Modelling choice of information acquisition*

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Abstract

This paper constructs a static model of information acquisition when the agent does not know exactly what pieces of information he is missing. The paper shows a representation of preferences over information channels by adapting the model of unforeseen contingencies by Dekel et al. (2001), which is an extension of Kreps (1979, 1992). One implication of the paper is that one can define the value of an information channel even if one cannot define the value of each piece of information the channel would deliver. Some possible applications of the results are discussed.

1 Introduction

There is no doubt that the recent development of communication technologies, including but not limited to telecommunication and transport technologies, has made communication far easier. In particular, the development of the internet has made communication between distant places drastically easy, cheap and rapid, which was difficult, expensive and slow only a few decades ago.

The economics literature has traditionally been paying much of its attention to situations in which economic agents have difficulties in acquiring information, while they know what pieces of information they are missing. However, on many occasions, we encounter innumerable pieces of information that are readily accessible but it is beyond our capacity to process all of them. Hence, like it or not, we can only pick up part of the abundant information available to us.

To further our analysis, we consider two separate cases. One is a case in which an agent knows exactly what pieces of information he is missing. The other is a case in which an agent does not know exactly what pieces

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of information he is missing. While the existing literature studies the first case in various contexts, the latter case is virtually unexplored as far as we understand.[1] This paper is aiming at modelling the latter case.

When an agent does not know exactly what pieces of information he is missing, it is impossible for him to select the exact piece(s) of information to acquire. Nevertheless, he may well be able to select an information channel that will deliver information to him. For example, we usually subscribe to a newspaper or a magazine without really knowing what pieces of information it will deliver. Or, when we use the search engine on the internet such as google or yahoo, we do not really know what pieces of information will be returned to us. It is obvious that we are not choosing *pieces* of information (e.g. exact contents of a newspaper), but instead, we are choosing *information channels* (e.g. a newspaper). The existing literature only focusses on the choice of pieces of information, while this paper focusses on the choice of information channels.

One line of studies related to this paper is that on the value of information. The literature has largely been inspired by a classical paper by Blackwell (1953), which gives a definition of *informativeness* that in turn provides a basis for a definition of value of information in a statistical decision problem. Hirshleifer (1971) presents an example in which more information makes the whole economy/society worse off. As a generalisation of Hirshleifer's example, Sulganik and Zilcha (1997) argues that more information may be disadvantageous because it may affect the opportunity sets in the economy. Although there have been a quite a few works that exhibit results in which more information may be disadvantageous, as far as we understand, all of them assume that the agents know exactly what piece of information they are missing (or can acquire). In particular, the literature assumes that the standard expected utility representation holds.

Another line of studies related to this paper is the literature of search models. There is an enormous amount of works in this literature, and thus, it is beyond the scope of this paper to review the works thoroughly. The literature in general assumes rational expectations, and thus, it assumes that it is the environmental limitation that prevents the agents from obtaining all opportunities including information, but not the limited human capability. In other words, the literature assumes that the agent knows exactly what opportunities or pieces of information they can acquire potentially, but cannot acquire all of them at once because of environmental limitations.

The other line of studies related to this paper is that of unforeseen contingencies.[2] The literature studies a situation in which an agent *does not* specify the full structure of the state space, although it does not necessarily mean that he *cannot* specify the full structure. Kreps (1979) develops a model that captures preference for flexibility, while Kreps (1992) reinterprets the model and its results in the context of unforeseen contingencies. In his model, the agent chooses a menu of deterministic options before the uncertainty is resolved, while the deterministic option is chosen after the

[1] Of course, this may well be due to our ignorance.

[2] See Dekel et al. (1997) for a literature review.

uncertainty is resolved. To evaluate different menus, the agent specifies a subjective state space that partitions the true state space; thus, the specification of the subjective state space reflects the agent's estimate of the true state space. Unfortunately, the subjective state space thus constructed is not unique, which in turn implies that the same preference may have multiple representations.

To cope with this problem, there are some extensions to Kreps (1992). Nehring (1999) considers Savage acts that give menus as consequences, while acts are replaced with subjective lotteries by deriving a subjective probability distribution; thus, he effectively considers lotteries over menus. With this structure, a unique representation of the preference is shown. Instead of considering lotteries over menus, Dekel et al. (2001) considers menus of lotteries, and shows that there is a unique state space for all representations of the ex ante preferences given that the ex post preferences have expected utility representations. This paper adapts the results by Dekel et al. (2001) to describe choice of information channels.

The rest of the paper proceeds as follows. Section 2 introduces some examples that involve information acquisition. In so doing, the well celebrated classical theorem by Blackwell (1953) is introduced. Section 3 exhibits a formal model of information acquisition when the agent does not know what pieces of information he is missing. Some representation results are shown there, while the examples presented in section 2 are reviewed in the light of the formal model. Section 4 discusses the implications and the limitations of the results. Section 5 concludes the paper.

2 Examples

In this section, we examine three examples to see how an agent's lack of complete knowledge about the missing information matters in describing his optimisation problem. The examples will be reexamined later in the light of the formal model that will be presented in the subsequent section.

2.1 Example 1: Statistical decision problem

Suppose there is a meteorologist, who will be given a reward depending on the accuracy of his weather forecast. More specifically, let $\mathcal{S} = \{1, 2, ..., S\}$ be the set of states of nature, and the random object of interest is a random variable whose realisations are described as r_s, while his payoff is $w_s = 100 - (r_s - f)^2$ for $s = 1, 2, ..., S$, where f is the forecast of the meteorologist.

Before announcing his forecast, the meteorologist has access to a number of data sources that are supposed to provide information about the state to realise. Let $\mathcal{J} = \{1, 2, ..., J\}$ be the set of data sources. Assume, however, he can pick up only one data source. For simplicity, assume that each source provides N sorts of data: For example, the data of data source j is $\mathbf{y}^j = (y_1^j, y_2^j, ..., y_N^j)$, i.e. N-dimensional vector. Each data source then can be characterised as an $S \times N$ stochastic matrix, $\mathbf{P}^j$, whose typical element $p_{s,n}^j$

is the probability of observing $\{y_n^j = y_{s,n}^j\}$, i.e. $y_{s,n}^j$ is the realisation of y_n^j in state s.

The meteorologist is an expected utility maximiser, and thus, when he chose data source k he is facing an optimisation problem:

$$\max_f E_Q \left\{ u \left(100 - (r - f)^2 \right) \middle| \mathbf{y}^j \right\}.$$

Let $V_Q(\mathbf{y}^j, u)$ denote the value function of the problem, where Q indicates the probability belief of the meteorologist. Then, he solves

$$\max_{j \in \mathcal{J}} E \left\{ V_Q(\mathbf{y}^j, u) \right\},$$

while $V_Q(j, u)$ denotes the value function of this problem.

Following Blackwell (1953), it is common to define *informativeness* as follows:

Definition: *j is said to be* more informative *than j', which is denoted by $j \succeq j'$ if there is an $N \times N$ stochastic matrix $\mathbf{Z}$ such that $\mathbf{P}^{j'} = \mathbf{P}^j \mathbf{Z}$.*

With the notion of informativeness, the following well known theorem is useful to characterise the solution to the above meteorologist's problem:

Theorem 1 (Blackwell, 1953): *$j \succeq j'$ if and only if $V_Q(j, u) \geq V_Q(j', u)$ for all Q, u.*

Hence, it can be said that the meteorologist should choose the most informative data source j^*. However, this characterisation and more fundamentally the structure of the problem assumes that the meteorologist has a thorough understanding of the environment. Namely, he can specify the exact structure of data source j, i.e. the $S \times N$ matrix $\mathbf{P}^j$ for all $j \in \mathcal{J}$.

There are a few cases in which the meteorologist does not know the exact structure of all possible data sources. First, the set $\mathcal{J}$, i.e. the collection of data sources the meteorologist recognises, may be rather small compared to the set of potentially available data sources. Second, the meteorologist may not know what sort of data a data source actually provides. In this case, he may not know if there are N sorts of data, or he may know that there are N sorts of data, but does not know what they are exactly. Third, the meteorologist may not be confident enough to assign specific probabilities $\mathbf{P}^j$.

The first case does not really raise a concern here, because the expected utility framework per se does not require the set $\mathcal{J}$ to include all possible data sources even if there are no environmental limitations. Under rational expectations in the sense that the agents know the true probability, however, $\mathcal{J}$ must indeed be exhaustive unless there is some environmental limitation.

On the other hand, the second case clearly violates the assumption of the above model. In this case, the meteorologist cannot specify $\mathbf{P}^j$. This paper provides a resolution to this case. However, the third case is not within the scope of this paper. This case is really concerning ambiguity, or Knightian uncertainty.

2.2 Example 2: Investment

Consider a firm who is making an investment decision. The firm is considering to expand its production, but the current location has not enough space, and thus, looking for another site to accommodate an additional production facility.

The investment consists of two stages: the first stage is really the investment decision per se, which involves a purchase of a land site and production facilities constructions (i.e. production capacity is determined), and in the second stage, the firm decides how much to produce. The first stage takes place before the uncertainty is resolved, while the second stage take place after the uncertainty is resolved. These two stages can be modelled as follows: the first stage is choice of a menu B, and the second stage is a choice $a \in B$.

However, before making an investment, the firm can collect information that has some relevance to the investment. There are J information sources that provide such information, while each source provides m^j pieces of information for all $j \in \mathcal{J} \in \{1, 2, ..., J\}$, i.e. source j provides $\mathbf{y}^j = (y_1^j, y_2^j, ..., y_{m^j}^j)$. Somehow, the firm can use only one information source.

Because there is a possibility that the products may not be sold out, there remains some unmodelled uncertainty at the end of second stage. We assume that the preference of the firm at the second stage by an expected utility $u(\cdot, s)$ given that state s realises (for the uncertainty modelled). Thus, we define the value function for the optimisation problem in state s by

$$v(B) = \max_{a \in B} u(a, s).$$

Then, if the firm is an expected utility maximiser also in the first stage, the optimisation problem of the firm at the end of the first stage is

$$V(\mathbf{y}^j) = \max_{B} E\left\{ v(B) \,\middle|\, \mathbf{y}^j \right\},$$

while that of the firm at the beginning of the first stage is

$$\max_{j \in \mathcal{J}} E\left\{ V(\mathbf{y}^j) \right\}.$$

As in example 1, the full expected utility framework assumes that the firm knows exactly that the information source provides information $\mathbf{y}^j = (y_1^j, y_2^j, ..., y_{m^j}^j)$ although it is still allowed that the joint probability distribution for the relevant random variables including $\mathbf{y}^j$ is subjective. Namely, the firm knows exactly what pieces of information it is lacking. Hence, the constraint such that the firm can only use one source at a time makes sense only when it is environmental, not due to limited human capability.

2.3 Example 3: Job search

Suppose an agent is looking for a job. First he acquires information about the job opportunities, and then chooses which one to apply. From the agent's point of view, job opportunities are uncertain. To describe such uncertainty, we introduce the following random variable that describe the job opening status of position k:

$$X^k = \begin{cases} 1 & \text{if there is an opening} \\ 0 & \text{otherwise.} \end{cases}$$

Physically, there exist K positions (finite), and we denote the set of those positions by $\mathcal{K}$. Let σ denote the σ-field generated by $(X^1, X^2, ..., X^K)$. Because of limitations of the environment (i.e. information technologies, transport technologies), however, the agent is unable to obtain all information. Namely, the agent's information set will not be σ. Instead, there are subsets of σ that the agent can acquire indexed by j: $\sigma^j \subset \sigma$ for all $j = 1, 2, ..., J$, while the collection of such sub σ-fields is denoted by $\mathcal{J} := \{\sigma^1, \sigma^2, ..., \sigma^J\}$.

After obtaining an information set σ^j, the agent chooses a position a (to apply for) in accord with his preference (expected utility):

$$v_j(\sigma^j) = \max_{a \in B(\sigma^j)} E\left\{u(a) \,\middle|\, \sigma^j\right\} \quad \text{s.t.} \quad B(\sigma^j) := \left\{k \in \mathcal{K} \,\middle|\, X^k \in \sigma^j\right\},$$

where $B(\sigma^j)$ is the choice set of agent h when he acquired information set σ^j and u is the utility function. The choice set $B(\sigma^j)$ is defined as such because one does not apply unless he knows the position. We may define a more restrictive choice set by adding a condition such that $X^k = 1$, but this does not change the structure at all since an application for a position that has no opening will never be chosen, and thus, the payoff structure itself is unaffected by this alteration. If the agent's preference is represented by an expected utility, the agent's problem concerning information acquisition is

$$\max_{j \in \mathcal{J}} E v_j(\sigma^j).$$

However, it is usually the case that the structure of σ or $\mathcal{J}$ is not really understood by the agent due to a limitation of human capability. Namely, he does not know exactly what positions are potentially available, and what pieces of information he can obtain from each information channel j. In other words, he does not know the exact structure of the state space, and thus, an expected utility framework cannot be applied directly.

3 Representation

In this section, we construct a model in which an agent does not know exactly what pieces of information he is missing, but recognises the information channels. We then show how we can represent a preference over the information channels by adapting the results in Dekel et al. (2001). We evaluate the model by referring to the examples we introduced in the previous section so as to make it more explicit what the real issues are.

3.1 The model

Consider an agent who is facing a static optimisation problem with uncertainty. First an agent acquires information. To do so, he chooses an information channel j in $\mathcal{J}$, the set of information channels available to the agent (and the ones he recognises), which is assumed to be a finite set. Also, the agent does not necessarily know exactly the pieces of information he would obtain through the chosen channel. $\mathcal{M}_j$ is the state space of information channel j perceived by the agent, while its typical element is denoted by m_j (there are M_j members in $\mathcal{M}_j$). We assume that $\mathcal{M}_j$ partitions the true state space of the information channel j. Namely, there is no surprise for the realisation of information from channel j. Furthermore, we denote the preference over the information channels by $\succ$, e.g. $j \succ j'$, which reads "information channel j is strictly preferred over channel j'."

After choosing an information channel and acquiring information from it, but before the uncertainty itself is resolved, he chooses a menu of options. In particular, let B_{m_j} denote the menu the agent chooses when information channel j was chosen and the contents of the information turns out to be m_j. Then, after the uncertainty is resolved he chooses an option $a \in B_{m_j}$, which is a lottery of a finite set C of n prizes (i.e. C is n-dimensional). Let $\Delta(C)$ denote a set of probability distributions on C. Hence, a typical element of $\Delta(C)$ is a, while a typical subset of it is B^j. At this stage, there may exist a further source of (unmodelled) uncertainty, but we just assume that such an uncertainty is evaluated with respect to expected utility unlike the uncertainty before.

The agent recognises S states *a priori* (finite) and the set of such states $\mathcal{S}$, which is a partition of the true state space. In fact, the agent perceives that $\mathcal{S}$ has the following structure:

$$\mathcal{S} = \prod_{m_j \in \mathcal{M}_j} \mathcal{S}_{m_j},$$

where $\mathcal{S}_{m_j}$ is the perceived space of (unresolved) states when he acquires information m_j. $\mathcal{S}_{m_j}$ has S_{m_j} members.

Although his recognition of the structure of the state space is limited, he has an *ex ante* preference $\succ_{m_j}$ over the corresponding menus $B \succ_{m_j} B'$ given information m_j. Here, the term ex ante refers to the resolution of the uncertainty. On the other hand, he has an *ex post* preference $\succ_s^*$ over a, which is represented by a state dependent expected utility function $u : \Delta(C) \times \mathcal{S} \mapsto \Re$ by assumption. The *ex ante* preference given information m_j, $\succ_{m_j}$, is represented by the following:

$$V_{m_j}(B) = U_{m_j}\left(\left(\sup_{a \in B} u(a,s)\right)_{s \in \mathcal{S}_{m_j}}\right), \forall m_j \in \mathcal{M}_j, \quad (1)$$

where $U_{m_j} : \Re^{S_{m_j}} \mapsto \Re$ is an aggregator function. Moreover, the preference over the information channels $\succ$ is represented by

$$v(j) = \varphi_j\left(\left(\sup_{B} V_{m_j}(B)\right)_{m_j \in \mathcal{M}_j}\right), \forall j \in \mathcal{J}, \quad (2)$$

where $\varphi_j : \Re^{M_j} \mapsto \Re$ is an aggregator function. With (1) and (2), we can define an aggregator function $\hat{\varphi} : \Re^{\mathcal{S}} \mapsto \Re$ such that

$$v(j) = \hat{\varphi}\left(\left(\sup_{a \in B_{m_j}} u(a,s)\right)_{s \in \mathcal{S}}\right), \forall j \in \mathcal{J}, \tag{3}$$

where B_{m_j} is the solution to $\sup_B V_{m_j}(B)$ if such a solution exists.

While $\mathcal{S}$ is referred to as the state space, it is merely an index set that partitions the true state space. Following Dekel et al. (2001), we define the subjective state space $\mathbf{P}(\mathcal{S}, u)$ as

$$\mathbf{P}(\mathcal{S}, u) := \{\succ_s^* |s \in \mathcal{S}\}, \tag{4}$$

and thus, $\mathbf{P}(\mathcal{S}_{m_j}, u) := \left\{\succ_s^* \middle| s \in \mathcal{S}_{m_j}\right\}$.

3.2 Representation of preferences

Clearly, our formulation for the ex ante preference given information m_j follows that of Dekel et al. (2001). Hence, their representation results for $\succ_{m_j}$ hold here, too. As we mentioned in the introduction, their representation identifies a unique subjective state space. While our ultimate aim is to identify a unique subjective state space $\mathbf{P}(\mathcal{S}, u)$, we first want to show a uniqueness result with respect to $\mathbf{P}(\mathcal{S}_{m_j}, u)$, the subjective state space given information m_j. It is clear that if the aggregator U_{m_j} is allowed to ignore certain states, we could never obtain such a uniqueness result since we can arbitrarily add or delete such states. Thus, we restrict our attention to relevant states, which we define as follows:

Definition: *Given a representation of the form (1) with $\mathbf{P}(\mathcal{S}_{m_j}, u)$ finite, a state $s \in \mathcal{S}_{m_j}$ is* ***relevant*** *if there exist B and B' such that either $B \succ_{m_j} B'$ or $B' \succ_{m_j} B$ and for any $s' \in \mathcal{S}_{m_j}$ with $\succ_s^* \neq \succ_{s'}^*$, $\sup_{a \in B} U_{m_j}(a, s') = \sup_{a \in B'} U_{m_j}(a, s')$.*

Next, we define a weak EU representation of $\succ_{m_j}$ as follows:

Definition: *A* ***weak EU representation*** *of $\succ_{m_j}$ is a triple $(\mathcal{S}_{m_j}, u, U_{m_j})$ such that*

(i) *V_{m_j} as defined in (1) is continuous and represents $\succ_{m_j}$,*

(ii) *Each $u(\cdot, s)$ is an expected utility function.*

(iii) *Every $s \in \mathcal{S}_{m_j}$ is relevant.*

(iv) *If $s, s' \in \mathcal{S}_{m_j}$, $s \neq s'$, then $\succ_s^* \neq \succ_{s'}^*$.*

Note that although the aggregator U_{m_j} is defined as a function on $\Re^{S_{m_j}}$, it is only relevant on the subspace $\mathcal{U}_*(\mathcal{S}_{m_j}, u) := \{(\sup_{a \in B} u(a,s))_{s \in \mathcal{S}_{m_j}} | B \subset \Delta(C)\}$. Moreover, we introduce the following definition to define uniqueness of the aggregator U_{m_j}:

Definition: *Let $R^i_{m_j} = (\mathcal{S}^i_{m_j}, u^i, U^i_{m_j})$, $i = 1, 2$ be weak EU representations of some preferences $\succ_{m_j}$. If the subjective state spaces of these representations are finite, then $R^1_{m_j}$ and $R^2_{m_j}$ are* ***essentially equivalent*** *if the following hold:*

(i) *The subjective state spaces are the same: $\mathbf{P}(\mathcal{S}^1_{m_j}, u^1) = \mathbf{P}(\mathcal{S}^2_{m_j}, u^2)$.*

(ii) *There is a bijection $h_{m_j} : \mathcal{S}^2_{m_j} \mapsto \mathcal{S}^1_{m_j}$ and functions $\gamma_{m_j} : \mathcal{S}^2_{m_j} \mapsto \Re_+$ and $\delta_{m_j} : \mathcal{S}^2_{m_j} \mapsto \Re$ such that for any $u^1_* \in \mathcal{U}_*(\mathcal{S}^1_{m_j}, u^1)$, the vector $g_{m_j}(u^1_*)$ defined by*

$$g_{m_j}(u^1_*)(s^2) = \gamma_{m_j}(s^2) u^1_*(h_{m_j}(s^2)) + \delta_{m_j}(s^2)$$

is contained in $\mathcal{U}_*(\mathcal{S}^2_{m_j}, u^2)$. The function $g_{m_j} : u_*(\mathcal{S}^1_{m_j}, u^1) \mapsto u_*(\mathcal{S}^2_{m_j}, u^2)$ is a bijection.

(iii) *Up to a monotonic transformation, $U^1_{m_j}(u^1_*) = U^2_{m_j}(g_{m_j}(u^1_*))$ for all $u^1_* \in \mathcal{U}_*(\mathcal{S}^1_{m_j}, u^1)$.*

Before stating a representation theorem, we introduce the following axioms:

Axiom 1 (Weak Order): *$\succ_{m_j}$ is asymmetric and negatively transitive.*

Axiom 2 (Continuity): *The strict upper and lower contour sets of $B \subset \Delta(C)$, $\{B' \subset \Delta(C) | B' \succ_{m_j} B\}$ and $\{B' \subset \Delta(C) | B \succ_{m_j} B'\}$, are open (in the Hausdorff topology).*

Axiom 3 (Nontriviality): *There is some B and B' such that $B \succ_{m_j} B'$.*

Axiom 4 (Indifference to Randomisation): *For every $B \subset \Delta(C)$, $B \sim_{m_j} conv(B)$, where $conv(B)$ denotes the convex hull of B.*

With the above axioms, we have the following representation theorem, which is a direct adaptation of Theorem 1 of Dekel et al. (2001) in our context:

Theorem 2 (Weak EU representation of $\succ_{m_j}$):

(a) *The ex ante preference $\succ_{m_j}$ has a weak EU representation if and only if it satisfies Axioms 1–4.*

(b) *If an ex ante preference $\succ_{m_j}$ has a weak EU representation with a finite state space, then all weak EU representations of $\succ_{m_j}$ are essentially equivalent.*

However, what we are really interested in is the representation of the preference over the information channels $\succ$. We assume that the preference $\succ$ is only formed simultaneously as the preferences over the menus $(\succ_{m_j})_{m_j \in \mathcal{M}}$, where $\mathcal{M} = \prod_{j\in\mathcal{J}} \mathcal{M}_j$. This assumption makes sense because the preference over the menus given information m_j, $\succ_{m^j}$, is really defined hypothetically within the agent's mind before he actually acquires information. This is why we call $\succ_{m_j}$ as ex ante preference. Also, although we do not explicitly indicate $\succ_{m_j}$ in the definition of representation of $\succ$ below, $\succ_{m_j}$ for all $m_j \in \mathcal{M}$ is implicitly included in the construction of $\succ$. With this in mind, we introduce the following definition:

Definition: *A* ***weak EU representation*** *of* $\succ$ *is* $(\mathcal{S}, \hat{\varphi}, (\varphi_j)_{j\in\mathcal{J}}, u, (U_{m_j})_{m_j\in\mathcal{M}})$ *such that*

(i) $\succ_{m_j}$ *has a weak EU representation for all* m_j.

(ii) v *as defined in* (2) *represents* $\succ$.

By construction, we defined $\hat{\varphi} : \Re^{\mathcal{S}} \mapsto \Re$ by (3) using (1) and (2). Note that it is only relevant on the subspace

$$\mathcal{U}_*(\mathcal{S}, u) := \left\{ \left(\sup_{a\in B_{m_j}} u(a,s) \right)_{s\in\mathcal{S}} \middle| B_{m_j} \text{ is the solution to } \sup_{B\subset\Delta(C)} V_{m_j}(B) \right\}.$$

Next, we define essential equivalence of representations of $\succ$ as follows:

Definition: *Let* $R^i = (\mathcal{S}^i, \hat{\varphi}^i, (\varphi^i_j)_{j\in\mathcal{J}}, u^i, (U^i_{m_j})_{m_j\in\mathcal{M}^i})$, $i = 1, 2$ *be weak EU representations of some preferences* $\succ$. *If the subjective state spaces of these representations are finite, then* R^1 *and* R^2 *are* ***essentially equivalent*** *if the following hold:*

(i) *The subjective state spaces are the same:* $\mathbf{P}(\mathcal{S}^1, u^1) = \mathbf{P}(\mathcal{S}^2, u^2)$.

(ii) *There is a bijection* $h : \mathcal{S}^2 \mapsto \mathcal{S}^1$ *and functions* $\gamma : \mathcal{S}^2 \mapsto \Re_+$ *and* $\delta : \mathcal{S}^2 \mapsto \Re$ *such that for any* $u^1_* \in \mathcal{U}_*(\mathcal{S}^1, u^1)$, *the vector* $g(u^1_*)$ *defined by*
$$g(u^1_*)(s^2) = \gamma(s^2) u^1_*(h(s^2)) + \delta(s^2)$$
is contained in $\mathcal{U}_*(\mathcal{S}^2, u^2)$. *The function* $g : u_*(\mathcal{S}^1, u^1) \mapsto u_*(\mathcal{S}^2, u^2)$ *is a bijection.*

(iii) *Up to a monotonic transformation,* $\hat{\varphi}^1(u^1_*) = \hat{\varphi}^2(g(u^1_*))$ *for all* $u^1_* \in \mathcal{U}_*(\mathcal{S}^1, u^1)$.

Also, we introduce the following axiom for $\succ$:

Axiom 5 (Weak Order): $\succ$ *is asymmetric and negatively transitive.*

Since $\mathcal{J}$ is finite, axiom 5 implies the existence of some function v such that
$$v(j) > v(j') \Longleftrightarrow j \succ j'.$$
Hence, the following theorem holds:

Theorem 3 (Weak EU representation of $\succ$):

(a) *The preference $\succ$ has a weak EU representation if and only if it satisfies Axiom 5 and every $\succ_{m_j}$ satisfies Axioms 1–4.*

(b) *If preference $\succ$ has a weak EU representation with a finite state space, then all weak EU representations of $\succ$ are essentially equivalent.*

(Proof)
Since Theorem 2 (a) holds for all m_j, the requirement (i) for a weak EU representation of $\succ$ trivially holds. Hence, by construction, $\sup_B V_{m_j}(B)$ is well defined for all $m_j \in \mathcal{M}_j$ given some finite set $\mathcal{M}_j$. Axiom 5 implies the existence of a function v such that $v(j) > v(j') \iff j \succ j'$ since $\mathcal{J}$ is finite. Hence, we can define some function $\varphi_j : \Re^{M_j} \mapsto \Re$ as in (2). This completes the proof of the sufficiency of (a). The necessity of (a) is trivial.

Theorem 2 (b) holds for all $m_j \in \mathcal{M}_j$ given some finite set $\mathcal{M}_j$ and $\mathcal{S}_{m_j}$. Since $\mathcal{S} = \prod_{m_j \in \mathcal{M}_j} \mathcal{S}_{m_j}$ by construction, the uniqueness argument for every m_j and thus for every $\mathcal{S}_{m_j}$, gives a uniqueness result to each partition of $\mathcal{S}$. Hence (b) holds. *Q.E.D.*

3.3 Examples revisited

In what follows, we examine the examples introduced in the previous section to see how they can be described within our framework.

3.3.1 Example 1: Statistical decision problem

When the agent does not know exactly what pieces of information he would acquire from the data sources, he does not possess a complete knowledge of the structure of the states of nature. Hence, the state space $\mathcal{S}$ is not the set of states of nature, but is only a perceived state space, and the subjective state space is defined as $\mathbf{P}(\mathcal{S}, u)$.

In this example, the agent directly chooses his forecast f, rather than choosing a menu first before choosing an option within the menu. Yet, the utility function u should be described as a state dependent utility here, i.e. $u(\cdot, s)$, because he does not specify the true state space, but is only evaluating with respect to his perceived state space.

Moreover, the agent does not compare different data sources j and j' in terms of informativeness, since he does not even specify the number of elements of random vectors $\mathbf{y}^j$ and $\mathbf{y}^{j'}$, not to mention the stochastic matrices $\mathbf{P}^j$ and $\mathbf{P}^{j'}$. However, his preference over the data sources can be represented by a value function v such that

$$v(j) = \varphi_j \left(\left(\sup_f V_{m_j}(f) \right)_{m_j \in \mathcal{M}_j} \right), \forall j \in \mathcal{J},$$

where $V_{m_j}(f) = U_{m_j} \left(\left(u \left(100 - (r_s - f)^2, s \right) \right)_{s \in \mathcal{S}_{m_j}} \right)$ for all $m_j \in \mathcal{M}_j$.

It is particularly important that one can define a (marginal) value of each piece of information when an expected utility representation holds, and thus, the value of an information channel can be defined as the sum of the (marginal) values of each piece of information in this context, although the (marginal) value of each piece of information may well change in accord with the order of (information) addition. On the other hand, it is impossible to define a value of a piece of information when one does not know exactly what pieces of information are missing. Nevertheless, one can define the value of an information channel even that is the case.

3.3.2 Example 2: Investment

This example fits into the model almost directly. When the firm does not know exactly what pieces of information it could acquire potentially, it only chooses an information source j, but does not really know what pieces of information it would deliver.

Our representation result implies that we can identify a unique subjective state space $\mathbf{P}(\mathcal{S}, u)$, while the firm's preference over information sources is represented by a value function v such that

$$v(j) = \varphi_j \left(\left(\sup_B V_{m_j}(B) \right)_{m_j \in \mathcal{M}_j} \right), \forall j \in \mathcal{J},$$

where $V_{m_j}(B) = U_{m_j} \left((\sup_{a \in B} u(a, s))_{s \in \mathcal{S}_{m_j}} \right), \forall m_j \in \mathcal{M}_j$. Namely, $v(j)$ is exactly the same as (2) and $V_{m_j}(B)$ is exactly the same as (1).

3.3.3 Example 3: Job search

In this example, the choice set $B(\sigma^j)$ changes in accord with the information the agent actually acquires. However, it is not trivial at all how to characterise the perception of the agent about the choice set a priori, when the agent does not know exactly what pieces of information he could acquire through each channel j.

One possibility is to define the *perceived* choice set as B_{m_j} given information m_j, which reflects the more coarse structure of the subjective state space compared to that of the true state space. Then, our representation results imply an identification of a unique subjective state space $\mathbf{P}(\mathcal{S}, u)$. Also, the preference over channels j is represented by a value function such that

$$v(j) = \varphi_j \left(\left(V_{m_j}(B_{m_j}) \right)_{m_j \in \mathcal{M}_j} \right), \forall j \in \mathcal{J},$$

where $V_{m_j}(B_{m_j}) = U_{m_j} \left(\left(\sup_{a \in B_{m_j}} u(a, s) \right)_{s \in \mathcal{S}_{m_j}} \right), \forall m_j \in \mathcal{M}_j$.

Note that the collection of the perceived choice sets $(B_{m_j})_{m_j \in \mathcal{M}}$ with $\mathcal{M} = \prod_{j \in \mathcal{J}} \mathcal{M}_j$ is really the agent's estimate of the (structure of the) choice sets as well as that of the information channels. It is possible that a certain position will never be recognised by the agent because he does not recognise any information channel that actually contains the information about that particular position. This is particularly important when we are to model recognition of products, especially those of new products. We shall come back to this point in the subsequent section when we discuss possible applications of our results.

4 Discussion

In what follows, we discuss the implications and the limits of our results. The key result is that a preference over informational channels has a representation in terms of a value function with a construction that identifies a unique subjective state space. The structure of the subjective state space really reflects the perception of the agent even if his perception is unable to identify the true state space. Hence, the result implies that the agent needs not be able to describe the exact pieces of information he would acquire through an information channel in order to have a preference over the information channels.

This implies that we can identify the value of an information channel ex ante even if we are unable to identify the value of each piece of information the information channel would deliver ex ante. Namely, even if we cannot tell a subjective value for a newspaper article before reading it, we can give a subjective value for a newspaper. In other words, we need not characterise the value of a newspaper as a sum of the values of its articles. In a multi-agent context, one may want to introduce the concept of price, but this may well be possible for a newspaper, but not for a particular article of it.

Our results may provide some insights concerning the roles of advertising, in particular, in the context of informative advertising. Usually, we do not choose to see a particular advertisement. Instead, we choose an information channel, and by doing so, we somewhat unexpectedly see an advertisement. Our model captures such a situation more naturally than a standard model that assumes an expected utility maximiser.

As is described in the job search example (i.e. example 3), our result may have an interesting implication for models in which the opportunity set changes in accord with information. For example, we may be able to model a situation in which one does not recognise a new product unless he comes across with an advertisement of it. Or, we may be able to describe the internet shopping sites as information channels: They typically have a rather long list of merchandise, while many of their merchandise are unknown to the consumer a priori. It may be interesting to see how long the list should be, because a list can be so long that it could exceed the consumer's information processing capacity.

Furthermore, our results may be used to model communication. Namely, our model describes to whom one would listen even if the agent does not know exactly what pieces of information he would acquire through communication. However, our model does not capture a decision problem concerning submission of information, i.e. to whom (or through which channel) the agent would submit information. To model both acquisition and submission of information simultaneously, namely communication per se, the model has to be a multi-agent one. Unfortunately, such an extension does not look very straightforward, and we would leave it for future studies.

Apart from the issue of multi-agent modelling, the interpretation of the results of the paper requires some care. While the preference $\succ$ over the informational channels is ex ante obviously, the same applies to the preferences $\succ_{m_j}$ over the menus: They are ex ante even if it is given information m_j. Here m_j is a description of information not given after the actual information acquisition, but is merely a perception of the agent a priori. Hence, the subjective state space $\mathbf{P}(\mathcal{S}, u)$ and also $\mathbf{P}(\mathcal{S}_{m_j}, u)$, the subjective state space given m_j, are both describing the perception of the agent before the actual information acquisition. In other words, when the agent actually chooses a menu after acquiring information, the structure of the optimisation problem would be different from $\sup_B V_{m_j}(B)$ because m_j does not fully describe the actual information the agent acquires, but is a more coarse description of it. Moreover, the structure of the state space $\mathcal{S} = \prod_{m_j \in \mathcal{M}_j} \mathcal{S}_{m_j}$ is subjectively determined within the agent's mind a priori. Thus, the structure of the preference is purely static in construction although decision making does have a sequential structure, i.e. first information acquisition takes place, then choice of a menu followed by choice of an option within the selected menu takes place.

Before concluding the paper, we would like to mention the issue of rationality. While the analyses that assume rational expectations tend to explain unusual economic phenomena by introducing some additional structure of the environment, behavioural economics focusses on the limitations of human capability, which is often termed as bounded rationality. It is probably true that both the environment and the human factors contribute to such anomalies and/or unusual phenomena. Hence, it is reasonable to accommodate both in an analysis, while drawing a fine line between the two so as to see what factor contributes how and to what extent.

In the context of information acquisition, it may be the environmental limitations such as telecommunication and/or transport technologies that have bigger impacts in some situations, while the human limitations to perceive the true structure of the economy may contribute more in other situations. For example, if a particular phenomenon persists despite the recent rapid development in telecommunication technologies eliminates the bulk of the environmental limitations, human factors would be more important. Hence, to identify a proper public policy, it is important to draw a fine line between the environmental and human factors.

To illustrate the issue, let us introduce the following example. It is widely believed that an earthquake insurance policy is expensive, and that is what many people raise as the major reason why they do not buy it.[3] However, in many cases, earthquake insurance policies are heavily subsidised by the government, and thus, at least actuarially, it is rather cheap. One may describe it as an irrational behaviour that reflects an irrational attitude toward

[3]A field study by Kunreuther et al. (1978) raised this issue. Yet, there has not been any agreed resolution to this puzzle yet.

risk. However, in the light of our framework, it may well be the case that people simply do not recognise the premium. It is possible that the insurance companies are not too keen to sell such policies since they are not really their own products, and thus, the amount of advertising is limited.

5 Conclusion

We have so far described a decision making problem under uncertainty, in which an agent who has an opportunity to acquire information by choosing an information channel, while he does not know exactly what pieces of information he is missing. By directly adapting the results of Dekel et al. (2001), we have shown that a preference over information channels has a weak EU representation, while a unique subjective state space $\mathbf{P}(\mathcal{S}, u)$ is identified.

The main implication of the result is that we can define the value of an information channel even when we cannot define a value of each piece of information due to the lack of knowledge about the contents of information the information channel delivers. This implication can be far reaching. The framework may yield many applications including informative advertising, endogenous communication, etc.

However, there are problems that should be resolved. First, the framework is static, and thus, we cannot deal with a dynamical model. Second, the framework is that of a single agent. It is not trivial how to extend the framework in a multi-agent context. These are beyond the scope of this paper, and we leave them as possible topics of future studies.

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