Defaults, Shortsales and the Social Costs of Volatility

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Abstract

This paper examines the welfare effects of credit and shortsales constraints and limited liability/minimum consumption guarantee in an overlapping generations (OLG) model with rational beliefs in the sense of Kurz (1994). To measure the social welfare, it instead adopts an ex post social welfare concept in the sense of Hammond (1981), since the standard Pareto criterion becomes inappropriate when heterogeneous beliefs are present. Simulation results indicate a trade-off between a larger opportunity set and a larger room for 'mistakes', and thus, the existence of a socially optimal level of various constraints.

1   Introduction

This paper examines the impacts of credit and shortsales constraints and limited liability (defaults) in an overlapping generations (OLG) model with securities. The standard economic thinking asserts that more flexibility or a larger opportunity set improves the welfare of the economic agents, and thus, loosening of constraints on securities positions is desirable. In particular, more choices of portfolios of securities would enhance the ability to transfer wealth across states and over time, and thus, the standard economic theory asserts that the welfare of the economy would improve. Moreover, limited liability would stablise the consumption stream of each agent by eliminating severe impoverishment, and thus, it would improve the welfare of the agents.

However, once we allow for heterogeneous beliefs, it is not trivial if these assertions can be supported. By allowing for heterogeneous beliefs, we essentially allow for agents to make systematic mistakes. Hence, a more liberal opportunity set may result in more frequent and/or serious mistakes. It follows that there is a potential trade-off between gains from trades and damages caused by mistakes arising from a more liberal opportunity set.

In the literature of general equilibrium incomplete markets models, the effects of defaults have been analysed also in the context of spanning, or

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ability to transfer wealth across states. Zame (1993) argues that defaults enhance such capability, and thus, they may improve the welfare of the economy. Dubey, Geanakoplos and Shubik (2005) describes banks as pools of funds or some sort of co-operatives, while introducing defaults. On the other hand, Kubler and Schmedders (2003) introduces defaults and collateral in an infinite horizon exchange economy. However, all these studies do not allow for heterogeneous beliefs, and thus, the potential trade-off has not been studied.

The current paper attempts to evaluate the benefits and damages of short-sales constraints on the securities as well as the limited liability in the form of minimum consumption guarantee while allowing for heterogeneous beliefs. The class of heterogeneous beliefs we focus on is that of *rational beliefs*. Kurz (1994) developed the concept of rational beliefs, which require that the beliefs be compatible with the empirical data in the sense that one cannot reject the belief as irrational by looking at the data. In this sense, the concept of rational beliefs shares the same spirit with the idea of rational expectations. However, the rationality requirement for rational beliefs is much looser than that for rational expectations. While rational expectations require the belief to be the true probability, rational beliefs are typically incorrect beliefs, and thus, the framework of rational beliefs admits people to make mistakes, while that is never the case with rational expectations.

A number of papers that utilise rational beliefs report that the economy could face a much higher volatility than in an economy with rational expectations, e.g. Kurz and Schneider (1996), Kurz and Motolese (2001), Wu and Guo (2004), Kurz, Jin and Motolese (2005), and Nakata (2007). One peculiar feature that should be noted is that the mere existence of heterogeneity of beliefs generates *endogenous uncertainty* on top of the exogenous uncertainty arising from the usual primitives or fundamentals. In other words, the states of beliefs become part of the fundamentals on top of the usual fundamentals.

Once heterogeneity of beliefs is present, the *ex ante* optimality and the *ex post* optimality do not coincide as is shown by Hammond (1981). It is clear that the *ex post* optimality is more natural, since it can incorporate regrets caused by bad decisions made based on an incorrect belief. Nonetheless, the choice of social probabilities for the *ex post* social welfare function is not trivial, since no one can learn the true probability through empirical data in a rational belief environment. We examine this issue in the current paper.

The remainder of the paper proceeds as follows. Section 2 explains the structure of the model, and also discusses how we should measure the welfare of the agents and/or the economy. Section 3 exhibits simulation results, and section 4 concludes the paper.

2   The Model

In this section, we first introduce a standard OLG model with financial assets albeit with heterogeneous beliefs. Then, we describe the structure of beliefs of the agents. After describing the young agent's problem and its optimality conditions, we define the competitive equilibrium of the economy,

while restricting our attention to Markovian economies. Finally we define the Markov rational belief equilibrium of the economy.

2.1 The Structure of the Model

2.1.1 The Structure of the Economy

The structure of the economy is essentially the same as that of Kurz and Beltratti (1997) and Kurz and Motolese (2001), except that we introduce short-sale constraints and defaults. Consider a standard OLG economy with H young agents in each generation which we denote by $h = 1, 2, ..., H$ (H is some finite positive integer). Also, there are H old agents in each period. There is a single perishable consumption good, whose price is normalised to unity in every period t. We assume that only young agents receive an endowment W_t^h ($t = 1, 2, ...$) of this consumption good, except that in the initial period ($t = 0$) old agents (in period 1; born in period 0) receive endowments of the stock specified below (θ_0^h with $\sum_{h=1}^{H} \theta_0^h = 1$). Furthermore, each young agent is a *replica* of the old agent who preceded him, where a *replica* refers to the preferences and the set of beliefs.[1] This makes us interpret the streams of agents as 'dynasties' or 'types'. Also, there is a single infinitely lived firm owned by the agents. Let P_t denote the stock price of the firm in period t and θ_t^h the shareholding of young agent h purchased in period t. We assume without loss of generality that the aggregate supply of shares is fixed to unity in every period. The firm's technology generates an exogenous random stream of returns $\{D_t\}_{t=0}^{\infty}$, and we call it the dividend stream. We assume that $D_t > 0$ for all t. For the agents, shareholding yields income from the dividend as well as a capital gain or loss. In addition, there is a market for a zero net supply, short term riskless debt instrument which we call a 'bill'.

To summarize, the economy has three markets: (a) a market for the consumption good with an aggregate supply equaling the total endowment and the total dividends, (b) a stock market with a total supply of unity, and (c) a market for a zero net supply, short term riskless debt instrument which we call a 'bill'. We list the notation as follows: for each agent h,

> C_t^{1h}: consumption of agent h when young in period t;
> C_{t+1}^{2h}: consumption of h when old in $t + 1$ (the agent was born in t);
> $d_{t+1} := D_{t+1}/D_t$: the random growth rate of dividends;
> θ_t^h: amount of stock purchases by young agent h in period t;
> B_t^h: amount of one-period bill purchased by young agent h in period t;
> W_t^h: endowment of young agent h in period t;
> P_t: the price of the stock in period t;
> $p_t := P_t/D_t$: the price/dividend ratio in period t;
> q_t: the price of the one-period bill in period t. This is a discount price.

Next, we specify the structure of the dividend process. Our specification

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[1] We explain what we mean by a set of beliefs when we specify the structure of beliefs.

follows that of [10], which is standard in the literature. Namely,

$$D_{t+1} = d_{t+1} D_t, \tag{1}$$

where the stochastic process $\{d_t\}_{t=1}^{\infty}$ is a stable and ergodic Markov process. Following [5], the state space of the process is $\mathcal{D} := \{d^H, d^L\}$, and the stochastic process $\{d_t\}_{t=1}^{\infty}$ is driven by an empirical transition probability matrix

$$\begin{bmatrix} \phi & 1-\phi \\ 1-\phi & \phi \end{bmatrix}. \tag{2}$$

With this specification, the dividends may well tend to rise over time; thus it is more convenient to focus on the growth rates of the economic variables. To this end, we define the following variables:

$w_t^h := W_t^h / D_t$:   the endowment/dividend ratio of young agent h;

$b_t^h := B_t^h / D_t$:   the bill/dividend ratio of young agent h in t;

$c_t^{1h} := C_t^{1h} / D_t$:   the ratio of consumption to aggregate capital income when young;

$c_{t+1}^{2h} := C_{t+1}^{2h} / D_{t+1}$:   the ratio of consumption to aggregate capital income when old.

In order to elucidate the sources of randomness of the economy, we assume that $w_t^h = w^h$ are constant for all h, t. Hence, the aggregate endowment of the consumption good $\sum_{h=1}^{H} W_t^h$ is proportional to the total dividend D_t in each period t.

2.1.2 The Structure of Beliefs

In what follows, we specify the structure of beliefs. Instead of fixing a belief over generations within each dynasty h, we assume that the effective belief Q_t^h, which defines a time-invariant transition probability, is random over time, and is governed by a probability measure μ^h. More specifically, we assume that every agent h's effective belief is an i.i.d. sequence that is governed by probability μ^h such that

$$\mu^h \{Q_t^h = Q_H^h\} = \alpha^h. \tag{3}$$

Namely, the effective belief of young agent h in period t is Q_H^h with a frequency of α^h and Q_L^h with a frequency of $1 - \alpha^h$. Moreover, we may say *optimistic* when $Q_t^h = Q_H^h$, and *pessimistic* when $Q_t^h = Q_L^h$. Furthermore, we assume that Q_H^h defines a time-invariant transition probability, and so does Q_L^h.

We note that the effective belief Q_t^h is really the theory with which young agent h in period t views the economy. Hence, when $Q_H^h \neq Q_L^h$, there are multiple theories that might be adopted by young agent h. In fact, the randomness of Q_t^h means that the actual theory in use is chosen randomly in each period. This is possible when an agent is ambiguous about the choice of theory.

Now, we explain why it is reasonable to assume such an ambiguity. To begin with, it is reasonable to say that no agent actually knows the truth, and that, every 'intelligent' agent knows that he does not know the truth. Hence, with the knowledge that he does not know the truth, each agent relies on a theory, which always employs some assumption(s) by definition. Because each agent knows that it is impossible for an assumption to be always correct (otherwise it is the truth itself, which is not an assumption by definition), he is uncertain or ambiguous about the choice of theory as long as there are multiple theories available. Namely, each agent is ambiguous in the sense that he is not very sure which theory is the most relevant amongst others, yet he ultimately relies on a theory at the time when he is making decisions. This observation motivates us to randomize Q_t^h rather than to fix it as a particular measure over time, while assuming that the set of theories $\mathcal{Q}^h = \{Q_H^h, Q_L^h\}$ is inherited over generations within the dynasty h.

In addition, it is common that institutional investors including financial institutions adopt some sort of quantitative/statistical model to determine their portfolio choices in practice, and has become increasingly so recently. This means that they adopt a particular probabilistic model to make a portfolio choice, although they do alter the models from time to time. Alterations of models may simply be changes in the parameters of the models, or they may even involve changes in the structure of the model itself. Although such changes are common, there hardly exists a fixed rule/model for model selections. This observation is therefore consistent with our construction of beliefs because each institution uses a probabilistic model out of several possible models in every period, whilst there remains ambiguity in the model selection process.

Our construction of beliefs is capable of describing a common situation in which the same investor sometimes becomes optimistic and sometimes becomes pessimistic even though the data at hand are the same. We stress that it is the belief of the agent that determines if he is optimistic or not, not the data. We do not adopt the view that a particular investor/institution always believes in a particular theory over the periods, and that a change in behaviour only occurs when the data changes. Rather, we allow for an agent to change his view (or mind) even though there is no change in data.

2.1.3 Young Agent's Problem

We now turn our attention to each individual agent's optimisation problem. We assume that each (young) agent in period t forms an effective belief Q_t^h as described above when he makes his decisions.[2] In particular, instead of assuming rational expectations or a common prior, we allow for heterogeneous beliefs.

Before describing the optimisation problem of young agents, let us summarize the timing of the model in each period.

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[2]We assume that each agent is an expected utility maximizer, whose preference is represented by the subjective belief Q_t^h and a von Neumann-Morgenstern utility function, although he is ambiguous about the choice of Q_t^h potentially.

The Timing in Each Period

1. Each young agent h forms a probability belief (effective belief) Q_t^h (or $\hat{Q}_t^h$).

2. d_t realises and transactions take place.

The optimisation problem of a young agent h in period t after observing the announcements (of others) is the following:

$$\max_{(\theta_t^h, B_t^h)} \quad E_{Q_t^h}\{u^h(C_t^{1h}, C_{t+1}^{2h}) \,|\mathcal{G}_t^h\}$$
$$\text{s.t.} \quad C_t^{1h} + P_t\theta_t^h + q_t B_t^h = W_t^h$$
$$C_{t+1}^{2h} = \max\{\theta_t^h \cdot (P_{t+1} + D_{t+1}) + B_t^h - T_{t+1}^h, \gamma D_t\},$$
$$\theta_t^h \geq \underline{\theta}, B_t^h \geq \underline{b}D_t,$$

where C_t^{1h} denotes the consumption of h when young in period t, C_{t+1}^{2h} the consumption of h when old in period $t+1$ (bearing in mind h was born in period t), T_t^h is the tax imposed on old agent h in period $t+1$, and $\mathcal{G}_t^h$ the information set of young agent h in period t at the time of portfolio choice. Note that the second constraint implies that the old agent may default, while sustaining a consumption of γD_t. In other words, the default by old agent h in period $t+1$ is defined by

$$\Delta_{t+1}^h = \max\{0, \gamma D_t - \theta_t^h \cdot (P_{t+1} + D_{t+1}) - B_t^h\}.$$

Moreover, the defaults of a generation are completely financed by the tax imposed on the same generation:

$$\sum_{h=1}^{H} T_{t+1}^h = \sum_{h=1}^{H} \Delta_{t+1}^h.$$

While there are many possible tax schemes that determine the individual tax T_{t+1}^h in general, in the simulation model below where $H = 2$, $T_{t+1}^h = \Delta_{t+1}^{(h)}$ must hold, where (h) reads 'not h'. Since we only analyse this case, in what follows, we assume $T_{t+1}^h = \Delta_{t+1}^{(h)}$.

To enable us to compute equilibria, we assume agent h's utility function to be of the CES form

$$u^h(C_t^{1h}, C_{t+1}^{2h}) = \frac{1}{1-\nu^h}(C_t^{1h})^{1-\nu^h} + \frac{\beta^h}{1-\nu^h}(C_{t+1}^{2h})^{1-\nu^h}, \quad \nu^h > 0,$$

where $\beta^h \in (0,1)$ is the discount factor and ν^h is the parameter that indicates the degree of relative risk aversion of agent h. Then, the first-order conditions (the Euler equations) for the optimisation problem of a young agent h in period t will be (apart from the complementary slackness conditions)

$$-P_t \cdot (C_t^{1h})^{-\nu^h} + \beta^h E_{Q_t^h}\{1_{\Delta_{t+1}^h}(C_{t+1}^{2h})^{-\nu^h} \cdot (P_{t+1} + D_{t+1}) \,|\mathcal{G}_t^h\} + \lambda_t^{h\theta} = 0,$$
$$-q_t \cdot (C_t^{1h})^{-\nu^h} + \beta^h E_{Q_t^h}\{1_{\Delta_{t+1}^h}(C_{t+1}^{2h})^{-\nu^h} \,|\mathcal{G}_t^h\} + \lambda_t^{hB} = 0,$$

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where
$$1_{\Delta_{t+1}^h} = \begin{cases} 1 & \text{if } \Delta_{t+1}^h = 0 \\ 0 & \text{otherwise.} \end{cases}$$

We can describe these conditions by using ratios $(p_t, q_t, d_t, c_t^{1h}, c_{t+1}^{2h}, b_t^h)$ instead of absolute values $(P_t, D_t, C_t^{1h}, C_{t+1}^{2h}, B_t^h)$ as follows:

$$p_t \cdot (c_t^{1h})^{-\nu^h} = \beta^h E_{Q_t^h}\{1_{\Delta_{t+1}^h}(c_{t+1}^{2h}d_{t+1})^{-\nu^h}(p_{t+1}+1)d_{t+1}\,|\mathcal{G}_t^h\} + \tilde{\lambda}_t^{h\theta},$$

$$q_t \cdot (c_t^{1h})^{-\nu^h} = \beta^h E_{Q_t^h}\{1_{\Delta_{t+1}^h}(c_{t+1}^{2h}d_{t+1})^{-\nu^h}\,|\mathcal{G}_t^h\} + \tilde{\lambda}_t^{hB},$$

$$c_t^{1h} = -p_t\theta_t^h - q_t b_t^h + w^h,$$

$$c_{t+1}^{2h} = \max\left\{\theta_t^h \cdot (p_{t+1}+1) + \frac{b_t^h}{d_{t+1}}, \frac{\gamma}{d_{t+1}}\right\},$$

where $\tilde{\lambda}_t^{h\theta} = \lambda_t^{h\theta}D_t^{\nu^h-1}$ and $\tilde{\lambda}_t^{hB} = \lambda_t^{hB}D_t^{\nu^h-1}$.

Now, we make the following assumption to make the simulations of this model tractable:

Assumption 2: *Each young agent h in period t believes that the economy is Markovian.*

By Assumption 2, each agent believes that the joint process $\{p_t, q_t, d_t\}_t$ is Markov. It follows that the conditions above will be rewritten as

$$p_t \cdot (c_t^{1h})^{-\nu^h} = \beta^h E_{Q_t^h}\{1_{\Delta_{t+1}^h}(c_{t+1}^{2h}d_{t+1})^{-\nu^h} \cdot (p_{t+1}+1)d_{t+1}\,|\,p_t, q_t, d_t\} + \tilde{\lambda}_t^{h\theta},$$

$$q_t \cdot (c_t^{1h})^{-\nu^h} = \beta^h E_{Q_t^h}\{1_{\Delta_{t+1}^h}(c_{t+1}^{2h}d_{t+1})^{-\nu^h}\,|\,p_t, q_t, d_t\}, + \tilde{\lambda}_t^{hB}$$

$$c_t^{1h} = -p_t\theta_t^h - q_t b_t^h + w^h,$$

$$c_{t+1}^{2h} = \max\left\{\theta_t^h \cdot (p_{t+1}+1) + \frac{b_t^h}{d_{t+1}}, \frac{\gamma}{d_{t+1}}\right\}.$$

It follows that the demand correspondences of the young will be time-invariant: for every h, t,

$$\theta_t^h = \theta_{Q_t^h}^h(p_t, q_t, d_t), \tag{4}$$

$$b_t^h = b_{Q_t^h}^h(p_t, q_t, d_t). \tag{5}$$

Observe that the demand is influenced by Q_t^h. This suggests that the distribution of the effective beliefs may have impacts on the equilibrium of the economy.

2.2 The Equilibrium

We have so far defined the optimisation problem of a young agent and derived its solution, i.e. the demand correspondences. In what follows, we define the equilibrium of the economy by introducing the market clearing conditions in addition to the optimality conditions of the young agents' problems.

2.2.1 The Definition of the Competitive Equilibrium

In addition to the optimality conditions for young agents' problems, the equilibria of the economy are characterised by the market clearing conditions: for every period t, the markets clear if

$$\sum_{h=1}^{H} \theta_{Q_t^h}^h(p_t, q_t, d_t) = 1, \tag{6}$$

$$\sum_{h=1}^{H} b_{Q_t^h}^h(p_t, q_t, d_t) = 0. \tag{7}$$

We therefore define a stable Markov competitive equilibrium of our economy as follows:

Definition: *Sequences of probability measures* $\{Q_t^1, Q_t^2, ..., Q_t^H\}_{t=1}^{\infty}$ *and a joint stochastic process* $\{p_t, q_t, (\theta_t^h, b_t^h), d_t; h = 1, 2, ..., H\}_{t=1}^{\infty}$ *with initial portfolios* $[(\theta_0^h, b_0^h), h = 1, 2, ..., H]$ *associated with the true probability measure* Π *constitute a* **stable Markov competitive equilibrium** *if*

1. $(p_t, q_t, \theta_t^h, b_t^h, d_t; h = 1, 2, ..., H)$ *satisfy conditions (4), (5), (6) and (7) for all t.*

2. Π *is a stable measure, and each sequence* $\{Q_t^h\}_{t=1}^{\infty}$ *constitutes a stable measure for all h.*[3]

By construction, the equilibrium prices will be a sequence generated by a time-invariant map as follows:

$$\begin{bmatrix} p_t \\ q_t \end{bmatrix} = \Phi_{\mathbf{Q}_t}(d_t), \quad \forall t, \tag{8}$$

where $\mathbf{Q}_t = (Q_t^1, Q_t^2, ..., Q_t^H)$.

It is clear from the equilibrium map (8) that the primitives of the economy are the dividend growth rate d_t and the effective beliefs $\mathbf{Q}_t$ given the preferences. It follows that although the economy appears to be a joint stochastic process of $(p_t, q_t, d_t, \mathbf{Q}_t)$, which is assumed to be Markov, it is sufficient to describe the joint stochastic process of $(d_t, \mathbf{Q}_t)$ since they determine the prices (p_t, q_t). In other words, the states of the prices are partitioned by the states of d_t and $\mathbf{Q}_t$, and consequently, at most $2 \times 2^H = 2^{1+H}$ prices will be observed in this economy.

2.2.2 The True Probability Measure and the Stationary Measure

In this paper we focus on stable Markov competitive equilibria, which are defined above. Because of the stochastic stability, there exists a stationary measure m that is induced from the true probability measure that governs the equilibrium prices (and allocations) over time. However, we stress that

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[3]See Kurz (1994) for the formal definition of stable measure.

this does not mean that the true measure is stationary. In fact, there are (infinitely) many stable measures from which the same stationary measure m can be induced, and the true measure is only one of them.

From the equilibrium map (8), we know that there are at most 2^{1+H} prices in the economy, which are determined by $(d_t, \mathbf{Q}_t)$. Hence, the equilibrium process of the economy is fully specified if we specify the true measure Π that governs $(d_t, \mathbf{Q}_t)$, which is a stable Markov process. However, we are only interested in the long term averages/frequencies of the economic variables; thus, we instead specify the stationary transition probability matrix Γ for the joint stochastic process $(d_t, \mathbf{Q}_t)$, which characterises the stationary measure m that is induced by the true probability measure Π.

In doing so, we require the stationary transition probability matrix to satisfy the following:

- the marginal distribution for d_t is specified by the stationary dividend (growth rate) process (2),

- the marginal distributions for Q_t^h specify Q_t^h to be i.i.d. with $m_{Q^h}\{Q_t^h = Q_H^h\} = \alpha^h$ for all h.

There are many matrices that satisfy these conditions. However, we specify the one that is found in [5] so that we can reinterpret their simulation results in the light of our construction of the beliefs and the economy.

2.2.3 Rational Belief Equilibrium

Now, we allow for heterogeneous beliefs unlike the REE. Nevertheless, we require every sequence of effective beliefs $\{Q_t^h\}_{t=1}^{\infty}$ to constitute a rational belief. The generic condition/definition is the following:

Definition: *A sequence of effective beliefs $\{Q_t^h\}_{t=1}^{\infty}$ constitutes a **rational belief** if it induces a stationary measure that is equivalent to the one under the true probability measure Π.*

In what follows, we explicitly show how to describe rational beliefs in our Markovian economy. Recall that the equilibrium map (8) is

$$\left[\begin{array}{c} p_t \\ q_t \end{array} \right] = \Phi_{\mathbf{Q}_t}(d_t), \quad \forall t.$$

Hence, the equilibrium is driven by a stable Markov process of $(d_t, \mathbf{Q}_t)$ as we noted above. Namely, we need to define a dynamical system on $(V^{\infty}, \mathcal{B}(V^{\infty}))$, where V is the state space of $(d_t, \mathbf{Q}_t)$, as a stable Markov process.

For the computation of the long-term frequencies or long-term averages of the economic variables, it is sufficient to specify a stationary transition matrix Γ that specifies the transition probabilities from $(d_t, \mathbf{Q}_t)$ to $(d_{t+1}, \mathbf{Q}_{t+1})$, i.e. Γ is on $V \times V$, which induces a stationary measure, and that, the stationary measure is the one that is induced by the true probability measure. Note that the true process may not be stationary, but it is enough for us to specify

its induced stationary measure that is fully characterised by the transition probability matrix Γ to compute the long-term frequencies.

On the other hand, we specified that the effective beliefs are determined randomly, either Q_H^h or Q_L^h. Hence, we define pairs of transition probability matrices that correspond to the pair of effective beliefs (Q_H^h, Q_L^h) as follows: young agent h in period t adopts a transition matrix F_t^h by the following rule

$$F_t^h = \begin{cases} F_H^h & \text{if } Q_t^h = Q_H^h; \\ F_L^h & \text{if } Q_t^h = Q_L^h. \end{cases} \tag{9}$$

Recall that Q_H^h is a measure on $(X^\infty, \mathcal{B}(X^\infty))$, where X is the state space of (p_t, q_t, d_t) for all t, which is larger than V, the state space of $(d_t, \mathbf{Q}_t)$. However, we are only interested in the long-term frequencies of the economic variables, and thus, we can ignore the states that occur with probability 0. Hence, it is sufficient for the transition probability matrices F_H^h and F_L^h to be on $V \times V$ rather than on $X \times X$.

With this and (9) in mind, we require the sequence of effective beliefs to satisfy the rationality condition, which is analogous to the one found in papers on rational beliefs (e.g. Kurz and Beltratti, 1997; Kurz and Motolese, 2001; Kurz and Schneider, 1996, etc.):

Rationality Condition: *The transition matrices F_H^h and F_L^h of each agent h must satisfy the following condition for the sequence of effective beliefs $\{Q_t^h\}_{t=1}^\infty$ to constitute a rational belief:*

$$\alpha^h \cdot F_H^h + (1 - \alpha^h) \cdot F_L^h = \Gamma, \quad \forall h. \tag{10}$$

Because the frequency of the event $\{Q_t^h = Q_H^h\}$ is α^h with respect to the true probability μ, agent h uses the transition probability matrix F_H^h with frequency α^h. Hence, the rationality condition (10) requires the sequence of beliefs $\{Q_t^h\}_{t=1}^\infty$ to be compatible with the data that is generated by the stationary transition probability matrix Γ. In other words, there is no way for the agents to reject the set of theories $\mathcal{Q}^h$ for being invalid by observing the data.

Now we define a Markov Rational Belief Equilibrium as follows:

Definition: *A* **Markov Rational Belief Equilibrium** *(RBE) is a stable Markov Competitive Equilibrium in which the sequences of effective beliefs $\{Q_t^h\}_{t=1}^\infty$ $(h = 1, 2, ..., H)$ satisfy (3) and the rationality condition (10).*

The definition of a Markov Rational Belief Equilibrium allows for heterogeneous beliefs. However, it requires the sequence of beliefs to constitute a rational belief. Hence, it is required that both the true equilibrium process of the economic variables and the subjective process of them must be stable, but not necessarily stationary. However, it is not obvious at all how communication and/or the non-stationarity of beliefs impact the equilibrium of the economy. In the subsequent section, we therefore develop a simulation model to examine the impacts of communication and/or the non-stationarity of the beliefs.

2.3 Welfare Measure

In our model, there are several public policy instruments that can affect the resource allocation. Two sets of short-sales constraints are determined by $\underline{\theta}$ (for the common stock) and $\underline{b}$ for the bond, while γ determines the default requirement.

The policy maker therefore can use these instruments to attain the social welfare optimal allocation. However, the definition of the social welfare optimum requires some care. We therefore examine the concepts of social welfare below.

Heterogeneity of beliefs invalidates the standard Pareto efficiency and/or social welfare criterion. This is because heterogeneous beliefs mean that agents hold wrong beliefs in general, and such wrong beliefs cause mistakes or regretful decisions. To take such regrets into account, it is probably reasonable to measure the welfare of the individuals and the society as a whole with respect to an *ex post* measure. An *ex post* social welfare function for one generation is defined by

$$\hat{E}V(u^1, u^2, ..., u^H),$$

where $\hat{E}$ is the expectation operator with respect to the social probability measure, u^h is the *ex post* utility of individual h (a random variable), and V is a von Neumann-Morgenstern social welfare function, which is a function of the *ex post* utilities of the individuals.

Hammond (1981) shows that an allocation based on an *ex post* social welfare function is not Pareto efficient in terms of individual's expected utilities unless all individuals agree on the probabilities and the *ex post* social welfare function takes a special form such that

$$\hat{E}\sum_{h=1}^{H} y^h u^h = \sum_{h=1}^{H} y^h \hat{E} u^h, \tag{11}$$

where y^h is some positive weight attached to individual h, which corresponds to the Negishi-Pareto weight if all individuals agree on the probabilities. It is clear that the individuals have different probability estimates in the rational belief framework, and thus, the *ex post* optimal allocation and the *ex ante* optimal allocation do not match.

Even if we assume that the *ex post* social welfare function takes the form as (11) above, the choice of the social probability measure is not trivial when we allow for heterogeneous beliefs. Nielsen (2006) proposes to use the stationary measure as the social probability measure for the *ex post* social welfare function defined by (11). Recall that all agents agree on the stationary measure, which is the empirical distribution of the observables. Other than the stationary measure, there will be no agreement amongst the agents. Moreover, there is no way to reject the probability beliefs of the agents by looking at the empirical data as long as they are rational beliefs, although they are typically incorrect, while simultaneously there is no way to uncover the true probability measure from the empirical data. We therefore argue from a practical point of view that the proposal of Nielsen (2006) is reasonable.

Hence, each agent h's ex post welfare is measured by

$$\hat{E}\left\{\frac{1}{1-\nu^h}(C_t^{1h})^{1-\nu^h}+\frac{\beta^h}{1-\nu^h}(C_{t+1}^{2h})^{1-\nu^h}\right\}=\frac{1+\beta^h}{1-\nu^h}(CE^h)^{1-\nu^h}$$

where $\hat{E}$ denotes the expectation with respect to the stationary measure and CE^h the ex post certainty equivalent of agent h. We define CE^h, because comparisons with respect to this are easier to interpret than the values of expected utilities themselves. Moreover, we break it down to two parts. Namely, we measure the ex post welfare when young,

$$\hat{E}\left\{\frac{1}{1-\nu^h}(C_t^{1h})^{1-\nu^h}\right\}=\frac{1}{1-\nu^h}(CE^{1h})^{1-\nu^h},$$

where CE^{1h} is young agent h's certainty equivalent. Also, we measure the ex post welfare when old,

$$\hat{E}\left\{\frac{1}{1-\nu^h}(C_t^{2h})^{1-\nu^h}\right\}=\frac{1}{1-\nu^h}(CE^{2h})^{1-\nu^h},$$

where CE^{2h} is old agent h's certainty equivalent. Since the appropriate values of Negishi-Pareto weights are not always very obvious, we just use these individual agents' certainty equivalents when we examine the effects of various public policy instruments on welfare below.

3   Simulations

In this section, we examine the effects of communication on the equilibrium of the financial economy. To do so, we develop a simulation model, which is the same as the one in Kurz and Beltratti (1997) and Kurz and Motolese (2001). Also, we classify the effective beliefs to help examine the effects of communication. Then, we exhibit simulation results of three different cases: (a) Rational Expectations Equilibrium, (b) Rational Belief Equilibrium reported in Kurz and Motolese (2001) and (c) another Rational Belief Equilibrium. Finally, we discuss the implications of the results.

3.1   The Simulation Model

First, we assume that the number of dynasties to be $H = 2$. Then, the number of states in each period is $2 \times 2^2 = 8$. It follows that we can define a map Φ^* between the state space of the index of the prices and the state space of (d_t, Q_t^1, Q_t^2) (which is indexed by numbers from 1 to 8 rather than by t):

$$\begin{bmatrix}1\\2\\3\\4\\5\\6\\7\\8\end{bmatrix}=\Phi^*\begin{bmatrix}d_1=d^H, & Q_1^1=Q_H^1, & Q_1^2=Q_H^2\\d_2=d^H, & Q_2^1=Q_H^1, & Q_2^2=Q_L^2\\d_3=d^H, & Q_3^1=Q_L^1, & Q_3^2=Q_H^2\\d_4=d^H, & Q_4^1=Q_L^1, & Q_4^2=Q_L^2\\d_5=d^L, & Q_5^1=Q_H^1, & Q_5^2=Q_H^2\\d_6=d^L, & Q_6^1=Q_H^1, & Q_6^2=Q_L^2\\d_7=d^L, & Q_7^1=Q_L^1, & Q_7^2=Q_H^2\\d_8=d^L, & Q_8^1=Q_L^1, & Q_8^2=Q_L^2\end{bmatrix}. \qquad (12)$$

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We assume that the 8×8 stationary transition probability matrix Γ has the following structure:

$$\Gamma = \begin{bmatrix} \phi A & (1 - \phi)A \\ (1 - \phi)B & \phi B \end{bmatrix},$$

where A and B are 4×4 matrices which are characterised by ten parameters $(\alpha^1, \alpha^2, a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4)$:

$$A = \begin{bmatrix} a_1, & \alpha^1 - a_1, & \alpha^2 - a_1, & 1 + a_1 - \alpha^1 - \alpha^2 \\ a_2, & \alpha^1 - a_2, & \alpha^2 - a_2, & 1 + a_2 - \alpha^1 - \alpha^2 \\ a_3, & \alpha^1 - a_3, & \alpha^2 - a_3, & 1 + a_3 - \alpha^1 - \alpha^2 \\ a_4, & \alpha^1 - a_4, & \alpha^2 - a_4, & 1 + a_4 - \alpha^1 - \alpha^2 \end{bmatrix}, \quad (13)$$

$$B = \begin{bmatrix} b_1, & \alpha^1 - b_1, & \alpha^2 - b_1, & 1 + b_1 - \alpha^1 - \alpha^2 \\ b_2, & \alpha^1 - b_2, & \alpha^2 - b_2, & 1 + b_2 - \alpha^1 - \alpha^2 \\ b_3, & \alpha^1 - b_3, & \alpha^2 - b_3, & 1 + b_3 - \alpha^1 - \alpha^2 \\ b_4, & \alpha^1 - b_4, & \alpha^2 - b_4, & 1 + b_4 - \alpha^1 - \alpha^2 \end{bmatrix}. \quad (14)$$

The proposed structure of the stationary transition probability matrix Γ satisfies the following properties required for the marginal distributions to hold:

- Marginal measures specify Q_t^h to be i.i.d. with probability α^h.

- The marginal measure for d_t is specified by (2).

Note that the joint distribution of (Q_t^1, Q_t^2) may depend on d_t.

Next, we specify the transition probability matrices that represent the beliefs of the agents. As we noted above, young agent h in period t uses F_H^h when his belief is Q_H^h, and F_L^h when Q_L^h. Because the rationality condition (10) must be satisfied, F_L^h is determined by F_H^h and Γ. Hence, we only specify F_H^h as follows:

$$F_H^h = \begin{bmatrix} \phi A_1(\lambda^h) & A_2(\lambda^h) \\ (1 - \phi)B_1(\lambda^h) & B_2(\lambda^h) \end{bmatrix}, \quad (15)$$

where

$$A_1(\lambda^h) = \begin{bmatrix} \lambda_1^h A^1 \\ \lambda_2^h A^2 \\ \lambda_3^h A^3 \\ \lambda_4^h A^4 \end{bmatrix}, \qquad A_2(\lambda^h) = \begin{bmatrix} (1 - \phi\lambda_1^h)A^1 \\ (1 - \phi\lambda_2^h)A^2 \\ (1 - \phi\lambda_3^h)A^3 \\ (1 - \phi\lambda_4^h)A^4 \end{bmatrix}, \quad (16)$$

$$B_1(\lambda^h) = \begin{bmatrix} \lambda_5^h B^1 \\ \lambda_6^h B^2 \\ \lambda_7^h B^3 \\ \lambda_8^h B^4 \end{bmatrix}, \qquad B_2(\lambda^h) = \begin{bmatrix} (1 - (1 - \phi)\lambda_5^h)B^1 \\ (1 - (1 - \phi)\lambda_6^h)B^2 \\ (1 - (1 - \phi)\lambda_7^h)B^3 \\ (1 - (1 - \phi)\lambda_8^h)B^4 \end{bmatrix} \quad (17)$$

where A^i is the ith row of matrix A and similarly for B^i $(i = 1, 2, 3, 4)$:

$$A^i = \begin{pmatrix} a_i, & \alpha^1 - a_i, & \alpha^2 - a_i, & 1 + a_i - (\alpha^1 + \alpha^2) \end{pmatrix}, \quad (18)$$
$$B^i = \begin{pmatrix} b_i, & \alpha^1 - b_i, & \alpha^2 - b_i, & 1 + b_i - (\alpha^1 + \alpha^2) \end{pmatrix}. \quad (19)$$

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There are eight parameters $(\lambda_1^h, \lambda_2^h, ..., \lambda_8^h)$ for every h, and it is clear that they determine how F_H^h deviates from Γ. In particular, when $\lambda_i^h > 1$, this means that the conditional probability of $\{d_{t+1} = d^H\}$ given state i with respect to Q_H^h is higher than the probability specified in Γ:

$$Q_H^h\{d_{t+1} = d^H | \text{ state } i\} > m\{d_{t+1} = d^H | \text{ state } i\}, \quad \text{if } \lambda_i^h > 1.$$

3.2 Anonymity

Before providing an interpretation of the transition probabilities, we pay attention to the issue of anonymity. By assumption 1, we assumed that the young agents announce their opinions truthfully, and thus, we put strategic concerns aside. However, to be consistent with this assumption and the competitive assumption as a whole, we need to sustain anonymity of each individual agent. In other words, each agent believes that he does not have any impact on the economy or on the equilibrium, although he actually has an impact on it.

Observe that the stationary transition probability matrix Γ can be computed by everyone, while young agent h in period t believes that the current economy is governed by Q_t^h (or equivalently by transition probability matrix F_t^h). By following the argument on this issue in [7], we argue that the anonymity is sustained by assuming the following: for every young agent h in period t,

$$\lambda_i^h = \lambda^h, \quad \text{for } i = 1, 2, ..., 8. \tag{20}$$

Recall that the parameters $(\lambda_1^h, \lambda_2^h, ..., \lambda_8^h)$ determine how F_H^h deviates from Γ. Thus, condition (20) implies that the deviation of F_H^h (and effectively that of F_L^h) from Γ is determined by a single parameter λ^h. It follows that the single parameter λ^h effectively represents F_H^h (and also F_L^h) relative to Γ. In other words, the degree of optimism or pessimism relative to Γ does not depend on the current state.

Although each young agent does not form a belief about effective beliefs, he understands that there is ambiguity about the choice of theories Q_t^h. Because he can compute Γ, he knows that his theory (effective belief) is represented by λ^h relative to Γ, and that, such a representation is not associated with the current state. Hence, condition (20) assures that each young agent h does not associate his own effective belief Q_t^h with the current state (p_t, q_t, d_t). It follows that anonymity is sustained under condition (20).

3.3 Simulation Results

In what follows, we show the simulation results. By following [5] or [7], we set $\phi = 0.43$, $d^H = 1.054$ and $d^L = 0.982$. Moreover, we set

$$
\begin{aligned}
(a_1, a_2, a_3, a_4) &= (0.5, 0.14, 0.14, 0.14), \\
(b_1, b_2, b_3, b_4) &= (0.5, 0.14, 0.14, 0.14), \\
(\alpha^h, \beta^h, w^h) &= (0.57, 0.9, 26), \forall h.
\end{aligned}
$$

Also, we set $\lambda^1 = \lambda^2 = \lambda$ in all cases. Moreover, we examine two different combinations of the coefficients of risk aversion, i.e. case A $(\nu^1, \nu^2) = (1.5, 2)$ and case B $(\nu^1, \nu^2) = (1.5, 2.5)$. Note that by introducing heterogeneity in risk aversion, we give the agents incentives to trade even without heterogeneity in beliefs, i.e. with rational expectations. Thus, agents may be heterogeneous with respect to risk aversion as well as beliefs, and heterogeneity in either sense would provide the agents with incentives to trade.

3.3.1 No Binding Constraints

First, we report the simulation results for the cases in which the credit limit and short-sale constraints do not bind. Moreover, the minimum consumption guarantee is set as $\gamma = 0.01$. Here, we examine the effects of the changes in λ, which represents the degree of deviation of F_H^h from Γ. Here, we examine case A.
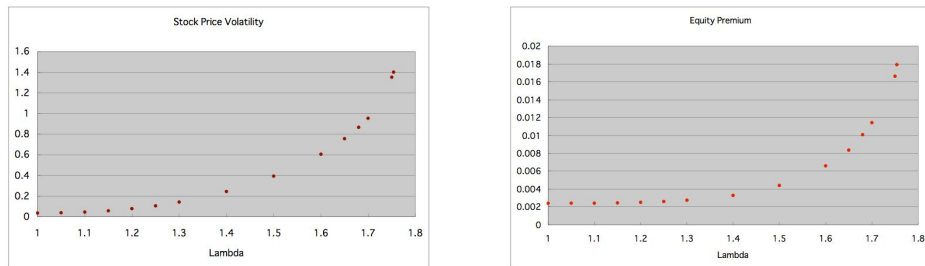
Figure 1: Effects of λ on Prices

Figure 1 reports the effects of changes in λ on the standard deviation of the price/dividend ratio p_t and the average equity premium (the horizontal axis measures λ). Obviously, they are all increasing in λ. Hence, the larger the discrepancy of F_H^h from Γ, the larger the standard deviations of the price/dividend ratio and the risk free interest rate, and the average equity premium.
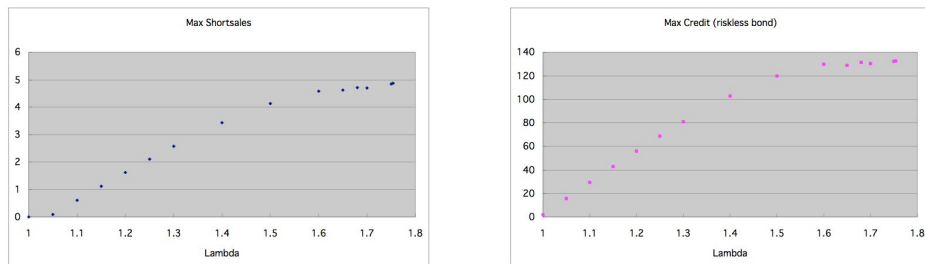
Figure 2: Effects of λ on Portfolios

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On the other hand, figure 2 reports the maximal short-sale position of the common stock and that of the bond for various values of λ (the horizontal axis measures λ as in figure 1). By and large, both of them are increasing in λ. However, they are concave in λ, i.e. the maximal shortsales of the assets increase rapidly in λ for small λ, but almost constant for large λ.

Note that the minimum consumption guarantee binds for all cases in which $\lambda \geq 1.65$. This may well be why the results shown in figure 2 are not necessarily monotonic in λ for cases $\lambda \geq 1.65$. To eliminate complexity arising from this concern, we set $\lambda = 1.65$ in all cases below.

3.3.2 Short-sale Constraints of the Common Stock

Next, we examine the effects of the short-sale constraints of the common stock. Here, the short-sale constraints of the bond are not binding in any state, while there is also no default.

The two graphs of figure 3 report the standard deviation of the price/dividend ratio and the average equity premium for various values of $|\underline{\theta}|$ (the horizontal axis of each graph measures $|\underline{\theta}|$ for case A. Note that a larger $|\underline{\theta}|$ means that the short-sale constraints of the common stock are looser, i.e. they are looser towards the right of each graph. It is clear that the standard deviation of the price/dividend ratio reaches its peak in the middle; it is increasing in $|\underline{\theta}|$ for tighter shortsales constraints (i.e. lower $|\underline{\theta}|$) and is decreasing in $|\underline{\theta}|$ for looser shortsales constraints.
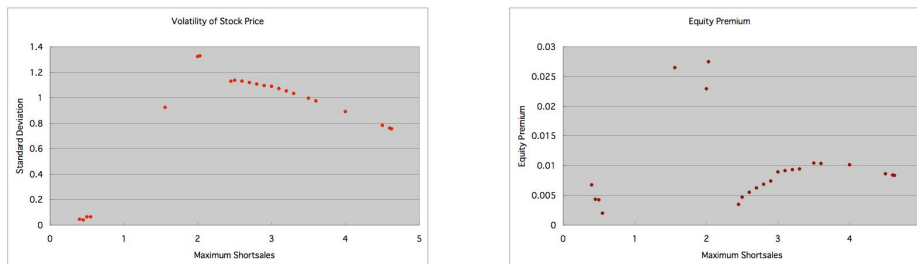

Figure 3: Effects of Shortsales Constraints on Prices: Case A

As for the average equity premium, it reaches its peak in the middle, too. However, it has a more complicated pattern than the standard deviation of the price/dividend ratio has. Apparently, there is discontinuity at around $|\underline{\theta}| = 1$ and $|\underline{\theta}| = 2$.

Figure 4 reports the standard deviation of the price/dividend ratio and the average equity premium for various values of $|\underline{\theta}|$ for case B. It is clear that the patterns observed in case A are retained in this case. Hence, the standard deviation of the price/dividend ratio and the average equity premium are highly correlated with each other, and the economy apparently becomes most volatile when the shortsales constraint $|\underline{\theta}|$ is set at an intermediate value.
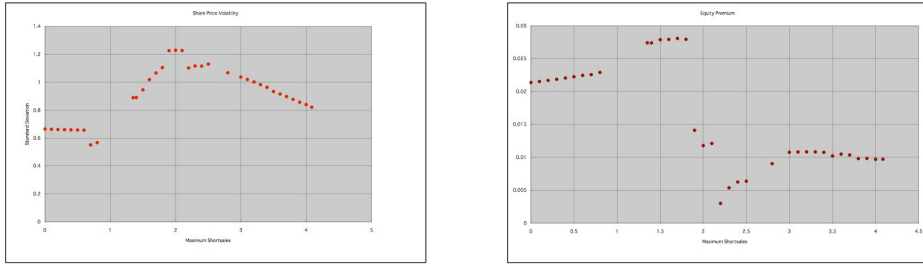
Figure 4: Effects of Shortsales Constraints on Prices: Case B

Next, we turn our attention to the effects on welfare. Figure 5 reports the results for case A. The graph on the left reports the each agent's life-long ex-post certainty equivalent, while the one on the right report the young agents' ex-post certainty equivalents and the old agents ex-post certainty equivalents.
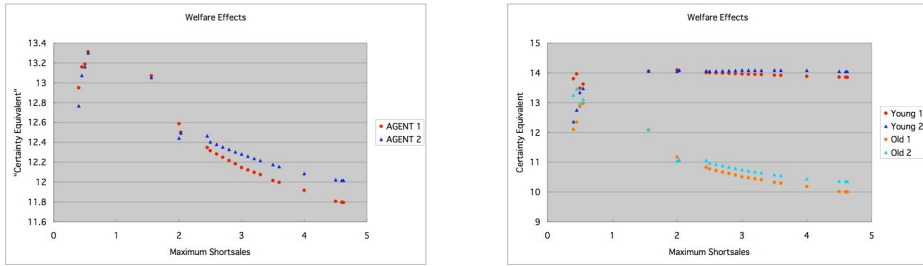

Figure 5: Welfare Effects of the Shortsales Constraints: Case A

It is clear from the graph on the left that there is an optimal level of shortsales constraint for both agents. Nevertheless, the results for very low values of $|\underline{\theta}|$ may well suffer from computation errors, and thus, we should not be conclusive about the existence of the optimal $|\underline{\theta}|$. Yet, we can safely claim that both agents would be better off by tightening the shortsales constraints as long as $|\underline{\theta}|$ is larger than 2. This result indicates that there is a trade-off between benefits of flexibility and more frequent and/or serious mistakes. Namely, agents benefit from larger opportunity sets for small values of $|\underline{\theta}|$, while they suffer from serious mistakes for larger for small values of $|\underline{\theta}|$.

Figure 6 reports the welfare effects of the shortsales constraints for case B. Unlike in case A, there is no obvious value of $|\underline{\theta}|$ that is optimal for both agents simultaneously. Nevertheless, both agents are better off by tightening the shortsales constraints as long as $|\underline{\theta}|$ is larger than unity.
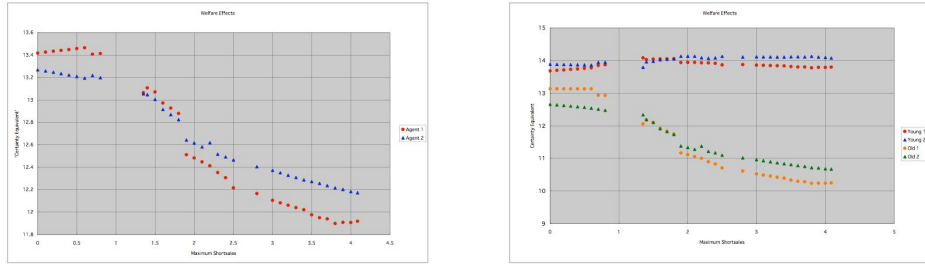
17

Figure 6: Welfare Effects of the Shortsales Constraints: Case B

3.3.3 Credit Limit

In what follows, we examine the effects of the credit limit (i.e. the shortsales constraint of the bond). Here, the short-sale constraints of the common stock are not binding in any state, while there is also no default.
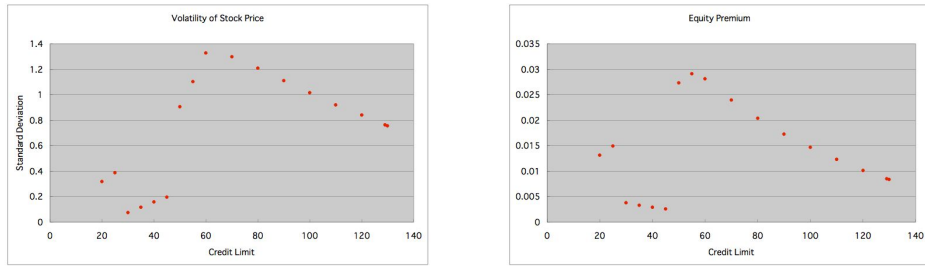

Figure 7: Effects of Credit Limit on Prices: Case A

Figure 7 reports the standard deviation of the price/dividend ratio and the average equity premium for various values of $|\underline{b}|$ in case A. Note that a larger $|\underline{b}|$ means that the credit limit is looser, i.e. it is looser towards the right. The results are clearly very similar to the effects of the shortsales constraints of the common stock: The standard deviation of the price/dividend ratio reached its peak in the middle, and so does the average equity premium.

Figure 8 reports the results for case B. It is clear that the results are very similar to the ones for case A; the standard deviation of the price/dividend ratio reached its peak in the middle, and so does the average equity premium. Hence, the standard deviation of the price/dividend ratio and the average equity premium are highly correlated with each other, and the economy apparently becomes most volatile when the credit limit $|\underline{b}|$ is set at an intermediate value. This finding is essentially the same as the one for shortsales constraints above.

Next, we turn our attention to the effects on the welfare. Figure 9 reports the results for case A. The graph on the left reports the each agent's life-long
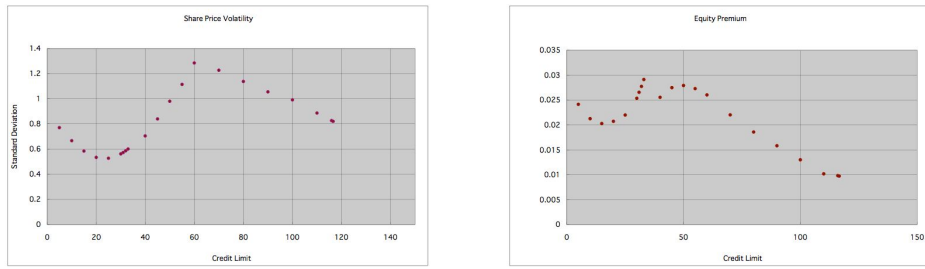
Figure 8: Effects of Credit Limit on Prices: Case B

ex-post certainty equivalent, while the one on the right report the young agents' ex-post certainty equivalents and the old agents ex-post certainty equivalents. Again, the results are by and large analogous to the ones for the shortsales constraints of the common stock. Namely, tighter constraints would make the agents better off for larger values of $|\underline{b}|$, while the opposite may hold true for smaller values of $|\underline{b}|$, although the latter claim is not very clear. Hence, the results indicate that the agents would suffer from more frequent or serious mistakes when the credit limit becomes looser.
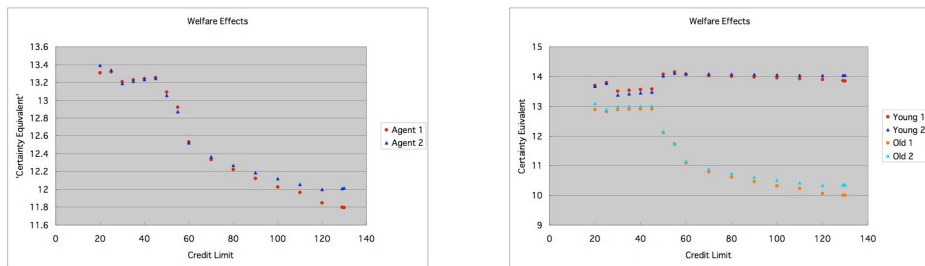
Figure 9: Welfare Effects of the Credit Limit: Case A

Figure 10 reports the results for case B. Again, the results are along the line of those for case A. Thus, the results suggest that a tighter credit limit would improve every agent's welfare by preventing frequent and serious mistakes when the credit limit is not very restrictive, while the benefit of more flexibility may exceed the damage caused by mistakes when the credit limit is very restrictive.
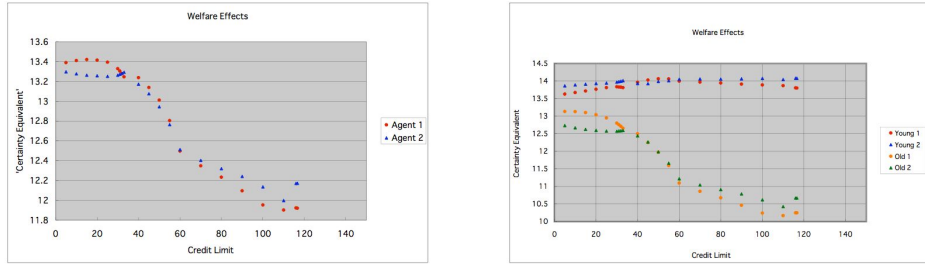
Figure 10: Welfare Effects of the Credit Limit: Case B

3.3.4 Minimum Consumption Guarantee

Finally, we examine the effects of the minimum consumption guarantee γ. Here, the short-sale constraints of the common stock and the bond are not binding in any state. Note that a larger γ means that the minimum consumption is higher, and that defaults are more likely to take place.
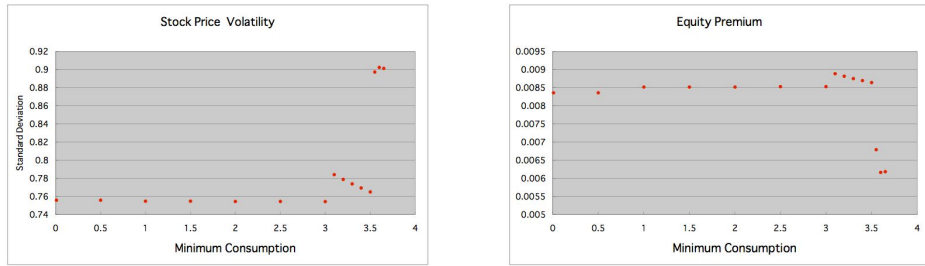

Figure 11: Effects of Minimum Consumption Guarantee on Prices: Case A

Figure 11 reports the standard deviation of the price/dividend ratio and the average equity premium for various values of γ in case A. Both graphs indicate that there is some discontinuity at about $\gamma = 3.5$: The standard deviation of the price/dividend ratio increasing drastically, while the average equity premium decreases rapidly.

Figure 12 reports the results for case B. Once again, the results are analogous to the ones in case A. Namely, the standard deviation of the price/dividend ratio and the average equity premium are not moving together, but are moving in opposing directions.

Next, we turn our attention to the effects of γ on the welfare of the agents. Figure 13 reports the results for case A. The graph on the left reports the each agent's life-long ex-post certainty equivalent, while the one on the right report the young agents' ex-post certainty equivalents and the old agents ex-post certainty equivalents. The graph on the left shows that the less risk averse agent (agent 1) benefits from a more generous minimum consumption
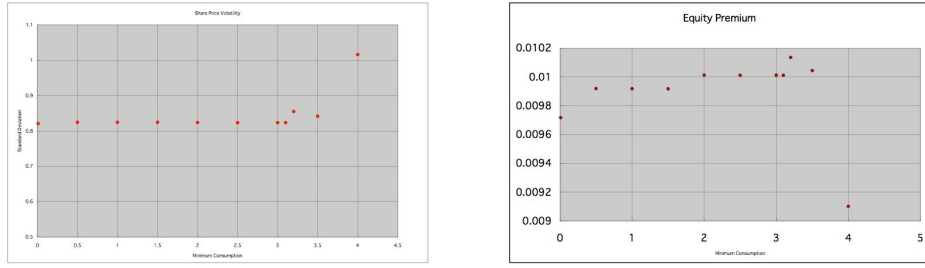
Figure 12: Effects of Minimum Consumption Guarantee on Prices: Case B

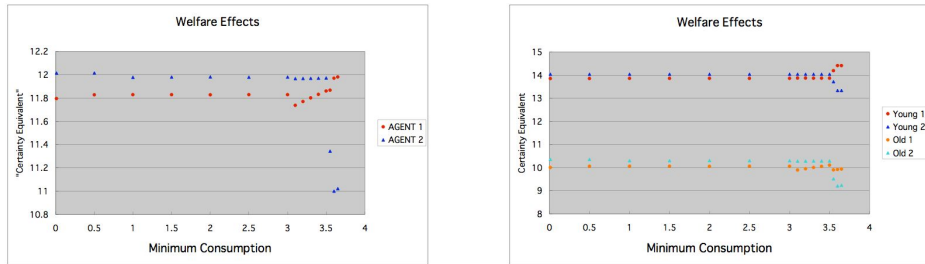guarantee, while the more risk averse agent (agent 2) suffers from it.

Figure 13: Welfare Effects of Minimum Consumption Guarantee: Case A

Figure 14 reports the results for case B. Once again, the results are analogous to the ones in case A. These results suggest that moral hazard in the form of excessive risk taking by the less risk averse agent triggered by the introduction of a more generous minimum consumption guarantee may become so substantial that damages to the welfare of the more risk averse agent become significant.

4 Conclusion

We have examined the impacts of the credit limit, shortsales constraint and the minimum consumption guarantee/limited liability through simulations. The simulation results indicate that a tighter credit limit improves everyone's welfare as long as it is at an intermediate level. Also, a similar conclusion can be drawn for the shortsales constraints. However, any marked universal improvement of welfare can be attained by introducing a more generous minimum consumption guarantee, but would trigger a significant transfer from more risk averse agents to less risk averse agents. Hence, our simulation results suggest that policies that restrain the opportunity sets (i.e. prevention) are more effective than those that correct the ex post outcomes (i.e. ex post
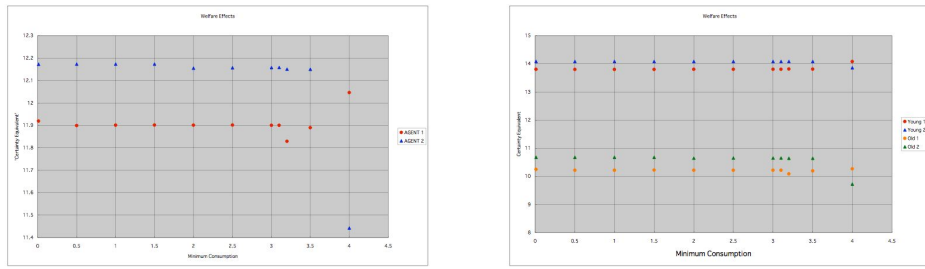
Figure 14: Welfare Effects of Minimum Consumption Guarantee: Case B

redistribution). In other words, regulations that restrict excess risk taking are socially more desirable than bailouts that compensate for the failures by less risk averse agents.

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