Redistribution and the wage-price dynamics: Optimal fiscal and monetary policy

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Abstract

When both prices and wages are subject to nominal frictions, an increase in input prices such as energy can initiate a wage-price dynamics, as both nominal wages and prices adjust slowly. High inflation in prices and wages reduces welfare as it generates distributional effects and affects aggregate demand. To analyze optimal policy in this environment, we consider a heterogeneous-agent model, with both wage and price stickiness. We derive joint optimal fiscal-monetary policy, using a rich set of fiscal tools. We first identify the set of fiscal tools, which implements nominal price and wage stability as an optimal outcome. Starting from this equivalence result, we identify the key instrument for implementing price and wage stability, which appears to be a time-varying wage subsidy. We call this policy a non-Keynesian stabilization policy, as it does not directly channel through aggregate demand. We finally compare our results to those obtained in a representative-agent environment

Keywords: Heterogeneous agents, wage-price spiral, inflation, monetary policy, fiscal policy.

JEL codes: D31, E52, D52, E21.

1 Introduction

Energy price shocks, like in the 70s and in the current conjuncture, have generated different dynamics in wages and prices. Indeed, on the one hand, workers bargain over nominal wages and their bargaining power depend on various institutional designs and market properties. On the other hand, firms facing price-adjustment costs may vary their prices at their own pace. This two-sided price and wage setting may generate heterogeneous dynamics in real wages and markups, and as a consequence, imply different price and wage inflation levels. The management

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of this joint wage-price dynamics, which can be the source of large adverse economic effects on output, inequality and welfare, is a topical economic question. Indeed, inflation generates distributional effects due to heterogeneous nominal exposures and other indirect channels (Doepke and Schneider, 2006, Kaplan et al., 2018, or Auclert, 2019, among others). These distributional effects are affected by fiscal policy, which can thus change the incentives of monetary policy to mitigate inflation (LeGrand et al., 2022). In particular, certain fiscal tools have recently been used to directly change inflation. Such tools belong to the so-called *unconventional fiscal policy* (D'Acunto et al., 2018), which is still a lively question (Dao et al., 2023). Designing a joint optimal monetary and fiscal policy is thus a relevant and salient economic question.

The goal of this paper is to derive fiscal and monetary policy in heterogeneous-agent (HA) models, with both price and wage nominal rigidities, generating both a price and a wage Phillips curve. Considering an HA model allows us to account for the distributional issues we mentioned above. We introduce energy in the production sector, and study optimal policy after an energy price shock, which is akin to a negative TFP shock in our economy. The fiscal policy we consider features a rich set of tools that include five fiscal instruments: a linear income tax, capital tax, a labor tax, employer social contributions and public debt – the reason for this choice will be clarified below. We derive three sets of results.

First, we first study the model with a standard monetary Taylor rules and standard fiscal rules à la Bohn (1998). We do not focus here on optimal policy and aim to identify the model responses in the context of standard non-optimal monetary-fiscal policy. Following an energy price shock, both price and wage inflation increase, which generates a drop in the real wage as price inflation is higher than the wage inflation. Such joint wage and price dynamics is often called a wage-price spiral in the literature (Lorenzoni and Werning, 2023). Compared to the representative-agent (RA) economy (see Erceg et al., 2000, Galí, 2015, chapter 6, and Lorenzoni and Werning, 2023 for additional results), the HA economy exhibits higher inflation responses, and hence a more pronounced wage-price spiral. This comes from the heterogeneity in the consumption drop after the shock.

Second, we prove that if the planner can optimally set the five fiscal instruments, then optimal price and wage inflation levels are always zero and the planner implements the flexible-price allocation. We have chosen these instruments as the minimal set (in the sense of the smallest cardinal) of fiscal tools that allows us to derive this equivalence result. This result is in the same vein as the ones of Correia et al. (2008), who consider a RA economy and of LeGrand et al. (2022), who consider a HANK model with a unique nominal friction – as in the majority of the HANK literature (see Kaplan et al., 2018 or Auclert et al., 2021 for a discussion). Including two types of nominal frictions is not a minor change. Indeed, nominal wage rigidity generates suboptimal real wage frictions and implies that households are out of their labor supply. A consequence is that introducing one nominal friction in addition to the one in LeGrand et al. (2022) requires two additional fiscal tools to restore the price-wage stability. These tools allow us to identify the channels through which inflation in wages or prices can be welfare improving when one or several fiscal are missing. When deriving the first-order conditions of the Ramsey planner, we make some effort to connect these conditions to the public finance approach (e.g., Saez and Stantcheva, 2016) to improve our understanding of policy tradeoffs in HA economies.

Third, we remove some of the five fiscal tools to identify economically and quantitatively the most important contributors to inflation stabilization. Our results indicate that the time-varying employer social contribution is the key instrument. Indeed, this tax allows the planner to modify the labor cost paid by the firm while letting the nominal wage of workers unchanged. So, even if nominal wages are sticky, decreasing this tax after a negative TFP shock allows the planner to reduce the labor cost and make it closer to the the marginal productivity of labor. In consequence, this reduces the room for monetary policy to decrease the real wage (and increase employment) through inflation. Removing this fiscal tool implies sizable wage and price inflation response to compensate for the missing instrument. It is noteworthy that these tools were massively used in Europe during the Covid-19 crisis to stabilize employment. In Germany, this was called *kurzarbeit*, while this was *activité partielle* policy in France for instance.[1] Hence, although additional work is indeed to complement our findings, time-varying automatic employment stabilizers (instead of automative stabilizers increasing directly aggregate demand) appear to be promising instruments to stabilize inflation and employment over the business cycle.[2]

Related literature. This paper belongs to the literature on optimal policy in heterogeneous agent model on one side, and on wage-price spirals on the other side. Deriving optimal policy in heterogeneous-agent models with aggregate shocks is a difficult theoretical and computational task. Some papers consider numerical methods to solve for optimal path of the instruments (Dyrda and Pedroni, 2022). Other papers rely on continuous-time techniques for the theoretical derivation of the first-order conditions of the planner (Nuño and Thomas, 2022 among others). Acharya et al. (2022) solve for optimal monetary policy using the tractability of the CARA-normal environment without capital. Bhandari et al. (2021) quantitatively solve for optimal policies in a new-Keynesian model with aggregate shocks. Yang (2022) solves for the optimal monetary policy by optimizing on the coefficients of a Taylor rule. McKay and Wolf (2022) derive a general quadratic-linear formulation to solve for optimal policy rules. In this paper, we use the tools of LeGrand and Ragot (2022a) and the improvements of LeGrand and Ragot (2022c) to solve for optimal fiscal and monetary policy with aggregate shocks. The gain of this approach is to allows to easily solve for optimal policy with many tools and with various nominal frictions. On the

[1] Note that any fiscal instrument affecting real labor cost would generate the same desirable outcome, but we consider the time-varying social contribution for its close connection to recent mechnisms.

[2] McKay and Reis (2021) study optimal automatic stabilizers in the context of optimal replacement rate. Their main focus is the trade-off between insurance and incentives in the presence of an aggregate demand effect. The mechanism we identify is different as it directly affects the gap between the real wage and the marginal productivity of labor.

theoretical side, the Lagrangian approach pioneered in Marcet and Marimon (2019) enables us to derive the first-order conditions of the Ramsey planner in an environment with both wage and price rigidities.

Regarding the literature on wage-price spirals, models including both price and wage stickiness have been studied in RA economies (Blanchard, 1986, Galí, 2015, chapter - chapter 6, or Blanchard and Gali, 2007 among others). Erceg et al. (2000) study optimal monetary policy in this environment. Recently, Lorenzoni and Werning (2023) analyze more deeply optimal policy and the real wage dynamics in this environment.

2 The environment

We consider a discrete-time economy populated by a continuum of size one of ex-ante identical agents. These agents are assumed to be distributed along a set J, with the non-atomic measure ℓ: $\ell(J) = 1$.[3]

2.1 Risk

We assume that the agents face an idiosyncratic productivity risk. The productivity process, denoted y, is assumed to take value in a finite set $\mathcal{Y}$ and to follow a first-order Markov chain with transition matrix $\boldsymbol{\pi} = (\pi_{yy'})_{y,y'}$. With wage w and labor supply l, an agent with productivity y earns the labor income wyl. In each period, the fraction of agents with productivity y is constant and denoted by n_y. We normalize average productivity to 1, i.e., such that $\sum_y n_y y = 1$. The history of idiosyncratic productivity shocks up to date t for an agent i is denoted by $y_i^t = \{y_{i,0}, \ldots, y_{i,t}\} \in \mathcal{Y}^{t+1}$, where $y_{i,\tau}$ is the date-τ productivity. The measure of idiosyncratic histories up-to-date t, denoted by θ_t, can be computed using the initial distribution and the transition matrix.

In addition to the previous idiosyncratic risk, agents face an aggregate risk z affecting the economic TFP, denoted by Z. We show in Section 2.9 that the aggregate risk can be interpreted as a shock on energy price.

2.2 Preferences

Households are expected-utility maximizers endowed with time-separable preferences and a constant discount factor $\beta \in (0, 1)$. In each period, households enjoy utility $U(c, l)$ from the consumption c of the unique consumption good of the economy and suffer from the disutility of providing the labor supply l. We further assume that in each period, the instantaneous utility is separable in consumption and labor: $U(c, l) = u(c) - v(l)$, where $u, v : \mathbb{R}_+ \to \mathbb{R}$ are twice

[3]We follow Green (1994) and assume that the law of large numbers holds.

continuously differentiable and increasing. Furthermore, u is concave, with $u'(0) = \infty$, and v is convex.

2.3 Labor taxes

For the sake of generality, and for a theoretical reason which we develop in Section 2.8 below, we introduce a rich set of linear labor tax, to have realistic wage bargaining process. First, we assume that unions bargain over the nominal wage rate, denoted by $\hat{W}_t$. Workers pay a linear labor tax τ_t^L on this income such that their post-tax income is $(1-\tau_t^L)\hat{W}_t$. Second, firms pay an additional labor tax, τ_t^S, which implies a wedge between the labor cost per unit of labor, $\tilde{W}_t$, paid by firms and the wage $\hat{W}_t$ bargained by workers. This additional tax can be thought of as an employer social contribution that does not appear on the payroll of workers. Formally, the labor cost $\tilde{W}_t$, the bargained wage $\hat{W}_t$ and the tax τ^S verify the following relationship: $\hat{W}_t = (1-\tau_t^S)\tilde{W}_t$. The tax τ_t^S will have an effect on labor demand that will be internalized by unions in their bargaining strategy. Similarly, the tax τ_t^L will have an effect on labor income that will also be internalized. The difference between the two taxes is that τ_t^S has a direct effect on employment for a given bargained wage $\hat{W}_t$ but not on the hourly wage income, whereas τ_t^L has a direct effect on hourly wage W_t for a given wage $\hat{W}_t$, but no direct effect on employment.[4]

2.4 Production

The specification of the production sector follows the New-Keynesian literature on price stickiness, adapted to the previous tax structure. The consumption good Y_t is produced by a unique profit-maximizing representative firm that combines intermediate goods $(y_{j,t}^f)_j$ from different sectors indexed by $j \in [0,1]$ using a standard Dixit-Stiglitz aggregator with an elasticity of substitution, denoted ε_P:

$$Y_t = \left[\int_0^1 {y_{j,t}^f}^{\frac{\varepsilon_P - 1}{\varepsilon_P}} dj\right]^{\frac{\varepsilon_P}{\varepsilon_P - 1}}.$$

For any intermediate good $j \in [0,1]$, the production $y_{j,t}^f$ is realized by a monopolistic firm and sold at price $p_{j,t}$. The profit maximization for the firm producing the final good implies:

$$y_{j,t}^f = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon_P} Y_t, \text{ where } P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon_P} dj\right)^{\frac{1}{1-\varepsilon_P}}.$$

The quantity P_t is the price index of the consumption good. Intermediary firms are endowed with a Cobb-Douglas production technology and use only labor. The production technology involves that $\tilde{l}_{j,t}$ units of labor are transformed into $y_{j,t}^f = Z_t \tilde{l}_{j,t}$ units of intermediate good. Since

[4] We denote direct effect the partial equilibrium effect of each variable. In general equilibrium (with endogenous income), these taxes affect all variables.

intermediate firms have market power, the firm's objective is to minimize production costs, subject to producing the demand $y^f_{j,t}$. The cost function $C_{j,t}$ of firm j is therefore $C_{j,t} = \min_{\tilde{l}_{j,t}} \{\tilde{w}_t \tilde{l}_{j,t}\}$, subject to $y^f_{j,t} = Z_t \tilde{l}_{j,t}$, where $\tilde{w}_t = \tilde{W}_t / P_t$ is the real overall wage rate. The maximization implies the following mark-up:

$$m_t = \frac{1}{Z_t} \tilde{w}_t. \tag{1}$$

In addition to the production cost, intermediate firms face a quadratic price adjustment cost à la Rotemberg when setting their price. Following the literature, the price adjustment cost is proportional to the magnitude of the firm's relative price change and equal to $\frac{\psi_P}{2} \left(\frac{p_{j,t}}{p_{j,t-1}} - 1 \right)^2$. We can thus deduce the real profit, denoted Ω_t at date t of firm j:

$$\Omega_{j.t} = \Big(\frac{p_{j,t}}{P_t} - m_t (1 - \tau^Y_t) \Big) \Big(\frac{p_{j,t}}{P_t} \Big)^{-\epsilon} Y_t - \frac{\psi_P}{2} \left(\frac{p_{j,t}}{p_{j,t-1}} - 1 \right)^2 Y_t - t^Y_t,$$

where t^Y_t is a lump-sum tax financing the subsidy τ^Y. Computing the firm j's intertemporal profit requires to define the firm's pricing kernel. We follow Bhandari et al. (2021) and assume a constant pricing kernel.[5] The firm j's thus sets its price schedule $(p_{j,t})_{t \geq 0}$ to maximize its intertemporal profit at date 0: $\max_{(p_{j,t})_{t \geq 0}} \mathbb{E}_0 [\sum_{t=0}^{\infty} \beta^t \Omega_{j,t}]$. The solution is independent of the firm type j and all firms in the symmetric equilibrium charge the same price: $p_{j,t} = P_t$. Denoting the price inflation rate as $\pi^P_t = \frac{P_t}{P_{t-1}} - 1$ and setting $\tau^Y = \frac{1}{\varepsilon_P}$ to obtain an efficient steady state, we obtain the standard equation characterizing the Phillips curve in our environment:

$$\pi^P_t (1 + \pi^P_t) = \frac{\varepsilon_P - 1}{\psi_P} (m_t - 1) + \beta \mathbb{E}_t \left[\pi^P_{t+1} (1 + \pi^P_{t+1}) \frac{Y_{t+1}}{Y_t} \right], \tag{2}$$

where:

$$Y_t = Z_t L_t \tag{3}$$

The real profit is independent of the firm's type and can be expressed as follows:

$$\Omega_t = \Big(1 - \frac{\psi_P}{2} (\pi^P_t)^2 \Big) Y_t - \tilde{w}_t L_t. \tag{4}$$

2.5 Labor market: Labor supply and Union wage decision

Following the New Keynesian sticky-wage literature, labor hours are supplied monopolistically by unions (Auclert et al., 2022). There is a continuum of unions of size 1 indexed by k and each union k supplies L_{kt} hours of labor at date t with nominal wage $\hat{W}_{kt}$. Union-specific labor supplies are then aggregated into aggregate labor supply by a competitive technology featuring a

[5] Our own computations also show us that the quantitative impact of the pricing kernel is limited.

constant elasticity of substitution ε_W:

$$L_t = \left(\int_k L_{kt}^{\frac{\varepsilon_W - 1}{\varepsilon_W}} dk \right)^{\frac{\varepsilon_W}{\varepsilon_W - 1}}. \tag{5}$$

The competitive aggregator demands the union labor supplies $(L_{kt})_k$ that minimize the total labor cost $\int_k \hat{W}_{kt} L_{k,t} dk$ subject to the aggregation constraint (5), where $\hat{W}_{kt}$ is the bargained nominal wage of the members of union k. The demand for labor of union k depends on the total labor cost paid by the firm $\tilde{W}_{kt}$: $L_{kt} = \left(\frac{\tilde{W}_{kt}}{\tilde{W}_t}\right)^{-\varepsilon_W}$, where $\tilde{W}_t = \left(\int_k \tilde{W}_{kt}^{1-\varepsilon_W} dk\right)^{\frac{1}{1-\varepsilon_W}}$ is the total nominal wage index. As the labor demand depends on relative wages, and $\frac{\tilde{W}_{kt}}{\tilde{W}_t} = \frac{\hat{W}_{kt}}{\hat{W}_t} \frac{1-\tau_t^S}{1-\tau_t^S} = \frac{\hat{W}_{kt}}{\hat{W}_t}$, total labor demand can be written as:

$$L_{kt} = \left(\frac{\hat{W}_{kt}}{\hat{W}_t} \right)^{-\varepsilon_W} L_t, \tag{6}$$

where $\hat{W}_t = \left(\int_k \hat{W}_{kt}^{1-\varepsilon_W} dk\right)^{\frac{1}{1-\varepsilon_W}}$ is the bargained nominal wage index. Each union k sets its wage $\hat{W}_{kt}$ so as to maximize the intertemporal welfare of its members subject to fulfilling the demand of equation (6). We assume the presence of quadratic utility costs related to the adjustment of the nominal wage and equal to $\frac{\psi_W}{2}(\hat{W}_{kt}/\hat{W}_{kt-1} - 1)^2 dk$. The objective of union k is thus:

$$\max_{(\hat{W}_{ks})_s} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \int_i \left(u(c_{i,s}) - v(l_{i,s}) - \frac{\psi_W}{2} \left(\frac{\hat{W}_{ks}}{\hat{W}_{ks-1}} - 1 \right)^2 \right) \ell(di),$$

subject to (6) and where $c_{i,t}$ and $l_{i,t}$ are the consumption and labor supply of agent i. The first-order condition with respect to W_{kt} thus writes as:

$$\pi_t^W(\pi_t^W + 1) = \frac{\hat{W}_{kt}}{\psi_W} \int_i \left(u'(c_{i,t}) \frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} - v'(l_{i,t}) \frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} \right) \ell(di) + \beta \mathbb{E}_t \left[\pi_{t+1}^W (\pi_{t+1}^W + 1) \right], \tag{7}$$

where the wage inflation rate is denoted by:

$$\pi_t^W = \frac{\hat{W}_{k,t}}{\hat{W}_{k,t-1}} - 1.$$

The labor supply l_{it} of agent i is the sum of her hours l_{ikt} supplied to union k, summed over all unions: $l_{it} = \int_k l_{ikt} dk$. Each union is assumed to request its members to supply an uniform number of hours, such that: $l_{ikt} = L_{kt}$. We thus deduce from (6):

$$\hat{W}_{kt} \frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} = \hat{W}_{kt} \frac{\partial \left(\int_k \left(\frac{\hat{W}_{kt}}{\hat{W}_t} \right)^{-\varepsilon_W} L_t dk \right)}{\partial \hat{W}_{kt}} = -\varepsilon_W L_{kt}. \tag{8}$$

To compute the derivative of consumption $\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}}$, it should observed that it is equal to the derivative of its net total income. The net total income of agent i writes as $(1-\tau_t^L)\hat{W}_{kt} y_{i,t} l_{i,t} / P_t$, where τ_t^L is the labor tax. Formally:

$$\begin{aligned}
\frac{1}{c_{i,t}}\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} &= \frac{1}{\hat{W}_{kt}} + \frac{1}{l_{i,t}}\frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} \\
&= \frac{1}{\hat{W}_{kt}} - \frac{\varepsilon_W}{\hat{W}_{kt}}\frac{L_{kt}}{l_{i,t}} \\
\hat{W}_{kt}\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} &= (1-\varepsilon_W)(1-\tau_t^L)\hat{W}_{kt}y_{i,t}l_{i,t}/P_t
\end{aligned} \tag{9}$$

We focus on the symmetric equilibrium where all unions choose to set the same wage $\hat{W}_{kt} = \hat{W}_t$, hence all households work the same number of hours, equal to $l_{it} = L_t$. Combining (7) with the partial derivatives (8) and (9), we deduce the following Phillips curve for wage inflation:

$$\pi_t^W(\pi_t^W+1) = \frac{\varepsilon_W}{\psi_W}\underbrace{\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}(1-\tau_t^L)\hat{w}_t\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)}_{\text{labor gap}}L_t + \beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W+1)\Big], \tag{10}$$

where $\hat{w}_t = \hat{W}_t/P_t$ is the real pre-tax wage.

2.6 Assets

The only asset is nominal public debt, whose supply size is denoted by B_t at date t, and which pays off the pre-determined before-tax nominal interest rate i_{t-1}. Public debt is issued by the government and assumed to be default free. The financial market clearing implies that the net total savings of households, denoted A_t, must equal public debt:

$$A_t = B_t. \tag{11}$$

The corresponding real before-tax (net) interest rate for public debt, denoted by $\tilde{r}_t$, is defined by:

$$\tilde{r}_t = \frac{1+i_t}{1+\pi_t^P} - 1. \tag{12}$$

2.7 Agents' program, resource constraints, and equilibrium definition

Each agent enters the economy with an initial endowment of public debt $a_{i,-1}$ and an initial productivity level $y_{i,0}$. The joint initial distribution over public debt and productivity levels is denoted Λ_0. In later periods, each agent learns her productivity level $y_{i,t}$, supplies labor, and earns her savings payoffs. Since the labor supply L_t is chosen by unions, the labor income is $(1-\tau_t^L)\hat{w}_t y_{i,t}L_t$. The before-tax real financial payoff amounts to $\tilde{r}_t a_{i,t-1}$.

We assume that agents pay two other taxes. First, a capital tax $\hat{\tau}_t^K$ is levied on interest payment and implies a net asset payoff $(1-\hat{\tau}_t^K)\tilde{r}_t a_{i,t-1}$. Second, an income tax τ_t^E is levied on total income, which implies a post-tax total income equal to $(1-\tau_t^E)\big((1-\tau_t^L)\hat{w}_t y_{i,t}L_t + (1-\hat{\tau}_t^K)\tilde{r}_t a_{i,t-1}\big)$.

We assume that the latter income tax τ_t^E is not internalized by the unions, as the latter cannot observe total income.[6]

Agents earn this net total income and use it together with their past savings to consume $c_{i,t}$ and save $a_{i,t}$. Their budget constraint can be expressed as follows:

$$c_{i,t} + a_{i,t} = a_{i,t-1} + (1 - \tau_t^E)\big((1 - \hat{\tau}_t^K)\tilde{r}_t a_{i,t-1} + (1 - \tau_t^L)\hat{w}_t y_{i,t} L_t\big). \tag{13}$$

To simplify the previous notation, we define the post-tax real interest and wage rates as:

$$r_t = (1 - \tau_t^E)(1 - \hat{\tau}_t^K)\tilde{r}_t, \tag{14}$$

$$w_t = (1 - \tau_t^E)(1 - \tau_t^L)\hat{w}_t = (1 - \tau_t^E)(1 - \tau_t^L)(1 - \tau_t^S)\tilde{w}_t. \tag{15}$$

The agent's program can be finally be written as:

$$\max_{\{c_{i,t},a_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{i,t}) - v(L_t)\right), \tag{16}$$

$$c_{i,t} + a_{i,t} = (1 + r_t)a_{i,t-1} + w_t y_{i,t} L_t, a_{i,t}, \tag{17}$$

and subject to the credit constraint $a_{i,t} \geq -\underline{a}$, and the consumption positivity constraint $c_{i,t} > 0$. The notation $\mathbb{E}_0$ is an expectation operator over both idiosyncratic and aggregate risks. The solution of the agent's program is a sequence of functions, defined over $([-\bar{a}; +\infty) \times \mathcal{Y}) \times \mathcal{Y}^t \times \mathbb{R}^t$ and denoted by $(c_t, a_t)_{t\geq 0}$, such that:[7]

$$c_{i,t} = c_t((a_{i,-1}, y_{i,0}), y_i^t, z^t),\ a_{i,t} = a_t((a_{i,-1}, y_{i,0}), y_i^t, z^t). \tag{18}$$

For the sake of simplicity, we will keep using the notation with the i-index. Denoting by $\nu_{i,t}$ the discounted Lagrange multipliers of the credit constraint, the Euler equation corresponding to the agent's program (16) is:

$$u'(c_{i,t}) = \beta \mathbb{E}_t\Big[(1 + r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t}. \tag{19}$$

2.8 Government and market clearing

The government has to finance an exogenous public good expenditure G_t, by raising a quite large number of taxes and by issuing one-period riskless public debt. First, the government raises three kinds of labor taxes: (i) a tax τ_t^S based on labor cost $\tilde{w}_t$ and paid by employers, (ii) a tax τ_t^L based on bargained wage $\hat{w}_t$ and paid by workers, and finally (iii) a tax τ_t^E based on total income and paid by workers. Importantly, the three labor instruments are independent and

[6] The justification of this tax is presented in the next section. Although playing a major theoretical role, it has a modest quantitative impact, as we illustrate below.

[7] See e.g. Miao (2006), Cheridito and Sagredo (2016), and Açikgöz (2018) for a proof of the existence of such functions.

not redundant. Indeed, on the one hand, τ_t^S creates a wedge between the labor cost and the bargained wage, while τ_t^L and τ_t^E create wedges between the bargained wedge and the net wage. On the other hand, τ_t^L is internalized by unions, while τ_t^E is not. These three taxes will play on different margins and will allow us derive our equivalence result below. Hence, they should be understood as theoretical tools needed to generate price and wage stability. Each tax will be removed in turn to consider more realistic fiscal settings and to assess how each fiscal instrument contributes to inflation volatility.

In addition to capital and labor taxes and to public debt, the government also fully taxes the firms' profits, Ω_t, which limits the distortions implied by profit distribution. We can now express the government budget constraint. The government has to finance public spending and the repayment of past public debt. Its resources consist of all labor taxes, capital taxes, corporate profits, and newly issued public debt. We obtain:

$$\begin{aligned} G_t + \frac{1+i_t}{1+\pi_t^P} B_{t-1} \leq \Omega_t + B_t + \tau_t^E \big((1-\hat{\tau}_t^K)\tilde{r}_t \int_i a_{i,t-1}\ell(di) + (1-\tau_t^L)\hat{w}_t L_t\big) \\ + \hat{\tau}_t^K \tilde{r}_t \int_i a_{i,t-1}\ell(di) + \tau_t^L \hat{w}_t L_t + \tau_t^S \tilde{w}_t L_t. \end{aligned}$$

We can simplify the previous government budget constraint using the financial market clearing (11), the post-tax interest rate $\tilde{r}_t$ (12), and the profit definition (4):

$$G_t + (1 + (1-\tau_t^E)(1-\hat{\tau}_t^K))B_{t-1} + (1-\tau_t^L)(1-\tau_t^E)\hat{w}_t L_t \leq \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big) Y_t + B_t,$$

which using post-tax rate definitions (14) implies:

$$G_t + r_t B_{t-1} + w_t L_t \leq \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big) Y_t + B_t - B_{t-1}, \tag{20}$$

We finally express the financial market clearing condition and the economy resource constraints:

$$\int_i a_{i,t}\ell(di) = A_t = B_t, \tag{21}$$

$$\int_i c_{i,t}\ell(di) + G_t = \left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right) Z_t L_t. \tag{22}$$

Equilibrium definition. We can finally formulate our definition of competitive equilibrium.

Definition 1 (Sequential equilibrium) *A sequential competitive equilibrium is a collection of individual functions $(c_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$, of aggregate quantities $(L_t, A_t, Y_t, \Omega_t, m_t)_{t\geq 0}$, of price processes $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t, i_t)_{t\geq 0}$, of fiscal policies $(\tau_t^L, \tau_t^S, \tau_t^E, \hat{\tau}_t^K, B_t)_{t\geq 0}$, and inflation dynamics $(\pi_t^W, \pi_t^P)_{t\geq 0}$ such that, for an initial wealth and productivity distribution $(a_{i,-1}, y_{i,0})_{i\in\mathcal{I}}$, and for an initial value of public debt verifying $B_{-1} = \int_i a_{i,-1}\ell(di)$, and for an initial value of the aggregate shock z_0, we have:*

1. *given prices, the functions $(c_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$ solve the agent's optimization program (16)–(17);*

2. *financial, and goods markets clear at all dates: for any $t \geq 0$, equations (21) and (22) hold;*

3. *the government budget is balanced at all dates: equation (20) holds for all $t \geq 0$;*

4. *firms' profits Ω_t and the mark-up m_t are consistent with equations (1) and (4);*

5. *the price inflation path $(\pi_t^P)_{t\geq 0}$ is consistent with the price Phillips curve (2), while the wage inflation path $(\pi_t^W)_{t\geq 0}$ is consistent with the wage Phillips curve (10);*

6. *the nominal and real rates $(\tilde{r}_t, i_t)_{t\geq 0}$ verify (12);*

7. *post tax rates $(w_t, r_t, \tilde{r}_t, \hat{w}_t, \tilde{w}_t)_{t\geq 0}$ are defined in equations (14)–(15).*

2.9 Interpretation the TFP shock as an energy price shock

We explain how the TFP shock can be interpreted as an energy price shock. We do so in a general case featuring capital. We consider a CRS production function $\tilde{F}$ using capital, labor, and energy. Energy is denoted E and its price is denoted by $\tilde{q}$. We thus have:

$$\tilde{F}(K, L, E) = \tilde{Z} K^{\alpha_K} L^{\alpha_L} E^{1-\alpha_K-\alpha_L},$$

where α_K and α_L are capital and labor shares respectively. We can easily generalize the construction of Section 2.4. The markup of equation (1) is denoted with a tilde and becomes: $\tilde{m}_t = \frac{1}{\tilde{Z}_t}\left(\frac{\tilde{r}_t^K+\delta}{\alpha_K}\right)^{\alpha_K}\left(\frac{\tilde{w}_t}{\alpha_L}\right)^{\alpha_L}\left(\frac{\tilde{q}_t}{1-\alpha_K-\alpha_L}\right)^{1-\alpha_K-\alpha_L}$, while factor prices are defined as follows:

$$\tilde{r}_t^K + \delta = \tilde{m}_t \alpha_K \tilde{Z}_t K_{t-1}^{\alpha_K-1} L_t^{\alpha_L} E_t^{1-\alpha_K-\alpha_L}, \tag{23}$$

$$\tilde{w}_t = \tilde{m}_t \alpha_L \tilde{Z}_t K_{t-1}^{\alpha_K-1} L^{\alpha_L-1} E_t^{1-\alpha_K-\alpha_L}, \tag{24}$$

$$\tilde{q}_t = \tilde{m}_t (1-\alpha_K-\alpha_L) \tilde{Z}_t K_{t-1}^{\alpha_K} L_t^{\alpha_L} E_t^{-\alpha_K-\alpha_L} \tag{25}$$

Using the expression (25) of $\tilde{q}_t$, we obtain:

$$E_t = \left(\frac{\tilde{m}_t(1-\alpha_K-\alpha_L)\tilde{Z}_t}{\tilde{q}_t}\right)^{\frac{1}{\alpha_K+\alpha_L}} K_t^{\frac{\alpha_K}{\alpha_K+\alpha_L}} L_t^{\frac{\alpha_L}{\alpha_K+\alpha_L}}. \tag{26}$$

We introduce the following notation:

$$Z_t = \tilde{Z}_t^{\frac{1}{\alpha_K+\alpha_L}} \left(\frac{\tilde{q}_t}{1-\alpha_K-\alpha_L}\right)^{1-\frac{1}{\alpha_K+\alpha_L}}, \tag{27}$$

$$\alpha = \frac{\alpha_K}{\alpha_K+\alpha_L}, \tag{28}$$

$$m_t = (\alpha_K+\alpha_L)\tilde{m}_t^{\frac{1}{\alpha_K+\alpha_L}}. \tag{29}$$

Substituting for the expression (26) of E_t into factor prices (23), we obtain:

$$\tilde{r}_t^K + \delta = m_t \alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha}, \tag{30}$$

where the second equality comes from rearrangement and the last from the definitions (27)–(29). Similarly for (24):

$$\tilde{w}_t = m_t (1-\alpha) Z_t K_{t-1}^{\alpha} L_t^{-\alpha}. \tag{31}$$

We have been able to rewrite factor prices $\tilde{r}_t$ and $\tilde{w}_t$ consistently with factor price definition. We now have to find a consistent definition of the production function. Adapting (3), we have:

$$\tilde{F}(K_{t-1}, L_t, E_t) = \frac{(\tilde{r}_t^K + \delta)K_{t-1} + \tilde{w}_t L_t + \tilde{q}_t E_t}{\tilde{m}_t},$$

or after substituting the expressions of $\tilde{F}$ and E_t and

$$\frac{(\tilde{r}_t^K + \delta)K_{t-1} + \tilde{w}_t L_t}{\tilde{m}_t} = Z_t(\alpha_K + \alpha_L)\tilde{m}_t^{\frac{1}{\alpha_K+\alpha_L}-1} K_{t-1}^{\alpha} L_t^{1-\alpha},$$

where we have used the definitions (27) of $Z_t$and (28) of α. Using the definition (29) of $\tilde{m}_t$, we finally obtain:

$$Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} = \frac{(\tilde{r}_t^K + \delta)K_{t-1} + \tilde{w}_t L_t}{m_t},$$

which is thus similar to (3). The function $F(K, L) = ZK^{\alpha}L^{1-\alpha}$ with Z and α defined in (27) and (28) is thus consistent with the new definitions of factor prices (30) and (31), the markup (29), as well as with the equation (3) connection output, factor prices and markups.

Interestingly, the TFP expression is $Z_t = \tilde{Z}_t^{\frac{1}{\alpha_K+\alpha_L}} \left(\frac{\tilde{q}_t}{1-\alpha_K-\alpha_L}\right)^{1-\frac{1}{\alpha_K+\alpha_L}}$ with $0 < \alpha_K + \alpha_L < 1$: an increase in energy prices (a higher $\tilde{q}_t$) can thus be interpreted as a drop in TFP Z_t. We will use this analogy in our quantitative exercise of Section 6.

Alternatively to a Cobb Douglas production function, one would consider a production function with Constant Elasticity of Substitution (CES) of the following form:

$$F(K_{t-1}, L_t, E_t) = Z_t \Big[(1-\epsilon)^{\frac{1}{\eta}} \Big(K_{t-1}^{\alpha} L_t^{1-\alpha}\Big)^{\frac{\eta-1}{\eta}} + \epsilon^{\frac{1}{\eta}} (E_t)^{\frac{\eta-1}{\eta}}\Big]^{\frac{\eta}{\eta-1}}$$

where ϵ is the energy share and η and the elasticity of substitution between energy inputs and value-added $V_t = K_{t-1}^{\alpha} L_t^{1-\alpha}$. When the elasticity of substitution is close to zero, small fluctuation in quantity of energy supplied can cause large spike in energy prices and marginal costs for firms. This ordering of the nests between energy, labor and capital is suggested by empirical evidence that the energy share in the economy follows closely the fluctuation in energy prices. Such CES expression is well suited for matching such patterns, as suggested in Hassler et al. (2021). We cover this production function in a forthcoming extension of this work.

3 Understanding the mechanisms in a simpler RA environment

Before solving for optimal fiscal and monetary policy in the general model, we consider here simpler environments, in which instruments follow simple rules. This will allow us to identify the key mechanisms at stake in this two-friction economy. We assume that the labor tax follows a rule à la Bohn (1998) that aims to stabilize public debt:

$$\tau_t^L = \tau_{ss}^L - c^B(B_t - B_{ss}), \tag{32}$$

where τ_{ss}^L and B_{ss} are the steady-state values of labor tax and public debt and $c_B > 0$ is the rule parameter. The higher c_B, the stronger the reaction of the labor tax τ^L top an increase in public debt. The other fiscal instruments are set to 0: $\tau_t^S = \tau_t^E = \hat{\tau}_t^K = 0$, what is the standard assumption in the literature.

The monetary policy follows a standard Taylor rule:

$$i_t = r_{ss} + \phi^\pi \mathbb{E}_t \pi_{t+1}^P, \tag{33}$$

where r_{ss} is the steady real interest rate and $\phi^\pi > 1$. Finally, we assume that the log of TFP follows an AR(1) process: $Z_t = e^{u_t}$, with:

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u,$$

where $\rho_u \in (0,1)$ is the persistence and ε_t^u is the TFP shock.

We further consider an economy featuring a representative agent – which corresponds to a unique productivity level, equal to $y = 1$ for all agents. We further take advantage of simplifying assumptions to derive analytical solutions and identify key mechanisms at stake. We later verify that the mechanisms we identify still hold in a more general representative-agent economy.

Our main assumption for analytical tractability is to consider a log-linearized version of the model in the case of myopic price setters (i.e., $\beta = 0$). In this case, the two Phillips curves (2) and (10) simplify as follows.

$$\begin{aligned}
\pi_t^W &= \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}(1-\tau_t^L)\hat{w}_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) L_t, \\
\pi_t^P &= \frac{\varepsilon_P - 1}{\psi_P}(m_t - 1).
\end{aligned}$$

Removing the expectation term reduces the dimensionality of the dynamic system, without changing the nature of the mechanisms, as we will check using numerical simulations. Our first result, presented in the next proposition, considers monetary policy without fiscal policy.

Proposition 1 (Representative agent) *We assume $\chi = \phi = \gamma = 1$ and no fiscal policy $G_{ss} = B_{ss} = c^B = 0$. Then, the dynamics of the real wage is $w_t = d_1 w_{t-1} + d_2 u_t$, where d_2 has* *the sign of:*

$$\left(\frac{\varepsilon_P - 1}{\psi_P} - \frac{\varepsilon_W - 1}{\psi_W}\right)(1-\rho_u) + 2\frac{\varepsilon_P - 1}{\psi_P}\frac{\varepsilon_W - 1}{\psi_W}(\phi^\pi - 1)\rho_u. \tag{34}$$

The proof is in Appendix A.1. The previous propositions characterizes the reaction of the real wage to a technology shock u_t. The direction of this reaction crucially depends on the persistence of the aggregate shock. When the TFP shock is very persistent (ρ_u close to 1), the term in (34) becomes approximately equal to $2\frac{\varepsilon_P-1}{\psi_P}\frac{\varepsilon_W-1}{\psi_W}(\phi^\pi - 1)$, which is always positive since $\phi^\pi > 1$. In other words, in the absence of fiscal response, when the TFP shock is persistent, the real wage increases in booms. When the TFP shock is transitory, the effect is not univocal any more. Indeed, when the TFP shock is transitory (ρ_u close to 0), the sign of (34) is given by the sign of $\frac{\varepsilon_P-1}{\psi_P} - \frac{\varepsilon_W}{\psi_W}$, i.e., to the sign of the difference between the slopes of the price and wage Phillips curves. When the slope of the price Phillips curve is higher than the wage one, the real wage will increase after a positive TFP shock. Oppositely, when the slope of the wage Phillips curve is higher, the real wage will decrease at impact. In general (when $0 < \rho_u < 1$), the response of the real wage to the TFP shock will result from the combination of the two previous effects.

Our second result concerns the response of wage to a TFP shock in the presence of fiscal policy.

Proposition 2 (Representative agent) *We assume $\chi = \phi = \gamma = 1$ and a fiscal policy given by rule (32). We further assume that the slopes of the Philipps curves and the fiscal response c^B are small. Then, the dynamics of the real wage is $w_t = d_1 w_{t-1} + d_3 b_{t-1} + d_2 u_t$, where d_2 has the sign of*

$$\left(\frac{\varepsilon_P - 1}{\psi_P} - \frac{\varepsilon_W - 1}{\psi_W}\right)(1-\rho_u) + \frac{\varepsilon_P - 1}{\psi_P}\left(2\frac{\varepsilon_W - 1}{\psi_W}(\phi^\pi - 1) - c^B\right)\rho_u \tag{35}$$

The proof is in Appendix A.2. The two state variables are $w_{t-1}$and b_{t-1}. The fiscal coefficient now enters the response of the real wage, as it affects the wage bargaining process. The fiscal response tends to decrease the response of the real wage to the TFP shock, especially when the shock is persistent. Indeed, the fiscal response implies an increase in labor tax, which tends to decrease the real net wage. Similarly, the fiscal response also affects both price and wage inflation rates, which becomes:

$$\begin{aligned}
\pi_t^P &= \frac{\varepsilon_P - 1}{\psi_P} w_t - \frac{\varepsilon_P - 1}{\psi_P} u_t + \frac{\varepsilon_P - 1}{\psi_P} c^B b_{t-1}, \\
\pi_t^W &= w_t - w_{t-1} + \pi_t^P.
\end{aligned}$$

4 The Ramsey problem

We now solve for the optimal policy in the general model, featuring heterogeneous agents and the full fiscal system. Following LeGrand et al. (2022), we assume that the planner maximizes a generalized social welfare function, where the weights on each period utility can depend on the current productivity of the agent. The objective of the planner is:

$$W_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(l_t^i) \right) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right]. \tag{36}$$

This expression embeds the utilitarian case, where $\omega(y) = 1$ for all y, and the motivation for this generalization is explained in the quantitative section below. The Ramsey planner's program can be written as:

$$\max_{\left(\tau_t^L, \tau_t^S, \tau_t^E, \pi_t^P, \pi_t^W, w_t, r_t, i_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t \geq 0}} W_0, \tag{37}$$

$$G_t + (1 + r_t) \int_i a_{i,t-1} \ell(di) + w_t L_t \leq \left(1 - \frac{\psi_P}{2} (\pi_t^P)^2 \right) Z_t L_t + \int_i a_{i,t} \ell(di), \tag{38}$$

$$\text{for all } i \in \mathcal{I}\text{: } c_{i,t} + a_{i,t} = (1 + r_t) a_{i,t-1} + w_t y_{i,t} L_t, \tag{39}$$

$$a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0, \ \nu_{i,t} \geq 0, \tag{40}$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1 + r_{t+1}) u'(c_{i,t+1}) \Big] + \nu_{i,t}, \tag{41}$$

$$\pi_t^W (\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} \frac{w_t}{1 - \tau_t^L} \int_i y_{i,t} u'(c_{i,t}) \ell(di) \right) L_t + \beta \mathbb{E}_t \Big[\pi_{t+1}^W (\pi_{t+1}^W + 1) \Big], \tag{42}$$

$$\pi_t^P (1 + \pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P} \left(\frac{1}{Z_t} \frac{w_t}{(1 - \tau_t^L)(1 - \tau_t^S)(1 - \tau_t^E)} - 1 \right) + \beta \mathbb{E}_t \left[\pi_{t+1}^P (1 + \pi_{t+1}^P) \frac{Z_{t+1} L_{t+1}}{Z_t L_t} \right], \tag{43}$$

$$(1 + \pi_t^W) \frac{w_{t-1}}{1 - \tau_{t-1}^L} = \frac{w_t}{1 - \tau_t^L} (1 + \pi_t^P), \tag{44}$$

$$\mathbb{E}_t [1 + r_{t+1}] = \mathbb{E}_t \left[\frac{1 + i_t}{1 + \pi_{t+1}^P} \right]. \tag{45}$$

and subject to the positivity of consumption choices, and initial conditions.

The constraints in the Ramsey program include: the governmental and individual budget constraints (38) and (39), the individual credit constraint (and related constraints on $\nu_{i,t}$) (40), the individual Euler equations (41), the Phillips curves (42) and (43), the relationship (44) between price and wage inflation rates, and the relationship (45) between real and nominal rates.

This economy faces different frictions, which are worth summarizing. The monetary economy features two sets of market imperfections. The first set is related to the goods market. Intermediary firms enjoy a monopoly power, which implies a price markup m_t that can differ from one. There is also a Rotemberg cost for price adjustment, which prevents firms from freely setting their price. Note that the good market imperfections are complementary: one vanishes when the other is absent, as can be seen from the price Phillips curve (2). The second set of imperfections is related to the labor market. The union implies that the labor supply of agents is not set optimally, while the Rotemberg cost for wages prevents unions from freely setting wages. Note that in the absence of Rotemberg cost, the labor supply still remains sub-optimal, as it remains set at the union level. Without Rotemberg cost, the equation characterizing the choice of the labor supply (common to all agents) would be $v'(L_t) = w_t \int_i y_{i,t} u'(c_{i,t}) \ell(di)$, while it would be $v'(l_{i,t}) = w_t y_{i,t} u'(c_{i,t})$, if agents were able to choose their individual labor supply $l_{i,t}$. This sub-optimal common labor choice will play a major role in our equivalence results below.

Roadmap. To decompose and identify the mechanisms at stake, we decompose the analysis in several steps. In all cases, we will assume that the economy starts in period 0 from the steady-state distribution and is then hit once by a negative persistent productivity shock. We hence focus on so-called MIT shocks. Moreover, we focus on the case where $G_t = 0$ for the theoretical results – so as to stress out the impact of the TFP shock and not of the financing of public spending. We will relax the latter assumption and allow $G_t > 0$ in the quantitative section below.

We will consider various economies, corresponding to different sets of instruments available for the planner.

1. We first characterize the flexible-price allocation, where all instruments $(\tau_t^E, \tau_t^L, \tau_t^S)$ are available a,d where all agents are constrained to supply the same hours ($l_{i,t} = L_t$) but without union market power.

2. We then characterize the optimal allocation, where all fiscal $(\tau_t^E, \tau_t^L, \tau_t^S)$ and monetary instruments (i_t) are available and where unions are present. We show that the planner reproduces the previous allocation, and implements price and wage stability. This our main equivalence result.[8]

3. We then characterize the optimal allocation, where unions are present but where only two fiscal instruments (τ_t^L, τ_t^S) and monetary instruments (i_t) are available. We show that the planner implements price stability ($\pi_t^P = 0$), but not nominal wage stability ($\pi_t^W \neq 0$).

4. We finally characterize the optimal allocation with only one fiscal (τ_t^L) instrument. The planner then deviates from both price and wage stability: $\pi_t^P, \pi_t^W \neq 0$.

Finally, the previous analysis is performed both for the heterogeneous-agent and the representative-agent economies. This will allow us to understand in each case the contribution of market incompleteness.

To simplify the derivation of first-order conditions, we use some aspects of the methodology of Marcet and Marimon (2019) used in LeGrand et al. (2022), which is sometimes called the

[8] We ensure that the steady state is the same for the cases 2-5 analyzed below.

Lagrangian method (Golosov et al., 2016), applied to incomplete-market environments. This methodology connects to the public finance literature – that we further explain in the different environments listed above.

The summary is provided in Table 1 in Section 4.2. Section 6 provides a quantification of the different mechanisms.

4.1 The flexible-price economy

The flexible-price economy features no price- and no wage-adjustment cost. In this economy, all workers are assumed to work the same number of hours and the planner is assumed to be able to directly choose this common labor supply.[9] The firms make no profit and we thus have $m_t = 1$. In addition, monetary policy has no role has price are fully flexible, and the real interest rate is determined in equilibrium.

To save some space, we provide the program in Appendix B.1, and focus here on the methodology and the main results. First, we denote by $\beta^t \lambda_{i,t}$ the Lagrange multipliers of the Euler equations (41) of agent i at date t. The Lagrange multiplier of the government budget constraint is $\beta^t \mu_t$. (38) with $\pi_t^P = 0$. We can then express the intertemporal Lagrangian of the program, denoted by $\mathcal{L}$. From this Lagrangian, we can define $\psi_{i,t}^{FP}$ as:

$$\psi_{i,t}^{FP} := \frac{\partial \mathcal{L}}{\partial c_{i,t}},$$

which is the value for the planner to transfer one extra unit of consumption good to agent i in period t.[10] To some extent, this quantity can be understood as the planner's version of the agent's marginal utility of consumption. We call this amount, the *social valuation of liquidity for agent i*. The expression of $\psi_{i,t}^{FP}$ is:

$$\psi_{i,t}^{FP} := \underbrace{\omega_t^i u'(c_{i,t})}_{\text{direct effet}} - \underbrace{(\lambda_{i,t} - (1 + r_t)\lambda_{i,t-1})\, u''(c_{i,t})}_{\text{effect on savings}}. \tag{46}$$

We add the upper-script FP to refer to flexible price, as the nature of the friction will change the expression of the valuation of liquidity for agents i. As can be seen in equation (46), this valuation consists of two terms. The first is the marginal utility of consumption $\omega_t^i u'(c_{i,t})$, which is the private valuation of liquidity for agent i multiplied by the current weight of agent i. The second term in (46) takes into consideration the impact of the extra consumption unit on saving incentives from periods $t-1$ to t and from periods t to $t+1$. An extra consumption unit makes the agent more willing to smooth out her consumption between periods t and $t+1$, and

[9] It is also possible to solve the model where the planner can différentiate hours across agents. The allocation is very different from the market one, and it it thus a useless benchmark.

[10] To simplify the notation, we keep the index i, but the sequential representation (referring to histories and not the identity of agent i) can be derived along the lines of equation (18).

thus makes her Euler equation (either nominal or real) more "binding". This more "binding" constraint reduces the utility by the algebraic quantity $u''(c_{i,t})\lambda_{i,t}$. The extra consumption unit at t also makes the agent less willing to smooth her consumption between periods $t-1$ and t and therefore "relaxes" the constraint of date $t-1$. This is reflected in the quantity $\lambda_{i,t-1}$.

This marginal valuation $\psi_{i,t}^{FP}$ has the same economic meaning as the *Generalized Social Marginal Welfare Weights* (GSMWW) introduced by Saez and Stantcheva (2016), which they denote as g_i. It is the marginal valuation, which allows one to assess the welfare effect of a marginal change in tax systems.[11] This quantity appears in planner's first-order conditions. For instance, the FOC with respect to the labor supply L_t is:

$$\int_i \omega_{i,t}\ell(di)v'(L_t) = Z_t \int_i y_{i,t}\psi_{i,t}^{FP}\ell(di), \tag{47}$$

which has to be compared to $v'(l_{i,t}) = w_t y_{i,t} u'(c_{i,t})$ when agents individually decide of their labor supply. As in the individual FOC, the planner equalizes the marginal cost of one extra unit of labor to the marginal benefit, but there are three differences. First, since the labor supply is common to all agents, the planner has to take into account all individual situations, and hence needs to aggregate over the whole population. Second, the planner does not value marginal consumption through marginal utility as agents but through the marginal valuation of liquidity $\psi_{i,t}^{FP}$. Finally, the planner does not value the marginal benefit of labor supply with the net wage w_t but but the marginal productivity Z_t.

In addition to $\psi_{i,t}^{FP}$, another key quantity is the Lagrange multiplier, μ_t, on the governmental budget constraint. The quantity μ_t represents the marginal cost in period t of transferring one extra unit of consumption to households. Therefore, the quantity $\psi_{i,t} - \mu_t$ can be interpreted as the "net" valuation of liquidity. This is from the planner's perspective, the benefit of transferring one extra unit of consumption to agent i, net of the governmental cost. We thus define:

$$\hat{\psi}_{i,t}^{FP} := \psi_{i,t}^{FP} - \mu_t. \tag{48}$$

The interpretation of first-order conditions is greatly clarified by expressing them using $\hat{\psi}_{i,t}$ rather than the multiplier on Euler equations, $\lambda_{i,t}$. For instance, the first-order condition with respect to the post-tax wage rate w_t, is:

$$\int_i \hat{\psi}_{i,t}^{FP} y_{i,t}\ell(di) = 0. \tag{49}$$

The planner sets the labor tax (and thus the real wage) so as to tradeoff on the one hand the resources obtained from raising taxes (equal to the shadow price multiplied by labor supply $\mu_t L_t$)

[11] The corresponding expression, following Saez and Stantcheva (2016) notation, in a static environment would thus be $\int_t g_i y_i \ell(di) = \mu$. In a dynamic setting, it appears that $\psi_{i,t}$ is not a sufficient statistics for agents i, and that the knowledge of the marginal utility of agent i is necessary to determine optimal policy (see equation (55) for instance). Note that compared to Saez and Stantcheva (2016), the elasticity of labor supply does not appear in the formula for taxation, because labour supply is determined by demand (as agents are not on their labor supply).

and on the other hand benefits of higher taxes, which depends on the productivity y_i for agent i, and on the marginal valuation $\hat{\psi}_{i,t}$.

The heterogeneous-agent model provides (with some obvious restrictions) some additional dynamic constraints on these valuation for the planner. For instance, we show that dynamics of this valuation for unconstrained agents is:

$$\hat{\psi}_{i,t}^{FP} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{i,t+1}^{FP} \right],$$

which can be seen as a generalized consumption Euler equation for the planner and not for agents. We derive all first-order conditions in Appendix B.1. We use this allocation to derive our equivalence results in the next section.

4.2 The sticky price economy with all instruments

We now solve for optimal policy in an economy plagued with two nominal frictions, where the planner has use all fiscal and monetary instruments. The Ramsey planner can be written as:

$$\max_{(\tau_t^L, \tau_t^S, \tau_t^K, B_t, T_t, \pi_t^P, \pi_t^W, w_t, r_t, \Omega_t, i_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i)_{t \geq 0}} W_0, \tag{50}$$

subject to equations (36)–(44). We can state our main equivalence result.

Proposition 3 (An equivalence result) – *In the HA economy, when all instruments $(\tau_t^E, \tau_t^S, \tau_t^L, \tau_t^K, B_t, i_t)$ are optimally chosen, the planner exactly reproduces the flexible-price allocation and the inflation on prices and wages is null in all periods.*

– *In the RA economy, when all instruments $(\tau_t^E, \tau_t^S, \tau_t^L, \tau_t^K, B_t, i_t)$ are optimally chosen, the planner implements the first-best allocation.*

Proposition 2 generalizes the equivalence result of Correia et al. (2008) and Correia et al. (2013) for representative agent economies and LeGrand et al. (2022) for heterogeneous-agent economy to the case where there are both sticky prices and sticky wages. Interestingly, compared to LeGrand et al. (2022), we need two additional instruments (τ_t^E, τ_t^S), whereas we introduce one additional nominal constraint. Indeed, we need one instrument to prevent wage inflation (which destroys resources) and another one to reproduce the flexible price labor supply and neutralize the market power of unions. In the presence of a sufficiently large fiscal system, monetary policy has no role but price stability. Importantly, the result requires the presence of two labor taxes. The first labor tax τ^S (internalized by the planner) enables the planner to "isolate" the pre-tax rate $\tilde{w}_t$ that is determined by the allocation (with a zero price inflation) from the union wage $\hat{w}_t$ that is determined by the inflation path $(\pi_t^W)_t$. Removing τ^S as an independent instrument imposes a constraint between the factor price $\tilde{w}_t$ and the wage inflation path. In other words, the planner would have to balance the effects of price inflation (determining $\tilde{w}_t$) and of wage inflation (determining $\hat{w}_t$). The second labor tax τ_t^E enables the planner to simultaneously set the labor supply optimally (as in equation (47)) and close the wage gap in the wage Phillips curve. Removing τ_t^E would imply that the planner would need to tradeoff two inefficiencies: (i) the sub-optimal labor supply due the market power of unions and (ii) the cost of wage inflation. Should one of these two instruments be removed, Proposition 3 would not hold anymore and the economy would feature positive inflation on wages or on prices.

Overall, the equivalence results of Proposition 3 rationalize our tax system, which is the minimal tax system for which price stability is optimal.[12]

4.3 The model with missing instruments in the HA economy

We now remove some instruments to assess their contribution to price and wage stability. The following proposition summarizes our results, which are derived in Appendices B.3–B.4.

Proposition 4 (Result τ^E) *In the HA economy:*

- *When $\tau_t^E = 0$, and the other instruments $(\tau_t^S, \tau_t^L, \tau_t^K, B_t, i_t)$ are optimally chosen, the planner implements $\pi_t^P = 0$ but $\pi_t^W \neq 0$.*
- *When $\tau_t^E = \tau_t^S = 0$ and the other instruments $(\tau_t^L, \tau_t^K, B_t, i_t)$ are optimally chosen, the planner implements $\pi_t^P \neq 0$ and $\pi_t^W \neq 0$.*

Proposition 4 characterizes the impact of removing τ^E and then τ^S as independent instruments for the planner. First, when we remove the income tax τ_t^E, the planner still implements price stability, but now wage inflation is not constant after a TFP shock. This comes from the fact that the planner cannot close the wage gap of the wage Phillips curve and optimally set the common labor supply. Due to union labor market power, closing the wage gap would imply an inefficient labor supply. The planner chooses to change the number of worked hours along the business cycle by allowing an non-zero wage inflation. The planner thus trades off a more efficient labor supply at the cost of quadratic wage adjustment.

Second, when we remove both τ_t^E and τ_t^S, both price and wage inflation move along the business cycles. Indeed, on addition to the previous mechanism for τ_t^E, removing τ^S prevents the planner from closing the price gap and to set the labor cost to marginal productivity of labor. The planner chooses to let price inflation optimally vary so as to change the cost of labor.

The expression of the social value of liquidity actually depends on the instruments of the planner. For instance, in the case where $\tau_t^E = \tau_t^S = 0$, such that the only instruments are

[12] More precisely, other tax systems could correspond to price and wage stability. For instance, it could be possible to consider time-varying consumption tax as in Correia et al. (2008). However, the number of independent instruments would not be smaller. We consider our tax system to be not unrealistic, at least in some countries.

$(\tau_t^L, \tau_t^K, B_t, i_t)$, the expression of the social valuation of liquidity for agent i is:

$$\psi_{i,t}^{ES} := \underbrace{\omega_t^i u'(c_{i,t})}_{\text{direct effet}} - \underbrace{\left(\lambda_{i,t} - (1+r_t)\lambda_{i,t-1}\right) u''(c_{i,t})}_{\text{effect on savings}} \\ \underbrace{- \frac{\varepsilon_W - 1}{\psi_W} \gamma_{W,t} w_t L_t y_{i,t} u''(c_{i,t})}_{\text{effect on wage inflation}} . \tag{51}$$

Compared to the expression (46) of $\psi_{i,t}^{FP}$ in the flexible price economy, the expression of $\psi_{i,t}^{ES}$ features a third effect that comes from the fact that the wage Phillips curve is constraint for the planner. Indeed, in this case, the planner does not close the gap and the wage Phillips curve is a constraint for the planner, which implies the presence of the corresponding Lagrange multiplier $\gamma_{W,t}$. If the planner increases the consumption of agents i in period t, this will change the incentives to work and thus the union incentives to affect the wage dynamics. This is captured in the third term of equation (46). Furthermore, this new expression of $\psi_{i,t}^{FP}$ still verifies Euler-like equation for unconstrained agents: $\hat{\psi}_{i,t}^{ES} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{i,t+1}^{ES} \right]$, where $\hat{\psi}_{i,t}^{ES} = \psi_{i,t}^{ES} - \mu_t^{ES}$.

Finally, these deviations to price or wage stability still need to be quantified in the quantitative section, so as to assess the economic relevance of the various instruments at play.

4.4 Comparing with the representative agent

Before providing quantitative results, it is worth providing similar results for the representative agent. In this case, the unions do not generate distortions by imposing the same labor supply to agents endowed with heterogeneous productivity levels. This implies that removing fiscal instruments will have different conclusions in the RA economy than in the HA economy. We summarize our findings in the next proposition. The proofs are provided in Appendix C.

Proposition 5 (An equivalence result) *In the RA economy:*

- *When all instruments* $(\tau_t^E, \tau_t^S, \tau_t^L, \tau_t^K, B_t, i_t)$ *are optimally chosen, the planner implements the first-best allocation.*
- *When* $\tau_t^E = \tau_{SS}^E$ *and* $(\tau_t^S, \tau_t^L, \tau_t^K, B_t, i_t)$ *are optimally chosen, the planner implements the first-best allocation.*

Proposition 5 shows that when all instruments are available to the planner, the latter can implement not only price and wage stability, but also the first-best allocation. In particular, the income tax may not be time-varying to implement this allocation (second item of the proposition), because its role is only to compensate for the steady-state market power of the union.

Table 1 summarizes the effect of missing instruments from Propositions 3–5.

Time-varying labor taxes	RA	HA
$\tau^L + \tau^S + \tau^E$	$\pi^P = 0$ and $\pi^W = 0$ (first-best alloc.)	$\pi^P = 0$ and $\pi^W =$ 0(flexible-price alloc.)
$\tau^L + \tau^S$	$\pi^P = 0$ and $\pi^W =$ 0(first-best alloc.)	$\pi^P = 0$ and $\pi^W \neq 0$
τ^L	$\pi^P \neq 0$ and $\pi^W \neq 0$	$\pi^P \neq 0$ and $\pi^W \neq 0$

Table 1: Price and wage inflation for different instruments, Representative Agent economy (RA) and Heterogeneous-agent economy (HA).

5 Simulating the dynamics of the economy

To investigate the optimal dynamics of the model, we perform the following experiment – which is standard in the New Keynesian RA literature, but which must be adapted to the HA case. We first solve for the optimal policy for a given set of instruments and consider the steady-state allocation – which is the long run allocation in the absence of any aggregate shock. We then initialize the model with this allocation and implement a period-0 transitory negative TFP shock. This procedure ensures that the transition is not affected by initial conditions. However, it raises two difficulties: (i) how can we find the optimal instruments at the steady state? and (ii) how can we simulate the HA model with aggregate shocks? We tackle these two issues in Sections 5.1 and 5.2, respectively.

5.1 Internally consistent Social Weights

The steady state crucially depends on the Social Welfare Function used in the Ramsey program. In addition, the steady state can also differ with the set of instruments under consideration. To overcome this difficulty and to start from the same steady state in all cases, we use the inverse optimal taxation approach, as in Heathcote and Tsujiyama (2021) and LeGrand and Ragot (2023). More precisely, we consider the same steady-state fiscal instruments, defined by $\tau^S = \tau^K = 0$, and $\tau^L > 0$, and estimate the weights of the SWF for each set of fiscal tools to ensure that this steady state is optimal. More formally, each instrument of the planner generates a first-order condition, which imposes one restriction on the SWF.[13]We then choose the SWF satisfying these restrictions, which is the closest one to the utilitarian SWF (where all weights are equal). We also verify that the SWF does not quantitatively affect the dynamics of the allocation at the first order.

[13]As in standard New Keynesian models, optimal steady-state price and wage inflation is 0, whatever the social welfare function. As a consequence, steady-state price stability does not impose any restriction on the SWF.

5.2 Truncation

The Ramsey problem in HA models cannot be solved with simple simulation techniques. Indeed, the Ramsey equilibrium is now a joint distribution across wealth and Lagrange multipliers, which is a high-dimensional object. While the steady-state values of Lagrange multipliers is already difficult to compute, the Ramsey solution actually requires the dynamics of this joint distribution. For this reason, we use the truncation method of LeGrand and Ragot (2022a) to determine the joint distribution of individual wealth and Lagrange multipliers.[14] The accuracy of optimal policies has been analyzed in LeGrand and Ragot (2022b) for both the steady state and the dynamics. In addition, an improvement to efficiently reduce the state space is provided in LeGrand and Ragot (2022c). We detail the calculations in Appendix, and refer to these papers for details about the method.

To find the steady-state values of the Lagrange multipliers and SWF for a given fiscal policy, we use the following algorithm:

1. Set a truncation structure (a maximum truncation length N) and set instrument values.
2. Solve the steady-state allocation of the full-fledged Bewley model with the given instrument values, using standard techniques.
3. Consider the truncated representation of the economy, i.e., aggregate over truncated histories.
4. Compute the steady-state Ramsey solution in truncated economy
   (a) Derive first-order conditions of the planner for each instrument in the truncated representation.
   (b) Compute the SWF weights that are the closest to 1, for which all the planner's FOCs hold.
   (c) Compute associated Lagrange multipliers.
   (d) The truncated representation, together with the fiscal instruments, the estimated SWF, and Lagrange multipliers is a steady-state optimal Ramsey allocation for the truncated representation.
5. Compute the optimal dynamics of instruments and allocation in the truncated economy using the first order conditions of the planner – as is standard in any finite state space model.

We check that the dynamics does not depend on the truncation length.

[14] Optimizing on simple rules in the spirit of Krusell and Smith (1998) is also hard to implement as their are many independent instruments.

6 Quantitative assessment

This section quantifies the inflation dynamics under various assumptions concerning the set of instruments available to the planner. The objective is to identify the most relevant instruments to stabilize inflation in HA models, among the ones presented in Table 1. The calibration is described in Section 6.1. Section 6.2 presents the inflation dynamics with exogenous fiscal and monetary rules, and compares HA and RA economies. Section 6.3 finally contrasts the outcomes of economies allowing for different sets of optimally chosen instruments.

6.1 The calibration and steady-state distribution

Preferences. The period is a quarter. The discount factor is $\beta = 0.99$, and the period utility function is: $\frac{c^{1-\sigma}-1}{1-\sigma} - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}$. The Frisch elasticity of labor supply is set to $\varphi = 0.5$, which is the value recommended by Chetty et al. (2011) for the intensive margin in HA models. The scaling parameter is $\chi = 0.01$, which implies an aggregate labor supply of roughly 1/3.

Technology and TFP shock. The production function is: $Y = ZL$. The TFP process is a standard AR(1) process, with $Z_t = \exp(z_t)$ and $z_t = \rho_z z_{t-1}$, for $t \geq 1$, and $z_0 < 0$ is the period 0 negative TFP shock. We set $\rho_z = 0.95$,which the standard quarterly persistence.

Idiosyncratic risk. We use a standard productivity process: $\log y_t = \rho_y \log y_{t-1} + \varepsilon_t^y$, with $\varepsilon_t^y \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_y^2)$. We calibrate a persistence of the productivity process $\rho_y = 0.994$ and a standard deviation of $\sigma_y = 0.06$. These values are consistent with empirical estimates (Krueger et al., 2018), and generates a steady-state Gini of wealth of 0.78, which is again in line with the data.[15] Finally, we use the Rouwenhorst (1995) procedure to discretize the productivity process into 10 idiosyncratic states with a constant transition matrix.

Steady state taxes and public debt. We first solve the model with constant exogenous taxes and explain below the choice of the Social Welfare Function (SWF). We first assume that steady-state taxes are 0, except for the labor tax τ^L: $\tau^E = \tau^S = 0$ and $\tau^L = 16\%$. This last value (together with the value of public debt explained below) implies that public spending over GDP is 15, which is close to the US value in 2007. The amount of public debt (which is the only asset here) is set to the annual value of 1.28. As public debt is the only asset in our economy, we target this amount to obtain an average Marginal Propensity to Consume (MPC) of 0.3.[16]

Monetary parameters. Following the literature and in particular Schmitt-Grohé and Uribe (2005), we assume that the elasticity of substitution is $\varepsilon_P = 6$ across goods and $\varepsilon_W = 21$ across

[15] The Gini of wealth is 0.78 using the SCF data in 2007, before the 2008 Great Recession.

[16] We thus adopt a liquid one-asset liquid wealth calibration to match a realistic MPC (Kaplan and Violante, 2022).

labor types. The price adjustment cost is set to $\psi_P = 100$, such that the slope of the price Phillips curve is $\frac{\varepsilon_P - 1}{\psi_P} = 5\%$ (see Bilbiie and Ragot, 2021, for a discussion and references). The wage adjustment cost is set to $\psi_W = 2100$, such that the slope of the wage Phillips curve is 1%, assuming wages to be stickier than prices.[17] Finally, as there is no inflation on prices or wages at the steady state: $\pi^P = \pi^W = 0$, these coefficients only matter in the dynamics.

Table 2 provides a summary of the model parameters.

Parameter	Description	Value	Target
	Preference and technology		
β	Discount factor	0.99	Quarterly calibration
σ	Curvature utility	2	
$\bar{a}$	Credit limit	0	
χ	Scaling param. labor supply	0.01	$L = 1/3$
φ	Frisch elasticity labor supply	0.5	Chetty et al. (2011)
	Shock process		
ρ_y	Autocorrelation idio. income	0.993	Krueger et al., 2018
σ_y	Standard dev. idio. income	6%	$Gini = 0.78$
ρ_z	Autocorrelation TFP shock	0.95	
	Tax system		
τ^L	Labor tax	16%	$G/Y = 15$
τ^S,τ^E,τ^K	Other tax	0%	
B/Y	Public debt over yearly GDP	128%	$MPC = 0.3$
G/Y	Public spending over yearly GDP	15%	Targeted
	Monetary parameters		
ε_p	Elasticity of sub. between goods	6	Schmitt-Grohé and Uribe (2005)
ψ_p	Price adjustment cost	100	Price PC 5%
ε_w	Elasticity of sub. labor inputs	21	Schmitt-Grohé and Uribe (2005)
ψ_w	Wage adjustment cost	2100	Wage PC 1%

Table 2: Parameter values in the baseline calibration. See text for descriptions and targets.

Truncation period. We now construct the truncated model. We use the refined truncation approach, with a number of length for the refinement equals to $N = 8$. We check that the results do not depend on the choice of the truncation length. As in LeGrand and Ragot (2022a), the truncation provides accurate results, thanks to the introduction of the ξs parameters, as

[17] We have performed sensitivity analysis regarding these coefficients. Our qualitative results appear not to be sensitive to these values, even if inflation and wage volatility increases with the slopes of Phillips curves.

explained in Section 5.

Calibration of the representative agent economy. In the next section, we will compare the dynamics of the HA economy to the one of the RA economy. The calibration of the RA economy considers the same preference parameters as in the HA economy. We denote with upper-script RA (HA) the allocation in the RA (HA) economy. In the RA economy, the steady-state labor supply L^{RA} (with $\pi^W = 0$) is determined by $v'(L^{RA}) = \frac{\varepsilon_W - 1}{\varepsilon_W}(1-\tau^L)u'(c^{RA})$. Due to consumption inequality and the convexity of marginal utility, the average marginal utility in the RA economy is lower than the one in the HA economy. As a consequence, for the same parameters $L^{HA} > L^{RA}$. To consider comparable economies, we set public debt (B^{RA}) and public spending (G^{RA}), in the RA economy such that public-debt-to GDP and public-spending-to-GDP are identical in the two economies: $B^{RA}/Y^{RA} = B^{HA}/Y^{HA}$ and $G^{RA}/Y^{RA} = G^{HA}/Y^{HA}$.

6.2 Dynamics with fiscal and monetary rules

We first simulate the model with ad-hoc fiscal and monetary rules to understand the mechanisms at stake in the two-friction economy. We compare the dynamics along two dimensions: (i) HA vs. RA and (ii) ad-hoc rules vs. optimal instruments. Concerning monetary policy, we introduce a standard Taylor rule, which depends on price inflation:

$$i_t = i_* + \phi_\pi \pi_t^P, \tag{52}$$

where i_t is the nominal interest rate between period t and period $t+1$. The constant $i_* = 1\%$ is the steady-state nominal rate, which is equal to the real interest rate, as steady-state inflation is 0. The parameter ϕ_π is the coefficient of the Taylor rule. As noted by Erceg et al. (2000) and Galí (2015), price determinacy generally requires the sum of the Taylor rule coefficients on both price and wage inflation to be larger than 1. In our case, $\phi_\pi > 1$ ensures price stability. We consider two values for this parameter: $\phi_\pi = 1.1$ and $\phi_\pi = 1.5$ to show the sensitivity of the dynamics to this coefficient.

Considering fiscal rules, we assume that tax rates are constant and set to their steady-state values. We only introduce an adjustment in the transfers related to the debt level à la Bohn (1998) to ensure debt sustainability. The fiscal rules thus involve $\tau_t^K = \tau_t^S = \tau_t^E = 0$ and:

$$\tau_t^L = \tau_*^L + \rho_B(B_t - B_*), \tag{53}$$

where we set $\rho_B = 0.08$, and then $\rho_B = 0.8$ to investigate the sensitivity of the economy dynamics to the fiscal rule. The value $B_*$is the steady-state level of public debt in either the RA or HA economy, while $\tau_*^L = 16\%$ is the steady-state value of this labor tax.

Figure 1 plots the Impulse Response Functions (IRFs) for the main variables in an economy with the previous rules, after a 1% negative TFP shock. Labor tax τ^L and wage and price inflation are reported in level deviations, while all other variables are reported in percent deviation from their steady-state values.

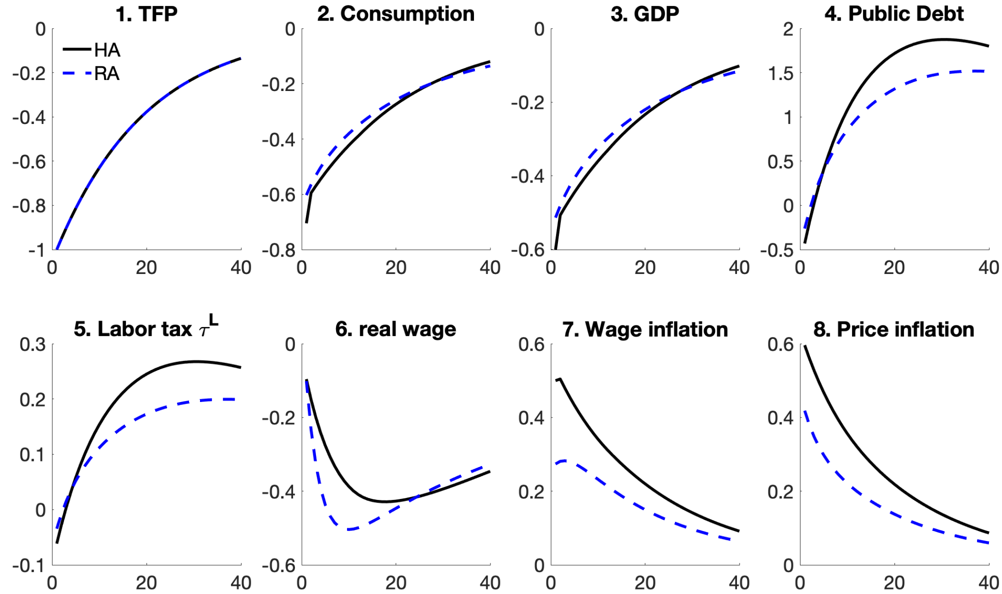

Figure 1: Impulse response functions of main variables after a negative TFP shock, for the model with a Taylor rule and simple fiscal rule. The labor tax and price and wage inflation are reported in level deviation (in percent), while all other ones are in proportional percentage deviations from steady state values. HA is the heterogeneous agent economy, RA is the representative agent economy.

The shock is a negative supply shock, akin a energy price shock. Two results are worth mentioning.

1. HA and RA models generate qualitatively similar results, but the HA model exhibits a stronger fall in consumption. Indeed, as the MPC is higher in the HA model than in the RA model, the fall in consumption and output is higher in HA model, due to both direct and indirect effect (Kaplan et al., 2018).

2. Both HA and RA models generate a price-wage spiral that corresponds to an increase in both price and wage inflation, associated to a decrease in the real wage. This "spiral" is of a larger magnitude in the HA model than in the RA one. Indeed, both price and wage inflation responses are of larger magnitude: twice larger for the wage inflation and 50% larger for price inflation. Since wage inflation is much larger, this also translates to a smaller drop in the real wage in the HA economy than in the RA one. The reason is quite subtle and relates to the heterogeneity of the drop in consumption. Although the average consumption drop is higher in HA than in RA economy, the HA consumption drop is more

severe (both in absolute and relative terms) for high-wealth agents than for low-wealth (credit-constrained) agents. In our simulation, credit-constrained agents suffer from a 0.1% consumption drop (which is roughly the real wage drop), whereas wealthy agents experience of 0.8% consumption drop. For the sake of comparison, note that the consumption drop in the RA economy amounts to 0.6%. This stronger drop in consumption for wealthy (and hence high-consumption) agents than for poor (and hence low-consumption) agents is due to the drop in real interest rate that adds to the drop in real wage. As a consequence, the average marginal utility increases *less* in the HA economy than in the RA economy. Hence, the labor gap increases *more* in the HA economy and so does wage inflation.

We now investigate the sensitivity of these results, changing both the monetary and fiscal rules, plotting the same variables. In addition to the baseline environments ($\phi^\pi = 1.1$ and $\rho^B = 0.08$) in black solid line, we consider alternative economies. In the second economy (blue dashed line), the Taylor rule is more sensitive to inflation ($\phi^\pi = 1.5$ and $\rho^B = 0.08$). In the third economy (red dotted line), the fiscal rule is more sensitive to public debt ($\phi^\pi = 1.1$ and $\rho^B = 0.8$).

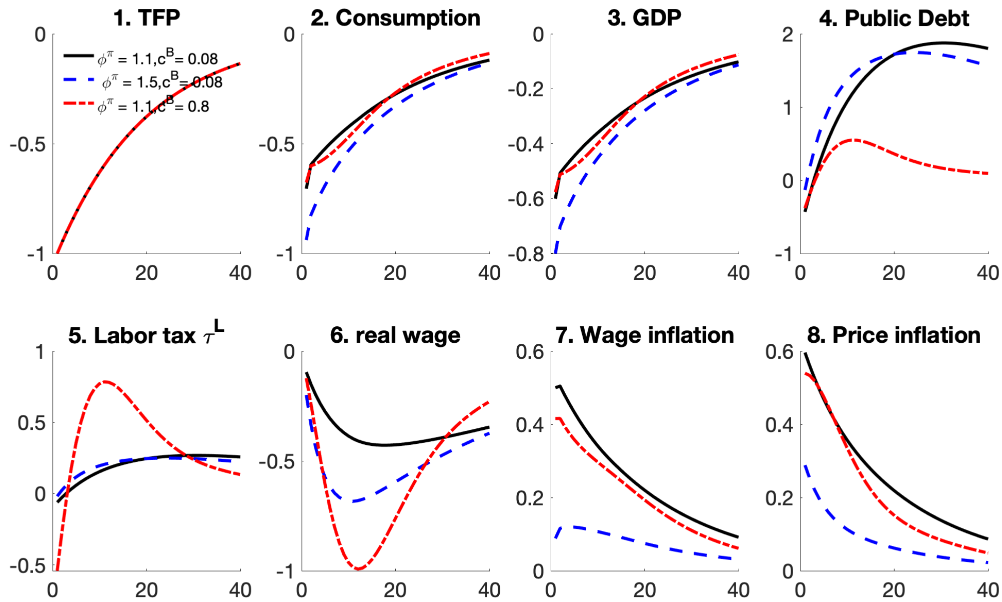

Figure 2: Impulse response functions of main variables after a negative TFP shock, for the model with a Taylor rule and simple fiscal rule. The labor tax and price and wage inflation are reported in level deviation (in percent), while all other ones are in proportional percentage deviations from steady state values. the HA economy for different monetary and fiscal rules.

We first observe that the monetary policy rule affects both the the allocation and the inflation dynamics. A more aggressive monetary policy (higher ϕ^π) generates a larger drop in consumption and a much smaller inflation reaction. The change in the fiscal rule translates to a more volatile response of the labor tax and a smoother public debt variation. The real wage drops much more but inflation and output reactions are barely affected.[18]

The sensitivity of both real allocation and inflation to monetary and fiscal rules obviously raises the question of the optimal monetary and fiscal policy after a TFP shock, which is the subject of the following sections.

6.3 Optimal fiscal and monetary policy

We next simulate the dynamics of the economy, when the both fiscal and monetary policy are optimally chosen. We simulate the economy with two sets of fiscal instruments, always assuming optimal monetary policy. These two economies correspond to the two last lines of Table 1. In the first economy, $(\tau_t^S, \tau_t^L)_t$ are optimally chosen, while in the second economy $\tau_t^S = 0$ and only $(\tau_t^L)_t$ is optimally chosen. These two economies feature $\tau_t^E = 0$. Indeed, as we will see, whereas the instrument τ_t^E is theoretically necessary to reproduce the flexible price allocation, its quantitative relevance is small.[19] We also report a third economy, where both monetary and fiscal tools follow the simple rules of equations (52) and (53), with $\phi^\pi = 1.5$ and $\rho^B = 0.08$. Figure 3 represents the outcomes of these three economies.

We describe the main lessons we can draw from Figure 3. We start with the economy featuring $\tau_t^E = 0$ and $(\tau_t^S, \tau_t^L)_t$ optimally chosen (black solid line).

1. This economy almost implements price and wage stability (panels 8 and 9). Wage inflation decreases, the magnitude is small. This reflects that the cost of wage inflation is too high compared to market power of unions. The planner barely uses the wage inflation to manipulate the labor supply

2. The real wage drops (panel 2) and almost parallels the drop in the marginal productivity of labor – equal to TFP (panel 1) in this economy.

3. Labor tax (τ_t^L) increases on impact (panel 7), whereas employer social contribution (τ_t^S) decreases (panel 6). As a consequence, the planner taxes households to subsidize labor. Public debt increases moderately (panel 5).

4. As a consequence, the number of hours worked increases (panel 4), while consumption decreases less than TFP (panel 3).

We now turn to the economy with $\tau_t^E = \tau_t^S = 0$ and $(\tau_t^L)_t$ optimally chosen (blue dotted line).

[18] The small effect of fiscal policy on output is due to the fact than we use the linear labor tax as a fiscal instrument to be consistent with the analysis. A fiscal rule based on a lumpsum transfer would generate a higher variation in output due to the higher MPC in the HA economy.

[19] The economy with optimal $(\tau_t^S, \tau_t^E, \tau_t^L)$ is very close to the one with optimal (τ_t^S, τ_t^L) and $\tau_t^E = 0$, as shown below. As a consequence, we only plot the latter to save some space.

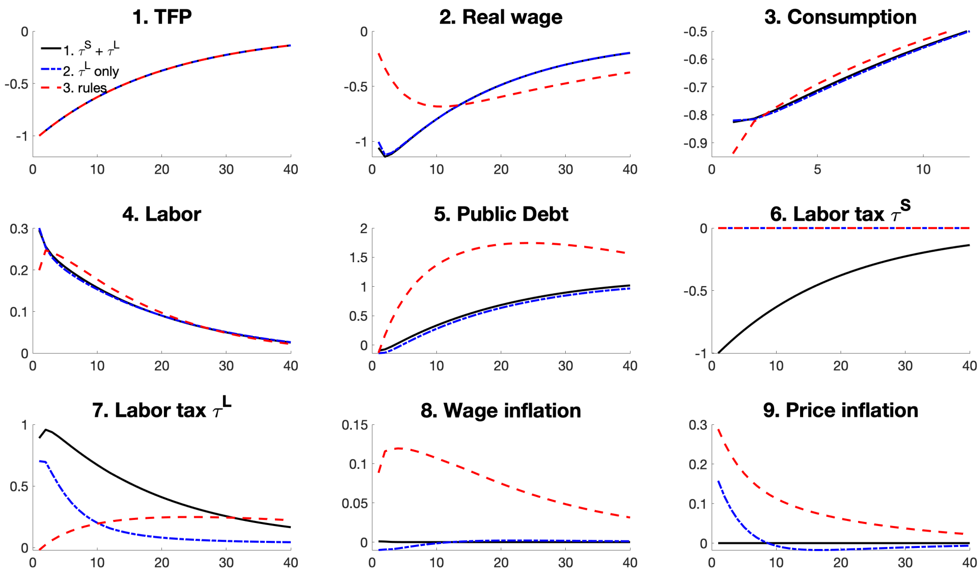

Figure 3: Impulse response functions of main variables after a negative TFP shock, for the model with optimal monetary and fiscal policy. Variable are reported in percentage proportional deviations from steady state. Labor taxes and inflation rates (Panels 6 to 9) are reported in percentage level deviations from steady state. The black solid lien corresponds to $\tau_t^E = 0$ and $(\tau_t^S, \tau_t^L)_t$ optimally chosen; the blue dashed line to $\tau_t^E = \tau_t^S = 0$ and $(\tau_t^L)_t$ optimally chosen; the red dotted line to monetary and fiscal rules (52) and (53), with $\phi^\pi = 1.5$ and $\rho^B = 0.08$.

1. The real allocation in this economy is very close to the previous one, with $\tau_t^E = 0$ and optimal $(\tau_t^S, \tau_t^L)_t$ (panels 2, 3, and 4).
2. However, the allocation is reached with a different fiscal-monetary policy mix.
3. By construction, employed social contribution is now constant (panel 6). Labor tax $(\tau_t^L)_t$ increases less than in the previous allocation (panel 7), but implements a similar path for public debt (panel 5).
4. The drop in real wage (panel 2) is very similar in both economies. However, this drop is not only generated by higher $(\tau_t^L)_t$. Indeed, both wage and price inflation rates (panels 8 and 9) markedly increase on impact but in a way that contributes to a drop in the real wage.Overall, the departure from price-wage stability is much larger in this economy compared to the one where $\tau_t^E = 0$ and optimal $(\tau_t^S, \tau_t^L)_t$.

We conclude from this experiment that the time-varying employer social contribution $(\tau_t^S)_t$ is a key instrument to ensure price stability.

Finally, we consider the economy with the standard monetary and fiscal rules of equations (52)–(53) (red dashed line).

1. This economy generates a very different real allocation. The real wage falls with a delay (panel 2), generating a smaller increase in the number of worked hours (panel 4) and a larger fall in consumption (panel 3).

2. Labor tax increases more progressively, which generates a larger increase in public debt (panel 5).

3. Finally, the fall in the real wage is the result of a wage inflation (panel 8), and an even bigger price inflation (Panel 9), that we previously qualified of *inflation spiral*.

6.4 Dynamic of gaps

To better understand the dynamics of inflation, we now focus on the economies with $\tau_t^E = 0$ and optimal (τ_t^S, τ_t^L) on the one hand and with $\tau_t^E = \tau_t^S = 0$ an optimal $(\tau_t^L)_t$ on the other. Figure 3 reports the dynamic of the labor gap defined in Equation (10) and of the price gap defined in Equation (2), together with the Lagrange multiplier on the budget of the State μ, and the Lagrange multiplier on the price Phillips curve γ_P and on the wage Phillips curve γ_W.

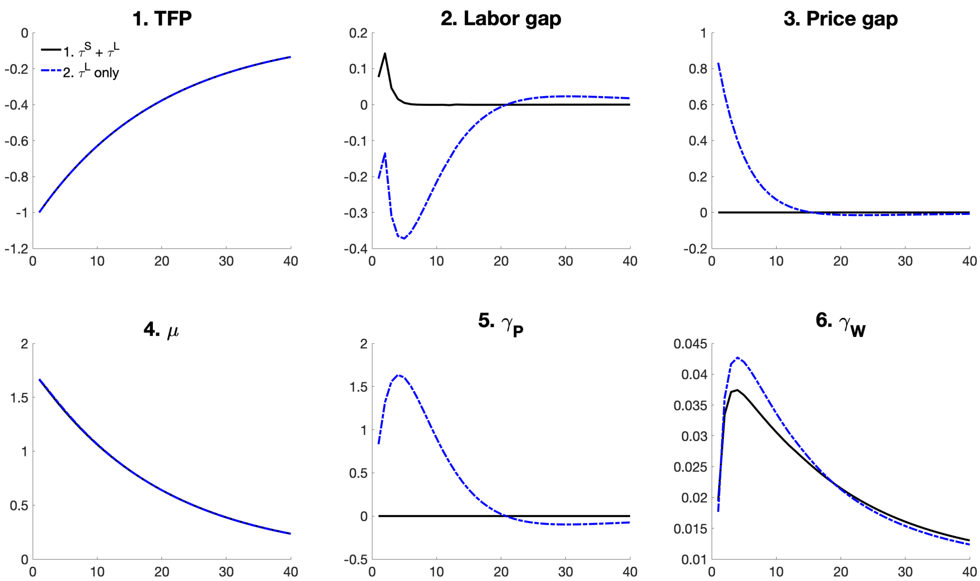

Figure 4: Dynamics of TFP, labor gap, price gap, and Lagrange multipliers for two economies ($\tau_t^E = 0$ and $\tau_t^E = \tau_t^S = 0$). TFP and μ are in percentage proportional deviation, while other variables are in percentage level deviations, as their steady-state value is 0.

We derive four observations. First, the labor gap slightly increases in the economy $\tau_t^E = 0$, whereas it decreases in the economy $\tau_t^E = \tau_t^S = 0$. The planner uses the time varying τ_t^S to almost close the labor gap and reduce the market power of unions. Second, the price gap increases sharply in the economy $\tau_t^E = \tau_t^S = 0$, whereas it is null in economy $\tau_t^E = 0$, as the instrument τ_t^S allows the planner to close the price gap by equalizing the labor cost paid by the firm and the marginal productivity of labor. Third, the Lagrange multiplier on the governmental budget constraint, μ, is very similar in the two economies, reflecting that in spite of very different inflation dynamics, the governmental marginal willingness to raise taxes is the same in the two economies. Finally, the price Phillips curve is only a constraint in the economy $\tau_t^E = \tau_t^S = 0$ (panel 5), whereas the wage Phillips curve is a constraint in both economies.

7 Conclusion

We derive joint optimal monetary-and-fiscal policy in an HA model with both sticky prices and sticky wages, after an energy price shock. Our main finding is that a sufficiently rich fiscal policy can efficiently stabilize both inflation and activity. The key instrument appears to be a time-varying wage subsidy, which stabilizes employment over the business cycle. Its primary goal is to reduce the gap between marginal labor productivity and the sticky labor cost, even though indirect effects on aggregate demand and inflation are also present. It is noteworthy that these tools have been recently been used in Europe to stabilize employment. In Germany, the so-called *kurzarbeit* device played this role, while in France, the *activité partielle* policy was a wage subsidy to reduce layoffs during the Covid-19 crisis. More quantitative work is needed to further investigate the role of these non-Keynesian automatic stabilizers.

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Appendix

A RA model with fiscal and Taylor Rules

A.1 With a Taylor rules and no fiscal rule

The model is characterized by the following set of equations:

$$
\begin{aligned}
& G + (1+r_t)\, B_{t-1} + w_t L_t \leq \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big) Z_t L_t + B_{t-1}, \\
& C_t + B_t = (1+r_t) B_{t-1} + w_t L_t, \\
& u'(C_t) = \beta \mathbb{E}_t \Big[(1+r_{t+1}) u'(C_{t+1})\Big], \\
& \pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\Big(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t u'(C_t)\Big) L_t + \beta \mathbb{E}_t \Big[\pi_{t+1}^W(\pi_{t+1}^W + 1)\Big], \\
& \pi_t^P(1+\pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P}\big(\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)} - 1\big) + \beta \mathbb{E}_t \Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\Big). \\
& \mathbb{E}_t r_{t+1} = \Big(1 - \tau_{t+1}^K\Big)\Big(\frac{\tilde{R}_t^N}{1+\pi_{t+1}^P} - 1\Big),
\end{aligned}
$$

together with $w_t = (1-\tau_t^L)\hat{w}_t = (1-\tau_t^L)\tilde{w}_t$ and $\Big(1+\pi_t^W\Big) w_{t-1}(1-\tau_t^L) = (1+\pi_t^P) w_t (1-\tau_{t-1}^L)$.

Steady State

We assume $\pi = 0 = \tau_{ss}^L = B_{ss} = 0, Z = 1, Y = L$. So $G + C = L$ and the main equation is:

$$
v'(L) = \frac{\varepsilon_W - 1}{\varepsilon_W} u'\left(L - G\right).
$$

We focus on the CRRA case $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ and $v(l) = \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}$. We then have $L_{ss}^{1/\varphi+\gamma} = \frac{\varepsilon_W - 1}{\varepsilon_W}\chi$, $1 + r_{ss} = \frac{1}{\beta}$ and $w_{ss} = 1$. Further assuming $\chi = \phi = \gamma = 1$, and myopic price setters, the linear system is defined by the following equations:

$$
\begin{aligned}
C_t &= L_t + u_t, \\
C_t &= \mathbb{E}_t C_{t+1} - (\phi^\pi - 1)\, \mathbb{E}_t \pi_{t+1}^P, \\
\pi_t^W &= \Big(C_t - u_t - w_t + C_t\Big)\frac{\varepsilon_W}{\psi_W}\left(L^{SS}\right)^2, \\
\pi_t^P &= \frac{\varepsilon_P - 1}{\psi_P}(w_t - u_t), \\
w_t &= \pi_t^W + w_{t-1} - \pi_t^P.
\end{aligned}
$$

Defining $A := \frac{\varepsilon_W - 1}{\psi_W}$, $B := \frac{1}{\gamma}(\phi^\pi - 1)$, and $D := \frac{\varepsilon_P - 1}{\psi_P}$, we obtain that the solution is $w_t = d_1 w_{t-1} + d_2 u_t$, with:

$$\begin{aligned} d_1 &= \frac{(A + D + 2) - \sqrt{(A + D)^2 + 8ABD}}{2(A + D + 1 - 2ABD)}, \\ d_2 &= \frac{(D - A)(1 - \rho_u) + 2ABD\rho_u}{A + D + 2 - (A + D + 1 - 2ABD)(d_1 + \rho_u)}, \end{aligned}$$

where we can check that $|d_1| < 1$. The coefficient d_2 has the sign of:

$$\left(\frac{\varepsilon_P - 1}{\psi_P} - \frac{\varepsilon_W - 1}{\psi_W}\right)(1 - \rho_u) + 2\frac{\varepsilon_P - 1}{\psi_P}\frac{\varepsilon_W - 1}{\psi_W}(\phi^\pi - 1)\rho_u,$$

which completes the proof of Proposition 1.

A.2 RA model with a Taylor rule a fiscal rule

We consider a steady state where $B = \tau^L = T = G = 0$ that we linearize for small values of τ^L. The system characterizing the economy is:

$$\begin{aligned} &C_t = \left(1 - \frac{\psi_P}{2}(\pi_t^P)^2\right) Z_t L_t - G, \\ &u'(C_t) = \beta \mathbb{E}_t\left[\frac{1 + i_t}{1 + \pi_{t+1}^P} u'(C_{t+1})\right], \\ &\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t u'(C_t)\right) L_t + \beta \mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W + 1)\right], \\ &\pi_t^P(1 + \pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P}\left(\frac{1}{Z_t}\frac{w_t}{1 - \tau_t^L} - 1\right) + \beta \mathbb{E}_t\left(\pi_{t+1}^P(1 + \pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\right), \end{aligned}$$

where we also add a linearized Bohn rule (recall the steady state debt B^{SS} is 0, and $\tau^{LSS} = 0$): $\tau_t^L = c^B B_{t-1}$. We thus have $i^{SS} = 1/\beta - 1 > 1$ and close to 1. Defining $b_t = B_t / C^{ss}$, and $c^b = c^B C^{SS}$ and introducing myopic price setters, we have $\tau_t^L = c^b b_t$ and:

$$\begin{aligned} C_t &= L_t + u_t, \\ C_t &= \mathbb{E}_t C_{t+1} - \frac{1}{\gamma}\left(i_t - \mathbb{E}_t \pi_{t+1}^P\right) \\ \pi_t^W &= \left(\frac{1}{\phi} L_t - w_t + \gamma C_t\right)\frac{\varepsilon_W}{\psi_W}\frac{1}{\chi}\left(L^{SS}\right)^{\frac{1}{\phi}+1} + \beta \mathbb{E}_t \pi_{t+1}^W, \\ \pi_t^P &= \frac{\varepsilon_P - 1}{\psi_P}(w_t - u_t + c^b b_{t-1}) + \beta \mathbb{E}_t \pi_{t+1}^P \\ w_t &= \pi_t^W + w_{t-1} - \pi_t^P \\ C_t + b_t &= (1 + r^{ss}) b_{t-1} + w_t + L_t, \\ i_t &= \phi^\pi \mathbb{E}_t \pi_{t+1}^P \end{aligned}$$

Further assuming $\gamma = \phi = \chi = 1$, and defining $A := \frac{\varepsilon_W - 1}{\psi_W}$, $B := \phi^\pi - 1$, and $D := \frac{\varepsilon_P - 1}{\psi_P}$, we get:

$$\begin{aligned}
C_t &= \mathbb{E}_t C_{t+1} - B\mathbb{E}_t \pi_{t+1}^P, \\
\pi_t^W &= 2AC_t - Au_t - Aw_t, \\
\pi_t^P &= Dw_t - Du_t + Dc^b b_{t-1}, \\
w_t &= \pi_t^W - \pi_t^P + w_{t-1}, \\
b_t &= (1 + r^{ss}) b_{t-1} + w_t - u_t, \\
u_t &= \rho_u u_{t-1} + \epsilon_t.
\end{aligned}$$

Combining the fourth and fifth equation and iterating one period, we obtain

$$0 = \left(1 - Dc^b\right) w_t + 2AC_{t+1} - (A + D + 1) w_{t+1} + \left(D - A + Dc^b\right) \rho_u u_t - Dc^b (1 + r^{ss}) b_{t-1}.$$

Using $2AC_t - 2A\mathbb{E}_t C_{t+1} = -2AB\mathbb{E}_t \pi_{t+1}^P$ (first equation), we deduce:

$$\begin{aligned}
&0 = w_{t-1} + 2AC_t - (A + D + 1) w_t + (D - A) u_t - Dc^b b_{t-1} \\
&0 - \left(1 - Dc^b\right) w_t - 2AC_{t+1} + (A + D + 1) w_{t+1} - \left(D\left(1 + c^b\right) - A\right) \rho_u u_t + Dc^b (1 + r^{ss}) b_{t-1},
\end{aligned}$$

which becomes after some manipulations:

$$\begin{aligned}
0 = {} & w_{t-1} - 2ABD (1 + r^{ss}) c^b b_{t-1} + Dc^b r^{ss} b_{t-1} \\
& + (A + D + 1 - 2ABD) w_{t+1} \\
& - \left(2 + A + D + 2ABDc^b\right) w_t \\
& + \left((D - A)(1 - \rho_u) - Dc^b \rho_u + 2ABD\left(\rho_u + c^b\right)\right) u_t.
\end{aligned} \tag{54}$$

We solve the system using a guess-and-verify approach and assumes that:

$$w_t = d_1 w_{t-1} + d_2 u_t + d_3 b_{t-1},$$

which implies after some manipulations:

$$\mathbb{E}_t w_{t+1} = (d_1 + d_3) d_1 w_{t-1} + d_3 (d_1 + d_3 + 1) b_{t-1} + (d_2 (\rho_u + d_1 + d_3) - d_3) u_t.$$

Plugging these two equations into (54) yields:

$$\begin{aligned}
0 = {} & w_{t-1} - 2ABD (1 + r^{ss}) c^b b_{t-1} + Dc^b r^{ss} b_{t-1} \\
& - \left(2 + A + D + 2ABDc^b\right) (d_1 w_{t-1} + d_2 u_t + d_3 b_{t-1}) \\
& + (A + D + 1 - 2ABD) ((d_1 + d_3) d_1 w_{t-1} + d_3 (d_1 + d_3 + 1) b_{t-1} + (d_2 (\rho_u + d_1 + d_3) - d_3) u_t) \\
& + \left((D - A)(1 - \rho_u) - Dc^b \rho_u + 2ABD\left(\rho_u + c^b\right)\right) u_t.
\end{aligned}$$

We obtain by identification the following system:

$$
\begin{aligned}
&0 = 1 - (2 + A + D + 2ABDc^b)d_1 + (A + D + 1 - 2ABD)\,(d_1 + d_3)\,d_1,\\
&0 = (A + D + 1 - 2ABD)\,(d_2\,(\rho_u + d_1 + d_3) - d_3) + (D - A)\,(1 - \rho_u)\\
&\quad - Dc^b\rho_u + 2ABD\left(\rho_u + c^b\right) - (2 + A + D + 2ABDc^b)d_2,\\
&0 = (A + D + 1 - 2ABD)\,d_3\,(d_1 + d_3 + 1) - 2ABD\,(1 + r^{ss})\,c^b + Dc^b r^{ss}\\
&\quad - (2 + A + D + 2ABDc^b)d_3.
\end{aligned}
$$

Assuming that $|ABDc^b| \ll 1$ and $|Dc^b r^{ss}| \ll 1$, we then obtain that $d_3 \simeq 0$, $d_1 = \frac{1}{A+D+1}$, as well as: $d_2 = \frac{(D-A)(1-\rho_u)+D(2AB-c^b)\rho_u}{(1+A+D)(1-\rho_u)+2ABD(\frac{1}{A+D+1}+1)\rho_u}$.

The dynamics of inflation. We obtain the following system:

$$
\begin{aligned}
\pi_t^P &= Dw_t - Du_t + Dc^b b_{t-1} = D\,(d_2 - 1)\,u_t,\\
\pi_t^W &= \pi_t^P + w_t = (D\,(d_2 - 1) + d_2)\,u_t.
\end{aligned}
$$

Price inflation is likely to increase on impact, while wage inflation can increase or decrease.

B Ramsey program for HA models

B.1 Flexible-price equilibrium

We here assume here that the planner must choose a common labor supply for all agents, in a flexible price economy: $\pi_t^P = \pi_t^W = 0$. The program is:

$$
\begin{aligned}
&\max_{\left(\tau_t^L,\tau_t^S,\tau_t^K,w_t,r_t,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega(y_t^i)\left(u(c_t^i) - v(L_t)\right)\ell(di)\right],\\
&G_t + (1 + r_t)\int_i a_{i,t-1}\ell(di) + w_t L_t + T_t \leq Z_t L_t + \int_i a_{i,t}\ell(di),\\
&\text{for all } i \in \mathcal{I}\text{: } c_{i,t} + a_{i,t} = (1 + r_t)a_{i,t-1} + w_t y_{i,t} L_t,\\
&\qquad a_{i,t} \geq -\overline{a}, \nu_{i,t}(a_{i,t} + \overline{a}) = 0,\ \nu_{i,t} \geq 0,\\
&\qquad u'(c_{i,t}) = \beta\mathbb{E}_t\Big[(1 + r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t}.
\end{aligned}
$$

The Lagrangian can be written as:

$$
\begin{aligned}
\mathcal{L} = &\ \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i \omega_t^i(u(c_{i,t}) - v(L_t))\ell(di) - \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i (\lambda_{i,c,t} - (1 + r_t)\lambda_{i,c,t-1})\,u'(c_{i,t})\ell(di)\\
&+ \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mu_t\left(Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1 + r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t - T_t\right).
\end{aligned}
$$

We recall that $\psi_{i,t} = \omega_t^i u'(c_{i,t}) - (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u''(c_{i,t})$. Compared to (46), we drop the FP subscript for the sake of simplicity. We compute the FOCs wrt four independent instruments: r_t, w_t, L_t and $(a_{i,t})_i$. The other instruments can be recovered from the constraints.

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1}u'(c_{i,t})\ell(di) = 0. \tag{55}$$

FOC wrt w_t.

$$\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) = 0.$$

FOC wrt L_t. Using the FOC on w_t:

$$\int_i \omega_{i,t}\ell(di)v'(L_t) = \mu_t Z_t = Z_t \int_i y_{i,t}\psi_{i,t}\ell(di).$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta\mathbb{E}_t\left[(1+r_{t+1})\hat{\psi}_{i,t+1}\right].$$

B.2 The HA economy with all instruments

The program is:

$$\max_{\left(\tau_t^L,\tau_t^S,\tau_t^E,\tau_t^K,\pi_t^P,\pi_t^W,w_t,r_t,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega(y_t^i)\left(u(c_t^i)-v(L_t)\right)\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t + (1+r_t)\int_i a_{i,t-1}\ell(di) + w_t L_t \leq \left(1-\frac{\psi_P}{2}(\pi_t^P)^2\right)Z_t L_t + \int_i a_{i,t}\ell(di),$$

$$\text{for all } i\in\mathcal{I}\text{: } c_{i,t}+a_{i,t} = (1+r_t)a_{i,t-1} + w_t y_{i,t}L_t,$$

$$a_{i,t}\geq -\overline{a}, \nu_{i,t}(a_{i,t}+\overline{a}) = 0,\ \nu_{i,t}\geq 0,$$

$$u'(c_{i,t}) = \beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t},$$

$$\pi_t^W(\pi_t^W+1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W-1}{\varepsilon_W}\frac{w_t}{1-\tau_t^E}\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)L_t + \beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W+1)\Big],$$

$$\pi_t^P(1+\pi_t^P) = \frac{\varepsilon_P-1}{\psi_P}\left(\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)(1-\tau_t^S)(1-\tau_t^E)} - 1\right) + \beta\mathbb{E}_t\Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_t L_t}\Big),$$

$$(1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L} = \frac{w_t}{1-\tau_t^L}(1+\pi_t^P).$$

We can set:

– τ_t^S such that $1-\tau_t^S = \frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)(1-\tau_t^E)}$, hence $\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)(1-\tau_t^S)(1-\tau_t^E)} - 1$ and $\pi_t^P = 0$.

– τ_t^E is a free parameter that can be deduced from π_t^W and the allocation. Hence, the wage Phillips curve is not a constraint.

– π_t^W only reduces utility and is an independent parameter that can be set through τ^L, hence $\pi_t^W = 0$

The program then reduces to the same one as in the flexible-price economy without union:

$$\begin{aligned}
&\max_{\left(\tau_t^L,\tau_t^S,\tau_t^E,\tau_t^K,w_t,r_t,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega(y_t^i)\left(u(c_t^i)-v(L_t)\right)\ell(di)\right],\\
&G_t + (1+r_t)\int_i a_{i,t-1}\ell(di) + w_tL_t + T_t \leq Z_tL_t + \int_i a_{i,t}\ell(di),\\
&\text{for all } i\in\mathcal{I}\text{: } c_{i,t}+a_{i,t} = (1+r_t)a_{i,t-1} + w_ty_{i,t}L_t,\\
&\qquad u'(c_{i,t}) = \beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t}.
\end{aligned}$$

B.3 The HA economy without τ_t^E

We impose $\tau_t^E = 0$. The program is otherwise the same as in Section B.2. In particular, τ_t^S only appears in the price Phillips curve. As consequence, this equation is not a constraint and τ_t^S is set, such that $\pi_t^P = 0$. Inflation indeed only destroys resources here. We then obtain the following program:

$$\begin{aligned}
&\max_{\left(\tau_t^L,B_t,T_t,\pi_t^P,\pi_t^W,w_t,r_t,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega(y_t^i)\left(u(c_t^i)-v(L_t)\right)\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2\right],\\
&G_t + (1+r_t)\int_i a_{i,t-1}\ell(di) + w_tL_t + T_t \leq Z_tL_t + \int_i a_{i,t}\ell(di),\\
&\text{for all } i\in\mathcal{I}\text{: } c_{i,t}+a_{i,t} = (1+r_t)a_{i,t-1} + y_{i,t}w_tL_t + T_t,\\
&\qquad u'(c_{i,t}) = \beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t},\\
&\pi_t^W(\pi_t^W+1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W-1}{\varepsilon_W}w_t\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)L_t + \beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W+1)\Big],
\end{aligned}$$

Because of $\tau_t^E = 0$, we cannot have simultaneously optimal labor supply and $\pi_t^W = 0$: the planner has to balance the relative costs of wage inflation with the suboptimal provision of labor supply.

The Lagrangian is:

$$\begin{aligned}
\mathcal{L} = &\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\pi_t^W)^2 \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u'(c_{i,t})\ell(di) \\
&- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_{W,t} - \gamma_{W,t-1})\pi_t^W (1+\pi_t^W) \\
&+ \frac{\varepsilon_W}{\psi_W}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_{W,t} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) L_t \\
&+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(Z_t L_t + \int_i a_{i,t}\ell(di) - G_t - (1+r_t)\int_i a_{i,t-1}\ell(di) - w_t L_t - T_t \right).
\end{aligned}$$

We recall that in this economy, we have $\psi_{i,t} = \omega_t^i u'(c_{i,t}) - (\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1})\, u''(c_{i,t}) - \frac{\varepsilon_W - 1}{\psi_W}\gamma_{W,t} w_t y_{i,t} u''(c_{i,t}) L_t$, where compared to (51), we laso drop the superscript.

FOC wrt π_t^W.

$$-\psi_W \pi_t^W - (\gamma_{W,t} - \gamma_{W,t-1})(2\pi_t^W + 1) = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1}\hat{\psi}_{i,t}\ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t})\ell(di) = 0.$$

FOC wrt w_t.

$$\int_i y_{i,t}\hat{\psi}_{i,t}\ell(di) = \gamma_{W,t}\frac{\varepsilon_W - 1}{\psi_W}\int_i y_{i,t} u'(c_{i,t})\ell(di).$$

FOC wrt L_t. Using the FOC wrt w_t:

$$-\int_i \omega_{i,t}\ell(di) v'(L_t) + \mu_t Z_t + \frac{\varepsilon_W}{\psi_W}\gamma_{W,t}\left(v''(L_t)L_t + v'(L_t)\right) = 0.$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1+r_{t+1})\hat{\psi}_{i,t+1} \right].$$

B.4 The HA economy without τ_t^E and τ_t^S

In this case, there is no obvious simplification and the program is:

$$\max_{\left(\tau_t^L,\tau_t^S,\tau_t^K,B_t,T_t,\pi_t^P,\pi_t^W,w_t,r_t,\Omega_t,\tilde{R}_t^N,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i \omega(y_t^i)\left(u(c_t^i)-v(L_t)\right)\ell(di)-\frac{\psi_W}{2}(\pi_t^W)^2\right],$$

$$G_t+(1+r_t)\int_i a_{i,t-1}\ell(di)+w_tL_t+T_t\leq\left(1-\frac{\psi_P}{2}(\pi_t^P)^2\right)Z_tL_t+\int_i a_{i,t}\ell(di),$$

for all $i\in\mathcal{I}$: $c_{i,t}+a_{i,t}=(1+r_t)a_{i,t-1}+w_ty_{i,t}L_t,$

$$a_{i,t}\geq-\overline{a},\nu_{i,t}(a_{i,t}+\overline{a})=0,\ \ \nu_{i,t}\geq 0,$$

$$u'(c_{i,t})=\beta\mathbb{E}_t\Big[(1+r_{t+1})u'(c_{i,t+1})\Big]+\nu_{i,t},$$

$$\pi_t^W(\pi_t^W+1)=\frac{\varepsilon_W}{\psi_W}\left(v'(L_t)-\frac{\varepsilon_W-1}{\varepsilon_W}w_t\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)L_t+\beta\mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W+1)\Big],$$

$$\pi_t^P(1+\pi_t^P)=\frac{\varepsilon_P-1}{\psi_P}\left(\frac{1}{Z_t}\frac{w_t}{(1-\tau_t^L)}-1\right)+\beta\mathbb{E}_t\Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Z_{t+1}L_{t+1}}{Z_tL_t}\Big),$$

$$(1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L}=\frac{w_t}{1-\tau_t^L}(1+\pi_t^P),$$

while the corresponding Lagrangian becomes:

$$\begin{aligned}
\mathcal{L}=&\ \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i\omega_t^i(u(c_{i,t})-v(L_t))\ell(di)-\frac{\psi_W}{2}(\pi_t^W)^2\\
&-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\int_i\left(\lambda_{i,c,t}-(1+r_t)\lambda_{i,c,t-1}\right)u'(c_{i,t})\ell(di)\\
&-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(\gamma_{W,t}-\gamma_{W,t-1})\pi_t^W(1+\pi_t^W)\\
&+\frac{\varepsilon_W}{\psi_W}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma_{W,t}\left(v'(L_t)-\frac{\varepsilon_W-1}{\varepsilon_W}w_t\int_i y_{i,t}u'(c_{i,t})\ell(di)\right)L_t\\
&-\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(\gamma_{P,t}-\gamma_{P,t-1})\pi_t^P(1+\pi_t^P)Z_tL_t+\frac{\varepsilon_P-1}{\psi_P}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\gamma_{P,t}\left(\frac{w_t}{(1-\tau_t^L)}-Z_t\right)L_t\\
&+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mu_t\Big((1-\frac{\psi_P}{2}(\pi_t^P)^2)Z_tL_t+\int_i a_{i,t}\ell(di)-G_t-(1+r_t)\int_i a_{i,t-1}\ell(di)-w_tL_t\Big)\\
&+\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Lambda_t\left((1+\pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L}-\frac{w_t}{1-\tau_t^L}(1+\pi_t^P)\right)
\end{aligned}$$

We now turn to the computation of the FOCs.

FOC wrt π_t^W.

$$-\psi_W\pi_t^W-(\gamma_{W,t}-\gamma_{W,t-1})(2\pi_t^W+1)+\Lambda_t\frac{w_{t-1}}{1-\tau_{t-1}^L}=0.$$

FOC wrt π_t^P.

$$-(\gamma_{P,t} - \gamma_{P,t-1})(2\pi_t^P + 1) - \mu_t \psi_P \pi_t^P - \frac{\Lambda_t}{Z_t L_t} \frac{w_t}{1 - \tau_t^L} = 0.$$

FOC wrt r_t.

$$\int_i a_{i,t-1} \hat{\psi}_{i,t} \ell(di) + \int_i \lambda_{i,c,t-1} u'(c_{i,t}) \ell(di) = 0.$$

FOC wrt w_t. Using the FOC wrt to τ^L, we have:

$$0 = \int_i y_{i,t} \hat{\psi}_{i,t} \ell(di) - \gamma_{W,t} \frac{\varepsilon_W - 1}{\psi_W} \int_i y_{i,t} u'(c_{i,t}) \ell(di).$$

FOC wrt L_t. Using the FOC wrt w_t:

$$\begin{aligned} 0 = &- \int_i \omega_{i,t} \ell(di) v'(L_t) + \mu_t \Big(1 - \frac{\psi_P}{2} (\pi_t^P)^2\Big) Z_t + \frac{\varepsilon_W}{\psi_W} \gamma_{W,t} \left(v''(L_t) L_t + v'(L_t)\right) \\ &- (\gamma_{P,t} - \gamma_{P,t-1}) \pi_t^P (1 + \pi_t^P) Z_t + \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} \left(\frac{w_t}{(1 - \tau_t^L)} - Z_t \right). \end{aligned}$$

FOC wrt $a_{i,t}$.

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{i,t+1} \right].$$

FOC wrt τ_t^L. We derive wrt $\frac{1}{1-\tau_t^L}$ and obtain:

$$0 = \frac{\varepsilon_P - 1}{\psi_P} \gamma_{P,t} L_t - \Lambda_t (1 + \pi_t^P) + \beta \mathbb{E}_t \left[\Lambda_{t+1} (1 + \pi_{t+1}^W) \right].$$

C The Ramsey program for the RA models

C.1 RA economy with all instruments

The first best is characterized by $v'(L_t) = Z_t u'(C_t)$ and $C_t = Z_t L_t$.

The situation is similar though simpler than in the HA economy. Indeed, if $\pi_t^W = 0$, we have $v'(L_t) = \frac{\varepsilon_W - 1}{\varepsilon_W} \frac{w_t}{1 - \tau_t^E} u'(C_t)$. We thus set $\tau_t^E = \frac{\varepsilon_W}{\varepsilon_W - 1}$, which gives $v'(L_t) = \frac{\varepsilon_W - 1}{\varepsilon_W} \frac{w_t}{1 - \tau_t^E} u'(C_t)$. If $\pi_t^P = 0$, we set $1 = (1 - \tau_t^E)(1 - \tau_t^S)$ and $\tau_t^L = 0$. This gives $Z_t = w_t$, $G_t + w_t L_t = Z_t L_t$ and $C_t = Z_t L_t$. We can thus decentralize the first-best allocation.

C.2 Representative agents without time-varying τ_t^E

We set $\tau_t^E = \frac{\varepsilon_W - 1}{\varepsilon_W}$ and the program is:

$$
\begin{aligned}
&\max_{\left(\pi_t^P, \pi_t^W, w_t, r_t, \tilde{R}_t^N, L_t, (a_{i,t})_i, \tau_t^L, \tau_t^K\right)_{t\geq 0}} W_0,\\
&(1 + r_t)\, B_{t-1} + w_t L_t = \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big) Z_t L_t + B_t,\\
&C_t = \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big) Z_t L_t - G_t,\\
&u'(C_t) = \beta \mathbb{E}_t\Big[1 + \Big(1 - \tau_{t+1}^K\Big)\left(\frac{\tilde{R}_t^N}{1 + \pi_{t+1}^P} - 1\right)\Big] u'(C_{t+1}),\\
&\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\Big(v'(L_t) - w_t u'(C_t)\ell(di)\Big) L_t + \beta \mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W + 1)\Big],\\
&\pi_t^P(1 + \pi_t^P) Z_t L_t = \frac{\varepsilon_P - 1}{\psi_P}\Big(\frac{1}{Z_t}\frac{w_t}{(1 - \tau_t^L)(1 - \tau_t^S)} - 1\Big) Z_t L_t + \beta \mathbb{E}_t\Big(\pi_{t+1}^P(1 + \pi_{t+1}^P) Z_{t+1} L_{t+1}\Big),\\
&r_t = \Big(1 - \tau_t^K\Big)\left(\frac{\tilde{R}_{t-1}^N}{1 + \pi_t^P} - 1\right),\\
&\Big(1 + \pi_t^W\Big) w_{t-1}(1 - \tau_t^L) = (1 + \pi_t^P) w_t (1 - \tau_{t-1}^L).
\end{aligned}
$$

Thus $r_t = \Big(1 - \tau_t^K\Big)\left(\frac{\tilde{R}_{t-1}^N}{1+\pi_t^P} - 1\right)$ and the Euler equations are not a constraint. As in the HA case, the instrument τ_t^S ensures $\pi_t^P = 0$. In addition letting $B_t = 0$, we get $w_t = Z_t$. The program simplifies into:

$$
\begin{aligned}
&\max_{\left(\pi_t^W, w_t, \tau_t^L, L_t\right)_{t\geq 0}} W_0,\\
&\pi_t^W(\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W}\Big(v'(L_t) - w_t u'(C_t)\ell(di)\Big) L_t + \beta \mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W + 1)\Big],\\
&\Big(1 + \pi_t^W\Big) w_{t-1}(1 - \tau_t^L) = (1 + \pi_t^P) w_t (1 - \tau_{t-1}^L).
\end{aligned}
$$

Setting $\pi_t^W = 0$ and setting $Z_{t-1}(1 - \tau_t^L) = Z_t(1 - \tau_{t-1}^L)$ implements the first best again.

C.3 The HA economy without time-varying τ_t^E and τ_t^S

We impose $\tau_t^E = \tau_{SS}^E$ and $\tau_t^S = \tau_{SS}^S$. The Phillips curves expressions are:

$$
\begin{aligned}
\pi_t^W(\pi_t^W + 1) &= \frac{\varepsilon_W}{\psi_W}\Big(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}\frac{w_t}{1 - \tau_{SS}^E}\int_i y_{i,t} u'(c_{i,t})\ell(di)\Big) L_t + \beta \mathbb{E}_t\Big[\pi_{t+1}^W(\pi_{t+1}^W + 1)\Big],\\
\pi_t^P(1 + \pi_t^P) &= \frac{\varepsilon_P - 1}{\psi_P}\Big(\frac{1}{Z_t}\frac{w_t}{(1 - \tau_t^L)(1 - \tau_{SS}^S)(1 - \tau_{SS}^E)} - 1\Big) + \beta \mathbb{E}_t\Big(\pi_{t+1}^P(1 + \pi_{t+1}^P)\frac{Z_{t+1} L_{t+1}}{Z_t L_t}\Big),
\end{aligned}
$$

with further more $(1 + \pi_t^W)\frac{w_{t-1}}{1-\tau_{t-1}^L} = \frac{w_t}{1-\tau_t^L}(1 + \pi_t^P)$. This shows that neither the price gap nor the wage gap can be closed. This economy features non-zero inflation for wages and prices.