Fair waste pricing:

An axiomatic analysis to the NIMBY problem

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Abstract

A waste disposal facility has to be sited in one of several districts that produce different amounts of wastes. The construction cost of the facility depends on where it is sited. When a district accepts the facility, it bears a disutility. The problem here is to choose a siting district and to share the construction cost with considering fair compensation to the siting district. We provide an axiomatic framework to normatively analyze this problem and seek for desirable decision rules. A *fair pricing rule* is a rule that selects a district so as to minimize the social loss, puts a negative price to wastes according to the loss, and gives a full compensation to the siting district. We show that this rule is the unique rule that satisfies certain efficiency, fairness, and robustness to strategic transfers of wastes. We also establish the nearly robustness of this rule to misrepresentation of disutility information.

Keywords: Cost sharing, Fair compensation, NIMBY (Not in my backyard), Market design, Mechanism design, Degree of manipulability.

JEL codes: D61, D62, D74, Q51, C71, H23, H41.

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1 Introduction

Consider a situation where a waste disposal facility (or any locally undesirable land use facility) has to be sited in one of several districts that produce different amounts of wastes. The construction cost of the facility depends on where it is sited. When a district accepts the facility, it bears a disutility. The problem here is to choose a siting district and to share the construction cost with considering fair compensation to the siting district. Following the terminology in environmental studies, we call this problem the "NIMBY" (Not in my backyard) problem. [1] The purpose of this paper is to develop a framework that normatively analyzes the NIMBY problem and to find allocation rules that solve the problem.

The NIMBY problem shares some aspects of existing fair allocation problems. First, it is a problem of allocating one indivisible object (the facility) among districts when monetary transfers are possible (Tadenuma and Thomson, 1993; Sakai, 2007). [2] Second, it is a problem of sharing cost (Moulin and Shenker, 1992). [3] Third, it is a problem of choosing one public alternative (a district) with monetary transfers (Moulin, 1985a,b). Finally, it is a traditional Pigouvian problem of fair compensation to the victims from negative externalities. We shall develop a model so as to incorporate these aspects. In our model, each district is characterized by a triplet: the amount of wastes it produces; a disutility function; a cost function. There, a *rule* is a function that determines the siting district and appropriate monetary transfers among districts, given the profile of the triplets. Our approach is axiomatic: we first define normatively desirable properties of rules and then seek for rules satisfying them. [4]

Our main theorem recommends to put a price to the dispose of wastes, so that the full compensation for the disutility of the siting district is possible. This rule is called a *fair pricing rule*. Under this rule, a siting district is chosen so as to minimize the social loss (the sum of the construction cost and the disutility of the siting district); the social loss is divided by the amount of total wastes, and the value so obtained is the price of disposing wastes; each non-siting district pays the multiplication of the price and the amount of its wastes; the siting district also pays in the same manner but it also receives the full compensation for its disutility; overall, each district's final utility level is equal to the multiplication of the price and the amount of its wastes. All fair pricing rules are equivalent in welfare in that they always yield the same utility allocations. In this sense, this recommendation is unique. Any fair pricing rule always chooses an allocation that belongs to the core. It is monotonic to any improvement of environments and is robust to strategic transfers of wastes. Furthermore, any rule

[1] For surveys of various NIMBY problems, we refer to Brion (1991), Rabe (1994), and Lesbirel (1998).

[2] A survey is provided by Thomson (Ch.10, 2005).

[3] Moulin (2002) offers a survey.

[4] There are some studies of auctioning an undesirable facility (Kunreuther and Kleindorfer, 1986; Kunreuther, Kleindorfer, Knez, and Yaksick, 1987; Kleindorfer and Sertel, 1994; Minehart and Neeman, 2002). Our model is much more structured than theirs, in that we deal with not only disutility, but also construction cost and the amounts of wastes. Our model is rather similar to Moulin's (1985b) quasi-linear social choice model than those auction models.

satisfying these properties is necessarily a fair pricing rule. Since the calculation of the price is quite simple, this axiomatization gives us a practical way to solve the NIMBY problem.

Although a fair pricing rule is not perfectly robust to misrepresentation of disutility information, it is nearly robust in the following senses: any manipulated outcome still belongs to the core; the welfare condition under manipulation is close to the welfare condition under true information. This nearly robustness result much contrasts with various existing impossibility results on implementability conditions such as strategy-proofness or Nash implementability.

This paper is organized as follows. Section 2 introduces the model. Section 3 defines axioms of rules. Section 4 studies fair pricing rules. Section 5 discusses our modeling and future works. Section 6 concludes the discussion. All proofs are relegated to the Appendix.

2 The model

Let $N \equiv \{1, 2, \ldots, n\}$ be the set of *districts*. Each $i \in N$ needs a waste disposal facility (or simply, a facility) for the amount of wastes $w_i \geq 0$. Let $w \equiv (w_i)_{i \in N}$ be the profile of waste parameters. For each $S \subseteq N$, let $W_S \equiv \sum_{i \in S} w_i$. Also, let $W \equiv W_N$, i.e., $W \equiv \sum_{i \in N} w_i$. Since we are not interested in the case with no wastes, we assume that $W > 0$. Let $\mathcal{W} \equiv \{w \in \mathbb{R}^N_+ : W > 0\}$ be the set of waste distributions.

The construction cost of the facility at $i \in N$ that deals with the amount of wastes W is $c_i(W) \geq 0$. We assume that $c_i : \mathbb{R}_+ \to \mathbb{R}_+$ is a weakly concave and strongly increasing function such that $c_i(0) = 0$. [5] We call it a *cost function*. Let $\mathcal{C}$ be the set of cost functions.

When a waste disposal facility whose scale is to deal with W amount of wastes is sited at $i \in N$, i bears a disutility $v_i(W) \geq 0$; otherwise, i bears no disutility. [6] We assume that $v_i : \mathbb{R}_+ \to \mathbb{R}_+$ is a weakly concave and strongly increasing function such that $v_i(0) = 0$. [7] We call it a *disutility function*. Let $\mathcal{V}$ be the set of disutility functions. [8]

[5]Weak concavity means that the cost of constructing one big facility is weakly less than the cost of constructing some small facilities that together have the same disposal ability to the big facility. This fact is often pointed out in the literature of environmental policies. See, for example, "1997 New Guideline for Waste Disposal" by the Ministry of Health and Welfare in Japan (now called the Ministry of Health, Labor, and Welfare).

[6]This "local undesirability" assumption is supported by some empirical evidences on the distributions of disutilities, which observe that people do not bear a disutility if a waste disposal facility is sited outside of their houses by about 3 miles (Hirshfeld, Vesilind, and Pas, 1992; Sasao, 2002).

[7]The weak concavity assumption is based on the following intuition: if a district does not have a facility, it bears a big disutility when it accepts a facility. However, when the district already has a facility, its disutility does not grow very much when the size of the facility becomes bigger. Minehart and Neeman (2002) also impose the same assumption in a simpler model.

[8]Our discussions and proofs hold without any substantial change even if cost-disutility functions are restricted to be strongly concave and/or weakly increasing and/or continuous. This issue is discussed in Section 5.4.

Each i's preference over waste-money pairs, $\mathbb{R}_+ \times \mathbb{R}$, is represented by the quasi-linear function $u_i : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ such that for each $(W, m_i) \in \mathbb{R}_+ \times \mathbb{R}$,

$$u_i(W, m_i) \equiv -v_i(W) + m_i.$$

Here, $v_i(W)$ is i's disutility when a facility that deals with W amount of wastes is sited at i, and m_i is i's net monetary transfer. [9] Note that $u_i(0, m_i) = m_i$.

Thus, each district $i \in N$ is characterized by a triplet $(w_i, v_i, c_i) \in \mathbb{R}_+ \times \mathcal{V} \times \mathcal{C}$. Let $(w, v, c) \equiv (w_i, v_i, c_i)_{i \in N} \in \mathcal{D} \equiv \mathcal{W} \times \mathcal{V}^N \times \mathcal{C}^N$ be the profile of these triplets.

An *assignment function* is a mapping $\sigma : N \to \{0, 1\}$ such that $|\sigma^{-1}(1)| = 1$. Let $\mathcal{A}$ be the set of assignment functions. Given $i \in N$, when $\sigma(i) = 1$, the facility is assigned to i; otherwise, the facility is not assigned to i. Given a waste distribution and a profile of cost functions $(w, c) \in \mathcal{W} \times \mathcal{C}^N$, an *allocation for* (w, c) is a triplet of the amount of total wastes $W = \sum_{i \in N} w_i$, an assignment function σ, and a monetary transfer vector m that covers the construction cost, namely,

$$(W, \sigma, m) \in \{W\} \times \mathcal{A} \times \mathbb{R}^N$$

such that [10]

$$\sum_{i \in N} m_i = -c_{\sigma^{-1}(1)}(W).$$

Let $X(w, c)$ be the set of allocations for (w, c). Note that, for two w, w' with $W = W'$, we have $X(w, c) = X(w', c)$. Given $(W, \sigma, m) \in X(w, c)$, every $i \in N$ obtains a bundle

$$(\sigma(i) \cdot W, m_i) \in \mathbb{R}_+ \times \mathbb{R}.$$

Note that

$$\begin{aligned} u_i(\sigma(i) \cdot W, m_i) = u_i(W, m_i) = -v_i(W) + m_i \quad &\text{if } \sigma(i) = 1, \\ u_i(\sigma(i) \cdot W, m_i) = u_i(0, m_i) = m_i \quad &\text{if } \sigma(i) = 0. \end{aligned}$$

We write $x = (W, \sigma, m)$ and $x_i = (\sigma(i) \cdot W, m_i)$ for each $i \in N$. Given $(w, v, c) \in \mathcal{D}$ and $x \in X(w, c)$, let $u(x) \equiv (u_i(x_i))_{i \in N} \in \mathbb{R}^N$.

A *rule* is a function ψ that associates with each profile $(w, v, c) \in \mathcal{D}$ an allocation for (w, c), $\psi(w, v, c) \in X(w, c)$. Given $x = \psi(w, v, c)$ and $i \in N$, we write $x_i = \psi_i(w, v, c)$. Two rules ψ and ϕ are *welfare equivalent* if for each $(w, v, c) \in \mathcal{D}$, $u(\psi(w, v, c)) = u(\phi(w, v, c))$.

Given a rule ψ and $(w, v, c) \in \mathcal{D}$, when $(W, \sigma, m) = \psi(w, v, c)$ and $j = \sigma^{-1}(1)$, we say that j is the $\psi(w, v, c)$-*accepter*.

[9] Thus, i pays m_i if $m_i \leq 0$, and receives m_i if $m_i \geq 0$.

[10] Note that the disutility part $v_{\sigma^{-1}(1)}(W)$ does not appear in the budget balancedness. This is natural, since this balancedness is about monetary transfers and is not about utility transfers. The same budget balancedness is introduced by Moulin (1985b) in the context of quasi-linear social choice with cost functions.

3 Axioms

Given $(w, v, c) \in \mathcal{D}$, an allocation $x \in X(w, c)$ is *(Pareto) efficient* for (w, v, c) if there exists no $y \in X(w, c)$ such that for each $i \in N$, $u_i(y_i) \geq u_i(x_i)$, with strict inequality holding for at least one i. For each $(W, \sigma, m) \in X(w, c)$ with $j = \sigma^{-1}(1)$, since

$$\sum_{i \in N} u_i(\sigma(i) \cdot W, m_i) = -v_j(W) + m_j + \sum_{i \neq j} m_i = -(v_j(W) + c_j(W)),$$

(W, σ, m) is *efficient* for (w, v, c) if and only if $j \in \arg\min_{i \in N}(v_i(W) + c_i(W))$. When $j \in \arg\min_{i \in N}(v_i(W) + c_i(W))$, we say that j is an *efficient district* at (w, v, c). Let $P(w, v, c)$ be the set of *efficient* allocations for (w, v, c).

Efficiency: For each $(w, v, c) \in \mathcal{D}$, $\psi(w, v, c) \in P(w, v, c)$.

Given $(w, v, c) \in \mathcal{D}$, an allocation $x \in X(w, c)$ is *individually rational* for (w, v, c) if for each $i \in N$, $u_i(x_i) \geq -(v_i(w_i) + c_i(w_i))$. Let $I(w, v, c)$ be the set of *individually rational* allocations for (w, v, c).

Individual rationality: For each $(w, v, c) \in \mathcal{D}$, $\psi(w, v, c) \in I(w, v, c)$.

Given $(w, v, c) \in \mathcal{D}$, a coalition $S \subseteq N$, and an allocation $x \in X(w, c)$, S *can block* x *at* (w, v, c) if the members of S can be made simultaneously better off by deviating from x, i.e., there exists $(\sigma', m') \in \mathcal{A} \times \mathbb{R}^S$ such that for some $j \in S$,

$$\begin{aligned} \sigma(j) &= 1, \\ \sum_{i \in S} m'_i &= -c_j(W_S), \\ u_i(x_i) &< u_i(\sigma(i) \cdot W_S, m'_i) \text{ for each } i \in S. \end{aligned}$$

Equivalently, we can state that S can block x at (w, v, c) if

$$\sum_{i \in S} u_i(x_i) < -\min_{j \in S}(v_j(W_S) + c_j(W_S)).$$

Let $C(w, v, c)$ be the set of allocations that cannot be blocked by any coalition at (w, v, c). We refer to C as the *core*.

Core property: For each $(w, v, c) \in \mathcal{D}$, $\psi(w, v, c) \in C(w, v, c)$.

Obviously, the *core property* implies *efficiency* and *individual rationality*. When an economist recommends a core allocation, no group of districts can propose a deviation plan against the recommendation. Thus, besides its appeal as a distributional requirement, the *core property* is quite important for a recommended allocation to be accepted.

The next is a weak monotonicity axiom with respect to improvements of cost-disutility functions. It states that, when the cost and disutility at a district decreased at every level of wastes and the change can lead Pareto improvement, then all districts should weakly gain.

Monotonicity: Let $(w,v,c) \in \mathcal{D}$ and $j \in N$. If $c'_j \in \mathcal{C}$ and $v'_j \in \mathcal{V}$ are such that for each $q > 0$, $c'_j(q) < c_j(q)$ and $v'_j(q) < v_j(q)$, and $v'_j(W) + c'_j(W) < \min_{i \in N}(v_i(W) + c_i(W))$, then

$$u_j(\psi_j(w,v,c)) \leq u'_j(\psi_j(w,v'_j,v_{-j},c'_j,c_{-j})),$$
$$u_i(\psi_i(w,v,c)) \leq u_i(\psi_i(w,v'_j,v_{-j},c'_j,c_{-j})) \text{ for each } i \neq j.$$

In the definition of *monotonicity*, we compare utility levels of different utility functions, $u_j(\psi_j(w,v,c))$ and $u'_j(\psi_j(w,v'_j,v_{-j},c))$. This makes sense, since under quasi-linearity, these values mean i's happiness evaluated in terms of money.

Since wastes are usually tradable, a group of districts may gain by strategically transferring their wastes among the group through side-payments. The next axiom states that no such a manipulation is possible. [11] Alternately, it says that social choice should be based on the true information on the distribution of wastes.

Reallocation-proofness: For each $(w,v,c) \in \mathcal{D}$ and each $S \subseteq N$, if $w' \in \mathcal{W}$ is such that $W'_S = W_S$ and for each $i \in N \setminus S$, $w_i = w'_i$, then

$$\sum_{i \in S} u_i(\psi_i(w,v,c)) = \sum_{i \in S} u_i(\psi_i(w',v,c)). \tag{1}$$

Consider the case that (1) does not hold with $\sum_{i \in S} u_i(\psi_i(w,v,c)) < \sum_{i \in S} u_i(\psi_i(w',v,c))$. In this case, when w is the true waste distribution, by realizing w' through transfers of wastes, group S can increase its group benefit from $\sum_{i \in S} u_i(\psi_i(w,v,c))$ to $\sum_{i \in S} u_i(\psi_i(w',v,c))$. And then, by appropriate side-payments among S, all members of S can increase its welfare. The parallel comment applies to the case with the opposite inequality. *Reallocation-proofness* excludes such strategic manipulation.

Reallocation-proofness is important even if transfers of wastes are prohibited. To check if districts do not transfer wastes, the government has to collect the correct information on the amounts of wastes for all districts, which usually costs a lot. However, when a rule is designed to be *reallocation-proof*, such a monitoring activity is not necessary. According to this aspect, *reallocation-proofness* can be interpreted as a cost-efficiency axiom.

4 Fair waste pricing

A natural way of solving the NIMBY problem is to put a price to disposing wastes. A problem is how to determine an appropriate price. For instance, consider an "unfair" pricing rule ψ such that for each $(w,v,c) \in \mathcal{D}$, whenever $(W,\sigma,m) \equiv \psi(w,v,c)$ and $j \equiv \sigma^{-1}(1)$, it holds that

$$j \in \arg\min_{i \in N}(v_i(W) + c_i(W)),$$
$$m_i = -w_i \cdot \frac{c_j(W)}{W} \quad \text{for each } i \in N.$$

[11] This axiom is introduced by Moulin (1985) in the context of quasi-linear social choice. He calls it "no advantageous reallocation". Our naming is due to Ju, Miyagawa, and Sakai (2007), who extensively study this axiom in a general setting of resource allocation problems.

Here, $\frac{c_j(W)}{W}$ can be read as the price of disposing wastes. This rule selects an efficient district j as the siting district, and allocates the construction cost $c_j(W)$ proportionally to wastes. However, since $u_j(\psi_j(w,v,c)) = -v_j(W) - w_j\frac{c_j(W)}{W}$, not only j pays $w_j\frac{c_j(W)}{W}$, but also it bears the disutility $v_j(W)$. That is, this rule does not give any compensation to the disutility of the siting district, and it violates *individual rationality*.

We next consider a rule that calculates a "fair" price, so that the full compensation for the disutility of the siting district is possible. A rule ψ is a *fair pricing rule* if for each $(w,v,c) \in \mathcal{D}$, whenever $(W,\sigma,m) \equiv \psi(w,v,c)$ and $j \equiv \sigma^{-1}(1)$, it holds that

$$\begin{aligned} j &\in \arg\min_{i\in N}(v_i(W) + c_i(W)), \\ m_j &= -w_j \cdot \frac{v_j(W) + c_j(W)}{W} + v_j(W), \\ m_i &= -w_i \cdot \frac{v_j(W) + c_j(W)}{W} \quad \text{for each } i \neq j. \end{aligned}$$

Then, for each $i \in N$,

$$u_i(\psi_i(w,v,c)) = -w_i \cdot \frac{v_j(W) + c_j(W)}{W}.$$

The *fair price at* (w,v,c) is then defined by

$$p(w,v,c) \equiv \frac{\min_{i\in N}(v_i(W) + c_i(W))}{W} > 0.$$

Thus, to calculate the fair price, this rule takes into account not only the construction cost, but also the disutility of the siting district. All fair pricing rules are welfare equivalent. In fact, whenever there is only one efficient district, all fair pricing rules select the same allocation. They satisfy all the axioms defined in the last section. Furthermore, whenever there are at least three districts, they are the only rules satisfying the axioms:

Theorem 1. Any fair pricing rule satisfies the *core property*, *monotonicity*, and *reallocation-proofness*.

Conversely, when $n \geq 3$, if a solution satisfies *individual rationality*, *monotonicity*, and *reallocation-proofness*, then it is a fair pricing rule.

Proof. See the Appendix. □

The three axioms in the characterization, *individual rationality*, *monotonicity*, and *reallocation-proofness*, are independent. Examples of rules satisfying the three axioms except for each one are offered in the Appendix C.

In applying a rule to solve real-life problems, we need to collect information on the amount of wastes, the construction cost, and the disutility of each district. The information on the amount of wastes and the construction cost can be usually checked or estimated, but disutility information is not so. So, we need to somehow obtain the disutility information. However, a general impossibility result by Holmstrom (1979) implies in this model that no *efficient* rule is strategy-proof. [12] Since a fair pricing rule is *efficient*, it is not strategy-proof. Also, since a fair pricing rule violates Maskin's monotonicity condition that is necessary for Nash implementation (Maskin, 1999), it is not Nash implementable. [13] However, these negative results do not mention anything about the degree of manipulability of a fair pricing rule. To analyze this issue, we consider the direct revelation game for a fair pricing rule.

Let ψ be a fair pricing rule. Given $(w, v, c) \in \mathcal{D}$, $v' \in \mathcal{V}^N$ is a *Nash equilibrium of the direct revelation game for ψ at (w, v, c)* if for each $i \in N$ and each $v''_i \in \mathcal{V}$,

$$u_i(\psi_i(w, v', c)) \geq u_i(\psi_i(w, v''_i, v'_{-i}, c)).$$

Let $\mathcal{N}(\psi, w, v, c)$ be the set of Nash equilibria of the direct revelation game for ψ at (w, v, c). [14] We shall investigate what happens in Nash equilibrium. The next theorem is positive. It states that the fair price under any possible manipulation is higher than the fair price under truthful revelation but the difference is not significant; any allocation attainable through Nash equilibrium still belongs to the core.

Theorem 2. Let ψ be a fair pricing rule. For each $(w, v, c) \in \mathcal{D}$ and each $v' \in \mathcal{N}(\psi, w, v, c)$, whenever $v_1(W) + c_1(W) \geq v_2(W) + c_2(W) \geq \cdots \geq v_n(W) + c_n(W)$,

(i) $p(w, v, c) = \dfrac{v_n(W) + c_n(W)}{W} \leq p(w, v', c) \leq \dfrac{v_{n-1}(W) + c_{n-1}(W)}{W}$,

(ii) $\psi(w, v', c) \in C(w, v, c)$.

Proof. See the Appendix. □

Note that the above game is easy to play for districts by virtue of the simple definition of a fair pricing rule. Indeed, since a fair pricing rule ψ does not depend on details of disutility functions, each district i only needs to report a real-value $v'_i(W)$ instead of the entire form of v'_i. Also, since ψ itself is the outcome function in the game, calculating the outcome is easy.

Theorem 2 implies the following welfare conditions in Nash equilibria:

Corollary 1. Let ψ be a fair pricing rule. For each $(w, v, c) \in \mathcal{D}$ and each $v' \in \mathcal{N}(\psi, w, v, c)$, whenever $v_1(W) + c_1(W) \geq v_2(W) + c_2(W) \geq \cdots \geq v_n(W) + c_n(W)$ and j is the $\psi(w, v', c)$-accepter,

(i) $v_j(W) + c_j(W) = v_n(W) + c_n(W)$,

(ii) $u_j(\psi_j(w, v, c)) \leq u_j(\psi_j(w, v', c))$ and

$$u_j(\psi_j(w, v', c)) - u_j(\psi_j(w, v, c)) \leq \big((v_{n-1}(W) + c_{n-1}(W)) - ((v_n(W) + c_n(W))\big)\big(1 - \frac{w_j}{W}\big),$$

(iii) $-w_i \cdot \dfrac{v_{n-1}(W) + c_{n-1}(W))}{W} \leq u_i(\psi_i(w, v', c)) \leq -w_i \cdot \dfrac{v_n(W) + c_n(W)}{W} = u_i(\psi_i(w, v, c))$.

[12] Strategy-proofness says that truth telling is always a dominant strategy. Ohseto (2000, Theorem 1) points out that Holmstrom's impossibility result works in economies with one indivisible object and money.

[13] Fujinaka and Sakai (2007a) observe that any standard fairness notion is incompatible with Nash implementation in various problems of indivisible goods allocation.

[14] We examine the existence of equilibria in Section 5.1.

Proof. Condition (i) states that $\psi(w, v', c)$ is *efficient* at (w, v, c). This holds, since by Theorem 2, $\psi(w, v', c) \in C(w, v, c)$.

Condition (i) and the possible range of $p(w, v', c)$ specified in Theorem 2 imply Conditions (ii) and (iii). $\square$

The message from Corollary 1 is clear: only an efficient district can gain by manipulation, the gain is not significant, and any other district may lose but the loss is not significant. Summarizing the discussions on strategic manipulation, we can conclude that a fair pricing rule is nearly robust to strategic manipulation, though it is not perfectly robust. Any manipulated fair price is close to the true fair price, the allocation attained by the manipulated price even belongs to the core, and the welfare condition at the allocation is close to the welfare condition at the true fair pricing allocation.

5 Discussions

5.1 Existence of equilibria

The set $\mathcal{N}(\psi, w, v, c)$ can be empty for some fair pricing rule ψ and $(w, v, c) \in D$. The reason is that, in some situations, a district has a non-compact set of "better" strategies at any strategy profile and it cannot find the best response. The following is an intuitive sketch of this story. Consider the case that $N = \{1, 2\}$, $v_1(W) > v_2(W)$ and $c_1(W) = c_2(W)$. Also, assume that district 1 has a priority to accept the facility in the case of ties. Whenever district 1's strategy v_1' is such that $v_1'(W) > v_2(W)$, district 2 can gain more as $v_2'(W)$ becomes larger with keeping

$$v_1'(W) > v_2'(W) > v_2(W). \tag{2}$$

This is because, whenever inequality (2) is kept, district 2 can accept the facility and receive a higher compensation for its disutlity by reporting higher disutilities. However, district 2 cannot find the best response here, since for every v_2' with $v_1'(W) > v_2'(W)$, there is v_2'' such that $v_1'(W) > v_2''(W) > v_2'(W)$. However, fortunately, there exists a fair pricing rule whose associated game always has a Nash equilibrium.

Proposition 1. There exists a fair pricing rule ψ such that for each $(w, v, c) \in D$, $\mathcal{N}(\psi, w, v, c) \neq \emptyset$.

Proof. See the Appendix B. $\square$

An alternative approach to establish the existence of equilibrium is to slightly relax Nash equilibrium to ε-Nash equilibrium. Fujinaka and Sakai (2006) consider allocations that can be supported by ε-Nash equilibrium for arbitrary small $\varepsilon > 0$ and call those allocations *most realizable*. They establish the existence of most realizable allocations in a simpler model. By the same approach as Fujinaka and Sakai, we can show that, for arbitrary fair pricing rule, most realizable allocations exist and all of them have the same desirable properties of Nash equilibrium allocations established by Theorem 2. Interested readers are invited to see Fujinaka and Sakai (2006). [15]

5.2 Transferable utility game

An alternative way of defining solutions for our problem is to invoke the tools of cooperative game theory. There is a natural way of defining the worth of a coalition. Let $a \equiv (w, v, c) \in \mathcal{D}$ and $S \subseteq N$. The *worth* of S at a, $g_a(S)$, is defined as the minimal social loss to deal with W_S among the coalition, that is,

$$g_a(S) \equiv -\min_{i \in S}(v_i(W_S) + c_i(W_S)).$$

We call g_a a *game* at a. By routine calculation, one can check that g_a is *convex*, that is, for each $S, T \subseteq N$, $g_a(S) + g_a(T) \leq g_a(S \cup T) + g_a(S \cap T)$. It is known that any convex games has a non-empty core and there are some interesting core selections such as the Shapley value (Shapley, 1953). [16] In fact, the solution that associates with each problem the utility allocation calculated by the Shapley value satisfies the *core property*. It also satisfies *monotonicity*, but not *reallocation-proofness*. Further properties of this solution and other solutions defined by the cooperative game approach are still unknown.

5.3 More than one facilities and a globally unwanted facility

One may wish to consider the problem of siting (possibly) more than one facility. This can be done by naturally generalizing our definition of allocations so that more than one facility can be built and the sum of monetary transfers equals the sum of all construction costs. However, under the concavity of cost-disutility functions, for each allocation that associates more than one facility, there exists an allocation that associates only one facility and weakly Pareto dominates the allocation. In this sense, under the concavity conditions, the assumption to construct only one facility does not lose generality. Studying the case with non-concave cases is left to the future research.

Another possible approach is to deal with a "globally unwanted" facility (e.g., a nuclear power plant). In this case, it would be natural to assume that people have a "single-dipped" preference over districts whose dip is the place they live in. [17] Dealing with this case is also left to the future research.

[15] Tadenuma and Thomson (1995) and Fujinaka and Sakai (2007b) analyze manipulation games of envy-free rules in the same model as Fujinaka and Sakai (2006) with the help of an assumption that implies the existence of Nash-like equilibria.

[16] For properties of convex games and other core selections, see Moulin (1988).

[17] For the definition of single-dippedness, see Ehlers (2002). He studies probabilistic resource allocation with single-dipped preferences.

6 Conclusion

We have formulated an axiomatic model to analyze the NIMBY problem. We introduced a fair pricing rule and showed that it is the unique rule satisfying a set of various desirable axioms. We also showed that this rule is nearly robust to strategic manipulation in revelation of disutility information. These results ensure significant advantages of a fair pricing rule to solve the NIMBY problem.

Besides its importance as a real-life problem, the NIMBY problem is a quite interesting topic to apply our knowledge and techniques of social choice theory and implementation theory in economic environments. As suggested in the previous section, there are still many things to be resolved by future works.

Appendix

A. Proofs of Theorems

Proof of Theorem 1

Lemma 1. Any fair pricing rule satisfies the *core property*, *monotonicity*, and *reallocation-proofness.*

Proof. Let ψ be a fair pricing rule. We only show that it satisfies the *core property*, since other properties are obvious.

Let $(w, v, c) \in \mathcal{D}$ and $S \subseteq N$. Let $k \in \arg\min_{i \in S}(v_i(W_S) + c_i(W_S))$ and $j \in \arg\min_{i \in N}(v_i(W) + c_i(W))$. Then,

$$\sum_{i \in S} u_i(\psi_i(w, v, c)) = -W_S \cdot \frac{(v_j(W) + c_j(W))}{W}.$$

It suffices to show that

$$W_S \cdot \frac{v_j(W) + c_j(W)}{W} \leq v_k(W_S) + c_k(W_S).$$

This equality is equivalent to

$$\frac{W_S}{W} \leq \frac{v_k(W_S) + c_k(W_S)}{v_j(W) + c_j(W)},$$

which is true by concavity of those functions. Thus, ψ satisfies the *core property.* □

The following somewhat technical condition, introduced by Ju, Miyagawa, and Sakai (2007), states that transfers of wastes do not change utilities unboundedly:

One-sided boundedness: For each $(w, v, c) \in \mathcal{D}$, there exists $i \in N$ such that $u_i(\psi_i(\,\cdot\,, v, c))$ is bounded below or above on the set $\{w' \in \mathbb{R}^N_+ : W' = W\}$.

Ju, Miyagawa, and Sakai (2007, Theorem 2) characterize the class of rules satisfying *reallocation-proofness* and *one-sided boundedness* in a general setting. Their result implies in our model that:

Lemma 2. Assume $n \geq 3$. If a rule ψ satisfies *reallocation-proofness* and *one-sided boundedness*, then there exist functions

$$A : \mathbb{R}_+ \times \mathcal{C}^N \times \mathcal{V}^N \to \mathbb{R},$$
$$B : \mathbb{R}_+ \times \mathcal{C}^N \times \mathcal{V}^N \to \mathbb{R}^N$$

such that for each $(w, v, c) \in \mathcal{D}$ and each $i \in N$,

$$u_i(\psi_i(w, v, c)) = w_i A(W, v, c) + B_i(W, v, c).$$

Lemma 3. *Individual rationality* implies *one-sided boundedness.*

Proof. Let ψ_i be an *individually rational* rule. Let $(w, v, c) \in \mathcal{D}$ and $\mathcal{W}(w) \equiv \{w' \in \mathbb{R}^N_+ : W = W'\}$. For each $i \in N$ and each $w' \in \mathcal{W}(w)$, since

$$-(c_i(W) + v_i(W)) \leq -(c_i(w'_i) + v_i(w'_i)),$$

by *individual rationality*,

$$-(c_i(W) + v_i(W)) \leq u_i(\psi_i(w', v, c)).$$

Thus, for each $i \in N$, $u_i(\psi_i(\,\cdot\,, v, c))$ is bounded from below by $-(c_i(W) + v_i(W))$ on $\mathcal{W}(w)$. Hence, ψ satisfies *one-sided boundedness.* □

Lemma 4. Assume $n \geq 3$. *Individual rationality*, *monotonicity*, and *reallocation-proofness* together imply *efficiency.*

Proof. Let ψ be a rule satisfying the three properties and let A, B be its associated functions. Let $(w, v, c) \in D$ and $k \in N$ be the $\psi(w, v, c)$-accepter. We shall show that $k \in \arg\min_{i \in N}(v_i(W) + c_i(W))$.

We first show that, for each $i \in N$, $B_i(W, v, c) \geq 0$. Otherwise, there exists $i \in N$ such that $B_i(W, v, c) < 0$. Let $w' \in \mathbb{R}^N$ be such that $w'_i = 0$ and $W' = W$. Then

$$u_i(\psi_i(w', v, c)) = B_i(W', v, c) = B_i(W, v, c) < 0 = -(v_i(w'_i) + c_i(w'_i)).$$

This contradicts *individual rationality.*

Let $j \in \arg\min_{i \in N}(v_i(W) + c_i(W))$ and $w'' \in \mathbb{R}^N_+$ be such that $w''_j = W'' = W$. By *reallocation-proofness*,

$$\sum_{i \in N} u_i(\psi_i(w'', v, c)) = \sum_{i \in N} u_i(\psi_i(w, v, c)) = -(v_k(W) + c_k(W)). \tag{3}$$

Then,

$$\begin{aligned} -(v_j(W) + c_j(W)) \leq u_j(\psi_j(w'', v, c)) &= -(v_k(W) + c_k(W)) - \sum_{i \neq j} u_i(\psi_i(w'', v, c)) \\ &= -(v_k(W) + c_k(W)) - \sum_{i \neq j} B_i(W, v, c) \leq -(v_k(W) + c_k(W)), \end{aligned}$$

where the first weak inequality follows from *individual rationality*, the first equality follows from (3), the second equality follows from the definitions of A, B, and the second weak inequality follows from the non-negativity of each B_i established in the last paragraph. Hence,

$$v_k(W) + c_k(W) \leq v_j(W) + c_j(W). \tag{4}$$

Since $j \in \arg\min_{i \in N}(v_i(W) + c_i(W))$, by (4), $k \in \arg\min_{i \in N}(v_i(W) + c_i(W))$. Thus ψ is *efficient.* □

Lemma 5. Assume $n \geq 3$. If a rule satisfies *individual rationality*, *monotonicity*, and *reallocation-proofness*, then it is a fair pricing rule.

Proof. Let ψ be a rule satisfying the three axioms and let A, B be its associated functions. By Lemma 4, ψ is *efficient*.

Step 1: *B* is constantly zero. Let $(w, v, c) \in \mathcal{D}$ and $j \in N$. We shall show that $B_j(W, v, c) = 0$. Let $k \neq j$ and let $w' \in \mathcal{W}$ be such that $w'_k = W' = W$. Since $W = W'$ and $w'_j = 0$, by Lemma 2,

$$u_j(\psi_j(w', v, c)) = B_j(W, v, c).$$

Thus, to prove $B_j(W, v, c) = 0$, it suffices to show that $u_j(\psi_j(w', v, c)) = 0$. Since $w'_j = 0$, by *individual rationality*, $u_j(\psi_j(w', v, c)) \geq 0$. Suppose, by contradiction, that $u_j(\psi_j(w', v, c)) > 0$.

Let $\beta \in (0, 1)$ be such that

$$\beta \cdot (v_k(W) + c_k(W)) < \min_{i \in N}(v_i(W) + c_i(W)).$$

Define v'_k, c'_k by

$$v'_k(q) \equiv \beta \cdot v_k(q) \text{ for each } q \geq 0,$$
$$c'_k(q) \equiv \beta \cdot c_k(q) \text{ for each } q \geq 0.$$

Since the change from (w', v, c) to $(w', v'_k, v_{-k}, c'_k, c_{-k})$ satisfies the hypothesis of *monotonicity*, by *monotonicity*,

$$u_j(\psi_j(w', v'_k, v_{-k}, c'_k, c_{-k})) \geq u_j(\psi_j(w', v, c)) > 0. \tag{5}$$

For each $i \in N \setminus \{j, k\}$, since $w'_i = 0$, by *individual rationality*,

$$u_i(\psi_i(w', v'_k, v_{-k}, c'_k, c_{-k})) \geq 0. \tag{6}$$

Since $w'_k = W$, by *individual rationality*,

$$u'_k(\psi_k(w', v'_k, v_{-k}, c'_k, c_{-k})) \geq -(v'_k(W) + c'_k(W)). \tag{7}$$

By the construction of v'_k and c'_k and *efficiency*, k is the $\psi(w', v'_k, v_{-k}, c'_k, c_{-k})$-accepter. Hence,

$$u'_k(\psi_k(w', v'_k, v_{-k}, c'_k, c_{-k})) + \sum_{i \neq k} u_i(\psi_i(w', v'_k, v_{-k}, c'_k, c_{-k})) = -(v'_k(W) + c'_k(W)).$$

However, by (5), (6), and (7),

$$u'_k(\psi_k(w', v'_k, v_{-k}, c'_k, c_{-k})) + \sum_{i \neq k} u_i(\psi_i(w', v'_k, v_{-k}, c'_k, c_{-k})) > -(v'_k(W) + c'_k(W)).$$

This is a contradiction.

Step 2: Concluding. Let $(w, v, c) \in \mathcal{D}$. By Step 1, for each $i \in N$,

$$u_i(\psi_i(w, v, c)) = w_i A(W, v, c).$$

By *efficiency*,

$$A(W, v, c) = -\frac{\min_{k \in N}(v_k(W) + c_k(W))}{W}.$$

Hence,

$$u_i(\psi_i(w, v, c)) = -w_i \cdot \frac{\min_{k \in N}(v_k(W) + c_k(W))}{W}.$$

Thus, when j is the $\psi(w, v, c)$-accepter, $m_j = v_j(W) - w_j \cdot \frac{\min_{k \in N}(v_k(W)+c_k(W))}{W}$ and for each $i \neq j$, $m_i = -w_i \cdot \frac{\min_{k \in N}(v_k(W)+c_k(W))}{W}$. □

Lemmas 1 and 5 together completes the proof of Theorem 1.

Proof of Theorem 2

Proof of Theorem 2: Consider any fair pricing rule ψ. Let $(w, v, c) \in \mathcal{D}$ and $v' \in \mathcal{N}(\psi, w, v, c)$. We only deal with cases such that for each $i \in N$, $w_i < W$. The case such that for some $i \in N$, $w_i = W$ can be dealt with by a much simpler way, so we omit it.

For each $i \in N$, let $V_i \equiv v_i(W)$, $C_i \equiv c_i(W)$, $V_i' \equiv v_i'(W)$, and $C_i' \equiv c_i'(W)$. Also, we write $V'' \equiv v_i''(W)$ for each $v_i'' \in \mathcal{V}$. Without loss of generality, we assume that $V_1 + C_1 \geq V_2 + C_2 \geq \cdots \geq V_n + C_n$. Let $(W, \sigma, m) \equiv \psi(v', c)$ and $j \equiv \sigma^{-1}(1)$. Note that $j \in \arg\min_{i \in N}(V_i' + C_i)$.

Claim 1. There is $k \neq j$ such that $V_k' + C_k = V_j' + C_j$: Suppose that the claim is not true. Then, since $j \in \arg\min_{i \in N}(V_i' + C_i)$, we have for each $i \neq j$, $V_i' + C_i > V_j' + C_j$. Let v_j'' be such that

$$\min_{i \neq j}(V_i' + C_i) > V_j'' + C_j > V_j' + C_j.$$

Since $W > w_j$ and $V_j'' > V_j'$,

$$u_i(\psi_i(w, v_j'', v_{-j}', c)) = -V_j + V_j''(1 - \frac{w_j}{W}) > -V_j + V_j'(1 - \frac{w_j}{W}) = u_i(\psi_i(w, v', c)).$$

Thus, j could gain by reporting v_j'', a contradiction.

Claim 2. $V_j' \geq V_j$: Suppose, by contradiction, that $V_j' < V_j$. Then,

$$u_j(\psi_j(w, v', c)) = -V_j + V_j' - \frac{w_j}{W}(V_j' + C_j) < -\frac{w_j}{W}(V_j' + C_j). \tag{8}$$

By Claim 1, we have $j, k \in \arg\min_{i \in N}(V_i' + C_i)$. Hence, for any v_j'' with $V_j'' > V_j'$,

$$u_j(\psi_j(w, v_j'', v_{-j}', c)) = -\frac{w_j}{W}(V_k' + C_k) = -\frac{w_j}{W}(V_j' + C_j). \tag{9}$$

Since (8) and (9) together imply that j could gain by reporting v''_j, this is a contradiction.

Claim 3. For each $i \in N, u_i(\psi_i(w, v', c)) \geq -\frac{w_i}{W}(V_i + C_i)$: By Claim 2,

$$u_j(\psi_j(w, v', c)) = -V_j + V'_j - \frac{w_j}{W}(V'_j + C_j)$$
$$= -\frac{w_j}{W}(V_j + C_j) + (V'_j - V_j)(1 - \frac{w_j}{W}) \geq -\frac{w_j}{W}(V_j + C_j).$$

Suppose, by contradiction, that there exists $i \neq j$ such that

$$-\frac{w_i}{W}(V'_j + C_j) = u_i(\psi_i(v', c)) < -\frac{w_i}{W}(V_i + C_i).$$

This implies that $V'_j + C_j > V_i + C_i$. Hence, i could gain by truthfully reporting v_i, since

$$u_i(\psi_i(v_j, v'_{-j}, c)) = -\frac{w_i}{W}(V_i + C_i).$$

This is a contradiction.

Claim 4. $j \in \arg\min_{i \in N}(V_i + C_i)$ and

$$\boldsymbol{u_i(\psi_i(w, v', c)) \geq -\frac{w_i}{W}(V_{n-1} + C_{n-1})} \textbf{ for each } \boldsymbol{i \in N} \textbf{ :}$$

By Claim 3,

$$u_{n-1}(\psi_{n-1}(w, v', c)) \geq -\frac{w_{n-1}}{W}(V_{n-1} + C_{n-1}) \tag{10}$$

and

$$u_n(\psi_n(w, v', c)) \geq -\frac{w_n}{W}(V_n + C_n). \tag{11}$$

Consider the case $n = j$. By definition, $j \in \arg\min_{i \in N}(v_i + c_i)$. Since $u_{n-1}(\psi_{n-1}(w, v', c)) = -\frac{w_{n-1}}{W}(V'_j + C_j)$, by the definition of a fair pricing rule and (10),

$$V'_j + C_j \leq V_{n-1} + C_{n-1}.$$

Hence, for each $i \neq j$,

$$u_i(\psi_i(w, v', c)) = -\frac{w_i}{W}(V'_j + C_j) \geq -\frac{w_i}{W}(V_{n-1} + C_{n-1}).$$

This and (11) together establish the claim in this case.

Next, consider the case $n \neq j$. Since

$$-\frac{w_n}{W}(V_n + C_n) \leq u_n(\psi_n(w, v', c)) = -\frac{w_n}{W}(V'_j + C_j),$$

$V_n+C_n \geq V'_j+C_j$. By Claim 2, $V'_j+C_j \geq V_j+C_j$. By definition, $V_j+C_j \geq V_n+C_n$, so

$$V_j + C_j = V'_j + C_j = V_n + C_n.$$

Therefore, $j \in \arg\min_{i\in N}(V_i + C_i)$, and for each $i \in N$,

$$u_i(\psi_i(w, v', c)) = -\frac{w_i}{W}(V_n + C_n) \geq -\frac{w_i}{W}(V_{n-1} + C_{n-1}).$$

Claim 5. $\boldsymbol{\frac{V_n+C_n}{W} \leq p(w, v', c) \leq \frac{V_{n-1}+C_{n-1}}{W}}$**:** Claim 4 implies $p(w, v', c) \leq \frac{V_{n-1}+C_{n-1}}{W}$. By Claims 2 and 4, $V'_j+C_j \geq V_j+C_j = V_n+C_n$. Thus, $\frac{V_n+C_n}{W} \leq p(w, v', c)$.

Claim 6. $\boldsymbol{\psi(w, v', c) \in \mathcal{C}(w, v, c)}$**:** We shall show that no $S \subseteq N$ can block $\psi(w, v', c)$ at (w, v, c). Let $S \subseteq N$. Since Claim 4 implies $\psi(w, v', c) \in P(w, v, c)$, it suffices to consider the case that $S \subsetneq N$.

Case 6-1. $\boldsymbol{j \in S}$**:** Since

$$\sum_{i\in S\setminus\{j\}} u_i(\psi_i(w, v', c)) = -\sum_{i\in S\setminus\{j\}} w_i \frac{v'_j(W) + c_j(W)}{W}$$

and

$$u_j(\psi_j(w, v', c)) = -v_j(W) + v'_j(W) - w_j \frac{v'_j(W) + c_j(W)}{W},$$

we have

$$\sum_{i\in S} u_i(\psi_i(w, v', c)) = -W_S \frac{v'_j(W) + c_j(W)}{W} - v_j(W) + v'_j(W).$$

Since $v'_j(W) \geq v_j(W)$ by Claim 2,

$$\sum_{i\in S} u_i(\psi_i(w, v', c)) \geq -W_S \frac{v_j(W) + c_j(W)}{W}. \tag{12}$$

Since $j \in \arg\min_{i\in S}(v_i(W_S) + c_i(W_S))$, the highest attainable utility of the coalition S is

$$-(v_j(W_S) + c_j(W_S)).$$

By concavity of v_j and c_j,

$$\frac{v_j(W) + c_j(W)}{W} \leq \frac{v_j(W_S) + c_j(W_S)}{W_S}. \tag{13}$$

By (12) and (13),

$$\sum_{i\in S} u_i(\psi_i(w, v', c)) \geq -(v_j(W_S) + c_j(W_S)). \tag{14}$$

Thus, S cannot block (w, v', c) at (w, v, c).

Case 6-2. $j \in N \setminus S$: Let $k \in \arg\min_{i \in S}(v_i(W_S) + c_i(W_S))$. Then, the highest attainable utility of the coalition S is

$$-(v_k(W_S) + c_k(W_S)).$$

By Claim 4,

$$\sum_{i \in S} u_i(\psi_i(w, v', c)) \geq -W_S \frac{v_{n-1}(W) + c_{n-1}(W)}{W}. \tag{15}$$

Since $j \notin S$,

$$v_{n-1}(W) + c_{n-1}(W) \leq v_k(W) + c_k(W),$$

and

$$\frac{v_{n-1}(W) + c_{n-1}(W)}{W} \leq \frac{v_k(W) + c_k(W)}{W}. \tag{16}$$

By concavity,

$$\frac{v_k(W) + c_k(W)}{W} \leq \frac{v_k(W_S) + c_k(W_S)}{W_S}. \tag{17}$$

Hence, by (16) and (17),

$$W_S \frac{v_{n-1}(W) + c_{n-1}(W)}{W} \leq v_k(W_S) + c_k(W_S).$$

Thus, by (15),

$$\sum_{i \in S} u_i(\psi_i(w, v', c)) \geq -(v_k(W_S) + c_k(W_S)).$$

Therefore, S cannot block (w, v', c) at (w, v, c).

Claims 3 and 6 together completes the proof Theorem 2. □

B. Proof of Proposition 1

Proof of Proposition 1. The proof is constructive. We will define a rule that determines tie-breaking very randomly. Let $(H_1, H_2, \ldots, H_n)$ be a partition of $\mathbb{R}$ such that for each $i \in N$, H_i is dense in $\mathbb{R}$. This partition is fixed throughout. Let ψ be the fair pricing rule that breaks a tie for each $(w, v, c) \in D$ by the following manner:

Let

$$M(w, v, c) \equiv \{i \in N : i \in \arg\min_{i \in N}(v_i(W) + c_i(W)) \text{ and } v_i(W) + c_i(W) \in H_i\}.$$

Case 1: If $M(w,v,c)=\emptyset$, then the district having the largest index among $\arg\min_{i\in N}(v_i(W)+c_i(W))$ is chosen as the $\psi(w,v,c)$-accepter. That is, the district $j\in\arg\min_{i\in N}(v_i(W)+c_i(W))$ satisfying

$$j>i \text{ for each } i\in\arg\min_{i\in N}(v_i(W)+c_i(W))\setminus\{j\}$$

accepts the facility.

Case 2: If $M(w,v,c)\neq\emptyset$, then the district having the largest index among M is chosen as the $\psi(w,v,c)$-accepter. That is, $j\in M(w,v,c)$ such that

$$j>i \text{ for each } i\in M(w,v,c)\setminus\{j\}$$

accepts the facility.

Obviously, ψ is well-defined. We shall show that, given $(w,v,c)\in D$, $N(\psi,w,v,c)\neq\emptyset$. The proof is constructive. Without loss of generality, assume that

$$v_1(W)+c_1(W)\geq v_2(W)+c_2(W)\geq\ldots\geq v_n(W)+c_n(W).$$

If $v_{n-1}(W)+c_{n-1}(W)=v_n(W)+c_n(W)$, then one can check that the truth teling, v, is a Nash equilibrium of the game.

Next, consider the case that $v_{n-1}(W)+c_{n-1}(W)<v_n(W)+c_n(W)$. Since H_n is dense in $\mathbb{R}$, there exists $v'_n\in V$ such that

$$v_{n-1}(W)+c_{n-1}(W)\leq v'_n(W)+c_n(W)\leq v_n(W)+c_n(W),$$
$$v'_n(W)+c_n(W)\in H_n.$$

Let v'_{n-1} be such that

$$v'_{n-1}(W)+c_{n-1}(W)=v_n(W)+c_n(W),$$

and for each $i<n-1$, let $v'_i\equiv v_i$. It is easy to see that v' is a Nash equilibrium of the game. □

C. Independence of axioms

We shall establish the independence of *individual rationality*, *monotonicity*, and *reallocation-proofness*. We omit easy proofs to show that rules provided below satisfy all the axioms except for one.

Example 1. Dropping *individual rationality*: For each $(w,v,c)\in D$, define $(\sigma,m)=\psi(w,v,c)$ by, letting $j\equiv\sigma^{-1}(1)$,

$$j\in\arg\min_{i\in N}(v_i(W)+c_i(W)),$$
$$m_j\equiv v_j(W)-\frac{v_j(W)+c_j(W)}{n},$$
$$m_i=-\frac{v_j(W)+c_j(W)}{n} \text{ for each } i\neq j.$$

Example 2. Dropping *monotonicity*: For each $(w, v, c) \in D$, define $(\sigma, m) = \psi(w, v, c)$ by, letting $j \equiv \sigma^{-1}(1)$ and $k \in \arg\min_{i \neq j}(v_i(W) + c_j(W))$,

$$j \in \arg\min_{i \in N}(v_i(W) + c_i(W)),$$

$$m_j \equiv v_j(W) - w_i \cdot \frac{v_k(W) + c_k(W)}{W} + (v_k(W) + c_k(W) - v_j(W) - c_j(W)),$$

$$m_i \equiv -w_i \cdot \frac{v_k(W) + c_k(W)}{W} \text{ for each } i \neq j.$$

Example 3. Dropping *reallocation-proofness*: For each $(w, v, c) \in D$, define $(\sigma, m) = \psi(w, v, c)$ by, letting $j \equiv \sigma^{-1}(1)$,

$$m_j \equiv v_j(W) - (v_j(w_j) + c_j(w_j)) + \frac{\sum_{k \in N}(v_k(w_k) + c_k(w_k)) - (v_j(W) + c_j(W))}{n},$$

$$m_i = -(v_j(w_j) + c_j(w_j)) + \frac{\sum_{k \in N}(v_k(w_k) + c_k(w_k)) - (v_j(W) + c_j(W))}{n} \text{ for each } i \neq j.$$

Note that the term $\frac{\sum_{k \in N}(v_k(w_k) + c_k(w_k)) - (v_j(W) + c_j(W))}{n}$ is non-negative by concavity of disutility-cost functions.

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