

# Endogenous Present-Biasedness and Policy Implementation\*

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First version: September 2005

This version: December 2006

## Abstract

We show that under a two-party political system the party in office tends to be present-biased and time-inconsistent. This may lead to inefficient procrastination of socially beneficial projects. However, procrastination needs not be indefinite. There exist equilibria in which the project is carried out, maybe even in finite time. The procrastination problem tends to get more serious as the cost of the project gets higher. When the cost is low, there is no procrastination problem. When the cost is high, the project can be procrastinated indefinitely, though there exist equilibria in which the project is implemented gradually, with the process going on for a long time.

*JEL* Classification: C70, D11, D60, D72, D78, D91

Keywords: endogenous present-biasedness, hyperbolic discounting, time-inconsistency, procrastination, multi-party political system, policy implementation

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\*We are grateful to Robert Staiger and participants of the seminars at Australian National University, Chukyo University, City University of Hong Kong, Fukushima University, Hitotsubashi University, Hong Kong University of Science and Technology, Chinese University of Hong Kong, Keio University, Tsukuba University, Seoul National University, Singapore Management University, Waseda University, Yokohama National University, 2005 APTS meeting, ETSG 2005 Seventh Annual Conference, and Spring 2006 Midwest Economic Theory meeting for helpful comments. The work described in this paper is partially supported by grants from the 21st Century Center of Excellence Project on the Normative Evaluation and Social Choice of Contemporary Economic Systems (Japan) and the Research Grants Council of the Hong Kong Special Administrative Region, China [Project No. CityU 1476/05H]. Edwin Lai acknowledges the support of the Department of Economics and Finance at City University of Hong Kong while writing this paper.

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# 1 Introduction

People often procrastinate doing things that generate lasting benefits but require the payment of an immediate cost, to the detriment of their long-term interests. Quitting bad habits, such as smoking and drinking, is one prominent example. Other examples include house-cleaning, studying for an examination, and writing a referee report. A recent literature (e.g., Akerlof, 1991 and O'Donoghue and Rabin, 1999) explains this phenomenon by focusing on the existence of present-biased preferences, which induce time-inconsistent behavior. As a present-biased individual considers trade-offs between two future periods, stronger relative weight is given to the earlier period as it approaches. This creates time-inconsistent behavior because an individual's relative preference for payoff at an earlier period over a later period strengthens as the earlier period approaches. Procrastination may therefore ensue as the present self cannot commit the future selves to future actions. A present-biased, time-inconsistent individual may procrastinate completing a task forever, even though it is in her best long-term interest to complete the task immediately.

Similarly, it is often observed that politicians procrastinate implementing socially beneficial policies that require incurring immediate costs, but generate long-lasting benefits. For example, politicians are reluctant to raise income taxes even though it may benefit citizens in the long-run by helping to reduce the government deficit and hence lower the long-term interest rate. The delay of trade liberalization, despite its long-term benefits to the country as a whole, can be explained by the fact that the costs of resource reallocation are incurred immediately while social benefits are spread far into the future. Another prominent example of government procrastination is that of pension reform. As Feldstein (2005) stated: "Many economists and policy analysts acknowledge the long-run advantages of shifting from a pay-as-you go [tax-financed] system to a mixed system [that combines pay-as-you-go benefits with investment-based personal retirement account] but believe that the transition involves unacceptable costs. This is often summarized by saying that the transition generation would have to pay 'double' — to finance the social security benefits of current retirees and to save for its own retirement." This might explain why many countries delay pension reforms.

In this paper, we provide a theory to explain government procrastination based on a model of endogenous present-biasedness, which is a consequence of a two-party political system. In our model, a party has the same intertemporal preferences as a typical citizen, which is characterized by geometric discounting, if the party believes it will be in office in every future period. Its discount factor between any two consecutive periods is constant, and its intertemporal utility function does not give rise to time-inconsistency. However, under a two-party political system, the government of each term becomes present-biased and time-inconsistent. Present-biasedness arises because a government's probability of getting elected in the future is less than one, and it takes social welfare more seriously when it is in office than when it is not in office. As a result, each government in a two-party system has incentives to procrastinate carrying out projects that should be undertaken immediately if social welfare is to be maximized.

We are not the first to identify the two-party political system as a source of present-biasedness. In their studies of government debt, Alesina and Tabellini (1990) and Amador (2003) argue that the government saves too little, or accumulates too much debt, due to the political uncertainty caused by a two-party system. In particular, Amador (2003) specifically mentions that time-inconsistency with which the government is faced is equivalent to the problem faced by a hyperbolic consumer. The distinguishing feature of our paper is that we examine the mechanism through which the government comes to have present-biased preferences in the two-party political system in more detail and in a more general setting than theirs.<sup>1</sup> Moreover, the policy implementation problem on which we focus in this paper is quite different from theirs.

The present-biasedness of the government may well prevent a socially beneficial project from being undertaken. Of course, it is not surprising that if the implementation cost is

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<sup>1</sup>In our model, the election outcome is characterized by a Markov process, such that the current ruling party will be re-elected with an arbitrarily fixed probability between 0 and 1. Whereas Alesina and Tabellini (1990) and Amador (2003) assume that every party has an equal probability of being elected in every election, which is a special case of ours (when the probability of being re-elected equals one half). Although Alesina and Tabellini (1990) mention in a footnote of their paper that the analysis can be extended to a similar framework to ours, they have not explored how the likelihood of being re-elected affects the government present-biasedness as much as we do in this paper.

high, the net present value of social benefit of the policy may be negative and so the policy should not be implemented. Interestingly, we show that even if the present discounted value of net social benefits is positive, and political parties have the same geometric discount factor as the citizens, a government may still procrastinate implementing the policy in a political system in which two parties compete for office. Suppose there is a socially beneficial project. Suppose further that the project is divisible. We demonstrate that, depending on the cost of the project relative to the discount factor, the present-biased governments may (i) carry out the project immediately exactly in accordance with citizens' interests, (ii) procrastinate somewhat, but still manage to complete the whole project in some period in finite time, (iii) undertake the project in stages, with the process continuing for a long time, or (iv) completely fail to undertake the socially beneficial project.

Indefinite procrastination of socially beneficial projects can sometimes be explained by a model of myopic government who cares more about current constituents and discounts heavily future unborn generations. That is, the government discounts future more heavily than the typical citizen but they both remain time-consistent. The government has incentives to procrastinate the project indefinitely if and only if the government discounts future sufficiently heavier than the citizens. Since the government remains time-consistent, the project is either completed immediately or procrastinated indefinitely depending on the government's discount factor. However, ours is not a model of myopia. Instead, it is a model of endogenous time-inconsistency of the political parties. The outcome of such a model is different from that of myopic government in that there exist equilibria in which, despite certain degree of procrastination, a socially beneficial project is carried out and completed in finite time. Thus, our analysis reveals the distinction between two sources of procrastination by governments. The first arises from the government being more impatient than the citizens, i.e. a myopic government. The second arises from endogenous present-biasedness as political parties face uncertainty about the prospect of being elected and take social welfare less seriously when they are out of office. In this paper, we focus on the second source, which is the more interesting one.

The above results can be applied more generally to situations in which a present-biased agent is faced with completing a divisible task that generates long-lasting benefits but requires incurring an immediate cost. This agent can be a person, a government or an organization. The agent is typically faced with a self-control problem. (See, for example, O'Donoghue and Rabin 2001.) Our main contribution to this literature is that we identify the existence of gradual implementation equilibria when the task is divisible. According to standard analysis, given that implementation cost is high, if the task is indivisible, a present-biased individual would procrastinate doing it indefinitely. We find that if the task is divisible, the agent can procrastinate less drastically by carrying it out gradually. Take the example of house-cleaning. If a present-biased individual can choose to partially clean her house in a day, she may just clean a little bit of it every day, yet never gets it completely cleaned; but she would never clean the house if she has to complete it in one single day.

Indeed, government projects or policies can be partially carried out within the term of a government. For example, a government can choose to partially liberalize the trade regime by cutting only some tariffs, or lowering those tariffs somewhat but not all the way to free-trade level. In the case of balancing the budget, a government can choose to reduce the deficit somewhat but not all the way to a balanced budget. According to our analysis, the possibility of partially carrying out the project allows the present-biased government to bypass the fate of indefinite procrastination of the project when the implementation cost is high. Seen in this light, this paper identifies a new source of gradualism in the literature on dynamic contribution to a public good, namely the endogenous present-biasedness arising from a two-party political system.<sup>2</sup>

In section 2, we lay down the basic assumptions and setup of the model. In section 3, we show how a two-party political system gives rise to present-biasedness of the party in office. We consider a socially beneficial project with immediate cost outlay and future flows of benefits. Given that two parties compete for office in each period, the party currently in office

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<sup>2</sup>Compte and Jehiel (2004), for example, attribute a positive correlation between a player's contribution and the other party's outside option value to gradualism in contribution games. Players gradually make contributions to prevent their respective partners from terminating the game.

plays a game with all future governments (including its future self) in choosing the fraction of the project to be implemented today. In section 4, we compute the subgame perfect equilibria corresponding to different implementation costs. In section 5, we summarize the results and conclude.

## 2 The Basic Setup of the Model

There are two political parties that seek power in the government. One of them is in office in period  $t \in \{0, 1, 2, \dots\}$ . Let each period be a term. The party in office makes policy decisions in accordance with its own preferences; therefore the objective function of the current government is the same as that of the party in office. The two parties have the same preferences over the same policy when faced with the same circumstances. Moreover, both of their preferences are characterized by geometric discounting and the same discount factor on future payoffs,  $\delta$ , which is also the same as that of the citizens.<sup>3</sup> The selection of the ruling party in each election is characterized by a Markov process, such that the current ruling party will be re-elected with a constant probability  $p \in (0, 1)$ . If  $p > 1/2$ , for example, the ruling party has a higher probability to be in office in the next period than the opposition party, which means that there is an incumbent advantage. We assume that the probability of a party being elected is independent of how the policy is implemented by the party or its rival. One can imagine that the project being considered is one among many that is being implemented by the government. Therefore, the implementation of any one of them has a negligible effect on  $p$ .

The policy that we consider is about undertaking a project that involves an immediate implementation cost of  $c$  but generates a constant benefit flow of 1 in the current period and every period thereafter. We assume that the project is divisible in the sense that it is feasible for only a fraction of the project to be carried out in a period so that a fraction  $a_t$  of the project undertaken in period  $t$  poses an immediate cost  $a_t c$  to society while generating benefit flows of  $a_t$  in each period. We assume that  $1/(1 - \delta) > c$ , so the project is worth

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<sup>3</sup>However, they have different views about payoffs than that of a typical citizen, as will be shown below.

carrying out from the citizens' point of view.

The flow of social welfare enjoyed by citizens in period  $t$  is given by

$$u_t = \sum_{i=0}^t a_i - a_t c. \quad (1)$$

The first term on the right-hand side shows the benefit that society enjoys in period  $t$  from the fraction of the project that has been completed, whereas the second term represents the costs that society incurs from the part of the project undertaken in period  $t$ . We assume that the party in office in period  $t$  puts a (normalized) weight of one on the flow of social welfare in period  $t$ , and so its one-shot payoff in period  $t$  equals  $u_t$ , while the opposition party puts a weight of  $\alpha \in [0, 1]$  on the flow of social welfare in the same period. This discounting is motivated by the presumption that, while the members of the opposition party is part of the citizenry and so they care about social welfare just like the average citizen, the opposition party also treats the success of the ruling party as unfavorable, perhaps because it undermines its political status. On the contrary, the ruling party in period  $t$  has a different view about payoff in period  $t$ : the members of the ruling party care about social welfare in that period as they are part of the citizenry; in addition, they have incentives to care more about social welfare in period  $t$  because a higher social welfare gives them higher political status.

### 3 Endogenous Present-Biasedness

In this section, we show that in a two-party political system, the party in office will possess present-biased preferences. By present-biasedness, we mean when we consider two consecutive periods, the discounting of well-being at the later period as of the earlier period is greatest when the earlier period is the present. In other words, the government of any period puts a disproportionately high weight on current payoff. In addition, we show that if  $p \geq 1/2$ , the party in office will possess a payoff function with generalized hyperbolic discounting, which is a subset of present-biased preferences. By generalized hyperbolic discounting, we mean one such that the discounting of period  $t + 1$ 's payoff at period  $t$  weakly diminishes as  $t$  increases. One way to motivate this is that the distinction between two consecutive periods is more salient between today ( $t = 0$ ) and tomorrow ( $t = 1$ ), but it becomes

less and less so further and further into the future.<sup>4</sup> To be more specific, let

$$U_t = \sum_{k=0}^{\infty} \beta_k u_{t+k} \quad (2)$$

represent the present discounted payoff function for the party in office in period  $t$ , which we call Government  $t$  henceforth. Then,  $U_t$  exhibits generalized hyperbolic discounting if the ratio of the two consecutive discount functions  $\beta_{k+1}/\beta_k$  weakly increases with  $k$ .<sup>5</sup>

Let  $p_k$  denote the probability that the current ruling party will also be in office  $k$  periods later. Since the ruling party will be in office in the next period with probability  $p$  and the opposition party will be in office with probability  $1 - p$ , the value of  $p_k$  evolves according to

$$\begin{aligned} p_{k+1} &= p \cdot p_k + (1 - p)(1 - p_k) \\ &= 1 - p + (2p - 1)p_k, \end{aligned} \quad (3)$$

with  $p_0 = 1$ . We can solve this difference equation explicitly when  $p \neq 1/2$  to obtain

$$p_k = \frac{(2p - 1)^k + 1}{2}. \quad (4)$$

When  $p = 1/2$ , we have  $p_0 = 1$  and  $p_k = 1/2$  for  $k \geq 1$ . Figure 1 depicts the transition of this probability when  $p > 1/2$ . If  $p > 1/2$ , the incumbent has an advantage in future elections in the sense that its probability of being elected is higher in all future periods than its rival. This advantage, however, diminishes as time goes by, which is reflected by the fact that  $p_k$  falls with  $k$  as shown by (4). If  $p = 1/2$ , the ruling party has no incumbent advantage nor disadvantage. The probability that the ruling party will be in office  $k$  periods later is  $p_k = 1/2$  for any  $k \geq 1$ . Finally, if  $p < 1/2$  (i.e. when there is an incumbent disadvantage) the probability  $p_k$  fluctuates, converging to  $1/2$  as  $k$  tends to infinity.

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<sup>4</sup>Akerlof (1991) gives an excellent discussion about the salience of the present for a present-biased individual.

<sup>5</sup>The instantaneous discount rate of the “usual” exponential discount function  $\beta_e(t) \equiv e^{-rt}$  in continuous time models is given by  $-\beta'_e(t)/\beta_e(t) = r$ , whereas that of hyperbolic discount function  $\beta_h(t) \equiv (1 + \alpha t)^{-\gamma/\alpha}$  is given by  $-\beta'_h(t)/\beta_h(t) = \gamma/(1 + \alpha t)$  that decreases with  $t$  (for hyperbolic discounting, see Loewenstein and Prelec, 1992, who call it *generalized* hyperbolic discounting contrary to our terminology). Phelps and Pollak (1968) develop an intertemporal utility function of the form:  $U_t = u_t + \beta \sum_{k=1}^{\infty} \delta^k u_{t+k}$  (where  $0 < \beta < 1$  and  $0 < \delta < 1$ ) to capture imperfect altruism for future generations. Laibson (1997) introduces this utility function with quasi-hyperbolic discounting to behavioral economics in order to capture important properties of hyperbolic discounting. Note that quasi-hyperbolic discounting is a special case of generalized hyperbolic discounting as  $\beta_{k+1}/\beta_k$  weakly increases with  $k$  ( $\beta_1/\beta_0 = \beta\delta$  and  $\beta_{k+1}/\beta_k = \delta$  for  $k \geq 1$ ).



To find the discounted payoff function of Government  $t$ , first we observe that  $u_{t+k}$  is independent of who is in office in period  $t+k$ , where  $k = 1, 2, 3, \dots$ . This is true because both parties have the same preferences when in office and are expected to behave in the same way when faced with the same circumstances. Therefore, Government  $t$  has no reason to distinguish between which of the two parties will be in office in period  $t+j$  when it anticipates the action taken by Government  $t+j$ , where  $j = 1, 2, \dots, k$ . Consequently, the history as of period  $t+k$ ,  $\{a_i\}_{i=0}^{t+k}$ , is independent of who has been in office up to that period.<sup>6</sup> So, the expected one-shot payoff of Government  $t$  in period  $t+k$  can be written as

$$p_k u_{t+k} + (1 - p_k) \alpha u_{t+k} = [\alpha + (1 - \alpha) p_k] u_{t+k}.$$

Therefore, the present discounted expected payoff for Government  $t$  is given by (2), where

$$\begin{aligned} \beta_k &\equiv \delta^k [\alpha + (1 - \alpha) p_k] \\ &= \delta^k \left[ \alpha + (1 - \alpha) \frac{(2p - 1)^k + 1}{2} \right] \end{aligned} \quad (5)$$

is the discount function applied to social welfare  $k$  periods from  $t$ . Note that  $\beta_0 = 1$  as  $p_0 = 1$ . Also note that if  $\alpha = 1$ , the discount function  $\beta_k$  reduces to the “usual” geometric discount function  $\delta^k$ . If a party does not discount the flow of social welfare when it is out of office, and if it knows that the rival party would behave exactly the same as it would when faced with the same circumstances, the party would behave as if it would always be in office. Therefore, the party would not be present-biased. Since we are interested in problems arising from present-biased preferences, we henceforth assume that  $\alpha < 1$  unless we explicitly state otherwise.

The payoff function represented by (2) and (5) exhibits generalized hyperbolic discounting if  $\beta_{k+1}/\beta_k$  weakly increases with  $k$ . It follows from (4) and (5) that when  $p \neq 1/2$ ,

$$\frac{\beta_{k+1}}{\beta_k} = \delta \left[ \frac{1 + \alpha + (1 - \alpha)(2p - 1)^{k+1}}{1 + \alpha + (1 - \alpha)(2p - 1)^k} \right]. \quad (6)$$

It can be readily verified from (6) that if  $p > 1/2$ , then  $\beta_{k+1}/\beta_k$  increases with  $k$  and converges to  $\delta$  as  $k$  tends to infinity. Thus, the government’s payoff function exhibits generalized

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<sup>6</sup>Regardless of which party will be in office in period  $t+k$ , it will inherit the same history and have the same expectation about the future as those of the other party. Since the two parties will be faced with the same circumstances, they will take the same action in period  $t+k$ .

hyperbolic discounting in this case. If  $p < 1/2$ , on the other hand, we see from (6) that  $\beta_{k+1}/\beta_k$  fluctuates around  $\delta$  as  $k$  increases such that it is less than  $\delta$  when  $k$  is even, is greater than  $\delta$  when  $k$  is odd, and converges to  $\delta$  as  $k$  tends to infinity. Moreover,  $\beta_{k+1}/\beta_k$  takes on the smallest value when  $k = 0$ , which implies that the discount rate is greatest in the current period, i.e., the government has a present-biased preferences as in the case where  $p \geq 1/2$ .

Finally, if  $p = 1/2$ , it follows from (5) that  $\beta_1/\beta_0 = \delta(1 + \alpha)/2 < \delta$  and  $\beta_{k+1}/\beta_k = \delta$  for  $k \geq 1$ . Therefore, the government's payoff function exhibits quasi-hyperbolic discounting (Laibson, 1997; see also footnote 5). The current ruling party discounts social welfare in the next period more heavily than the discounting brought about by the discount factor  $\delta$  as it will be out of office with probability  $1/2$ . Since the probability of being in office stays the same from the next period onward, that is, the ruling party never enjoys incumbent advantage nor disadvantage in future elections, discounting between future consecutive periods is stationary.

In a similar multi-party political environment as ours, Amador (2003) shows that if all political parties including the current ruling party have equal probabilities of being elected in the next election, the preferences of the ruling party is characterized by quasi-hyperbolic discounting. His model therefore corresponds to ours in the case where  $p = 1/2$ .<sup>7</sup>

We record the above findings in the following proposition.

**Proposition 1** *A two party political system leads to present-biased preferences of the government. The preferences of the government is characterized by generalized hyperbolic discounting if the probability that the current ruling party is re-elected is greater than or equal to one half.*

Under such circumstances, Government  $t$  would be time-inconsistent: its relative preference for payoff at an earlier period over a later period gets stronger as the earlier period approaches.

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<sup>7</sup>Our argument can easily be generalized to the case of multi-party political system with more than two parties. We demonstrate our argument in the case of two parties to avoid the discussion of issues such as coalition formation to gain a majority, which are not of central interest in our analysis.

## 4 Policy Implementation

In this section, we analyze the policy choice of a government. It has been shown that an individual with a quasi-hyperbolic payoff function exhibits time-inconsistent behavior, which includes inefficient procrastination of costly actions that generate a future stream of large benefits (see, for example, O'Donoghue and Rabin, 1999). In the current setting, the government, or the party in office, has a present-biased payoff function. Therefore, it is faced with a time-inconsistency problem, and we expect that it may procrastinate. Indeed, we find that procrastination sometimes occurs, and the problem gets worse as implementation cost gets higher. However, even when it does happen, procrastination needs not be indefinite. Although the government sometimes procrastinates implementing socially beneficial projects, there exist equilibria in which the project is undertaken, and may even be completed in finite time. Specifically, we show that (i) the entire project is carried out immediately in period 0 if the cost of the project is small; (ii) there may be some finite delay in undertaking the project if the cost is in the intermediate range; and (iii) if the cost is high, the project may never be carried out, though there may also exist other equilibria in which the project is carried out gradually over many periods of time.

Given the history  $\{a_i\}_{i=0}^{t-1}$ , Government  $t$  with the payoff function given in (2) chooses  $a_t$  under the constraint  $a_t \geq 0$  and  $\sum_{i=0}^t a_i \leq 1$ . The action of Government  $t$  unambiguously affects those of future governments, and Government  $t$ 's expectation about the actions of future governments affects its behavior. This policy implementation problem can therefore be considered as a game that the present government plays with future governments (including its future self), each of which lasts for one period.

Define the present discounted value of net benefits of the project that is carried out in period  $t$  as evaluated in period 0 as  $B_t \equiv \sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c$ . Now, we rewrite Government 0's discounted payoff function given in (2) for  $t = 0$ , making use of the fact that the fraction  $a_t$  of the project undertaken in period  $t$  yields an expected payoff  $a_t B_t$ :

$$U_0 = \sum_{t=0}^{\infty} a_t B_t. \quad (7)$$

Since (7) is linear with respect to  $\{a_t\}_{t=0}^\infty$ , it is in the best interest of Government 0 to have the project carried out in a period where the present value of its net benefits is greatest. Let us define  $t^*$  by

$$t^* \in \arg \max_{t \in \mathbf{Z}_+} B_t,$$

where  $\mathbf{Z}_+$  denotes the set of non-negative integers. Then, it is clear that the best sequence of  $\{a_t\}_{t=0}^\infty$  is that  $a_t = 1$  if  $t = t^*$  and  $a_t = 0$  if  $t \neq t^*$ .<sup>8</sup>

To find  $t^*$ , we compare the present values of the expected payoffs for two consecutive periods  $t$  and  $t + 1$ . Government 0 (weakly) prefers having the project undertaken in period  $t$  to having it done in period  $t + 1$  if and only if

$$\begin{aligned} B_t &\geq B_{t+1} \\ \Leftrightarrow \beta_t &\geq (\beta_t - \beta_{t+1})c \\ \Leftrightarrow \frac{\beta_{t+1}}{\beta_t} &\geq \frac{c-1}{c}. \end{aligned} \tag{8}$$

The second inequality is easy to interpret: Government 0 is better off undertaking the project in period  $t$  rather than in  $t + 1$  if and only if the reduction in payoff by postponing the project by one period,  $\beta_t$ , is at least as high as the saving in cost by doing so,  $(\beta_t - \beta_{t+1})c$ . Equation (8) will be the key equation determining the time pattern of policy implementation.

If neither party discounts social welfare when it is out of office, i.e.,  $\alpha = 1$ , then  $\beta_t = \delta^t$  and inequality (8) holds for any  $t$  since it reduces to  $1/(1 - \delta) \geq c$ . Then, Government 0 prefers having the project undertaken in period  $t$  to having it postponed to the next period, no matter what  $t$  is. This implies that  $t^* = 0$ , and so it is in Government 0's best interest to carry out the entire project within its term. Note that, since  $\beta_t = \delta^t$ , the government's payoff function is exactly the same as that of the citizens. Therefore, in this case, the government's action maximizes the welfare of the citizens. We summarize this finding in the following proposition.

**Proposition 2** *Suppose that neither party discounts social welfare when it is out of office, i.e.,  $\alpha = 1$ . Then no government would procrastinate a socially beneficial project.*

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<sup>8</sup>Generically,  $t^*$  is uniquely determined.

On the other hand, if every party discounts social welfare when it is out of office, i.e.,  $\alpha < 1$ , then postponing the project may be preferable for the current government since, by doing so, the reduction in cost can outweigh the loss in benefit.

We have shown that if  $p > 1/2$ , then  $\beta_{t+1}/\beta_t$  strictly increases with  $t$  and that it converges to  $\delta$  as  $t$  tends to infinity. This is shown in Figure 2. Since  $\delta > (c - 1)/c$ , there exists a threshold value of  $t$  such that (8) holds if and only if  $t$  is greater than or equal to the threshold value. As Government 0 prefers having the project undertaken in period  $t$  to having it done in period  $t + 1$  for all  $t$  greater than or equal to the threshold value, whereas it prefers postponing the project from  $t$  to  $t + 1$  for any  $t$  smaller than the threshold value, we infer that Government 0 prefers having the project undertaken in this threshold period, i.e.,

$$t^* = \min \left\{ t \in \mathbf{Z}_+ \mid \frac{\beta_{t+1}}{\beta_t} \geq \frac{c-1}{c} \right\}.$$

Figure 2 also shows  $\beta_{t+1}/\beta_t$  for the cases  $p = 1/2$  and  $p < 1/2$ . If  $p = 1/2$ , then  $\beta_1/\beta_0 = \delta(1 + \alpha)/2$  and  $\beta_{k+1}/\beta_k = \delta$  for  $k \geq 1$ . Since  $(c - 1)/c < \delta$ , we see from (8) that  $t^* = 0$  if  $\delta(1 + \alpha)/2 \geq (c - 1)/c$  and  $t^* = 1$  otherwise. Finally, if  $p < 1/2$ , then  $t^* = 0$  if  $c$  is sufficiently small so that  $\beta_1/\beta_0 \equiv \delta[\alpha + (1 - \alpha)p] \geq (c - 1)/c$  and  $t^* \geq 1$  otherwise. In this case,  $\beta_{t+1}/\beta_t$  fluctuates around  $\delta$  as  $t$  increases and converges to  $\delta$  as  $t$  tends to infinity, such as taking on the smallest value at  $t = 0$ , the largest value at  $t = 1$ , and the second smallest value at  $t = 2$ , and so on. Thus, it is not straightforward to determine the exact value of  $t^*$  unless  $\beta_3/\beta_2 \geq (c - 1)/c$  in which case  $\beta_{t+1}/\beta_t \geq (c - 1)/c$  if and only if  $t \geq 1$  so that  $t^* = 1$ . As the following analysis reveals, what is important in the analysis of policy implementation is whether or not the current ruling party has an incentive to procrastinate, i.e., whether or not  $t^* = 0$ . Therefore, we do not pursue any further the computation of  $t^*$  when  $p < 1/2$ .

As can be seen, common to all cases is the fact that  $\beta_1/\beta_0$  is the smallest of all  $\beta_{t+1}/\beta_t$  for  $t \in \mathbf{Z}_+$ , which means that there is present-biasedness as long as  $p < 1$ . The smaller is the fraction  $\beta_1/\beta_0 \equiv \delta[\alpha + (1 - \alpha)p]$  relative to  $\delta$ , the greater is the present-biasedness. The present-biasedness increases as  $\alpha$  or  $p$  decreases.

The policy implementation equilibrium outcome  $\{a_t\}_{t=0}^\infty$  depends on the cost of the project  $c$  relative to the discount functions  $\{\beta_t\}_{t=0}^\infty$ , which in turn depend on  $p$  and  $\alpha$ .

As we shall show shortly, the characteristics of the subgame perfect equilibrium of the policy implementation game differ greatly across three cases sorted by the relative size of the implementation cost  $c$ .

We define  $\bar{\beta} \equiv (\sum_{k=0}^{\infty} \beta_k - 1) / \sum_{k=0}^{\infty} \beta_k$  and show below that  $\beta_1$  and  $\bar{\beta}$  are two critical values of  $(c-1)/c$  that delineate three distinct cases. It is readily verified that  $\beta_1 < \bar{\beta} < \delta$  as this relationship is equivalent to  $\sum_{k=0}^{\infty} \beta_1^k < \sum_{k=0}^{\infty} \beta_k < \sum_{k=0}^{\infty} \delta^k$ , the proof of which is relegated to the Appendix. The first case we consider is characterized by  $(c-1)/c \leq \beta_1$ . It is shown that the government prefers to carry out the project immediately in this case. The second case is the one in which the cost of the project is in the intermediate range:  $\beta_1 < (c-1)/c \leq \bar{\beta}$ . Every government prefers having the project undertaken by a future government, though the net benefits from carrying out the project immediately is non-negative, i.e.,  $B_0 = \sum_{k=0}^{\infty} \beta_k - c > 0$ . Finally, in the case where  $\bar{\beta} < (c-1)/c < \delta$ , every government strongly prefers postponing the project as  $B_0 < 0$  while  $B_t > 0$  for some  $t > 0$ .

#### 4.1 Low Implementation Cost

We begin with the case in which the cost of the project is small such that  $(c-1)/c \leq \beta_1$ . Note that this inequality implies that equation (8) holds for all  $t \geq 0$  while  $(c-1)/c < \bar{\beta}$  implies that  $B_0 > 0$ . The latter says that it is worthwhile for Government 0 to carry out the project, while the former says that the optimal timing of implementation  $t^*$  is period 0.

The situation is depicted in Figure 2. It is evident from the figure that if the cost is so small that  $(c-1)/c \leq \beta_1$ , inequality (8) holds for all  $t$ , so that  $t^* = 0$ . This result holds regardless of the value of  $p$  since, for any  $p$ ,  $\beta_{t+1}/\beta_t$  takes on the smallest value when  $t = 0$ , as we have shown in the discussion below equation (6).

The uppermost schedule in Figure 3 depicts  $B_t$  when implementation cost is small, ignoring the fact that this present value is only defined on  $\mathbf{Z}_+$  for the sake of exposition. As we have seen,  $B_t$  is decreasing in  $t$ , taking on the highest value at  $t = 0$ . Therefore, the government of any period will undertake the entire remainder of the project if there is any left. The unique subgame perfect equilibrium is that  $a_0 = 1$  and  $a_t = 1 - \sum_{i=0}^{t-1} a_i$  for any  $t = 1, 2, \dots$ ;

Government 0 carries out the entire project despite having present-biased preferences.

**Proposition 3** *If the cost of the project is small so that  $(c - 1)/c \leq \beta_1$ , the entire project is carried out in period 0.*

## 4.2 Intermediate Implementation Cost

We turn to the case in which the cost of the project is in the intermediate range:  $\beta_1 < (c - 1)/c \leq \bar{\beta}$ .

Since  $(c - 1)/c \leq \bar{\beta}$  is equivalent to  $B_0 \geq 0$ , the government of any period derives a non-negative payoff from the part of the project it undertakes. However, as Figure 4 shows, the inequality  $\beta_1 < (c - 1)/c$  implies that (8) holds if and only if  $t$  is sufficiently larger than zero. In particular, (8) does not hold for  $t = 0$ ; so each government would be better off postponing the project to some future period. Figure 4 shows the situation in which  $t^* = 2$  (in the case where  $p > 1/2$ ). In this example, the government of any period wishes that the project be undertaken two periods later. It appears that the project is at risk due to the time-inconsistency problem.

Despite the governments' incentive to procrastinate, however, there exists a subgame perfect equilibrium with cyclical strategies, in which the project is successfully completed. Cyclical strategy to complete an indivisible task with an immediate cost and an infinite stream of delayed benefits has been introduced by O'Donoghue and Rabin (2001) in the case of quasi-hyperbolic discounting. Here, we compute the cyclical strategy equilibria under generalized hyperbolic discounting when the task is divisible.<sup>9</sup> In the following calculation, we shall demonstrate that cyclical strategy can implement the policy in our framework of present-biased preferences.

As the middle schedule in Figure 3 indicates, the present value of the net benefit  $B_t$  increases with  $t$  until  $t^*$  is reached and then decreases with  $t$  to 0 as  $t$  tends to infinity. For the case in which  $(c - 1)/c < \bar{\beta}$ , let us define  $\hat{t}$  as

$$\hat{t} = \min \{t \in \mathbf{Z}_+ \mid B_0 > B_t\}.$$

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<sup>9</sup>Matsuyama (1990) proposes cyclical strategy of the same type in a trade liberalization game.

As Figure 3 shows,  $0 < t^* < \hat{t} < \infty$  if  $(c-1)/c < \bar{\beta}$ . If  $(c-1)/c = \bar{\beta}$ , however,  $B_t > B_0$  for any  $t \geq 1$ . Therefore, we define  $\hat{t} = \infty$  in that case. Now, since  $B_t \geq B_0$  for  $t < \hat{t}$  while  $B_t < B_0$  for any  $t \geq \hat{t}$ , we see that  $\hat{t} - 1$  is the maximum tolerable delay, from Government 0's point of view, before the present discounted value of payoffs falls below that arising from immediate implementation. If the project has to be delayed  $\hat{t}$  periods, the current government would rather carry out the entire project immediately.

There are  $\hat{t}$  subgame perfect equilibria such that for any  $\tilde{t} \in \{0, 1, \dots, \hat{t} - 1\}$ ,

$$a_t = \begin{cases} 1 - \sum_{i=0}^{t-1} a_i & \text{if } t = \tilde{t} + j\hat{t}, \text{ for } j = 0, 1, 2, \dots \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

In other words, Government  $\tilde{t}$  is the first government that is to carry out the project, and this government is to carry out the entire project. If Government  $\tilde{t}$  fails to complete the project (in an out-of-equilibrium path), future governments every  $\hat{t}$  periods later are to carry out the entire remainder of the project if there is any left. Thus, in equilibrium, Government  $\tilde{t}$ , which is the only government that actually carries out the project, completes it.<sup>10</sup> We summarize this in the following proposition.

**Proposition 4** *If the cost of the project is in the intermediate range ( $\beta_1 < (c-1)/c \leq \bar{\beta}$ ), there are  $\hat{t}$  subgame perfect equilibria such that the entire project is carried out in one of the periods  $\{0, 1, \dots, \hat{t} - 1\}$ .*

**Proof:** Regardless of its own action, every government receives payoffs from the part of the project that previous governments have carried out. To simplify exposition, we shall ignore this stream of payoffs when we examine the decision of the government. When  $t = \tilde{t} + j\hat{t}$  where  $j = 0, 1, 2, \dots$ , given that a fraction  $1 - \sum_{i=0}^{t-1} a_i$  of the project remains to be undertaken, the government of that period would obtain the payoff  $U_t = \left(1 - \sum_{i=0}^{t-1} a_i\right) B_0$  if it conforms to the equilibrium strategy. If it deviates by carrying out the fraction  $a_t \in [0, 1 - \sum_{i=0}^{t-1} a_i]$ , on the other hand, it would obtain the payoff  $a_t B_{\hat{t}}$ . Since  $a_t < 1 - \sum_{i=0}^{t-1} a_i$ , the former payoff

<sup>10</sup>In a knife-edge case where  $(c-1)/c = \bar{\beta}$ , each government is indifferent between undertaking any feasible fraction of the project and not undertaking it, regardless of how much of the project is left. Therefore, the equilibrium strategy of Government  $t$  is to choose  $a_t \in [0, 1 - \sum_{i=0}^{t-1} a_i]$ . That is, it may choose to undertake any feasible fraction of the project, including zero. One possible outcome is that the implementation process may end prematurely leaving a fraction of the project not undertaken.



is greater than the latter payoff if  $B_0 \geq B_{\hat{t}}$ , which is true by the definition of  $\hat{t}$ . Therefore, we conclude that Government  $t$  conforms to the equilibrium strategy when  $t = \tilde{t} + j\hat{t}$ , for  $j = 0, 1, 2, \dots$ .

Next, we show that Government  $t$  also conforms to the equilibrium strategy (of not doing anything) when  $t \neq \tilde{t} + j\hat{t}$ . Let  $s \in \{1, \dots, \hat{t} - 1\}$  denote the number of periods that have to elapse before the remainder of the project (if any) is to be undertaken. Then, for any given  $\sum_{i=0}^{t-1} a_i$ , the payoff for Government  $t$  when it conforms to the equilibrium strategy equals

$$\left(1 - \sum_{i=0}^{t-1} a_i\right) B_s,$$

whereas the payoff when it deviates by conducting  $a_t \in (0, 1 - \sum_{i=0}^{t-1} a_i]$  of the remaining project equals

$$\left(1 - \sum_{i=0}^t a_i\right) B_s + a_t B_0.$$

The former payoff is greater than the latter payoff if and only if

$$a_t B_s > a_t B_0,$$

$$\Leftrightarrow B_s > B_0,$$

which is satisfied for  $s < \hat{t}$ .

**Q.E.D.**

When  $t = \tilde{t} + j\hat{t}$ , where  $j = 0, 1, 2, \dots$ , Government  $t$  figures that if it does not carry out the entire remainder of the project, it would not be undertaken until  $\hat{t}$  periods later. But since  $\hat{t} - 1$  is the maximum tolerable delay before the payoff falls below that arising from immediate implementation, Government  $t$  would rather complete the entire remainder within its term. When  $t \neq \tilde{t} + j\hat{t}$ , a government is willing to wait for  $\hat{t} - 1$  periods or less, since  $\hat{t} - 1$  is the maximum tolerable delay. A government will wait if the project is to be carried out in  $\hat{t} - 1$  periods or less; but it will not wait if the project is to be carried out  $\hat{t}$  periods later.<sup>11</sup>

The following corollary immediately follows from the fact that  $t^* < \hat{t}$ .

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<sup>11</sup>Take the example of  $\hat{t} = 3$  and  $\tilde{t} = 2$ . Government 0 does not carry out the project as it prefers waiting for Government 2 to complete the project. Government 1 does not undertake the project for the same reason. Government 2, however, carries out the entire project since otherwise it would have to wait three

**Corollary 1** *If the cost of the project is in the intermediate range, there exists a subgame perfect equilibrium in which the entire project is carried out in a future period  $t^*$ , which is the optimal timing for Government 0.*

We have shown that despite the time-inconsistency problem, the project can be successfully carried out in finite time. The citizens wish that the project be carried out immediately since they possess the “usual” geometric discounting. However, though immediate implementation is one equilibrium, there are other equilibria too. So, there may be procrastination. Indeed, given that Government 0 can openly announce a proposed future timing of implementation of the policy, the “focal point” may be that the project is carried out in period  $t^*$ , the most preferred time by Government 0.<sup>12</sup> Therefore, we conclude that procrastination is likely to arise if the cost of the project is in the intermediate range.

### 4.3 High Implementation Cost

We finally consider the case in which  $\bar{\beta} < (c - 1)/c < \delta$ . In this case, we have  $B_0 < 0$ , so that the government of any period would obtain negative payoff from carrying out any positive fraction of the project. Nevertheless, every government wishes that the project be undertaken sometime in the future since  $B_t$  is positive if  $t$  is large enough. To see this claim, we note that

$$B_t = \beta_t \left[ \sum_{k=0}^{\infty} \frac{\beta_{t+k}}{\beta_t} - c \right]. \quad (10)$$

As we have seen in Section 3, the behavior of present-biased preferences is very similar to that of geometric discounting far off in the future, i.e.,  $\beta_{k+1}/\beta_k$  converges to  $\delta$  as  $k$  tends to infinity. Thus,  $\beta_{t+k}/\beta_t = \prod_{i=0}^{k-1} (\beta_{t+i+1}/\beta_{t+i})$  approaches  $\delta^k$  as  $t$  gets larger and larger, and hence the expression in square brackets on the right-hand side of (10) converges to  $\sum_{k=0}^{\infty} \delta^k - c$  as  $t$  tends to infinity. Since  $\sum_{k=0}^{\infty} \delta^k - c > 0$  under the assumption  $1/(1 - \delta) > c$ , we have  $B_t = \beta_t \left[ \sum_{k=0}^{\infty} \frac{\beta_{t+k}}{\beta_t} - c \right] > 0$  when  $t$  exceeds a certain level.

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more periods for the project to be undertaken, which yields a lower payoff. If a part of the project is left to be undertaken in period 3 (which is out of the equilibrium path), the subsequent governments would conform to the prescribed strategies, according to the same logic as above.

<sup>12</sup>For a discussion of focal point equilibrium, see Schelling (1960).

It follows from  $B_0 < 0$  that if the current government expects all future governments to refrain from carrying out the project, it should also stay out of the project. That is, no government wants to be the last to undertake the project. The strategy profile in which  $a_t = 0$  for any  $t$  is a subgame perfect equilibrium.

**Proposition 5** *If the cost of the project is so high that  $(c - 1)/c > \bar{\beta}$ , there is a subgame perfect equilibrium in which the project will not be carried out, to the detriment of the citizens' interest.*

This proposition is certainly bad news for the citizens. Although the project is socially beneficial, there is a possibility of indefinite procrastination. Does there exist any subgame perfect equilibrium in which some governments at least carry out part of the project? The cyclical strategies that we have considered in the last subsection would not work here since the government that is supposed to carry out the entire project certainly prefers obtaining zero payoff by doing nothing to obtaining a negative payoff by conforming to the prescribed cyclical strategy.

No government would want to carry out the project to completion since it would incur a net loss from undertaking the last part of it. Suppose that, contrary to our original assumption, the project is indivisible, then the project will never get done. Thus, we have the following proposition.

**Proposition 6** *If the implementation cost is so high that  $(c - 1)/c > \bar{\beta}$ , and if the project is not divisible, then the socially beneficial project never gets implemented.*

Under circumstances where partial completion of the project is not feasible, there is indefinite procrastination.

Indeed, if the project is to be implemented at all, it must be spread out over time to assure a non-negative payoff for every government. Moreover, the policy implementation process must continue indefinitely, since otherwise the government that completes the project would suffer a loss from the part of the project it undertakes. The following analysis presents such a “gradual implementation equilibrium.”

We shall show that a gradual implementation equilibrium exists if  $\sum_{i=0}^{\infty} B_i > 0$ , i.e., the simple sum of all current and future net benefits is positive. As Figure 3 shows, the schedule of  $B_t$  shifts down as  $c$  increases. The following lemma implies that  $B_i > 0$  for all  $i \geq 1$  when  $(c-1)/c = \bar{\beta}$ , and hence  $\sum_{i=0}^{\infty} B_i > 0$  if  $c$  is sufficiently small while satisfying  $(c-1)/c > \bar{\beta}$ .

**Lemma 1** *If  $\alpha < 1$ , then  $B_t > \beta_t B_0$  for any  $t \geq 1$ .*

The proof of Lemma 1 is relegated to the Appendix. Under the usual geometric discounting such that  $\beta_t = \delta^t$ ,  $B_t$  would be equal to  $\beta_t B_0$ . Under the present-biased preferences, however, the current government puts a disproportionately high weight on the cost incurred in the current period, and so  $B_0$  is disproportionately small.

Now, consider the stationary action profile such that  $a_t = a(1-a)^t$  for some constant  $a \in (0, 1)$ . According to this action profile, every government undertakes the fraction  $a$  of the remainder of the project, and this process continues indefinitely. Consequently, the relevant payoff for Government  $t$  as evaluated in period  $t$  equals

$$\sum_{i=0}^{\infty} [a(1-a)^i B_i]. \quad (11)$$

**Lemma 2** *Suppose  $\sum_{i=0}^{\infty} B_i > 0$ . Then, there exists  $\bar{a} \in (0, 1)$  such that for any  $a \in (0, \bar{a})$ , Government  $t$ 's relevant payoff given by (11) is positive.*

**Proof:** We first notice that  $\sum_{i=0}^{\infty} (1-a)^i B_i$  converges to  $\sum_{i=0}^{\infty} B_i > 0$  as  $a \rightarrow 0$ . Thus, there exists an  $\bar{a}$  such that for any  $a \in (0, \bar{a})$ ,  $\sum_{i=0}^{\infty} (1-a)^i B_i > 0$ , and hence  $\sum_{i=0}^{\infty} a(1-a)^i B_i > 0$ .

**Q.E.D.**

Can this gradual implementation scheme with  $a \in (0, \bar{a})$  be supported as a subgame perfect equilibrium? The answer is “yes” as the following strategy profile is subgame perfect.

$$a_t = \begin{cases} a(1-a)^t & \text{if there has been no deviation from } a_i = a(1-a)^i \text{ for all } i \leq t-1 \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

Hence, we can state the following proposition.

**Proposition 7** *If the cost of the project is sufficiently high that  $(c-1)/c > \bar{\beta}$  but small enough that  $\sum_{i=0}^{\infty} B_i > 0$ , there is a subgame perfect equilibrium in which every government*

*carries out a constant fraction of the remainder of the project so the implementation process goes on indefinitely.*

**Proof:** We show here that the strategy profile (12) is subgame perfect. It follows from Proposition 5 that we need only show that no government has incentives to deviate from the prescribed actions when there has been no deviation in the past. If there has been no deviation, Government  $t$  is to choose  $a_t = a(1-a)^t$ , obtaining a positive payoff from its action (Lemma 2). If Government  $t$  chooses some other level of  $a_t$ , on the other hand, the equilibrium path would switch to the “punitive equilibrium” described in Proposition 5, making the present value of future payoffs zero. Since the one-shot payoff from choosing a positive  $a_t$  for Government  $t$  is negative, the discounted sum of payoffs would be non-positive if the government chooses an  $a_t$  that is not equal to  $a(1-a)^t$ . Hence, Government  $t$  is better off conforming to the equilibrium path than choosing any other levels of  $a_t$ . Therefore, Government  $t$  will choose  $a_t = a(1-a)^t$  if there has been no deviation before period  $t$ .

**Q.E.D.**

As long as the cost of the project is not so large, there also exist some non-stationary, subgame perfect equilibria in which the policy implementation process goes on indefinitely. Consider an action profile  $\{a_t\}_{t=0}^{\infty}$  such that  $a_t > 0$  and

$$\sum_{i=0}^{\infty} a_{t+i} B_{t+i} > 0,$$

for any period  $t$ . It is easy to see that a trigger strategy similar to the above implements this action profile.

Summarizing the above results, we note that inefficient procrastination of the government is a result of the discrepancy between the socially optimal timing of implementation of the project and the optimal timing from the point of view of the government. When the implementation cost is small, the project is worth completing for both the citizens and for the current government, and there is no discrepancy between the optimal timing of implementation for the citizens and that for the government. When the implementation cost

is intermediate, the project is again worth completing for both the citizens and the current government. However, the citizens and the government disagree on the optimal timing of implementation — the government wants it to be later. When the implementation cost is high, the project is not worth undertaking for any government acting alone, even though it is socially beneficial. In this case, indefinite procrastination is a subgame perfect equilibrium, though the project can also be gradually implemented when  $c$  is not too high.

Incidentally, the analysis we have presented can also be applied more generally to a present-biased agent faced with completing a divisible task. The agent can be an individual, a firm or a government. Propositions 6 and 7 together now constitute a new result in the literature on procrastination of completing a task by a present-biased agent. It says that if the task is divisible, the procrastination problem can be somewhat alleviated, though the implementation process will drag on indefinitely. Although it is not worthwhile for any present self to undertake any fraction of the task by her action alone, it can still be worthwhile for the present self to carry out a fraction of the task when there is an expectation that future selves would also undertake some fractions of it. The present self can “cooperate” with future selves by agreeing on a “focal point” subgame perfect equilibrium so as to get the task carried out gradually. The divisibility of the task allows a present-biased individual to bypass the fate of indefinite procrastination of the task. The existence of “gradual implementation equilibria” is a new result in the literature.

## 5 Concluding Remarks

We have shown that under a two-party political system the party in office tends to be present-biased and time-inconsistent. This may lead to inefficient procrastination of socially beneficial projects. Procrastination arises because a party’s chance of being in office in any future period is less than one, and that it discounts social welfare when it is not in office. Amador (2003) also includes a model of multiple parties competing for office leading to present-biased government. It is notable that his modeling of the present-biased government is a special case of ours: there is no incumbent advantage or disadvantage in being elected

in the next election, and a party puts zero weight on social welfare when not in office. Thus, his government is characterized by quasi-hyperbolic discounting instead of generalized hyperbolic discounting. Moreover, the emphasis of his paper is quite different from ours. While we undertake detailed analysis of policy implementation of a divisible project with immediate cost and delayed benefits, he focuses on the sovereign debt problem.

Our paper also contributes to the literature on procrastination of a present-biased agent when she is faced with a divisible task that requires incurring an immediate cost and yields delayed benefits. We investigate the problem by way of analyzing the implementation of a socially beneficial project by a present-biased government. We find that the procrastination problem tends to get more serious as the cost of the project gets higher. When the cost is low, there is no procrastination problem. When the cost is intermediate, there is likely to be some procrastination. When the cost is high, the project can be procrastinated indefinitely, though there exist equilibria in which the project is implemented gradually. The existence of gradual implementation equilibria in the face of a divisible task is a new result in the literature.

A possible extension of this research is to endogenize the probability of a party being elected and the weight a party puts on social welfare. It is also worthwhile to investigate the implications of asymmetry among different parties of the probabilities of being elected.

# Appendix

## Proof of Lemma 1:

To prove Lemma 1, it suffices to show that  $\beta_{t+k}/\beta_t > \beta_k$ , or  $\beta_{t+k} > \beta_t\beta_k$ , for any  $t \geq 1$  and  $k \geq 1$ , since  $B_t = \beta_t [\sum_{k=0}^{\infty} (\beta_{t+k}/\beta_t) - c]$  and  $B_0 = \sum_{k=0}^{\infty} \beta_k - c$ . Recall equation (5) and define

$$f(\alpha) \equiv \alpha + (1-\alpha) \frac{(2p-1)^{t+k} + 1}{2} - \left[ \alpha + (1-\alpha) \frac{(2p-1)^t + 1}{2} \right] \left[ \alpha + (1-\alpha) \frac{(2p-1)^k + 1}{2} \right].$$

It is easy to see that  $\beta_{t+k} > \beta_t\beta_k$  if and only if  $f(\alpha) > 0$ .

Now,

$$\begin{aligned} f(0) &= \frac{(2p-1)^{t+k} + 1}{2} - \frac{(2p-1)^t + 1}{2} \cdot \frac{(2p-1)^k + 1}{2} \\ &= \frac{1}{4} [1 - (2p-1)^t] [1 - (2p-1)^k], \end{aligned}$$

which is positive as  $-1 < 2p-1 < 1$ . In addition,  $f(1) = 0$ . Moreover, since

$$f''(\alpha) = -2 \left( 1 - \frac{(2p-1)^t + 1}{2} \right) \left( 1 - \frac{(2p-1)^k + 1}{2} \right) < 0,$$

the function  $f$  is a concave function. Thus, we have shown that  $f(\alpha) > 0$  for any  $\alpha \in [0, 1)$ .

## Proof of the claim that $\sum_{k=0}^{\infty} \beta_1^k < \sum_{k=0}^{\infty} \beta_k < \sum_{k=0}^{\infty} \delta^k$ :

We first note that  $\beta_1^0 = \beta_0 = \delta^0 = 1$  when  $k = 0$ , and that  $\beta_1 < \delta$  when  $k = 1$ . For  $k \geq 2$ , we use the equation

$$\beta_k = \beta_1 \sum_{i=1}^{k-1} \frac{\beta_{i+1}}{\beta_i} \tag{13}$$

to show that  $\beta_1^k < \beta_k < \delta^k$ . It is obvious that we need only show these inequalities in order to prove the claim.

The first inequality is easy to show. Indeed, it follows immediately from (13) and  $\beta_1 < \beta_{i+1}/\beta_i$  that  $\beta_1^k < \beta_k$ . If  $p \geq 1/2$ , it is also straightforward to derive the second inequality  $\beta_k < \delta^k$ , since the fact that  $\beta_0 = \delta^0$ ,  $\beta_1 < \delta$ , and  $\beta_{i+1}/\beta_i \leq \delta$  for any  $i \geq 1$  together with (13) imply that  $\beta_k < \delta^k$  for any  $k \geq 1$ .

In the case where  $p \leq 1/2$ , however, the above proof does not apply since  $\beta_{i+1}/\beta_i > \delta$  for any odd  $i$  as Figure 2 shows. So, in this case, we use the inequality

$$\frac{\beta_{i+1}}{\beta_i} \cdot \frac{\beta_{i+2}}{\beta_{i+1}} = \frac{\delta^2 [1 + \alpha + (1-\alpha)(2p-1)^{i+2}]}{1 + \alpha + (1-\alpha)(2p-1)^i} < \delta^2,$$



which is valid when  $i$  is even, to show that  $\beta_k < \delta^k$  for any  $k \geq 1$ :

$$\beta_k = \sum_{j=0}^{(k/2)-1} \frac{\beta_{2j+1}}{\beta_{2j}} \cdot \frac{\beta_{2(j+1)}}{\beta_{2j+1}} < \delta^{2(k/2)} = \delta^k$$

for an even  $k$ , and

$$\beta_k = \left[ \sum_{j=0}^{(k-3)/2} \frac{\beta_{2j+1}}{\beta_{2j}} \cdot \frac{\beta_{2(j+1)}}{\beta_{2j+1}} \right] \frac{\beta_k}{\beta_{k-1}} < \delta^{2((k-1)/2)} \cdot \delta = \delta^k$$

for an odd  $k$ .

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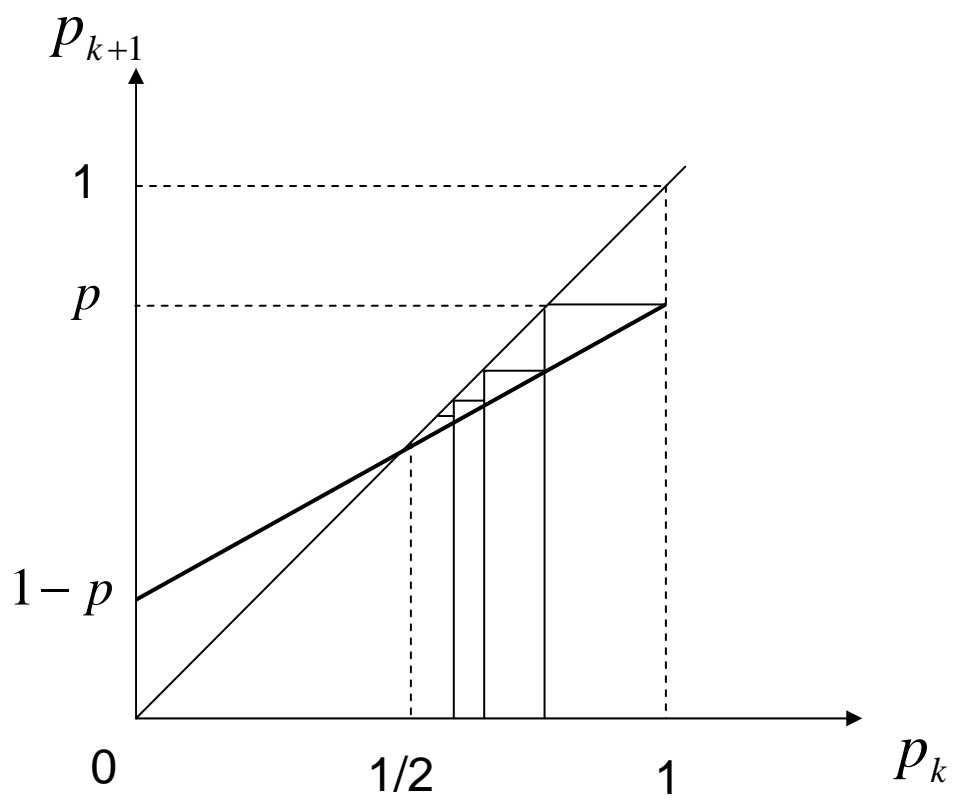


Figure 1. The Transition of the Probability of being in Office when  $p > 1/2$

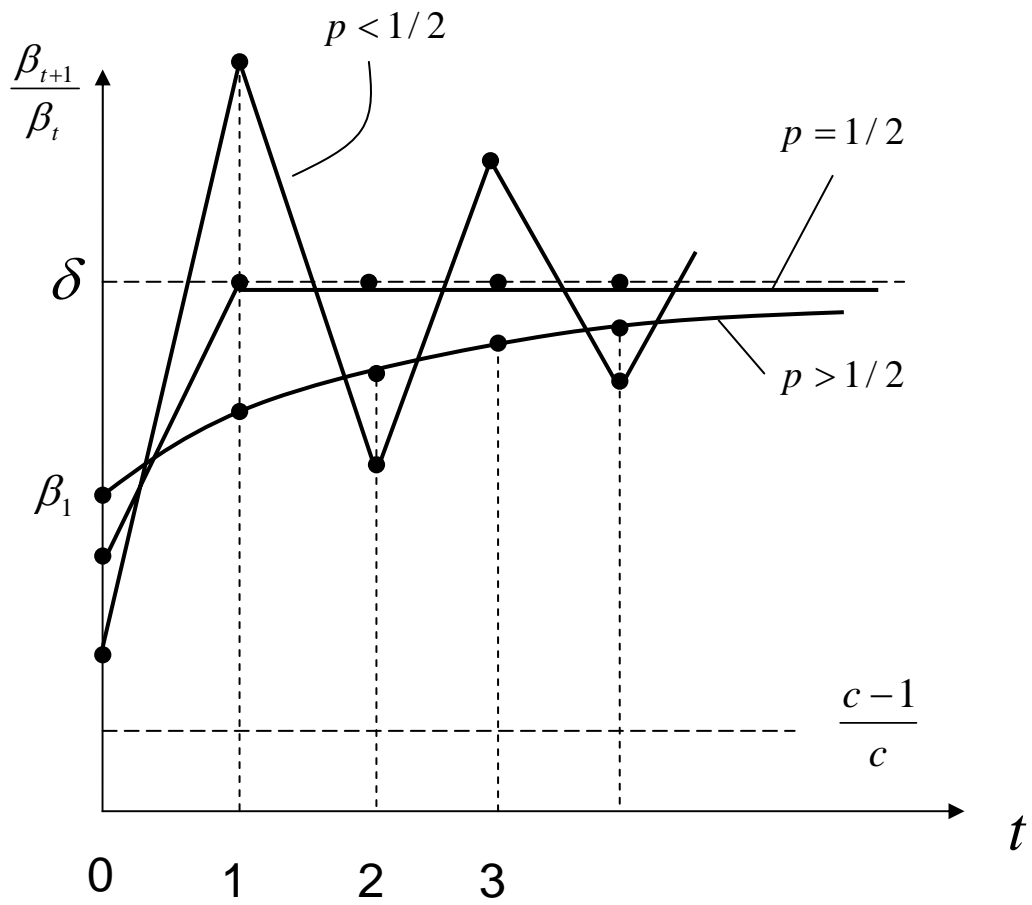


Figure 2. The Case of Low Implementation Cost with different values of  $p$

$$\sum_{k=0}^{\infty} \beta_{t+k} - \beta_t c \equiv B_t$$

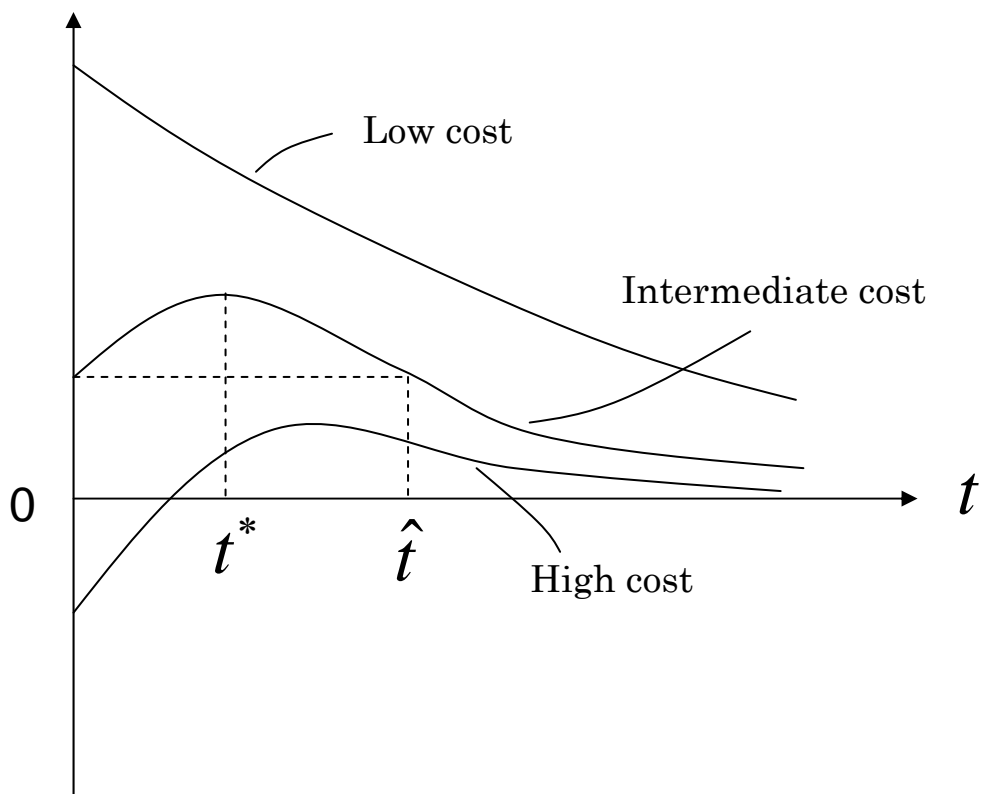


Figure 3. Present Value of the Net Benefits of a project that is done at period  $t$ , as evaluated at period 0 (treating  $t$  as continuous variable)

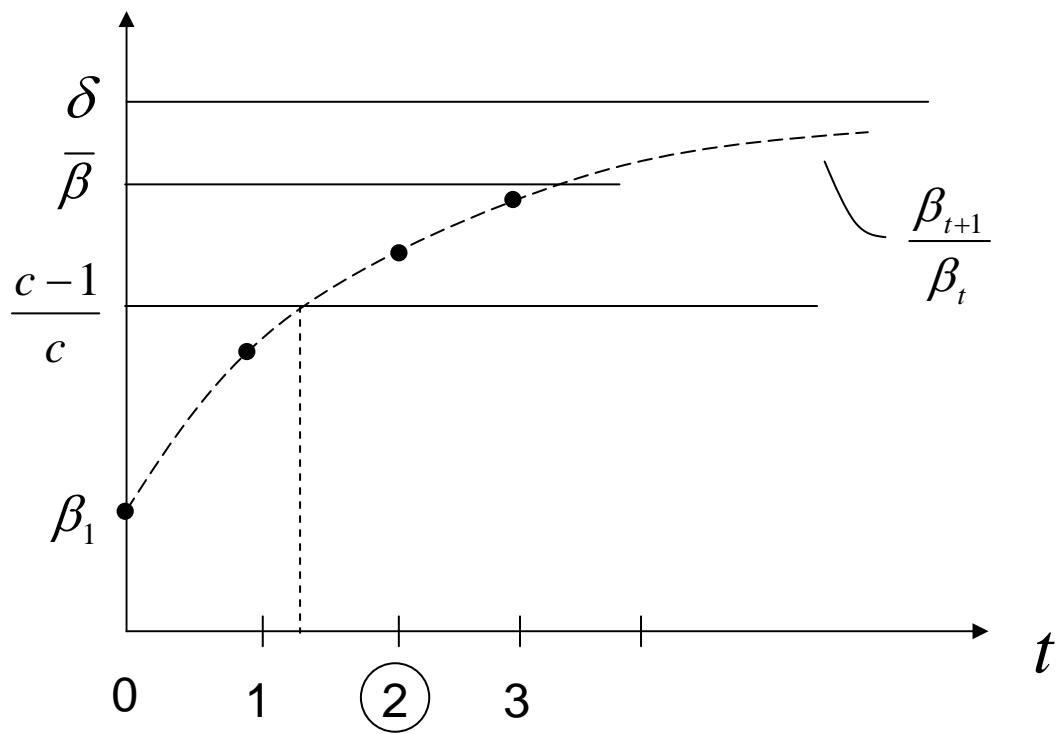


Figure 4. The Case of Intermediate Implementation Cost when  $p > 1/2$