A Referendum Experiment with Participation Quorums*

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Abstract

This paper analyzes a *yes-no* referendum in which its outcome is valid only if the voter turnout is greater than a predetermined level. Such a participation quorum is argued to induce the minority group of voters to abstain strategically with the intention to spoil the outcome by achieving a low voter turnout. We first construct a game-theoretic model to derive a theoretical prediction about the relationship between quorums and voting outcomes. It is shown that there exist multiple equilibria, and that strategic abstention *can* happen if such a participation quorum is imposed. To examine which type of outcome is more likely to be realized, we then conduct a laboratory experiment. We observe that (*i*) if the quorum is small, all voters go to the poll, and (*ii*) if the quorum is large, voters in the *ex-ante* majority group go to the poll whereas voters in the ex-ante minority group tend to abstain. As a result, it is less likely that the *ex-post* minority group wins the referendum, but it frequently happens that the voting outcome is made invalid due to low voter turnout when the quorum is large.

Keywords: referendum, participation quorum, voter turnout, laboratory experiment

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1 Introduction

Imposing participation quorums is observed in national referendums of, for example, Italy, Portugal, Romania, Slovenia and Slovakia (mentioned by Côrte-Real and Pereira, 2004) and local referendums of Japan and the U.S. Most of them require their voter turnout rates to be at least 50% for the validity of their outcomes. The main idea behind such a quorum requirement is statistical in the sense that the vote distribution realized in a referendum is a fair sample of the opinion of the whole population only when its voter turnout is sufficiently large. However, this statement is true only when voters behave sincerely. In fact, theoretical works which assume strategic voters in non-cooporative games, such as Aguiar-Conraria and Magalhães (2010b), and Hizen and Shinmyo (2011), show that imposing such a quorum requirement can induce strategic abstention which tries to spoil the outcome rather than going to the poll to lose the referendum, and that such behaviors may distort the outcome in favor of the minority.[1] Empirical works, such as Murata (2006), and Aguiar-Conraria and Magalhães (2010a), confirm that imposing a participation quorum decreases the voter turnout.

Following these theoretical and empirical literatures, this paper examines experimentally the effect of participation quorums on voting behaviors and outcomes. Laboratory experiments enable us to observe directly how institutional rules work by controlling all the other factors. In particular, our experiment enables us to obtain data regarding not only 50% as in the most actual referendums but also other levels of quorums. Hence, we can analyze not only the effect of the *presence* of participation quorum, but also the relationship between the *level* of quorum and voter turnout.

Our experiment is related to the following two literatures of voting experiments, which analyze voter turnout and vote coordination respectively. The literature on voter turnout describes an election as a costly participation game between two groups, and compare their experimental results with the theoretical predictions (see Schram and Sonnemans (2008) for a survey). Among them, the effect of voting rules is examined by Schram and Sonnemans (1996). They compare voter turnout between the plurality rule and proportional representation.

The literature on vote coordination analyzes a three-way race among two majority candidates and one minority candidate. They consider what kind of information helps

[1] Other theoretical approaches include an axiomatic approach (Côrte-Real and Pereira, 2004), a group-based model (Herrera and Mattozzi, 2010), and a non-strategic voter model (Zwart, 2010). Maniquet and Morelli (2011) also assume strategic voters under population uncertainty. All of them obtain negative results against participation quorums.

the coordination between two split-majority voters against one minority group (see Rietz (2003) for a survey). Among them, the effect of voting rules is examined by Gerber, Morton and Rietz (1998). They compare how often the minority candidate wins between straight voting and cumulative voting.

The participation quorum in referendums is the voting rule that we deal with in this paper. We focus on how it affects the voter turnout and, as a result, how often the outcome is made invalid or minority voters win. In our experiment, subjects are divided into two groups randomly, and the expected number of members is greater for one group (called *ex-ante majority*) than the other (called *ex-ante minority*). Each subject knows her own group but does not know how the other subjects are divided into the two groups.

As a preparation for the experiment, we first construct a game-theoretic model of a referendum with participation quorums. Our model yields multiple equilibria, which include a full-turnout equilibrium, a full-abstention equilibrium, equilibria in which one group goes to the poll whereas the other group abstains, and mixed-strategy equilibria. Our experiment works as a device of equilibrium selection so that it tells us which type of equilibrium outcome is more likely to be realized for each level of quorum.

We observe that (*i*) if the quorum is small, all voters go to the poll, and (*ii*) if the quorum is large, voters in the ex-ante majority go to the poll whereas voters in the ex-ante minority tend to abstain. As a result, it is less likely that the *ex-post* minority wins the referendum, but it frequently happens that the voting outcome is made invalid due to low voter turnout when the quorum is large. Therefore, when politicians design referendums with participation quorums, the possibility of large quorums inducing strategic abstention must be taken into account.

This paper is organized as follows. Section 2 constructs a model. Section 3 derives equilibria. Section 4 describes our experimental design. Experimental observations are provided in Section 5. Section 6 concludes. Appendix includes the instruction used in our experiment.

2 The Model

In this section, we describe a yes-no referendum with a participation quorum as a static game of incomplete information in order to examine the relationship between the level of quorum and voting behaviors. Our experiment is designed based on this model.

Our model is closely related to the model of Hizen and Shinmyo (2011). They assume that, if the outcome is invalid due to low voter turnout, alternative *no* is selected; that is, the status quo is *no*. In this paper, as explained below, we assume symmetry between alternatives, but introduce asymmetry only in the expected number of members between two groups.

2.1 Basic Structure

A yes-no referendum is held among m (> 0) voters. Voters are independently given preferences regarding the subject of the referendum: each voter is given "*yes*" (abbreviated as "y") with probability $s \in (0, 1)$ and "*no*" (abbreviated as "n") with probability $1 - s$. The number of voters m and the probability s are common knowledge among voters, but each voter does not know the realized preferences of the other voters. Each voter has one vote. After their preferences are given, voters simultaneously and non-cooperatively vote for *yes* or *no*, or abstain. That is, the pure strategy of each voter is a function from her own preference to her voting behavior, $\{y, n\} \to \{y, n, a\}$, where "a" represents abstention. Mixed strategies are also allowed.

Let m_i $(i = y, n, a)$ denote the number of voters who have chosen i in the ex-post sense. The outcome of the referendum is valid only if the voter turnout is greater than or equal to a predetermined number. In other words, the outcome is valid if $m_y + m_n \geq [rm]$, and is invalid otherwise, where $r \in [0, 1]$ is the turnout rate that must be achieved for the validity of the outcome, and $[x]$ represents the smallest integer greater than or equal to x. For the computational ease of pivot probabilities, as in Hizen and Shinmyo's (2011) analysis, we impose the following two assumptions on the values of m and r:

Assumption 1: *m is an odd number;*

Assumption 2: *$[rm]$ is an odd number.*

When the outcome is valid, alternative *yes* is selected as the outcome if $m_y > m_n$, whereas alternative *no* is selected if $m_y < m_n$. If $m_y = m_n$, either alternative *yes* or *no* is selected equiprobably.

Each voter whose preference is *yes* (*no*, respectively) enjoys a benefit of 1 if alternative *yes* (*no*) is selected as the outcome, whereas the benefit is 0 if alternative *no* (*yes*) is selected. Since only the relative relationship between the benefits matters for voters' decisions, this normalization of benefits (i.e., 0 and 1) does not affect the equilibrium analysis but makes calculations easier. If the outcome is invalid, each voter receives a benefit of $v \in (0, 1)$. That is, the invalid outcome is worse than her preferred outcome but is better than the non-preferred outcome. We assume that going to the poll costs nothing. Under this assumption, the only reason why voters abstain is to spoil the outcome by decreasing the voter turnout. Each voter chooses her strategy to maximize her expected benefit by taking care of how her vote affects the outcome.

If every voter selects a strategy that maximizes her expected benefit given the other voters' strategies, then the strategy profile is called a *Bayesian Nash equilibrium*. A Bayesian Nash equilibrium is called *symmetric* if all the voters choose the same strategy. In the analysis, we focus on symmetric Bayesian Nash equilibria.

Voting for the non-preferred alternative is a weakly dominated strategy for any voter, in that voting for her preferred alternative has the same effect as voting for the non-preferred alternative with respect to increasing the voter turnout, but voting for her preferred alternative produces the preferred result when her vote changes the winner from one alternative to the other. Our analysis is also focused on equilibria in which voters do not use weakly dominated strategies; that is, voters either go to the poll to vote for their preferred alternative or abstain.

2.2 Pivot Probabilities

Let us describe the pivot probabilities for each vote. Let σ_i $(i = y, n)$ denote the probability that a voter with preference i chooses action i. Then, $1-\sigma_i$ is the probability of abstention. A vote for alternative i can affect the outcome in the following three ways. The first is to validate the outcome with alternative i being selected. This happens with certainty if, except for one vote, $m_y + m_n = [rm] - 1$ and $m_i \geq m_j$ $(j \neq i,\ j = y, n)$ hold, and with probability 1/2 if, except for one vote, $m_y + m_n = [rm] - 1$ and $m_i = m_j - 1$ hold. However, the latter event never occurs under Assumption 2. Hence, this probability is written as

$$p_i = \sum_{k=0}^{\frac{[rm]-1}{2}} \frac{(m-1)!}{k!([rm]-1-k)!(m-[rm])!} \pi_i^{[rm]-1-k} \pi_j^k \pi_a^{m-[rm]},$$

where $\pi_y = s\sigma_y$, $\pi_n = (1-s)\sigma_n$, and $\pi_a = 1 - \pi_y - \pi_n$.

The second is to validate the outcome with alternative j being selected. This happens with certainty if, except for one vote, $m_y + m_n = [rm] - 1$ and $m_j \geq m_i + 2$ hold, and with probability 1/2 if, except for one vote, $m_y + m_n = [rm] - 1$ and $m_j = m_i + 1$ hold. The latter event never occurs under Assumption 2. Hence, this probability is written as

$$q_i \quad = \quad \sum_{k=0}^{\frac{[rm]-1}{2}-1} \frac{(m-1)!}{k!([rm]-1-k)!(m-[rm])!} \pi_i^k \pi_j^{[rm]-1-k} \pi_a^{m-[rm]}.$$

The third is for a vote for i to change the winner from alternative j to alternative i when the outcome is valid even without that vote. This happens with probability 1/2 if, except for one vote, $m_y + m_n \geq [rm]$ and either $m_i = m_j - 1$ or $m_y = m_n$ hold. By Assumptions 1 and 2, this probability is written as

$$t_i = \frac{1}{2} \sum_{k=\frac{[rm]-1}{2}}^{\frac{m-1}{2}-1} \frac{(m-1)!}{k!(k+1)!(m-2k-2)!} \pi_i^k \pi_j^{k+1} \pi_a^{m-2k-2}$$
$$+ \frac{1}{2} \sum_{k=\frac{[rm]-1}{2}+1}^{\frac{m-1}{2}} \frac{(m-1)!}{k!k!(m-2k-1)!} \pi_y^k \pi_n^k \pi_a^{m-2k-1}.$$

Given these probabilities, voters with preference i vote for i only if

$$(1-v)p_i + t_i \geq v q_i. \tag{1}$$

3 Theoretical Analysis

In this section, we derive symmetric Bayesian Nash equilibria of the above model, in which nobody uses weakly dominated strategies. We first deal with a benchmark where quorums are not imposed or are ineffectively small. Then, we examine how quorums affect voting behaviors if they are sufficiently large.

3.1 Ineffectively Small Quorums

If participation quorums are not imposed (i.e., $r = 0$), the outcome is always valid. Then, voters care only about the pivot probability whereby their votes will change the winner (i.e., $p_i = q_i = 0$ and $t_i > 0$, $i = y, n$). This is the well-known two-candidate election, in which voting for the most-preferred alternative is optimal even for strategic voters.

For $r \in (0, 1/m]$, one vote is sufficient to validate the outcome. In other words, a vote changes the outcome from invalid to valid only when all other voters abstain. Therefore, when a vote for i validates the outcome, j ($j \neq i$) is never the outcome (i.e., $q_i = 0$) so that the incentive of abstention does not exist. We obtain our first proposition.

Proposition 1 *For $r \leq 1/m$, the unique Bayesian Nash equilibrium is* $(\sigma_y = 1, \sigma_n = 1)$.

Since all voters go to the poll and vote for their preferred alternatives, the outcome, or the distribution of votes for *yes* and *no*, reflects voters' preferences exactly for such small quorums.

3.2 Effectively Large Quorums

Next, let us consider effectively large quorums. We can divide the range of effectively large quorums into two intervals, $r \in (1/m, (m-1)/m]$ and $r \in ((m-1)/m, 1]$. We first consider the interval $r \in (1/m, (m-1)/m]$. Then, the second interval, in which full turnout is required for the validity of the outcome, is examined.

3.2.1 Interval $r \in (1/m, (m-1)/m]$

For this range of quorum, one vote is not sufficient to validate the outcome, nor is full turnout required. Therefore, regardless of the behavior of one voter, the outcome is invalid if all other voters abstain, whereas the outcome is valid if all other voters go to the poll. This implies that both zero turnout, $(\sigma_y = 0, \sigma_n = 0)$, and full turnout, $(\sigma_y = 1, \sigma_n = 1)$, are realized in equilibrium.

Suppose that *yes*-voters go to the poll whereas *no*-voters abstain (i.e., $(\sigma_y = 1, \sigma_n = 0)$). Under this strategy profile, validating the outcome must be accompanied by *yes* being selected as the outcome (i.e., $p_y, q_n > 0$ and $p_n = q_y = 0$). In addition, any valid outcome necessarily implies that *yes* is selected (i.e., $t_i = 0$, $i = y, n$). Therefore, *yes*-voters go to the poll without worrying about their votes resulting in *no* being selected as the outcome, whereas each *no*-voter can only spoil the outcome by abstaining. That is, this strategy profile constitutes an equilibrium. Similarly, $(\sigma_y = 0, \sigma_n = 1)$ is also an equilibrium. The same logic suggests that neither $(\sigma_y = 0, \sigma_n \in (0, 1))$ nor $(\sigma_y \in (0, 1), \sigma_n = 0)$ constitutes an equilibrium because the group members using a mixed strategy will switch to voting for their preferred alternative with certainty.

Next, let us consider the remaining strategy profiles. Suppose $(\sigma_y = 1, \sigma_n \in (0, 1))$. This strategy profile is incentive compatible if there exists a value of $\sigma_n \in (0, 1)$ that satisfies equation (1) with either equality or inequality for $i = y$ and with equality for

$i = n$. These two conditions are combined as follows:

$$\frac{p_y + t_y}{p_y + q_y} \geq \frac{p_n + t_n}{p_n + q_n} = v. \tag{2}$$

The equality (i.e., the incentive constraint for *no*-voters) determines the value of σ_n as a function of four parameters, m, s, v, and r. For such a value of σ_n to constitute an equilibrium, the value of σ_n must be between 0 and 1, and also must satisfy the inequality (i.e., the incentive constraint for *yes*-voters). When σ_n converges to 0, p_y and q_n converge to a positive value (i.e., $\frac{(m-1)!}{([rm]-1)!(m-[rm])!} s^{[rm]-1}(1-s)^{m-[rm]}$), whereas other pivot probabilities converge to 0, which implies that fraction $(p_y + t_y)/(p_y + q_y)$ converges to 1 while fraction $(p_n + t_n)/(p_n + q_n)$ converges to 0. Since pivot probabilities are continuous in σ_n, therefore, at least for sufficiently small values of v, we can find $\sigma_n \in (0, 1)$ that satisfies equation (2).

Does this type of Bayesian Nash equilibrium exist for any set of parameter values? We can show by construction that it does not. For example, suppose that s is sufficiently small. Then, fraction $(p_y + t_y)/(p_y + q_y)$ in equation (2) is greater than fraction $(p_n + t_n)/(p_n + q_n)$ only if $\sigma_n \leq s/(1-s)$ or if σ_n is close to 1.[2] In the case of $\sigma_n \leq s/(1-s)$, t_n is small relative to p_n and q_n because the expected level of voter turnout is low. Hence, fraction $(p_n + t_n)/(p_n + q_n)$ is sufficiently smaller than 1. In the case that σ_n is close to 1, on the other hand, t_n is much greater than p_n and q_n because almost all voters are expected to go to the poll. Hence, fraction $(p_n + t_n)/(p_n + q_n)$ is greater than 1. As a result, if v is close to 1, equation (2) does not hold.

Finally, let us consider $(\sigma_y \in (0, 1), \sigma_n \in (0, 1))$. This strategy profile constitutes an equilibrium if there exists a pair $(\sigma_y, \sigma_n) \in (0, 1) \times (0, 1)$ that satisfies equation (1) with equality for $i = y, n$. These two conditions are combined as follows:

$$\frac{p_y + t_y}{p_y + q_y} = \frac{p_n + t_n}{p_n + q_n} = v. \tag{3}$$

As mentioned above, given a value of $\sigma_y \in (0, 1)$, the convergence of σ_n to 0 leads to fraction $(p_y + t_y)/(p_y + q_y)$ converging to 1, whereas fraction $(p_n + t_n)/(p_n + q_n)$ converges

[2]The case of $\sigma_n \leq s/(1-s)$ implies that $\pi_y \geq \pi_n$. Under such a condition, we have $p_y \geq p_n$ and $t_y \leq t_n$. For sufficiently small s, the expected level of voter turnout is low (i.e., $\pi_y + \pi_n = s + (1-s)\sigma_n \leq s + (1-s)\frac{s}{1-s} = 2s$), and so p_y and p_n are much greater than t_y and t_n. On the other hand, the case in which σ_n is close to 1 implies that $\pi_y < \pi_n$ for sufficiently small s. Under such a condition, we have $p_y < p_n$ and $t_y > t_n$. Since the expected level of voter turnout is close to 100%, t_y and t_n are much greater than p_y and p_n. Both cases result in $p_y + t_y \geq p_n + t_n$. Note that the denominators of the two fractions satisfy $p_y + q_y = p_n + q_n < 1$ for any $\sigma_i \in (0, 1)$ $(i = y, n)$ and parameter values.

to 0. The opposite is also true: given a value of $\sigma_n \in (0,1)$, the convergence of σ_y to 0 leads to fraction $(p_y+t_y)/(p_y+q_y)$ converging to 0 whereas fraction $(p_n+t_n)/(p_n+q_n)$ converges to 1. Therefore, for each set of parameter values, we can find a pair $(\sigma_y, \sigma_n) \in (0,1)\times(0,1)$ that satisfies the first equality in equation (3). Then, the question is whether such a pair also satisfies the second equality for each value of v. As shown below, the answer is that it does not necessarily.

Suppose that v is sufficiently small. Then, the numerators of the two fractions $p_i + t_i$ $(i = y, n)$ must be sufficiently small. The convergence of the two numerators to 0 requires that both σ_y and σ_n converge to 0. However, this must also be accompanied by the convergence of the denominators to 0. Hence, we need to determine the limit of the two fractions. Suppose that we let σ_y and σ_n converge to 0 keeping either $\pi_y > \pi_n$ or $\pi_y < \pi_n$ but satisfying $|p_y - p_n| = |t_n - t_y| > 0$ so that the first equality in equation (3) holds. However, since t_i is of higher order than p_i and q_i with respect to σ_y or σ_n, this condition does not hold for sufficiently small values of σ_y and σ_n. Hence, let σ_y and σ_n converge to 0 keeping $\pi_y = \pi_n$. Then, the first equality in equation (3) holds for any value of σ_y. In the limit, for $i = y, n$, we have

$$\lim_{\sigma_y \to 0|_{\pi_y=\pi_n}} \frac{p_i + t_i}{p_i + q_i} = \frac{\sum_{k=0}^{\frac{[rm]-1}{2}} \frac{1}{k!([rm]-1-k)!}}{2\sum_{k=0}^{\frac{[rm]-1}{2}} \frac{1}{k!([rm]-1-k)!} - \frac{1}{\left[\left(\frac{[rm]-1}{2}\right)!\right]^2}} > \frac{1}{2}.$$

Therefore, the second equality in equation (3) does not hold for sufficiently small values of v. Intuitively, if the invalid outcome is not attractive, members of at least one group will go to the poll with certainty. We have the following proposition:

Proposition 2 *For $r \in (1/m, (m-1)/m]$,*

(i) $(\sigma_y = 1, \sigma_n = 1)$, $(\sigma_y = 0, \sigma_n = 0)$, $(\sigma_y = 1, \sigma_n = 0)$, and $(\sigma_y = 0, \sigma_n = 1)$ are Bayesian Nash equilibria for any parameter values;
(ii) $(\sigma_y = 1, \sigma_n \in (0,1))$, $(\sigma_y \in (0,1), \sigma_n = 1)$, and $(\sigma_y \in (0,1), \sigma_n \in (0,1))$ are Bayesian Nash equilibria for a subset of parameter values;
(iii) $(\sigma_y = 0, \sigma_n \in (0,1))$ and $(\sigma_y \in (0,1), \sigma_n = 0)$ are never Bayesian Nash equilibria.

3.2.2 Interval $r \in ((m-1)/m, 1]$

Finally, let us consider what happens if full turnout is required for the validity of the outcome (i.e., $[rm] = m$). When even one voter can spoil the outcome by abstaining under $r > (m-1)/m$, full turnout is more difficult to realize because any voter whose preferred alternative is less likely to win will abstain. This incentive leads to the following proposition:

Proposition 3 *For* $r > (m-1)/m$,

(i) $(\sigma_y = 0, \sigma_n = 0)$, $(\sigma_y = 1, \sigma_n = 0)$, *and* $(\sigma_y = 0, \sigma_n = 1)$ *are Bayesian Nash equilibria for any parameter values;*
(ii) $(\sigma_y = 1, \sigma_n = 1)$, $(\sigma_y = 1, \sigma_n \in (0,1))$, $(\sigma_y \in (0,1), \sigma_n = 1)$, *and* $(\sigma_y \in (0,1), \sigma_n \in (0,1))$ *are Bayesian Nash equilibria for a subset of parameter values;*
(iii) $(\sigma_y = 0, \sigma_n \in (0,1)$ *and* $(\sigma_y \in (0,1), \sigma_n = 0)$ *are never Bayesian Nash equilibria.*

Strategy profiles $(\sigma_y = 1, \sigma_n = 1)$ and $(\sigma_y \in (0,1), \sigma_n \in (0,1))$ are examined below. See Appendix for the other strategy profiles.

There are two differences from the case of $r \in (1/m, (m-1)/m]$. First, as suggested above, full turnout $(\sigma_y = 1, \sigma_n = 1)$ can happen only for a subset of parameter values under $r > (m-1)/m$. Let us examine this case. Since every vote is necessary for the validity of the outcome, we have $t_y = t_n = 0$ for such r's. Hence, the incentive constraint for full turnout is written as

$$max \left\{ \frac{q_y}{p_y}, \frac{q_n}{p_n} \right\} \leq \frac{1-v}{v}. \tag{4}$$

For what range of parameter values, v and s, is this condition easier to satisfy given m and r? Since the right-hand side converges to infinity when v converges to 0, inequality (4) holds for most values of s if v is sufficiently small. Small values of v mean small benefits from invalid outcomes, which induces voters to go to the poll.

The left-hand side of inequality (4) is smaller when the values of q_y/p_y and q_n/p_n are closer to each other. The reason for this is that q_y/p_y is decreasing in s whereas q_n/p_n is increasing in s (which comes from the fact that p_y and q_n are increasing in s whereas p_n and q_y are decreasing in s). Therefore, suppose that $q_y/p_y = q_n/p_n$, which holds at $s = 1/2$. Then, for $i = y, n$, we have

$$\frac{q_i}{p_i} = 1 - \frac{1}{\left[\left(\frac{m-1}{2}\right)!\right]^2 \sum_{k=0}^{\frac{m-1}{2}} \frac{1}{k!(m-1-k)!}}. \tag{5}$$

This formula is increasing in m and converges to 1 when m converges to infinity. Therefore, inequality (4) does not hold for sufficiently large values of v. Even when $m = 3$, for example, inequality (4) does not hold for $v > 3/4$.

The second difference from the case of $r \in (1/m, (m-1)/m]$ is that the strategy profile $(\sigma_y \in (0,1), \sigma_n \in (0,1))$ constitutes an equilibrium for a measure-zero set of parameter values. The incentive constraint for this strategy profile is

$$\frac{q_y}{p_y} = \frac{q_n}{p_n} = \frac{1-v}{v}.$$

The first equality holds if and only if $\pi_y = \pi_n$. Then, equation (5) also holds for any such pair of σ_y and σ_n. Hence, only when m and v equate $(1-v)/v$ with the right-hand side of equation (5), this strategy profile constitutes an equilibrium.

4 Experimental Design

In this section, we describe our experimental design. As shown in Section 3, there exist multiple equilibria except for $r \leq 1/m$. We conduct a laboratory experiment to analyze which equilibrium outcome is more likely to be realized for each set of parameter values.

4.1 Parameter Values

In our experiment, we specify the parameter values of the above model as follows. The total number of voters is $m = 13$. We call alternative *yes* (*no*, respectively) as alternative A (B), and we also call the group of voters who are given preference A (B) as group A (B). The probability of each voter being assigned to group A is either $s = 0.51$ (*close race*) or $s = 0.6$ (A-*dominance*). The benefit from the invalid outcome is $v = 0.5$. Quorums $\lceil rm \rceil$ are set to be 1, 3, 5, 7, 9, 11 and 13.

4.2 Theoretical Predictions

For the above parameter values, Propositions 1 and 2 lead to the following corollary:

Corollary 1 *In both cases of $s = 0.51$ and $s = 0.6$:*
(i) For $[rm] = 1$, the unique Bayesian Nash equilibrium is that all voters go to the poll.
(ii) For $[rm] = 3, 5, 7, 9, 11$, there exist the following symmetric pure-strategy Bayesian Nash equilibria: (1) all voters go to the poll; (2) all voters abstain; (3) A-voters go to the poll whereas B-voters abstain; and (4) A-voters abstain whereas B-voters go to the poll.

Here, we examine the remaining cases, that is, symmetric pure-strategy equilibria for $[rm] = 13$ and symmetric mixed-strategy equilibria for each value of $[rm]$. In the former case, Proposition 3 says that at least the following three equilibria exist: (1) all voters abstain; (2) A-voters go to the poll whereas B-voters abstain; and (3) A-voters abstain whereas B-voters go to the poll. Now we consider whether all voters go to the poll in equilibrium for $[rm] = 13$. Since each A-voter has the stronger incentive to go to the poll than each B-voter, we only have to examine whether a B-voter would like to go to the poll or abstain when all the other voters go to the poll. Under $[rm] = 13$, the full turnout is required for the validity of the outcome. Hence, a vote for alternative B can affect the outcome in the following two ways. First, it validates the outcome and leads to B's win, whose probability is denoted by $p_B(s)$. Second, it validates the outcome and leads to A's win, whose probability is denoted by $q_B(s)$, where

$$p_B(s) = \sum_{k=0}^{6} \frac{12!}{k!(12-k)!}(1-s)^{12-k}s^k,$$
$$q_B(s) = \sum_{k=0}^{5} \frac{12!}{k!(12-k)!}(1-s)^{k}s^{12-k}.$$

Each B-voter goes to the poll if

$$(1-0.5)p_B(s) + (0-0.5)q_B(s) \geq 0,$$

which is simplified as

$$p_B(s) \geq q_B(s).$$

We can calculate $p_B(0.51) \approx 0.59$, $q_B(0.51) \approx 0.41$, $p_B(0.6) \approx 0.33$ and $q_B(0.6) \approx 0.67$. Therefore, we obtain

Corollary 2 *For $[rm] = 13$, there exist the following symmetric pure-strategy Bayesian Nash equilibria according to the value of s:*

(i) For both $s = 0.51$ *and* $s = 0.6$*, (1) all voters abstain; (2) A-voters go to the poll whereas B-voters abstain; and (3) A-voters abstain whereas B-voters go to the poll.*
(ii) Only for $s = 0.51$*, all voters go to the poll.*

Part (*ii*) of Corollary 2 implies that each voter of the ex-ante minority group tries to spoil the outcome by abstaining if the expected size of her group is so small that it seems hard to win the referendum.

Regarding symmetric mixed-strategy equilibria, Propositions 2 and 3 tell that there can be the following three types of equilibria according to the parameter values: (1) all voters use mixed strategies; (2) A-voters use a mixed strategy whereas B-voters go to the poll with certainty; and (3) A-voters go to the poll with certainty whereas B-voters use a mixed strategy. In the third case, for example, the equilibrium probability of each B-voter going to the poll is calculated in Table 1.

[Table 1 here]

4.3 Experimental Procedures

We had 6 sessions on November 1, 2007 at Hokkaido University, Japan. Subjects were recruited on campus. Most of them were first-year undergraduate students from various academic disciplines, without any experience of political-science or economics experiments. Each session had 13 subjects in one classroom, and subjects took seats sufficiently apart from each other. Each subject joined one session only. Three sessions were held at one time, and each session spent about 80 to 90 minutes.

When subjects read the instruction, which is in Appendix, a tape recording of an experimenter reading the instruction aloud was kept playing so that all the subjects could read it at the same pace. The instruction was written with abstract words; that is, we did not use words such as referendum, vote, win, or any others that may by themselves give subjects a feeling of obligation to go to the poll. After the instruction, 5 minutes were spent for subjects to ask questions and consider how to make decisions in the experiment. Then, 20 rounds were held. Of the 6 sessions, sessions 1, 2 and 3 had $s = 0.5$ while sessions 4, 5 and 6 had $s = 0.6$.

In each round, experimenters distributed a card to each subject, in which the subject's group (either group A or B) and "the required number of subjects who choose 1", which corresponds to quorum $[rm]$, were written. The quorum $[rm]$ took numbers 1 and 13 twice, 3, 5, 9 and 11 three times, and 7 four times respectively in random order round by round. Each subject circled either "0" or "1" printed on the card, and submitted it to an experimenter. Choosing 0 and 1 in the experiment corresponds to abstaining and going to the poll in the theoretical model, respectively. Each subject also wrote her decision 0 or 1 in her record sheet so that she could remember her decision history. Experimenters collected the cards from 13 subjects and counted how many subjects chose 1 in each group and also in total. Then, subjects' earnings were determined according to the rules of the model in Section 2. Payoffs 0, 0.5 and 1 in the model were replaced with 0, 100 and 200 yen in the experiment (1 yen was 0.00871 dollars on November 1, 2007). An experimenter announced how many subjects chose 1 in each group and in total respectively, and also announced the earnings for subjects of each group. Another experimenter typed the information in an Excel table which was projected on an overhead screen. Each subject copied the result and her payoff from the screen on her record sheet.

After the final round, subjects answered questionnaires while experimenters prepared for payments. Then, subjects received the sum of earnings obtained through 20 rounds in cash one by one, and left the classroom. The earnings of each subject ranged from 1,600 yen to 3,000 yen, and the average was 2,314 yen.

5 Experimental Results

In this section, we provide our experimental results. We focus on voter turnout, voting outcomes, and individual voting strategies.

5.1 Voter Turnout

We analyze the effects of quorums (i.e., $[rm]$), groups (i.e., A or B) and the probability of each subject being assigned to group A (i.e., s) on the subjects' aggregate behaviors. Figures 1(a) and 1(b), which are drawn from the data in Table 2, describe the relationship between quorums and the turnout rate for each group in each session under $s = 0.51$ and $s = 0.6$, respectively. For both $s = 0.51$ and $s = 0.6$, when the quorum is 38% ($[rm] = 5$) or smaller, the turnout rate is high in both groups A and B. As the quorum becomes larger, the turnout rate decreases, but the turnout rate of group A keeps relatively high whereas that of group B decreases to a large extent. In particular, the turnout rate of group B for $s = 0.6$ has a jump at 54% of quorum ($[rm] = 7$) from 70-90% to 10-30%. That is, the ex-ante minority voters abstain more aggressively when the expected number of members is sufficiently different between the two groups: in such a case, it seems hard for the ex-ante minority voters to win, and so sufficiently large quorums give them the incentive to spoil the outcome by abstaining.

[Table 2 and Figure 1 here]

These observations from Figures 1(a) and 1(b) are confirmed by the log-linear model selection. We examine the correlations of the above four variables, that is, s, quorums, groups, and voting behaviors. Table 3 describes the AIC (Akaike information criterion) statistics of possible multi-order models. The model with the smallest AIC statistic includes the following three multi-order effects, that is, (1) s, quorums and groups, (2) s, groups and voting behaviors, and (3) quorums and voting behaviors ($AIC = 18.315$).[3] We can summarize our observations as follows.

[Table 3 here]

Observation 1

(i) The larger quorum decreases the turnout.

(ii) The turnout is greater in group A than group B.

(iii) Group A's turnout is greater for $s = 0.6$ than for $s = 0.51$. Group B's turnout is greater for $s = 0.51$ than for $s = 0.6$.

Support

(i) We conduct a two-way analysis of variance (parametric test) and the Friedman test (nonparametric test) so that the differences between sessions with the same value of s are taken into account. We first calculate the turnout rate of each group of each session for each quorum from Table 2. Then, we obtain 12 sets (i.e., two groups in each of six sessions) of 7 turnout rates. We divide them into four sets according to groups (A or B) and the value of s (0.51 or 0.6). Table 4 shows the statistics of the two tests for each set of data. Both tests tell that the differences in turnout rates between quorums are statistically significant at the 5% level or lower for each data set.

[Table 4 here]

[3] Although subjects were divided into two groups independently of quorums in our experimental design, the actual realizations of group division happened to be different between quorums, which created the first multi-order effect (i.e., s, quorums and groups).

(ii) We conduct the one-tailed t-test (parametric test) and the Wilcoxon signed-rank test (nonparametric test) for paired data. We divide our data of turnout rates calculated in Support (i) into two sets according to the value of s (0.51 or 0.6). In each data set, we compare the turnout rates between groups A and B for each quorum of each session: there are 21 pairs (i.e., 7 quorums for each of three sessions with the same value of s). The t-statistics (p-values) of the one-tailed t-test are 4.73 (0.000) for $s = 0.51$ and 6.434 (0.000) for $s = 0.6$. The Wilcoxon-statistics (p-values) are 15 (0.000) for $s = 0.51$ and 0 (0.000) for $s = 0.6$. That is, both tests tell that the difference in turnout rates between groups is statistically significant at lower than the 1% level.

(iii) We conduct the one-tailed t-test and the Wilcoxon signed-rank test for paired data. We first calculate the average of the turnout rates among the three sessions with the same value of s for each group and for each level of quorum. Next, we divide these data into two sets according to groups. In each data set, we compare the turnout rates between $s = 0.51$ and $s = 0.6$ for each level of quorum. The t-statistics (p-values) of the one-tailed t-test are 3.471 (0.007) for group A and 3.391 (0.007) for group B. The Wilcoxon-statistics (p-values) are 1 (0.016) for group A and 0 (0.008) for group B. That is, both tests tell that the difference in turnout rates between $s = 0.51$ and $s = 0.6$ is statistically significant for both groups at the 2% level or lower.

Note that, if we compare the turnout rates of session 6 ($s = 0.6$) with the average turnout rates of sessions 1, 2 and 3 ($s = 0.51$), the t-statistics (p-values) of the one-tailed t-test are 0.437 (0.339) for group A and 0.752 (0.240) for group B. The Wilcoxon-statistics (p-values) are 13 (0.469) for group A and 11 (0.344) for group B. That is, the results in session 6 are not significantly different from those in the three sessions with $s = 0.51$. Some differences between sessions with the same value of s must be given attention.

[Table 5 here]

Observation 1 is also confirmed by logistic regressions with individual data. Table 5 shows the estimated coefficients of treatment variables. The dependent variable is whether each voter voted (1) or abstained (0) in each round of each session. Variable "quorum" takes values 1, 3, 5, 7, 9, 11 and 13 in regressions (1) and (2), whereas dummy variables for quorums 3, 5, 7, 9, 11 and 13 are used in regressions (3) and (4) where quorum level 1 is their baseline.

We can see in Table 5 that the larger quorum decreases the probability of turnout. That is, the coefficient of "quorum" is negative for regressions (1) and (2), and the absolute value of the coefficient of quorum dummy variable is increasing in the size of quorum in regressions (3) and (4).

We can also see that the probability of turnout decreases when subjects are assigned to group B (i.e., the coefficient of "group B dummy" is negative), and this tendency is strengthened if the ex-ante majority is more advantageous in its expected size (i.e., the coefficient of "$s = 0.6$ dummy $\times$ group B dummy" is also negative).

5.2 Voting Outcomes

Group A is the ex-ante majority in the sense that the expected number of members is greater for group A than group B. However, which group is the ex-post majority depends on how 13 subjects are actually divided into the two groups.

[Table 6 here]

The upper part of Table 6(a) describes how often the ex-post majority won, the ex-post minority won, two groups were in a tie, and the outcome was made invalid, according to the probability of each subject being assigned to group A (i.e., s). The lower part divides data according to quorums. From this table, we obtain

Observation 2

(i) Ex-post minority groups hardly win.

(ii) Invalid outcomes happen frequently when the quorum is 9 (69%) or larger, but hardly happen for smaller quorums.

Support

There is no clear difference in their voting outcomes between $s = 0.51$ and $s = 0.6$. So let us see the sum of all sessions. The ex-post minority won only 3 of 120 rounds (2.5%) in total. Table 6(b) tells that all the three wins by the ex-post minority happened when group B had 7 members but many of them abstained expecting that they were the minority. On the other hand, referendums were made invalid in 43 of 120 rounds (35.8%). The lower part of Table 6(a) tells that 41 of 43 invalid outcomes happened when the quorum was 9 (69%) or larger.

Although these results depend on not only subjects' behaviors but also the realizations of the random division of subjects into two groups, it seems that we need to care about referendums being made invalid by strategic abstention when the quorum is large, but that we do not need to care about the ex-post minority's win so seriously.

5.3 Individual Strategies

Next, we focus on how individual subjects behaved according to the level of quorum and the group to which they were assigned. Of the 78 subjects (6 sessions of 13 subjects), 75 subjects can be regarded to have used one of the following five strategies according to the group that they belong to: (1) vote under every quorum (we call this behavior *vote*); (2) vote under small quorums but randomize between voting and abstaining under large quorums (i.e., *vote/randomize*); (3) randomize under every quorum (i.e., *randomize*); (4) randomize under small quorums but abstain under large quorums (i.e., *randomize/abstain*); and (5) vote under small quorums but abstain under large quorums (i.e., *vote/abstain*).

Table 7(a) provides some subjects' voting behaviors observed in the experiment. For example, subject 5 in session 1 employed *vote* when he or she was assigned to group A while *randomize/abstain* when assigned to group B. Even if a voter abstains several times, we regard him or her to employ *vote* rather than *randomize* if a voter who randomizes between voting and abstaining equiprobably can result in such an observed behavior only with probability 5% or lower.

[Table 7 here]

Table 7(b) describes how many subjects chose each voting strategy. The row represents subjects' behaviors when they were assigned to group A, while the column represents those when assigned to group B. From this table, we obtain

Observation 3
(i) For $s = 0.6$*, subjects tend to choose "vote" in group* A *while "vote/abstain" in group* B*.*
(ii) For $s = 0.51$*, subjects tend to choose "vote" in group* A *while "vote/randomize" and "vote/abstain" in group* B*.*

Support
For $s = 0.6$, 21 of 38 subjects (55.3%) chose *vote* in group A and *vote/abstain* in group B. For $s = 0.51$, on the other hand, subjects' behaviors disperse more widely. In addition to the cell of *vote* in A and *vote/abstain* in B (9 subjects), the cell of *vote* in A and *vote/randomize* in B gathers 8 subjects.

From this observation, we can conclude the following. (i) For small quorums, the full turnout is most likely to be realized. (ii) For large quorums, if group A is expected to be sufficiently larger than group B (i.e., $s = 0.6$), the strategy profile in which voters vote in group A but abstain in group B is most likely to be realized. (iii) For large quorums, if group A is not expected to be sufficiently larger than group B (i.e., $s = 0.51$), voters in group B behave asymmetrically to each other, but *vote/randomize* and *vote/abstain* are employed more frequently than other behaviors.

6 Conclusion

We conducted a referendum experiment with participation quorums. From our observations, we can say that large quorums induce ex-ante minority voters to abstain so that referendums result in invalid outcomes frequently. Of course, the much greater number of voters in the real referendums makes it difficult for each voter to affect the outcome by abstaining, and hence a strong leadership or a sufficiently reliable expectation about the other voters' behaviors is required for the strategic abstention to happen. Although whether voters actually abstain in the real referendums depends on the voting environment, our experiment shows that such an incentive of strategic abstention does exist.

Appendix

Proof of Proposition 3

Here, we deal with the strategy profiles that were not examined in the main text.

The same logic as in the case of $r \in (1/m, (m-1)/m]$ implies that strategy profiles $(\sigma_y = 0, \sigma_n = 0)$, $(\sigma_y = 1, \sigma_n = 0)$, and $(\sigma_y = 0, \sigma_n = 1)$ are Bayesian Nash equilibria, whereas strategy profiles $(\sigma_y = 0, \sigma_n \in (0,1))$ and $(\sigma_y \in (0,1), \sigma_n = 0)$ are not.

Let us consider $(\sigma_y = 1, \sigma_n \in (0,1))$. This strategy profile constitutes an equilibrium if

$$\frac{q_y}{p_y} \leq \frac{q_n}{p_n} = \frac{1-v}{v}.$$

Note that q_y/p_y is increasing in π_n, while q_n/p_n decreasing. When π_n converges to 0, q_y/p_y converges to 0 while q_n/p_n converges to infinity. Hence, for sufficiently small values of v, this incentive condition holds. If we try to make q_n/p_n as small as possible while satisfying

$q_y/p_y \leq q_n/p_n$, we must have $q_y/p_y = q_n/p_n$, which is attained by $\pi_n = s/(1-s)$. For such a value of π_n, q_n/p_n is equal to the right-hand side of equation (5). Hence, this strategy profile is not an equilibrium for sufficiently large values of v. Similar logic applies to $(\sigma_y \in (0,1), \sigma_n = 0)$. *Q.E.D.*

Instruction

Next, we provide an English translation of the Japanese instruction used in the sessions with $s = 0.51$.

Instruction

Enclosures in Your Envelope

- instruction (this booklet)
- a sample of the record sheet (blue)
- a sample of card 1 (blue)
- a sample of card 2 (blue)
- a piece of paper written a number

Please raise your hand if something above is missing.

Explanation of the Experiment

This experiment is held for research about decision making. The amount of rewards you receive at the end of the experiment is determined by the decisions of you and other participants.

Participant Number

There is a piece of paper written "Your participant number is (   )." in your envelope. This is your participant number. This number is used when you make decisions. Please keep it at hand so that you do not lose it. Because the experiment is held anonymously by using participant numbers, your decisions and rewards are never known to any other participants.

Organization of the Experiment

The experiment consists of 20 rounds from Round 1 to Round 20. Each round is independent of any other rounds. That is, decisions and results of the previous rounds are not carried over to the next round.

What to Do in Each Round

Each round of the experiment proceeds in the following order.

(1) Grouping and Decisions
There are 13 participants including you in this classroom. In each round, each participant is independently assigned to **group A with probability 51% and group B with probability 49%**. You are informed of your own group, but you are not informed of how the other 12 participants are divided into the two groups. From the rule that "each participant is independently assigned to group A with probability 51% and group B with probability 49%," however, we can derive the probabilities about how the other 12 participants, except you, are divided into the two groups as in the following table.

[Table A1 here]

At the beginning of each round, you receive a card from an experimenter. Please look at *the sample of card 1*. It is blue, but we use pink ones in the experiment. We are using different colors to avoid using sample cards in the experiment.

Please look at the part below the title "Sample of Card 1". In the first line is written "**Round 1**". This means what round of 20 rounds is the current round. As the experiment proceeds, this changes to "Round 2", "Round 3", ..., and "Round 20".

In the second line is written "**Your Participant Number: (   )**". You write your participant number in the parentheses. For your practice, please write in your participant number now. Have you done? If not, please raise your hand. Hereafter, whenever you have any problem, please raise your hand. An experimenter comes to you.

Please look at *the sample of the record sheet*. It is blue, but we use pink ones in the experiment. We are using different colors to avoid using sample sheets in the experiment. In the upper-right part is written "***Your Participant Number: (   )***". Please write your participant number in the parentheses now. Have you done?

Please look at the sample of card 1. In the third line is written "**Your group is A.**" This means that you are assigned to group A in this round. It is written "Your group is B." if you are assigned to group B. You record your group name in the leftmost cell "***Your Group (A or B)***" of your record sheet. For your practice, please record it now. Because "**Round 1**" and "**Your group is A.**" are written in the sample of card 1, please write "A" in the cell "***Your Group (A or B)***" in the row of "***Round 1***". Have you done?

Please look at the sample of card 1. In the fourth line is written "**Required Number of Participants: 5**". Please write "5" in the cell "***Required Number of Participants***" in the row of "***Round 1***". Have you done? This "5" may change round by round, or it may be the same as the previous round. We explain what it means later.

Please look at the sample of card 1. In the fifth line and below are written "**Your Decision (Circle 0 or 1)**" and "**0   1**". You choose and circle either 0 or 1. You also record the number you have chosen in the cell "***Your Decision (0 or 1)***" on your record sheet. Now suppose that you choose "0." Please circle "0" on the sample of card 1. Have you done? Furthermore, please write "0" in the cell "***Your Decision (0 or 1)***" of the row of "***Round 1***" on the sample of the record sheet. Have you done? Next, we explain how your earnings you receive at the end of the experiment are determined according to this decision.

(2) Determining Your Earnings
After all participants finish writing down in the card and the record sheet, experimenters collect the cards. Now experimenters actually collect the sample of card 1. Please hand it over to them. Experimenters sum up participants' decisions written on the cards, and they count

how many paricipants have chosen 1
(1) in group A,
(2) in group B, and
(3) in total (that is, the sum of (1) and (2)).

For example, as is written on the sample of card 1, suppose that

<u>you are assigned to group A, and "the required number of participants" is 5.</u>

Then, your earnings in this round are determined as follows.

Case 1: If the number of participants who have chosen 1 is
- greater than or equal to 5 in total, and
- greater in group A than group B,

then your earnings are **200 yen**.

Case 2: If the number of participants who have chosen 1 is
- greater than or equal to 5 in total, and
- greater in group B than group A,

then your earnings are **0 yen**.

Case 3: If the number of participants who have chosen 1 is
- greater than or equal to 5 in total, and
- the same between the two groups,

then your earnings are **100 yen**.

Case 4: If the number of participants who have chosen 1 is
- smaller than 5 in total,

then your earnings are **100 yen**.

When you are assigned to group B, on the other hand, your earnings are 0 yen in Case 1 and 200 yen in Case 2. That is, when the number of participants who have chosen 1 is greater than or equal to "the required number of participants," the participants of the group with more members choosing 1 than the other group earn 200 yen while the participants of the other group earn 0 yen.

After experimenters aggregate the participants' decisions written on the cards, they type it in an Excel table, and it is projected on the screen in front of the classroom. For example, suppose the following.

Round 1
Required Number of Participants: 5.
The number of participants who have chosen 1 is 4 in group A, 3 in group B, and 7 in total.

Then, on the screen is projected the following.

[Table A2 here]

In this case, "the number of participants who have chosen 1" is 4 in group A, 3 in group B and 7 in total, and so it reaches the "required number of participants." Moreover, because "the number of participants who have chosen 1" is greater in group A than group B, the participants of group A earn 200 yen while the participants of group B earn 0 yen. You look at the screen and write "4", "3" and "7" in the cells ***Number of Participants Who Have Chosen 1: "Group A", "Group B" and "in Total"*** respectively and also "200" in the cell "***Your Earnings (200, 100 or 0)***" in the row of "***Round 1***" on your record sheet. For your practice, please write them in the sample of the record sheet now. Have you done? Note that, regardless of each participant's choice either 0 or 1, all the members of group A earn 200 yen while all the members of group B earn 0 yen.

If all the participants finish writing in their record sheet, this round ends and we proceed to the next round. The above procedures of the experiment can be summarized as follows.

Summary of What to Do in Each Round

Round ( )
Your Participant Number: ( )
Your group is ( ).
Required Number of Participants: ( )
Your Decision (Circle 0 or 1)

0 1

(1) Receive a card from an experimenter.
(2) Write your participant number in the parentheses of "**Your Participant Number: ( )**" on the card.
(3) Look at "**Your group is ( ).**" and "**Required Number of Participants: ( )**" written on the card. Then, copy them in the cells "***Your Group (A or B)***" and "***Required Number of Participants***" respectively on your record sheet.
(4) Make a decision about whether to choose 0 or 1. If you decide, circle "**0**" or "**1**" on the card, and record it in the cell "***Your Decision (0 or 1)***" on your record sheet.

(5) Experimenters collect the cards and count the number of participants who have chosen 1. Then, they fill in the cells "Number of Participants Who Have Chosen 1: Group A, Group B, and in Total" and "Earnings: Group A and Group B" on the screen.
(6) Look at the information on the screen, and copy it in the cells "***Number of Participants Who Have Chosen 1: Group A, Group B and in Total***" and "***Your Earnings (200, 100, 0)***" on your record sheet.
(7) This round ends. The next round begins, and you receive a new card. This is repeated 20 times.

Let us see another example. Please look at *the sample of card 2*. Suppose that you have received the sample of card 2 in round 2.

First, please write your participant number in the parentheses of "**Your Participant Number: ( )**" in the second line. Have you done? Next, please look at "**Your group is B.**" in the third line and "**Required Number of Participants: 4**" in the fourth line. Then, write "B" in the cell "***Your Group (A or B)***", and "4" in the cell "***Required Number of Participants***" of the row of "***Round 2***" on the sample of record sheet. Have you done? Note that, in any round, each participant is assigned to group A with probability 51% and group B with 49%. This way of dividing participants in the two groups never changes through 20 rounds.

If you have done the above, it is the time to make a decision. You consider whether to choose 0 or 1. Now suppose that you have decided "1." Please circle "1" at the bottom of the sample of card 2. Have you done? At the same time, please write "1" in the cell "***Your Decisions (0 or 1)***" on the sample of the record sheet. Have you done? After every participant finishes writing down, experimenters collect the cards. Now experimenters actually collect the sample of card 2. Please hand it over to them.

Experimenters sum up the decisions written on the cards. Suppose that the result is as follows.

The number of participants who have chosen 1 is 1 in group A, 2 in group B, and 3 in total.

Then, an experimenter fills in the table projected on the screen as follows.

[Table A3 here]

This time, the number of participants who have chosen 1 is 3 in total. Because this is smaller than 4 ("the required number of participants"), all the participants of both groups earn 100 yen. So please write "1" in the cell "***Group A***" of the "***Number of Participants Who Have Chosen 1***", "2" in "***Group B***", "3" in "***in Total***", and "100" in "***Your Earnings (200, 100, 0)***". Have you done?

Earnings

We conduct 20 rounds. After the 20th round, you sum up your earnings from Round 1 to Round 20, and write it in the cell "***Sum of Your Earnings from Round 1 to Round***

20" on your record sheet. This is the amount of money you receive at the end of the experiment.

After this instruction, experimenters collect the sample of record sheet, and distribute the record sheet used in the experiment. Next, you have 5 minutes to make sure of your understanding of the rules of the experiment and also to consider how to make decisions. Then, we start Round 1.

This is the end of the instruction. If you have any questions, please raise your hand. An experimenter comes to you. Please do not talk with anyone else until the experiment ends and you leave the classroom.

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Quorum	s=0.51	s=0.6
13	0.885	NA
11	0.681	0.782
9	0.493	0.546
7	0.321	0.343
5	0.168	0.170
3	0.044	0.040

Table 1. Equilibrium Probabilities of Group-B Voters Going to the Poll

Note: This table applies to the symmetric mixed-strategy Bayesian Nash equilibria in which group-A voters go to the poll with certainty while group-B voters use mixed strategies.

Session	1				2				3				total (s=0.51)			
Group	A		B		A		B		A		B		A		B	
Quorum	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain
13	11	3	1	11	8	2	5	11	9	7	3	7	28	12	9	29
11	17	6	4	12	14	7	9	9	13	7	4	15	44	20	17	36
9	12	1	6	20	14	4	10	11	16	7	4	12	42	12	20	43
7	22	6	11	13	17	0	28	7	23	5	13	11	62	11	52	31
5	19	1	14	5	10	3	25	1	19	1	16	3	48	5	55	9
3	21	0	15	3	22	1	16	0	19	2	18	0	62	3	49	3
1	13	0	11	2	12	0	13	1	10	0	16	0	35	0	40	3

Session	4				5				6				total (s=0.6)			
Group	A		B		A		B		A		B		A		B	
Quorum	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain
13	15	3	0	8	16	2	0	8	9	6	3	8	40	11	3	24
11	27	1	0	11	20	2	3	14	16	10	3	10	63	13	6	35
9	23	4	1	11	27	2	2	8	18	3	5	13	68	9	8	32
7	29	2	2	19	29	1	7	15	28	2	7	15	86	5	16	49
5	23	0	10	6	23	0	12	4	20	0	18	1	66	0	40	11
3	21	1	15	2	16	0	21	2	22	0	15	2	59	1	51	6
1	18	1	6	1	16	0	8	2	13	0	13	0	47	1	27	3

Table 2. The Aggregate Data

Model	G^2	df	p-value	AIC
4-way effects	-	-	-	32.000
All 3-way effects	7.774	6	0.255	27.774
Three 3-way effects				
SQG,SQB,SGB	14.660	12	0.261	22.660
SQG,SQB,QBG	47.470	7	0.000	65.470
SQG,SGB,QGB	13.265	12	0.350	21.265
SQB,SGB,QGB	32.091	12	0.001	40.091
Two 3-way effects				
SQG,SQB,GB	55.311	13	0.000	61.311
SQG,SGB,QB	22.315	18	0.218	18.315
SQG,QGB,SB	53.938	13	0.000	59.938
SQB,SGB,QB	39.820	18	0.002	35.820
SQB,QGB,SG	57.350	13	0.000	63.350
SGB,QGB,SQ	37.564	18	0.004	33.564
One 3-way effect				
SQG,SB,QB,GB	61.725	19	0.000	55.725
SQB,SG,QG,GB	64.502	19	0.000	58.502
SGB,SQ,QG,QB	45.434	24	0.005	29.434
QGB,SQ,SG,SB	63.834	19	0.000	57.834
All 2-way effects	71.227	25	0.000	53.227

Table 3. The Log-Linear Model Selection

Note: S, Q, G and B abbreviate s, quorums, group assignment and voting behaviors respectively. G^2 represents the likelihood ratio chi square.

	Group A		Group B	
s=0.51	5.346	(0.007)	50.201	(0.000)
	12.893	(0.045)	16.036	(0.014)
s=0.6	3.082	(0.046)	73.354	(0.000)
	13.714	(0.033)	16.821	(0.010)

Table 4. Test Statistics of the Effect of Quorums on Turnout

Note: In each cell, the upper figure is the F-value of the analysis of variance, and the lower figure is the Chi-square of the Friedman test. P-values are in the parentheses.

	(1)	(2)	(3)	(4)
s=0.6 dummy	1.119	*0.130*	1.122	*0.100*
	(.306)	*(1.038)*	(.307)	*(1.077)*
Quorum	-0.380	-0.452		
	(.025)	(.030)		
Quorum 3 dummy			*-0.053*	*-0.028*
			(.498)	*(.534)*
Quorum 5 dummy			-0.818	-0.847
			(.460)	(.493)
Quorum 7 dummy			-2.595	-2.959
			(.427)	(.463)
Quorum 9 dummy			-3.428	-3.945
			(.438)	(.482)
Quorum 11 dummy			-3.751	-4.432
			(.440)	(.487)
Quorum 13 dummy			-3.892	-4.546
			(.454)	(.503)
Group B dummy	-1.712	-2.115	-1.672	-2.082
	(.205)	(.241)	(.204)	(.243)
S=0.6 dummy x group B dummy	-1.673	-1.794	-1.780	-1.963
	(.300)	(.340)	(.305)	(.350)
Round	0.048	0.058	0.051	0.062
	(.013)	(.014)	(.013)	(.014)
Female dummy	*-0.337*	*-0.460*	*-0.384*	*-0.418*
	(.232)	*(.995)*	*(.235)*	*(1.043)*
Constant	4.231	5.774	3.752	5.090
	(.326)	(.768)	(.473)	(.860)
Subject specificity		included		included

Table 5. Coefficients of Logistic Regressions on Voter Turnout

Note: The dependent variable is whether to vote (1) or abstain (0) for each subject in each round. Standard errors are in parentheses. The Italic figures express that they are not significant at the 10% level. The other estimates are significant at the 1% level except for quorum 5 dummy which is significant at the 10% level in both regressions (3) and (4). In regressions (3) and (4), quorum 1 (i.e., one vote validates the referendum) is the baseline for quorum dummies. Session dummies are also included in all the regressions but omitted from the table.

s	Ex-post majority's win	Ex-post minority's win	Tie	Invalid	Sum
0.51	30	2	4	24	60
0.6	40	1	0	19	60
Sum	70	3	4	43	120

Quorum	Ex-post majority's win	Ex-post minority's win	Tie	Invalid	Sum
13	0	0	0	12	12
11	1	0	0	17	18
9	5	0	1	12	18
7	18	2	2	2	24
5	16	1	1	0	18
3	18	0	0	0	18
1	12	0	0	0	12
Sum	70	3	4	43	120

Table 6(a). The Number of Results according to the Value of s and Quorums

Session	s	Round	Quorum	# of Subjects		Votes	
				A	B	A	B
1	0.51	15	5	6	7	6	4
3	0.51	1	7	6	7	5	2
5	0.6	19	7	6	7	6	2

Table 6(b). The Cases of the Ex-Post Minority’s Win

	Session 1, Subject 5				Session 1, Subject 8				Session 2, Subject 7			
Group	A		B		A		B		A		B	
Quorum	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain	Vote	Abstain
13	0	0	0	2	1	0	0	1	1	0	1	0
11	3	0	0	1	2	0	0	1	0	2	0	1
9	2	0	0	0	0	0	0	3	1	0	1	1
7	2	0	1	1	1	0	0	3	0	0	2	2
5	1	0	1	1	2	0	1	0	0	2	1	0
3	2	0	1	0	2	0	1	0	2	0	1	0
1	2	0	0	0	1	0	1	0	0	0	2	0
Behavior	Vote		Randomize/Abstain		Vote		Vote/Abstain		Randomize		Vote/Randomize	

Table 7(a). Examples of Individual Behaviors

Note: Each number expresses how many times each subject voted or abstained in each group under each quorum.

s=0.51

Group A/B	Vote	Vote/ Randomize	Randomize	Randomize/ Abstain	Vote/ Abstain	Sum
Vote	4	8	1	2	9	24
Vote/Randomize	0	0	1	0	3	4
Randomize	1	2	0	0	1	4
Randomize/Abstain	0	0	0	0	0	0
Vote/Abstain	0	0	0	0	5	5
Sum	5	10	2	2	18	37

s=0.6

Group A/B	Vote	Vote/ Randomize	Randomize	Randomize/ Abstain	Vote/ Abstain	Sum
Vote	2	2	1	5	21	31
Vote/Randomize	0	1	1	1	2	5
Randomize	0	0	0	0	0	0
Randomize/Abstain	0	0	0	1	0	1
Vote/Abstain	0	0	0	1	0	1
Sum	2	3	2	8	23	38

Table 7(b). Classification of Individual Behaviors

Note: Two subjects for s=0.51 and one subject for s=0.6 were not classified in any of these behaviors, who were not included here.

Number of Group A Members	Number of Group B Members	Probability (%)
0	12	0.02
1	11	0.24
2	10	1.37
3	9	4.75
4	8	11.13
5	7	18.53
6	6	22.50
7	5	20.08
8	4	13.06
9	3	6.04
10	2	1.89
11	1	0.36
12	0	0.03

Table A1. Group Divisions of Twelve Participants Except for You and Its Probabilities

	Required Number of Participants	Number of Participants Who Have Chosen 1			Earnings (Yen)	
		Group		In Total	Group	
		A	B		A-members	B-members
Round 1	**5**	**4**	**3**	**7**	**200**	**0**
Round 2						
-- omitted --	-- omitted --	-omitted-	-omitted-	-- omitted --	-- omitted --	-- omitted --
Round 20						

Table A2. Screen 1

	Required Number of Participants	Number of Participants Who Have Chosen 1			Earnings (Yen)	
		Group		In Total	Group	
		A	B		A-members	B-members
Round 1	**5**	**4**	**3**	**7**	**200**	**0**
Round 2	**4**	**1**	**2**	**3**	**100**	**100**
-- omitted --	-- omitted --	-omitted-	-omitted-	-- omitted --	-- omitted --	-- omitted --
Round 20						

Table A3. Screen 2

Figure 1(a). Quorums and Turnout for s=0.51

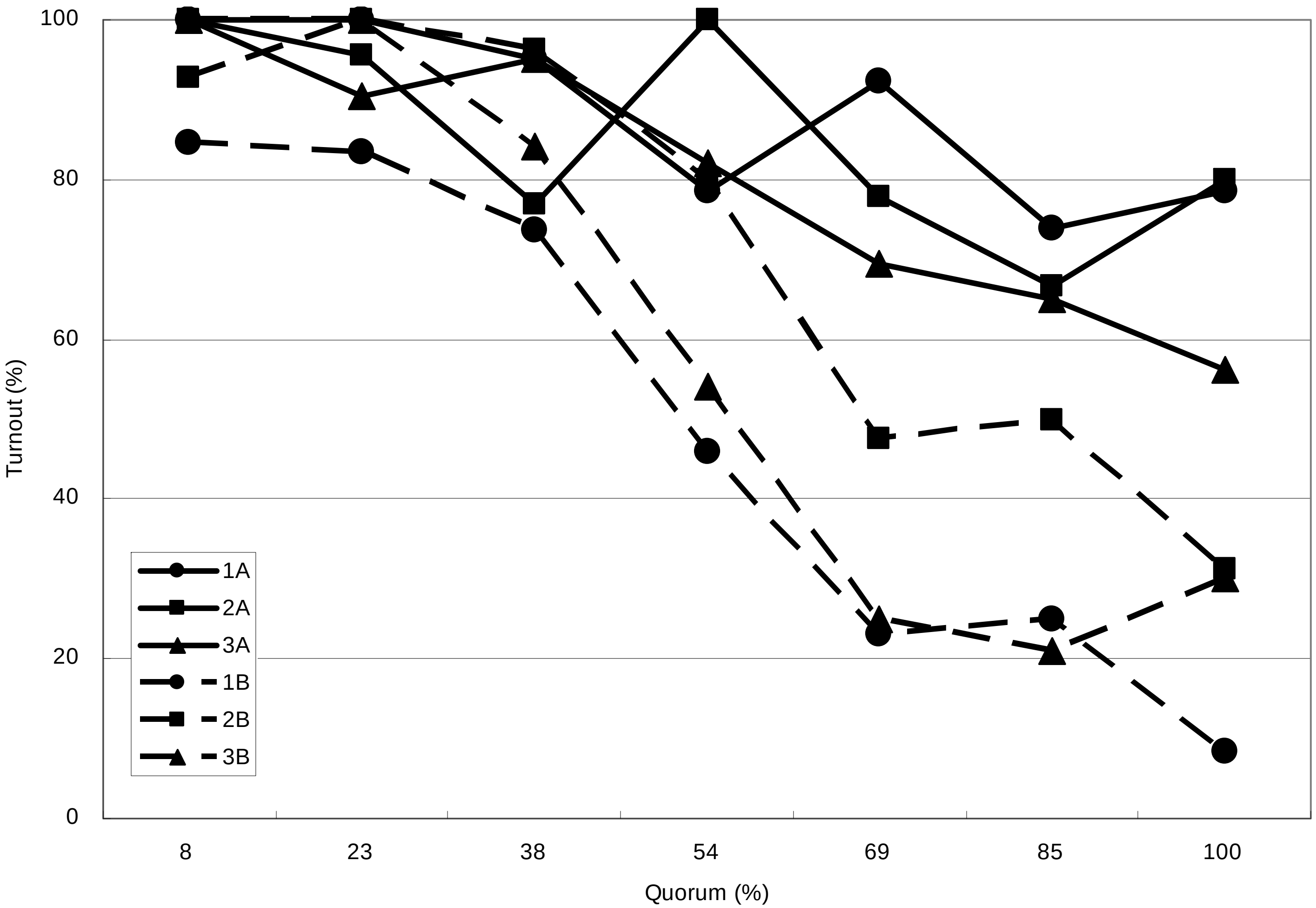

Figure 1(b). Quorums and Turnout for s=0.6