Bidding Strategies in Parallel Internet Auctions[*]

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Abstract

This paper proposes a simple model for multiple second-price auctions which run parallel to each other, in the sense that though they might not begin or end at the same time, they have certain periods of overlap. We characterize the equilibrium bidding strategy of the buyers and the equilibrium price of each auction. Last-minute bids arise naturally in the equilibrium. We show that, except for the last auction, the maximum price a buyer is willing to pay is less than his valuation, and that the *ex ante* expected transaction prices are identical for all auctions. A simple empirical test is also performed to verify the theoretical implication of the model.

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1 Introduction

A recent surge in the importance of online auctions has given rise to much academic interest in their theoretical investigation. Research on online auctions calls for its own theoretical modeling, rather than relying on traditional auction theory, not only because they have different rules regulating bidding behavior and price determination, but also because the environments in which online auctions are conducted are often different from that of traditional auctions. One of the differences is that in traditional auctions the buyers usually participate in only one auction at any given time, while in online auctions buyers can almost costlessly switch between auctions selling identical items. Consequently, there is the possibility of cross-bidding behavior. That is, buyers may switch between auctions to arbitrage the difference in expected payoffs.

The purpose of the paper is to propose a theoretical model for what we call "parallel auctions" which, though not necessarily beginning or ending at the same time, have overlap in certain periods. We characterize buyers' bidding behavior, together with the equilibrium prices in all auctions. It is shown that although the buyers can participate in more than one auction at any time, in the equilibrium they will never cross-bid. Instead, they will only compete in the auction which ends first among all the auctions and, if they lose, then enter the auction which ends first among remaining auctions. The implication for this is that the standing price of an auction will start to rise only after it becomes the one that will end first among all auctions. Moreover, in each auction (except for the last one), since a bidder has the option to buy the item in latter auctions, the maximum amount he is willing to pay is strictly less than his valuation, and depends on the number of future auctions. We also show that buyers will fully take into consideration the option value of participating in future auctions so that, although the expected valuation of active buyers decreases over auctions, the expected transaction prices are identical for all auctions.

An important implication of our model is that, when the arrival of new auctions is stochastic, buyers will wait until the last moment of an auction to submit their bids. This gives a rationale for the "last-minute-bid" behavior which is well-recorded in the online auction literature. Most of the literature explaining this behavior relies on correlation of the bidders' valuations (so that delay of bid avoids early revelation of private information) or the coordination of bidders.[1] Our result suggests that correlation or coordination are not necessarily the reasons for last-minute bid: As long as there are parallel auctions, last-minute-bid behavior arises even if valuations are independent, and without bidders' coordination.

We collect online auction data from eBay to verify the implication of the theoretical model. Specifically, we observe the standing prices of 187 parallel auctions selling an identical object, and test (a) whether the standing price of an auction rises significantly only after it becomes the first auction to end; and (b) whether the rate of increases in standing price is particularly large at the last moment of an auction. Our empirical results strongly support the theoretical prediction.

Our theoretical model is closely related to the sequential auctions model (Milgrom and Weber, 2000; Wang, 2006), where auctions are held sequentially and one object is sold in each auction. Our model is different from the sequential auction model in two main aspects.

[1] See, for example, Roth and Ockenfels (2002), Stryszowska (2004), Bajari and Hortaçsu (2004), Ockenfels and Roth (2006), Ockenfels, Reiley and Sadrieh (2006) and Wang (2006).

First, in the sequential auction model, all buyers are present from the first auction and stay until they win one object and leave. In our model the buyers enter sequentially. This is a better approximation of online auctions. Second, in the sequential auction model, an auction starts only after the previous one is ended, and at any time there is only one active auction so that there is no room for cross-bidding. In our model the auctions overlap in time. Our model is also related to the literature on competing auctions (Peters and Severinov, 1997, 2006), where multiple auctions are held simultaneously, with each auction selling an identical object. Their model differs from ours in that, first, since all auctions start and end at the same time, buyers will necessarily cross-bid. (Our result implies that the bidders never cross-bid even if they are allowed to.) Second, since all bidders are present at the time when the auctions begin, and stay until the winner in every auction is known, there is no issue of entry and exit.

2 The Model

In this section, we present a model which is a simplified version of eBay auctions. Our purpose is to characterize the bidder's strategy and the implied equilibrium prices of each auction. The equilibrium is derived via backward induction. That is, we first derive the equilibrium in the final period, then the equilibrium at any earlier period.

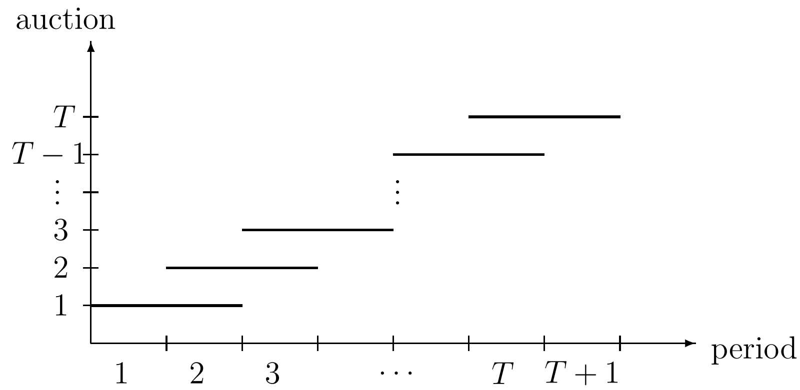

Figure 1: Timing of Auctions

2.1 The Environment

We model eBay auctions as ascending auctions.[2] Assume that T English auctions, each with one identical object to be sold, will start sequentially but with overlaps. Every auction lasts for two periods, with auction i starting at the beginning of period $t = i$ and ending at $t = i + 1$. (i=1, $\cdots$, T.) Therefore, auction i has a one-period overlap with auctions $i - 1$ (in period $t = i$) and $i + 1$ (in period $t = i + 1$), respectively, and in each period there are only two active auctions. The timing of auctions is illustrated in Figure 1.

One buyer enters in each period. The valuation of the object to a buyer, v, is his private information, but is drawn from $[0, \bar{v}]$ independently according to the commonly known density function $f(v)$, which is assumed to be non-degenerate. Let $F(v)$ be the distribution function of $f(v)$. We use $v_t^{(2)}$ to denote the second-highest valuation among the buyers in period t. Every buyer is risk-neutral and needs only one object. It is costless to bid. There is

[2] Zeithammer and Adams (2006) empirically test the bidding behavior on eBay and find the data are better described by ascending auctions rather than real-bid auctions.

no discount between periods. A buyer's payoff is his valuation less the transaction price if he wins the object, and is zero otherwise. The buyers' objective is to maximize their expected payoffs. Since all buyers have inelastic unit demand, the winning bidder leaves.

Auctions are conducted in the following way. At the beginning of a period, the standing price of the new auction is at zero, while the standing price of the auction which began in the previous period is its standing price at the end of the previous period. A buyer can choose to bid on both auctions, one of the auctions, or none of them. According to eBay's price determination rule, the standing price is set to be the minimum for the highest bidder to win, and is essentially a second-price auction. The standing price of an auction remains at zero until one of the buyers place bids. Thus a good scenario for the bidder's strategy and price determination is that in each period, the bidder decides whether to bid in an auction and, if yes, indicates the maximum he is willing to bid in the auction. Standing price in that auction for that period is the second highest among all bids. When no active buyers chooses to submit a new bid on any auction, the period ends.

Our goal is to derive the equilibrium prices of every auction in the two periods it stays active. Let p_t^i denote the equilibrium standing price of auction i at the end of period t.[3] Since auction $t-1$ ends in period t, p_{t-1}^t is its final transaction price. In addition, let $\pi_t^i(v, p_t^{t-1}, p_t^t)$ denote the expected payoff of the highest bidder (whose valuation of the object is v) in auction i at the end of period t, when the standing prices of auctions $t-1$ and t are p_t^{t-1} and p_t^t, respectively.

Obviously, in every auction there must be a bidder who wins the auction and leaves.

[3] Note that since an auction lasts for only two periods, given any t the value of i can only be $t-1$ or t.

For if in any equilibrium no one places any bid, a bidder must find it profitable to bid the minimum increment. Since in every period $t \geq 2$, one bidder wins (and leaves) and one bidder arrives, it must be that in every period $t \geq 2$, there are only two bidders.

2.2 Equilibrium in Period $T+1$

The price of auction T at the beginning of period $T+1$ is its standing price at the end of period T, p_T^T. Since there is only one auction, the environment is identical to an English auction with starting price p_T^T. Both bidders will bid up to their valuations of the object, and therefore the equilibrium transaction price for auction T is $p_{T+1}^T = \max(p_T^T,\, v_{T+1}^{(2)})$. The winner of auction T is the buyer with the higher valuation in period $T+1$.

2.3 Equilibrium in Period T

We begin our analysis by computing the expected payoffs of the two active auctions in period T. Since, at the end of period T, the highest bidder in auction $T-1$ will win the auction for sure,

$$\pi_T^{T-1}(v,\, p_T^{T-1},\, p_T^T) = v - p_T^{T-1}.$$

The expected payoff of the highest bidder in auction T at the end of period T , $\pi_T^T(v,\, p_T^{T-1},\, p_T^T)$, is more involved. It depends on the realization probabilities of the following three events. First, the entrant in period $T+1$ has a higher valuation than his. In that case he loses and has zero payoff. This occur with probability $\int_v^{\bar{v}} f(x)dx$. Second, the entrant in period $T+1$ has a valuation lying between 0 and p_T^T. In that case $p_{T+1}^T = p_T^T$, and his payoff is

$v - p_T^T$. This occurs with probability $\int_0^{p_T^T} f(x)dx$. Finally, the entrant of period $T+1$ has a valuation lying between p_T^T and v. In that case he will win with a price equal to the entrant's valuation. This occurs with probability $\int_{p_T^T}^{v} f(x)dx$. As a result,

$$\pi_T^T(v, p_T^{T-1}, p_T^T) = \int_0^{p_T^T} f(x)(v - p_T^T)dx + \int_{p_T^T}^{v} f(x)(v-x)dx = \int_{p_T^T}^{v} F(x)dx. \qquad (1)$$

The second equality in (1) is by integration by parts.

Active buyers in period T can bid in both auction $T-1$ and auction T. However, we will show that in equilibrium the bidders choose to bid only in auction T. This can be seen by observing that (1) is a decreasing function of p_T^T. That is, bidding up the price of auction T in period T only decreases a buyer's expected payoff in that auction. Since the price of auction T rises only if some bidders chooses to bid in it, this implies that refraining from bidding in auction T is a dominant strategy for every bidder in period T, which in turns implies that $p_T^T = 0$ in equilibrium. In fact, this result holds not only in period T, but also in every other period:

Lemma 1. *In any period t, it is a dominant strategy for all buyers only to place bids in auction $t-1$ (or in the case when $t=1$, not to bid at all), and the equilibrium price of auction t in period t, p_t^t, is 0.*

Proof. Suppose $p_t^t = \bar{p} > 0$ and bidder B is the highest bidder in auction t at the end of period t. Note that if B is the highest bidder in auction t, then (since a bidder only needs one item) he is not bidding in auction $t-1$, and therefore will lose auction $t-1$ and enters period $t+1$. (Remember that auction t ends in period $t+1$, not period t.) Let us consider the following alternative bidding strategy: *Instead of bidding in auction t in period t, bidder*

B *waits until period* $t+1$ *to bid* $\bar{p}$ *in auction* t. There are two possibilities. First, if the period-($t+1$) entrant bids above $\bar{p}$ in auction t, then B loses under both the original strategy and the alternative strategy. That means the two strategies give the same payoff to B in this case. Second, if the maximal price that period-($t+1$) entrant is willing to bid is lower than $\bar{p}$, then (because the starting price of auction t in period $t+1$ is its standing price at the end of period t) under the original strategy bidder B will win by paying $\bar{p}$. However, under the alternative strategy bidder B will win by paying the entrant's bid, which is lower than $\bar{p}$. That is, bidder B's payoff will be strictly higher than the original strategy in this case. Combining the two cases, we know that the alternative strategy will yield the same payoff (namely, zero) as the original strategy if B loses in period $t+1$, and strictly higher payoff if he wins. We thus show that any strategy which results in a strictly positive value of p_t^t is dominated by one which refrains from bidding in auction t in period t. Consequently, in any period t the bidders only compete in auction $t-1$, and $p_t^t = 0$. $\square$

The intuition for the result is clear: In period t, being the highest bidder in auction t does not have any advantage. This is because, as long as the standing price p_t^t is less than his valuation v, whether one will win auction t in period $t+1$ depends solely on the valuation of the entrant in period $t+1$, which is independent of p_t^t. Moreover, bidding up the value of p_t^t in order to be the highest bidder of auction t in period t has its disadvantage. It increases the expected price he needs to pay if he wins in the next period. In other words, bidding up p_t^t only increases the expected price a bidder needs to pay in period $t+1$ if he wins, without increasing the probability that he will win in that period. Thus refraining from bidding in auction t is a dominant strategy for any bidder in period t.

According to Lemma 1, the equilibrium price of auction t is zero in period t. The expected payoff of bidding in auction t in period t is thus equal to the expected payoff of being the highest bidder in the next period at price zero. Moreover, this expected payoff is not affected by the price of auction $t-1$. Consequently, $\pi_t^t(\cdot)$ depends neither on p_t^t nor on p_t^{t-1}, and we can denote the bidder's expected payoff of auction t in period t as

$$S_t(v) \equiv \pi_t^t(v,\, p_t^{t-1},\, 0),$$

which depends only on the bidder's own valuation of the object.

Given the value of p_T^{T-1} and that $p_T^T = 0$, a bidder's decision of whether to bid in auction $T-1$ or auction T is simple: He should bid in auction $T-1$ if and only if $\pi_T^{T-1}(v,\, p_T^{T-1},\, 0) > \pi_T^T(v,\, p_T^{T-1},\, 0)$. Note that

$$\frac{\partial\Big(\pi_T^{T-1}(v,\, p_T^{T-1},\, p_T^T) - \pi_T^T(v,\, p_T^{T-1},\, p_T^T)\Big)}{\partial v} = 1 - F(v) > 0. \tag{2}$$

The expected payoff of auction $T-1$ relative to that of auction T is increasing in the buyer's valuation. This implies that, given the values of p_T^{T-1} and p_T^T, the higher a buyer's valuation, the more he prefers auction $T-1$ relative to auction T. Together with Lemma 1, we know that the winner of auction $T-1$ must be the highest-valuation bidder in period T. In order that he wins, p_T^{T-1} must rise to a level so that the second-highest-valuation bidder is just indifferent between auctions $T-1$ and T, and forced to enter the latter. The standing price of auction T at the end of period $T-1$, p_T^{T-1}, therefore must satisfy

$$\pi_T^{T-1}(v_T^{(2)},\, p_T^{T-1},\, 0) = \pi_T^T(v_T^{(2)},\, p_T^{T-1},\, 0); \;\; \textit{i.e.},$$
$$p_T^{T-1} = v_T^{(2)} - \int_0^{v_T^{(2)}} F(x)dx.$$

Note that $p_T^{T-1}(v_T^{(2)}) = v_T^{(2)} - \int_0^{v_T^{(2)}} F(x)dx$ is a function which maps the second-highest valuation of buyers in period T to the equilibrium standing price of auction $T-1$. The equilibrium standing price of auction $T-1$ is thus a function of only the highest losing bidder's valuation at period T. Moreover, since a buyer keeps bidding on auction $T-1$ until he is indifferent between the two active auctions, the function $p_T(v) \equiv v - \int_0^v F(x)dx = v - S_T(v)$ is actually a buyer's bidding function. That is, given a bidder's valuation v, the maximum he is willing to pay for auction $T-1$ is $p_T(v)$. Finally, since $F(\cdot)$ is non-degenerate, it must be that $p_T^{T-1} < v_T^{(2)}$. We thus have the following lemma.

Lemma 2. *The equilibrium transaction price of auction $T-1$, p_T^{T-1}, depends only on the highest-losing bidder's valuation, $v_T^{(2)}$, and that $p_T^{T-1} < v_T^{(2)}$.*

In a standard English auction, the equilibrium price is the valuation of the second-highest bidder. Lemma 2 shows that, when the bidders have the option to buy the same item in another auction that follows, the equilibrium price is strictly lower than that in a standard English auction, by an amount exactly equal to the expected payoff of participating in the latter auction. The equilibrium in period T is summarized in Proposition 1.

Proposition 1. *In period T, buyers only place bids in auction $T-1$, and thus $p_T^T = 0$. A buyer with valuation v is willing to pay up to $p_T(v) = v - \int_0^v F(x)dx$. The buyer with the higher valuation wins auction $T-1$ with price $p_T^{T-1} = p_T(v_T^{(2)})$.*

2.4 Equilibrium in Period $t < T$

Given the bidding function $p_T(v)$ and the expected payoff of the future auction $S_T(v)$ in period T, we can proceed to derive the equilibrium in the previous periods. First we derive equilibrium in period $T-1$.

2.4.1 Period $T-1$

By Proposition 1, a buyer's expected payoff of entering future auctions at the end of period $T-1$, $S_{T-1}(v)$, consists of two components. First, if the period-T entrant has a valuation lower than his, then he will win auction $T-1$ with a price p_T^{T-1}, which depends on the entrant's valuation. Second, if the period-T entrant has a higher valuation, he will lose and enters period $T+1$, whose expected payoff is $S_T(v)$. Consequently,

$$S_{T-1}(v) = \int_0^v \Big(v - p_T^{T-1}(x)\Big) f(x)dx + \int_v^{\bar{v}} S_T(v) f(x)dx. \tag{3}$$

The first term on the right-hand-side of (3) is the expected payoff of the bidder when he wins auction $T-1$. The second term is his expected payoff when he loses auction $T-1$ in period T and therefore enters period $T+1$. Substituting the price function $p_T^{T-1}(v)$ into (3), and using integration by parts, we have

$$\begin{aligned} S_{T-1}(v) &= \Big[v - p_T^{T-1}(x)\Big] F(x)\Big|_0^v + \int_0^v F(x)\frac{dp_T^{T-1}(x)}{dx} + S_T(v)\Big[1 - F(v)\Big] \\ &= \left[\int_0^v F(x)dx\right] \cdot F(v) + \int_0^v F(x)\Big[1 - F(x)\Big]dx + \int_0^v F(x)dx\Big[1 - F(v)\Big] \\ &= \int_0^v F(x)\Big(2 - F(x)\Big)dx. \end{aligned} \tag{4}$$

Since auction $T-2$ ends in period $T-1$, the payoff of being the highest bidder in this auction is $\pi_{T-1}^{T-2}(v,\, p_{T-1}^{T-2},\, p_{T-1}^{T-1}) = v - p_{T-1}^{T-2}$. Again, since (by Lemma 1) $p_{T-1}^{T-1} = 0$ in equilibrium, we can see that $\pi_{T-1}^{T-2}(v,\, p_{T-1}^{T-2},\, 0) - \pi_{T-1}^{T-1}(v,\, p_{T-1}^{T-2},\, 0)$ is an increasing function of v. Similar to the reasoning for auction $T-1$ in Section 2.3, the winner of auction $T-2$ is the buyer with the highest valuation in period $T-1$, and its standing price p_{T-1}^{T-2} must rise to the level at which the second-highest-valuation buyer is indifferent between getting the item by paying p_{T-1}^{T-2} and entering the next auction. Thus the equilibrium price is

$$p_{T-1}^{T-2}\Big(v_{T-1}^{(2)}\Big) = v_{T-1}^{(2)} - S_{T-1}\Big(v_{T-1}^{(2)}\Big) = v_{T-1}^{(2)} - \int_0^{v_{T-1}^{(2)}} F(x)\Big(2 - F(x)\Big)dx.$$

2.4.2 The General Case

In general, using induction on i we can show that

$$\begin{aligned} S_{T-i}(v) &= \int_0^v \Big(v - p_{T-i+1}^{T-i}(x)\Big) f(x)dx + \Big(1 - F(v)\Big) S_{T-i+1}(v) \\ &= \left[v - p_{T-i+1}^{T-i}(x)\right] F(x)\Bigg|_0^v + \int_0^v \left\{F(x)\frac{dp_{t-i+1}^{T-i}(x)}{dx}\right\}dx + \Big(1 - F(v)\Big) S_{T-i+1}(v) \\ &= S_{T-i+1}(v) + \int_0^v F(x)\left[1 - \frac{dS_{T-i+1}(x)}{dx}\right]dx. \end{aligned}$$

We can further substitute $S_{T-i+1}(v)$ with $S_{T-i+2}(v)$, which can in turn be written in terms of $S_{T-i+3}(v)$, and so on. Eventually, $S_{T-i}(v)$ can be written only in terms of $S_{T-1}(v)$ which, by (4), is $\int_0^v F(x)\Big(2 - F(x)\Big)dx$. After some algebra, we have

$$S_{T-i}(v) = \sum_{j=0}^{i} \int_0^v F(x)\Big[1 - F(x)\Big]^j dx = \int_0^v \left[1 - \Big(1 - F(x)\Big)^{i+1}\right]dx.$$

Note that $S_{T-i}(v) > S_{T-i+1}(v)$ for all i=1, $\cdots$, $T-1$. That is, expected payoff of entering future auctions is lower.

Similar to the case in Section 2.4.1, it is easy to see that the expected payoff of the current auction relative to that of entering future auctions is an increasing function of a buyer's valuation. (That is, $\pi_t^{t-1} - \pi_t^t$ is increasing in v.) This implies that in any period t, the highest valuation buyer will outbid the other bidder and wins auction $t-1$. The transaction price of auction $t-1$ must rise to the level so that buyer with the second highest valuation is just indifferent between bidding on the current auction and entering future auctions. Consequently, the equilibrium price of auction $t-1$ at the end of period t must satisfy

$$p_{T-i}^{T-i-1}\left(v_{T-i}^{(2)}\right) = v_{T-i}^{(2)} - S_{T-i}\left(v_{T-i}^{(2)}\right) = v_{T-i}^{(2)} - \int_0^{v_{T-i}^{(2)}} \left[1 - \left(1 - F(x)\right)^{i+1}\right] dx, \tag{5}$$

which depends only on the valuation of the loser.

The *ex post* transaction price of an auction given in equation (5) depends on the random draw of the buyers' valuations. However, we can compute the *ex ante* expected transaction price for every auction. Since in any period t, it is always the buyer with higher valuation to win auction $t-1$ and the buyer with lower valuation to lose and enter auction t, $v_t^{(2)}$ is not only the value of the lower-valuation bidder in period t, but is actually the lowest valuation of *all* buyers entering from period 1 to t. This fact makes it easy to calculate the distribution function of $v_t^{(2)}$, and the *ex-ante* expected transaction price of auction $T-i-1$ is

$$\begin{aligned} E\left(p_{T-i}^{T-i-1}\right) &= \int_0^{\bar{v}} \left(v_{T-i}^{(2)} - S_{T-i}\left(v_{T-i}^{(2)}\right)\right) g\left(v_{T-i}^{(2)}\right) dv_{T-i}^{(2)} \\ &= G\left(v_{T-i}^{(2)}\right)\left(v_{T-i}^{(2)} - S_{T-i}(v_{T-i}^{(2)})\right)\Big|_0^{\bar{v}} - \int_0^{\bar{v}} \left(1 - S'_{T-i}(v_{T-i}^{(2)})\right) G\left(v_{T-i}^{(2)}\right) dv_{T-i}^{(2)}; \end{aligned} \tag{6}$$

where $g(\cdot)$ and $G(\cdot)$ are the density and distribution functions of the order-statistic of the lowest valuation among $T-i$ bidders, respectively.

Since

$$S_{T-i}(v) = \int_0^v \left[1 - \Big(1 - F(x)\Big)^{i+1}\right] dx,$$

we know that

$$S'_{T-i}(v) = 1 - \Big(1 - F(v)\Big)^{i+1}.$$

Also, the distribution function of the order-statistic of the lowest valuation among $T - i$ bidders is

$$G\Big(v_{T-i}^{(2)}\Big) = 1 - \Big(1 - F(v_{T-i}^{(2)})\Big)^{T-i}.$$

Equation (6) therefore becomes

$$\begin{aligned} E\Big(p_{T-i}^{T-i-1}\Big) =& \bar{v} - S_{T-i}(\bar{v}) - \int_0^{\bar{v}} \left[1 - F\Big(v_{T-i}^{(2)}\Big)\right]^{i+1} \cdot \left[1 - \Big(1 - F(v_{T-i}^{(2)})\Big)^{T-i}\right] dv_{T-i}^{(2)}. \\ =& \bar{v} - \int_0^{\bar{v}} \left[1 - \Big(1 - F(v)\Big)^{i+1} + \Big(1 - F(v)\Big)^{i+1} - \Big(1 - F(v)\Big)^{T+1}\right] dv \\ =& \bar{v} - \int_0^{\bar{v}} \left[1 - \Big(1 - F(v)\Big)^{T+1}\right] dv \\ =& \int_0^{\bar{v}} \Big(1 - F(v)\Big)^{T+1} dv. \end{aligned} \tag{7}$$

The expected transaction price of auction $T - i - 1$, $E(p_{T-i}^{T-i-1})$, is thus independent of i, implying that the expected transaction price of every auction is the same. There is no tendency for the transaction price either to rise or fall across auctions. Also note that the maximum value a bidder is willing to bid for auction t (i.e., his bidding function) in period $t + 1$ is

$$p_{t+1}(v) = v - S_{t+1}(v). \tag{8}$$

Since $S_t(v)$ is an decreasing function of t, we know from (8) that $p_{t+1}(v)$ is increasing in t. That is, a buyer is willing to bid higher, all else equal, in later auctions.

Theorem 1. *The dominant bidding strategy of a buyer with valuation v in period t is to only bid in auction $t-1$, so that $p_t^t(v) = 0$ for $1 \le t \le T$. The bidding strategy of the bidder is $p_t(v) = v - S_t(v)$ for $2 \le t \le T$; where*

$$S_t(v) = \int_0^v \left[1 - \left(1 - F(x)\right)^{T-t+1}\right] dx.$$

In each period t $(2 \le t \le T)$, the buyer with higher valuation wins auction $t-1$ with a price $p_t\left(v_t^{(2)}\right) = v_t^{(2)} - S_t\left(v_t^{(2)}\right)$. The lower-valuation buyer loses in auction $t-1$ and enters auction t. Then expected transaction price of every auction is $\int_0^{\bar{v}} \left(1 - F(v)\right)^{T+1} dv$.

2.5 Properties of the Equilibrium

There are several properties implied by Theorem 1. First, the auction is efficient in the sense that the items are sold to the T highest-valuation bidders.

Second, the equilibrium price is not the second-highest-valuation of bidders, as is the case for the standard English auction. This is because the auctions that end later carry a positive option value. Buyers are thus not willing to bid up to their valuations (as in the English auction) for the item. Rather, the maximum a bidder is willing to pay is his valuation minus the option value of participating in future auctions. The existence of other auctions selling the same items decreases the expected payoff of the seller. In our model, the number (and order) of auctions is fixed throughout. However, in reality, new auctions might open at any moment which *end earlier than any existing auctions.* If the arrival of a new auction is stochastic rather than deterministic as in our model, a buyer would like to wait until the last minute of an auction before submitting a bid because he wants to wait for more information on the number of future auctions. This gives a rationale for the last-minute bid behavior in online auctions.

Third, even if two auctions have certain overlap, they run like sequential auctions: Buyers first compete in the auction that ends earlier, and enter the auction which ends next only when they lose in the earlier auction. As a result, the price of the auction which ends in later period will always stay at zero, and will start to rise only when it becomes the auction which ends first. Moreover, by our result in the previous paragraph this tendency of rising price is stronger, the closer the auction moves towards its end.

Fourth, although buyers in the parallel auctions bid in a way as if the auctions were held sequentially, the two auction formats are not equivalent. In particular, the bidding strategies are different. We will compare the two types of auctions in more detail in Section 3.

3 Comparison to Sequential Auctions

The main difference between our model and the sequential auction model is that, in the latter model, all bidders are present right from the beginning, with the winner in each auction exiting sequentially. Those who do not win will participate in subsequent auctions. Since bidders observe equilibrium price for each completed auction, they will learn more about other bidders' valuations after each auction is completed. An important issue in the sequential auctions is thus information revelation over auctions. In our model, the bidders enter sequentially. Every time an entrant enters, nothing beyond the common prior is known about his valuation. Expected payoff of future auctions only depends on future entrants, not on any of the current rival buyers. Information update is thus not an issue because the two competing bidders in an auction will never meet again later (one of them wins and exits).

Even when informational update is not an issue, the bidding strategy of the bidders is different between sequential and parallel auctions. For example, consider the case of two auctions and three buyers. Suppose the valuations of the bidders are independently drawn from $[0, 1]$ uniformly. Then from (8), the bidding strategy of the bidder in our parallel auction model for auction 1 (in period 2) is $p(v) = v - v^2/2$; while in the sequential auction model, the bidding strategy is $p(v) = v/2$. (See Proposition 15.3 and Example 15.2 of Krishna, 2002.)

Despite the difference in strategy, qualitatively our results are similar to the sequential auctions. First, the price a bidder is willing to pay is higher in later auction: In our model, it is easy to see that $p_{t+1}^t(v) > p_t^{t-1}(v)$ for all t=1, $\cdots$, $T-1$ and for all v. This property also holds for sequential auctions. Second, the expected transaction prices are the same across auctions for both sequential and parallel auctions.[4] Third, the expected transaction price in our model is actually identical to the ex ante expected transaction price of the sequential auctions with T objects and $T + 1$ bidders. The proof is straightforward: In both models the expected transaction price for the last auction is the expected valuation of the losing bidder. Since the expected transaction prices in both models are identical across auctions, the expected transaction prices are the same for both types of auction.[5]

[4] For proof of this fact in the sequential auction, see Section 15.1.3 of Krishna (2002).

[5] Note that this is true only for ex ante expected transaction price. Since there is informational update in the sequential auction, the expected transaction price of an object when some objects have already been sold (and whose prices are known) will depend on the realized transaction prices of sold objects.

4 Empirical Evidence

One important implication of our theoretical model is that the buyers start to bid in an auction only when it becomes the first auction to end. This in turn implies that the standing price of an auction starts to rise only when it becomes the auction which ends first. Moreover, when new auctions arrive randomly as in the case of eBay auctions, buyers tend to wait until the last moment (to make sure that the auction in which he bids is indeed the one to end first) of an auction to submit their bids. In this section, we perform a simple empirical test to verify these implications.

4.1 Regression Equation

Consider an auction i, which lasts for a duration of T_i days. (All time lengths are measured in the unit of days in this section.) Use p_{it} to denote its *standing price at t days since the begin of the auction* ($t \in [0, T_i]$). Auction i becomes the first auction to end at time F_i ($F_i \in [0, T_i]$). We would like to test whether the rate of increase in auction i's standing prices varies before and after time F_i. In addition, we test for the existence of *bidding at the last moment* Δ. For example, a *last-minute bid* is one which is submitted when $t > T_i - \Delta$, where in our regression Δ is either 1 hour or one minute or one second.

The regression equation is

$$p_{it} = \beta_1 t + \beta_2 [t - F_i]_+ + \beta_3 [t - (T_i - \Delta)]_+ + \xi_i + \varepsilon_{it}, \tag{9}$$

where $[t - F_i]_+ \equiv \max\{t - F_i, 0\}$ is the time since the auction becomes the first-to-end auction; $[t - (T_i - \Delta)]_+ \equiv \max\{t - (T_i - \Delta), 0\}$ is the time in the last moment of an auction; ξ_i is

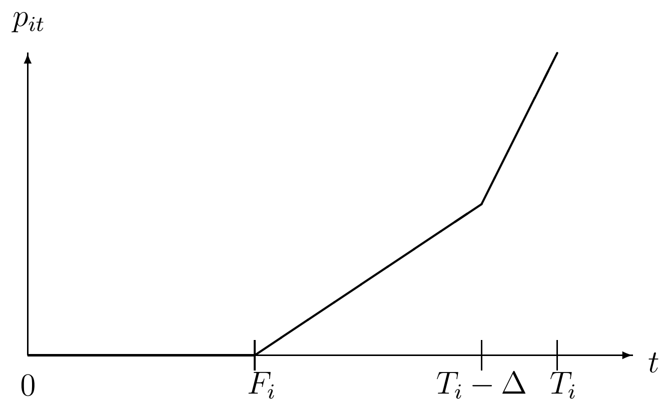

Figure 2: Regression Equation

the fixed effect of an auction, which captures any effect which may influence the willingness to pay for the items, such as the shipping term, seller's reputation, starting bid, the number of other auctions conducted at the same time, etc.. Figure 2 demonstrates a possible shape of the regression equation.

By our theoretical model, a buyer places a bid in auction i only for $t \in (F_i, T_i]$. Therefore, we expect $\beta_1 = 0$ and $\beta_2 > 0$. Moreover, since the arrival of a new auction is stochastic in eBay, buyers do not know the option value of future auctions exactly. They therefore delay submitting their bids in order to obtain more information on the number of future auctions. Consequently, we expect $\beta_3 > 0$.[6]

[6] We view our regression as a test for the signs of β_1, β_2, and β_3. We do not consider heterogeneity of these parameter as in a random coefficient model.

4.2 Data

The data are collected from U.S. eBay auctions between January 1 and September 17, 2007. We collect the bidding history for all auctions selling an identical item: *Casio G-Shock World Time Data Memory Watch G2900F*. In the estimation, we only include auctions which are shipped from the U.S. because shipping costs are much higher for cross-border transactions, and buyers probably do not treat a watch shipped from the U.S. the same as one shipped from a different country. Nine auctions ended by *Buy-It-Now* rather by bidding, and are excluded in our regression.

There are 178 auctions that remain during our research period. The average duration of an auction is 4.38 days. We observe a total of 892 submitted bids. On average, an auction receives 5.01 bids. The maximal number of bids in an auction is 19, while 26.4% of auctions receive no bid. We also record the standing prices at the beginning and the end of an auction. Consequently, we observe 1248 standing prices in total. See Table 1 for the descriptive statistics of the data.

4.3 Results

The estimation results are presented in Table 2. Specification (A) only tests the effect of being a first-to-end auction. In the other three specifications (B) – (D), we also estimate the effect of bidding at the last moment. We consider three different lengths of the last moment: Δ equals "one hour", "one minute", or "one second".

These estimation results are consistent with our theoretical predictions. The estimates

Table 1: Descriptive statistics of the data

Variable	Mean	Standard Deviation	Minimum	Maximum
p_{it}	18.00	15.38	0.00	66.00
t	2.48	1.72	0.00	10.00
$[t - F_i]_+$	0.98	1.14	0.00	8.92

Notes: The number of observations is 1248.

Table 2: Regression results

Parameter	Specification A	Specification B Δ=1 hour	Specification C Δ= 1 minute	Specification D Δ= 1 second
β_1	0.129	-0.680	-0.477	-0.253
	(0.426)	(0.432)	(0.437)	(0.442)
β_2	10.245*	8.956*	9.720*	10.072*
	(0.910)	(0.863)	(0.873)	(0.893)
β_3		190.254*	8889.009*	341423*
		(19.232)	(1001.725)	(44105)
within-R^2	0.389	0.452	0.423	0.402

Notes: Huber-White robust standard errors are in parentheses. The superscript * represents significance at 1% level.

of β_1 and β_2 are robust to the inclusion of last-moment-bidding effects. We find β_1 to be insignificantly different from zero in all four specifications, suggesting that buyers do not submit bids when an auction is not the first-to-end auction. On the contrary, the estimates of β_2 are significantly positive. The standing prices of an auction start to grow when it becomes the first one to end. Note that β_2 remains significantly positive after we control for last-minute bids in specifications (B) – (D). This strongly indicates that the reason the price of an auction starts to rise is that it has become the first auction to end, not because it is about to end.

The ascending rate of standing prices increases drastically ($\hat{\beta}_3 > 0$) at the last moment. Comparing the estimates of β_3 in Specifications (B), (C), and (D), we find the rising rate of standing prices to accelerate as the closing time of an auction approaches. Since the arrival of new auctions in eBay is stochastic, our result indicates that many buyers wait until the final moment of an auction to determine how much to bid (i.e., to determine how much is the value of $S_t(v)$).

5 Conclusion and Extension

In this paper, we propose a theoretical model of parallel auctions, and characterize the equilibrium bidding strategy and price in each auction. Last-minute bidding is shown to be consistent with equilibrium behavior because of the uncertainty about future auctions. Data collected from eBay confirm our theoretical implications.

A strong assumption we impose on the model is that only one bidder enters in every period. If, more generally, more than one entrant enters, informational update will become an important issue. Moreover, the informational updating process will be substantially more complicated in our model than in the sequential auctions. This is because the identity of the winner will matter. For example, suppose a buyer in period 5 faces two other buyers in an auction. The information he possesses, when the other two buyers were entrants of periods 2 and 3, will be different to when they were, say, entrants of periods 2 and 4. This is because the period-3 entrant has lost twice while the period-4 entrant only once. More information is therefore revealed in the former case. Since the identity of buyers has to be taken into consideration when more than two buyers are active in each period, the informational updating process will be very complicated.

If we assume that the new entrants can observe all the past bids, as they sometimes do in the online auctions, then our results will extend to the case with more than one entrant. We can easily show that it is still a dominant strategy for a bidder to refrain from bidding on auction t in period t. Moreover, since all bidders have the same information of past bids, the bidding function in (8) will become

$$p_{t+1} = v - E\left[S_{t+1}(v)\Big|\, \Omega_{t+1}\right];$$

where Ω_{t+1} is the commonly available information in period $t+1$, revealed by past bids. However, since in every period only one bidder wins and leaves but more than one bidder enters, the number of bidders will increase over auctions, as will the expected transaction prices.

When the new entrants cannot observe past bids, then the information they possess will be different from the bidders who lost in previous auctions. In that case it is difficult to characterize the bidding function and, therefore, equilibrium prices. While we believe that the key results in this paper remain true in the more general setting, rigorous proof awaits future research.

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