Small and Partial Views derived from Limited Experiences*

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Abstract

This paper develops a theory of inductive derivations of personal views from experiences focusing on limited trials and partial memories. The theory allows us to discuss how a player might simplify his view of a complicated situation. Two new concepts of *marked information pieces* and *tested actions* are introduced for this purpose. Each player accumulates local memories as long-term term memories up to marked pieces and tested actions, and constructs a view. At unmarked pieces, he behaves following his prescribed regular behavior pattern without thinking about it. A derived view is typically small, partial, and sometimes prejudiced relative to the original game situation. This paper studies the emergence of such views. One result is the characterization of a Nash equilibrium relative to the relevant domain of the objective game based on the limited and partial views of the individual players.

1. Introduction

Game theory has been successful in describing how to play a game when the rules are well described and well understood by the players. However, the theory does not describe the origin of the beliefs of the players about the rules and structure of the game. A new theory called *inductive game theory* can be used to unearth the origin and emergence of individual beliefs/knowledge on the structure of a socio-economic situation. The theory is "*inductive*" in the sense that it describes how a player (an individual person) builds his beliefs/knowledge from his experiences.

Kaneko-Matusi [10] initiated this theory in the context of "festival games" studying the issues of discrimination and prejudice. In this theory, prejudices emerge from

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limited experiences and limited understandings of the game situation. Kaneko-Kline [9] gave a more entire discourse on inductive game theory touching on various steps of induction, e.g., memories, experiences, derivations of views, and their behavioral implications. That paper gave positive results on how the beliefs of a player might be inductively derived from his experiences. On the negative side, it pointed out how a player's observational and memory capabilities might cause difficulties in the various steps of his long inductive processes. Success at avoiding those difficulties relied on the choice of an appropriate domain of accumulated experiences and memory function. In one successful case of that paper, however, the domain may become large and unmanageable for a player in a typical game situation.

In the present paper, we will focus on much smaller domains of accumulated experiences, which are made possible by the introduction of the notions of "marked (information) pieces" and "tested actions". These notions allow us to use inductive game theory to study how players might simplify and manage complex social game situations. We start with one motivating example, which we will use throughout the paper to illustrate our results. After this example, we will give a brief historical review on related works.

Coordination Game with Precision Γ_C^k (1): Suppose that an objective extensive game is played by two players and has k rounds. In each round, each player i chooses a pair of actions (a_i, b_i) from $\{0,1\} \times \{1,2,3\}$. The a-type action a_i is more payoff relevant than the b-type action b_i for both players. At each round if they succeed in coordinating both actions, they get the highest payoff. However, the payoffs from coordinating the a-type actions are rapidly decreasing as the rounds proceed. The game structure is depicted for the case of $k = 1$ in Fig.1.1. For our purpose, the information structure in the extensive game can be arbitrary, but we assume that each player can distinguish his previous actions at each round.

A possible profile of actions is written as a vector $((a_{11}, b_{11}), ..., (a_{1k}, b_{1k})), ((a_{21}, b_{21}), ..., (a_{2k}, b_{2k}))$, where (a_{it}, b_{it}) is the action taken by player $i = 1, 2$ at round $t = 1, ..., k$. Since each endnode of the game can be expressed as a profile of actions, we can specify the payoff to player i as

$$\begin{aligned} & h_i((a_{11}, b_{11}), ..., (a_{1k}, b_{1k})), ((a_{21}, b_{21}), ..., (a_{2k}, b_{2k})) \\ = & \sum_{t=1}^{k} (1 - |a_{1t} - a_{2t}|)/10^{t-1} + \sum_{t=1}^{k} (2 - |b_{1t} - b_{2t}|)/10^4. \end{aligned}$$

It would be better for each player to be precise for the choice of an a-type action in the beginning, but he doesn't need to be precise at later rounds or for the choice of the b-type action.

Consider the size of a game with $k = 5$ rounds. The first round ends with $6 \times 6 = 6^2$ possible nodes, and in the second round, the same size tree is concatenated to each endnode of the first round. At the end of the second round, we have $6^2 \times 6^2$ nodes.

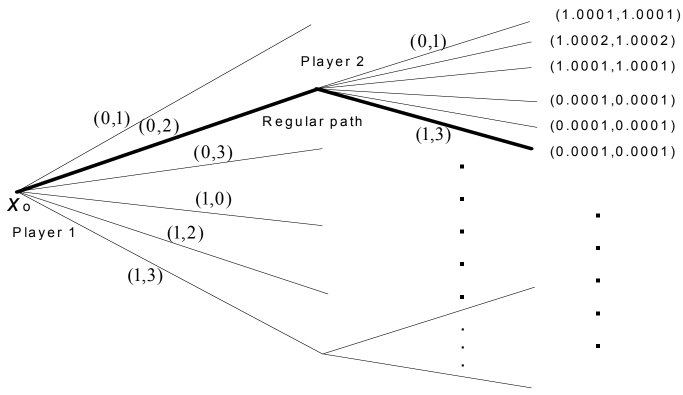

Figure 1.1:

Continuing in the same manner, the number of endnodes in the entire game is

$$6^{2\times 5} = (6^5)^2 = 7,776^2 = 60,466,176. \tag{1.1}$$

Here we see that a simple game can become gigantic and potentially unmanageable for a player when the number of rounds increases. It is fair to say that this situation with $k = 5$ may still be regarded as not large in the standard game theoretic sense as well as in the ordinary social sense. One question we address is this paper is how a player might simplify this kind of a game in order to understand it and make a strategic decision.

Our starting point for inductive game theory is that a player has no *a priori* knowledge about the game structure before he plays it. He will learn about the payoff structure, number of rounds, sequence of moves, and actions only from playing the game. In order to learn the five round game in its entirety, for example, a player would need to experience over 60 million paths! In many social situations, individuals have far fewer experiences. Even if one did have such a large set of experiences, he would surely forget some parts. Additionally, it is very questionable that he would be able to observe all the aspects of the game from playing it, e.g., the strategies and payoffs of the other players might not be observed by him.

In a situation with a "large" structure, a player focuses on what he views as important. First, he marks the information that he views as important, which we will call "marked information pieces". He might also try out only some of his available actions to induce different experiences, which we will call "tested actions." These tests are made by a player as a deviation from what we call his "regular behavior". The notion of "regular behavior" plays a crucial role in inductive game theory. If a player wants to learn about the situation by changing his own actions, he needs to assume that the behaviors of the other players have some regularity. Otherwise, he could not conclude whether his trial or other players' (irregular) behaviors caused a change in his observations.

The regular behavior in a social situation is specified as a strategy profile in our theory. This prescribes an action to be taken by a player in each contingency that might arise. For example, in the five round coordination game Γ_C^5, the (information-independent) regular behavior might be for player 1 to choose ((0,2),(1,1),(1,2),(0,3),(0,1)) and for player 2 to choose ((1,3),(1,1),(0,2),(0,2),(0,3)). The path induced by this regular behavior in the first round is the dark line in Fig.1.1. Over the five rounds, these give player 1 a payoff of 0.1017. The best that player 1 could hope for is found by mimicking the strategy of player 2, which gives him and his opponent 1.1121 each.

Consider a possible analysis by player 1. Suppose that he presumes that the only important pieces of information in this game are the first round choice and the payoff information. He now marks the root piece and any endpieces he experiences. How many endpieces he experiences will depend on how many experiments he makes. Suppose that he will only experiment with his a-type action at the root piece. His tested actions then include the regular behavior (0,2) as well as (1,2) in the first round. In rounds two to five, he sticks to his regular behavior ((1,1),(1,2),(0,3),(0,1)). To find how his experiments affect his payoff, he plays the five round game at least two times. After he collects his experiences from several plays, he may try to build a simple view of the five round game. Since he only regards the first round as important, his view will involve only that round. In addition, we will presume throughout this paper that his view is only a one player game in that player 2 follows his regular behavior and is effectively regarded as an environment of player 1. Then, he may construct the simple game of Fig.1.2, where he may still know that 6 available actions but knows only the resulting outcomes from two actions. This game is much simpler than the objective game since it involves only 2 endpieces as opposed to more than 60 million. It requires only one comparison for player 1 to determine that he should change his regular behavior to improve his payoff to 1.1017, which is close to the best possible payoff 1.1121.

In this paper we will discuss how a player might inductively derive such a view from such a complicated extensive form game. We also discuss under what conditions the regular behavior of the players is found to be optimal in their inductively derived views. The main contribution of this paper is to show that an inductively derived view may become small and partial relative to the entire situation by introducing marking pieces and tested actions. To show this, we will introduce a measure on the size of a view in Section 8, which is simply the number of endnodes of an extensive game. With this measure we can discuss how much the derived view is reduced relative to the objective situation.

For the basic idea of inductive game theory, we can find various people treating the same problem in the history, while a little has been discussed in the recent game theory. Inductive game theory is related to Plato's Republic [18], as well as the works

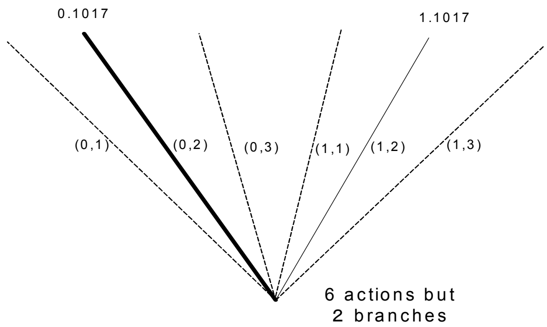

Figure 1.2:

of Bacon [1] and Hume [6]. Those works questioned and explored our limitations on beliefs/knowledge. Inductive game theory succeeds this tradition, and, in fact, it is also related to literature in sociology and social psychology in that human inductive behavior depends critically on the social context (cf, Mead [17] for such discussions and Collins [3] for more recent developments of those sociological ideas).

Our theory also shares some of the motivations behind the "bounded rationality" of Simon [19] and the "standards of behavior" of von Neumann-Morgenstern [21]. Yet, it is methodologically quite different and has a more basic and close connection with the epistemic logic approach to game theory (cf., Kaneko [7]). This basic connection is that inductive game theory provides experiential foundations for the basic beliefs/knowledge in epistemic logic.

We have deviated from the standard literature of game theory and economics (cf., Luce-Raiffa [14] for a classical textbook and Osborne-Rubinstein [16] for a recent textbook). This deviation is needed since we want to focus on the origin and emergence of beliefs/knowledge in individual experiences. This may sound similar to the case-based decision theory of Gilboa-Schmeidler [5] and inductive logic of Carnap [2]. Our theory is different since it focuses on social structures and is methodologically more finitistic, particularly in this paper.[1] It also differs from evolutionary game theory where a player does not think about the social structure, and "memories" are remaining only in the distribution of strategies (cf. Weibull [20]).

Now, we should go further into the basic description of the theory.

[1]Kaneko [8] discussed methodological difficulties arising from infinities involved in game theory and economics, and argued that more finitistic treatments of social problems are needed. Finite, small and partial views considered here have the same spirit.

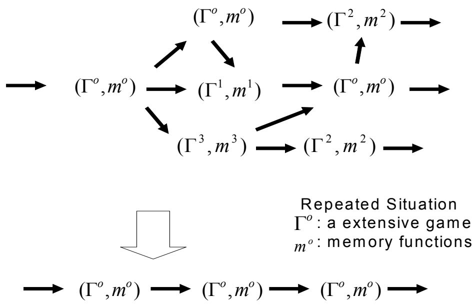

Figure 2.1:

2. Description of Inductive Game Theory

For discussions of the partial views obtained from limited experiences, we need some understanding of inductive game theory. Here, we give a short version of the discourse given in Kaneko-Kline [9], while focusing on the direction this paper takes.

1: Situations: Inductive game theory starts with a basic system of social situations as described in the top part of Fig.2.1. Focussing on one social situation such as (Γ^o, m^o) and making a simplifying assumption that behaviors of players are separated from other situations, the target social situation is represented as a sequence like the one in the bottom part of the figure.

A specific situation $(\Gamma^o, \mathfrak{m}^o)$ consists of an extensive game Γ^o with memory functions $\mathfrak{m}^o = (\mathfrak{m}_1^o, ..., \mathfrak{m}_n^o)$. The memory function $\mathfrak{m}_i^o$ for each player i is a new concept to game theory introduced in [9] and it describes local (short-term) memories occurring within one play of Γ^o. This memory function is separated from the description of Γ^o and is needed to describe the basic ingredients for a player's subjective view.

The description of an extensive game we use is essentially the same as one in the sense of Kuhn [13]. One difference is that an information piece, rather than an information set, is attached to each node. We do not treat the concept of an information set as a basic concept in our theory. Information sets fit well with players who are fully cognizant of the game structure, but not with those who learn by their experiences. We will discuss more about our treatment of information as pieces in Section 3.

2: Regular Behaviors and Limited Trial Deviations: The reader may wonder how a player can, at first, behave without any *a priori* knowledge on the structure of the social situation. In fact, we often behave without knowledge on the structure of the situation. We simply make a choice or follow some learned rule of behavior. Players typically behave following prescribed regular behavior patterns and occasionally make some trial deviations. An extreme example is a default action, i.e., when a player meets in a new decision situation, he chooses the first available action.

If players use only their regular behaviors, then they would only get a very narrow view of the situation. In order to learn more about the structure of the game, they make some trial deviations. In Kaneko-Kline [9], the unilateral trial domain and several others were considered. In many games, like the coordination game with five rounds Γ_C^5 of Section 1, the unilateral trial domain becomes quite large and unmanageable. In the present paper, we consider partial trial domains made possible by our introduction of marked pieces and tested actions.

3: Limited Accumulations of Experiences as Long-term Memories: In addition to the time constraints, a player is constrained by his cognitive ability to store his experiences as memories. Some experiences are more likely to remain as long-term memories, and some disappear from his mind. A player retains only a small set of long-term memories relative to his full potential with unlimited trials and storage capacity.

The transformation from short-term memories to long-term ones has a lot of steps. In Kaneko-Kline [9], these steps were summarized as an *informal* theory. In the present paper we consider memories up to marked pieces and tested actions. Taking these constraints into account, we adopt a specific memory function called the *marked exact perfect recall memory function.* This memory function describes a player who recalls exactly his part of his marked and tested paths within each play.

4: Induction from Long-term Memories to a View: A player constructs his view from the long-term memories stored in his mind. Kaneko-Kline [9] gave a general definition of inductively derived views, and showed that the inductive derivation of such a view is sometimes impossible and sometimes generates many views, which are due the general setting in [9]. In this paper, we avoid those difficulties by adopting the exact perfect recall memory function, and can use a more stringent definition of the inductively derived view than in [9]. This enables us concentrate our research on small and partial views derived from experiences.

It is a contribution of the paper that the introduction of marked pieces and tested actions make the inductively derived view is much smaller and more partial than the original game situation. This suggests that a player may be able to consider his behavior in a large game situation. Therefore, we have a natural connection to the next step.

5: Behavioral Use of an Inductively Derived View: After a player constructs a view, he can use it to determine if his behavior is optimal. Kaneko-Kline [9] obtained an experiential characterization of Nash equilibrium for the full objective game with n-players. In this paper, due to the smallness and partiality of the views, there may be a great distance between subjective optimality and objective optimality. Nevertheless, we show that the subjectively optimal behaviors can be regarded as objectively optimal in a restricted sense. In this way, we can give a corresponding connection to Nash equilibrium.

We will give a measure of the sizes of personal view as well as of the objective game, which will be introduced in Section 8. This enables us to discuss the smallness and partiality of personal views relative to the objective game in a meaningful manner.

3. Extensive Games, Memory Functions, and Behavior

This section summarizes the basic theory of extensive games, and introduces memory functions. An extensive game Γ and a memory function $\mathfrak{m}_i$ are used both for the description of an objective situation and for a subjective personal view. We use the convention of describing the objective situation as $(\Gamma^o, \mathfrak{m}^o)$ and a personal view for a generic player i as $(\Gamma^i, \mathfrak{m}^i)$.

3.1. Extensive Game with Partial Actions

Our definition of an extensive game for the objective description is, more or less, equivalent to that of Kuhn [13]. The main differences are in the treatment of information and memories. These differences are important for the coherent development of inductive game theory. We discuss them after giving the definition of an extensive game. Then, we will modify the definition to describe a personal view of player i.

For notational simplicity, we sometimes make use of a function with the empty domain, which we call an *empty function*. When the empty domain and some (possibly nonempty) region are given, the empty function is uniquely determined.

Definition 3.1 (Extensive Games). A *marked extensive game* $\Gamma = ((X, <), (\lambda, W), M, \{(\varphi_x, A_x)\}_{x \in X}, (\pi, N), h)$ is defined as follows:

K1(*Game Tree*): $(X, <)$ is a finite tree;

K11: X is a finite non-empty set of nodes, and $<$ is a partial ordering over X;

K12: $\{x \in X : x < y\}$ is totally ordered with $<$ for any $y \in X$;[2]

K13: X is partitioned into the set X^D of *decision nodes* and the set X^E of *endnodes* so that every node in X^D has at least one successor, and every node in X^E has no successors;[3]

K14: X has the smallest element x_0, called the *root*.[4]

[2] The binary relation $<$ is called a *partial ordering* on X iff it satisfies (i)(irreflexivity): $x \not< x$; and (ii)(transitivity): $x < y$ and $y < z$ imply $x < z$. It is a *total* ordering iff it is a partial ordering and satisfies (iii)(totality): $x < y$, $x = y$ or $y < x$ for all $x, y \in X$.

[3] We say that y is a *successor* of x iff $x < y$, and that y is an *immediate* successor of x, denoted by $x <^I y$, iff $x < y$ and there is no $z \in X$ such that $x < z$ and $z < y$.

[4] A node x is called the *smallest* element iff $x < y$ or $x = y$ for all $y \in X$.

K2(*Information Function*): W is a finite set of information pieces and $\lambda : X \to W$ is a surjection with $\lambda(x) \neq \lambda(z)$ for any $x \in X^D$ and $z \in X^E$;

K3(*Marked Pieces*): M is an arbitrary (possibly empty) subset of W;

K4(*Available Action Sets*): A_x is a finite set of *available actions* for each $x \in X$;

K41: $A_x = \emptyset$ for all $x \in X^E$;

K42: for all $x, y \in X^D$, $\lambda(x) = \lambda(y)$ implies $A_x = A_y$;

K43: for any $x \in X$, φ_x is a *bijection* from the set of immediate successors of x to A_x;

K5(*Player Assignment*): N is a finite set of players and $\pi : W \to 2^N$ is a player assignment with the following condition;

K51: $|\pi(w)| = 1$ if $w \in \{\lambda(x) : x \in X^D\}$ and $\pi(w) = N$ if $w \in \{\lambda(x) : x \in X^E\}$;

K6(*Payoff functions*): $h = \{h_i\}_{i \in N}$, where $h_i : \{\lambda(x) : x \in X^E\} \to R$ is a payoff function for player $i \in N$.

By K42, we can write A_w for A_x when $\lambda(x) = w$.

The main differences between Kuhn's [13] definition and ours are:

(1) the use of information pieces rather than information sets consisting of nodes having the same information;

(2) some information pieces are marked (see K3 above).

The difference (1) is essential to capture players who do not have full knowledge of the social situation. We assume that an information piece is symbolically given, for example, it is a sentence or simply a symbol. In one play of the game, a player obtains a sequence of information pieces along the realized path. This is the source for information about the game structure. The difference (2) is the source for smallness and partiality of personal views. Now, let us discuss the differences (1) and (2) in detail.

In our interpretation, when a player arrives at a node x, he receives the information piece $w = \lambda(x)$. From the objective point of view, information sets consisting of nodes can be derived from the above definition, i.e., a pair (w, i) of an information piece w and a player i determines the corresponding information set $\{x : \lambda(x) = w \text{ and } i \in \pi(x)\}$. However, the player is not presumed to be able to carry out such a derivation before he forms his beliefs about the structure of the game. He has too little knowledge about the game structure to know which nodes fit where. Here, we emphasize that nodes and, *a fortiori,* information sets are hypothetical constructs. On the other hand, the information pieces are tangible to a player.

The set M of marked (information) pieces is additional to an extensive game of Kuhn [13] and it is used to describe the part of the game that each player regards as important. While the set M is arbitrary here, some requirement will be given in Section

4. The roles of marked pieces will become clearer when we talk about a memory function and trials.

An extensive game is said to be *fully marked* iff $W = M$. Given the set M of marked pieces, we denote $M_i = \{w \in M : i \in \pi(w)\}$ as the set of player i's marked pieces. From the marked pieces, we can define the set of marked nodes:

$$X^M = \{x \in X : \lambda(x) \in M\} \text{ and } X_i^M = X^M \cap X_i, \tag{3.1}$$

where $X_i = \{x \in X : i \in \pi \cdot \lambda(x)\}$ is the set of *nodes for player* i.

In this paper, we assume throughout that:

(3iA): the objective game $\Gamma^o = ((X^o, <^o), (\lambda^o, W^o), M^o, \{(\varphi_x^o, A_x^o)\}_{x \in X}, (\pi^o, N^o), h^o)$ is given as an extensive game satisfying all the above conditions.

Furthermore, each piece of information w contains a minimal content of information:

M1: the set A_w^o of available actions;

M2: the value $\pi^o(w)$ of the player assignment π^o if w is a decision piece;

M3: his own payoff $h_i^o(w)$ (as a numerical value) if w is an endpiece.

The information about the available actions, player assignment and payoff contained in M1, M2 and M3 is interpreted as written on each piece w. These assumptions are made to simplify the description.

For the subjective use of an extensive game, we replace condition K43 by the following K43′ :

K43′: for any $x \in X$, φ_x is an *injection* from the set of immediate successors of x to A_x.

Here, some actions in A_x may not be associated with any immediate successors. The reason for this is to allow for limited trials. All the available actions are still assumed to be perceived by the player who moves at node x (i.e., M1), but their resulting paths are not considered or understood by him unless he tries them. In the coordination game Γ_C^6 with precision in Section 1, according to M1, player 1 is informed that he has six available actions at the root node, but he has tried only two actions $(0, 2)$ and $(1, 2)$. Hence, in this personal view, the root has only two immediate successor nodes, which is depicted in Fig.1.2. When we require only condition K43′ for Γ, we call Γ as an extensive game *with partial actions*.

A basic question for inductive game theory is how a player derives a view on the social situation from his accumulated experiences. In this paper, we assume that

(3iiA): a personal view $\Gamma^i = ((X^i, <^i), (\lambda^i, W^i), M^i, \{(\varphi_x^i, A_x^i)\}_{x \in X^i}, (\pi^i, \{i\}), h^i)$ is a one-player extensive game with partial actions.

Since this is a 1-person game, only the payoff function for player i is listed.

Kaneko-Kline [9] considered extensive games in a weaker sense relaxing the root node assumption and requiring φ_x in K43 to be only a function. This generalization was to allow more general personal views that might include other players.

3.2. Memory and Behavior

In the standard game theory, the memory capability of a player is expressed in terms of information sets.[5] In inductive game theory, however, memory is the source for a personal view. We need to separate the memory capability of a player from information pieces and give an explicit definition of memory. Here, we define a memory function and a strategy for a player. The memory function describes a player's short-term memory.

Let us return to an extensive game Γ either with K43 or K43$'$. For each node x, the injection φ_x associates an action with an immediate successor of x. Using this, we define: $x <_a y$ iff $x < y$ and either $\varphi_x(y) = a$ or ($\varphi_x(x') = a$ and $x' < y$ for some immediate successor x' of x). This means that action a at node x leads to node y. When x is an endnode, φ_x is the empty function and is a bijection since A_x is empty by K41.

To discuss the personal memory capacity, the history up to an information piece within the game Γ is important. We start with a definition of a path. The *path* to a node x is defined as the unique sequence of nodes $x_1, ..., x_{m+1} = x$ satisfying $\{x_1, ..., x_m\} = \{y \in X : y < x\}$ and $x_t < x_{t+1}$ for $i = 1, ..., m$. Now, we define the history of information pieces received and actions taken up to x. The (objective) *history to* x is given as:

$$\theta(x) = \langle(\lambda(x_1), a_1), ..., (\lambda(x_m), a_m), \lambda(x)\rangle, \tag{3.2}$$

where $\langle x_1, ..., x_{m+1}\rangle$ is the path to x and $x_t <_{a_t} x_{t+1}$ for $t = 1, ..., m$. By K2 and K43 (or K43$'$), $\theta(x)$ is uniquely determined, and is the entire history up to x.

Now, we focus on a node x for player i, and the history of his information pieces and actions up to x. Let $\langle x_{k_1}, ..., x_{k_l}, x_{k_{l+1}}\rangle$ be the subsequence of nodes for player i in the path $\langle x_1, ..., x_{m+1}\rangle$, i.e., the subsequence is defined by the property that $i \in \pi \cdot \lambda(x_{k_t})$ for $t = 1, ..., l+1$. We define the *history to* x *relative to player* i as:

$$\theta(x)_i = \langle(\lambda(x_{k_1}), a_{k_1}), ..., (\lambda(x_{k_l}), a_{k_l}), \lambda(x_{k_{l+1}})\rangle, \tag{3.3}$$

where each a_{k_t} is the action taken at x_{k_t} to connect to x. Since $x = x_{m+1}$ belongs to player i, we have $x_{k_{l+1}} = x$. Finally, we define the *marked history relative to player* i, which we denote by $\theta(x)_i^M$. The history $\theta(x)_i^M = \langle(\lambda(y_{t_1}), a_{t_1}), ..., (\lambda(x_{t_s}), a_{t_s}), \lambda(y_{t_{s+1}})\rangle$ is the maximal subsequence of $\theta(x)_i$ with $\lambda(y_{t_k}) \in M$ for all $k = 1, ..., s+1$.

Kaneko-Kline [9] discussed a variety of memory functions, some of which included information pieces of other players. In this paper, we restrict attention to the marked

[5] Traditionally, the memories about his previous moves as well as the information he obtained previously are described by information sets in Γ. In this treatment, memories themselves are interpreted as being written in an information set (cf., Kuhn [13] and Kline [12]).

exact-perfect recall memory function.

Definition 3.2 (Marked Exact-Perfect Recall Memory Function): The MR-memory function $\mathfrak{m}_i^{MR}$ of player i is given as follows. For each node $x \in X_i = \{x \in X : i \in \pi \cdot \lambda(x)\}$,

$$\mathfrak{m}_i^{MR}(x) = \begin{cases} \{\theta(x)_i^M\} & \text{if } \lambda(x) \in M \\ \\ \{\langle \lambda(x) \rangle\} & \text{if } \lambda(x) \notin M. \end{cases} \tag{3.4}$$

In the following, we sometimes refer to the sequence $\theta(x)_i^M$ in $\mathfrak{m}_i^{MR}(x)$ as a *memory thread.*

This memory function gives player i perfect recall about his own marked pieces and tested actions within Γ. At unmarked pieces, he recalls nothing about the past and receives only the current information piece. Note that he has no observations of other players' pieces, and no memories of them.

Now, we have completed the extensive game description and the memory function describing short-term memories. These are used both for the description of the objective situation and the personal view discussed in Section 1. In this paper,

(3iB): the *objective description* is given as a pair $(\Gamma^o, \mathfrak{m}^o) = (\Gamma^o, (\mathfrak{m}_1^{MR}, ..., \mathfrak{m}_n^{MR}))$ of an n-person game Γ^o and a profile of memory functions $\mathfrak{m}^o = (\mathfrak{m}_1^o, ..., \mathfrak{m}_n^o)$;

(3iiB): a *personal view* of player i is a pair $(\Gamma^i, \mathfrak{m}^{MR})$, where Γ^i is a 1-player fully marked extensive game with partial actions and $\mathfrak{m}^{MR}$ is the exact-perfect memory function.

Recall that by "fully marked", we mean that all pieces in Γ^i are marked by player i.

We use a behavior pattern to describe the behavior of players in the recurrent objective situation $(\Gamma^o, \mathfrak{m}^o)$ and also to describe a subjective strategy in a personal view $(\Gamma^i, \mathfrak{m}^i)$ for player i. While the objective situation is repeated such as in the bottom figure of Fig.2.1, we do not treat the behavior as a strategy over the repeated play, but rather as a strategy, or behavior for one play.

Let $\Gamma = ((X, <), (\lambda, W), M, \{(\varphi_x, A_x)\}_{x \in X}, (\pi, N), h)$ be an an extensive game with partial actions and let $\mathfrak{m}_i^{MR}$ be the marked memory function for player $i \in N$. We say that a function σ_i defined over the set of decision nodes $X_i^D = \{x \in X^D : i \in \pi \cdot \lambda(x)\}$ is a *behavior pattern* (*strategy*) iff it satisfies: for all $x, y \in X_i^D$,

$$\sigma_i(x) = \varphi_x(x') \text{ for some immediate successor } x' \text{ of } x; \tag{3.5}$$

$$\mathfrak{m}_i^{MR}(x) = \mathfrak{m}_i^{MR}(y) \text{ implies } \sigma_i(x) = \sigma_i(y). \tag{3.6}$$

The first states that σ_i prescribes an "available" action to each decision node. The second requires σ_i to depend only upon local memories, and is often called the measurability condition. We say that an n-tuple of strategies $\sigma = (\sigma_1, ..., \sigma_n)$ is a *profile of* *behavior patterns*. We denote, by (σ_i', σ_{-i}), the profile obtained from σ by substituting σ_i' for σ_i in σ.

We allow for multiple interpretations of a behavior pattern. When the player has no idea about how to behave, for example, in some new game for society, his behavior is based on a default choice. A behavior pattern for a slightly experienced player is best interpreted as a norm or standard of behavior learned or copied by watching other players in the same or a similar game. With these interpretations, we can avoid the difficulty that a player may have no structural beliefs/knowledge of the game but still has a well defined behavior pattern.

A final remark is that we assume no chance moves. We avoid chance moves because their introduction gives certain conceptual problems and will create some unnecessary complications. Without chance moves, a profile of behavior patterns $\sigma = (\sigma_1, ..., \sigma_n)$ determines a unique path $x_1, ..., x_{m+1}$ in the extensive game Γ.

4. Trials, Unilateral Domains and Memory Kits

The source for a player's personal view is his accumulated long-term memories of past experiences. In Section 4.2, we formulate the partial unilateral trial domain of accumulation and define a corresponding memory kit. The domain of accumulation is based on some behavioral and cognitive postulates. First, in Section 4.1, we briefly discuss an informal theory of postulates that suggest some partial unilateral trial domain of accumulation [deleted fragment here]. See Kaneko-Kline [9] for an informal theory treating the complete unilateral trial domain. Recall that the point of the present paper is to treat small and partial domains when the unilateral trial domain becomes too large to manage.

4.1. Informal Theory of Behavior, Trials and Memorization

The informal postulates we give are divided into two types: *behavioral* and *cognitive*. The postulates in this paper have to do with the marked pieces and limited trials. We start our informal theory with the behavioral postulates.

(1): Postulates for Behavior and Trials: The first postulate is the rule-governed behavior of each player in the recurrent situation $..., (\Gamma^o, \mathfrak{m}^o), ..., (\Gamma^o, \mathfrak{m}^o), ...$

Postulate BH1 (Regular Behavior): Each player, i, typically behaves regularly following his behavior pattern σ_i^o.

As already stated, player i may have adopted his regular behavior for some time without thinking about it. Without assuming regular behavior for the players, a player may not be able to extract any causal pattern from his experiences. In essence, learning requires some regularity.

After having followed some social regular behavior for some time, players might wonder what would happen if they changed their behavior. To learn the pattern as well as responses, a player may make some trial deviations. We postulate that such trial deviations take place in the following manner.

Postulate BH2 (Occasional Deviations): Once in a while (infrequently), each player unilaterally makes a trial deviation σ_i from his regular behavior σ_i^o, and then returns to his regular behavior.

Early on, such deviations may be unconscious or not well thought out. Nevertheless, a player might find that a deviation leads to a better outcome, and he may start making deviations consciously in the future. Once he has become conscious of his behavior/deviation, he might make more and/or different trials. In fact, this postulate allows a player to make no trial deviation. This partiality will be stated in the following two postulates.

Postulate BH3(Marked Pieces): Each player, i, makes a trial only at a marked information piece.

Postulate BH4 (Limited Trials): Each player, i, experiments over only some possible behaviors.

The objective situation may be too large for a player with limited cognitive abilities to handle. Consequently, each player marks some parts as being important and leave the others unmarked. At these marked pieces, a player will consciously consider trials, which may be only some subset of the full set of available actions. Formally, these two postulates will be captured by restricting a trial strategy set given in Section 4.2.

(2): Cognitive Postulates: Each player may learn something through his regular behavior and deviations. What he obtains in an instant is described by his short-term memory, $\mathfrak{m}_i(x)$. For the transition from short-term memories to long-term memories, there are various possibilities. Here we list some postulates based on bounded memory abilities of a player.

The first postulate states that if a short-term memory does not occur frequently enough, it will disappear from the mind of a player. We give this cognitive bound on a player in postulate form as follows:

Postulate CP1 (Forgetfulness): If experiences (short-term memories) are infrequent for player i, then they would disappear from a player's mind.

The word "infrequent" is relative to player i as well as his consciousness. In contrast to the behavior postulates, this postulate is more subjective.

In the face of the cognitive bound of forgetfulness on player i, only some memories become lasting. We assume that the first type of memories that become lasting are the regular ones since they occur quite frequently. The process of making a memory last by repetition is known as habituation.

Postulate CP2 (Habituation): A short-term (local) memory becomes lasting as a long-term memory in the mind of player i by habituation, i.e., if he experiences something frequently enough, it remains in his memory as a long-term memory.

By CP2, when all players follow their regular behavior patterns, the short-term memories experienced by them will become long-term memories by habituation. The remaining possibilities for long-term memories are the memories of trials made by some players. We postulate that a player may consciously spend some effort to memorize the outcomes of his own trials.

Postulate CP3 (Conscious Memorization Effort at Marked Pieces): Each player makes a conscious effort to memorize his marked pieces as well as the results of his own trials at them.

Postulate CP3 means that when a player makes a trial deviation at a marked piece, he also makes a conscious effort to record his experience in his long-term memory. These memories are more likely to remain if they are repeated frequently enough relative to his trials. Since the players are presumed to behave independently, the trial deviations involving multiple players will occur infrequently, even relative to one player's trials. Thus, the memories associated with multiple players' trials do not remain as long-term memories.

We have assumed that when a player receives an unmarked piece, he follows his regular behavior without becoming conscious about it. We interpret CP3 as additionally meaning that an experience at such an unmarked piece does not appear in the mind of the player as a long-term memory, while such experiences along the regular path must be frequent enough.

As we noted above, this is an informal theory, and may be compatible with various formulations of the individual accumulated long-term memories. Nevertheless, here, we interpret Postulates CP1 to CP4 and BH1 to BH3 as suggesting that we can concentrate on a partial domain of accumulation of player which will be described formally in Section 4.2.

Since the above informal theory is highly finitistic, in particular, it includes forgetfulness, we think that simulation studies of this informal theory and others may lead to future progress with inductive game theory.

4.2. Collecting Information at Marked Pieces and Partial UT-domains

Kaneko-Kline [9] gave an informal theory suggesting a unilateral trial domain of accumulation. Here, we focus on partial unilateral trial domains which are suggested by the informal theory given above.

Consider the objective situation $(\Gamma^o, \mathfrak{m}^o) = (\Gamma^o, (\mathfrak{m}_1^{MR}, ..., \mathfrak{m}_n^{MR}))$, where the players follow some given regular behavior patterns $\sigma^o = (\sigma_1^o, ..., \sigma_n^o)$. First, we assume the following very basic condition on the set of marked pieces M^o and $\sigma^o = (\sigma_1^o, ..., \sigma_n^o)$:

R0(Regular Marked Endnode): $\lambda^o(x) \in M^o$ for the endnode x induced by $\sigma^o = (\sigma_1^o, ..., \sigma_n^o)$.

Recall that any profile of behavior patterns $\sigma = (\sigma_1, ..., \sigma_n)$ determines a unique path $x_1, ..., x_{m+1}$ to an endnode x_{m+1}. We define the *marked cane domain* for player i relative to σ as

$$D_i^{Mc}(\sigma) = \{x_1, ..., x_{m+1}\} \cap X_i^{oM}, \tag{4.1}$$

where $X_i^{oM} = \{x \in X^o : i \in \pi^o(x) \text{ and } \lambda^o(x) \in M^o\}$. This is the sequence of marked nodes for player i on the path determined by the behavior profile σ. A role of marked information pieces is to allow player i to make trial deviations at those pieces. To describe this role, we define a *trial strategy set* Σ_i to be a subset of the set Σ_i^o of all strategies for i satisfying the following three conditions:

T0(Regular Behavior): $\sigma_i^o \in \Sigma_i$;

T1(Deviations at Marked Pieces Only): if $\sigma_i^o(x) \neq \sigma_i(x)$ for some $\sigma_i \in \Sigma_i$, then $\lambda^o(x)$ is marked;

T2(Closed with respect to Marking): Let x be an endnode, i.e., $x \in X^{oE}$. If $y <_{\sigma_i(y)} x$ for some marked node $y \in X_i^{oM}$ and some $\sigma_i \in \Sigma_i$, then x is also marked.

The set Σ_i expresses permissible trial deviations for player i. Condition T0 is a basic requirement. Condition T1 means that a trial deviation can be made only at a marked piece. Condition T2 states that if an endnode follows a marked node by some trial strategy in Σ_i, then that endnode is marked by player i.

A trivial example for a trial strategy set Σ_i is the singleton set $\{\sigma_i^o\}$, which is the smallest one and gives no freedom for player i to make trials. The largest one is given as the set Σ_i^{oM} of all strategies σ_i satisfying, for all $x \in X_i^o$,

$$\sigma_i(x) \in A_x^o \text{ if } \lambda^o(x) \in M^o \text{ and } \sigma_i(x) = \sigma_i^o(x) \text{ if } \lambda^o(x) \notin M^o. \tag{4.2}$$

In this case, player i has the freedom to choose any action at a marked node, but keeps to his regular action at unmarked nodes. In general, a set Σ_i satisfying T0-T2 is a subset of Σ_i^{oM}.

We can now present the domain for accumulating memories in this paper. Consider a pair (σ^o, Σ_i), where $\sigma^o = (\sigma_1^o, ..., \sigma_n^o)$ is the regular behavior profile for $(\Gamma^o, \mathfrak{m}^o)$ and Σ_i is a trial strategy set for player i. Player i's *partial UT-domain for* (σ^o, Σ_i) is given as:

$$D_i^U(\sigma^o, \Sigma_i) = \bigcup_{\sigma_i \in \Sigma_i} D_i^{Mc}(\sigma_{-i}^o, \sigma_i). \tag{4.3}$$

When Σ_i is the smallest set $\{\sigma_i^o\}$, $D_i^{Mc}(\sigma_{-i}^o, \Sigma_i)$ coincides with the cane domain $D_i^{Mc}(\sigma^o)$. When $\Sigma_i = \Sigma_i^{oM}$, the domain of accumulation $D_i^U(\sigma^o, \Sigma_i^{oM})$ is the set of marked nodes that are reachable by unilateral deviations at marked nodes. The UT-domain discussed in Kaneko-Kline [9] is the special case where $M_i^o = W_i^o$ and $\Sigma_i = \Sigma_i^o$.

The domain $D_i^U(\sigma^o, \Sigma_i)$ is the set of nodes reachable from the regular behaviors and trials. Since nodes are hypothetical constructs from the viewpoint of the outside observer, we need to translate this set in terms the constructs tangible for player i himself. That is, we translate $D_i^U(\sigma^o, \Sigma_i)$ to the corresponding memory kit. Let us denote $D_i^U(\sigma^o, \Sigma_i)$ by D_i for simplicity. We say that T_{D_i} is a *memory kit* for domain D_i iff

$$T_{D_i} = \bigcup_{x \in D_i} \mathfrak{m}_i^o(x); \tag{4.4}$$

The kit T_{D_i} represents the entire set of memory threads remaining in the mind of player i. In Section 5, it will be used to construct a skeleton for a personal view.

Coordination Game with Precision (2): Let us see the concepts developed above in the coordination game with precision (Γ_C^5, m^o). To present marking and testing in a systematic manner, we give the following constraints on Σ_i and M^o. Let p and r positive integers, where p is the number of marked nodes in each path and r is the number of tested action at each marked node. Now, let Σ_i^{pr} be a set of information-independent strategies in Σ_i^{oM} satisfying $\sigma_i^o \in \Sigma_i^{pr}$ and

(1-M): for each path $\{x_1, ..., x_{11}\}$ to endnode x_{11}, $\left|\{x \in \{x_1, ..., x_{11}\} : x \in X_i^{oM}\}\right| = p$;

(2-M): $|\{\sigma_i(x) : \sigma_i \in \Sigma_i^{pr}\}| = r$ for all decision nodes $x \in X_i^{oM}$.

Note that by the assumption of information-independence, each strategy in Σ_i^{pr} is expressed as a sequence of actions such as $((a_{i1}, b_{i1}), ..., (a_{i5}, b_{i5}))$. In (Γ_C^5, m^o), p can be $1, ..., 6$, and r can be $1, ..., 6$. When $p = 1$, player i makes no trials. This case is too trivial.

Consider one case with $p = 2$ and $r = 2$ which corresponds to example given in the introduction. We assume that only player i's first decision nodes and the endnodes are marked. We also specify a regular pair of (information-independent) strategies:

$$((a_{11}^o, b_{11}^o), ..., (a_{15}^o, b_{15}^o)) = ((0, 2), (1, 1), (1, 2), (0, 3), (0, 1)) \tag{4.5}$$

$$((a_{21}^o, b_{21}^o), ..., (a_{25}^o, b_{25}^o)) = ((1, 3), (1, 1), (0, 2), (0, 2), (0, 3)) \tag{4.6}$$

Now, we consider player 1's test strategy set Σ_1^{pr} and the resulting memory kit. Suppose that Σ_1^{pr} consists of two (information-independent) strategies:

$$\Sigma_1^{pr} = \{((a_{11}, b_{11}^o), (a_{12}^o, b_{12}^o), ..., (a_{15}^o, b_{15}^o)),\ a_{11} \in \{0, 1\}\}. \tag{4.7}$$

That is, player 1 makes only one additional trial of deviation for the a-type action at round 1 and follows his regular behavior otherwise. In this case, the memory kit $T_{D_1} = \bigcup_{x \in D_1} \mathfrak{m}_1^o(x)$ of player 1 is given as

$$\{\langle\lambda^o(x_0)\rangle\} \cup \{\langle(\lambda^o(x_0),(a_{11},2),\lambda^o(z^1(a_{11})\rangle : a_{11} \in \{0,1\}\}, \tag{4.8}$$

where $z^1(0)$ and $z^1(1)$ are the endnodes induced by his a-type actions 0 and 1 at round 1, provided that otherwise the players follow the behaviors of (4.5) and (4.6). Thus, the memory kit T_{D_1} consists of only three memory threads.

Contrary to the tininess of (4.8), this set of threads could become gigantic when we adopt the full set W^o for M^o and the set of all strategies $\Sigma_i = \Sigma_i^o$. In this case, the trial deviations for player i are all sequences $((a_{11}, b_{11}), ..., (a_{15}, b_{15}))$. Hence, the cardinality of $D_1 = D_1^U(\sigma^o, \Sigma_1)$ is the sum of reachable nodes and is given as

$$1 + 6^1 + 6^2 + 6^3 + 6^4 + 6^5 = \frac{1}{5}(6^6 - 1) = 9331. \tag{4.9}$$

This is the same as the cardinality of the memory kit T_{D_1}.

5. Inductively Derived Views

Now, we are in a state to consider how player i constructs his personal view from his memory kit $T_{D_i} = \bigcup_{x \in D_i} \mathfrak{m}_i^o(x)$ given by (4.4). It will be shown that the definition of induction given in this paper is almost deterministic. The standard notion of induction involves bold and sophisticated reasoning from a finite number of experiences (memories) to a general rule (view), which suggests that there might be infinitely many candidates for a view. Our definition of the deterministic induction method becomes possible by the adoption of the (marked) exact-memory function for a player and the restricted domain of accumulation. This allows us to focus on the problem of small and partial views.

The memory kit T_{D_i} provides all the ingredients for a player's induction. Our definition of induction is as follows.

Definition 5.1 (Inductively Derived View). A pair $(\Gamma^i, \mathfrak{m}^i) = (((X^i, <^i), (\lambda^i, W^i), M^i, \{(\varphi_x^i, A_x^i)\}_{x \in X^i}, (\pi^i, N^i), h^i), \mathfrak{m}^i)$ is an *inductively derived view* (an *i.d.view* for short) from the memory kit T_{D_i} iff

P1(Construction of an Extensive Game): Γ^i is a fully marked 1-person extensive game with partial moves and $N^i = \{i\}$ satisfying:

(a)(Preservation of the Informational Structure): the set $\Theta(\Gamma^i)$ of histories in Γ^i coincides with T_{D_i};

(b)(Action Sets): for each node $x \in X^i$, $A_x^i = A^o_{\lambda^i(x)}$, i.e., the set A_x^i of available actions at each x node in Γ^i is given as the set $A^o_{\lambda^i(x)}$ of available actions at information piece $\lambda^i(x)$ in Γ^o;

(c)(Correct Payoffs): $h^i \cdot \lambda^i(x) = h_i^o \cdot \lambda^i(x)$ for each endnode x in Γ^i;

P2(Construction of a Memory Function): $\mathfrak{m}^i = \mathfrak{m}^{MR}$ is the marked exact perfect memory function for Γ^i.

Let us explain the above definition and give remarks on several points. Player i constructs Γ^i from the accumulated long-term memories T_{D_i}. First, this game is required to be fully marked, i.e., $M^i = W^i$. Since player i has constructed this game in his mind, he is conscious of every information piece in the game Γ^i. Second, this game is an extensive game with partial actions, i.e., only K43′ is assumed in order to accommodate the limited trials of a player.

Recall that the ingredients available to player i are given in the memory kit T_{D_i}. Player i uses some additional hypothetical nodes X^i to construct a tree with the binary relation $<^i$. Then he attaches an information piece $\lambda^i(x)$ from T_{D_i} to each node x and consequently all pieces in W^i come from T_{D_i}. By M1 in Section 3, the attached piece $\lambda^i(x)$ has the information about the available actions $A^o_{\lambda^i(x)}$. Condition P1a requires that the set $\Theta(\Gamma^i)$ of histories in Γ^icoincides with the kit T_{D_i}. Condition P1b requires the set A^i_x of available actions in Γ^i to be $A^o_{\lambda^i(x)}$. Note also that Γ^i is a 1-player game for player i himself, which is compatible with M2.

It follows from our assumptions on the memory function and domain of accumulation that the endnodes in X^i will always be assigned endpieces from W^o and the decision nodes in X^i will always be assigned decision pieces from W^o. Consequently, we can require, in P1c, that the payoff for $\lambda^i(x)$ in Γ^i is the payoff for $\lambda^i(x)$ in Γ^o. The availability of the payoff information comes from M3 - the payoff of player i is written on each endpiece.

Condition P2 simply requires the memory function $\mathfrak{m}^i = \mathfrak{m}^{MR}$ in Γ^i. Kaneko-Kline [9] gave a more general formulation of P2, but we do not need such a general formulation since we assume the exact perfect recall memory functions for the objective situation and the subjective description.

Now, consider the existence and uniqueness of an i.d.view. Both are answered in the affirmative in the following theorem.

Theorem 5.1 (Existence and Uniqueness up to Isomorphisms). Let $D_i = D_i^U(\sigma^o, \Sigma_i)$ be a partial UT-domain.

(1): There exists an inductively derived view $(\Gamma^i, \mathfrak{m}^i)$ from T_{D_i}.

(2): Let $(\Gamma, \mathfrak{m})$ and $(\Gamma', \mathfrak{m}')$ be two i.d.views of player i from T_{D_i}. The two views are isomorphic in the sense that there is a bijection $\psi : X \to X'$ such that for all $x, y \in X$,

(a) $\psi(x) <' \psi(y)$ if and only if $x < y$; (b) $\lambda(x) = \lambda'(\psi(x))$;

(c) $A_x = A'_{\psi(x)}$; (d) $\varphi_x(y) = \varphi'_{\psi(x)}(\psi(y))$;

(e) $h_i(x) = h'_i(\psi(x))$; (f) $\mathfrak{m}(x) = \mathfrak{m}'(\psi(x))$.

Proof.(1): Under the assumptions of this paper, the memory kit T_{D_i} has enough information to construct an i.d.view. First, we let $X^i = T_{D_i}$. Indeed, if a sequence $\langle (w_1, a_1), ..., (w_k, a_k), w_{k+1} \rangle$ is in T_{D_i}, any *initial segment* $\langle (w_1, a_1), ..., (w_l, a_l), w_{l+1} \rangle$ $(l < k)$ is also in T_{D_i}. For any two sequences $\langle \xi, v \rangle$ and $\langle \eta, w \rangle$ in T_{D_i} we write $\langle \xi, v \rangle <^i \langle \eta, w \rangle$ iff $\langle \xi, v \rangle$ is an initial segment of $\langle \eta, w \rangle$. Then, $(T_{D_i}, <^i)$ can be regarded as a tree satisfying K1. Next, we define W^i to be the set of information pieces that appear in T_{D_i}. Then, let $M^i = W^i$; the game is fully marked, let $N^i = \{i\}$; the game is a 1-person game involving only player i, and let $\mathfrak{m}^i = \mathfrak{m}^{MR}$ to satisfy P2.

The components $\lambda^i, \{A^i_x\}_{x \in X^i}$, and h^i are then simply derived from $X_{D_i} = T_{D_i}$ using M1 and M3. Specifically, for each $x = \langle (w_1, a_1), ..., (w_k, a_k), w_{k+1} \rangle$ in T_{D_i}, we let λ^i (x) $= w_{k+1}$, $A^i_x = A^o_{w_{k+1}}$, and the payoffs are defined by h^o_i.

The treatment of $\{\varphi^i_x\}_{x \in X^i}$ is a little more delicate, but under our assumptions it is uniquely defined. If $x = \langle (w_1, a_1), ..., (w_k, a_k), w_{k+1} \rangle$ is in T_{D_i} and it has no immediate successors, then φ^i_x is the empty function. Otherwise, $x = \langle (w_1, a_1), ..., (w_k, a_k), w_{k+1} \rangle$ has an immediate successor $y = \langle (w_1, a_1), ..., (w_k, a_k), (w_{k+1}, a_{k+1}), w_{k+2} \rangle$, and then $\varphi^i_x(y) = a_{k+1}$.

(2): Let $\theta(x)$ and $\theta'(x)$ be the objective histories up to nodes x in Γ and x' in Γ'. By P1a, $\Theta(\Gamma) = \Theta(\Gamma')$. Additionally, since Γ and Γ' are both rooted trees, θ is a bijection from X to $\Theta(\Gamma)$ and θ' is a bijection from X' to $\Theta(\Gamma')$. Hence, we can define $\psi(x) = \theta'^{-1} \circ \theta(x)$ for any node x in Γ. Thus, $\psi(x)$ is well defined and is a bijection from X to X'. This bijection satisfies (a) to (f). ■

The specific i.d.view $(\Gamma^i, \mathfrak{m}^i)$ given in the proof of Theorem 5.1.(1) is constructed directly from the memory kit T_{D_i} treating each memory thread in T_{D_i} as a node. For this reason, we call this i.d.view the *memory-thread representation*.

We can construct another i.d.view from D_i simply using D_i as the set of nodes in the personal view. We call this view the *original-node representation*. While this latter view is more convenient in the subsequent analysis, it is not natural from the viewpoint of player i since he observes only the information pieces, rather than the nodes in D_i. This view is obtained simply by restricting the objective extensive game $(\Gamma^o, \mathfrak{m}^o)$ to D_i. All the components of $(\Gamma^i, \mathfrak{m}^i)$, except $\{(\varphi^i_x, A^i_x)\}_{x \in X^i}$, are obtained from $(\Gamma^o, \mathfrak{m}^o)$ by restricting attention to $X^i = D_i$.

Formally, those components are defined as follows: $<^i = (<^o) \cap (D_i \times D_i)$, λ^i is the restriction of λ^o to D_i, $W^i = M^i = \{\lambda^i(x) : x \in D_i\}$, $\pi^i(x) = N^i = \{i\}$ for all $x \in D_i$ and $h^i(x) = h^o_i(x)$ for all $x \in D_i \cap X^{oE}$. The tree $(X^i, <^i)$ may not inherit the immediate successorship in $(X^o, <^o)$. Let $x \in X^i$ be the immediate successor $y \in X^i$ with respect to $<^i$. When it inherits the immediate successorship from Γ^o, we define $\varphi^i_x(y) = \varphi^o_x(y)$. If not, we find a node y^o in X^o so that it is the immediate successor of x and also a predecessor of y with respect of $<^o$. Then, we define $\varphi^i_x(y) = \varphi^o_x(x)$. Now, we have the original node representation $(\Gamma^i, \mathfrak{m}^i)$. We can verify that this is an i.d.view, too.

Theorem 5.1.(2) states that this original-node representation is isomorphic to the memory-thread representation given in the proof of Theorem 5.1.(1).

Coordination Game (Γ_C^5, m^o) **with Precision (3)**: Recall that we specified a regular pair of (information-independent) strategies in (4.5) and (4.6). These strategies give payoffs $(0.1017, 0.1017)$ to the players. Incidentally, the Nash equilibrium payoffs in the objective game $\Gamma^o = \Gamma_C^5$ are $(1.1121, 1.1121)$.

First, consider the size of the i.d.view when M^o is the full set W^o and Σ_1 is the set of all information independent strategies in Γ_C^5. In this case, as calculated in (4.9), the memory kit T_{D_1} has 9331 memory threads. Since any i.d.view is isomorphic to the memory-thread representation, any i.d.view has also 9331 nodes. In fact, we will argue in Section 8 that the number of endnodes is more convenient to consider. In this example, the number of endnodes of any i.d.view Γ^1 is $6^5 = 7,776$. This number is already much smaller than the number of endnodes $(6^5)^2 = 60,466,176$ of the objective situation (Γ^o, m^o).

Second, let us go to the case $(p, r) = (2, 2)$ considered in Section4 and the introduction. In that case, the memory kit T_{D_i} is given as (4.8), which consists of 3 memory threads. The inductively derived view $(\Gamma^1, \mathfrak{m}^1)$ is constructed from those 3 memory threads. It is extremely simple having only 2 branches and 2 endnodes, depicted in Fig. 1.2. In this view, one endnode gives the payoff 0.1017 and the other gives 1.1017.

The point of this example is to show that player 1 obtains a very small partial i.d.view from marking only a few seemingly important parts of the game and restricting possible actions. If marking is adequate, it gives almost the best payoff, i.e., 1.1017 relative to the Nash payoff 1.1121. However, it depends upon a choice of marking and possible actions. Maybe, some player only marks unimportant pieces, and he does not improve his payoff by making more trials. Additionally, testing the wrong actions might not help either. In this sense, we can regard the set of tested actions as a type of marking, where a player marks the actions to be tested that he regards as important.

6. Strategic Use of an Inductively Derived View

After player i derives his view $(\Gamma^i, \mathfrak{m}^i)$ from the memory kit T_{D_i}, he may think about an optimal strategy in that view, and he may also think about how to use it in the objective situation. The scenario is described in Fig.6.1, where the steps of finding an optimal strategy and bringing it back correspond to the lower part of the figure.

Let $(\Gamma^i, \mathfrak{m}^i)$ be an i.d.view from the memory kit T_{D_i} in the objective game $(\Gamma^o, \mathfrak{m}^o)$ with the regular behavior patterns $\sigma^o = (\sigma_1^o, ..., \sigma_n^o)$. To define a subjectively optimal behavior strategy, we consider a strategy τ_i in the view $(\Gamma^i, \mathfrak{m}^i)$. Each strategy τ_i in $(\Gamma^i, \mathfrak{m}^i)$ determines a unique path to an endnode x in $(\Gamma^i, \mathfrak{m}^i)$, and defines the endpiece $\lambda^i(x)$, which we denote by $\lambda^i(\tau_i)$.

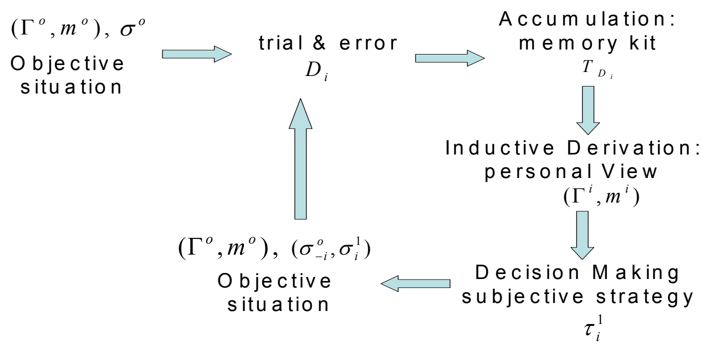

Figure 6.1:

Now, player i looks for a strategy τ_i^1 in $(\Gamma^i, \mathfrak{m}^i)$ that optimizes his subjective payoff h_i^i, i.e.,

$$h^i(\lambda^i(\tau_i^1)) \geq h^i(\lambda^i(\tau_i)) \text{ for all strategies } \tau_i \text{ in } (\Gamma^i, \mathfrak{m}^i). \tag{6.1}$$

Since $(\Gamma^i, \mathfrak{m}^i)$ is a finite 1-person game, he has an optimal strategy τ_i^1. Now, we meet four questions:

Q61: how does player i bring τ_i^1 back to the objective situation $(\Gamma^o, \mathfrak{m}^o)$?

Q62: what relationships does the optimal strategy have to the regular behavior σ_i^o?

Q63: what is the conceptual status of the optimal strategy relative to Nash equilibrium?

Q64: what is the size of the above optimization problem (6.1)?

We will discuss these questions in the remainder of this paper. In this section, we consider only Q61 and Q62. The question Q63 will be discussed in Section 7 and Q64 will be discussed in Section 8.

Let us start with Q61. Observe that player i first looks for an optimal plan τ_i^1 in the i.d.view $(\Gamma^i, \mathfrak{m}^i)$. After he finds one, his next problem is how to bring τ_i^1 back to the objective situation $(\Gamma^o, \mathfrak{m}^o)$. The strategy τ_i^1 is a plan in $(\Gamma^i, \mathfrak{m}^i)$, but the player would like to use it to modify his regular behavior σ_i^o by τ_i^1.

Since $(\Gamma^i, \mathfrak{m}^i)$ may have different nodes to those in $(\Gamma^o, \mathfrak{m}^o)$, it may appear that the player would not be able to apply his strategy τ_i^1 easily to $(\Gamma^o, \mathfrak{m}^o)$. However, strategies are effectively defined over the set of memories by (3.6), so this is not a problem. All memory threads in $(\Gamma^i, \mathfrak{m}^i)$ are those in T_{D_i}, which are memory threads for player i in

$(\Gamma^o, \mathfrak{m}^o)$. This fact enables us to make the following definition: for all $x \in X_i^o$,

$$\sigma_i^1(x) = \begin{cases} \tau_i^1(y) & \text{if } \mathfrak{m}_i^o(x) = \mathfrak{m}^i(y) \text{ and } y \in X^i; \\ \sigma_i^o(x) & \text{otherwise.} \end{cases} \tag{6.2}$$

Note that for each $x \in X_i^o$, the node y in the above if-clause is uniquely determined by P1a and $\mathfrak{m}^i = \mathfrak{m}^{MR}$. This definition states that player i follows τ_i^1 in $(\Gamma^o, \mathfrak{m}^o)$ whenever a local memory described in his personal view $(\Gamma^i, \mathfrak{m}^i)$ occurs, and otherwise he resorts to σ_i^o. This is the answer to Q61.

Now, we suppose that only player i adjusts his behavior in the above manner. The profile of regular behavior patterns $\sigma^o = (\sigma_1^o, ..., \sigma_n^o)$ changes into $(\sigma_{-i}^o, \sigma_i^1)$ and we have the following observation, which is stated as a theorem for a purpose of comparisons with later theorems. This is our answer to Q62.

Theorem 6.1 (Optimality brought back). Let $(\Gamma^i, \mathfrak{m}^i)$ be an i.d.view from the kit T_{D_i} with the trial strategy set Σ_i. Then, a strategy τ_i^1 in $(\Gamma^i, \mathfrak{m}^i)$ is optimal in the sense of (6.1) if and only if σ_i^1 is optimal in Σ_i relative to σ_{-i}^o in the objective game $(\Gamma^o, \mathfrak{m}^o)$, i.e.,

$$h_i^o(\lambda^o(\sigma_i^1, \sigma_{-i}^o)) \geq h_i^o(\lambda^o(\sigma_i, \sigma_{-i}^o)) \text{ for all } \sigma_i \in \Sigma_i. \tag{6.3}$$

Proof. By Theorem 5.1, all i.d.views are game theoretically isomorphic. Hence, we can assume that $(\Gamma^i, \mathfrak{m}^i)$ is the original node representation defined in the end of Section 5.1. Also, player i's regular behavior is fixed for the unmarked nodes, and the other players' regular behaviors σ_{-i}^o are entirely fixed. Hence, the set of nodes reachable by some $\sigma_i \in \Sigma_i$ are relevant in (6.3) and are the same as $D_i = D_i^U(\sigma^o, \Sigma_i) = \bigcup_{\sigma_i \in \Sigma_i} D_i^{Mc}(\sigma_{-i}^o, \sigma_i)$. Now, consider any $\sigma_i \in \Sigma_i$. We denote the restriction of σ_i to D_i by τ_i. Then, the endnode defined by $(\sigma_i, \sigma_{-i}^o)$ in $(\Gamma^o, \mathfrak{m}^o)$ is the same as the endnode defined by τ_i in $(\Gamma^i, \mathfrak{m}^i)$. Hence, we have $h_i^o(\lambda^o(\sigma_i, \sigma_{-i}^o)) = h^i(\lambda^i(\tau_i))$. Thus, (6.3) is equivalent to (6.1). ■

This theorem states that an i.d.view helps player i to choose an optimal behavior in the objective situation. This objective optimality is relative only to the trial strategy set Σ_i. This theorem is not the end of inductive game theory. Once player i brings back his optimal strategy σ_i^1 to the objective situation, he starts playing σ_i^1 as his regular behavior. Here we see that the regular behavior can now be regarded as optimal in a restricted sense. The situation has returned to the top row of Fig.6.1, and his new regular behavior may generate new experiences for other players.

$$\cdots \longrightarrow \sigma^o \longrightarrow (\sigma_i^1, \sigma_{-i}^o) \longrightarrow (\sigma_j^1, \sigma_i^1, \sigma_{-ij}^o) \longrightarrow \cdots \tag{6.4}$$

If some other player j constructs a new i.d.view and finds an optimal strategy τ_j^o, then the process continues. When he brings σ_j^1 back to the objective situation, the regular behavior is changed again. In this case, the behavior σ_i^1 of player i may no longer be optimal objectively in the sense of (6.3). We presume that this process of analysis does not proceed period to period, but each step needs several periods. In this way, the i.d.views and behavior of the players are emerging. In the next section we look at stationary outcomes of this process.

7. Partial Nash Equilibrium and the Restricted Objective Game

As the players are moving through the steps in the diagram of Fig.6.1, the i.d.views and regular behavior patterns might be changing. This process may continue for a long time. Once the behavior patterns become "stationary", the state can be regarded as a Nash equilibrium. By the word "stationary" we mean that each player reaches an i.d.view from his test set where he does not modify his behavior anymore. This stationary regular behavior patterns a Nash equilibrium, which is seen from Theorem 6.1 in the game having the restricted domain experienced by the players. This result is stated as the first theorem of this section. We then rewrite this theorem as Theorem 7.2, which is tailored to suit our exploration into the complexity of an i.d.view as well as of a Nash equilibrium. This will be the subject of Section 8.

Theorem 7.1 (Decomposition of Nash Equilibrium). Let $(\Gamma^i, \mathfrak{m}^i)$ be an i.d.view from the kit T_{D_i} with the trial strategy set Σ_i for all $i \in N^o$. Then, the profile $\sigma^o = (\sigma_1^o, ..., \sigma_n^o)$ of regular behaviors is a Nash equilibrium in the strategy sets $\Sigma_1 \times \cdots \times \Sigma_n$ in $(\Gamma^o, \mathfrak{m}^o)$, i.e., it satisfies (6.3) for each $i \in N^o$, if and only if for all $i \in N^o$, the restricted strategy τ_i^1 in $(\Gamma^i, \mathfrak{m}^i)$ defined by

$$\tau_i^1(x) = \sigma_i^o(y) \text{ if } \mathfrak{m}^i(x) = \mathfrak{m}_i^o(y) \tag{7.1}$$

is optimal in the sense of (6.1) in $(\Gamma^i, \mathfrak{m}^i)$.

We do not give a proof of this result since it is a direct consequence of Theorem 6.1.

Observe that the restricted strategy τ_i^1 is well-defined by the conditions for $(\Gamma^i, \mathfrak{m}^i)$. This theorem is a variant of the characterization concept of Nash equilibrium in Kaneko-Kline [9], except that it takes a trial strategy set Σ_i into account. The present point is the equivalence result in the restricted sense.

It would be easier to restate Theorem 7.1 from the viewpoint of the objective-node representation of an i.d.view. Recall that the node representation $(\Gamma^i, \mathfrak{m}^i)$ is obtained by restricting the objective game $(\Gamma^o, \mathfrak{m}^o)$ to the partial UT-domain $D_i^U(\sigma^o, \Sigma_i) = \bigcup_{\sigma_i \in \Sigma_i} D_i^{Mc}(\sigma_{-i}^o, \sigma_i)$. Consider the union of those $D_i^U(\sigma^o, \Sigma_i)$'s:

$$D^U = \bigcup_{i \in N^o} D_i^U(\sigma^o, \Sigma_i). \tag{7.2}$$

Now, we have the restriction of $(\Gamma^o, \mathfrak{m}^o)$ to the union D^U, which we will denote by $(\Gamma^{or}, \mathfrak{m}^{or})$. This game $(\Gamma^{or}, \mathfrak{m}^{or})$ is an extensive game with partial actions. We have the restriction σ^{or} of the profile σ^o to D^U. Then Theorem 7.1 is restated as follows:

Theorem 7.2 (Nash Equilibrium in the Restricted Game). Let $(\Gamma^i, \mathfrak{m}^i)$ be an i.d.view from the kit T_{D_i} with the trial strategy set Σ_i for all $i \in N^o$. Then, the restriction σ^{or} of $\sigma^o = (\sigma_1^o, ..., \sigma_n^o)$ to D^U is a Nash equilibrium in the game $(\Gamma^{or}, \mathfrak{m}^{or})$ if and only if for all $i \in N^o$, the restricted strategy τ_i^1 defined by (7.1) is optimal in the sense of (6.3) in $(\Gamma^i, \mathfrak{m}^i)$.

Proof. The domain D^U is simply the union of $D_i^U(\sigma^o, \Sigma_i)$'s. Each $D_i^U(\sigma^o, \Sigma_i)$ is the set of marked nodes obtained by trials in Σ_i for player i from σ^{or}. The utility maximization in the sense of (6.3) in $(\Gamma^i, \mathfrak{m}^i)$ is equivalent to the utility maximization of player i in the definition of Nash equilibrium for player i. Hence, when we require the optimality in sense of (6.3) in $(\Gamma^i, \mathfrak{m}^i)$ for all players i, $\sigma^{or} = (\sigma_1^{or}, ..., \sigma_n^{or})$ is a Nash equilibrium in the restricted game $(\Gamma^{or}, \mathfrak{m}^{or})$, and *vice versa.* ■

By this theorem, we can reduce the problem of evaluating a partial Nash equilibrium into the size of $(\Gamma^{or}, \mathfrak{m}^{or})$. We will discuss the size of this game and other games in Section 8.

8. Sizes of Inductively derived Views and Restricted Games

In Section 5.2, we saw how an i.d.view is typically very small relative to the objective game. There, we employed the definition of the size of the view as the number of endnodes - - the width of the tree. In fact, this definition reflects the complexity of the optimization problem for a player with a given i.d.view. On the other hand, since the number of all nodes in the i.d.view is exactly the same as the number of memory threads in the memory kit T_{D_i}, an alternative definition of the size of the i.d.view is the number of all nodes in the view. In what follows, we argue that the number of endnodes is an effective measure of the size of a view.

8.1. Sizes of Trees

Since we are interested in measuring the "complexity" of an i.d.view and/or decision making, the reader might recall the computational complexity in logic and computer science. The concept is to measure the size of a Turing machine to compute a given mathematical problem (of instances) (see Garey-Johnson [4] for general discussions of this issue, and see McLennan-Tourky [15] and its references for game theoretical applications). This notion of computational complexity does not distinguish between two problem instances in a meaningful manner. Our target, on the other hand, is to consider the "size" of an instance of an i.d.view or the game. An idea similar to ours was discussed in Kaneko-Suzuki [11] to measure the necessary size of inferences in epistemic logic.

First, we show the following lemma, which was given in a context of epistemic logic in Kaneko-Suzuki [11].

Lemma 8.1 (Kaneko-Suzuki [11]). Let T be a finite tree with the nonterminal nodes $1, ..., k$ with m_t branches at each nonterminal node t.

(1): The total number of nodes is given as

$$\sum_{t=1}^{k} m_t + 1. \tag{8.1}$$

(2): Let $n_1, ..., n_\ell$ be the non-end nodes with $m_{n_t} > 1$ for all $t = 1, ..., \ell$. Then the number of endnodes of T is given as

$$\sum_{t=1}^{\ell} m_{n_t} - (\ell - 1). \tag{8.2}$$

Under the assumption that each decision node has at least two branches, the following theorem gives the precise lower and upper bounds of the total number of nodes in a tree, which are characterized by the number of endnodes. This result is interpreted as meaning that the number of endnodes $\eta(T)$ reflects well the total number of nodes $\tau(T)$.

Theorem 8.2 (Lower and Upper Bounds for the Total Number of Nodes). Let T be a finite tree where each decision node has at least 2 branches. Let $\eta(T)$ be the number of endnodes in T, and let $\tau(T)$ be the number of all nodes in T. Then we have

$$\eta(T) + 1 \leq \tau(T) \leq 2\eta(T) - 1. \tag{8.3}$$

Furthermore, these bounds are taken by some trees T_1 and T_2 with at least 2 branches such that $\tau(T_1) = \tau(T_2) = \tau(T)$.

Proof. Consider an arbitrary tree T' having the number of endnodes $\eta(T)$. Then, $\eta(T')$ is given as the formula of (8.3), i.e., $\sum_{t=1}^{k} m_t + 1 - k$. Now, keeping $\sum_{t=1}^{k} m_t + 1 - k$ as constant $\eta(T)$, we change k, i.e., T' is changed by k. Observe that $\sum_{t=1}^{k} m_t + 1$ is minimized at $k = 1$. We can find a tree with $k = 1$, and denote this tree by T_1. Then it holds that:

$$\eta(T) + 1 = \eta(T_1) + 1 = \tau(T_1).$$

Again, keeping $\sum_{t=1}^{k} m_t + 1 - k$ as constant $\eta(T)$, we change k. Observe that $\sum_{t=1}^{k} m_k + 1$ is maximized, when m_t's are all 2. We find a tree T_2 so that $\eta(T_2) = \eta(T)$ and $k = \eta(T) - 1$. Then, $\tau(T_2) = \sum_{t=1}^{\eta(T)-1} m_t + 1 = 2(\eta(T) - 1) + 1 = 2\eta(T) - 1$. In Fig.8.1, examples of T_1 and T_2 are given for the number of endnodes 4. ■

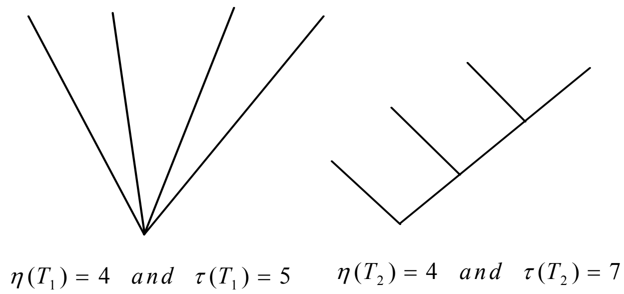

Figure 8.1:

The point of Theorem 8.2 is that the total number of endnodes $\eta(T)$ is not very different from the total number of nodes $\tau(T)$. As a measure to captured rough magnitudes of trees, we can employ either $\eta(T)$ or $\tau(T)$. Hence, even the number of memory threads in T_{D_i} is reflected by the number of endnodes of an i.d.view Γ^i. For calculation convenience, we have used the total number of endnodes $\eta(T)$.

The above theorem is stated under the assumption that each decision node of those trees has at least two actions. When a tree T has s nodes with single actions, (8.3) is modified into

$$\eta(T) + 1 + s \leq \tau(T) \leq 2\eta(T) - 1 + s, \tag{8.4}$$

since an addition of such a node increases the total number $\tau(T)$ of nodes by one. It is natural to assume that at each marked piece, a player makes a new trial. Then, s becomes zero. In general, s would be a small number in our context.

8.2. Sizes of the Optimization Problem using an I.D.View

Here we consider the question Q64 of Section 6 which is about the size of the optimization problem of a player. Since $(\Gamma^i, \mathfrak{m}^i)$ is a 1-person game with a tree structure, a strategy is a complete plan of contingent actions and is already quite complicated. Looking at the optimization problem (6.1) carefully, however, we find that a relevant variable in the optimization problem is an endpiece. Under the assumption that the pieces attached to the endnodes are all distinct, we can focus on endnodes rather than endpieces. Suppose also that an optimal endpiece is chosen while player i himself does not yet know its optimality. The verification of its optimality requires the player to compare its payoff with the payoffs from the other endnodes. The number of these verifications is $\eta(\Gamma^i) - 1$.

Of course, this is the most fortunate case for finding an optimal endnode. Thus, the number of endnodes is one measure of the size of the optimization problem (6.1).

We can think about this argument more formally using a certain game theoretical concept. We denote the realization equivalence relation over the set of strategies in $(\Gamma^i, \mathfrak{m}^i)$ by $\sim$, i.e., $\tau_i' \sim \tau_i''$ iff $\lambda^i(\tau_i') = \lambda^i(\tau_i'')$. Recall that $\lambda^i(\tau_i)$ is the endpiece induced by τ_i in $(\Gamma^i, \mathfrak{m}^i)$. Each strategy τ_i' defined an equivalence class $[\tau_i'] = \{\tau_i'' : \tau_i' \sim \tau_i''\}$. Since each strategy is characterized by the induced endpiece, the number of equivalence classes is the same as the number of endpieces in Γ^i. Thus, one possible measure of the size of the optimization problem (6.1) is the number of pieces of Γ^i minus 1. Under the assumption that all endpieces are distinct, it becomes $\eta(\Gamma^i) - 1$.

Although the number of endnodes in the i.d.view can be regarded as representing the size of the optimization problem, this optimization process may not be easily separated from the inductive processes. Thus, it is not a good idea to take the number $\eta(\Gamma^i) - 1$ as the exact number for optimization. Rather, we should take the number of endnodes as a rough measure representing the size of the entire inductive processes including optimization. When the value of this measure is large, we have the implication that the inductive derivation as well as its use for optimization could be difficult. Small sizes are required for individual player to be able to make a successful induction and use the result effectively.

Coordination Game (Γ_C^5, m^o) with Precision (4): Now, let us return to the coordination game Γ_C^5 with precision, Consider the size of the i.d.view when $M^o = W^o$ and $\Sigma_i = \Sigma_i^o$. In this case, the number of nodes is 9331, and the number of endnodes is $6^5 = 7,776$. Literally speaking, the optimization problem (6.1) needs at least $6^5 - 1$ comparisons to choose an optimal strategy. In our ordinary life, this number of comparisons is enormously large and, perhaps, exceeds our individual capacities. Nevertheless, the situation we are considering is a rather common one that we may face often, and that we manage to deal with somehow.

One way we manage to deal with such situations is to simplify them by marking and limiting our tests. As we saw, the number of endnodes to consider becomes suddenly very small when we give restrictions on marked pieces and tested actions.

8.3. Relative Sizes of I.D.Views to the Restricted Games and Original Game

Using the measure η, let us consider the relative sizes of an i.d.view to the original game Γ^o and/or to the restricted game Γ^{or} defined in Section 7. First, we show that the restricted game Γ^{or} is obtained by summing up individual i.d.views $\{(\Gamma^i, \mathfrak{m}^i)\}_{i \in N^o}$.

Theorem 8.3 (Size of the Restricted Game). The number of endnodes $\eta(\Gamma^{or})$ of the restricted game Γ^{or} is given as

$$\eta(\Gamma^{or}) = \sum_{i \in N^o} (\eta(\Gamma^i) - 1) + 1 = \sum_{i \in N^o} (\eta(D_i^U) - 1) + 1. \tag{8.5}$$

Proof. Since Γ^{or} is obtained by restricting Γ^o to $D^U = \bigcup_{i\in N^o} D_i^U$, it suffices to evaluate the size of $D^U = \bigcup_{i\in N^o} D_i^U$. Since each Γ^i is isomorphic to D_i^U, we have the second equality. Hence, we should show that the number of endnodes $D^U = \bigcup_{i\in N^o} D_i^U$ is given as in $\sum_{i\in N^o}(\eta(D_i^U) - 1) + 1$. The endnode induced by the regular behavior $\sigma^o = (\sigma_1^o, ..., \sigma_n^o)$ is commonly included in D_i^U and D_j^U $(i \neq j)$. Any other endnode in D_i^U is specific to a deviation by player i. Hence, the unique endpoint is commonly included in $D_1^U, ..., D_n^U$. Thus, we have $\eta(D^U) = \sum_{i\in N^o}(\eta(D_i^U) - 1) + 1$. ■

Theorem 8.3 holds for any marked set M^o and trial strategy set Σ_i, and states that $\eta(\Gamma^{or})$ is almost the sum of the sizes of $\eta(\Gamma^i)$'s. On the other hand, $\eta(\Gamma^o)$ is fixed and independent of M^o and the trial strategy set Σ_i. Typically, $\eta(\Gamma^o)$ is much greater than $\eta(\Gamma^{or})$. The maximum number for $\eta(\Gamma^{or})$ is obtained by the fully marked case $M^o = W^o$ where Σ_i is the maximum set Σ_i^o for all i. Even in this case, $\eta(\Gamma^{or})$ may be far smaller than $\eta(\Gamma^o)$. One example was discussed in Section 5.2, where $\eta(\Gamma^o) = 6^{5\times 2}$ and $\eta(\Gamma^{or}) = 2 \times 6^5 - 1$. We extend this calculation result in the following theorem.

Theorem 8.4 (Sizes of the Objective Game). Assume that $\Gamma^o = \Gamma^o(\ell)$ is a layered game with ℓ decision layers $(\ell \geq 1)$, i.e., in every path to an endnode, the players move in the same ordering $(i_1, ..., i_\ell)$, and the action set of player i_k at the k-th layer $(k = 1, ..., \ell)$ is independent of the history. Let $\Gamma^o(\ell)$ be fully marked and fully tested, i.e., $W^o = M^o$,and $\Sigma_i = \Sigma_i^o$ for all $i \in N^o$. Then number of endnodes $\eta(\Gamma^o)$ is given as

$$\eta(\Gamma^o(\ell)) = \prod_{i\in N^o} \eta(\Gamma^i). \tag{8.6}$$

Proof. We prove the assertion by induction over ℓ.

In the induction base case of $\ell = 1$, we have $\eta(\Gamma^o(1)) = \eta(\Gamma^{i_1})$. Also, any player $i \neq i_1$ experiences only the regular endnode, and so view consists of only one node. Hence, $\eta(\Gamma^o(1)) = \prod_{i\in N^o} \eta(\Gamma^i(1))$.

Now, suppose the induction hypothesis that (8.6) holds up to a natural number ℓ. Then, we consider the case of $\ell + 1$. This game is denoted by $\Gamma^o(\ell + 1)$. We ignore payoffs since we consider the sizes of trees only. Then, by the inductive hypothesis, $\eta(\Gamma^o(\ell)) = \prod_{i\in N^o} \eta(\Gamma^i(\ell))$.

First, consider the case where $i_{\ell+1}$ differs from any of $i_1, ..., i_\ell$. Then, $\eta(\Gamma^{i_{\ell+1}}(\ell + 1))$ is the number of actions for player $i_{\ell+1}$. Also, $\eta(\Gamma^i(\ell)) = \eta(\Gamma^i(\ell+1))$ for all $i \neq i_{\ell+1}$, and $\eta(\Gamma^{i_{\ell+1}}(\ell)) = 1$. Since the number $\eta(\Gamma^o(\ell+1))$ is obtained by $\eta(\Gamma^o(\ell)) \times \eta(\Gamma^{i_{\ell+1}}(\ell+1))$, we have, using the induction hypothesis for ℓ,

$$\begin{aligned} \eta(\Gamma^o(\ell + 1)) &= \eta(\Gamma^o(\ell)) \times \eta(\Gamma^{i_{\ell+1}}(\ell + 1)) \\ &= \prod_i \eta(\Gamma^i(\ell)) \times \eta(\Gamma^{i_{\ell+1}}(\ell + 1)) = \prod_{i\in N^o} \eta(\Gamma^i(\ell + 1)). \end{aligned}$$

Second, consider the case where $i_{\ell+1} = i_t$. Then, the above argument should be modified slightly for player $i_{\ell+1}$. We denote the number actions at decision $\ell + 1$ by $|A_{\ell+1}|$. Then, $\eta(\Gamma^{i_{\ell+1}}(\ell+1)) = \eta(\Gamma^{i_{\ell+1}}(\ell)) \times |A_{\ell+1}|$, and $\eta(\Gamma^i(\ell+1)) = \eta(\Gamma^i(\ell))$ for all $i \neq i_{\ell+1}$. Hence, we have, using the induction hypothesis for ℓ,

$$\begin{aligned}\eta(\Gamma^o(\ell+1)) &= \eta(\Gamma^o(\ell)) \times |A_{\ell+1}| = \prod_i \eta(\Gamma^i(\ell)) \times |A_{\ell+1}| \\ &= \prod_{i \neq i_{\ell+1}} \eta(\Gamma^i(\ell)) \times (\eta(\Gamma^{i_{\ell+1}}(\ell)) \times |A_{\ell+1}|) = \prod_{i \in N^o} \eta(\Gamma^i(\ell+1)).\end{aligned}$$

■

9. Conclusions

We have focussed on the small and partial views that players with limited cognitive abilities might derive from playing large and complicated games. This analysis follows the scenario of inductive game theory spelled out in Kaneko-Kline [9]. That scenario gave experiential foundations for noncooperative game theory, particularly, extensive game theory, but in [9], potential difficulties and obstacles are analyzed. In this paper, we avoided obstacles by focusing on the (marked) exact perfect recall memory function $\mathfrak{m}_i^{MR}$ and on the (marked) unilateral domain for trial and error. The introduction of "marked pieces" and "tested actions" was essential for making further progress in large and complicated games. We have found a path to allow players to apply the standard tools of game theory to their small and partial views.

The introduction of "marked" and "tested actions" has removed some theoretical difficulties, particularly, when an extensive game theory is used in inductive game theory. Without these new notions, the number of necessary trials might become quickly unmanageable as the objective game situation gets larger, which was shown by using the example in Section 5.2. In such a large game, the required periods for accumulating a sufficient number of experiences would be too long. Another difficulty is that even if a player has accumulated sufficient experiences, the number of ingredients (memory threads) could be too large to cook (construct) a subjective view. Finally, even if such a complicated view is successfully derived, it might be unmanageable from the viewpoint of decision making as there would be too many alternatives to check.

In Section 8, we introduced a measure for evaluating the sizes of i.d.views, the restricted game and the objective game. According to this measure, we could show how i.d.views may increase in size quite quickly and become unmanageable even in situations that are regarded as simple and straightforward in standard game theory. We argued in favour of the reduction of the objective situation into a small view by the introduction of marked pieces and tested actions from a player's viewpoint. In

Section 7, we considered this type of reduction from the objective point of view. In fact, this procedure has some similarity to practices by game theorists and economists who construct small models by choosing seemingly important players, information and actions from a large socioeconomic situation. If one treats the original large situation directly, it would be impossible to analyze.

It may be asked how marked pieces and tested actions are chosen as important variables for a player. So far, the choice of those variables is still exogenous in our theory. Recall that it is our starting assumption that a player has no *a priori* knowledge of the structure of the objective situation. Under this assumption, the player has no *a priori* criteria for the choices of marked pieces and tested actions. This is similar to a scientist in the beginning of his research. An accumulation of experiences will help, and communication with other researchers may help more. Thus, some criteria for the choice of marked pieces and tested actions may be emerging by continuing trial and error, and by communicating with other players.

In this paper, we have focused on a personal view which is a 1-person game. We could extend this to a many player game where each player observes the behavior of other players and treat them as following only their regular behavior patterns. Then we would get essentially the same results given in the present paper. However, an interesting development happens when a player starts to think about the decision making of other players. Then, he sees the game as a social situation and might ask why the other players behave the way they do and how he and they might improve on their present social outcomes.

For this extension, there are several ways to proceed. One way for a player to learn about the motivations and behavior of other players is for him to step into the shoes of the other players by taking on their "roles". Then, he uses this extended set of memories to extend his i.d.view. This would give him a larger and more accurate view of the social situation. Another way to proceed is for the players to communicate their personal experiences to each other. For this to be successful, "marking" and small views become important as the objects for communication must be abstract enough to be understood by others. We believe that an analysis of communication and experience by "role-playing" has a great potential to extend the scope of our inductive game theory and we plan to undertake this analysis in a separate paper.

For those issues, the sociological notions of "*social roles and expectations*" (see Collins [3], Chap.7) may become important. In a social situation, a "player" is often identified to be a social role rather than a specific individual. For example, the department head is a social role in a university, which is taken by one person at a time, and that person is expected to behave in a certain manner. In this case, different people could take the same role in the same or similar situations. They may collect different experiences, *a fortiori*, different views of the situation. A former department head can also instruct the present head on how to behave.

In sum, we have used inductive game theory to show how players with limited cognitive abilities might successfully construct simplified views in the face of complicated social situations. This has been one step in the direction of tailoring game theory to address social and economic problems in a better way. We believe that this paper may open vast new fields for studying social interactions.

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