Directed search versus frictional transfers in the marriage market

Aloysius Siow

University of Toronto

Presentation at the University of Tokyo, 2013

This lecture presents the CS frictionless transferable utility marriage matching models, a directed search non-transferable utility marriage matching model and a CS marriage matching model with frictional transfers. The CS model is identified with the marriage distribution from one marriage market. Male and female spousal preferences are not identified. The directed search model needs multi-market data for identification. Male and female preferences are identified. The CS model with frictional transfers needs multi-market data for identification. Male and female preferences are identified. An empirical application to the marriage market of the famine born cohorts in China is presented.

- Gary Becker introduced the transferable utility frictionless matching marriage model. This model have two central assumptions:

  1. Market parcipants have constant marginal utility of incomes.

  2. There is frictionless matching and transfers are used to clear the marriage market.

- The Choo Siow (CS) model and its extensions, DLMS, CSW, Graham and GS, provide an empirical framework for estimating transferable utility frictionless matching marriage models.

- A central concern of these studies are issues of identification.

- The CS model is identified without the analyst having to impose apriori ranking of potential partners by observed characteristics.

- Empirical researchers often do not observe marital transfers The CS class deals with this problem.

- CS is identified with one cross section marriage market data. Without adding other restrictions, its extensions require data from multiple markets.

- Male and female preferences are not identified in CS. They are identified in some extensions.

- Other researchers have investigated non-transferable utility and frictional matching marriage models.

- An important class is the directed search models first introduced by Moen. Individuals choose which sub-marriage market to participate in. A feature of these models, is that transfers do not adjust to clear the marriage market. Rather, the marriage market clears by the probability of being unmatched in different sub-marriage markets.

- ABM provides an empirical framework to estimate these directed search models. In their demand for spousal attributes, ABM also uses the random additive utility framework of McFadden which is central to the CS class.

- The objective of this paper is to contrast the directed search models with the CS class of models. In order to do so, we extend the CS class to allow for frictional transfers. With frictional transfers, the giver values the transfer differently from the receiver. We maintain the frictionless matching assumption.

- Are the two classes of model can be distinguished with marriage matching data alone.

1 The CS benchmark

- Let the utility of male g of type i who marries a female of type j be:

$$V_{ijg} = \widetilde{\alpha}_{ij} - \tau_{ij} + \varepsilon_{ijg}, \qquad \text{where} \tag{1}$$

$\widetilde{\alpha}_{ij}$: Systematic gross return to male of type i married to female of type j.

τ_{ij}: Equilibrium transfer made by male of type i to spouse of type j.

ε_{ijg} : idiosyncratic i.i.d. random variable specific to g.

1. V_{ijg} is known as the additive random utility model. It consists of a systematic component which is common to husbands in $\{i, j\}$ marriages and an idiosycratic component which is specific to g.

2. Neither systematic return not transfer, i.e. systematic component, depends on the specific woman chosen.

3. The idiosyncratic component, ε_{ijg}, is also independent of any particular woman of type j. I.e. no particular woman can hold the man up for additional transfers. Is this a strong assumption?

- The payoff to g from remaining unmarried, denoted by $j = 0$, is:

$$V_{i0g} = \widetilde{\alpha}_{i0} + \varepsilon_{i0g} \tag{2}$$

where ε_{i0g} is also an i.i.d. random variable.

Individual g will choose according to:

$$V_{ig} = \max_j [V_{i0g}, .., V_{ijg}, .., V_{iJg}] \tag{3}$$

Before he sees his idiosyncratic realizations, the probability that g will choose type j female is:

$$\begin{aligned} \Pr(V_{ijg} - V_{ikg} &\geq 0 \; \forall k = 0, .., J) = \\ \Pr(\widetilde{\alpha}_{ij} - \tau_{ij} - (\widetilde{\alpha}_{ik} - \tau_{ik}) &\geq \varepsilon_{ikg} - \varepsilon_{ijg} \; \forall k = 0, .., J) \end{aligned}$$

For large m_i, we can estimate $\Pr(V_{ijg} - V_{ikg} \geq 0 \; \forall k = 0, .., J)$ as:

$$\Pr(V_{ijg} - V_{ikg} \geq 0 \; \forall k = 0, .., J) = \frac{\mu_{ij}^d}{m_i} \tag{4}$$

Let ε_{ijg} be a Type 1 extreme value random variable. It has density:

$$h(\varepsilon_{ijg}) = e(^{-\varepsilon_{ijg}}) \exp(e(^{-\varepsilon_{ijg}})), \ -\infty < \varepsilon_{ijg} < \infty$$

Using (4),

$$\begin{aligned} \Pr(V_{ijg} - V_{ikg} &\geq 0 \ \forall k = 0, .., J) = \\ \frac{\mu_{ij}^d}{m_i} &= \frac{\exp(\widetilde{\alpha}_{ij} - \tau_{ij})}{\sum_{k=0}^J \exp(\widetilde{\alpha}_{ik} - \tau_{ik})} \end{aligned}$$

Obtain quasi-demand function for j type spouse relative to remaining unmarried is:

$$\begin{aligned} \ln \mu_{ij}^d - \ln \mu_{i0}^d &= \widetilde{\alpha}_{ij} - \widetilde{\alpha}_{i0} - \tau_{ij} \\ &= \alpha_{ij} - \tau_{ij} \end{aligned} \tag{5}$$

Assume women have the same additive random utility functions for husbands. Then we can derive the quasi supply function of j type spouse to $\{i, j\}$ market:

$$\ln \mu_{ij}^s - \ln \mu_{0j}^s = \gamma_{ij} + \tau_{ij} \tag{6}$$

where $\gamma_{ij} = \widetilde{\gamma}_{ij} - \gamma_{0j}$ is the systematic payoff to a type j woman married to a type i man relative to remaining unmarried.

- Marriage market clearing for all $\{i.j\}$ submarkets:

$$\mu_{ij} = \mu_{ij}^{d} = \mu_{ij}^{s} \tag{7}$$

- CS marriage matching function obtained by summing (5) and (6), and imposing (7):

$$\ln \mu_{ij} - \frac{\ln \mu_{i0} + \ln \mu_{0j}}{2} = \frac{\alpha_{ij} + \gamma_{ij}}{2} = \pi_{ij}$$

$$\frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}} = \Pi_{ij}$$

- CS calls Π_{ij} the total gains to an $\{i, j\}$ marriage.

- The marriage matching function has constant returns to scale.

- It can fit any observed marriage matching distribution.

- It can be estimated by $\frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}}$. With a single marriage market, the model is just identified. Note that there is only $I \times J$ endogenous data points and there are $I \times J$ parameters, Π.

- With multiple marriage marriage markets, the model will be overidentified.

- α_{ij} and γ_{ij} are not separately identified even if you have multiple market data.

Figure 1: Sichuan number of individuals by age, 1990

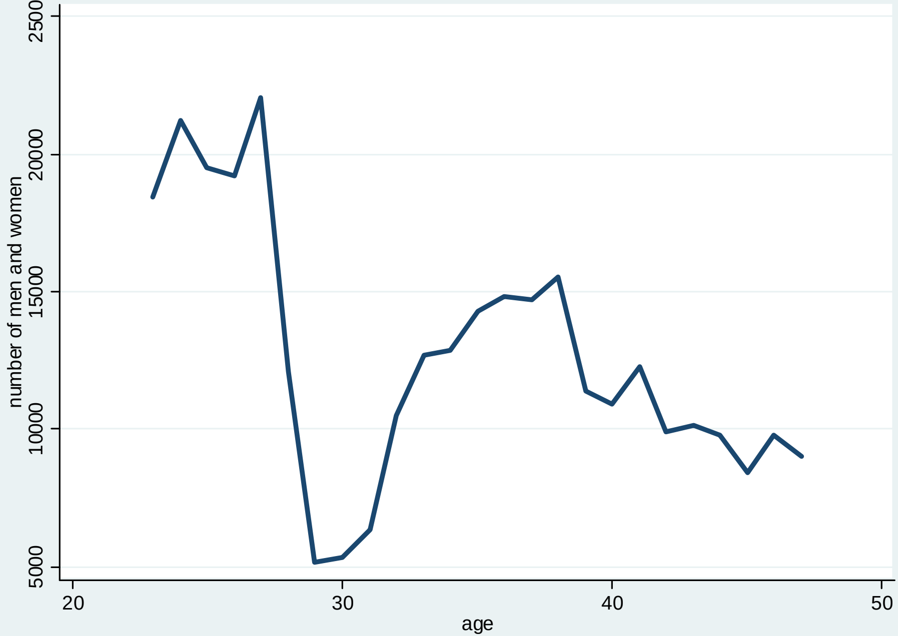


Figure 2: Sichuan sex ratios by female age, 1990

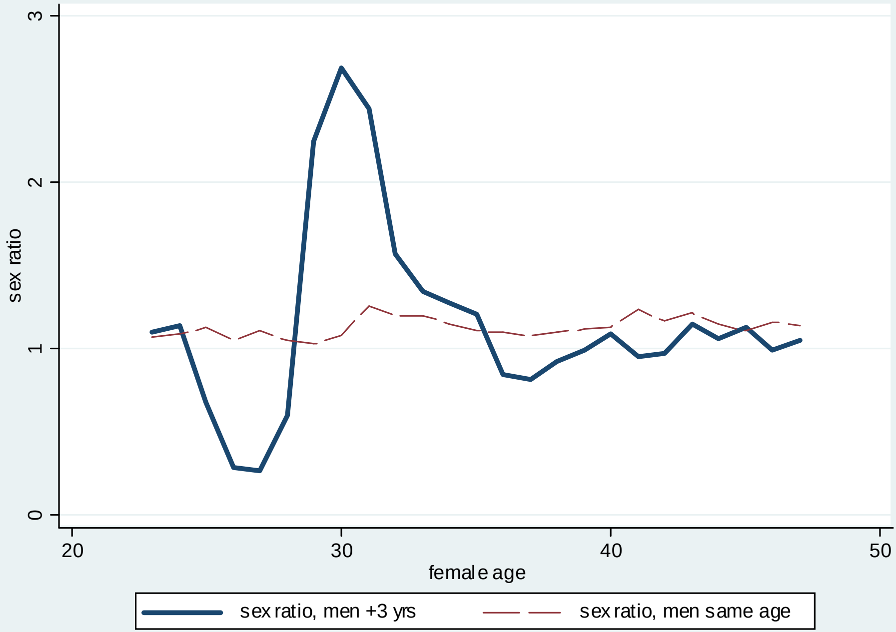


Figure 3: Sichuan marriage rates, 1990, 1982

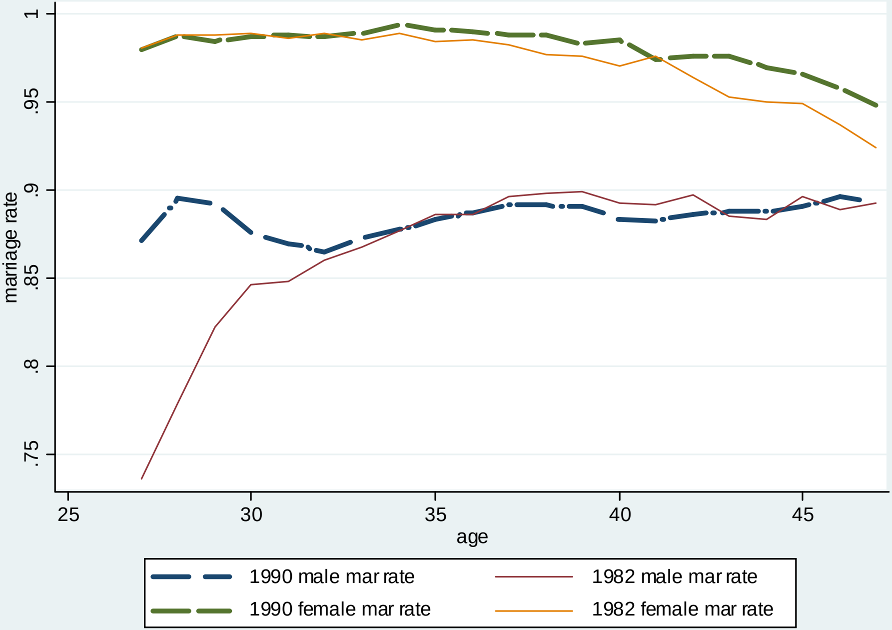


Figure 8: Sichuan total gains, 1982

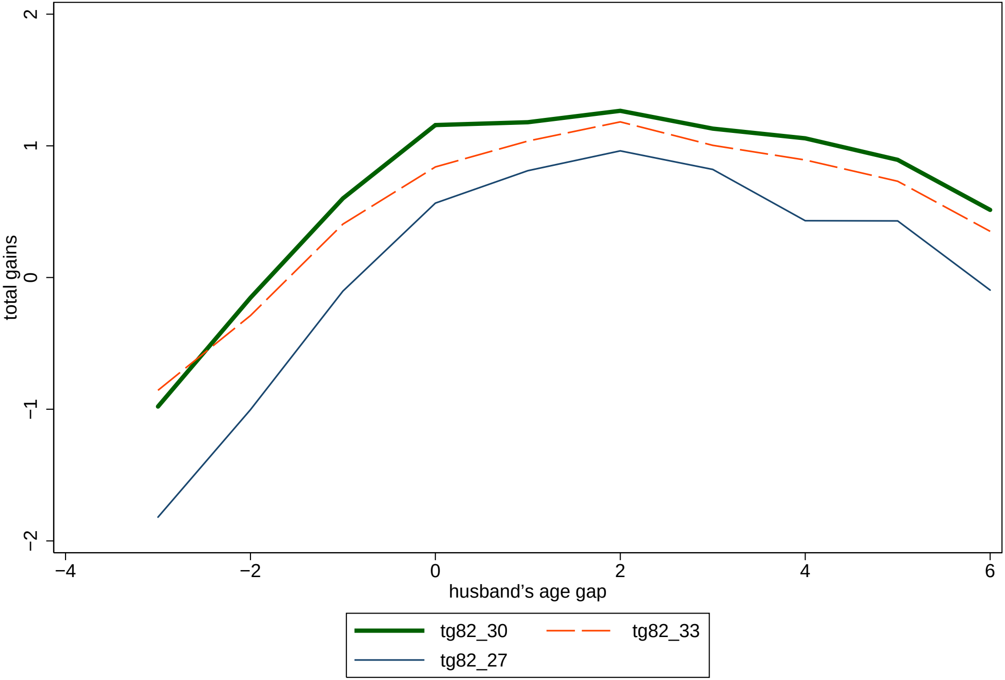


Figure 9: Sichuan total gains, 1990

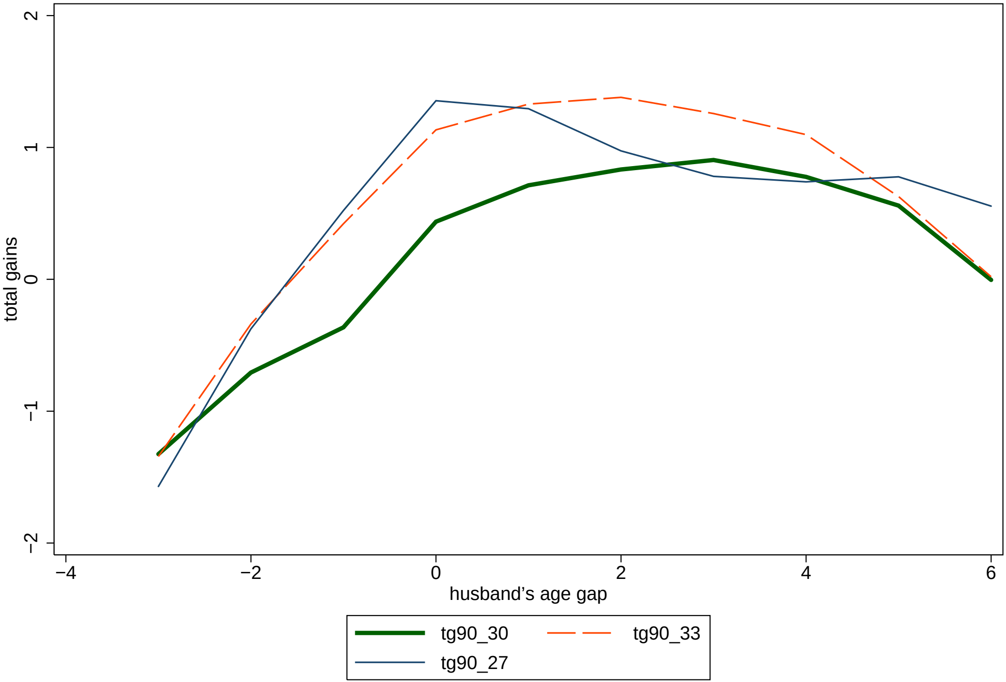
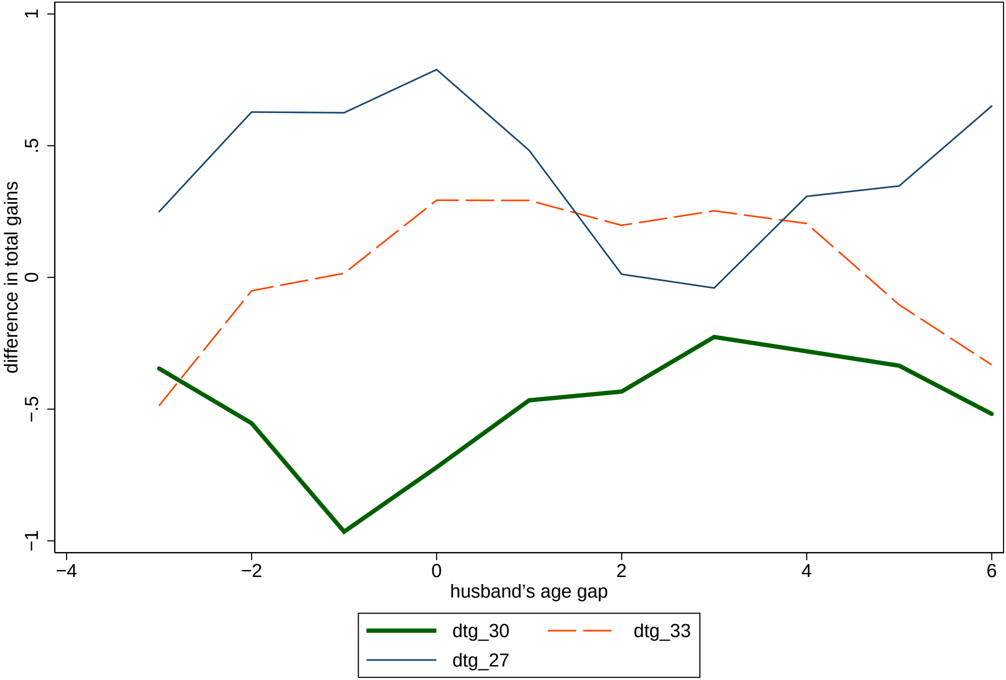


Figure 10: Sichuan 1990−1982 total gains



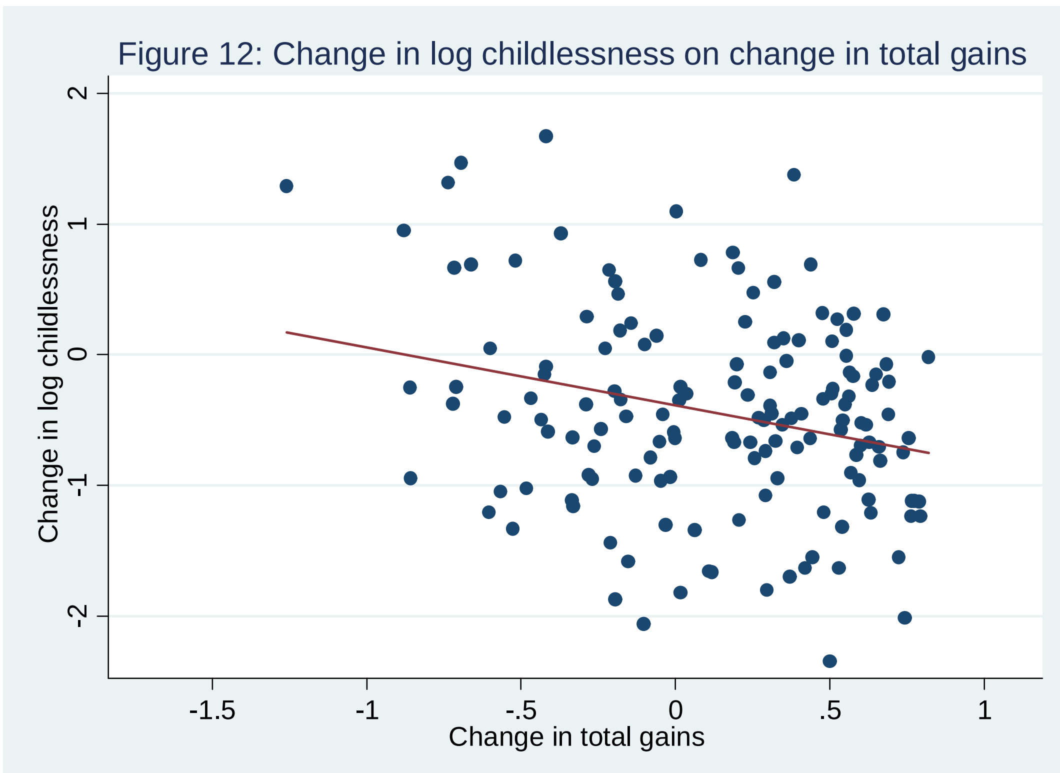


Figure 12: Change in log childlessness on change in total gains
Change in log childlessness
2
1
0
-1
-2
-1.5
-1
-.5
0
.5
1
Change in total gains

Figure 13: 1990 Sichuan male marriage rates

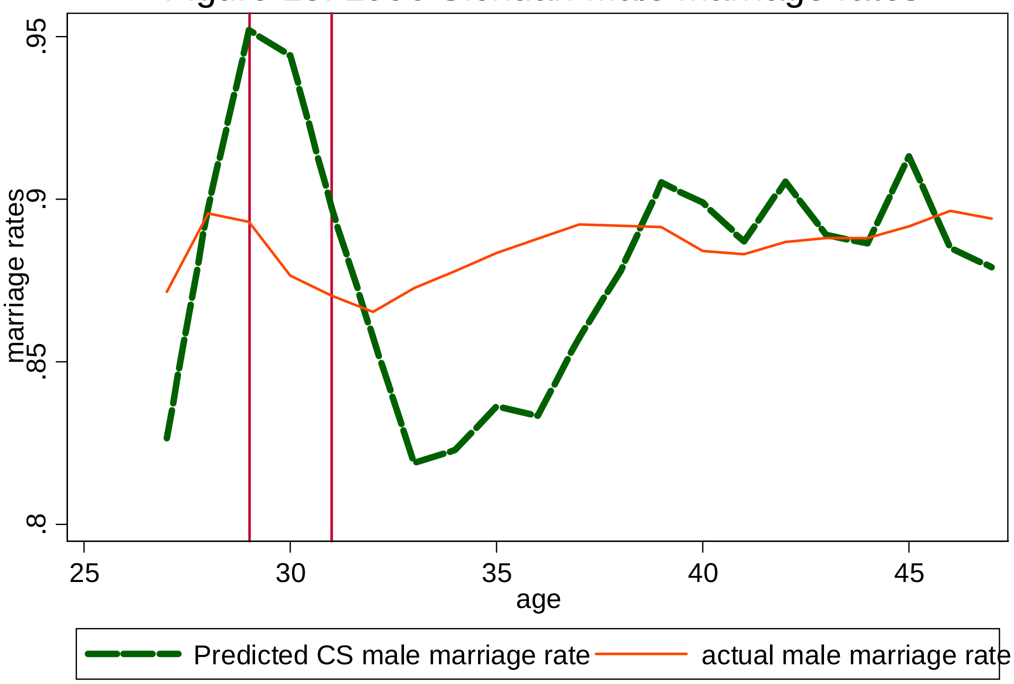
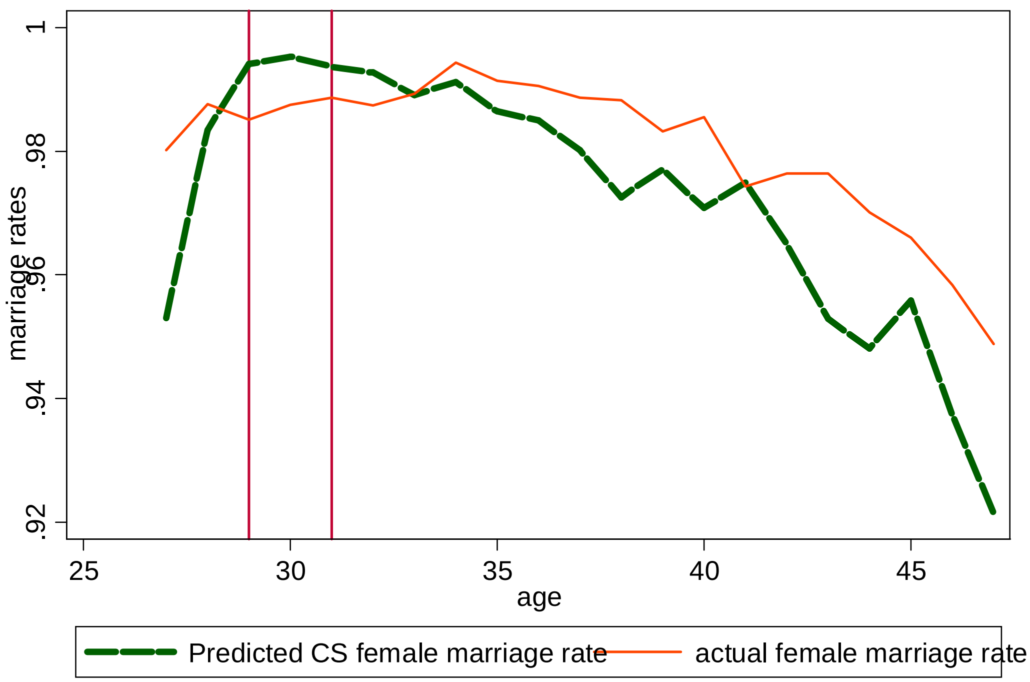


Figure 14: 1990 Sichuan female marriage rates



1.1 Positive assortative matching

- Consider the following case of the CS model where i denotes the one dimensional marriage marketing level of men and j denote the one dimensional marriage marketing level of women. So different types of individuals are individuals with different levels of by marriage marketing attainment.

- Sytematic marital output π_{ij} is increasing in i and j:

$$\pi_{ij} = \frac{\alpha_{ij} + \gamma_{ij}}{2}$$
$$\pi_{i+1,j} > \pi_{ij}$$
$$\pi_{i,j+1} > \pi_{ij}$$

- There is complementarity in spousal educational attainment. For all $\{i, j\}$:

$$\pi_{i+1,j+1} + \pi_{ij} > \pi_{i+1,j} + \pi_{i,j+1} \tag{8}$$

Consider two men with different levels of education and two women with different levels of education. Complementarity means that the sum of systematic marital output is higher with positive assortative matching than with negative assortative matching by education.

- Recalling the CS marriage matching function:

$$\ln \mu_{ij} - \frac{\ln \mu_{i0} + \ln \mu_{0j}}{2} = \pi_{ij} - \pi_{ij}^{0}$$

implies

$$l_{ij} = \ln \frac{\mu_{i+1,j+1}\mu_{ij}}{\mu_{i+1,j}\mu_{i,j+1}} = \pi_{i+1,j+1} + \pi_{ij} - \pi_{i+1,j} - \pi_{i,j+1} \tag{9}$$

The left hand side of (9) is known as the local odds ratio of the marriage distribution μ. Equation (9) says that the local odds ratio measures the degree of local complementarity of the systematic marital output function.

- The above result is remarkable in the sense that the local odds ratio do not depend on the distributions of men and women by educational attainment, i.e. M and F. This validates some demographers who study marriage matching which ignores population supplies.

- If there is complementarity in spousal educational attainment, i.e. (8) holds, then

$$l_{ij} = \ln \frac{\mu_{i+1,j+1}\mu_{ij}}{\mu_{i+1,j}\mu_{i,j+1}} > 0 \; \forall i, j$$

- Generalization to general additive random utility model (Graham; Galichon Salanie):

Let $H(\varepsilon_{ijg})$ and $G(\epsilon_{ijk})$ be the cumulative distribution functions of ε_{ijg} and ϵ_{ijk} respectively, where ε_{ijg} are the idiosyncratic payoff to man g (woman k) of type i (j) in an $\{i, j\}$ marriage.

Assume H(ε_{ijg}) and G(ϵ_{ijk}) are both strictly increasing on the entire real line with continuous, bounded derivatives.

Then sign of $\pi_{i+1,j+1} + \pi_{ij} - \pi_{i+1,j} - \pi_{i,j+1}$ *is equal to the sign of* l_{ij}.

- The generalization raises the question as to what other properties of π also generalizes beyond the Type 1 extreme value distribution for the additive idiosyncratic payoffs to marriage.

2 Directed search with non-transferable utility (ABD)

I types of men and J types of women.

m_i^k is supply of type i men and f_j^k is supply of type j woman in marriage market k.

α_{ij}^n and α_{ij}^s are mean gross gains to type i men for marriage vs cohabitation with type j woman.

γ_{ij}^n and γ_{ij}^s are mean gross gains to type j woman for marriage vs cohabitation with type i men.

Mean gross gain to remaining unattached is zero.

In each marriage market k, all individuals first choose a relationship sub-market, n or s to join.

ω_{ij}^{rk} type i men join sub-market r with type j woman in marriage market k where

$$m_i^k = \sum_{r=n,s} \sum_j \omega_{ij}^{rk}$$

Ω_{ij}^{rk} type j woman join sub-market r with type i men in marriage market k and

$$f_j^k = \sum_{r=n,s} \sum_i \Omega_{ij}^{rk}$$

μ_{ij}^{nk} and μ_{ij}^{sk} are the number of marriage vs cohabitation (i, j) relationships in marriage market k.

Number of type (i, j, r) relationships in marriage market k:

$$\mu_{ij}^{rk} = \phi(\omega_{ij}^{rk}, \Omega_{ij}^{rk}; \eta) \tag{10}$$

Individuals who do not find a relationship will remain unattached and get a utility of zero.

Let the expected utility of a type i man g in marriage market k entering relationship r with a type j woman be:

$$EU_{ij}^{rkg} = P_{ij}^{rk} e^{\alpha_i^r + \varepsilon_{ij}^{rkg}}$$

ε_{ij}^{rkg} is a standard Type 1 extreme value random variable.

Take logs and then utility maximizing type i men will generate a quasi demand for relationship n relative to s with type j woman:

$$\ln \frac{\omega_{ij}^{nk}}{\omega_{ij}^{sk}} = \ln p_{ij}^{nk} + \alpha_{ii}^{n} - (\ln p_{ij}^{sk} + \alpha_{ij}^{s}) \tag{11}$$

$$p_{ij}^{rk} = \frac{\mu_{ij}^{rk}}{\omega_{ij}^{rk}} \tag{12}$$

$$\ln \frac{\mu_{ij}^{nk}}{\mu_{ij}^{sk}} = 2 \ln p_{ij}^{nk} + \alpha_{ij}^{n} - (2 \ln p_{ij}^{sk} + \alpha_{ij}^{s}) \tag{13}$$

Similarly utility maximizing type j woman will generate a quasi supply for relationship n relative to s with type i men:

$$\ln \frac{\Omega_{ij}^{nk}}{\Omega_{ij}^{sk}} = \ln P_{ij}^{nk} + \gamma_{ij}^{n} - (\ln P_{ij}^{sk} + \gamma_{ij}^{s}) \tag{14}$$

$$P_{ij}^{rk} = \frac{\mu_{ij}^{rk}}{\Omega_{ij}^{rk}} \tag{15}$$

$$\ln \frac{\mu_{ij}^{nk}}{\mu_{ij}^{sk}} = 2 \ln P_{ij}^{nk} + \alpha_{ij}^{n} - (2 \ln P_{ij}^{sk} + \alpha_{ij}^{s}) \tag{16}$$

Summing (13) and (16):

$$\ln \frac{\mu_{ij}^{nk}}{\mu_{ij}^{sk}} = \frac{\alpha_{ij}^{n} + \gamma_{ij}^{n}}{2} - \frac{\alpha_{ij}^{s} + \gamma_{ij}^{s}}{2} + \ln \frac{p_{ij}^{nk} P_{ij}^{nk}}{p_{ij}^{sk} P_{ij}^{sk}} \tag{17}$$

Assuming a constant returns to scale meeting function:

$$\begin{aligned} p_{ij}^{rk} &= \frac{\Omega_{ij}^{rk}}{\omega_{ij}^{rk}} g(\frac{\omega_{ij}^{rk}}{\Omega_{ij}^{rk}}) \\ P_{ij}^{rk} &= g(\frac{\omega_{ij}^{rk}}{\Omega_{ij}^{rk}}) \end{aligned}$$

So (17) becomes:

$$\ln \frac{\mu_{ij}^{nk}}{\mu_{ij}^{sk}} = \frac{\alpha_{ij}^{n} + \gamma_{ij}^{n}}{2} - \frac{\alpha_{ij}^{s} + \gamma_{ij}^{s}}{2}$$
$$+ \ln \frac{\Omega_{ij}^{nk}}{\omega_{ij}^{nk}} \left[g(\frac{\omega_{ij}^{nk}}{\Omega_{ij}^{nk}}) \right]^2 - \ln \frac{\Omega_{ij}^{sk}}{\omega_{ij}^{sk}} \left[g(\frac{\omega_{ij}^{sk}}{\Omega_{ij}^{sk}}) \right]^2$$

Do comparative statics of $\ln \frac{\mu_{ij}^{nk}}{\mu_{ij}^{sk}}$ w.r.t. sex ratio.

- In each marriage market, ABM can observe $\{\mu_{ij}^{nk}, \mu_{ij}^{sk}, m_i^k, f_j^k$ They say they can identify η, γ_{ij}^r and α_{ij}^r. So you will need multiple marriage markets. (to be done. is this easier to do with only one type of man and one type of woman)

- No choice of remaining unattached. If individuals choose not to participate, then no way to estimate number of willing participants. With many type of individuals, this model explains the varying fractions of unattached individuals by type as due to search frictions. (need a more formal statement)

3 CS with frictional transfers

Let τ_{ij}^{rk} be the equilibrium transfer which the type i man makes to the type j woman in relationship r in market

k. ϕ_{ij}^{rk} is an indicator function which takes the value 1 if $\tau_{ij}^{rk} \geq 0$ and 0 otherwise. ϕ_{ij}^{rk} indicates who is a net payer for the relationship.

If $\phi_{ij}^{rk} = 1$, the value of the transfer to the woman is $\beta_{ij}^{r} \tau_{ij}^{rk}$ where $1 > \beta_{ij}^{r} > 0$. When the man is the net payer, the woman values the transfer at less than what it costs the man. If $\phi^{rk} = 0$, the value of the transfer to the woman is $\tau_{ij}^{rk} / \beta_{ij}^{r}$. When the woman is the net payer, the man values the transfer at less than what it costs the woman.

Why is this plausible? Since transfers in relationships may not be monetary , giver and receiver may value transfers differently. This different valuations does raise two interesting questions:

Does the competitive solution still coincide with a planner's solution ala Galichon and Salanie? We will have a surprising answer to this question.

How does frictions in marital transfers reduce the marriage rate?

Making the standard Type 1 extreme value assumption for the additive idiosyncratic payoff to a choice, the quasi demand by type i men for type j woman in relationship r becomes:

$$\ln \frac{\mu_{ij}^{rk}}{\mu_{i0}^{k}} = \alpha_{ij}^{r} - \tau_{ij}^{rk} \tag{18}$$

Utility maximizing woman will generate a quasi demand for relationship r:

$$\begin{aligned} \ln \frac{\mu_{ij}^{rk}}{\mu_{0j}^{k}} &= \gamma_{ij}^{r} + \phi_{ij}^{rk}\beta_{ij}^{r}\tau_{ij}^{rk} + (1-\phi_{ij}^{rk})\frac{\tau_{ij}^{rk}}{\beta_{ij}^{r}} && (19) \\ &= \gamma_{ij}^{r} + \Phi_{ij}^{rk}\tau_{ij}^{rk} && (20) \end{aligned}$$

where $\Phi_{ij}^{rk} = \phi_{ij}^{rk}\beta_{ij}^{r} + (1-\phi_{ij}^{rk})(\beta_{ij}^{r})^{-1}$.

Assuming market equilibrium and solving out τ_{ij}^{rk} :

$$\Phi_{ij}^{rk} \ln \frac{\mu_{ij}^{rk}}{\mu_{i0}^{k}} + \ln \frac{\mu_{ij}^{rk}}{\mu_{0j}^{k}} = \Phi_{ij}^{rk}\alpha_{ij}^{r} + \gamma_{ij}^{r} = \pi_{ij}^{rk} \qquad (21)$$

(21) is an equilibrium relationship. Let $\Phi_{ij}^{nk} = \Phi_{ij}^{sk} + \delta_{ij}^{k}$. Then

$$(1 + \Phi_{ij}^{sk}) \ln \frac{\mu_{ij}^{nk}}{\mu_{ij}^{sk}} + \delta_{ij}^{k} \ln \frac{\mu_{ij}^{nk}}{\mu_{i0}^{k}} = \pi_{ij}^{nk} - \pi_{ij}^{sk}$$

$$\ln \frac{\mu_{ij}^{nk}}{\mu_{ij}^{sk}} = \frac{\pi_{ij}^{nk} - \pi_{ij}^{sk} - \delta_{ij}^{k}\alpha_{ij}^{n}}{1 + \Phi_{ij}^{sk}} + \frac{\delta_{ij}^{k}\tau_{ij}^{nk}}{1 + \Phi_{ij}^{sk}}$$

If $\delta_{ij}^{k} \neq 0$, then changes in τ_{ij}^{nk}, which will be affected by changes in the sex ratio in marriage market k, m_i^k/f_j^k, will affect $\ln \frac{\mu_{ij}^{nk}}{\mu_{ij}^{sk}}$. I.e. the sex ratio in a marriage market affects the bargaining power between genders and also the types of relationships which are form.

If $\delta^k_{ij} = 0$, then $\ln \frac{\mu^{nk}_{ij}}{\mu^{sk}_{ij}}$ is invariant to changes in the sex ratio if the changes in sex ratio are sufficiently small so that τ^{sk}_{ij} do not change signs as the sex ratio changes. Even if τ^{rk}_{ij} do not change signs as the sex ratio changes, $\delta^k_{ij} \neq 0$ if $\beta^n_{ij} \neq \beta^s_{ij}$.

With data on multiple marriage markets, and assuming the changes in sex ratios across marriage markets are sufficiently small so that $\Phi^{rk}_{ij} = \Phi^{rk}_{ij} = \Phi^r_{ij}$, we can identify Φ^r_{ij} and π^r_{ij}:

$$\Phi^r_{ij} = \left[\ln \frac{\mu^{rk'}_{ij}\mu^k_{0j}}{\mu^{k'}_{0j}\mu^{rk}_{ij}}\right]\left[\ln \frac{\mu^{rk}_{ij}\mu^{k'}_{i0}}{\mu^{k'}_{i0}\mu^{rk}_{ij}}\right]^{-1}$$

If $\Phi^r_{ij} < 1$ then $\tau^{rk}_{ij} > 0$. If $\Phi^r_{ij} > 1$, then $\tau^{rk}_{ij} < 0$.

- There are still lots of overidentifying restrictions with multiple marriage markets because we only increased the number of parameters over CS by 2.

- Can we also identify gender preferences for the type of relationships, n versus s?

- The CS model assumes $\beta_{ij}^r = 1$. In this case, (21) becomes:

$$\ln \frac{\mu_{ij}^{rk}}{\sqrt{\mu_{i0}^k \mu_{0j}^k}} = \frac{\alpha_{ij}^r + \gamma_{ij}^r}{2} = \pi_{ij}^r$$

Also:

$$\ln \frac{\mu_{ij}^{nk}}{\mu_{ij}^{sk}} = \pi_{ij}^n - \pi_{ij}^s \tag{22}$$

- π_{ij}^r is identified from data from one marriage market. α_{ij}^r and γ_{ij}^r are not separately identified. Data from other marriage markets are overidentifying.

- It is reasonable to assume that $\Phi_{ij}^{rk} = \Phi_{ij}^{rk} = \Phi_{ij}^r$. The reason is that if the sex ratios across marriage markets are so different such that the gender of the net giver changes across markets, it is likely that other parameters of the model are also changing across markets.

- If $\ln \frac{\mu_{ij}^{nk}}{\mu_{ij}^{sk}}$ is inversely correlated with the sex ratio, $\ln \frac{m_i^k}{f_j^k}$, then CS is wrong. More than that, you can likely reject the CS class ala Graham above [needs proof].

- Important identifying assumption of frictional transfer model compared with directed search: With many types of students, this model explains the varying fractions of unattached students by type as due to voluntary choice. No one is unattached involuntarily. What if some students are unattached involuntarily. Then it becomes an identification problem because we cannot compute the number of voluntarily unattached.

3.1 Chiappori, Salanie and Weiss (CSW)

They extend CS to heteroscedastic idiosyncratic errors and derive the marriage matching function using stability arguments.

Applying to our context, let the additive idioyscratic component of a particular type i man g in relationship r in marriage market k with a type j woman be $\sigma_i^r \varepsilon_{ijg}^{rk}$ where ε_{ijg}^{rk} is a Type 1 extreme value random variable where $\sigma_i^r > 0$. Let the systematic return to remaining unmarried by zero.

Before he sees his idiosyncratic realizations, the probability that g will choose relationship r with type woman j is:

$$\Pr(V_{ijg}^{rk} - V_{ihg}^{r'k} \geq 0 \; \forall h, r' = 0, n, s) = \Pr(\frac{\alpha_{ij}^r - \tau_{ij}^{rk}}{\sigma_i^r} - \frac{\alpha_{ih}^r - \tau_{ih}^{r}}{\sigma_i^{r'}}$$

Then

$$\mathrm{Pr}(V_{ijg}^{rk} - V_{ijg}^{r'k} \geq 0 \; \forall h, r' = 0, n, s) = \frac{\mu_{ij}^{rk}}{m_i^k} = \frac{\exp(\alpha_{ij}^{r} - \tau_{ij}^{rk}}{\sum_{r'} \exp(\alpha_{ih}^{r'} - \tau_{i}^{r}}$$

Obtain quasi-demand function of type i men in marriage market k for r type relationship with type j woman relative to remaining unattached is:

$$\begin{aligned} \ln \mu_{ij}^{rk} - \ln \mu_{i0}^{k} &= \frac{\alpha_{ij}^{r} - \tau_{ij}^{rk}}{\sigma_i^r} - \frac{0}{\sigma_i^0} \\ \sigma_i^r(\ln \mu_{ij}^{rk} - \ln \mu_{i0}^{k}) &= \alpha_{ij}^{r} - \tau_{ij}^{rk} \end{aligned}$$

Similarly, let the additive idioyscratic component of a particular woman q of type j in relationship r in marriage market k with a type i man be $\Sigma_j^r \varepsilon_{ijq}^{rk}$ where ε_{ijq}^{rk} is a Type 1 extreme value random variable where $\Sigma_j^r > 0$. Again let the systematic return to remaining unmarried be zero.

Obtain quasi-supply function of type j woman in marriage market k for r type relationship with type i men relative to remaining unattached is:

$$\Sigma_j^r(\ln \mu_{ij}^{rk} - \ln \mu_{0j}^k) = \gamma_{ij}^r + \tau_{ij}^{rk}$$

Market equilibrium implies

$$\frac{\sigma_i^r}{\Sigma_j^r} \ln \frac{\mu_{ij}^{rk}}{\mu_{i0}^k} + \ln \frac{\mu_{ij}^{rk}}{\mu_{0j}^k} = \frac{\alpha_{ij}^r + \gamma_{ij}^r}{\Sigma_j^r} = \pi_{ij}^r \tag{23}$$

Comparing (21) and (23), let

$$\Phi_{ij}^{rk} = \Phi_{ij}^r = \frac{\sigma_i^r}{\Sigma_j^r}$$

Then CSW is observationally equivalent to CS with frictional transfers. From Galichon and Salanie, this means that the equilibrium outcome of CS with frictional transfers is equivalent to the utilitarian social planner solving the CSW marriage matching problem.

The observation equivalence between CS with frictional transfers and CSW also raises a normative problem. We cannot tell what the social planner‘s objective function is from observed marriage matching data.

As discussed in the frictional transfer model, CSW is overidentified with multiple marriage markets.

4 References

Arcidiacono, Beauchamp, McElroy (2010). “Terms of Endearment: An Equilibrium Model of Sex and Matching”. NBER working paper.

Brandt, Siow and Vogel (2009), “Large shocks and small changes in the marriage market”. IZA discussion paper.

Chiappori, Salanie and Weiss (2011). “Partner choice and the college marriage premium”. Columbia working paper.

Eugene Choo and Aloysius Siow (2006), “Who Marries Whom and Why”. *JPE*.

Colin Decker, Elliott H. Lieb, Robert M. McCann, and Benjamin K. Stephens (2012), “Unique Equilibria and Substitution Effects in a Stochastic Model of the Marriage Market” *JET* forthcoming.

Alfred Galichon and Bernard Salanie (2012), “Cupid’s Invisible Hand: Social Surplus and Identification in Matching Models”. Columbia working paper.

Aloysius Siow (2009), “Testing Becker’s Theory of Positive Assortative Matching”. University of Toronto working paper.

Bryan Graham (2011), “Econometric Methods for the Analysis of Assignment Problems in the Presence of Complementarity and Social Spillovers”. *Handbook of Social Economics*.