, migration costs are symmetric, $t_{ij} = t_{ji}$, and migration costs satisfy the triangle inequality, $t_{ij} \leq t_{ih} + t_{hj}$. Such migration costs include the costs of selling and buying/renting houses and any psychological costs of renewing social networks. The main focus of our paper is on the impacts of existence and changes in such migration costs on labor market outcomes and welfare.

We assume that only unemployed workers search jobs. A firm-worker pair in region i produces output y_i, where without loss of generality, we assume that a region with a larger number is associated to higher productivity, $y_{i+1} \geq y_i$. A worker exits the economy according to a Poisson process with rate δ (> 0). Moreover, a worker generates a new worker according to a Poisson process with rate β (> 0), and the new worker enter the economy as an unemployed worker in the same region as her/his parent.

[6] We will show later that an unemployed worker may move only when she/he gets employed. While unemployed, she/he has no incentive to move.

[7] Alternatively, we can assume that workers can search only locally, which can be named as the "move then search" regime. In our framework, workers can move between regions while searching jobs, implying that this regime is possible. In addition, workers can search jobs outside of the region of their residence, implying that the "search then move" regime is also possible. However, as will be shown later, only the "search then move" regime emerges in equilibrium. See Molho [14] for the comparison between the "move then search" regime to the "search then move" regime regarding the equilibrium unemployment rate.

2.1 Matching framework

Because we build our argument based on a competitive search model, the overall job search market is divided into sub-markets, of which each is characterized by wage rate, and hence, by migration pattern, which is known as the block recursivity (Menzio and Shi [12]; Shi [22]). Job matches that are accompanied by migration from region i to region j are generated by a Poisson process with rate $M_{ij} = m(u_{ij}, v_{ij})$, where u_{ij} and v_{ij} are the number of unemployed workers who are searching jobs in region j while living in region i and the number of vacancies directed at such job searchers. We call this sub-market as sub-market ij. $m(\cdot,\cdot)$ is the matching function defined on $\mathbf{R}_+ \times \mathbf{R}_+$, and assumed to be strictly increasing in both arguments, twice differentiable, strictly concave, and homogeneous of degree one. We also assume that $m(\cdot,\cdot)$ satisfies $0 \leq M_{ij} \leq \min[u_{ij}, v_{ij}]$, $m(u_{ij}, 0) = m(0, v_{ij}) = 0$ and the Inada condition for both arguments.

In each sub-market, worker-job matching occurs at the rate of $p_{ij} = p(\theta_{ij}) = M_{ij}/u_{ij} = m(1, \theta_{ij})$ for a job searcher, and $q_{ij} = q(\theta_{ij}) = M_{ij}/v_{ij} = m(1/\theta_{ij}, 1)$ for a firm seeking to fill a vacancy. θ_{ij} is the measure of labor market tightness in sub-market ij defined as $\theta_{ij} = v_{ij}/u_{ij}$. From the assumptions regarding $m(\cdot,\cdot)$, we obtain that $p_{ij}u_{ij} = q_{ij}v_{ij}$, $\mathsf{d}p_{ij}/\mathsf{d}\theta_{ij} > 0$ and $\mathsf{d}q_{ij}/\mathsf{d}\theta_{ij} < 0$ for any $\theta_{ij} \in (0, +\infty)$. We can also see that $\lim_{\theta_{ij}\to 0} p_{ij} = 0$, $\lim_{\theta_{ij}\to\infty} p_{ij} = \infty$, $\lim_{\theta_{ij}\to 0} q_{ij} = \infty$, and $\lim_{\theta_{ij}\to\infty} q_{ij} = 0$. Moreover, we assume that the elasticity of the firm's contact rate with respect to the market tightness, $\eta_{ij} \equiv -(\theta_{ij}/q_{ij})dq_{ij}/d\theta_{ij} = 1 - (\theta_{ij}/p_{ij})dp_{ij}/d\theta_{ij}$, is constant and common across all submarkets ($\eta_{ij} = \eta$, $\forall i, j$).[8]

2.2 Asset value functions

Let ρ (> 0) denote the discount factor and define r as $r = \delta + \rho$. When locating in region i, the asset value functions for an employed worker, $W_i(w)$, for an unemployed worker, U_i for a firm with a filled position, $J_i(w)$, for a firm with a vacancy, V_i, are respectively given by

$$rW_i(w) = w, \tag{1}$$

$$rJ_i(w) = y_i - w, \tag{2}$$

$$rU_i = b + \sum_{h=1}^{H} p_{ih} \left(W_h(w_{ih}) - U_i - t_{ih} \right), \tag{3}$$

$$rV_{ij} = -k + q_{ij} \left(J_j(w_{ij}) - V_{ij} \right). \tag{4}$$

[8] This assumption leads to a set of functions that include the Cobb-Douglass function, which is very standard in the literature of theoretical and empirical search models (See Petrongolo and Pissarides [18]).

b and k represent the unemployment benefit and the cost of posting a vacancy, respectively. We assume that $y_i > b$, $\forall i$. Note here that the wage rate may differ between sub-markets and hence the asset values $W_i(w)$ and $J_i(w)$ may also differ: we may observe that $w_{ij} \neq w_{ij'}$, $W_i(w_{ij}) \neq W_i(w_{ij'})$, and $J_i(w_{ij}) \neq J_i(w_{ij'})$ $(j \neq j')$. In (3), the second term is the sum of expected gains in the asset values from finding jobs net of migration costs.

2.3 Equilibrium

Because this is a competitive search model, that is, search is directed and firms post wages, the job search market in each region is divided into sub-markets according to the migration pattern in making matches: the number of vacancies. An unemployed worker in region i chooses sub-markets to search for jobs in order to maximize her asset value. In so doing, she/he can search for jobs in multiple sub-markets.[9] In equilibrium, the asset value in each sub-market in region i takes the same value U_i.

A firm posting a vacancy determines its wage to post while anticipating the market response: it regards U_i as given and takes the relationship between w_{ij} and θ_{ij} that is determined by (3) into consideration. The firm's decision is described as

$$\max_{w_{ij},\theta_{ij}} V_{ij} \qquad \text{s.t. (3), where } U_i \text{ is treated as given.}$$

By using (1), (2), and (4), this optimization is written as

$$\max_{w_{ij},\theta_{ij}} -k + q_{ij}\left(\frac{y_j - w_{ij}}{r} - V_{ij}\right) \tag{5}$$
$$\text{s.t. } rU_i = b + \sum_{h=1}^{H} p_{ih}\left(\frac{w_{ih}}{r} - U_i - t_{ih}\right), \text{ where } U_i \text{ is treated as given.}$$

The related first-order conditions are given by

$$0 = -q_{ij} - \lambda p_{ij},$$
$$0 = \frac{dq_{ij}}{d\theta_{ij}}\left(\frac{y_j - w_{ij}}{r} - V_{ij}\right) - \lambda\frac{dp_{ij}}{d\theta_{ij}}\left(\frac{w_{ij}}{r} - U_i - t_{ij}\right).$$

We assume the free entry and exit of firms, which drives the asset value of posting a vacancy to zero: $V_{ij} = 0$.

The first-order conditions then yield the wage rate posted by a firm:

$$w_{ij} = \eta y_j + (1-\eta)\, r\,(U_i + t_{ij}). \tag{6}$$

[9] From the assumption of the Poisson process, the probability that an unemployed worker obtains multiple offers at a time is zero.

Thus, for a given market tightness, the wage rate becomes higher as the productivity ,y_j, the asset value of an unemployed worker, U_i, and the migration cost, t_{ij}, rise. A higher y_j enables for a firm to offer a higher wage rate whereas a higher U_i or t_{ij} requires a firm to compensate more in order to attract job applicants. Plugging (6) into the zero-profit condition, $V_{ij} = 0$, we obtain

$$rk = q_{ij} (1 - \eta) (y_j - rU_i - rt_{ij}) . \tag{7}$$

Of course, there may be some region j where $y_j - rt_{ij} - rU_i \leq 0$. In such a case, no vacancy is posted and $p_{ij} = 0$.

We focus on the steady state. Because the total population may change, the population in each region may also change over time. Here, the steady state requires that the unemployment rate in each region, un_i, is constant. The dynamics of unemployment rate is given by $\dot{un}_i = \beta - un_i (\delta + \sum_h p_{ih})$, where a dot represents the derivative with respect to time. This yields the steady state level of unemployment rate as

$$un_i = \frac{\beta}{\delta + \sum_{h=1}^{H} p_{ih}}. \tag{8}$$

Once the asset value of an unemployed worker, U_i, is given, other endogenous variables are well determined: (7) uniquely determines the market tightness, θ_{ij}. Then, (6) and (8) give the wage and unemployment rates, w_{ij} and un_i, respectively. The asset values other than U_i are determined accordingly.

The asset value of an unemployed worker, (3), can be rewritten[10]

$$rU_i = b + \sum_{h=1}^{H} \left[p_{ih} \left(\frac{y_h}{r} - U_i - t_{ih} \right) - k\theta_{ih} \right], \tag{9}$$

which implicitly determines U_i. We can show the following proposition.

Proposition 1 *The steady state equilibrium exists and is unique.*

Proof. See Appendix B. ■

3 Equilibrium properties

3.1 Migration patterns

In equilibrium, we can confirm that unemployed workers, while searching a job, don't have incentive to migrate:

[10] See Appendix A for the derivation of (9).

Proposition 2 *The difference between the asset value of an unemployed worker in region i and that in region i' is smaller than the migration costs between the two regions:*

$$t_{ii'} \geq |U_i - U_{i'}|, \qquad \forall i, i' \in H,$$

where the equality holds true if and only if $t_{ii'} = 0$.

Proof. See Appendix C. ■

Thus, we know that migration takes place only when unemployed workers find jobs. The probability of such migration depends on the difference between the social gains from making a match $(y_j - rt_{ij} - rU_i)$, which is the output of a match minus the value of an unemployed worker and the related migration costs:

Proposition 3 *The job finding rate accompanied by migration from region i to region j rises as the social gains from a match increase:*

$$p_{ij} > p_{ij'}, j' \neq j \qquad \text{if and only if } y_j - rt_{ij} - rU_i > y_{j'} - rt_{ij'} - rU_i.$$

Proof. See Appendix D. ■

A destination with high productivity and low migration cost attracts more employed workers from a particular region (i.e., for a job searcher in region i, the job finding rate in region j, p_{ij}, is higher than that in region j', $p_{ij'}$, if and only if $y_j - y_{j'} > r\left(t_{ij} - t_{ij'}\right)$). Moreover, a destination attracts people more from a region with low migration cost and low asset value of an unemployed worker (i.e., in region$j$, the job finding rate from region i, p_{ij}, is higher than that from region i', $p_{i'j}$, if and only if $t_{i'j} - t_{ij} > U_i - U_{i'}$). The net migration from region i to j is positive when the productivity is lower and the asset value of an unemployed worker is higher in region i than those in region j (i.e., $p_{ij} > p_{ji}$ if and only if $y_j - y_i > rU_i - rU_j$).

3.2 Effects of labor market integration and spillover effects of productivity shocks through migration

We examine the effects of regional labor market integration. In our framework, the labor market integration is described by a reduction in migration cost t_{ij}.

Proposition 4 *A reduction in the migration cost from region i to region j,* t_{ij}*, (i) increases the asset value of an unemployed worker in region i,* U_i*, (ii) increases the job finding rate from region i to region j,* p_{ij}*, but decreases that from i to* $j' \neq j$*,* $p_{ij'}$ $(j' \neq j)$*, (iii) lowers the wage* *rate when finding a job in region j from region i, w_{ij}, but raises the wage rate when finding a job in other regions, $w_{ij'}$ $(j' \neq j)$, (iv) has ambiguous effects on the unemployment rate in region i, un_i.*

Proof. See Appendix E. ■

A reduction in the migration cost t_{ij} raises the gains for a job searcher in region i from job match in region j, increasing the asset value, U_i. From (7), we can see that a reduction in t_{ij} directly increases θ_{ij} (direct effect) and it also affect θ_{ij} through changes in U_i (indirect effect). Although the direct effect positively affects θ_{ij} and raises p_{ij} and the indirect effect has opposite impacts, the direct effect dominates the indirect effect in region j. In other regions, we observe no direct effect, implying that $p_{ij'}$ $(j' \neq j)$ unambiguously declines. The wage rate w_{ij} is lower for a lower t_{ij} because firms need to pay lower compensation in order to attract job searchers from region i to j, which in turn, implies that firms in other regions need to pay higher wages in order to attract workers from region i. Although a lower migration cost, t_{ij}, implies a higher job finding rate to region j, p_{ij}, it leads to lower job finding rates to other regions, $p_{ij'}$ $(j' \neq j)$, through changes in U_i. The former effect lowers the unemployment rate in region i, un_i, whereas the latter effect raises it. When $y_j - rt_{ij}$ is sufficiently large, a change in t_{ij} significantly affects U_i and hence it becomes possible that the latter effect dominates the former. Put differently, a better access from region i to a region with good job opportunities may reduce job placement flows to other regions and increase the unemployment rate in region i. This is counter-intuitive since we normally expect that such a better access would lower the unemployment rate in the source region. The spillover effects on the job finding rate in other regions give rise to this intriguing result.

Moreover, due to responses of migration flows, a productivity shock in a particular region spills over to other regions.

Proposition 5 *A rise in the productivity in region j , y_j, (i) increases the asset value of an unemployed worker in region i $(i \neq j)$, U_i, (ii) increases the job finding rate from region i to region j, p_{ij}, but decreases that from i to $j' \neq j$, $p_{ij'}$ $(i \neq j', j' \neq j)$ (iii) raises not only the wage rate when finding a job in region j from region i, w_{ij}, but also the wage rate when finding a job in other regions, $w_{ij'}$, (iv) has ambiguous effects on the unemployment rate in region i, un_i.*

Proof. See Appendix E. ■

A productivity improvement in region j raises the employment flows from all regions into region j, $p_{ij}, \forall i$, which increases the asset values of an unemployed worker in these sending regions, U_i.

However, it lowers the employment flows to other regions (region j', ($j' \neq j$)), $p_{ij'}$. In contrast, it increases the wage rate in all regions while such effect is the most prominent in the region where the shock arises. With a higher productivity in region j, firms in region j can afford to post higher wages, and firms in other regions need to pay higher wages in order to attract workers. The effect on the unemployment rate, un_i, is again ambiguous because of the opposing effects of changes in p_{ij} and changes in $p_{ij'}$ on un_i. This is in contrast to the results of standard job search models with no migration cost, where a positive productivity shock always lowers the unemployment rate (see Rogerson et al [21], for instance).

4 Inefficiency arising from the migration cost

Now we characterize the efficiency of equilibrium. We use the social surplus as the efficiency criterion, which is standard in job search models (See Pissarides [19]). The social surplus is the sum of total output and value of leisure minus the costs of posting vacancies and migration.

We start by deriving the socially optimal allocation. The social planner maximizes the social surplus subject to the laws of motion of regional population and unemployment:

$$\max_{\theta_{ij},N_i,u_i} \int_0^\infty \sum_{i=1}^{H} \left[y_i \left(N_i - u_i\right) - u_i \sum_{h=1}^{H} \left(k\theta_{ih} + p_{ih} t_{ih}\right) + bu_i \right] e^{-\rho\tau} d\tau$$

$$\text{s.t. } \dot{N}_i = \left(\beta - \delta\right) N_i + \sum_{h=1}^{H} u_h p_{hi} - u_i \sum_{h=1}^{H} p_{ih}$$

$$\text{and } \dot{u}_i = \beta N_i - u_i \left(\sum_{h=1}^{H} p_{ih} + \delta \right).$$

Changes in regional population arise from the natural changes ($\left(\beta - \delta\right) N_i$) and the social changes (differences between in-migration $\sum u_h p_{hi}$ and out-migration $u_i \sum p_{ih}$). Inflows to the unemployment pool are newcomers to the economy and outflows from it are those landing jobs.

Proposition 6 *Define D_{ij} as*

$$D_{ij} \equiv \frac{\left(1-\eta\right) q_{ij}}{r + \eta \sum_{h=1}^{H} p_{ih}} \left[r\left(U_i - U_j\right) + \eta \sum_{h=1}^{H} p_{ih} \left(U_h - U_j\right) \right]. \tag{10}$$

Equilibrium market tightness θ_{ij} is socially optimal if and only if $D_{ij} = 0$ in equilibrium. Iff $D_{ij} > 0$, θ_{ij} is larger than the optimal tightness. The opposite holds true iff $D_{ij} < 0$.

Therefore, the equilibrium market tightness θ_{ij} and the job finding rate p_{ij} is inefficiently low when the destination region has a relatively high asset value of an unemployed worker, U_j, or when the source region has a relatively low U_i. A high asset value of an unemployed worker in the destination region implies that in-migration of job searchers to the region is socially beneficial. However, firms ignore such benefits of migration in opening their vacancies, leading to inefficiently low market tightness. In contrast, when the asset value of an unemployed worker in the source region is low, out-migration of job searchers from the region is socially beneficial. Again, firms ignore such benefits in opening vacancies, resulting in too low market tightness.

If the migration cost is the same for all migration patterns ($t_{ij} = t, i \neq j, \forall i, j$), then, migration from any region i to region N is always too small and that to region 1 is always too large, and there exists a threshold region $\widehat{j}(i)$ for which migration to region $j > \widehat{j}(i)$ is too small and migration to region $j \leq \widehat{j}(i)$ is too large.[11]

Moreover, Proposition 2 implies that $t_{ij} = 0, \forall i, j$ implies that $U_i = U_j \; \forall i, j$ and hence $D = 0$:

Corollary 7 *If $t_{ij} = 0, \forall i, j$, equilibrium is socially optimal.*

Thus, we know that when no migration cost exists, our framework becomes a standard competitive search model, of which equilibrium is socially optimal (see Moen [13] and Rogerson et al [21], among others).

5 Tentative quantitative analysis

In this section, we demonstrate that our framework can be used to quantify the impacts of regional integration on labor markets and welfare. Here, we calibrate our model to Japanese prefectural data, and provide counterfactual analysis regarding changes in migration costs.

We use the data on Japanese prefectures for the year 2000.[12] Our focus is on the unemployment rate: the overall unemployment rate of these 46 prefectures is 0.467, and the unemploy-

[11] We can prove the result as follows. We readily know that $U_i = U_j$ if $y_i = y_j$. Moreover, (13) proves that

$$\frac{dU_j}{dy_j} - \frac{dU_i}{dy_j} = \frac{p_{jj}}{r + \sum_h p_{jh}} - \frac{p_{ij}}{r + \sum_h p_{ih}}.$$

Proposition 3 implies that $p_{ii} = p_{jj} > p_{ij} = p_{ji}$ and $p_{ih} = p_{jh}$ if $y_i = y_j$, which lead to

$$\left.\frac{dU_j}{dy_j} - \frac{dU_i}{dy_j}\right|_{y_j = y_i} > 0.$$

Hence, the continuity of U_i with respect to y_j, $\forall i, j$, proves that $U_i > U_j$ if $y_i > y_j$. From the assumption that $y_N > \cdots > y_{i+1} > y_i > \cdots > y_1$, we know that $U_N > \cdots > U_{i+1} > U_i > \cdots > U_1$. From (10), we readily know that $D_{iN} < 0$ and $D_{i1} > 0$ for all i, and there exists a threshold region $\widehat{j}(i)$ for which $D_{ij} < 0$ for $j > \widehat{j}(i)$ and $D_{ij} < 0$ for $j \leq \widehat{j}(i)$.

[12] We excluded Okinawa prefecture and used the data on the remaining 46 prefectures. This is because Okinawa

ment rate of each prefecture ranges from 0.296 (Shimane prefecture) to 0.700 (Osaka prefecture) (Population Census, Ministry of Internal Affairs and Communications). Figure 1 plots the unemployment rate against the output per capita measured by the real Gross Prefectural Domestic Product per capita.

[Figure 1 around here]

Dots in this figure represent the observed data, from which we can recognize there exists considerable differentials in regional unemployment rates. The degree of dispersion can be measured by the coefficient of variation: $CV = (1/\overline{un})\sqrt{(1/46)\sum_{i=1}^{46}(un_i - \overline{un})^2}$ where $\overline{un}$ is the average of regional unemployment rates. CV for the year 2000 is 0.172, which is somewhat lower than that in the United States.[13] We will examine the extent to which migration costs affect the overall unemployment rate and the dispersion of regional unemployment rates.

5.1 Calibration

In the following analysis, we employ a Cobb-Douglas form of the matching function, which is given by $m(u_{ij}, v_{ij}) = \mu_j u_{ij}^{\eta} v_{ij}^{1-\eta}$, where μ_j and η are constants satisfying that $\mu_j > 0$ and $0 < \eta < 1$. As surveyed by Petrongolo and Pissarides [18], the Cobb-Douglas matching function is very standard in the literature of theoretical and empirical search models. Note that we extend the basic model by assuming that the matching function has a regional specific component, μ_j. Moreover, we also allow for the cost of opening a vacancy, k, to differ from region to region. We specify the migration cost, t_{ij}, as a linear function of the distance between prefectures i and j, that is, $t_{ij} = tz_{ij}$, where z_{ij} is the distance between regions and t is a positive constant. In this exercise, we focus on the steady state regarding the total population, and hence we assume that $\beta = \delta$. Moreover, we normalize the total population, N, to one.

In the benchmark case, we set the value of the discount rate, ρ, as 0.0174, which comes from the average annual interest rate of 10-year national bond of Japan during the year 2000 (which is taken on February 20, 2013 from http://www.mof.go.jp/jgbs/reference/interest_rate/data/jgbcm_2000-2009.csv, Ministry of Finance). In the existing studies such as Coen-Pirani [6], Lkhagvasuren

prefecture consists of islands and locates extremely distant from other prefectures, making it as an outlier. In fact, the distance between it and the neighboring prefecture is around 650km whereas in most cases, the distance between neighboring two prefectures is less than 100km. Note here that the distance between prefectures is measured by the distance between the locations of prefectural governments.

[13] Lkhagvasuren [8] reported that between January 1976 and May 2011, the coefficient of variation of cross-state unemployment rates in the United States ranges from 0.175 to 0.346 with an average of 0.237.

[8], and Kennan and Walker [9], this value is set to 0.04 to 0.05. We will check the robustness of our results against the cases of moderate value of ρ ($\rho = 0.03$) and high value of ρ ($\rho = 0.05$).

The values of the job separation rate, δ, the regional output per capita, y_i, and the distance between regions, z_{ij}, are taken from the Japanese data: δ is set to 0.16, which is the annual job separation rate in Japan for the year 2000 (Survey on Employment Trends, Ministry of Health, Labour and Welfare). We employ the per capita gross prefectural domestic product (in million yen, Prefectural Accounts, Department of National Accounts, Cabinet Office) as y_i. z_{ij} is measured by the distance (in $100km$) between the locations of prefectural governments (which is taken on February 20, 2013 from http://www.gsi.go.jp/KOKUJYOHO/kenchokan.html, Geographical Information Authority of Japan).

We estimated the Japanese matching function to obtain γ and μ_i to obtain $\gamma = 0.66$, of which details are reported in Appendix G. The cost of providing a vacancy, k_i, is determined by (11) for $i = j$, where the data on wage is taken from the per employed worker prefectural compensation of employees (in million yen, Prefectural Accounts, Department of National Accounts, Cabinet Office). The remaining two parameters, the migration cost, t, and the flow utility of an unemployed worker, b, are chosen by targeting the coefficient of variation of unemployment rate and the national unemployment rate, which results in $t = 0.149$ and $b = 14.365$. Table 1 summarizes the parameter values.

[Table 1 around here]

Calibration results are reported in Table 2 and described also in Figure 1.

[Table 2 around here]

In Table 2, we also report the results under different discount rates ($\rho = 0.03, 0.05$). For higher values of discount rate, the national unemployment rate is stable whereas the regional unemployment rate gets somewhat more dispersed. In Figure 1, triangles represents the calibration result in the benchmark case. From these table and figure, we know that whereas the national unemployment rate and the overall dispersion of regional unemployment rates are well replicated in our benchmark case, there exist discrepancies between calibrated and observed unemployment rates for each region. For instance, the calibrated unemployment rate is higher than the observed one for Tokyo (that has the largest population in Japan) whereas the opposite holds true for Osaka (that has the second largest population), which would be a limitation of our analysis.

5.2 Counterfactual analysis

In order to see the quantitative impacts of migration costs, we consider the case with no migration cost and compare the resulting unemployment rate to that under the benchmark case described in the previous section. We set $t = 0$ while keeping other parameters fixed to the values in the benchmark case to run a counterfactual simulation. The results are reported in Table 2 and Figure 1. As shown in Table 2, in the benchmark case, the national unemployment rate drops by 0.006 points from 0.0467 to 0.0409, which corresponds to 12.4 percent decrease. The coefficient of variation declines by 0.025 points from 0.172 to 0.147, which corresponds to 14.5 percent decrease. These decreases are considerable degree of changes. If we employ a higher value of discount rate than in the benchmark case, changes are even larger. In Figure 1, squares represent the results of counterfactual experiment. We can recognize that regional unemployment rates decline in many prefectures and are less dispersed compared to the benchmark case. Note here that there exist several prefectures (e.g., Osaka) where unemployment rates are higher under the case with no migration cost than under the benchmark case. This reflects the existence of the negative effects of declines in migration costs on unemployment that is explained in Section 3.2. Summarizing, regional labor market integration leads to significant declines in national unemployment and in dispersion of regional unemployment.

6 Concluding remarks

In this paper, we developed a multi-region job search model and analyzed the impacts of migration costs both qualitatively and quantitatively. By qualitative analysis, we showed that shocks to a particular region, such as a productivity shock or improvement in access to another region, cause spillover effects to other regions through migration responses. We also prove that equilibrium is inefficient in the presence of migration costs. Quantitatively, we calibrated our framework to Japanese prefectural data and demonstrated by a counterfactual simulation that disappearance of migration cost would lower national unemployment and dispersion of regional unemployment rate by more than 10 percent.

We briefly mention the limitations and possible extensions of our model. First, in order to concentrate our attention on the analysis of migration patterns, we ignored one important dimensions related to migration and labor market integration. As shown by Miyagiwa [11] and Wildasin [24], labor market integration enhances the human capital accumulation and specialization. Although incorporating these into our framework would not change the efficiency results because human capital investment decision is know to be efficient in a competitive search model (Acemoglu and Shimer [2]), it would amplify the effects of migration: a region receiving large migration or having better access from other regions enjoys the benefits of larger human capital or deeper specialization whereas such benefits are absent in a region experiencing out-migration or having poor access from other regions.

Second, we represented the migration cost as a function of distance between regions in the quantitative analysis. However, this is of course a coarse approximation: a region having better transportation infrastructure such as a hub airport may be easier to move to and from than a region without it, for example. Indeed, Nakajima and Tabuchi [16] discussed that there exist a case where one should exclude distance when estimating migration costs: a case with no uncertainty of employment and migration takes place based on utility differentials. Fortunately, our framework does not correspond to such a case. Still, it would be worth exploring a better description of migration costs than ours.

Finally ,our framework can be extended to represent the relationships between countries. For instance, we can consider an expansion of the European Union (EU). We would then be able to examine the possible impacts of accession of a new member country on each member country's labor market and the overall EU labor market. All these are important topics for future investigation.

Appendices

Appendix A: *Derivation of the asset value of an unemployed worker, (9).*

The wage equation (6) is rewritten

$$(1-\eta)(y_j - rt_{ij} - rU_i) = y_j - w_{ij}.$$

Using this, we can rearrange the zero-profit condition (7) as

$$rk = q_{ij}(y_j - w_{ij}). \tag{11}$$

Plugging this, (1), and $q_{ij} = p_{ij}/\theta_{ij}$ into (3), we obtain

$$rU_i = b + \sum_{h \in \widehat{H}} \left[p_{ih} \left(\frac{y_h}{r} - U_i - t_{ih} \right) - k\theta_{ih} \right].$$

Appendix B: *Proof of Proposition 1.*

Define Γ_i as

$$\Gamma_i(U_i) \equiv rU_i - b - \sum_{h=1}^{H} \left[p_{ih} \left(\frac{y_h}{r} - U_i - t_{ih} \right) - k\theta_{ih} \right].$$

If $\Gamma_i(U_i) = 0$ has a unique solution for all i, we know that there exists a unique steady state equilibrium. Equation (7) is rearranged as

$$k = \frac{dp_{ij}}{d\theta_{ij}} \left(\frac{y_j}{r} - U_i - t_{ij} \right), \tag{12}$$

which, combined with the Inada condition of the matching function, implies that θ_{ij} and p_{ij} are positive when U_i is equal to zero and that θ_{ij} and p_{ij} converge to zero as U_i goes to $y_j/r - t_{ij}$. Hence, even though $\Gamma_i(U_i)$ is kinked at $U_i = y_j/r - t_{ij}$, it is continuous. Furthermore, letting $\overline{U}_i$ denote $\max[y_i/r, \max_j[y_j/r - t_{ij}]]$, we readily know that

$$\Gamma(0) < 0,$$
$$\Gamma(\overline{U}_i) = r\overline{U}_i - b \geq y_i - b > 0.$$

Thus, $\Gamma_i(U_i) = 0$ has at least one solution, which shows the existence.

$\Gamma_i(U_i)$ may not be differentiable at $U_i = y_j/r - t_{ij}$. However, except for these points, it is differentiable, and by differentiating $\Gamma(U_i)$ with respect to U_i, we obtain

$$\begin{aligned}\frac{d\Gamma_i(U_i)}{dU_i} &= r + \sum_h p_{ih} - \sum_h \frac{\partial \left[p_{ih} \left(y_h/r - U_i - t_{ih} \right) - k\theta_{ih} \right]}{\partial \theta_{ih}} \frac{\partial \theta_{ih}}{\partial U_i} \\ &= r + \sum_h p_{ih} > 0,\end{aligned}$$

where the second equality comes from (12). Combined with the continuity of $\Gamma_i(U_i)$, this proves that the solution of $\Gamma_i(U_i) = 0$ is unique.

Appendix C: *Proof of Proposition 2.*

From (1) and (3), we have

$$rU_i = b + \sum_{h=1}^{H} \left[p_{ih} \left(\frac{y_h}{r} - U_i - t_{ih} \right) - k\theta_{ih} \right],$$

which yields

$$U_i = \frac{b + \sum_h \left[p_{ih} \left(y_h/r - t_{ih} \right) - k\theta_{ih} \right]}{r + \sum_h p_{ih}}.$$

From (7), we know that $\theta_{ij} = \arg\max U_i$, $\forall i, j \in H$. Hence, we readily know that

$$U_{i'} = \frac{b + \sum_h \left[p_{i'h} \left(y_h/r - t_{i'h} \right) - k\theta_{i'h} \right]}{r + \sum_h p_{i'h}} \geq \frac{b + \sum_h \left[p_{ih} \left(y_h/r - t_{i'h} \right) - k\theta_{ih} \right]}{r + \sum_h p_{ih}}.$$

This implies that

$$
\begin{aligned}
U_i - U_{i'} &\leq \frac{b + \sum_h \left[p_{ih}\left(y_h/r - t_{ih}\right) - k\theta_{ih}\right]}{r + \sum_h p_{ih}} - \frac{b + \sum_h \left[p_{ih}\left(y_h/r - t_{i'h}\right) - k\theta_{ih}\right]}{r + \sum_h p_{ih}} \\
&= \frac{\sum_h p_{ih}\left(t_{i'h} - t_{ih}\right)}{r + \sum_h p_{ih}} \\
&\leq \frac{\sum_h p_{ih} t_{ii'}}{r + \sum_h p_{ih}} \\
&\leq t_{ii'},
\end{aligned}
$$

where the second inequality comes from the triangle inequality $t_{i'j} \leq t_{ii'} + t_{ij}$. Similar arguments show that $U_{i'} - U_i \leq t_{ii'}$.

Appendix D: *Proof of Proposition 3.*

Suppose temporarily that U_i is fixed. Differentiation of (7) with respect to $y_j - rU_i - rt_{ij}$ yields

$$
\begin{aligned}
0 &= q'_{ij}\left(y_j - rU_i - rt_{ij}\right) \frac{\partial \theta_{ij}}{\partial\left(y_j - rU_i - rt_{ij}\right)} + q_{ij} \\
&= \theta_{ij} q'_{ij}\left(y_j - rU_i - rt_{ij}\right) \frac{\partial \theta_{ij}}{\partial\left(y_j - rU_i - rt_{ij}\right)} + p_{ij}.
\end{aligned}
$$

Plugging (7) into this, we obtain

$$
\begin{aligned}
0 &= \frac{rk\theta_{ij} q'_{ij}}{q_{ij}(1-\eta)} \frac{\partial \theta_{ij}}{\partial\left(y_j - rU_i - rt_{ij}\right)} + p_{ij} \\
&= -\frac{rk\eta}{1-\eta} \frac{\partial \theta_{ij}}{\partial\left(y_j - rU_i - rt_{ij}\right)} + p_{ij},
\end{aligned}
$$

which implies that

$$
\frac{\partial \theta_{ij}}{\partial\left(y_j - rU_i - rt_{ij}\right)} = \frac{1-\eta}{\eta} \frac{p_{ij}}{rk} > 0.
$$

Hence, we obtain

$$
\begin{aligned}
\frac{\partial p_{ij}}{\partial\left(y_j - rU_i - rt_{ij}\right)} &= p'_{ij} \frac{\partial \theta_{ij}}{\partial\left(y_j - rU_i - rt_{ij}\right)} \\
&= p'_{ij} \frac{1-\eta}{\eta} \frac{p_{ij}}{rk} > 0.
\end{aligned}
$$

This implies that $p_{ij} > p_{ij'}, j' \neq j$ if and only if $y_j - rU_i - rt_{ij} > y_{j'} - rU_i - rt_{ij'}$.

Appendix E: *Proof of Propositions 4 and 5.*

We start by deriving the effect on the asset value of an unemployed worker, U_i. y_j and t_{ij} affect U_i only through changes in $y_j - rt_{ij}$. Differentiating (9) with respect to $y_j - rt_{ij}$ and using (12), we obtain

$$\frac{\partial U_i}{\partial(y_j - rt_{ij})} = \frac{p_{ij}}{r + \sum_{h\in\widehat{H}_i} p_{ih}} > 0. \tag{13}$$

We readily see that $\partial U_i/\partial y_j = \partial U_i/\partial(y_j - rt_{ij}) > 0$ and $\partial U_i/\partial t_{ij} = -r\partial U_i/\partial(y_j - rt_{ij}) < 0$. The effects on the job finding rate, p_{ij}, also appears through changes in $y_j - rt_{ij}$. Differentiation of (7) with respect to $y_j - rt_{ij}$, combined with (13), yields

$$\frac{\partial p_{ij}}{\partial (y_j - rt_{ij})} = \frac{(1-\eta)^2 p_{ij} q_{ij}}{\eta k}\left(1 - \frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}}\right) > 0, \tag{14}$$

$$\frac{\partial p_{ij'}}{\partial (y_j - t_{ij})} = -\frac{(1-\eta)^2 p_{ij'} q_{ij'}}{\eta k}\frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}} < 0,$$

which lead to $\partial p_{ij}/\partial y_j > 0$, $\partial p_{ij}/\partial t_{ij} < 0$, $\partial p_{ij'}/\partial y_j < 0$, and $\partial p_{ij'}/\partial t_{ij} > 0$. From (6), and by using (13), we obtain the effects on the wage rate:

$$\frac{\partial w_{ij}}{\partial y_j} = \eta + (1-\eta)\frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}} > 0,$$

$$\frac{\partial w_{ij'}}{\partial y_j} = (1-\eta)\frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}} > 0,$$

$$\frac{\partial w_{ij}}{\partial t_{ij}} = (1-\eta)\, r\left(1 - \frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}}\right) > 0,$$

$$\frac{\partial w_{ij'}}{\partial t_{ij}} = -(1-\eta)\, r\frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}} < 0.$$

Finally, from (14), we can see that

$$\begin{aligned}
\sum_{h=1}^{H}\frac{\partial p_{ih}}{\partial (y_j - rt_{ij})} &= \frac{(1-\eta)^2 p_{ij} q_{ij}}{\eta k} - \sum_{h=1}^{H}\frac{(1-\eta)^2 p_{ih} q_{ih}}{\eta k}\frac{p_{ij}}{r + \sum_{h=1}^{H} p_{ih}} \\
&= \frac{(1-\eta)^2 p_{ij}}{\eta k}\left[\frac{(r + \sum_h p_{ih})\, q_{ij} - \sum_h p_{ih} q_{ih}}{r + \sum_h p_{ih}}\right] \\
&= \frac{(1-\eta)^2 p_{ij}}{\eta k}\frac{r q_{ij} + \sum_h p_{ih}\,(q_{ij} - q_{ih})}{r + \sum_h p_{ih}}.
\end{aligned}$$

When $y_j - rt_{ij}$ is sufficiently large, the market tightness θ_{ij} is also large and q_{ij} is small, under which $\sum_h \partial p_{ih}/\partial (y_j - rt_{ij})$ is likely to be negative. Because the unemployment rate, un_i, is given by (8), this raises un_i.

Appendix F: *Proof of Proposition 6.*

The present-value Hamiltonian is defined as

$$\begin{aligned}
H = &\sum_{i=1}^{H}\left[y_i\,(N_i - u_i) - u_i\sum_{h=1}^{H}(k\theta_{ih} + p_{ih}t_{ih}) + bu_i\right]e^{-\rho\tau} \\
&+ \sum_{i=1}^{H}\mu_i^N\left[(\beta-\delta)\, N_i + \sum_{h=1}^{H} p_{hi}u_h - u_i\sum_{h=1}^{H} p_{ih}\right] + \sum_i \mu_i^u\left(\beta N_i - u_i\sum_{h=1}^{H} p_{ih} - \delta u_i\right).
\end{aligned}$$

Note here that the control variables are θ_{ij}, and the state variables are N_i and u_i. The first-order conditions are

$$ke^{-\rho\tau} = p'_{ij}\left(\mu_j^N - \mu_i^N - \mu_i^u - t_{ij}e^{-\rho\tau}\right) = (1-\eta)\, q_{ij}\left(\mu_j^N - \mu_i^N - \mu_i^u - t_{ij}e^{-\rho\tau}\right) \tag{15}$$

$$\mu_i^N = \frac{y_i e^{-\rho\tau} + \beta\mu_i^u}{r-\beta} \tag{16}$$

$$0 = -\left[y_i + \sum_{h=1}^{H}\left(k\theta_{ih} + p_{ih}t_{ih}\right) - b\right]e^{-\rho\tau} + \sum_{h=1}^{H}\mu_h^N p_{ih} - \mu_i^N\sum_{h=1}^{H}p_{ih} - \mu_i^u\left(\sum_{h=1}^{H}p_{ih} + r\right) \tag{17}$$

Equations (15) and (17) yield

$$\mu_i^u = -\frac{\left(y_i - b + \eta\sum_h p_{ih}t_{ih}\right)e^{-\rho\tau} + \eta\sum_h p_{ih}\left(\mu_i^N - \mu_h^N\right)}{r + \eta\sum_h p_{ih}} \tag{18}$$

Moreover, (16) is rearranged as

$$\mu_i^N - \mu_j^N = \frac{\left(y_i - y_j\right)e^{-\rho\tau} + \beta\left(\mu_i^u - \mu_j^u\right)}{r-\beta}. \tag{19}$$

Plugging (16), (17) and (19) into (15), we obtain

$$\begin{aligned} k &= (1-\eta)\, q_{ij}\left\{\frac{\left(y_i - b + \eta\sum_h p_{ih}t_{ih}\right) + \eta\sum_h p_{ih}\left[\left(y_i - y_h\right) + \beta\left(\mu_i^u - \mu_h^u\right)e^{\rho\tau}\right]/(r-\beta)}{r + \eta\sum_h p_{ih}}\right. \\ &\left. + \frac{\left(y_j - y_i\right) + \beta\left(\mu_j^u - \mu_i^u\right)e^{\rho\tau}}{r-\beta} - t_{ij}\right\} \\ &= \pi_{ij} - \beta D_{ij}, \end{aligned}$$

where π_{ij} and D_{ij} are defined as

$$\pi_{ij} \equiv (1-\eta)\, q_{ij}\left[\frac{y_j}{r} - t_{ij} - \frac{b + \eta\sum_h p_{ih}\left(y_h/r - t_{ih}\right)}{r + \eta\sum_h p_{ih}}\right], \tag{20}$$

$$D_{ij} \equiv (1-\eta)\, q_{ij}\frac{\left[y_i - b - \left(r + \eta\sum_h p_{ih}\right)t_{ij}\right] - r\left(\mu_j^u - \mu_i^u\right)e^{\rho\tau} - \eta\sum_h p_{ih}\left(\mu_j^u - \mu_h^u - t_{ih}e^{-\rho\tau}\right)e^{\rho\tau}}{r + \eta\sum_h p_{ih}} - k.$$

In equilibrium, because $p_{ij} = \theta_{ij}q_{ij}$, (7) is rewritten

$$rk\theta_{ij} = (1-\eta)\, p_{ij}\left(y_j - rU_i - rt_{ij}\right).$$

Summing up the both sides of it for $j = 1...H$, we obtain

$$rk\sum_{j=1}^{H}\theta_{ij} = (1-\eta)\sum_{j=1}^{H}p_{ij}\left(y_j - rU_i - rt_{ij}\right),$$

which is rearranged as

$$\eta \sum_{j=1}^{H} p_{ij} \left(\frac{y_j - rU_i - rt_{ij}}{r} \right) = \frac{\eta}{1-\eta} k \sum_{j=1}^{H} \theta_{ij}.$$

Plugging (1), (6) and the above equation into (3), the asset value of an unemployed worker in equilibrium can be rewritten as

$$\begin{aligned} rU_i &= b + \sum_{j=1}^{H} p_{ij} \left[\frac{\eta y_j + (1-\eta) r (t_{ij} + U_i)}{r} - U_i - t_{ij} \right] \\ &= b + \eta \sum_{j=1}^{H} p_{ij} \frac{y_j - rU_i - rt_{ij}}{r} \\ &= b + \frac{\eta}{1-\eta} k \sum_{j=1}^{H} \theta_{ij}. \end{aligned} \tag{21}$$

The second equality implies that

$$U_i = \frac{b + \eta \sum_h p_{ih} (y_h / r - t_{ih})}{r + \eta \sum_h p_{ih}}. \tag{22}$$

Using this, we can rewrite the zero-profit condition (7) as

$$k = (1-\eta) q_{ij} \left(\frac{y_j}{r} - t_{ij} - \frac{b + \eta \sum_h p_{ih} (y_h / r - t_{ih})}{r + \eta \sum_h p_{ih}} \right). \tag{23}$$

Plugging (22) into π_{ij} of (20), we can see that in equilibrium,

$$\pi_{ij} = (1-\eta) q_{ij} \left(\frac{y_j}{r} - U_i - t_{ij} \right),$$

which, combined with (7), implies that $\pi_{ij} = k$ holds true in equilibrium. Moreover, from (15), we obtain

$$\sum_h p_{ih} \left(\mu_h^N - \mu_i^N \right) = \frac{k}{1-\eta} \sum_h \theta_{ih} e^{-\rho\tau} + \sum_h p_{ih} \left(\mu_i^u + t_{ih} e^{-\rho\tau} \right).$$

Substituting this and (21) into (17), we know that in equilibrium,

$$\begin{aligned} \mu_i^u &= -\frac{(y_i - b + \eta \sum_h p_{ih} t_{ih}) e^{-\rho\tau} - [\eta/(1-\eta)] k \sum_h \theta_{ih} e^{-\rho\tau} - \eta \sum_h p_{ih} (\mu_i^u + t_{ih} e^{-\rho\tau})}{r + \eta \sum_h p_{ih}} \\ &= -\frac{y_i - rU_i}{r} e^{-\rho\tau} \end{aligned}$$

Using this and (21), we can rewrite D_{ij} of (20) as

$$\begin{aligned} D_{ij} = \frac{(1-\eta) q_{ij}}{r + \eta \sum_h p_{ih}} \Bigg\{ &-b + \left(r + \eta \sum_h p_{ih} \right) \left(\frac{y_j}{r} - t_{ij} \right) + \frac{\eta}{1-\eta} k \left(\sum_h \theta_{ih} - \sum_h \theta_{jh} \right) \\ &- \eta \sum_h p_{ih} \left[z_{ih} + \frac{\eta}{1-\eta} \frac{k}{r} \left(\sum_h \theta_{jh'} - \sum_h \theta_{hh'} \right) \right] \Bigg\} - k. \end{aligned}$$

From (23), this can be further rewritten as

$$D_{ij} = \beta \frac{(1-\eta)\, q_{ij}}{r + \eta \sum_{h=1}^{H} p_{ih}} \frac{\eta}{1-\eta} \left[k \sum_h \theta_{ih} - k \sum_h \theta_{jh} - \frac{\eta}{r} \sum_h p_{ih} \left(k \sum_h \theta_{jh'} - k \sum_h \theta_{hh'} \right) \right].$$

Finally, from (21), we obtain D_{ij} in equilibrium as

$$D_{ij} = \frac{(1-\eta)\, q_{ij}}{r + \eta \sum_h p_{ih}} \left[r\,(U_i - U_j) + \eta \sum_h p_{ih}\,(U_h - U_j) \right].$$

Therefore, the equilibrium is socially optimal if and only if $D_{ij} = 0$. Moreover, from the second-order condition of firm's optimization (5), the equilibrium market tightness is larger than the social optimum if and only if $D_{ij} > 0$, and the opposite holds true if and only if $D_{ij} < 0$.

Appendix G: *Estimation of the matching function.*

Data

Our spatial units are Japanese prefectures. Monthly Report of Public Employment Security Statistics contains number of active job applicants and active job openings, and job placements in every month. To remove seasonal volatility, we aggregate monthly data into annual by taking average. In the analysis, Okinawa prefecture is excluded.

Table 1 shows descriptive statistics.

Table 1: Descriptive statistics

Variables	Observations	Periods	Mean	SD	Max	Min
Unemployment rate	46	2000	0.045	0.011	0.094	0.030
Number of job placements	230	1996-2000	1906.18	1387.06	11019.33	487.58
Number of active job applicants	230	1996-2000	41561.85	39099.55	200184.58	6892.00
Number of active job openings	230	1996-2000	20625.29	16585.44	107374.92	4668.25

Empirical strategy

As we discussed in Section 5, we specify the matching function as a Cobb-Douglas form as follows,

$$e_{it} = \mu_i \nu_{it}^{\eta} u_{it}^{1-\eta},$$

where e_{it} is the number of job placements, ν_{it} is the number of active job openings, and u_{it} is the number of job applicants in prefecture i in period t. After taking natural logarithm, we estimate the equation by the fixed effect model, and obtain the point estimates of η and μ_i as the prefectural fixed effects.

Results

The results are shown in Table 2. The coefficient for ln(Number of job applicants) is positively

Table 2: Results in the first step

Dependent: ln(Number of job placements)	(1)
ln(Number of job applicants)	0.334**
	(0.053))
Adj R^2	0.160
Observations	230

significant. Further, the null hypothesis that all the fixed effects are zero is rejected at the 0.1 % level. The coefficient implies that the point estimate of η is 0.66.

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Parameters	*Value*	*Description*
δ	0.16	Job separation rate
ρ	0.0174	Discount rate
γ	0.66	Parameter of the matching function
μ_i	region specific	Regional fixed components of the matching function
y_i	region specific	Regional output per capita
k_i	region specific	Regional specific cost of posting a vacancy
t	0.149	Migration cost per distance
z_{ij}	specific between regions	Distance between regions i and j
b	14.365	Flow utility of unemployment
N	1 (normalization)	Total number of workers

Table 1. Parameters of the benchmark model.

Notes: The value of ρ comes from the Japanese long-term interest rate. The values of δ, y_i, and z_{ij} are taken from the Japanese data. We estimated the Japanese matching function to obtain γ and μ_i. The equation (11) for $i = j$ determines k_i. We normalize the total population, N, to one. The remaining two parameters, t and b are chosen by targeting the data listed in Table 2.

	Data	Benchmark ($\rho = 0.0174$)	Moderate ρ ($\rho = 0.03$)	High ρ ($\rho = 0.05$)
Calibration targets				
National unemp. rate, un_N	0.0467	0.0467	0.0466	0.0466
Unemp. rate differences, CV	0.172	0.172	0.179	0.194
Counterfactual (no migration cost)				
National unemp. rate, un_N		0.0409	0.0402	0.0392
		(−12.4)	(−13.7)	(−15.8)
Unemp. rate differences, CV		0.147	0.146	0.144
		(−14.5)	(−18.4)	(−25.7)

Table 2. Calibration and counterfactual results.

Notes: Percentage changes are in parentheses.

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Figure 1. Regional output per capita and unemployment rates.

Notes: Dots represents the observed data. Triangles and squares describe the benchmark and counterfactual cases, respectively.