Occupational Mobility and Consumption Insurance*

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Abstract

Evidence from the Consumer Expenditure Survey shows that over the 1980-1992 period, between-group consumption inequality for occupation groups has remained stable, while within-group consumption inequality has increased. Meanwhile, evidence from the Panel Study of Income Dynamics shows that involuntary occupational mobility has increased. Within a model with limited commitment, I show that under certain conditions a rise in occupational mobility increases the desire for insurance for low income individuals more than that of high income individuals, thereby increasing consumption inequality within that group. Numerical experiments suggest that taking account of occupational mobility is quantitatively important to account for the evolution of between- and within-group consumption inequality.

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1 Introduction

In this paper, I explore the impact of the increase in occupational mobility on between- and within-group consumption inequality over the 1980-1992 period in the United States. Evidence from the Consumer Expenditure Survey (CEX) over the 1980-1992 period shows that consumption inequality between occupation groups has remained stable, while within-group consumption inequality has increased.[1] Meanwhile, evidence from the Panel Study of Income Dynamics (PSID) shows that involuntary occupational mobility has increased over the same period. I study the relationship between occupational mobility and between- and within-group consumption inequality in a model with limited enforcement of contracts where agents face involuntary occupation switches as well as within-group income shocks. Numerical experiments suggest that taking account of occupational mobility is quantitatively important to account for the evolution of between- and within-group consumption inequality.

In the literature, a lot of effort has been devoted to measuring and accounting for the extent to which households insure their idiosyncratic income risk.[2] Based on the theoretical work by Kehoe and Levine (1993, 2001), Kocherlakota (1996) and Alvarez and Jermann (2000), Krueger and Perri (2006) was the first and, to my knowledge, the only work to quantitatively examine how the increase in idiosyncratic income risk observed in the United States translates into an idiosyncratic variation in consumption in a model with limited commitment. The authors find that their limited commitment model greatly underpredicts the increase in within-group consumption inequality observed in the CEX data, after controlling for various household characteristics including household head's occupation. A simplifying assumption in their paper is that the household characteristics are permanent. Consequently, the authors assume away occupational risk.

However, as observed in the PSID data, individuals may switch their occupations involuntarily. Therefore, in contrast to Krueger and Perri (2006), I take into account involuntary occupational mobility in addition to idiosyncratic income risk within

[1] For precise definition of between- and within-group inequality, see Section 2.3.

[2] See, among others, Mace (1991), Cochrane (1991), Nelson (1994), Attanasio and Davis (1996), Gervais and Klein (2006), Krueger and Perri (2006), Blundell et al. (2008) and Heathcote et al. (2008).

occupation groups.[3] I find that under certain conditions, which are stated below, a rise in occupational mobility leads to an increase in consumption inequality within occupation groups in a model with limited commitment.

I use data from the PSID to document occupational mobility of household heads over the 1980-1992 period. Since it is extremely costly to incorporate many occupation groups in a quantitative model, I use only two occupations in defining groups: professional/managerial specialty and others. Following Kambourov and Manovskii (2006), I define occupational mobility as the fraction of currently employed individuals who report a current occupation different from their previously reported occupation. With this definition, average occupational mobility between professional/managerial specialty and other occupation groups for the 1980-1985 period is 8.0%, and increases to 10.2% for the 1986-1992 period. In this paper, however, I focus my attention on the impact of occupation switches that are exogenous to individuals and thus can be considered as shocks. In reality, individuals can choose to change their occupation or anticipate changes in their occupation. For example, individuals at firms may know the timing of their promotion. Thus, as a proxy for exogenous occupation switch risk, I examine occupation switches due to involuntary job losses defined as a plant closing, an employer going out of business, and a layoff. Occupational mobility due to involuntary job losses increases from 0.6% in 1980-1985 to 0.8% in 1986-1992.

In order to measure the effects of involuntary occupational mobility on consumption insurance, I decompose the variance of log income and consumption into observable and unobservable components. For the observable components, I focus my attention on the variance accounted for by occupation groups and refer to this component as between-group inequality. I refer to the component of the overall variance not accounted for by observable attributes as within-group inequality.

Using income data from the PSID and expenditure data from the CEX, I document changes in between- and within-group income and consumption inequality between the periods 1980-1985 and 1986-1992. Between these two periods, between-group income inequality mildly increases, whereas within-group income inequality exhibits

[3] Fluctuations in group-specific mean income are another potentially important income risk that households face. Attanasio and Davis (1996) examine consumption insurance against relative wage movements among birth cohorts and education groups. Their estimation results suggest almost no consumption insurance against the group-specific income risk. Thus, as in Krueger and Perri (2006), I omit this type of group-specific income risk for simplicity.

a much larger increase. Meanwhile, between-group consumption inequality remains stable, while within-group consumption inequality increases.

To study the conceptual link between occupational mobility and between- and within-group consumption inequality, I construct a stylized pure exchange economy with limited enforcement of contracts along the lines of Kehoe and Levine (1993). I assume that there are two occupation groups, one with high mean income and one with low mean income. In addition, each occupation group has two idiosyncratic income states so that there are a total of four income states. There are four agents, each of whom faces two types of idiosyncratic income shocks, namely involuntary occupation switches and within-group income shocks. This simple structure allows me to analytically characterize constrained efficient symmetric stationary Markov allocations, as well as to isolate a simple mechanism through which a rise in occupational mobility leads to an increase in within-group consumption inequality.

When occupational mobility increases, the value of autarky (agent's outside option) for the low-income agent decreases more than that for the high-income agent in the high income occupation, in the presence of persistent within-group income shocks. According to the characterization result, both (all) of the agents in the high income occupation are constrained, which means that the agents contribute some of their resources to risk sharing. Given the changes in the value of autarky, the low-income agent in the high income occupation increases his/her contributions to risk sharing more than the high-income agent. As a result, consumption inequality within the high income occupation increases.

The next step is to study the quantitative effect of occupational mobility on the evolution of between- and within-group consumption inequality. To do so, I use a model of a production economy with limited enforcement of contracts and a continuum of agents. In this environment, I compute a stationary competitive equilibrium with solvency constraints that are not too tight along the lines of Alvarez and Jermann (2000) for the periods corresponding to 1980–1985 and 1986–1992. Unfortunately, due to computational difficulty, I am not able to numerically compute the stationary equilibrium with the empirical estimates of between-group income inequality.[4] However, numerical experiments with various values of between-group income inequality sug-

[4]For the computational difficulty, see Section 4.2.2.

gest that the increase in within-group consumption inequality would be at least twice or three times as large in the model with occupational mobility as that in the model without occupational mobility.

The rest of the paper is organized as follows. Section 2 presents empirical evidence of occupational mobility and between- and within-group income and consumption inequality. Section 3 examines constrained efficient allocations in a stylized pure exchange economy with limited enforcement of contracts. Section 4 presents a quantitative model, describes the benchmark parameterization, and reports the main results. Section 5 concludes.

2 Empirical Evidence

2.1 Data

I use data from the PSID since the survey provides detailed panel data on respondent's income and characteristics such as age, race, sex, education, and occupation. It is imperative to have the panel dimension in order to estimate occupational mobility and income process. However, the PSID does not collect detailed information on household expenditure, collecting only food and some housing-related (rent and property tax) expenditure on a regular basis. In order to address this problem, I report changes in consumption inequality using data from the CEX where more detailed expenditure information is available.

2.1.1 PSID

My benchmark PSID sample runs from 1980 to 1992.[5] The unit of analysis is a household. I define after-tax income as after-tax labor earnings plus transfers. In the PSID, income data collected in year t refer to the previous calendar year $t-1$. The PSID stopped imputing income taxes in 1992, which affects the 1991 and 1992 income

[5]The sample period starts in 1980 since consumption data from the CEX are not available prior to 1980. It is now possible to extend the sample period to 1997 since the PSID data for 1994–1997 have been made available. However, because the PSID changed frequency of survey interviews from yearly to once every two years in 1997, I cannot extend the analysis to the post 1997 period in a consistent manner.

measures. To construct income measures for those years, I use the *Cross-National Equivalent File* that imputes federal and state income taxes for the PSID data using the National Bureau of Economic Research *TAXSIM* package.[6]

I restrict the original sample to households with reliable data for labor earnings and main characteristics including education and occupation. Table 8 in Appendix A reports the step-by-step selection of my PSID sample. First, I exclude households experiencing major family composition change, in particular changes in the head or the spouse, over the sample period. It is because income reported in year t, for example, may not correspond to household characteristics reported in $t-1$ if the family composition changes dramatically from $t-1$ to t. I exclude households with female heads in order to focus on the evolution of income inequality and occupational mobility not driven by the increase in female labor force participation rates in the 1980s. [7] I exclude households with missing reports on region of residence, education, and occupation (conditional on being employed) as they are necessary to construct between- and within-group inequality in the next section. I exclude households with topcoded income and food expenditures assigned by the PSID. I also eliminate households with income and consumption outliers.[8] The PSID consists of two different subsamples: the first is representative of the US population; the second is a supplementary low income subsample (SEO sample). For the analysis below, I exclude the SEO sample. To focus on households whose head is of working age, I exclude households whose head is less than 30 or more than 65 years of age.

In what follows, I examine how households insure against income changes due to occupational mobility and other changes not explained by observable characteristics. For the benchmark analysis, I exclude households whose head is classified as 'armed services and protective service workers' in the one-digit occupation code. This occupation is likely to feature distinct occupational mobility and income shocks, and accounts for less than 3% of the sample. I also exclude households whose head is

[6]The *Cross-National Equivalent File* is created and maintained by the Department of Policy Analysis and Management at Cornell University.

[7]Many previous studies using the PSID data exclude female-headed households. For example, see Blundell et al. (2008).

[8]Following Blundell et al. (2008), I consider income and consumption as an outlier if an annual income is below 100 dollars or below total food expenditure, if income growth is above 500% or below -80%, or if total food expenditure is zero or missing.

unemployed for 5 consecutive years. This criterion drops only 2% of the remaining sample. As a result, my PSID sample contains 2,881 households and 22,299 observations.

2.1.2 CEX

The Consumer Expenditure Survey provides detailed information on household expenditures. In the survey, each household is interviewed once every three months over five consecutive quarters. In the second through fifth interview, households report their expenditures for the last three months from the time of each interview. To compute consumption inequality, I use data on annual expenditures, summing up the four observations of the quarterly expenditures. Unlike the PSID in which income data refer to a calendar year, annual expenditures in the CEX refer to the last twelve months from the fifth interview that can take place in any month of a year. Following Blundell et al. (2008), I assign an observation to a given year if the fifth interview of the given household is conducted between July of the given year and June of the following year.

My CEX sample runs from 1980 to 1992. I impose basically the same sample selection as the one described above. Table 9 describes the step-by-step sample selection. Table 10 shows that the mean characteristics of the CEX sample closely match those of the PSID sample.

2.2 Occupational Mobility

In this section, I document household head's occupational mobility for the 1980-1985 period and the 1986-1992 period, using PSID data. As mentioned before, computational considerations lead me to use only two occupations in defining groups: 1. professional/managerial specialty; 2. others. For those who are unemployed at the point of interview, I refer to their most recent occupation by exploiting PSID's long panel dimension. A timing issue arises for changes in occupation. Although people may change their job at any time of the year, for the benchmark analysis below, I refer to an occupation at the time of the interview as the status for the whole interview year.[9] Furthermore, I only consider respondent's main occupation even though people may have multiple jobs at a time.

Tables 1 and 2 report the transition matrix of household head's mobility over the occupation groups for two subperiods: 1980-1985 and 1986-1992. I compute transition probabilities over the sample period as follows. Take the transition from Group 1 to Group 2, for example. First, compute the fraction of household heads making the transition in each year. Then calculate the average fraction over the sample period. I take the average fraction as the transition probability for the transition from Group 1 to Group 2. I repeat the same computation for all transitions.

I consider two concepts of occupational mobility, labelled Mobility 1 and Mobility 2. Mobility 1 counts all the household heads who change their occupation. However, involuntary switches are of more interest when considering occupational mobility as income risk. The PSID provides information on the reason why a household head left his previous job, which is categorized as follows: 1. company folded/changed hands/moved out of town, employer died/went out of business; 2. strike, lockout; 3. laid off, fired; 4. quit, resigned, retired, pregnant, needed more money, just wanted a change in jobs, was self-employed; 5. no previous job; 6. promotion; 7. other, transfer, any mention of armed services; 8. job was completed, seasonal work, was a temporary job. Following Cochrane (1991), I consider job loss due to 1 - 3 to be involuntary. Then Mobility 2 only counts occupation switches due to involuntary job loss. Hence, note that 'staying in the same group' in Mobility 2 contains those who change their occupation voluntarily as well as those who stay in the same group.

Table 1 reports a transition matrix for Mobility 1 for two subperiods, 1980-1985 and 1986-1992, while Table 2 reports a transition matrix for Mobility 2. These two tables show that household heads do change their occupation over time. Although Mobility 2 features much less occupation switches, the numbers are still substantial (0.9% from Group 1 to Group 2, 0.6% from Group 2 to Group 1 for 1980-1992). In both tables, transition probabilities increase between the periods 1980-1985 and 1986-1992. Following Kambourov and Manovskii (2006), define occupational mobility as the fraction of currently employed individuals who report a current occupation different from their most previous report of an occupation. For Mobility 1, the value

[9] More than 80% of the interviews were conducted between March and May for 1980-1992.

Table 1: Occupational Mobility 1 (1980-1985, 1986-1992)

	1979-1985		1985-1992	
	1	2	1	2
1	0.9121	0.0879	0.9002	0.0998
2	0.0766	0.9234	0.0899	0.9101

Table 2: Occupational Mobility 2 (1980-1985, 1986-1992)

	1979-1985		1985-1992	
	1	2	1	2
1	0.9920	0.0080	0.9900	0.0100
2	0.0057	0.9943	0.0065	0.9935

increases from 8.0% to 10.2%. For Mobility 2, the value increases from 0.6% to 0.8%

2.3 Between- and Within-Group Inequality

To measure the effects of occupational mobility on consumption insurance in detail, I decompose income and consumption inequality into observable and unobservable components. Furthermore, for the observable components, I focus my attention on inequality accounted for by occupation and refer to this component as between-group inequality. I refer to the component of overall inequality not accounted for by observable attributes as within-group inequality. In this section, I document the evolution of between- and within-group income and consumption inequality over the 1980-1992 period, using income data from the PSID and expenditure data from the CEX.

My income measure is after-tax labor income plus transfers. I measure consumption by expenditures on nondurables, services, and small durables (such as household equipment) plus imputed services from housing and vehicles (ND+).[10] I deflate in-

[10] I follow Krueger and Perri (2006) to construct this ND+ measure. I have also computed between- and within-group consumption inequality with nondurable goods expenditures and food expenditures, respectively. Though the magnitude of the increase in within-group consumption inequality differs across these consumption measures, overall patterns are similar. Therefore, for brevity, I only report results with the ND+ measure.

come and consumption measures by the relevant CPI and by an adult equivalence scale taken to be the square root of the family size.[11] Income and consumption inequality are weighted by sample weights throughout this section.

I decompose income and consumption inequality into observable and unobservable components by a standard variance decomposition.[12] First, I regress the logarithm of income and consumption on a constant, region of residence, marital status, the number of family members, the number of earners other than household head and spouse, household head's age, household head's and spouse's (if present) race, education, and occupation. I run the cross-sectional regressions separately year by year, allowing coefficients to vary over time. Regression equations are as follows,

$$\begin{aligned} \ln y_{it} &= \gamma_{0t}^y + z_{it}'\gamma_{1t}^y + d_{it}^1\alpha_t^y + u_{it}^y \\ \ln c_{it} &= \gamma_{0t}^c + z_{it}'\gamma_{1t}^c + d_{it}^1\alpha_t^c + u_{it}^c \end{aligned}$$

where y_{it} and c_{it} are, respectively, household i's income and consumption at t, d_{it}^1 is the professional/manager dummy, and z_{it} is a vector of the regressors except for a constant and the professional/manager dummy. The orthogonality of the OLS estimator implies

$$\begin{aligned} Var(\ln y_{it}) &= Var(\hat{\gamma}_{0t}^y + z_{it}'\hat{\gamma}_{1t}^y + d_{it}^1\hat{\alpha}_t^y) + Var(\hat{u}_{it}^y) \\ Var(\ln c_{it}) &= Var(\hat{\gamma}_{0t}^c + z_{it}'\hat{\gamma}_{1t}^c + d_{it}^1\hat{\alpha}_t^c) + Var(\hat{u}_{it}^c). \end{aligned}$$

In this paper, instead of the quantitative contributions of all observable attributes to the overall inequality, I focus my attention on the contributions of occupation. Thus I measure between-group income and consumption inequality by $Var(d_{it}^1\hat{\alpha}_t^y)$ and $Var(d_{it}^1\hat{\alpha}_t^c)$. I measure within-group income and consumption inequality by $Var(\hat{u}_{it}^y)$ and $Var(\hat{u}_{it}^c)$.

2.3.1 Results

Figure 1 shows the evolution of between-group income and consumption inequality from its 1980 value, while Figure 2 shows the evolution of within-group income and

[11] The square root of the family size is a widely used adult equivalence scale. See, for example, Blundell et al. (2008).

[12] For example, see Katz and Autor (1999).

Figure 1: Between-Group Inequality

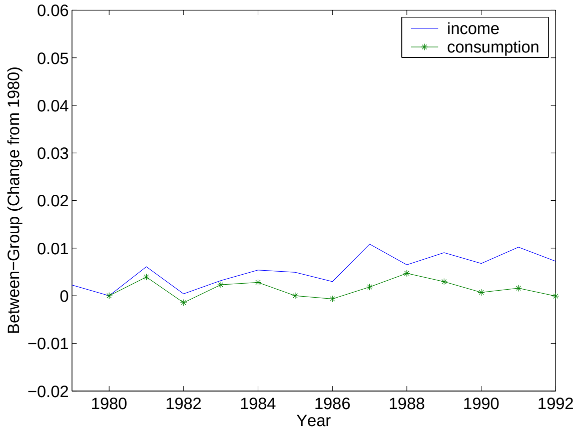


Figure 2: Within-Group Inequality

consumption inequality. In both figures, the line with asterisk on data points represents consumption inequality (measured by ND+), and the line without asterisk represents income inequality. As high-frequency variation in time series of income and consumption inequality is likely due to measurement error in the PSID and the CEX, I present average inequality for two sub-periods, 1980-1985 and 1986-1992 in Table 3.

In Table 3, first note that consumption inequality is smaller than income inequality in both between- and within-group components. This observation is consistent with households partially insuring against their idiosyncratic income risk. Between the periods 1980-1985 and 1986-1992, between-group income inequality mildly increases from 0.0072 to 0.0116, while within-group income inequality exhibits a much larger increase from 0.2052 to 0.2320. Meanwhile, between-group consumption inequality remains stable, while within-group consumption inequality increases from 0.1286 to 0.1457. However, the increase in within-group consumption inequality is smaller than within-group income inequality.

3 Pure Exchange Economies

In this section, I examine a qualitative mechanism that links occupational mobility and between- and within-group consumption inequality in a stylized model of pure

Table 3: Between- and Within-Group Inequality (1980-1992)

	1980-1985	1986-1992	Change
Between-group inequality			
Income (PSID)	0.0072	0.0116	0.0045
	$[0.0057, 0.0096]$	$[0.0096, 0.0140]$	
Consumption (ND+, CEX)	0.0040	0.0043	0.0003
	$[0.0033, 0.0061]$	$[0.0036, 0.0059]$	
Within-group inequality			
Income (PSID)	0.2052	0.2320	0.0268
	$[0.1988, 0.2134]$	$[0.2205, 0.2358]$	
Consumption (ND+, CEX)	0.1286	0.1457	0.0171
	$[0.1232, 0.1338]$	$[0.1411, 0.1495]$	

Notes. Numbers in square brackets refer to the 90 percent confidence interval of the corresponding number computed by a bootstrap procedure with 399 repetitions.

exchange economy. Key features of the model are two occupation groups with different mean income levels, two idiosyncratic income states within each occupation, and limited enforcement of contracts.

In Section 3.1, I set up a model with four agents and analytically characterize constrained efficient symmetric stationary Markov allocations (Propositions 1 and 2). Numerically computing constrained efficient symmetric stationary Markov allocations, I find that a rise in occupational mobility leads to an increase in within-group consumption inequality for the high-mean income group.

3.1 A Model with Four Agents

I consider a pure exchange economy. Time is discrete. There are four agents, $i \in \{1, 2, 3, 4\}$. Every agent lives for infinitely many periods. There are two occupation groups, each of which consists of two agents. Every period, agents are endowed with perishable consumption goods. The endowments follow a stochastic process that I describe below.

There are two sources of income shocks in the model: occupational mobility and idiosyncratic income shocks. The process y_t represents occupational mobility and z_t

represents idiosyncratic income shocks. Stochastic processes y_t and z_t are independent and both are symmetric stationary Markov chains with symmetric transition matrices. For all t, y_t and z_t take on only two values, $\{1,2\}$ and $\{h,l\}$, respectively. Let ρ and γ denote the probability of staying in the same state for y_t and z_t, respectively. Agents face idiosyncratic income shocks, that is, $0<\gamma<1$. For occupational mobility, I also consider the case with no such shocks, so $0 \le \rho \le 1$. Let $\theta_t=(y_t,z_t)$ and $\Theta = \{(y,z)|y\in\{1,2\}, z\in\{h,l\}\}$: the set Θ is the range of θ_t for every t. Since y_t and z_t are independent, θ_t also follows a symmetric stationary Markov chain with a symmetric transition matrix denoted by $\Pi=(\pi(\theta'|\theta))$. Let $\theta^t=(y^t,z^t)=(y_0,\ldots,y_t,z_0,\ldots,z_t)$ denote a history up to period t and $\pi(\theta^t)$ the probability of θ^t. The distribution of θ_0 is discrete uniform: $\pi(\theta_0)=1/4$ for all $\theta_0\in\Theta$.[13] Let ω_t^i denote an endowment process of Agent i. The endowment process is adapted to the stochastic process θ_t. Furthermore, $\omega_t^i(\theta^t)=\omega^i(\theta_t)$ and

$$\begin{cases} \omega^1=\omega_{1h},\ \omega^2=\omega_{1l},\ \omega^3=\omega_{2h},\ \omega^4=\omega_{2l} & \text{if } \theta_t=(1,h) \\ \omega^1=\omega_{1l},\ \omega^2=\omega_{1h},\ \omega^3=\omega_{2l},\ \omega^4=\omega_{2h} & \text{if } \theta_t=(1,l) \\ \omega^1=\omega_{2h},\ \omega^2=\omega_{2l},\ \omega^3=\omega_{1h},\ \omega^4=\omega_{1l} & \text{if } \theta_t=(2,h) \\ \omega^1=\omega_{2l},\ \omega^2=\omega_{2h},\ \omega^3=\omega_{1l},\ \omega^4=\omega_{1h} & \text{if } \theta_t=(2,l) \end{cases} \tag{1}$$

where $\omega_{1h}>\omega_{2l}>0$, $\omega_{2l}<\omega_{1l}<\omega_{1h}$, and $\omega_{2l}<\omega_{2h}<\omega_{1h}$. Let us interpret the situation as follows. If $\theta_t=(1,h)$, then Agent 1's state is $(1,h)$, Agent 2's state is $(1,l)$, and so on. The same interpretation applies to the cases of $\theta_t=(1,l)$, $\theta_t=(2,h)$, and $\theta_t=(2,l)$. Agent 1 and Agent 2 (3 and 4) are in the same group and Agent 1 and Agent 3 (2 and 4) always have the same idiosyncratic income state. Equation (1) guarantees that agents face occupational mobility when ρ is strictly less than one. That is, $(\omega_{1h}+\omega_{1l})/2 > (\omega_{2h}+\omega_{2l})/2$. Throughout this section, I do not impose any order between ω_{1l} and ω_{2h}. Assumption (1) also guarantees that the amount of aggregate resources available in the economy remains constant over time.

Agents have identical preferences represented by the following standard expected utility function,

$$\sum_{t=0}^{\infty}\sum_{\theta^t}\beta^t\pi(\theta^t)u(c_t^i(\theta^t)),$$

[13]With the uniform distribution, agents are ex ante identical.

where $\beta \in (0,1)$ is a discount factor. The period utility function u is given by

$$u(c) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1 \\ \ln c & \text{if } \sigma = 1. \end{cases}$$

Let $V^i(\theta_t)$ denote the value of autarky of Agent i at θ_t, which is defined as follows:

$$\begin{aligned} V^i(\theta_t) &= \sum_{\tau=t}^{\infty} \sum_{\theta^\tau|\theta^t} \beta^{\tau-t} \pi(\theta^\tau|\theta^t) u(\omega_\tau^i) \\ &= \sum_{\tau=t}^{\infty} \sum_{\theta^\tau|\theta_t} \beta^{\tau-t} \pi(\theta^\tau|\theta_t) u(\omega_\tau^i). \end{aligned}$$

Note that V^i is well-defined since θ_t and ω_t^i follow stationary Markov processes.

There is no storage technology or capital in this environment. Thus, the resource constraint reads:

$$\sum_{i=1}^{4} c_t^i(\theta^t) \leq \sum_{\theta \in \Theta} \omega_\theta \quad (\forall t)(\forall \theta^t).$$

Agents can walk away from their contract. If agents do so, then they are excluded from any risk sharing arrangement. With no storage technology or capital, agents must revert to autarky upon default. Hence participation constraints are formulated as

$$\sum_{\tau=t}^{\infty} \sum_{\theta^\tau|\theta^t} \beta^{\tau-t} \pi(\theta^\tau|\theta^t) u(c_\tau^i(\theta^\tau)) \geq V^i(\theta_t) \quad (\forall i)(\forall t)(\forall \theta^t). \tag{2}$$

An allocation is *feasible* if it satisfies the resource and participation constraints as well as the nonnegativity constraints on consumption. An allocation is *constrained efficient* if it is feasible and there is no other feasible allocation that Pareto dominates it. Any constrained efficient allocation is a solution to the following planner's problem with appropriate choice of weights $\langle \lambda^i \rangle_{i=1}^4$.

$$\max_{\langle c_t^i(\theta^t) \rangle_{i,t,\theta^t}} \sum_{i=1}^{4} \lambda^i \sum_{t=0}^{\infty} \sum_{\theta^t} \beta^t \pi(\theta^t) u(c_t^i(\theta^t))$$

subject to

$$\begin{aligned}\sum_{\tau=t}^{\infty}\sum_{\theta^\tau|\theta^t}\beta^{\tau-t}\pi(\theta^\tau|\theta^t)u(c^i_\tau(\theta^\tau)) &\geq V^i(\theta_t) \quad (\forall i)(\forall t)(\forall \theta^t),\\ \sum_{i=1}^{4} c^i_t(\theta^t) &\leq \sum_{\theta\in\Theta}\omega_\theta \quad (\forall t)(\forall\theta^t),\\ c^i_t(\theta^t) &\geq 0 \quad (\forall i)(\forall t)(\forall\theta^t).\end{aligned}$$

Note that the solution to the planner's problem exists uniquely, and it is characterized by first order conditions and complementary slackness conditions since the planner's problem is a convex problem.[14]

3.1.1 Constrained Efficient Symmetric Stationary Markov Allocations

This section examines properties of constrained efficient allocations under the endowment process specified above. As it is well known, constrained efficient allocations can feature either full, partial, or no consumption insurance. Since neither full nor no consumption insurance is consistent with the empirical evidence reported in the literature, I focus my attention on the case of partial insurance and study how consumption inequality changes with the persistence of occupational mobility, ρ.

I focus on constrained efficient allocations of the following simple structure and call it a *symmetric stationary Markov allocation*:

$$c^i_t(\theta^t) = \begin{cases} c_{1h} & \text{if } \omega^i_t(\theta^t) = \omega_{1h} \\ c_{1l} & \text{if } \omega^i_t(\theta^t) = \omega_{1l} \\ c_{2h} & \text{if } \omega^i_t(\theta^t) = \omega_{2h} \\ c_{2l} & \text{if } \omega^i_t(\theta^t) = \omega_{2l}. \end{cases}$$

[14] Suppose that $\pi(\theta_0) > 0$ for all $\theta_0 \in \Theta$. Define $\mathcal{S}$ and $\mathcal{C}^i$ by $\mathcal{S} = \prod_{t=0}^{\infty}\mathbb{R}^{4^{t+1}}$ and $\mathcal{C}^i = \{(c^i_0, c^i_1, \ldots) \in \mathcal{S} : (\forall t)(\forall\theta^t \in \Theta^t) \quad c^i_t(\theta^t) \geq 0\}$, respectively. The set $\mathcal{C}^i$ is agent i's consumption set. Define $\mathcal{C} = \prod_{i=1}^{4}\mathcal{C}^i$. Then the objective function in the above planner's problem is defined over $\mathcal{C}$. Let $\Phi \subset \mathcal{C}$ denote the constraint set of the problem. For each $t \geq 0$, endow $\mathbb{R}^{4^{t+1}}$ with the Euclidean metric. Endow $\mathcal{S}$ with the product topology. With this topology, one can show that the objective function is (upper-semi) continuous on $\mathcal{C}$ and the constraint set Φ is compact. Furthermore, Φ is convex and the objective function is strictly concave on Φ. Therefore, the maximum exists uniquely.

Let $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ denote a symmetric stationary Markov allocation. In general, constrained efficient allocations are history dependent and, therefore, do not take this simple form.[15] However, I present sufficient conditions for symmetric stationary Markov allocations to be constrained efficient (Proposition 1 in Section 3.1.1) and show that there exists a range of parameter values under which the sufficient conditions are satisfied (Proposition 3 in Appendix). For any symmetric stationary Markov allocation $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$, define V_θ^m by

$$V_\theta^m = u(c_\theta) + \beta \sum_{\theta' \in \Theta} \pi(\theta'|\theta) V_{\theta'}^m.$$

With a little abuse of notation, θ here represents the agent's endowment ω_θ. Thus, V_θ^m is the present discounted value that agents with ω_θ obtain in a risk sharing mechanism characterized by $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$. Note that, under assumption (1), autarky is also a symmetric stationary Markov allocation. Thus, let V_θ^a denote the value of autarky, which is defined by,

$$V_\theta^a = u(\omega_\theta) + \beta \sum_{\theta' \in \Theta} \pi(\theta'|\theta) V_{\theta'}^a.$$

With this notation, the participation constraint (2) for symmetric stationary Markov allocations can be written as follows: $V_\theta^m \geq V_\theta^a$ for all $\theta \in \Theta$.

I examine properties of constrained efficient allocations with some but not perfect risk sharing (partial risk sharing). An allocation is said to feature perfect risk sharing if it is the first best allocation under some initial weights, which is the unique solution to the planner's problem with the initial weights and without imposing the participation constraints (2). I say that perfect risk sharing is attainable from initial weights $\langle \lambda^i \rangle_{i=1}^4$ if the first best allocation under the initial weights is feasible. Since agents are ex ante identical, I consider the case of equal initial weights over agents ($\lambda^i = 1/4$ for all i). Under equal weights, the first best allocation is $c_t^i(\theta^t) = c_{FB} = \sum_\theta \omega_\theta / 4$ for all i, t, and θ^t. Hence, the first best allocation is feasible if and only if $u(c_{FB})/(1-\beta) \geq V_{1h}^a$.[16] An allocation is said to feature no risk sharing if it is autarkic. I say that no risk sharing is attainable if autarky is the only feasible

[15] Several researchers have found that the solution to the planner's problem stated above has a recursive structure if one expands the state space. See, for example, Marcet and Marimon (1998) or Rustichini (1998) for a recursive method to solve the planner's problem.

[16] All propositions in this section also hold with asymmetric initial weights. (See Remark 2 in

allocation (and, therefore, it is constrained efficient). Proposition 1 presents a set of sufficient conditions for symmetric stationary Markov allocations to be constrained efficient when agents face occupational mobility and some but not perfect risk sharing is attainable in the economy.

Proposition 1 *Let $\rho \in [0, 1)$. Suppose that perfect risk sharing is not attainable. Suppose that a feasible symmetric stationary Markov allocation $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ satisfies the following:*

- *some risk sharing:*
$$V_{2l}^m > V_{2l}^a. \tag{3}$$
- *resource constraint:*
$$c_{1h} + c_{1l} + c_{2h} + c_{2l} = \omega_{1h} + \omega_{1l} + \omega_{2h} + \omega_{2l}. \tag{4}$$
- *participation constraint:*
$$V_{1h}^m = V_{1h}^a, \quad V_{1l}^m = V_{1l}^a, \quad \text{and} \quad V_{2h}^m = V_{2h}^a. \tag{5}$$
- *non-degenerate distribution*
$$c_{1h} > \{c_{1l}, c_{2h}\} > c_{2l} \tag{6}$$
- *within-group inequality*
$$\frac{u'(c_{1h})}{u'(c_{1l})} \geq \frac{u'(c_{2h})}{u'(c_{2l})}. \tag{7}$$

Then the symmetric stationary Markov allocation is constrained efficient.

Proof. See Appendix A1. □

All the conditions in Proposition 1, except for (6), are also necessary for optimality. As for (6), the second inequality is part of the necessary conditions.

Appendix A1 for details.) Given $\langle \lambda^i \rangle_{i=1}^4$, the first best allocation is, for all i, t, θ^t, $c_t^i(\theta^t) = c_{FB}^i$ such that $u'(c_{FB}^i)/u'(c_{FB}^j) = \lambda^j/\lambda^i$ for all i, j. Therefore, the first best allocation is feasible if and only if $\min_i\{u(c_{FB}^i)/(1-\beta)\} \geq V_{1h}^a$. Note that if $u(\sum_\theta \omega_\theta/4)/(1-\beta) < V_{1h}^a$, then the first best allocation under any initial weight is not feasible.

Proposition 2 *Let $\rho \in [0,1)$. Suppose that a symmetric stationary Markov allocation $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ is constrained efficient and features some but not perfect risk sharing. Then the allocation satisfies the conditions (3), (4), (5) and (7) in Proposition 1, and $c_{2l} < \min\{c_{1h}, c_{1l}, c_{2h}\}$.*

Proof. See Appendix A2. □

Note that conditions (3), (4), and (5) pin down a feasible symmetric stationary Markov allocation and, thus, determine the structure of risk sharing arrangements. More precisely, under conditions (3) - (5), agents only insure against the worst income state ω_{2l}.[17] In particular, note that the participation constraint binds for the agent at $(1,l)$, the lowest income state in the high-mean income group. In Proposition 1, conditions (6) and (7) are imposed to guarantee constrained efficiency of such risk sharing arrangements. Condition (7) requires within-group consumption variability in the high-mean income group to be sufficiently low, allowing agents to focus on the insurance against the worst income state ω_{2l}. In Appendix A, I show that, for sufficiently high ρ, the condition is satisfied when, given the persistence of within-group income shocks, within-group income inequality for the high-mean income group is small relative to within-group consumption inequality for the low-mean income group (Proposition 3).

3.1.2 Numerical Example

In this section, I numerically examine how between- and within-group consumption inequality vary with the probability of switching occupation, denoted by $q = 1 - \rho$. For this exercise, I use the Theil index as the index is additively decomposable into between- and within-group components.[18] Let n denote the population size and y_i the income (or consumption) of agent i. The Theil index, T, is defined by

$$T = \sum_{i=1}^{n} s_i \log(n s_i),$$

[17] To see this, note that, for any feasible allocation, the following statement is true: if the participation constraint holds with equality for i at t and θ^t, then $c_t^i(\theta^t) \le \omega_t^i(\theta^t)$. The inequality becomes strict when there is some risk sharing. Therefore, conditions (3) and (5) imply that $c_{1h} < \omega_{1h}$, $c_{1l} < \omega_{1l}$, and $c_{2h} < \omega_{2h}$.

[18] Through simulations, Kuga (1980) shows that the experimental rankings of the Theil index are similar to those of the Gini index.

where $s_i = y_i/(n\overline{y})$. Here $\overline{y}$ is the arithmetic mean of $\{y_i\}_{i=1}^n$. Suppose that the population is partitioned into k subgroups. Then it holds that

$$T_{total} = T_{between} + \sum_{j=1}^{k} g_j T_j,$$

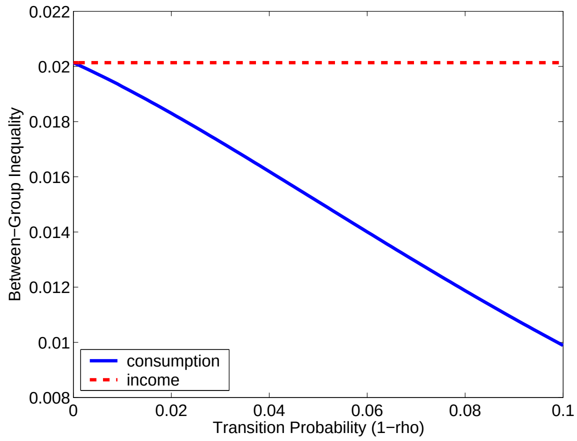
where g_j is the share of group j's income in total income (or consumption), T_j is the Theil index for group j, and $T_{between}$ is the Theil index for the allocation in which each group member is assigned the corresponding group-mean income (or consumption).

I set parameter values as follows: $\beta = 0.65$, $\sigma = 1.0$, $\gamma = 0.9$ and $(\omega_{1h}, \omega_{1l}, \omega_{2h}, \omega_{2l}) = (1.0, 0.5, 0.9, 0.1)$. The subjective discount factor β is set to 0.65 so that perfect risk sharing is not attainable. In light of Proposition 1, one can find a constrained efficient allocation by solving Equations (4) and (5) for a symmetric stationary Markov allocation and checking its optimality by the conditions stated in the proposition. In Figures 3 - 5, the solid line represents the Theil index of the constrained efficient allocation (consumption inequality) for $q \in [0, 0.1]$, while the dashed line represents that of autarky (income inequality). (Recall that $q = 1 - \rho$, the occupation switch probability.) Figure 3 shows between-group inequality. Figures 4 and 5 show within-group inequality for the high and low mean income groups, respectively.

Changes in occupational mobility affect both between- and within-group components of consumption inequality. Figure 3 shows that between-group consumption inequality decreases as occupational mobility increases. Without occupational mobility ($q = 0$), between-group consumption inequality takes the same value as between-group income inequality. This observation comes with no surprise given the fact that the Theil index is additively decomposable and the fact that obtaining the allocation $(\omega_{1h}, \omega_{1l}, \overline{c}_{2h}, \overline{c}_{2l})$ from $(\omega_{1h}, \omega_{1l}, \omega_{2h}, \omega_{2l})$ involves no between-group transfers. Figures 4 and 5 show that within-group consumption inequality for the high mean income group increases in q, while that for the low mean income group decreases in q.

For the increase in within-group consumption inequality for the high mean income group, the persistence of within-group income shocks and within-group consumption inequality for the low-mean income group play a crucial role. Since within-group income shocks are persistent, the agent with the low income state in the high-mean income group $(1, l)$ is more likely to transit to $(2, l)$ than the agent at $(1, h)$ in the incident of occupation switch. When occupational mobility increases, therefore, the agent at $(1, l)$ becomes worse off than the agent at $(1, h)$, given that within-group consumption inequality in the low-mean income group is strictly positive. Now recall that agents in the high-mean income group are constrained, thus giving up some of their resources. If the value of autarky for the agent at $(1, l)$ decreases sufficiently more than that for the agent at $(1, h)$, the agent at $(1, l)$ agrees to give up more resources than the agent at $(1, h)$. As a result, within-group consumption inequality for the high-mean income group increases as occupational mobility rises.

Figure 3: Between-Group Inequality



4 Quantitative Exercise

In this section, I extend the model by including a production sector and examine the quantitative impact of involuntary occupational mobility on between- and within-group consumption inequality. In Section 2.3, I divide the sample period into two subperiods, 1980-1985 and 1986-1992. For each subperiod, I compute a stationary competitive equilibrium and simulate it to compute between- and within-group consumption inequality.

Figure 4: Within-group inequality (high)

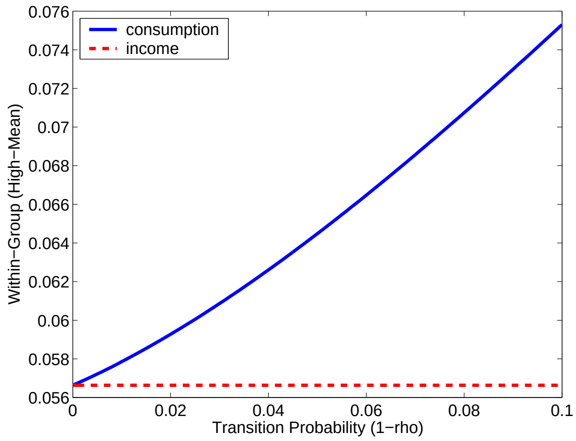
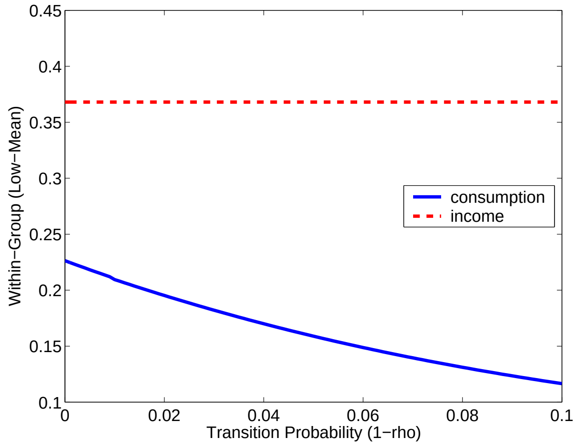


Figure 5: Within-group inequality (low)



4.1 The Model

I introduce occupational mobility in a model economy developed by Krueger and Perri (2006). The economy is a production economy. The representative firm produces a single good that can be used for consumption or investment in physical capital K. The aggregate resource constraint is as follows,

$$C_t + K_{t+1} = AK_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t, \tag{8}$$

where L_t is aggregate labor, K_t the aggregate capital stock, C_t aggregate consumption, A is a technology parameter, α is the capital income share, and δ is the depreciation rate.

There is a continuum of households of measure 1. I consider two occupation groups: 1. professional/managerial specialty; 2. the other. Every household belongs to either one of these two groups by their head's occupation. Households face a stochastic labor endowment process $\{\alpha_t u_t\}$, where $\exp(\alpha_t)$ and $\exp(u_t)$ represent the mean income of the occupation group to which household head belongs and the within-group idiosyncratic component, respectively. Household heads can change their occupation over time. Both α_t and u_t follow stationary Markov chains, with the support $\{\alpha_1, \alpha_2\}$ for α_t and the finite support U for u_t. These two stochastic processes are independent. Let $\theta_t = (\alpha_t, u_t)$ and $\Theta = \{(\alpha_t, u_t) | \alpha_t \in \{\alpha_1, \alpha_2\}, u_t \in$

$U\}$. The stochastic process θ_t also follows a stationary Markov chain with Θ as its support. Let $\pi(\theta'|\theta)$ denote the transition probabilities of the Markov chain. Let $\theta^t = (\alpha^t, u^t) = (\alpha_0, \ldots, \alpha_t; u_0, \ldots, u_t)$ denote a history up to period t. Also, $\pi(\theta^t|\theta_0) = \prod_{s=1}^{t} \pi(\theta_s|\theta_{s-1})$. Let Φ_0 denote the initial distribution over types (θ_0, a_0) where a_0 is household's initial asset holdings. Total labor supply is given by

$$L_t = \int \sum_{\theta^t} \exp(\alpha_t + u_t)\pi(\theta^t|\theta_0) d\Phi_0. \tag{9}$$

Household preferences are represented by the same utility function described in Section 3.

Households trade Arrow securities subject to borrowing constraints denoted by $B_t(\theta^t, \theta_{t+1})$. Let $q_t(\theta^t, \theta_{t+1})$ denote the prices for Arrow securities. Households face the following problem:

$$\max_{\{c_t\},\{a_{t+1}\}} u(c_0(a_0, \theta_0)) + \sum_{t=1}^{\infty} \sum_{\theta^t|\theta_0} \beta^t \pi(\theta^t|\theta_0) u(c_t(a_0, \theta^t)) \tag{10}$$

subject to

$$c_t(a_0, \theta^t) + \sum_{y_{t+1}} q_t(\theta^t, \theta_{t+1}) a_{t+1}(a_0, \theta^t, \theta_{t+1}) = \exp(w_t + \alpha_t + u_t) + a_t(a_0, \theta^t) \quad (\forall t)(\forall \theta^t) \tag{11}$$

$$a_{t+1}(a_0, \theta^t, \theta_{t+1}) \geq B_{t+1}(\theta^t, \theta_{t+1}) \quad (\forall t)(\forall \theta^t)(\forall \theta_{t+1}), \tag{12}$$

where $\theta^t|\theta_0$ means that θ^t is a possible continuation from θ_0, and $\exp(w_t)$ is the economy wide wage per efficiency unit of labor.

The constraints $B_t(\theta^t, \theta_{t+1})$ are specified as solvency constraints that are not too tight. First, if households default, they start the next period with neither assets or liabilities. In addition, after default, households do not have access to the markets for Arrow securities, but they are allowed to save (but not borrow) at a state-uncontingent interest rate r_d. Let $U_t^d(\theta_t)$ denote the continuation value of default from θ^t. Then,

$$U_t^d(\theta_t) = \max_{\{c_t, b_{t+1}\}} u(c_t) + \sum_{s=t+1}^{\infty} \sum_{\theta^s|\theta^t} \beta^{s-t} \pi(\theta^s|\theta^t) u(c_s)$$

subject to

$$\begin{aligned} c_s + \frac{b_{s+1}}{1+r_d} &= \exp(w_s + \alpha_s + u_s) + b_s \quad (\forall s \geq t)(\forall y^s|y^t) \\ b_{s+1} &\geq 0 \quad (\forall s \geq t) \end{aligned}$$

subject to $b_t = 0$. Next, define the continuation utility $V_t(\theta^t, a)$ of a household with history θ^t and current asset holdings a_t at time t as follows.

$$V_t(\theta^t, a_t) = \max_{\{c_t, a_{t+1}\}} u(c_t) + \sum_{s=t+1}^{\infty} \sum_{\theta^s|\theta^t} \beta^{s-t} \pi(\theta^s|\theta^t) u(c_s)$$

subject to (11) and (12). Then, the solvency constraints $\{B_{t+1}(\theta^t, \theta_{t+1})\}$ that are not too tight solve the following equation:

$$V_{t+1}(\theta^{t+1}, B_{t+1}(\theta^t, \theta_{t+1})) = U^d_{t+1}(\theta_{t+1}) \quad (\forall(\theta^t, \theta_{t+1})). \tag{13}$$

If the economy is in the steady state with aggregate quantities (K_t, L_t) and prices (r_t, w_t) constant, the Markov property of θ_t and the assumption that defaulting households start with neither assets nor liabilities imply that $U^d_t(\theta_t) = U^d(\theta_t)$, $V_t(\theta^t, a) = V(\theta_t, a)$, and $B_{t+1}(\theta^t, \theta_{t+1}) = B(\theta_{t+1})$. Thus, one can reformulate the household problem recursively with (θ, a) as state variables.

I am now in a position to define stationary competitive equilibrium with solvency constraints that are not too tight.

Definition 1 *A stationary competitive equilibrium with solvency constraints that are not too tight is a list of a value function V, policy functions for the household problem $\{c, a'\}$, solvency constraint B, aggregate quantities $\{K, L\}$, prices $\{w, r, q\}$ and a measure Φ defined over $(\Theta \times \mathbb{R}, \mathcal{B})$ where $\mathcal{B}$ is the Borel σ-algebra on $\Theta \times \mathbb{R}$ such that*

1. *Given prices and solvency constraints, V solves the following functional equation, and c and a' are the associated policy functions.*

$$V(\theta, a) = \max\{u(c) + \beta \sum_{\theta' \in \Theta} \pi(\theta'|\theta) V(\theta', a'(\theta, a, \theta'))\}$$

subject to

$$\begin{aligned} c + \sum_{\theta' \in \Theta} q(y'|y) a'(\theta, a, \theta') &\leq \exp(w + \alpha + u) + a \\ a'(\theta, a, \theta') &\geq B(\theta') \quad (\forall \theta' \in \Theta). \end{aligned}$$

2. *Solvency constraints are not too tight.*

$$V(\theta, B(\theta)) = U^d(\theta) \quad (\forall \theta \in \Theta).$$

3. *Given prices, the representative firm maximizes profits.*

$$\begin{aligned} w &= (1-\alpha)A\Big(\frac{K}{L}\Big)^{\alpha} \\ r &= \alpha A\Big(\frac{K}{L}\Big)^{\alpha-1} - \delta \end{aligned}$$

4. *All markets clear.*

$$\begin{aligned} \int c(\theta, a)d\Phi + \delta K &= AK^{\alpha}L^{1-\alpha} \\ L &= \int \exp(\alpha + u)d\Phi \\ K &= \frac{1}{1+r}\int \sum_{\theta'} a'(\theta, a, \theta')d\Phi. \end{aligned}$$

5. *Φ is stationary, that is, $\Phi(Z) = \int_{\Theta\times\mathbb{R}} Q((\theta, a), Z)d\Phi(\theta, a)$ for all $Z \in \mathcal{B}$ where Q is the transition kernel generated by transition probabilities $\pi(\theta'|\theta)$ and policy functions (c, a').*

4.2 Benchmark Parameterization

This section explains the selection of the benchmark parameter values. For preference and technology parameters, I use the same values as those used in Krueger and Perri (2006), while I parameterize occupational mobility and within-group income process based on the empirical evidence from the PSID data. However, since the policy function iteration often fails to converge with sufficient precision when two occupation groups overlap heavily, I need to set between-group income inequality larger than the empirical estimates reported in Section 2.3.[19] Though it makes the numerical results not directly comparable to the empirical estimates from the CEX data, the benchmark results and the sensitivity analysis with respect to between-group income inequality are useful to examine the quantitative effect of occupational mobility on the evolution of between- and within-group consumption inequality.

[19]For benchmark values of between-group income inequality, see Section 4.2.2.

4.2.1 Preference and Technology Parameters

I set the degree of relative risk aversion σ to 1, which implies that $u(c) = \log(c)$, and a wage rate is normalized to 1. I set technology parameters (A, α, δ) and the subjective discount factor β to the values used in Krueger and Perri (2006) who calibrate their model to the US economy in the 1980's. The authors obtain their benchmark parameter values using the following empirical evidence: a capital income share of 30%, an after-tax real return on physical capital of 4% per annum (McGrattan and Prescott (2003)), a capital-to-output ratio of 2.6 (the authors' calculation of the average wealth (including financial wealth and housing wealth) using CEX data for 1980-1981). The resulting parameter values that I also use for my model are: $\alpha = 0.3$, $A = 0.9637$, $\delta = 0.0754$ and $\beta = 0.959$.

4.2.2 Between-Group Income Inequality and Occupational Mobility

Since the policy function iteration often fails to converge with sufficient precision when two occupation groups overlap heavily, I need to set between-group income inequality larger than the empirical estimate reported in Section 2.3.[20] For this numerical exercise, I use between-group income inequality 10 times as large as the empirical estimates: the benchmark values of between-group inequality are 0.072 for 1980-1985 and 0.116 for 1986-1992.

I set the transition probabilities of switching occupation groups using the empirical estimates of occupational mobility due to involuntary job loss presented in Section 2.2, because occupation switch is exogenous in the current model. Table 4 shows the benchmark values of the transition probabilities for involuntary occupation switches.

[20] To compute the stationary equilibrium with solvency constraints that are not too tight, one needs to solve for both endogenous borrowing limits and policy functions in the policy function iteration. When two occupation groups overlap heavily, it becomes very hard to identify all the income states with binding borrowing constraints, which makes the policy function iteration unstable.

Table 4: Occupational Mobility

	1979-1985		1985-1992	
	1	2	1	2
1	0.9920	0.0080	0.9900	0.0100
2	0.0057	0.9943	0.0065	0.9935

Notes. Occupation groups are defined as follows: 1. professional/managerial occupation; 2. the rest.

4.2.3 Within-Group Income Process

The within-group income process is as follows.

$$\begin{aligned} u_{it} &= \eta_{it} + \epsilon_{it} \\ \eta_{it} &= \rho\eta_{it-1} + \xi_{it} \end{aligned}$$

where ξ_{it} and ϵ_{it} represent persistent and transitory shocks, respectively. Here ϵ_{it} and ξ_{it} are independent, serially uncorrelated, and normally distributed random variables.

For the benchmark analysis, I set autocorrelation ρ to 0.95, which is an intermediate value of the point estimates presented in Storesletten et al. (2004).[21] Then I estimate σ_η^2 and σ_ϵ^2 by the following two (unconditional) moment conditions. I compute empirical moments using the PSID data. Table 5 reports the resulting point estimates.

$$\begin{aligned} Cov(u_{it}, u_{it-1}) &= \rho\sigma_\eta^2 \\ Var(u_{it}) &= \sigma_\eta^2 + \sigma_\epsilon^2 \end{aligned}$$

In order to numerically compute and simulate the stationary equilibrium, I discretize the AR(1) process for the persistent component to a five-state Markov chain

[21] For the benchmark, Krueger and Perri (2006) use the autocorrelation value of 0.9989 based on the estimation results reported by **?**. However, since Krueger and Perri (2006) use the **?** procedure calibrating to the unconditional variance of the persistent component σ_η^2 to discretize the AR(1) process, the resulting Markov chain features autocorrelation lower than 0.9989. In fact, since I use the Tauchen (1986) procedure that tends to approximate the original AR(1) process better than **?** with a small number of grid points, the Markov chain used in this numerical experiment features higher autocorrelation than the one used in Krueger and Perri (2006).

Table 5: Estimation of the Income Process

	$\hat{\sigma}_\eta^2$	$\hat{\sigma}_\epsilon^2$
1980-1985	0.1612	0.0415
1986-1992	0.1763	0.0438

by the Tauchen (1986) procedure. In doing so, I calibrate the range of the support for the Markov chain to $\hat{\sigma}_\eta^2$.

Due to the computational burden of having both persistent and transitory shocks in the within-group component as well as occupational mobility, I omit the transitory income shocks in the benchmark analysis. Although it may affect quantitative results, the comparison between the cases with and without occupational mobility is likely to be valid since, as the qualitative analysis suggests, it is not the transitory shocks but rather the persistent shocks that determine the effect of occupational mobility on within-group consumption inequality.

4.3 Benchmark Results

Table 6 presents benchmark results. The top panel reports between-group consumption inequality for two periods corresponding to 1980-1985 and 1986-1992 as well as changes between the two periods. The middle panel reports within-group consumption inequality for the two periods. The bottom panel reports equilibrium interest rates for the models. In each panel, the row labelled 'CEX' reports the estimates from the CEX data. The row labelled 'Model (Mobility)' reports the benchmark results with occupational mobility. The row labelled 'Model (No mobility)' reports results of the model without occupational mobility.

Note that due to the computational difficulty described in Section 4.2.2, I set the benchmark values of between-group income inequality 10 times larger than the empirical estimate from the PSID data. Thus, the following two comparisons are meaningful in Table 6: 1. within-group consumption inequality for the CEX and that for the model without occupational mobility[22]; 2. all the statistics for the model with

[22] Within-group consumption inequality observed in the CEX data and that for the model without

and without occupational mobility.

Under the benchmark parameter values, the model with occupational mobility features lower between-group consumption inequality and higher within-group consumption inequality than the model with no occupational mobility. Since the model without occupational mobility overpredicts between-group consumption inequality (by assumption) and largely underpredicts within-group consumption inequality, these results suggest that occupational mobility can help account for between- and within-group consumption inequality observed in the data.

The last column of Table 6 reports the change in given statistics between the two periods. First, consistent with Krueger and Perri (2006), the model without occupational mobility largely underpredicts the increase in within-group consumption inequality relative to the empirical evidence from the CEX data: the increase in within-group consumption inequality is 0.0023 units in the model without occupational mobility and 0.0171 units in the CEX data. In the model with occupational mobility, though it is not comparable to the CEX data as noted above, the increase in within-group consumption inequality is 0.0160 units, about 7 times larger than the increase in within-group consumption inequality in the model without occupational mobility.[23] Lastly, the equilibrium interest rates decrease between the two periods because the increase in income risk raises household's precautionary saving motive.

4.4 Sensitivity to Between-Group Income Inequality

Table 7 reports results from the sensitivity analysis with respect to between-group income inequality. All four panels report results for the model with occupational mobility. In each panel, the row labeled 'PSID×10 (benchmark)' reports the benchmark results with occupational mobility where between-group income inequality is set to be 10 times larger than the empirical estimates from the PSID data. Rows labeled 'PSID×12' and 'PSID×14' report results with between-group income inequality 12 and 14 times larger than the PSID estimates, respectively.

The top panel of Table 7 reports between-group consumption inequality for the

occupational mobility are comparable, because between-group income inequality does not affect within-group consumption inequality in the model without occupational mobility.

[23]For sensitivity of this result, see Section 4.4.

Table 6: Results from the Parameterized Production Economy

	1980-1985	1986-1992	Change
Between-group consumption inequality			
CEX	0.0040	0.0043	0.0003
Model (Mobility)	0.0208	0.0302	0.0094
Model (No mobility)	0.0702	0.1106	0.0404
Within-group consumption inequality			
CEX	0.1286	0.1457	0.0171
Model (Mobility)	0.0386	0.0546	0.0160
Model (No mobility)	0.0125	0.0148	0.0023
Equilibrium interest rates (%)			
Model (Mobility)	3.91%	3.81%	-0.10%
Model (No mobility)	4.07%	4.05%	-0.02%

Notes. The top panel reports between-group consumption inequality for the 1980-1985 and 1986-1992 periods as well as changes between the two periods. The middle panel reports within-group consumption inequality for the two periods. The bottom panel reports equilibrium interest rates for the models. In each panel, the row labeled 'CEX' reports the estimates from the CEX data. Model (Mobility) reports the benchmark results with occupational mobility. Model (No mobility) reports results without occupational mobility.

three cases and confirms that it increases as between-group income inequality increases. Note that the change in between-group consumption inequality between the two periods also increases as between-group income inequality increases. The second panel shows that within-group consumption inequality appears to be sensitive to between-group income inequality: within-group consumption inequality increases by 0.0160 units in the benchmark case and it increases by 0.0204 units in the case where between-group income inequality is 14 times larger than the PSID estimate.

To examine the sensitivity of within-group consumption inequality to between-group income inequality in detail, I report within-group consumption inequality for the low- and high-mean income occupation groups in the third and fourth panels, respectively.[24] First of all, note that the change in within-group consumption inequality for the high-mean income occupation (professional/managerial) is not sensitive to between-group income inequality. The increase of 0.0094 units is 4 times

[24]Within-group consumption inequality for an occupation group refers to the variance of residuals from the cross-sectional regression of consumption conditional on being in the occupation group.

Table 7: Sensitivity to Between-Group Income Inequality

	1980-1985	1986-1992	Change
Between-group consumption inequality			
PSID×10 (benchmark)	0.0208	0.0302	0.0094
PSID×12	0.0252	0.0369	0.0117
PSID×14	0.0297	0.0434	0.0137
Within-group consumption inequality			
PSID×10 (benchmark)	0.0386	0.0546	0.0160
PSID×12	0.0430	0.0614	0.0184
PSID×14	0.0473	0.0678	0.0204
Within-group consumption inequality			
Low mean income occupation			
PSID×10 (benchmark)	0.0428	0.0627	0.0199
PSID×12	0.0492	0.0729	0.0237
PSID×14	0.0556	0.0826	0.0270
Within-group consumption inequality			
High mean income occupation			
PSID×10 (benchmark)	0.0327	0.0422	0.0095
PSID×12	0.0343	0.0437	0.0094
PSID×14	0.0358	0.0452	0.0094

Notes. All four panels report results for the model with occupational mobility. In each panel, the row labeled 'PSID×10 (benchmark)' reports the benchmark results with occupational mobility. Rows labeled 'PSID×12' and 'PSID×14' report results with between-group income inequality 12 and 14 times larger than the PSID estimates, respectively.

larger than the increase in within-group consumption inequality for the model without occupational mobility. Within-group consumption inequality for the low-mean income occupation group is sensitive to between-group income inequality, because, in the model, households decrease their consumption only gradually when they switch from the highest income state to any lower income states, which increases the maximum value of consumption in the low-mean income occupation proportionally as between-group income inequality increases.

5 Conclusion

In this paper, I examine the impact of the rise in involuntary occupational mobility on between- and within-group consumption inequality in a model with limited enforcement of contracts. To study a qualitative mechanism that links between occupational mobility and between- and within-group consumption inequality, I use a stylized pure exchange economy with two occupation groups with different mean incomes, two income states in each group, and limited enforcement of contracts. In the model, agents face two types of shocks, namely involuntary occupation switch and within-group income shocks. I show that when within-group income shocks are persistent and within-group consumption inequality is sufficiently large, a rise in occupational mobility increases the desire for insurance for low income individuals more than that of high income individuals, thereby increasing consumption inequality within that group. Numerical experiments suggest that taking account of involuntary occupational mobility is quantitatively important to account for the evolution of between- and within-group consumption inequality.

Data Appendix

Table 8: Sample Selection in the PSID

	Observations deleted	Remaining observations
Original data set		162297
Interviewed prior to 1979	58357	103940
Change in family composition	9909	94031
Change in marital status	2181	91850
Female head	26327	65523
Missing values and topcoding	3434	62089
Income and consumption outliers	4576	57513
Poverty subsample	23149	34364
Aged less than 30 or more than 65	11143	23221
Armed services, protective workers	756	22465
Unemployed for 5 years	507	21958

Notes. "Missing values and topcoding" excludes households if household's region of residence is missing, household head's or spouse's (if present) education or occupation (conditional on working) is missing, household's income is topcoded, or food expenditure is assigned by the PSID. "Income and consumption outliers" excludes households if an annual income is below \$ 100 or below total food expenditure, an income growth is above 500% or below -80%, or total food expenditure is zero or missing.

Table 9: Sample Selection in CEX

	Observations deleted	Remaining observations
Original data set		167919
Incomplete income respondents	32863	135056
Zero food consumption	674	134382
Only food consumption	94	134288
Missing interviews	72219	62069
Inconsistent characteristics	4141	57928
Missing main characteristics	5014	52914
Change in marital status	572	52342
Non-positive, missing annual income	1172	51170
Non-positive, missing labor income	12553	38617
Positive labor income with zero weeks worked	1	38616
Income less than food	516	38100
Aged less than 30 or more than 65	7427	30673
Armed services, protective workers	226	30477
Unemployed for 2 years	1426	29021
Observed after 1992	12836	16185

Notes. "Missing main characteristics" excludes CU's if the reference person's or spouse's (if present) sex, race, education, or occupation (conditional on working) is missing.

Table 10: Comparison of means, PSID and CEX

	1980		1983		1986		1990		1992	
	PSID	CEX	PSID	CEX	PSID	CEX	PSID	CEX	PSID	CEX
Age	45.10	45.10	45.29	43.89	44.82	44.68	44.77	44.01	44.88	44.19
Family size	3.34	3.44	3.25	3.15	3.12	3.09	3.03	3.07	3.00	2.99
White	0.92	0.89	0.94	0.89	0.94	0.86	0.93	0.86	0.94	0.86
HS graduate	0.33	0.34	0.33	0.33	0.29	0.30	0.30	0.30	0.29	0.30
College dropout	0.17	0.22	0.19	0.21	0.21	0.22	0.23	0.26	0.22	0.24
College graduate	0.27	0.22	0.29	0.29	0.31	0.32	0.31	0.31	0.34	0.32
Prof/manager	0.42	0.36	0.45	0.48	0.47	0.43	0.46	0.41	0.48	0.41
Northeast	0.23	0.22	0.23	0.21	0.23	0.21	0.23	0.22	0.23	0.22
Midwest	0.33	0.25	0.31	0.28	0.30	0.24	0.30	0.23	0.31	0.25
South	0.28	0.33	0.27	0.32	0.27	0.30	0.28	0.31	0.28	0.31
Family income	30661.61	24925.56	31250.80	25910.69	33614.28	28902.75	35146.83	27747.13	34651.19	27628.43
Food expenditure	4849.99	4533.56	4654.27	4149.82	4639.02	4248.59	4269.48	3934.67	4322.17	3806.65

A. Appendix

A1. Existence of constrained efficient symmetric stationary Markov allocations

Proposition 3 below shows that the sufficient conditions stated in Proposition 1 are satisfied when occupational mobility are sufficiently persistent (ρ is sufficiently close to 1) and idiosyncratic income risk in the high mean income group is sufficiently low (ω_{1h} and ω_{1l} are sufficiently close). The following lemma plays a key role to prove Proposition 3.

Lemma 1 *Suppose that there exists a non-autarkic solution, denoted by* $(\bar{c}_{2h}, \bar{c}_{2l})$*, to the following system of equations.*

$$\begin{cases} (1-\beta\gamma)(u(\omega_{2h}) - u(\bar{c}_{2h})) - \beta(1-\gamma)(u(\bar{c}_{2l}) - u(\omega_{2l})) = 0 \\ \bar{c}_{2h} + \bar{c}_{2l} - (\omega_{2h} + \omega_{2l}) = 0. \end{cases} \tag{14}$$

Then there exists $\bar{\rho} \in [0, 1)$ *such that for* $\rho \in [\bar{\rho}, 1)$*, the system of equations consisting of (4) and (5) has a non-autarkic solution* $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ *that tends to* $(\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})$ *as* $\rho \to 1$.

Proof. See Appendix A3. □

As Proposition 2 states, any constrained efficient symmetric stationary Markov allocation with some but not perfect risk sharing must satisfy (4) and (5). Hence Lemma 1 implies that the type of allocations must converge to $(\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})$ as $\rho \to 1$. It is this strong convergence result that helps identify sufficient conditions for existence in Proposition 3.

Note that the first equation of (14) is equivalent to $V^m_{2h} = V^a_{2h}$ with $\rho = 1$. Thus, there exists a non-autarkic solution to (14) if and only if there exists a solution to (14) that satisfies $V^m_{2l} > V^a_{2l}$ with $\rho = 1$. The condition means that some risk sharing is attainable in the low mean income group when there is no occupational mobility ($\rho = 1$). The necessary and sufficient condition for the existence of a non-autarkic solution to (14) can also be written as $u'(\omega_{2h})/u'(\omega_{2l}) < \beta(1-\gamma)/(1-\beta)$.

Proposition 3 *Suppose that* $u(c_{FB})/(1-\beta) < V_{1h}^a$ *at* $\rho = 1$ *and* $u'(\omega_{2h})/u'(\omega_{2l}) < \beta(1-\gamma)/(1-\beta)$. *Let* $(\bar{c}_{2h}, \bar{c}_{2l})$ *be the non-autarkic solution to the system (14) as in Lemma 1. Furthermore, suppose that* $\omega_{1h} > \{\omega_{1l}, \bar{c}_{2h}\} > \bar{c}_{2l}$ *and*

$$\frac{u'(\omega_{1h})}{u'(\omega_{1l})} > \frac{u'(\bar{c}_{2h})}{u'(\bar{c}_{2l})}. \tag{15}$$

Then, there exists $\bar{\rho} \in [0, 1)$ *such that for* $\rho \in [\bar{\rho}, 1]$, *the non-autarkic symmetric stationary Markov allocation* $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ *satisfying (4) and (5) is constrained efficient. (At* $\rho = 1$, *the allocation* $(\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})$ *satisfies (4) and (5) and is constrained efficient.)*

Proof. Immediate from Proposition 1 and Lemma 1. Note that the non-autarkic solution $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ is continuous in ρ in the neighborhood of $\rho = 1$. Optimality of $(\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})$ at $\rho = 1$ can be checked in the same manner as in the proof of Proposition 1. □

Proposition 3 tells us that constrained efficient allocations take the simple form of symmetric stationary Markov allocations when occupational mobility are sufficiently persistent (ρ is sufficiently close to 1) and idiosyncratic income risk in the high mean income group is sufficiently low so that condition (15) holds. The results, in particular the optimality of the allocation $(\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})$ at $\rho = 1$, hold since condition (15) rules out many allocations. First, under condition (15), autarky in the high mean income group, $(\omega_{1h}, \omega_{1l})$, is the unique solution to the system of equations consisting of $V_{1h}^m = V_{1h}^a$ with $\rho = 1$ and $c_{1h} + c_{1l} = \omega_{1h} + \omega_{1l}$.[25] Note that if there were another solution with some risk sharing to the system of equations, denoted by $(\bar{c}_{1h}, \bar{c}_{1l})$, then the allocation $(\bar{c}_{1h}, \bar{c}_{1l}, \bar{c}_{2h}, \bar{c}_{2l})$ would Pareto dominate $(\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})$. Therefore,

[25] The system of equations is equivalently written as follows.

$$\begin{cases} (1-\beta\gamma)(u(\omega_{1h}) - u(\bar{c}_{1h})) - \beta(1-\gamma)(u(\bar{c}_{1l}) - u(\omega_{1l})) = 0 \\ \bar{c}_{1h} + \bar{c}_{1l} - (\omega_{1h} + \omega_{1l}) = 0. \end{cases}$$

To prove the statement, first suppose that there exists a non-autarkic solution to the above system. Then, $u'(\omega_{1h})/u'(\omega_{1l}) < \beta(1-\gamma)/(1-\beta)$. In the meantime, Fact 1 in Appendix 3 tells us that $u'(\bar{c}_{2h})/u'(\bar{c}_{2l}) > \beta(1-\gamma)/(1-\beta\gamma)$. Combining these two inequalities yields

$$\frac{u'(\bar{c}_{2h})}{u'(\bar{c}_{2l})} > \frac{\beta(1-\gamma)}{1-\beta\gamma} > \frac{u'(\omega_{1h})}{u'(\omega_{1l})},$$

contradicting the assumption that $u'(\omega_{1h})/u'(\omega_{1l}) > u'(\bar{c}_{2h})/u'(\bar{c}_{2l})$. □.

the nonexistence of such allocations confirms constrained efficiency of $(\omega_{1h}, \omega_{1l}, \overline{c}_{2h}, \overline{c}_{2l})$ at $\rho = 1$. Second, under the current specification of the period utility function, condition (15) and $\omega_{1h} > \omega_{1l}$ imply that

$$1 < \frac{\omega_{1h}}{\omega_{1l}} < \frac{\overline{c}_{2h}}{\overline{c}_{2l}}.$$

Therefore, $\overline{c}_{2h} > \overline{c}_{2l}$, which means that perfect risk sharing is not attainable in the low mean income group when $\rho = 1$.

One may see Proposition 3 from the agent's point of view since constrained efficient allocations considered can be decentralized (with transfers):[26] with endowment processes satisfying (15), agents in the high mean income group cannot share their idiosyncratic income risk since the risk is too small. In other words, no borrowing-lending contract can provide some risk sharing and, at the same time, prevent defaults since the value of autarky is too high. Hence, when agents are in the high mean income group and face occupational mobility, they only insure against occupational mobility. Meanwhile, agents who belong to the low mean income group insure only against the idiosyncratic income risk. As a result, the constrained efficient (equilibrium) allocation exhibits the simple structure.

A2. Proofs

A2.1. Proof of Proposition 1

Solution to the planner problem is characterized by the first order conditions and the complementary slackness conditions associated with participation constraints and resource constraints. Let $\pi(\theta^t)$ denote the probability of θ^t. Let $\beta^t \pi(\theta^t) \varphi_t^i(\theta^t)$ denote a Lagrange multiplier associated with the participation constraint for Agent i at history θ^t. Following Marcet and Marimon (1998), I define a (modified) cumulative multiplier $\psi_t^i(\theta^t)$ recursively by $\psi_t^i(\theta^t) = \psi_{t-1}^i(\theta^{t-1}) + \varphi_t^i(\theta^t)$ for all t and $\psi_{-1}^i = \lambda^i$. Then the first order condition reads:

$$\frac{u'(c_t^i(\theta^t))}{u'(c_t^j(\theta^t))} = \frac{\psi_t^j(\theta^t)}{\psi_t^i(\theta^t)} = \frac{\psi_{t-1}^j(\theta^{t-1}) + \varphi_t^j(\theta^t)}{\psi_{t-1}^i(\theta^{t-1}) + \varphi_t^i(\theta^t)}. \tag{16}$$

[26] Kehoe and Levine (1993, 2001), Kocherlakota (1996), and Alvarez and Jermann (2000) provide decentralization results for the type of constrained efficient allocations consider here.

Note that conditions (3), (4) and (5) pin down a feasible symmetric stationary Markov allocation $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$. In this proof, I check if the first order condition (16) is satisfied with the symmetric stationary allocation $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ and *nonnegative* Lagrange multipliers $\{\varphi_t^i\}_{i,t}$. As shown below, conditions (6) and (7) guarantee the nonnegativity of Lagrange multipliers.

Suppose that $\theta_t = (1, h)$. Recall that when $\theta_t = (1, h)$, Agents 1, 2, 3, and 4 receive ω_{1h}, ω_{1l}, ω_{2h}, and ω_{2l}, respectively. Since $V_{2l}^m > V_{2l}^a$, the complementary slackness condition implies that $\varphi_t^4(\theta_t) = 0$. There are three cases to consider. (In what follows, I suppress t and θ_t in $\varphi_t^i(\theta_t)$.)

1. $\theta_{t-1} = (1, l)$**:** I check if the first order conditions hold with nonnegative Lagrange multipliers for the transition from $\theta_{t-1} = (1, l)$ to $\theta_t = (1, h)$. First, for Agents 1 and 4,

$$\frac{\psi_{t-1}^4}{\psi_{t-1}^1} = \frac{u'(c_{t-1}^1)}{u'(c_{t-1}^4)} = \frac{u'(c_{1l})}{u'(c_{2h})} > \frac{u'(c_{1h})}{u'(c_{2l})} = \frac{u'(c_t^1)}{u'(c_t^4)} = \frac{\psi_t^4}{\psi_t^1} = \frac{\psi_{t-1}^4}{\psi_{t-1}^1 + \varphi^1}.$$

From the left, the first equality is the first order condition at $t-1$, the second equality holds since $\theta_{t-1} = (1, l)$, the strict inequality holds since $c_{1h} > \{c_{1l}, c_{2h}\} > c_{2l} > 0$, the third equality holds since $\theta_t = (1, h)$, the fourth equality is the first order condition at t, and the last equality holds since $\varphi_t^4 = 0$. Therefore, $\varphi^1 > 0$. Similarly,

$$\frac{\psi_{t-1}^4}{\psi_{t-1}^3} = \frac{u'(c_{2l})}{u'(c_{2h})} > \frac{u'(c_{2h})}{u'(c_{2l})} = \frac{\psi_{t-1}^4}{\psi_{t-1}^3 + \varphi^3}.$$

Therefore, $\varphi^3 > 0$. Note that

$$\frac{\psi_{t-1}^4}{\psi_{t-1}^2} = \frac{u'(c_{1h})}{u'(c_{2h})} \geq \frac{u'(c_{1l})}{u'(c_{2l})} = \frac{\psi_{t-1}^4}{\psi_{t-1}^2 + \varphi^2},$$

where the weak inequality follows from the assumption $u'(c_{1h})/u'(c_{1l}) \geq u'(c_{2h})/u'(c_{2l})$. Therefore, $\varphi^2 \geq 0$.

2. $\theta_{t-1} = (2, h)$**:** As in Case 1, $\varphi^1 > 0$ and $\varphi^2 > 0$ due to the first order conditions and the fact that $c_{1h} > \{c_{1l}, c_{2h}\} > c_{2l} > 0$. As for φ^3, note that

$$\frac{\psi_{t-1}^4}{\psi_{t-1}^3} = \frac{u'(c_{1h})}{u'(c_{1l})} \geq \frac{u'(c_{2h})}{u'(c_{2l})} = \frac{\psi_{t-1}^4}{\psi_{t-1}^3 + \varphi^3}.$$

Therefore, $\varphi^3 \geq 0$.

3. $\theta_{t-1} = (2,l)$**:** $\varphi^1 \geq 0$ since $c_{1h} > c_{2l}$. As for φ^2, note that

$$\frac{\psi^4_{t-1}}{\psi^2_{t-1}} = \frac{u'(c_{2h})}{u'(c_{1h})} > \frac{u'(c_{1l})}{u'(c_{2l})} = \frac{\psi^4_{t-1}}{\psi^2_{t-1} + \varphi^2},$$

where the strict inequality holds because $c_{1h} > c_{2h}$ and $c_{1l} > c_{2l}$. Therefore, $\varphi^2 > 0$. As for φ^3, note that

$$\frac{\psi^4_{t-1}}{\psi^3_{t-1}} = \frac{u'(c_{1l})}{u'(c_{1h})} > \frac{u'(c_{2h})}{u'(c_{2l})} = \frac{\psi^4_{t-1}}{\psi^3_{t-1} + \varphi^3},$$

where the strict inequality holds because $c_{1h} > c_{1l}$ and $c_{2h} > c_{2l}$. Hence $\varphi^3 > 0$.

The Lagrange multiplier $\varphi^i_t(\theta^t)$ is also well defined and non-negative at $t = 0$ since initial weights $\langle \lambda^i \rangle^4_{i=1}$ are the same for all agents. The above and the complementary slackness conditions imply that $V^m_{1h} = V^a_{1h}$, $V^m_{1l} = V^a_{1l}$, and $V^m_{2h} = V^a_{2h}$. Apparently, the same argument applies to $\theta_t = (1,l)$, $(2,h)$, and $(2,l)$.

Therefore, one can conclude that the symmetric stationary Markov allocation $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ and the associated nonnegative Lagrange multipliers $\{\varphi^i_t\}_{i,t}$ satisfy the first order condition (16) and the complementary slackness condition for all t and θ^t. Hence the symmetric stationary Markov allocation satisfying all the conditions in Proposition 1 is constrained efficient. □

Remark 1. Marcet and Marimon (1998) show that the solution to the planner problem has a feedback representation with $(\langle \psi^j_{t-1} \rangle^4_{j=1}, \theta_t)$ as state variables. The feedback representation is,

$$\begin{cases} c^i_t(\theta^t) = c^i(\langle \psi^j_{t-1} \rangle^4_{j=1}, \theta_t), \quad \varphi^i_t(\theta^t) = \varphi^i(\langle \psi^j_{t-1} \rangle^4_{j=1}, \theta_t), \\ \psi^i_t = \psi^i_{t-1} + \varphi^i(\langle \psi^j_{t-1} \rangle^4_{j=1}, \theta_t), \quad \theta_t \text{ is a first-order Markov chain.} \end{cases}$$

For the current problem, $\varphi^i_t((\theta^{t-1}, \theta_t)) = \psi^4_{t-1} \left(\frac{u'(c^4_t(\theta^t))}{u'(c^i_t(\theta^t))} \right) - \psi^i_{t-1} = \varphi^i(\langle \psi^j_{t-1} \rangle^4_{j=1}, \theta_t)$ for $i \in \{1,2,3\}$, when $\theta_t = (1,h)$. The term $u'(c^4_t(\theta^t))/u'(c^i_t(\theta^t))$ depends only on θ_t because the consumption allocation is symmetric stationary Markov and endowments are deterministically linked across agents as specified above. By this argument, it is clear that $\varphi^i_t(\theta^t)$ can be written as a function of $\langle \psi^j_{t-1} \rangle^4_{j=1}$ and θ_t for all $\theta_t \in \Theta$. Therefore, the consumption-multiplier pair has the recursive structure proven by Marcet and Marimon (1998). □

Remark 2. Symmetric stationary Markov allocations can be optimal under asymmetric initial weights. However, initial weights must satisfy the following conditions. Let $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ be an allocation satisfying (4) and (5), and suppose that $\theta_0 = (1, h)$. The first order conditions hold at $t = 0$ with nonnegative Lagrange multipliers $\langle \varphi_0^i \rangle_{i=1}^4$ if and only if initial weights $\langle \lambda^i \rangle_{i=1}^4$ satisfy the following:

$$\frac{u'(c_0^1)}{u'(c_0^4)} = \frac{u'(c_{1h})}{u'(c_{2l})} \leq \frac{\lambda_4}{\lambda_1}, \quad \frac{u'(c_0^2)}{u'(c_0^4)} = \frac{u'(c_{1l})}{u'(c_{2l})} \leq \frac{\lambda_4}{\lambda_2}, \quad \text{and} \quad \frac{u'(c_0^3)}{u'(c_0^4)} = \frac{u'(c_{2h})}{u'(c_{2l})} \leq \frac{\lambda_4}{\lambda_3}.$$

Equivalently,

$$\lambda^4 \geq \max\{\frac{u'(c_{1h})}{u'(c_{2l})}\lambda^1, \frac{u'(c_{1l})}{u'(c_{2l})}\lambda^2, \frac{u'(c_{2h})}{u'(c_{2l})}\lambda^3\}.$$

Equal weights ($\lambda^i = 1/4$ for all i) satisfy this condition. However, it is not the only case in which the condition holds. For other θ_0's, similar conditions on initial weights must be satisfied. To summarize, the following weights work.

$$\langle \lambda^i \rangle_{i=1}^4 \text{ such that } \begin{cases} \lambda^4 \geq \max\{\frac{u'(c_{1h})}{u'(c_{2l})}\lambda^1, \frac{u'(c_{1l})}{u'(c_{2l})}\lambda^2, \frac{u'(c_{2h})}{u'(c_{2l})}\lambda^3\} & (\text{for } \theta_0 = (1, h)) \\ \lambda^3 \geq \max\{\frac{u'(c_{1l})}{u'(c_{2l})}\lambda^1, \frac{u'(c_{1h})}{u'(c_{2l})}\lambda^2, \frac{u'(c_{2h})}{u'(c_{2l})}\lambda^4\} & (\text{for } \theta_0 = (1, l)) \\ \lambda^2 \geq \max\{\frac{u'(c_{2h})}{u'(c_{2l})}\lambda^1, \frac{u'(c_{1h})}{u'(c_{2l})}\lambda^3, \frac{u'(c_{1l})}{u'(c_{2l})}\lambda^4\} & (\text{for } \theta_0 = (2, h)) \\ \lambda^1 \geq \max\{\frac{u'(c_{2h})}{u'(c_{2l})}\lambda^2, \frac{u'(c_{1l})}{u'(c_{2l})}\lambda^3, \frac{u'(c_{1h})}{u'(c_{2l})}\lambda^4\} & (\text{for } \theta_0 = (2, l)). \end{cases}$$

In words, the weight of the agent with ω_{2l} at $t = 0$ must be high enough. Otherwise, the planner would find it optimal to extract more resources from the agent with ω_{2l} at $t = 0$ since the agent's participation constraint is yet to bind.

As long as the agent with the lowest income at $t = 0$ is assigned a weight high enough, symmetric allocations can be optimal even with asymmetric initial weights. It is because the planner's ability to reallocate resources can be highly restricted by the participation constraints and the endowment process is symmetric. In fact, Proposition 2 tells us that constrained efficient symmetric stationary Markov allocations with some but not perfect risk sharing must satisfy $V_\theta^m = V_\theta^a$ for all $\theta \in \Theta \setminus \{(2, l)\}$ and $\sum_\theta c_\theta = \sum_\theta \omega_\theta$. These conditions are enough to pin down an allocation. This fact indicates that such allocations can be optimal only in special cases. It is indeed the case as shown in Proposition 3. □

A2.2. Proof of Proposition 2

Let us first prove that if a feasible symmetric stationary Markov allocation is constrained efficient and features some risk sharing, then it must be that $V^m_{2l} > V^a_{2l}$ (Condition (3)). I prove the following contrapositive of the statement.

Suppose that a feasible symmetric stationary Markov allocation features $V^m_{2l} = V^a_{2l}$. Then it is either autarky or not constrained efficient.

First of all, note that if $V^m_{2l} = V^a_{2l}$, then $c_{2l} \leq \omega_{2l}$. The inequality becomes strict if and only if $V^m_\theta > V^a_\theta$ for some $\theta \in \Theta \setminus \{(2,l)\}$. Therefore, if $c_{2l} = \omega_{2l}$, then $V^m_\theta = V^a_\theta$ for all θ, which implies that the allocation is autarky. Next, let us consider the case of $c_{2l} < \omega_{2l}$. In this case, there exists $\theta \in \Theta \setminus \{(2,l)\}$ such that $V^m_\theta > V^a_\theta$ and $c_\theta > \omega_\theta$. Then, one can construct a new feasible allocation $(c'_{1h}, c'_{1l}, c'_{2h}, c'_{2l})$ from $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ by moving some resources from c_θ to c_{2l}. Since the period utility function is strictly concave, the new allocation $(c'_{1h}, c'_{1l}, c'_{2h}, c'_{2l})$ obtains a higher value of the planner's objective function (with equal initial weights) than the original allocation $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$. Therefore, if a feasible symmetric stationary Markov allocation features $V^m_{2l} = V^a_{2l}$ and $c_{2l} < \omega_{2l}$, then it is not constrained efficient.

From the proof of Proposition 1, it is clear that the conditions (4) and (7) are not only sufficient but also necessary for optimality. As for (5), the first order condition and the complementary slackness condition imply (5) provided $c_{2l} < \min\{c_{1h}, c_{1l}, c_{2h}\}$. Let us prove this statement here and I conclude this proof by confirming $c_{2l} < \min\{c_{1h}, c_{1l}, c_{2h}\}$. Recall the first order condition. (See the proof of Proposition 1 for notation.)

$$\frac{u'(c^i_t(\theta^t))}{u'(c^j_t(\theta^t))} = \frac{\psi^j_t(\theta^t)}{\psi^i_t(\theta^t)} = \frac{\psi^j_{t-1}(\theta^{t-1}) + \varphi^j_t(\theta^t)}{\psi^i_{t-1}(\theta^{t-1}) + \varphi^i_t(\theta^t)}. \tag{17}$$

Note that the transition probability from the state $(2,l)$ to any other state is strictly positive. Suppose that Agent 1 is in the state $(2,l)$ in period $t-1$ and transits to the state $(1,l)$ in period t. Equation (17) implies that $\psi^1_{t-1}(\theta^{t-1}) < \psi^3_{t-1}(\theta^{t-1})$ and $\psi^1_t(\theta^t) > \psi^3_t(\theta^t)$. (Recall that whenever Agent 1 is in the state $(2,l)$, Agent 3 is in the state $(1,l)$, and *vice versa*.) In order for the above two inequalities to hold, it must be that $\varphi^1_t(\theta^t) > 0$. Then the complementary slackness condition implies that the participation constraint for Agent 1 at t holds with equality. Since symmetric stationary Markov allocations are Markovian, it means that participation constraints for those who are currently in the state $(1, l)$ always hold with equality. The same argument applies to the participation constraints for $(1, h)$ and $(2, h)$.

In the rest of the proof, I show that any constrained efficient symmetric stationary Markov allocation with some but not perfect risk sharing features $c_{2l} < \min\{c_{1h}, c_{1l}, c_{2h}\}$. First, c_{2l} cannot exceed c_{1h}, c_{1l}, or c_{2h}. To see this, suppose that $c_\theta < c_{2l}$ for some $\theta \in \Theta \setminus \{(2, l)\}$. Then the first order condition and the complementary slackness condition imply that $V^m_{2l} = V^a_{2l}$. It contradicts the fact that $V^m_{2l} > V^a_{2l}$ shown above. Hence, $c_{2l} \leq \min\{c_{1h}, c_{1l}, c_{2h}\}$.

There are essentially two cases other than $c_{2l} < \min\{c_{1h}, c_{1l}, c_{2h}\}$, namely, $c_{1h} > c_{1l} \geq c_{2h} = c_{2l}$ and $c_{1h} = c_{1l} > c_{2h} = c_{2l}$. (The rest of the proof goes through with only notational modifications when c_{1h}, c_{1l}, and c_{2h} are interchanged.) First, consider the former case. Suppose that $\theta_{t-1} = (1, h)$ and $\theta_t = (2, h)$. Then Agent 1 transits from $(1, h)$ to $(2, h)$, and Agent 2 transits from $(1, l)$ to $(2, l)$. The following holds:

$$\frac{\psi^2_{t-1}}{\psi^1_{t-1}} = \frac{u'(c^1_{t-1})}{u'(c^2_{t-1})} = \frac{u'(c_{1h})}{u'(c_{1l})} < \frac{u'(c_{2h})}{u'(c_{2l})} = \frac{u'(c^1_t)}{u'(c^2_t)} = \frac{\psi^2_t}{\psi^1_t} = \frac{\psi^2_{t-1} + \varphi^2_t}{\psi^1_{t-1} + \varphi^1_t},$$

where, from the left, the first equality is the first order condition at $t-1$, the second equality holds because $\theta_{t-1} = (1, h)$, the inequality holds because $c_{1h} > c_{1l}$ and $c_{2h} = c_{2l}$, the third equality holds because $\theta_t = (2, h)$, the fourth equality is the first order condition at t, and the last equality holds by definition of cumulative multipliers $\psi^i_t(\theta^t)$.[27] Since the Lagrange multiplier must be nonnegative, $\varphi^2_t > 0$, which implies that the participation constraint for $(2, l)$ binds. It contradicts the assumption that there is some risk sharing. Second, consider the case in which $c_{1h} = c_{1l} > c_{2h} = c_{2l}$. In this case, we have $V^m_{1h} = V^m_{1l}$ and $V^m_{2h} = V^m_{2l}$. Again, the first order condition and the complementary slackness condition tell us that $V^m_{1h} = V^a_{1h}$ and $V^m_{1l} = V^a_{1l}$. But it is impossible since $V^a_{1h} > V^a_{1l}$. Therefore, it must be that $c_{2l} < \min\{c_{1h}, c_{1l}, c_{2h}\}$. $\square$

A2.3. Proof of Lemma 1

The following is the system of equations consisting of (4) and (5), together with the definition of V^a_θ and V^m_θ. (With group-specific income shocks, Proposition 2 shows that the constrained efficient symmetric stationary Markov allocation solves the system of equations.)

[27] Refer to the proof of Proposition 1 for the first order conditions and the cumulative multipliers.

$$\begin{cases} V^a_\theta = u(\omega_\theta) + \beta \sum_{\theta' \in \Theta} \pi(\theta'|\theta) V^a_{\theta'} \quad (\forall \theta \in \Theta), \\ V^m_\theta = u(c_\theta) + \beta \sum_{\theta' \in \Theta} \pi(\theta'|\theta) V^m_{\theta'} \quad (\forall \theta \in \Theta), \\ V^m_\theta = V^a_\theta \quad (\forall \theta \in \Theta \setminus \{(2,l)\}) \\ \sum_{\theta \in \Theta} c_\theta = \sum_{\theta \in \Theta} \omega_\theta. \end{cases} \tag{18}$$

The above system simplifies to the following four equations. Define F by $F = (F_1, F_2, F_3, F_4)$.

$$\begin{aligned} F_1(c_{1h}, c_{1l}, c_{2h}, c_{2l}; \rho) &= a(u(\omega_{1h}) - u(c_{1h})) + b(u(\omega_{1l}) - u(c_{1l})) \\ &\quad + d(u(\omega_{2h}) - u(c_{2h})) - e(u(c_{2l}) - u(\omega_{2l})) = 0 \\ F_2(c_{1h}, c_{1l}, c_{2h}, c_{2l}; \rho) &= b(u(\omega_{1h}) - u(c_{1h})) + a(u(\omega_{1l}) - u(c_{1l})) \\ &\quad + e(u(\omega_{2h}) - u(c_{2h})) - d(u(c_{2l}) - u(\omega_{2l})) = 0 \\ F_3(c_{1h}, c_{1l}, c_{2h}, c_{2l}; \rho) &= d(u(\omega_{1h}) - u(c_{1h})) + e(u(\omega_{1l}) - u(c_{1l})) \\ &\quad + a(u(\omega_{2h}) - u(c_{2h})) - b(u(c_{2l}) - u(\omega_{2l})) = 0 \\ F_4(c_{1h}, c_{1l}, c_{2h}, c_{2l}; \rho) &= c_{1h} + c_{1l} + c_{2h} + c_{2l} - (\omega_{1h} + \omega_{1l} + \omega_{2h} + \omega_{2l}) = 0 \end{aligned}$$

where

$$\begin{aligned} a &= \frac{e}{\beta(1-\gamma)(1-\rho)} - 3\beta\rho\gamma - 2\beta^2\rho^2 - \beta^3\gamma\rho \\ &\quad -2\beta^2\rho^2 - 4\beta^3\rho^2\gamma^2 + 2\beta^3\rho^2\gamma + 4\beta^2\rho\gamma^2 + 2\beta^3\gamma^2\rho + 4\beta^2\rho^2\gamma \\ b &= \beta(1-\gamma)(2\rho + 2\beta\gamma - 4\beta\rho\gamma) - \frac{\rho e}{1-\rho} \\ d &= \beta(1-\rho)(2\gamma + 2\beta\rho - 4\beta\rho\gamma) - \frac{\gamma e}{1-\gamma} \\ e &= \beta(1-\gamma)(1-\rho)(1 - \beta^2 + 2\beta^2\rho + 2\beta^2\gamma - 4\beta^2\rho\gamma). \end{aligned}$$

I use the following fact to prove Lemma 1.

Fact 1 *Suppose that there exists a non-autarkic solution, denoted by $(\bar{c}_{2l}, \bar{c}_{2h})$, to the following system of equations.*

$$(1-\beta\gamma)(u(\omega_{2h}) - u(\bar{c}_{2h})) - \beta(1-\gamma)(u(\bar{c}_{2l}) - u(\omega_{2l})) = 0, \tag{19}$$

$$\bar{c}_{2h} + \bar{c}_{2l} - (\omega_{2h} + \omega_{2l}) = 0. \tag{20}$$

Then, $-(1-\beta\gamma)u'(\bar{c}_{2h}) + \beta(1-\gamma)u'(\bar{c}_{2l}) < 0$.

Proof. Note the following.

$$\begin{aligned}\frac{u(\omega_{2h})-u(\bar{c}_{2h})}{u(\bar{c}_{2l})-u(\omega_{2l})} &< \frac{u'(\bar{c}_{2h})(\omega_{2h}-\bar{c}_{2h})}{u'(\bar{c}_{2l})(\bar{c}_{2l}-\omega_{2l})}\\ &= \frac{u'(\bar{c}_{2h})}{u'(\bar{c}_{2l})},\end{aligned}$$

where the inequality holds by the strict concavity of u and the equality holds due to Equation (20).

Equation (19) and the above yield,

$$\begin{aligned}&\frac{\beta(1-\gamma)}{1-\beta\gamma}<\frac{u'(\bar{c}_{2h})}{u'(\bar{c}_{2l})}\\ \Leftrightarrow\quad &-(1-\beta\gamma)u'(\bar{c}_{2h})+\beta(1-\gamma)u'(\bar{c}_{2l})<0.\quad\square\end{aligned}$$

Proof of Lemma 1. Substitute $\rho=1$ in $F(c_{1h},c_{1l},c_{2h},c_{2l};\rho)$. Then, $a=(1-\beta\gamma)(1-\beta)(1+\beta-2\beta\gamma)$, $b=\beta(1-\gamma)(1-\beta)(1+\beta-2\beta\gamma)$, $d=0$, and $e=0$. Solutions to the system $F(c_{1h},c_{1l},c_{2h},c_{2l};1)=0$ satisfies $c_{1h}=\omega_{1h}$, $c_{1l}=\omega_{1l}$, and

$$(1-\beta\gamma)(u(\omega_{2h})-u(c_{2h}))-\beta(1-\gamma)(u(c_{2l})-u(\omega_{2l}))=0, \tag{21}$$

$$c_{2h}+c_{2l}-(\omega_{2h}+\omega_{2l})=0. \tag{22}$$

Equations (21) and (22) are identical to Equations (19) and (20), respectively. Therefore, the non-autarkic solution is $(\omega_{1h},\omega_{1l},\bar{c}_{2h},\bar{c}_{2l})$.[28] Jacobian $|J|$ of $F(c_{1h},c_{1l},c_{2h},c_{2l};\rho)$ with respect to $(c_{1h},c_{1l},c_{2h},c_{2l})$ at $(\omega_{1h},\omega_{1l},\bar{c}_{2h},\bar{c}_{2l};1)$ is as follows,

$$\begin{aligned}|J| &= (-au'(\omega_{1h}))\{(-au'(\omega_{1l}))(-au'(\bar{c}_{2h}))-au'(\omega_{1l})bu'(\bar{c}_{2l})\}\\ &+ (-1)(-bu'(\omega_{1l}))\{(-bu'(\omega_{1h}))(-au'(\bar{c}_{2h}))-bu'(\omega_{1h})bu'(\bar{c}_{2l})\}\\ &= \{-(1-\beta\gamma)u'(\bar{c}_{2h})+\beta(1-\gamma)u'(\bar{c}_{2l})\}\\ &\times \{(1-\beta\gamma)^2-\beta^2(1-\gamma)^2\}(1-\beta)^3(1+\beta-2\beta\gamma)^3u'(\omega_{1h})u'(\omega_{1l}).\end{aligned}$$

By Fact 1, $|J|<0$. Hence the Implicit Function Theorem tells us that there exits a neighborhood of $\rho=1$ in which the solution to the system $F=0$ is continuous in ρ. $\square$

[28]The other solution is autarky $(\omega_{1h},\omega_{1l},\omega_{2h},\omega_{2l})$.

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