Money is the Root of Asset Bubbles*

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Abstract

This paper examines how monetary expansion causes asset bubbles. When there is no monetary expansion, a bubbly asset is not created due to a hold-up problem. Monetary expansion increases buyers' income, and then, dealers are willing to buy a worthless asset from sellers, in hopes of selling it to buyers who may not know that it is worthless—a bubble now occurs.

Keywords: Bubbles, dealers, higher-order uncertainty, money

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1 Introduction

As advocated by many policymakers and observers, e.g., Masaaki Shirakawa, the former governor of Bank of Japan (see Okina et al. 2001) and Krugman (2015), bubbles occur when "too much money is chasing too few investment opportunities." While such a view is widely accepted not only among policymakers but also among the public, literally no theoretical work is done in the economics academia to examine the exact mechanism that links too much money and bubbles.[1]

The objective of this paper is to fill this gap. We propose a new framework in which too much money causes bubbles. Specifically, we incorporate a finite-horizon asset-bubble framework of Awaya et al. (2022) into a workhorse model in monetary theory by Lagos and Wright (2005). In each period, a decentralized asset market opens where agents use money as a payment instrument. Sellers produce an asset that buyers may wish to obtain. Agents have higher-order uncertainty, that is, a lack of common knowledge. Even if all agents know that the asset is worthless, there is a situation where dealers intermediate the trade between buyers and sellers with knowing the asset is worthless but without knowing that buyers know that the asset is worthless. Then, dealers may have incentives to buy the asset from sellers in hopes of selling it to buyers.

In this framework, too much money is modeled as a consequence of a policy, i.e., *monetary expansion*, with which the central bank issues money at the beginning of the decentralized market, and buyers receive the newly issued money as a lump-sum transfer. Hence, monetary expansion leads to a large amount of buyers' money holdings. The tractability of Lagos and Wright (2005) allows us to analyze such a monetary policy with a sound micro-founded theory of money.

[1]Barlevy (2018) points out the importance of knowing how (including, but not limited to) monetary policies affect bubbles.

Our main result is, informally,

Theorem. *Monetary expansion causes asset bubbles.*

To show this, first observe that without monetary expansion, asset bubbles do not occur. This is due to a *hold-up problem*. Since buyers know that dealers do not consume the asset, dealers have to accept even very little payment once they obtain the asset and trade with buyers. Anticipating this, buyers then do not have incentives to bring money to the decentralized market, and as a result, dealers are unwilling to buy the asset from sellers—bubbles never occur.

We next observe that with sufficiently large monetary expansion, asset bubbles occur. Since dealers know that buyers now have a sufficient amount of money, dealers speculate that buyers will pay a large amount of money if buyers do not know that the asset is worthless. Thus, dealers are willing to buy the asset from the sellers even if they know that it is worthless—a bubble now occurs.

Related Literature

This paper, of course, relates to the literature on both bubbles and money.

For the literature on bubbles, there are several approaches to studying bubbles (see, for example, Brunnermeier and Oehmke (2013) for a survey). Ours belongs to the one using higher-order uncertainty. Allen et al. (1993) is the first to take the approach and show the existence of bubbles in a finite-horizon model. The model is further refined and developed in a sequence of papers by Conlon (2004, 2015), Doblas-Madrid (2012), Liu and Conlon (2018), Liu et al. (2021), Awaya et al. (2022) and Dong et al. (2022). None of these models has money explicitly, and we use a New Monetarist framework to add money to Awaya et al. (2022). On top of this, another innovation relative to this literature is to show that bubbles can occur in a robust equilibrium with a finite-state space even when prices are publicly observable.

This paper also belongs to the New Monetarist literature. See Lagos et al. (2017)

and Rocheteau and Nosal (2017) for recent surveys. Among New Monetarist models, environments in which money and assets with uncertain return coexist have been studied by, for example, Rocheteau (2011) and Geromichalos et al. (2022). These models only consider first-order uncertainty. Mattesini and Nosal (2016) and Lagos and Zhang (2019, 2020) also incorporate intermediaries into the Lagos and Wright (2005) environment. Other than the fact that they do not consider asymmetric information, another important difference is the intermediation mode. In their models, dealers are a platform that offers a marketplace to investors. In our model, dealers are *middlemen* who buy the asset from their own accounts and resell it to buyers.

Organization of the Paper

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 shows that there is no equilibrium in which bubbles occur without monetary expansion, while Section 4 shows that there is an equilibrium in which bubbles occur with sufficiently large monetary expansion. Section 5 discusses some implications of our result.

2 Model

Time is discrete, continues forever, and is denoted by $t = 0, 1, \cdots$. Each period is divided into two subperiods as in the infinite-horizon monetary framework of Lagos and Wright (2005). In the first, agents interact in a decentralized asset market (DM). In the second, they interact in a frictionless centralized market (CM). There are three types of infinitely lived agents: sellers, dealers, and buyers. They discount future at a common rate $\beta \in (0, 1)$.[2] Their types are fixed over time, and the measure of each type is normalized to one.

[2]For simplicity, there is no discounting between the DM and the CM.

We assume there is an intrinsically useless, durable and uncounterfeitable object—*money*. Money is assumed to be divisible, and let M_t be its supply when the DM of period t opens (before monetary expansion that will be described below).

At the beginning of the DM of each period t, the central bank issues τM_t units of money, where $\tau \geq 0$. Only buyers receive the newly issued money through a lump-sum transfer, and hence each buyer obtains τM_t units in the DM.[3] In the subsequent CM, the central bank eliminates the newly issued fraction of money supply through a lump-sum tax τM_t. Hence, $M_t = M_{t+1}$ for each t. Assume that this monetary expansion is common knowledge.

The assumption that the central bank takes back money in the CM allows us to isolate the effect of monetary expansion from *inflation*—if the central bank did not take back the money, such monetary expansion would result in inflation. In general, inflation reduces the liquidity of buyers, which works against the original effect of monetary expansion of increasing the liquidity.

We will compare the case where $\tau = 0$ (no monetary expansion) to the case where τ is sufficiently large.

2.1 Centralized market

In the CM, money and a perishable good are traded. All agents enjoy $U(x)$ from consuming x units of the good. Assume that $U'(x) > 0$ and $U''(x) < 0$ for each $x > 0$. The good is numeraire and produced one-for-one using labor, and hence its price and the real wage are equal to one. Agents suffer disutility ℓ from working for ℓ hours.

[3]The assumption that only buyers get a transfer is for simplicity, and our result survives if sellers or dealers also get transfers. This kind of monetary policy is examined in the literature. See, for example, Molico (2006) and Wallace (2014).

2.2 Decentralized market

2.2.1 Economic Environment

In the DM, there exists an *indivisible* asset.[4] Each seller can produce only one unit of the asset at cost $c > 0$. Neither sellers nor dealers enjoy utility from consuming the asset, but buyers obtain utility $u > c$ with some probability and 0 with the remaining probability. Irrespective of whether it is consumed by buyers, the asset perishes after the DM.

In the DM, first, each seller meets a dealer for sure, trades the asset with the dealer, and leaves the DM. Then, the dealer meets a buyer for sure, trades the asset with the buyer, and both the dealer and the buyer leave the DM. Thus, trade between sellers and buyers is sequential and must go through dealers. For sellers and dealers, their counterparties are drawn randomly from all dealers and all buyers, respectively. Money is used as the payment instrument, and we employ generalized Nash bargaining to determine the terms of trade.[5]

We assume that buyers *can* observe the price between sellers and dealers. We will discuss its implication in Section 5.

2.2.2 Information Structure

We will construct an information structure that induces asset bubbles. Following the notion of *strong bubbles* by Allen et al. (1993), asset bubbles are defined as follows.

Definition 1. *An asset bubble occurs* if the asset is traded for a positive amount of money, the price of money is positive, and all agents know that the consumption value

[4]We can obtain similar implications with a divisible asset as well.

[5]Like Awaya et al. (2022), we consider a game form where a fictitious third party suggests the exchange ratio following the Nash bargaining solution, and then each agent either accepts or rejects the trade. The trade occurs only when both accept. While the agents have private information, the terms of trade that the third party proposes do not depend on it. The key to our result is that the bargaining solution is Pareto efficient.

of the asset for buyers is 0.

The information structure in the DM is (a simplified version of) the finite-horizon asset-bubble model of Awaya et al. (2022). All parameters describing utilities, costs, etc., are common knowledge except for the asset value (i.e., consumption value) for buyers. To describe the information structure, we introduce three states[6] : ω_{SD}^{u}, ω_{E}^{0}, and ω_{SD}^{0}. The superscripts indicate whether the asset value for buyers is u or 0. At ω_{SD}^{u}, it is u; at the other states, it is zero. The subscripts signify who knows the asset value for buyers. At ω_{SD}^{u} and ω_{SD}^{0}, only sellers and dealers know it; at ω_{E}^{0}, every agent knows it. The set of states is

$$\Omega = \{\omega_{SD}^{u}, \omega_{E}^{0}, \omega_{SD}^{0}\}.$$

The state of each period realizes at the beginning of the DM in that period, and drawn independently across periods.

The information is the same among the agents of each type. Sellers' and dealers' partitions are the same:

$$\mathcal{P}_{SD} = \{\{\omega_{SD}^{u}\}, \{\omega_{E}^{0}, \omega_{SD}^{0}\}\}.$$

The first element, $\{\omega_{SD}^{u}\}$, corresponds to the case where the buyers' asset value is u, and sellers and dealers know it. The second element, $\{\omega_{E}^{0}, \omega_{SD}^{0}\}$, corresponds to the case where the buyers' asset value is zero, and sellers and dealers do not know whether buyers know it. Buyers' partition is

$$\mathcal{P}_{B} = \{\{\omega_{SD}^{u}, \omega_{SD}^{0}\}, \{\omega_{E}^{0}\}\}.$$

The first element, $\{\omega_{SD}^{u}, \omega_{SD}^{0}\}$, corresponds to the case where buyers do not know whether the asset value is u or 0 for them. The second element, $\{\omega_{E}^{0}\}$, corresponds to the case where the asset value is 0, and buyers know it. An agent can distinguish

[6] In the original Awaya et al. (2022) model, at least five states are required to generate a bubble. We thank Gadi Barlevy who pointed out later that only three states are sufficient. The specification adopted in this paper is the one pointed out by him. See also Liu et al. (2021).

any two states if those states belong to a different element of his or her partition, but cannot otherwise.

The common prior distribution over Ω does not depend on time and is denoted by μ. Assume that $\mu(\omega) > 0$ for each $\omega \in \Omega$. At states ω_{SD}^{u} and ω_{SD}^{0}, buyers believe that the asset value is u with probability

$$\psi_B = \frac{\mu(\omega_{SD}^{u})}{\mu(\omega_{SD}^{u}) + \mu(\omega_{SD}^{0})}.$$

At state ω_{SD}^{u}, dealers know that buyers do not know the asset value, and hence they believe that they can sell the asset to buyers for sure. At states ω_{E}^{0} and ω_{SD}^{0}, dealers believe that buyers do not know the asset value with probability

$$\psi_D = \frac{\mu(\omega_{SD}^{0})}{\mu(\omega_{E}^{0}) + \mu(\omega_{SD}^{0})}.$$

We summarize how each state plays a role for the existence of bubbles. State ω_{SD}^{u} creates gains from trade of the asset. State ω_{E}^{0} establishes a situation where all agents know that the asset value for buyers is zero. State ω_{SD}^{0} constructs a case where the asset value is zero and buyers do not know it. These states are necessary ingredients of bubbles.

3 No Monetary Expansion

First we will show that

Proposition 1. *There is no equilibrium in which an asset bubble occurs if there is no monetary expansion ($\tau = 0$).*

The reason is a *hold-up problem.* Note that dealers do not enjoy utility from consuming the asset. Since the bargaining between dealers and buyers is efficient, the dealers must give up the asset, *regardless of the amount of buyers' money holdings.* Since holding money across periods is costly, buyers do not have any incentives to bring money.

Thus, when $\tau = 0$, buyers do not have money in the DM. Then, dealers do not have any incentives to buy the asset from sellers, and therefore there is no room for bubbles to exist.

The no-bubble result seems very different from the result of Awaya et al. (2022) who show bubbles occur for all parameters. The key difference is the timing of the bargaining. In Awaya et al. (2022), buyers can produce goods on the spot for dealers and hence buyers' payments are unconstrained when buyers bargain with dealers. In the current model, buyers' payments are constrained to the amount of money they bring to the DM when they bargain with dealers. In other words, buyers can commit smaller amounts of payment by bringing smaller amounts of money. Anticipating this, dealers do not buy the asset from sellers.

4 Monetary Expansion

In this section, we identify a necessary and sufficient condition for which an asset bubble occurs. Throughout the section we assume that the gains from trade are sufficiently large so that

$$\frac{\theta_2 \psi_B u}{c} \geq \underline{\tau} \equiv \max\left\{\frac{1}{\psi_D}, \frac{1}{\beta[\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)]}\right\}. \tag{1}$$

As we will see later, it turns out that there is no equilibrium in which an asset bubble occurs if (1) is violated. If we assume (1), we have

Theorem 1. *There is an equilibrium in which asset bubbles occur at a state if and only if monetary expansion is sufficiently large ($\tau \geq \underline{\tau}$).*

Of course, Proposition 1 is a special case of the theorem. Note that (1) is more likely to be satisfied when the interest rate $(1-\beta)/\beta$ is lower.

4.1 Bellman Equations

Let V_t^S, V_t^D, and V_t^B be the DM value functions of sellers, dealers, and buyers, respectively. Similarly, we denote the CM value functions by W_t^S, W_t^D, and W_t^B. For each type $i \in \{S, D, B\}$, the Bellman equation for the CM is

$$\begin{aligned} W_t^i(m_t^i) &= \max_{x_t^i, \ell_t^i, \widehat{m}_{t+1}^i} \left\{ U(x_t^i) - \ell_t^i + \beta V_{t+1}^i(\widehat{m}_{t+1}^i) \right\} \\ &\qquad \text{s.t. } x_t^i = \phi_t(m_t^i - \widehat{m}_{t+1}^i) + \ell_t^i - \frac{\tau M_t}{3}, \end{aligned}$$

where m_t^i and $\widehat{m}_{t+1}^i$ are money holdings when the CM opens and closes, and ϕ_t is the price of money. Note that since buyers receive money in the next DM (i.e., money expansion), the one they choose in the CM, $\widehat{m}_{t+1}^B$, differs from the one they actually hold in the DM, m_{t+1}^B. Here, we have $-\tau M_t/3$ in the budget constraint because measure 1 of buyers receive τM_t units of money in the DM and the central bank takes back that fraction of money from measure 3 of all agents through a lump-sum tax.

Assuming an interior solution for labor ℓ_t^i,

$$\begin{aligned} W_t^i(m_t^i) &= \phi_t m_t^i - \frac{\tau M_t}{3} + \max_{x_t^i} \left\{ U(x_t^i) - x_t^i \right\} \\ &\quad + \max_{\widehat{m}_{t+1}^i} \left\{ -\phi_t \widehat{m}_{t+1}^i + \beta V_{t+1}^i(\widehat{m}_{t+1}^i) \right\}. \end{aligned}$$

The optimal consumption in the CM is pinned down by

$$U'(x_t^i) = 1.$$

The optimal money holdings when the CM closes are determined independently of money holdings when the CM opens. Thus, we have the history independence of money holdings, and all agents of each type have the same unit of money at the end of the CM. Moreover, the CM value function is linear with slope ϕ_t:

$$\frac{dW_t^i(m_t^i)}{dm_t^i} = \phi_t.$$

Trade between Dealers and Buyers

Now we derive the terms of trade in seller-dealer meetings and dealer-buyer meetings, given money holdings of each type of agents. We start with dealer-buyer meetings and then consider seller-dealer meetings.

Consider DM trade between dealers and buyers. Let $a_{2,t}$ and $p_{2,t}$ be the amounts of the asset and money traded, respectively. At ω_{SD}^{u} and ω_{SD}^{0}, buyers do not know the asset value. Then, from the linearity of the CM value functions, dealers' surplus is

$$W_t^D(m_t^D + p_{2,t}) - W_t^D(m_t^D) = \phi_t p_{2,t},$$

and buyers' surplus is

$$\psi_B u a_{2,t} + W_t^B(m_t^B - p_{2,t}) - W_t^B(m_t^B) = \psi_B u a_{2,t} - \phi_t p_{2,t}.$$

Therefore, the terms of trade are determined by

$$\max_{a_{2,t}\in\{0,1\},p_{2,t}} (\phi_t p_{2,t})^{\theta_2}(\psi_B u a_{2,t} - \phi_t p_{2,t})^{1-\theta_2}$$
$$\text{subject to } a_{2,t} \le a_t^D \text{ and } p_{2,t} \le m_t^B,$$

where $\theta_2 \in (0,1)$ is dealers' bargaining power and a_t^D is the amount of dealers' asset holdings. Note that, implicitly, we also have incentive constraints that agents' surpluses must be nonnegative, $\phi_t p_{2,t} \ge 0$ and $\psi_B u a_{2,t} - \phi_t p_{2,t} \ge 0$.

If $\phi_t m_t^B > 0$, the solution to the bargaining problem between dealers and buyers takes the following form:

$$\begin{aligned} a_{2,t}(a_t^D, m_t^B) &= a_t^D \\ p_{2,t}(a_t^D, m_t^B) &= \begin{cases} m_t^B & \text{if } \phi_t m_t^B < \theta_2 \psi_B u a_t^D, \\ \frac{\theta_2 \psi_B u a_t^D}{\phi_t} & \text{if } \phi_t m_t^B \ge \theta_2 \psi_B u a_t^D. \end{cases} \end{aligned}$$

Note that the amount of the asset traded is *independent* of the amount of buyers' money holdings. This is because dealers do not enjoy utility from consuming the asset, and therefore we can make buyers better off without hurting dealers by increasing the amount of the asset traded.[7] If $\phi_t m_t^B = 0$, the terms of trade $[a_{2,t}(a_t^D, m_t^B), p_{2,t}(a_t^D, m_t^B)]$ are any pairs $(a_{2,t}, p_{2,t})$ satisfying the constraints. At state ω_E^0, buyers know that the asset value for buyers is 0, and thus $a_{2,t}(a_t^D, m_t^B)$ is any of 0 and a_t^D, and $p_{2,t}(a_t^D, m_t^B)$ is 0 if $\phi_t > 0$ and any number between 0 and m_t^B if $\phi_t = 0$.

Trade between Sellers and Dealers

Next consider DM trade between sellers and dealers. For states ω_E^0 and ω_{SD}^0, let $a_{1,t}^0$ and $p_{1,t}^0$ be the amounts of asset and money traded. At states ω_E^0 and ω_{SD}^0, dealers believe that buyers do not know the asset value with probability ψ_D. Then, sellers' surplus is

$$-ca_{1,t}^0 + W_t^S(m_t^S + p_{1,t}^0) - W_t^S(m_t^S) = \phi_t p_{1,t}^0 - ca_{1,t}^0,$$

and dealers' surplus is

$$\psi_D \phi_t p_{2,t}(a_{1,t}^0, M_t^B) + W_t^D(m_t^D - p_{1,t}^0) - W_t^D(m_t^D) = \phi_t[\psi_D p_{2,t}(a_{1,t}^0, M_t^B) - p_{1,t}^0].$$

The terms of trade are determined by

$$\max_{a_{1,t}^0 \in \{0,1\}, p_{1,t}^0} (\phi_t p_{1,t}^0 - ca_{1,t}^0)^{\theta_1} \{\phi_t[\psi_D p_{2,t}(a_{1,t}^0, M_t^B) - p_{1,t}^0]\}^{1-\theta_1}$$
$$\text{subject to } p_{1,t}^0 \leq m_t^D,$$

where $\theta_1 \in (0,1)$ is sellers' bargaining power. Again, implicitly, we also have incentive constraints that agents' surpluses must be nonnegative. This bargaining problem is more complicated than those in the models based on Lagos and Wright (2005) especially because dealers must take buyers' money holdings into account, when they trade with sellers.

[7] The assumption that dealers do not obtain utility from assets is standard in the study of over-the-counter (OTC) markets initiated by Duffie et al. (2005). Moreover, since $a_{2,t}(a_t^D, m_t^B) \in \{0,1\}$, our result holds when dealers obtain small utility from assets.

Define $w^0(M_t^B) = \theta_1 \phi_t \psi_D p_{2,t}(1, M_t^B) + (1-\theta_1)c$, which is the value of money traded in the above problem if (i) the constraint, $p_{1,t}^0 \leq m_t^D$, does not bind and (ii) there are gains from trade between sellers and dealers, $\phi_t \psi_D p_{2,t}(1, M_t^B) - c > 0$. We divide the argument into three cases.

Case 1: Suppose that $\phi_t \psi_D p_{2,t}(1, M_t^B) > c$, that is, the gains from trade between sellers and dealers are positive. Then, the solution to the bargaining problem between sellers and dealers is

$$a_{1,t}^0(m_t^D, M_t^B) = \begin{cases} 1 & \text{if } \phi_t m_t^D > c, \\ 0 \text{ or } 1 & \text{if } \phi_t m_t^D = c, \\ 0 & \text{if } \phi_t m_t^D < c, \end{cases}$$

$$p_{1,t}^0(m_t^D, M_t^B) = \begin{cases} \frac{w^0(M_t^B)}{\phi_t} & \text{if } \phi_t m_t^D \geq w^0(M_t^B), \\ m_t^D & \text{if } c < \phi_t m_t^D < w^0(M_t^B), \text{ or } \phi_t m_t^D = c \text{ and } a_{1,t}^0(m_t^D, M_t^B) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Case 2: Suppose that $\phi_t \psi_D p_{2,t}(1, M_t^B) = c$, that is, there are no gains from trade between sellers and dealers. Then,

$$a_{1,t}^0(m_t^D, M_t^B) = \begin{cases} 0 \text{ or } 1 & \text{if } \phi_t m_t^D \geq c, \\ 0 & \text{if } \phi_t m_t^D < c, \end{cases}$$

$$p_{1,t}^0(m_t^D, M_t^B) = \begin{cases} \frac{c}{\phi_t} & \text{if } \phi_t m_t^D \geq c \text{ and } a_{1,t}^0(m_t^D, M_t^B) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Case 3: Suppose that $\phi_t \psi_D p_{2,t}(1, M_t^B) < c$, that is, there are losses from trade between sellers and dealers. Then, $a_{1,t}^0(m_t^D, M_t^B) = 0$, and $p_{1,t}^0(m_t^D, M_t^B) = 0$ if $\phi_t > 0$ and $p_{1,t}^0(m_t^D, M_t^B)$ is any number between 0 and m_t^D if $\phi_t = 0$.

For state ω_{SD}^u, let $a_{1,t}^u$ and $p_{1,t}^u$ be the amounts of asset and money traded, respectively, and define $w^u(M_t^B) = \theta_1 \phi_t p_{2,t}(1, M_t^B) + (1-\theta_1)c$. Then, we can obtain the terms of trade $[a^u_{1,t}(m^D_t, M^B_t), p^u_{1,t}(m^D_t, M^B_t)$ by setting $\psi_D = 1$ in $[a^0_{1,t}(m^D_t, M^B_t), p^0_{1,t}(m^D_t, M^B_t)]$ derived above.

Bellman Equations for the DM

Now, we define the Bellman equations for the DM. For sellers, it is

$$\begin{aligned} V^S_t(m^S_t) &= W^S_t(m^S_t) + \phi_t\{\mu(\omega^u_{SD})[p^u_{1,t}(M^D_t, M^B_t) - ca^u_{1,t}(M^D_t, M^B_t)] \\ &\quad +[\mu(\omega^0_E) + \mu(\omega^0_{SD})][p^0_{1,t}(M^D_t, M^B_t) - ca^0_{1,t}(M^D_t, M^B_t)]\}. \end{aligned}$$

Sellers always trade with dealers.

For dealers,

$$\begin{aligned} V^D_t(m^D_t) &= W^D_t(m^D_t) + \phi_t\mu(\omega^u_{SD})\{p^u_{2,t}[a^u_{1,t}(m^D_t, M^B_t), M^B_t] - p^u_{1,t}(m^D_t, M^B_t)\} \\ &\quad +\phi_t\{\mu(\omega^0_{SD})p_{2,t}[a^0_{1,t}(m^D_t, M^B_t), M^B_t] - [\mu(\omega^0_E) + \mu(\omega^0_{SD})]p^0_{1,t}(m^D_t, M^B_t)\}. \end{aligned}$$

Dealers buy the asset from sellers, and at states ω^u_{SD} and ω^0_{SD}, can sell it to buyers.

Finally, for buyers,

$$\begin{aligned} V^B_t(m^B_t) &= W^B_t(m^B_t) + \mu(\omega^u_{SD})\{ua^u_{1,t}(M^D_t, M^B_t) - \phi_t p_{2,t}[a^u_{1,t}(M^D_t, M^B_t), m^B_t]\} \\ &\quad +\mu(\omega^0_{SD})\{-\phi_t p_{2,t}[a^0_{1,t}(M^D_t, M^B_t), m^B_t]\}. \end{aligned}$$

Buyers purchase the asset from dealers with probability $\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})$, and then, cannot enjoy utility from the purchased asset with probability $\mu(\omega^0_{SD})$.

4.2 Equilibrium

We will derive an equilibrium where an asset bubble occurs. Let $z_t = \phi_t M_t$, which is called the *value of money in period t*.

Note that as is usual in Lagos-Wright models, it is costly to hold money, and so agents who do not use money in the DM do not buy money in the CM. In our model, this means that neither sellers nor buyers have incentives to buy a positive amount of money in the CM, and hence each dealer has M_t units of money at the beginning of period t. Then, we have $\phi_t M_t^D = z_t$. Moreover, since buyers have τM_t units of money in the DM, $\phi_t M_t^B = \tau z_t$.

Lemma 1. *Dealers' money holdings satisfy $m_t^D \leq c/\phi_t$ in any equilibrium with $\phi_t > 0$.*

This is because in all the cases in the previous section, choosing $m_t^D > c/\phi_t$ does not increase $a_{1,t}^0$ and so bringing more money only incurs additional cost.

For an asset bubble to occur, we must have $a_{1,t}^0(M_t, \tau M_t) = 1$, and this can occur only in Cases 1 or 2 in the previous section. The condition can be rewritten as

$$\frac{\phi_t p_{2,t}(1, \tau M_t)}{c} \geq \frac{1}{\psi_D}. \tag{2}$$

With this condition, we also have $a_{1,t}^u(M_t, \tau M_t) = 1$ because $\psi_D < 1$.

Observe that in order for $a_{1,t}^0(M_t, \tau M_t) = 1$ to occur, we must have $m_t^D \geq c/\phi_t$ and thus $m_t^D = c/\phi_t$. Thus, $z_t = c$ and $p_{1,t}^0(M_t, \tau M_t) = p_{1,t}^u(M_t, \tau M_t) = c/\phi_t$ for each t.

Now,

$$p_{2,t}(1, \tau M_t) = \begin{cases} \tau M_t & \text{if } \tau < \frac{\theta_2 \psi_B u}{c}, \\ \frac{\theta_2 \psi_B u}{\phi_t} & \text{if } \tau \geq \frac{\theta_2 \psi_B u}{c}. \end{cases}$$

If $\tau < \theta_2 \psi_B u/c$, then (2) is rewritten as

$$\tau \geq \frac{1}{\psi_D}.$$

If $\tau \geq \theta_2 \psi_B u/c$, it is

$$\frac{\theta_2 \psi_B u}{c} \geq \frac{1}{\psi_D}.$$

Therefore, (2) holds if and only if

$$\min\left\{\tau, \frac{\theta_2 \psi_B u}{c}\right\} \geq \frac{1}{\psi_D}.$$

Finally, dealers must have incentives to bring money to the DM, that is, we must have the following condition:

$$\phi_t M_{t+1} \leq \beta\{\phi_{t+1} M_{t+1} + [\mu(\omega_{SD}^u) + \mu(\omega_{SD}^0)]\phi_{t+1} p_{2,t+1}(M_{t+1}, \tau M_{t+1}) - \phi_{t+1} M_{t+1}\}. \tag{3}$$

The left-hand side, $\phi_t M_{t+1}$, is the cost of bringing money to the DM. In the right-hand side, $\phi_{t+1} M_{t+1}$ is the resale value of money, $[\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})]\phi_{t+1} p_{2,t+1}(M_{t+1}, \tau M_{t+1})$ is the benefit from trade using money, and $-\phi_{t+1} M_{t+1}$ is the payment. We have $M_t = M_{t+1}$ and $\phi_t M_t = c$, and hence, (3) is rewritten as

$$\frac{\phi_{t+1} p_{2,t+1}(M_{t+1}, \tau M_{t+1})}{c} \geq \frac{1}{\beta[\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})]}.$$

Thus, (3) holds if and only if

$$\min\left\{\tau, \frac{\theta_2 \psi_B u}{c}\right\} \geq \frac{1}{\beta[\mu(\omega^u_{SD}) + \mu(\omega^0_{SD})]}.$$

Occurrence of Asset Bubbles

We will explain how an asset bubble occurs in this equilibrium. Consider state ω^0_E. At this state, the asset value for buyers is 0, and every agent knows it. Hence, the fundamental value of asset is 0. However, since dealers' knowledge is

$$\mathcal{P}_{SD} = \{\{\omega^u_{SD}\}, \{\omega^0_E, \omega^0_{SD}\}\},$$

dealers do not know that buyers know that the asset value is 0. Moreover, with monetary expansion, buyers have too much money in the sense that they can buy the same unit of the asset with any smaller amount of money than the one injected by the monetary expansion. To obtain such money from buyers, dealers buy the asset from sellers in hopes of selling it to buyers, but buyers do not purchase it from dealers because buyers know that the asset is worthless. That is, an asset bubble occurs in trade between sellers and dealers, and bursts in trade between dealers and buyers.

This shows the proposition.

5 Discussion

Observability of Prices

Prices in trade between sellers and dealers are the same across all the states, so buyers do not learn the asset value from its price. Thus, our result survives irrespective of whether past prices are observable or not. This is in sharp contrast to Awaya et al. (2022) who need to assume past prices are unobservable to establish robust bubbles. More precisely, in their Appendix A, Awaya et al. (2022) show that bubbles occur when past prices are observable but this is true only for some knife-edge parameters. In the current paper, bubbles occur in an open, nonempty set of parameters.

Intermediation Mode

In our model, intermediaries (i.e., dealers) do trade using their own accounts and make profits by flipping. There is of course another mode of intermediaries. In particular, platforms (or brokers) just connect buyers and sellers, or investors and inter-dealer markets, and make profits by brokerage fees. This difference is crucial.

In our model, dealers must hold the asset when they trade with buyers and thus their holding of the asset is sunk. This opens the room for the hold-up problem—buyers do not bring money and hence there is no bubble without monetary expansion. Such a hold-up would never occur if intermediaries are brokers. When there is monetary expansion, bubbles occur because dealers *buy* the asset even when they know that it is worthless. If intermediaries are brokers, trade never occurs if buyers know that the asset is worthless.

Policy Implications

Is it better to burst bubbles by tightening monetary policy? In our model, creation of worthless assets—and hence bubbles—is just a waste, but *ex ante* (without knowing which state to realize) creation of asset is welfare-improving. Therefore, such a bubblebursting policy is (i) welfare-improving *ex post* in which the central bank knows the state when it determines the policy because it can prevent costly production of the worthless object, but (ii) detrimental *ex ante* in which the central bank does not know the state when it determines the policy. In this sense, our crude model echoes the view of Bernanke and Gertler (2012) who write: "Trying to stabilize asset prices per se is problematic for a variety of reasons, not the least of which is that it is nearly impossible to know for sure whether a given change in asset values results from fundamental factors, non-fundamental factors, or both."[8]

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[8] Some papers point out that bubbles are detrimental. For example, Dong et al. (2022) show that bubbles create misallocation of talent. Grossman and Yanagawa (1993) and Guerrón-Quintana et al. (2020) demonstrate that asset bubbles crowd out investment. In Allen et al. (2022), bubbles cause costly default. See Barlevy (2018) for further discussion. None of these channels are present in the current paper.

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