Evolutionary Implementation of Optimal Number and Size of Cities

Preliminary Draft

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Abstract

With a model of system of cities and one rural area, we consider evolutionary implementation (due to Sandholm [Evolutionary implementation and congestion pricing, Review of Economic Studies 69 (2002), 667-689]) of optimal number and size of cities in which the planner leads the economy to a social optimum in the long-run without forcing lumpy migrations and without the knowledge of preferences. We show that the planner can achieve evolutionary implementation of a social optimum by internalizing the externalities evaluated at current prices and population distribution in each period. Unlike the original concept of Sandholm, we consider a migration dynamic with forward looking expectations to deal with multiplicity of local optima.

JEL classification: C73, D04, D62, R12.

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1 Introduction

This paper intends to contribute to the theory of system of cities initiated by Henderson (1974), which is a classical topic of urban economics. The theory of system of cities is concerned with optimal city size distribution and difference between market outcome and optimal one.[1] Generally, an optimal city size is determined so that positive agglomeration economies and negative congestion externalities are

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[1] See Abdel-Rhaman and Anas (2004) for a survey of this field.

balanced. Hence, if the positive [resp. negative] externalities dominate at a market equilibrium, the city size is too small [resp. large]. However, an equilibrium at which the positive externalities dominate will not be stable because a marginal increase of city population raises the utility of the city residents and hence, the population increases further. Therefore, a strong agreement in this literature is that the number of cities is too small and the sizes of cities are too large at a stable market equilibrium. In this case, the planner needs to reduce the sizes of the existing cities and increase the number of cities to fix the inefficiencies. However, since the equilibrium is stable, the planner cannot induce marginal migrations and therefore, he needs to force *lumpy* migrations to construct a new city. Hence, it has been argued that agents such as local governments and city developers that directly control city populations are necessary to achieve a social optimum.[2]

However, as Abdel-Rhaman and Anas (2004) argue, it is not realistic to think that city developers can completely control the spatial structure of economy. In the above argument, the planner cannot lead the economy to a social optimum because we are considering *one shot policies*. If, instead, we consider a period-by-period policy such that the planner constantly stimulates the economy, it may be possible to achieve a social optimum without directly controlling population. Such a policy is compatible with the standard framework of this literature because though a base model is static, we consider a dynamic environment in which agents behave according to a migration dynamic and settle down at equilibrium strategies (about where to locate) in the long-run. We argue that if the planner internalizes the externalities evaluated at current prices and population distribution in each period, he can lead the economy to a social optimum. However, since there are generally multiple local optima, the economy does not necessarily converge to a global optimum under myopic dynamics.[3] To tackle this problem, we incorporate forward looking expectations to migration dynamic in the spirit of Matsui and Matsuyama (1994) by introducing a friction in the sense that agents cannot migrate in every period.[4] Under the resulting dynamic, which is called *perfect foresight*

[2]In the New Economic Geography (NEG) literature, it has been demonstrated that a market can yield *under* agglomeration for low inter-city trade costs when there exists intra-city congestion (e.g., Helpman, 1998; Tabuchi, 1998; Pflüger and Südekum, 2008). If the market yields under agglomeration, the planner needs to increase the sizes of some existing cities and reduce the number of cities, but he still needs to force a lumpy migration because the equilibrium is stable.

[3]This point is also mentioned by Hadar and Pines (2004).

[4]While a myopic dynamic is usually assumed in the literature, dynamics with forward looking

dynamic, it follows that the economy converges to only a global maximizer.

When we design a policy, we also address the point that the planner cannot determine whether a current spatial configuration is optimal or not without knowing the preferences of people, to which a little attention has been paid in the literature.[5] Specifically, we formulate a base model as a *population game* in which there exists a continuum of players and consider *evolutionary Nash implementation* introduced by Sandholm (2002, 05b). If the planner succeeds in evolutionary implementation of a social optimum, the economy converges to the optimum from any initial state. The planner tries to achieve this with the *first-best policy* in which he internalizes externalities evaluated at current prices and population distribution in each period. Since the externalities are evaluated at the *current* observable states, the planner can carry it out without the knowledge of preferences.

This implementation concept is well-suited to our large economy because it does not require the agents to play Nash strategies at once and unlike standard revelation mechanisms such as the VCG mechanism, it does not ask the planner to collect messages. In other words, in view of the fact that our economy has a large number of people, our mechanism is not labor-intensive for the planner because he is relieved of the burden of collecting messages and computing allocations to each message. Moreover, from the players' point of view, the requirement in terms of their cognitive skills is moderate since the mechanism allows the players to make mistakes in forecasting strategies of the opponents in learning periods.

To make the argument clear, we consider the most primitive and simplest possible model in the literature: there are $K \in \mathbb{N}$ identical cities and one rural area[6]; there is no trade among the cities and the rural area and there is only one industry; agglomeration economies are incorporated by the Marshallian externalities in aggregate production functions; and congestion externalities are present in com-

expectations have been considered for the "history versus expectations" issue in the NEG literature. To my best knowledge, either the dynamic due to Matsui and Matsuyama (1994) or the one due to Krugman (1991) and Fukao and Bénabou (1993) at which agents can migrate anytime but incur a migration cost is used. We employ the dynamic of Matsui and Matsuyama (1994) mainly because it fits well with our framework.

[5]Hadar and Pines (2004) consider a policy in which the planner uses only information available to him. But they do not touch on implementation issues.

[6]Anas and Xiong (2005) consider a model that has two cities and two industries (manufacture and service). They argue that if one city diversifies over the industries and the other city specializes in one industry at optimum, a forced lumpy migration will not be necessary. However, such a result does not follow if the two cities are identical in terms of industrial structure.

muting. Hence, in the first-best policy we will consider, the planner does the following in each period: instruct the firms to pay the marginal productivities for labor to fix the agglomeration externalities; levy the Pigouvian congestion taxes to commuters to fix the congestion externalities; and distribute surplus or finance loss through lump sum transfer. The marginal productivities of labor and the Pigouvian taxes are evaluated at current population distribution. Under this policy, we show that the planner can lead the economy to a social optimum without directly controlling populations and without the knowledge of preferences, as long as he knows the aggregate production function and the commuting cost, and can observe current prices and population distribution.

One might recall the *Henry George Theorem* which states that the loss in production due to the marginal cost pricing is exactly financed by the revenue from the Pigouvian congestion tax and the land rent in each city when the number of cities is optimal. We can interpret that in the first-best policy, the planner tries to reach the population distribution such that the Henry George Theorem holds without the knowledge of preferences. To my best knowledge, this work is the first attempt to implement social optima in a model of system of cities.

The rest of this paper is organized as follows. Section 2 states the basic setup of the model, introduces myopic evolutionary dynamics, and defines equilibrium and social optimum. Some characterizations of equilibria, stability analysis, and comparison between equilibrium and optimum are also carried out. Section 3 explains the concept of evolutionary Nash implementation and after introducing the perfect foresight dynamic, states the main result. The relation of our analysis to the Henry George Theorem is also discussed. Finally, Section 4 concludes. The proof omitted from the text is provided in Appendix.

2 The Model

We consider an economy that has K potential cities and one rural area where K is a finite natural number. Our economy is a system of isolated cities, so there is no trade among the cities and rural area. There is unit mass of homogeneous households in the economy. Each household has the action space $S = \{0, 1, 2, ..., K\}$ where 0 represents the action of residing in rural area and $k \in S \setminus \{0\}$ represents the action of residing in city k. Let $n_k \in [0, 1]$ be the mass of households taking the action $k \in S$. Then, $\Delta = \left\{ n \in \mathbb{R}_{+}^{|S|} : \sum_{k \in S} n_k = 1 \right\}$ is the set of distributions of households over the actions.[7] We call $n \in \Delta$ a *strategy distribution*.

Each city is composed of one residential area and one central business district (CBD). A representative firm produces consumption good at the CBD. The aggregate production function is given by $f(n_k) = h(n_k)n_k$ where $h : [0,1] \to \mathbb{R}_+$ is a C^1 and strictly increasing function. We treat the consumption good as numéraire. $h(\cdot)$ is assumed to be external to the firm, hence the wage rate is given by $w_k = h(n_k)$. $h(\cdot)$ captures positive agglomeration externalities.

Each household in city k lives in the residential area, commute to the CBD, and inelastically supply one unit of labor. When they commute, they incur the commuting cost $T(n_k)$ where $T : [0,1] \to \mathbb{R}_+$ is a C^1 and strictly increasing function. Then, since the (per capita) commuting cost is strictly increasing in the city population, there are negative congestion externalities in commuting. The households in cities consume housing in the residential area. We assume that total supply of housing is fixed at $H > 0$. Then, since the households are homogeneous, the per capita housing consumption in city k is H/n_k. The household budget constraint is

$$c_k + T(n_k) + R_k H/n_k = w_k, \tag{1}$$

where c_k is the consumption of numéraire and R_k is the land rent. The rent revenue is assumed to be thrown away.[8] However, when the planner seeks a social optimum, he will use it to finance his policy. The payoff from living city k is

$$\begin{aligned} U_k(n) &= c_k + v(A_c) + u(H/n_k) \\ &= w_k - T(n_k) - R_k H/n_k + v(A_c) + u(H/n_k), \end{aligned} \tag{2}$$

where $v(A_c)$ is the utility from city amenity A_c (Robeck, 1982) and $u(\cdot)$ is a C^1 and strictly quasi-concave function. Note that the land rent R_k is given by $u'(H/n_k)$.

On the other hand, there is no consumption and no housing in rural area. The households in rural area derive their utilities from only rural amenity A_r. Hence, the payoff from residing in rural area is $U_0(n) = v(A_r)$.

[7] $|S|$ denotes the cardinality of S.

[8] This assumption is just for simplicity. We also could assume that the rent revenues are equally distributed to the households.

2.1 Population Game

We assume that $u(\cdot)$ is parameterized by a real vector γ.[9] Also, it turns out that only $v \equiv v(A_r) - v(A_c)$ matters for our analysis. Then, we call $\theta = (v, \gamma)$ the *type* of the households.[10] We assume that θ is an element of a finite set Θ in the Real space.[11] In the following analysis, we assume that *the planner knows $h(\cdot)$ and $T(\cdot)$, and can observe n and $\{R_k\}_{k\in S\setminus\{0\}}$, but he does not know θ.*

We represent a *population game* by $G = (U, \theta)$ in which the set of players is the unit mass of households, the action space is S, the payoff vector is U, and the type space is Θ.[12] A *Nash equilibrium* of G is a strategy distribution $n^* \in \Delta$ such that

$$n_k^* > 0 \Rightarrow k \in \arg\max_{j\in S} U_j(n^*) \quad \text{for all } k \in S. \tag{3}$$

That is, if there is a positive mass of households who take action k, then the action is a best response under the strategy distribution n^*.

Example 1 (Full agglomeration equilibrium)**.** Suppose $h(n_k) = \exp(\varepsilon n_k)$ as in Fujita and Thisse (2002, Ch 8), $T(n_k) = tn_k, u(H/n_k) = H/n_k$, and $v < 1$. Since $R_k = u'(H/n_k) = 1$, $U_k(n) = \psi(n_k) + v(A_c)$ for $k \in S \setminus \{0\}$ where $\psi(n_k) = h(n_k) - T(n_k)$. In this specification, the magnitude of agglomeration economies is captured by ε while that of congestion externalities is captured by t. Now, suppose all households locate in city k^*. We call this state *full agglomeration in one city*. Then, no household in city k^* has incentive to change his location if $\psi(1) \geq \max\{\psi(0), v\}$. Hence, full agglomeration in one city is a Nash equilibrium if $t \leq e^{\varepsilon} - 1$, or the positive agglomeration economies are large relative to the negative congestion effects. □

2.2 Myopic Evolutionary Dynamics

As we discussed in the introduction, it is difficult for each household to play a Nash strategy at once in large economies. Hence, we consider a situation in which

[9] Here, we invoke the fact that a continuous function $f : \mathbb{R} \to \mathbb{R}$ can be uniformly approximated by polynomials if the domain is restricted to a compact set.

[10] Recall that the households are assumed to be homogeneous. Hence, all the households have the same type. We can introduce heterogeneity by dividing the population into subpopulations according to type.

[11] I argue that this assumption is not so restrictive for v intuitively. Suppose v is measured in unit of dollar. Generally, v takes any values such as \$1.2345. But if we round off fractions and consider only values such as \$1.23 as in our daily life, we may assume that v takes only finite number of values as long as v has upper and lower bounds.

[12] Sandholm (2001) calls a game that has a continuum of players *population game*. In this game, Nash equilibrium is defined with respect to a strategy distribution rather than individual strategies.

the households play a game repeatedly and gradually learn to play equilibrium strategies. The resulting state is interpreted as a long-run equilibrium. To describe a learning process, we consider *evolutionary dynamic* $g : \Delta \to \mathbb{R}^{|S|}$ that defines a motion of strategy distribution. Given a game G, Sandholm (2001) considers a class of myopic evolutionary dynamics that satisfy the following conditions:

(LC) g is Lipschitz continuous,
(FI) Δ is forward invariant under $\dot{n} = g(n)$,
(PC) $g(n) \neq 0 \Rightarrow g \cdot U > 0$,
(NC) $g(n) = 0 \Rightarrow n$ is a Nash equilibrium of G.

Condition (LC) is a technical assumption to ensure that there exists a unique solution trajectory for each initial condition. Intuitively, this condition requires that there is no jump in changes of the strategy distributions. Condition (FI) is also a technical assumption to ensure that any solution trajectory does not leave Δ. To interpret condition (PC), we note that under condition (FI),

$$g \cdot U = \sum_{k \in S} g_k(n) \left(U_k(n) - \frac{1}{|S|} \sum_{j \in S} U_j(n) \right). \tag{4}$$

If the households behave rationally, each term in the summation should be positive: if an action yields a more than average payoff, then the mass of households taking the action should increase. Condition (PC) requires that this is true only in aggregate. Condition (NC) states that if there is a profitable deviation, some households change their strategies. By Proposition 4.3 in Sandholm (2001), conditions (FI) and (PC) imply that the converse is also true. Therefore, under conditions (FI), (PC), and (NC), n is a rest point of g if and only if n is a Nash equilibrium of G.

Sandholm defines that if an evolutionary dynamic satisfies the above conditions, it is *admissible* for G. To see whether myopic evolutionary dynamics are useful for our model, we consider this class of dynamics instead of taking a particular dynamic. Specific examples of the admissible dynamics include the projection dynamic and the Brown-von Neumann-Nash (BNN) dynamic.[13]

To study stability of equilibria of G in terms of the above class of dynamics, we utilize the fact that G is a potential game which is defined as follows: Let

[13] See Sandholm (2005a) for more examples. One important remark is that the replicator dynamic, which is often used in the NEG models, is not admissible. Under the replicator dynamic, the boundary states are always rest points and hence, it does not satisfy condition (NC).

$\overline{\Delta} = \left\{ n \in \mathbb{R}_+^{|S|} : 1 - \eta < \sum_{k \in S} n_k < 1 + \eta \right\}$ where $\eta > 0$. G is a *potential game* if it has a *potential function* that is a C^1 function $p : \overline{\Delta} \to \mathbb{R}$ such that

$$\frac{\partial p(n)}{\partial n_k} - \frac{\partial p(n)}{\partial n_\ell} = U_k(n) - U_\ell(n) \quad \text{for all } k, \ell \in S. \tag{5}$$

p is defined on $\overline{\Delta}$ so that the derivative of p on the boundary of Δ is well-defined. The following lemma shows that our game is a potential game.

Lemma 1. *G is a potential game.*

Proof. $W(n) = n_0 v(A_r) + \sum_{k=1}^{K} \int_0^{n_k} \left[h(y) - T(y) - u'(H/y)H/y + u(H/y) \right] dy + n_k v(A_c)$ is a potential function. □

Then, the following results are useful for us:

Lemma 2 (Sandholm, 2001)**.** *(i) n is a Nash equilibrium of G if and only if n satisfies the Kuhn-Tuker conditions for the maximization of W. (ii) The set of Nash equilibria of G is globally attractive under any admissible dynamic for G. (iii) Suppose there is no continuum of equilibria. Then, every Nash equilibrium of G that locally maximizes W is asymptotically stable under any admissible dynamic for G.*

Therefore, Nash equilibria of G and their stability can be characterized by using a potential function W. In particular, Lemma 2 (i) and (ii) imply that if W is strictly concave, Nash equilibrium of G is unique and is globally attractive under any admissible dynamic for G.

Example 2 (Stability of full agglomeration equilibrium)**.** Consider the setting in Example 1. By Lemma 2 (iii), full agglomeration in one city is an asymptotically stable equilibrium if W is locally maximized there. Then, since

$$dW = \sum_{k \in S \setminus \{0\}} (\psi(n_k) - v) dn_k,$$

it is asymptotically stable if $\psi(1) > \max\{\psi(0), v\}$, or equivalently, $t < e^{\varepsilon} - 1$. □

2.3 Social Optimum

In our paper, we consider the following social welfare function:

$$SW(n; \theta) = \sum_{k \in S} n_k U_k(n). \tag{6}$$

The planner would like to achieve a population distribution $n \in \Delta$ that maximizes the above function under the resource constraints:

$$n_k c_k + n_k T(n_k) \leq f(n_k) \quad \text{for all } k \in S \setminus \{0\}. \tag{7}$$

We choose $\{n_k\}_{k \in S \setminus \{0\}}$ with the constraints $\sum_{k \in S \setminus \{0\}} n_k \leq 1$ and $n_k \geq 0$ for all $k \in S \setminus \{0\}$. Then, the Kuhn-Tucker condition is

$$f'(n_k) - T(n_k) - n_k T'(n_k) - u'(H/n_k)H/n_k + u(H/n_k) - v - \mu \leq 0 \tag{8}$$

for $k \in S \setminus \{0\}$ where $\mu \geq 0$ is the Lagrange multiplier for the constraint $\sum_{k \in S \setminus \{0\}} n_k \leq 1$. The condition holds with equality whenever $n_k > 0$. The condition tells us that all the externalities must be internalized at a social optimum: in cities, the households receive the marginal productivity $f'(n_k)$ for their labor supplies and additionally pay the value of congestion externalities $n_k T'(n_k)$ for commuting.

Example 3. Consider the setting in Example 1. Then, (8) is written as $\psi(n_k) + n_k \psi'(n_k) - v - \mu \leq 0$. Thus, if $\psi(1) + \psi'(1) < \max\{\psi(0), v\}$, the Kuhn-Tucker condition is violated, hence full agglomeration in one city is not optimal. Therefore, by Example 2, *full agglomeration in one city is a nonoptimal asymptotically stable equilibrium if* $\max\{\psi(0), v\} < \psi(1) < \max\{\psi(0), v\} - \psi'(1)$, *or equivalently,* $\frac{e^{\varepsilon} + \varepsilon e^{\varepsilon} - 1}{2} < t < e^{\varepsilon} - 1$. □

In the above example, the planner needs to reduce the city population, but since the equilibrium is stable, he cannot induce marginal migrations by one shot policy as discussed before and hence, it has been argued that we need developers that directly control city populations to achieve a social optimum. In the next section, we design an environment in which the planner internalizes externalities evaluated at current prices and population distribution in each period and achieve a social optimum in the long-run. When we construct such an environment, we also address the point that the planner cannot compute optimal population distributions without the knowledge of the preferences.

3 Evolutionary Implementation

We consider evolutionary Nash implementation (of a social optimum) proposed by Sandholm (2002, 05b). For this purpose, we consider an environment in which the planner carries out a policy to achieve a social optimum and the households play the associated game. Specifically, we define a game $G^* = (U^*, \theta)$ in which

$$U_0^*(n) = v(A_r) + \Omega, \tag{9}$$
$$U_k^*(n) = w_k^* - T(n_k) - \tau_k^* - R_k H/n_k + v(A_c) + u(H/n_k) + \Omega, \quad \text{for } k \in S \setminus \{0\}, \tag{10}$$

where $w_k^* = f'(n_k)$, $\tau_k^* = n_k T'(n_k)$, and $\Omega = \sum_{k\in S\setminus\{0\}} f(n_k) - w_k^* n_k + \tau_k^* n_k + R_k H$. Under this game, the planner instructs the firms to pay the marginal productivities w_k^* for labors; levies the congestion taxes τ_k^* to commuters; and distributes surplus or finances loss through the lump-sum transfer Ω. Also, the planner takes the land markets as given to extract information from the prices. Observe that this policy is feasible without knowledge of the type. We call this scheme the *first-best policy*.

To define evolutionary implementation, we introduce a *social choice correspondence* $\phi : \Theta \rightrightarrows \Delta$ which specifies a set of strategy distributions $n \in \Delta$ for each type $\theta \in \Theta$. Then, Sandholm (2002, 05b) defines that the first-best policy *globally implements* a social choice correspondence ϕ if for all $\theta \in \Theta$, $\phi(\theta)$ is globally attractive under any admissible dynamic for G^*.[14]

Naturally, the planner would like to implement the *efficient social choice correspondence* ϕ^* defined as follows:

$$\phi^*(\theta) = \arg\max_{n\in\Delta} SW(n; \theta). \tag{11}$$

Since the planner takes the land markets as given, $R_k = u'(H/n_k)$. Therefore, we can readily see that G^* is a potential game for which SW is exactly a potential function. Thus, the set of Nash equilibria of G^* is globally attractive under any admissible dynamic for G^* by Lemma 2 (ii) and every element of $\phi^*(\theta)$ is a Nash equilibrium of G^* by Lemma 2 (i). Hence, if every equilibrium of G^* is a social optimum, the planner can achieve evolutionary implementation of a social optimum in the Sandholm's sense. However, since there generally exist nonoptimal equilibria, it is possible that the economy converges to a suboptimal equilibrium.

For illustration, let $h(n_k) = \exp(-\varepsilon/n_k)$ as in Henderson (1987), $T(n_k) = tn_k$, $u(H/n_k) = H/n_k$, $K = 2$, $\varepsilon = 0.3$, $t = 0.6$, and $v = 0.2$. Figure 1 depicts the contour plot of SW as a function of n_1 and n_2 for this example in which the function takes smaller value on darker area. We can see that SW attains the global maximum at the full dispersion equilibrium ($n_1 = n_2 = 1/2$), but it is locally maximized at no city equilibrium

[14]Sandholm considers the *price scheme* in which the planner levies taxes to each action, though our first-best policy cannot be described as a price scheme.

($n_0 = 1$) and equilibria of partial agglomeration in one city ($n_0 > 0$ and $n_k = 1 - n_0$ for some $k \in \{1, 2\}$). Hence, these suboptimal equilibria are asymptotically stable by Lemma 2 (iii) and therefore, $\phi^*(\theta)$ is not globally attractive.

One straightforward way to avoid this problem is to assume that SW is strictly concave as in Sandholm (2002, 05b). However, by doing so, we have to restrict our attention to cases in which full agglomeration in one city will never be optimal since the cities are identical. Therefore, it is not desirable in our context. Then, instead of resorting to properties of SW, we incorporate forward looking expectations to an evolutionary dynamic to remove this problem. Roughly speaking, while every local maximizer of the potential function is attractive under myopic admissible dynamics, only global maximizer(s) is attractive under the dynamic we will consider.

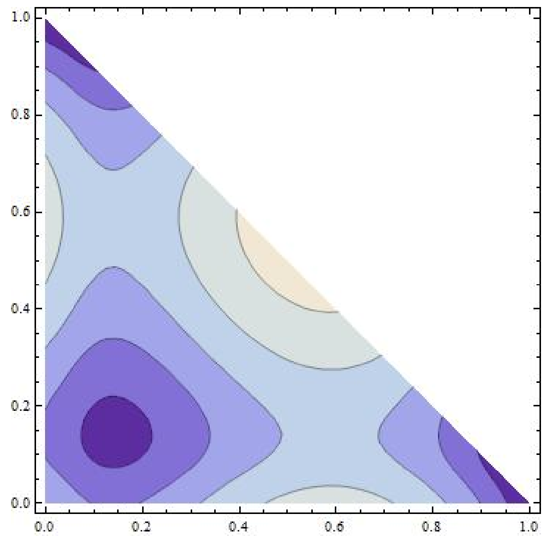

Figure 1: Contour Plot of SW

3.1 Perfect Foresight Dynamic

While the admissible evolutionary dynamics are myopic, in this section, we incorporate forward looking expectations to evolutionary dynamic in the spirit of Matsui and Matsuyama (1995). Specifically, we introduce a friction in migration decisions in the sense that the households have to commit to their location decisions for a random period of time. Opportunities to migrate follows the Poisson process with a rate parameter $\lambda > 0$ and we assume that the process is independent over the households and there is no aggregate uncertainty so that the average duration of commitment is $1/\lambda$. Let $\rho > 0$ be the rate of time preference. We call $\delta \equiv \rho/\lambda$ the *degree of friction*. Oyama (2009) considers the same kind of friction to study the "history versus expectations" issue in the New Economic Geography literature and in the following, we largely adopt his formulation.

We assume that each household has perfect foresight and therefore, he compares the present values of expected lifetime payoff from each action $k \in S$ to make a location decision at equilibrium. Since the duration of commitment is exponentially distributed with mean $1/\lambda$, given the first-best policy, the discounted lifetime payoff from action $k \in S$ at time t is computed as

$$\begin{aligned} V_k(t) &= (\lambda + \rho) \int_0^\infty \int_t^{t+s} e^{-\rho(z-t)} U_k^*(n(z)) dz \lambda e^{\lambda s} ds \\ &= (\lambda + \rho) \int_t^\infty e^{-(\lambda+\rho)(s-t)} U_k^*(n(s)) ds. \end{aligned} \tag{12}$$

Now, define the set $\mathcal{B}(t) = \left\{\alpha \in \Delta : \alpha_k > 0 \Rightarrow k \in \arg\max_{j \in S} V_j(t)\right\}$. This is the best response correspondence when the payoff from action k is given by $V_k(t)$. Then, we consider the following differential inclusion:

$$\dot{n}(t) \in \lambda(\mathcal{B}(t) - n(t)). \tag{13}$$

Given an initial state $n^0 \in \Delta$, $n(\cdot)$ is a solution trajectory of (13) if there exists $\alpha(t) \in \mathcal{B}(t)$ such that $\dot{n}(t) = \lambda(\alpha(t) - n(t))$ for almost all $t \geq 0$. In particular, a solution trajectory is called a *perfect foresight path* if it is Lipschitz continuous and it does not leave Δ. By Oyama *et al*. (2008), there exists a perfect foresight path for each initial state $n^0 \in \Delta$. Moreover, it follows that n is a rest point of (13) if and only if n is a Nash equilibrium of G^*.

We are interested in stability of equilibria of G^* in terms of the above perfect foresight dynamic. However, since we are considering a differential inclusion, solution trajectory is not necessarily unique for each initial state. Hence, we employ the stability concept proposed by Matsui and Matsuyama (1995) which is weaker than the standard one. We say that a set $A \subseteq \Delta$ is *accessible* from $n \in \Delta$ if there exists a prefect foresight path with $n^0 = n$ that converges to A.[15] $A \subseteq \Delta$ is *globally accessible* if it is accessible from any $n \in \Delta$. Then, we redefine evolutionary implementation as follows[16]:

[15]We define the distance between a point $n \in \Delta$ and a set $A \subseteq \Delta$ by $d(n, A) = \min_{y \in A} \|n - y\|$.

[16]Sandholm (2007) considers *stochastic evolutionary implementation* to deal with cases in which

Definition 1. *The first-best policy* globally implements *a social choice correspondence ϕ if for all $\theta \in \Theta$, $\phi(\theta)$ is globally accessible.*

If we restrict our attention to the accessibility, the analysis of stability is reduced to that of potential function as in the myopic case. Indeed, Oyama (2009) shows that if a strategy distribution is the unique global maximizer of a potential function, then it is globally accessible for small degrees of friction. Then, modifying his analysis so that it is applicable to our case, we can obtain the following result.

Proposition 1. *There exists $\bar{\delta} > 0$ such that the first-best policy globally implements the efficient social choice correspondence ϕ^* for all $\delta \in (0, \bar{\delta}]$.*

Proof. See Appendix. □

3.2 Henry George Theorem

Before closing the analysis, we discuss the relation of our argument to the Henry George Theorem which is a very important theorem in this field. Although the number of cities is a natural number in reality, suppose that it is allowed to take any nonnegative real number. Then, the *Henry George Theorem* states that the loss incurred by the firm due to the marginal cost pricing is exactly financed by the revenues from the congestion tax and the land rent in each city when the number of cities is optimal.[17] Hence, by comparing the loss and the revenue, the planner might be able to compute the optimal number of cities. However, it is not possible if he does not have the knowledge of preferences. Let $w^* = f'(n^*)$, $\tau^* = n^* T'(n^*)$, and $R^* = u'(H/n^*)$ where n^* solves $f(n^*) - w^* n^* + \tau^* n^* + R^* H = 0$. That is, n^* is the city population such that the Henry George Theorem holds. Since there is a rural area in our model, the optimal number of cities is given by

$$\begin{cases} 1/n^* & \text{if } h(n^*) - T(n^*) - R^* H/n^* + u(H/n^*) \geq v, \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

If the planner knew that it was efficient to construct cities, he could judge whether the current number of cities was optimal or not by computing Ω evaluated at current

a social welfare function is not strictly concave. In his model, opportunities to revise a strategy arrive randomly as in our model and hence, his setting is essentially the same as ours. However, he considers the logit choice rule and uses properties of the Markov process. Our framework is technically convenient because it allows us to assume that the players are nonatomic.

[17] See, for example, Fujita and Thisse (2002, Ch 4).

prices and population distribution. If it is zero, then the current state is optimal. However, since the planner generally does not know v, the planner may reach a fallacious conclusion. Moreover, even if the planner knows v, it is not possible for him to compute the optimal number of cities using the theorem because he needs to know the formula of R_k, that is, $u'(H/n_k)$ to calculate the city population such that $\Omega = 0$. In the first-best policy we considered, the planner seeks the city population such that the Henry George Theorem holds without the knowledge of preferences although the theorem does not exactly hold at optimum since the number of cities is restricted to be a natural number.

4 Conclusion

We considered evolutionary implementation of optimal number and size of cities in which the planner leads the economy to a social optimum in the long-run without forcing lumpy migrations and without the knowledge of preferences. We showed that the planner can achieve evolutionary implementation of a social optimum by internalizing the externalities evaluated at current prices and population distribution in each period. Such an implementation issue has received little attention in the literature and this result enables us to reconsider the unsatisfactory conclusion that we need developers to achieve a social optimum.

Subjects for future research can easily be found in view of the fact that our model is fairly simple. In particular, our model is a system of isolated identical cities (and rural area). As Anas and Xiong (2005) and Tabuchi and Thisse (2006) show, if we allow trade between cities and consider several industries, a rich set of stable equilibria such as urban hierarchy arise. Hence, it is worth investigating whether our implementation result is robust to more general settings.

Appendix

A.1 Proof of Proposition 1

The proof is almost the same as that of Oyama (2009), but since he considers the case in which a global maximizer of potential function is unique, we need to slightly modify his argument. Also, we need to ensure that his result is true for all $\theta \in \Theta$. Fix $\theta \in \Theta$ and let $d(n, \phi^*(\theta)) = \min_{n^* \in \phi^*(\theta)} \|n - n^*\|$. By Lemma C.7 of Oyama

(2009), for each $n^* \in \phi^*(\theta)$, there exists $B(\varepsilon(n^*))$, the open ball of n^* with diameter $\varepsilon(n^*) > 0$, such that any perfect foresight path with the initial state $n^0 \in B(\varepsilon(n^*))$ converges to $\phi^*(\theta)$ (i.e., $d(n(t), \phi^*(\theta)) \to 0$). Moreover, by Lemma C.4′ below, for any $\varepsilon > 0$, there exists $\bar{\delta} = \bar{\delta}(\varepsilon) > 0$ such that for all $\delta \in (0, \bar{\delta}]$ and for all $n^0 \in \Delta$, there exists a perfect foresight path such that $d(n(t), \phi^*(\theta)) < \varepsilon$ for some $t \geq 0$. Then, letting $\varepsilon^* = \min_{n^* \in \phi^*(\theta)} \varepsilon(n^*)$, for all $\delta \in [0, \bar{\delta}(\varepsilon^*))$ and for all $n^0 \in \Delta$, there exists a perfect foresight path $n(\cdot)$ such that $n(t) \in B(\varepsilon(n^*))$ for some $t \geq 0$ and for some $n^* \in \phi^*(\theta)$. Then, by the preceding argument, $n(t) \to \phi^*(\theta)$.

We have shown that for each $\theta \in \Theta$, there exists $\bar{\delta}(\theta) > 0$ such that $\phi^*(\theta)$ is globally accessible for all $\delta \in (0, \bar{\delta}(\theta)]$. Hence, it remains to show that $\inf_{\theta \in \Theta} \bar{\delta}(\theta) > 0$. But this immediately follows since Θ is assumed to be finite. □

Finally, we prove Lemma C.4′, which is a modified version of Lemma C.4 of Oyama (2009). Fix $\theta \in \Theta$ and consider the game G^* and its potential function $SW(\cdot\,; \theta)$. Then, consider the following optimal control problem:

$$\begin{aligned} \max \quad & J(n(\cdot)) = (\lambda + \rho) \int_0^\infty e^{-\rho t} SW(n(t); \theta) dt \\ \text{s.t.} \quad & n(\cdot) \text{ is Lipschitz continuous, does not leave } \Delta, \text{ and} \\ & \text{there exists } \alpha(t) \in \Delta \text{ such that } \dot{n}(t) = \lambda(\alpha(t) - n(t)) \text{ for almost all } t \geq 0. \end{aligned} \tag{15}$$

It follows that every solution to the above problem is a perfect foresight path. Then, in our context, Lemma C.4 of Oyama (2009) is stated as follows:

Lemma C.4′. *Fix $\theta \in \Theta$. Then, for any $\varepsilon > 0$, there exists $\bar{\delta} = \bar{\delta}(\varepsilon) > 0$ such that for all $\delta \in (0, \bar{\delta}]$ and for all $n^0 \in N$, if $n(\cdot)$ is an optimal solution to the problem* (15)*, then there exists $t \geq 0$ such that $d(n(t), \phi^*(\theta)) < \varepsilon$.*

Proof. Assume the contrary. That is, there exists $\varepsilon > 0$ such that for all $\bar{\delta} > 0$, there exists a solution $n(\cdot)$ to the problem (15) for some λ and ρ with $\delta = \lambda/\rho \in (0, \bar{\delta}]$ and for some $n^0 \in \Delta$ such that $d(n(t), \phi^*(\theta)) \geq \varepsilon$ for all $t \geq 0$. Pick some $n^* \in \phi^*(\theta)$. Given such an $\varepsilon > 0$, let $c = c(\varepsilon) > 0$ be such that

$$c = \max_{n \in \Delta} SW(n; \theta) - \max \left\{ SW(n; \theta) : d(n, \phi^*(\theta)) \geq \varepsilon \right\};$$

$T = T(\varepsilon) \geq 0$ be such that

$$SW(e^{-t} n^0 + (1 - e^{-t}) n^*; \theta) \geq \max_{n \in \Delta} SW(n; \theta) - c/2$$

for all $t \geq T$; and $\bar{\delta} = \bar{\delta}(\varepsilon) > 0$ be such that

$$\left(1 - e^{-\bar{\delta}T}\right) 2M < e^{-\bar{\delta}T} c/2,$$

where $M > 0$ is a constant such that $|SW(n;\theta)| \leq M$ for all $n \in \Delta$. Given such a $\bar{\delta} > 0$, let $n(\cdot)$ be a solution path with $\delta \in (0, \bar{\delta}]$ and $n^0 \in N$ such that $d(n(t), \phi^*(\theta)) \geq \varepsilon$ for all $t \geq 0$, as assumed. Let $y(t)$ be a candidate path for a solution of the problem (15) such that $y(t) = e^{-\lambda t} n^0 + (1 - e^{-\lambda t}) n^*$. Then, by the same argument as the original proof of Oyama (2009), we reach the contradiction that $J(y(\cdot)) > J(n(\cdot))$. □

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