Accounting for the slowdown in output growth after the Great Recession: A wealth preference approach*

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Abstract

Previous studies have argued that output growth in advanced economies declined during the Great Recession and remained low afterward. To explain the slowdown in output growth, we develop a simple model that incorporates wealth preferences and downward nominal wage rigidity into a standard monetary growth model. Our model demonstrates that output initially grows at the same rate as productivity and slows endogenously afterward. Persistent stagnation takes place even if productivity continues to grow at a steady rate. Applying our model to the US and Japanese data, we show that it successfully explains the slowdown in output growth along with the declines observed in the real interest rate and inflation.

JEL Classification: E41, E47, O10

Keywords: Secular stagnation, Wealth preferences, Liquidity preferences, the Great Recession, Downward nominal wage rigidity

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1 Introduction

The persistent decline in output growth in advanced economies after the Great Recession has formed one of the most well-known debates in macroeconomics. In his "secular stagnation hypothesis," Summers (2014) argues that while long-run US output was expected to grow steadily before the Great Recession, the actual output after the Great Recession failed to catch up with the expected output trend. Figure 1 plots two estimated trends for log real GDP per capita in the US and Japan along with the actual data. The upper panel shows the US output trends together with the actual output.[1] The linear trend (the dot-dashed line) is estimated from the log real GDP from 1990:Q1 to 2007:Q1 and extrapolated after 2007:Q1. While output is expected to grow based on the data up to 2007, the actual real GDP after 2007:Q1 did not grow as fast as the expected output trend. Using the data that includes the period after 2007:Q1, we estimate the cubic trend of output (the dashed line). The cubic trend falls below the linear trend, exhibiting a slowdown in output growth.[2] A similar observation can be made for Japan in the 1990s, as shown in the lower panel of the figure. We estimate the linear trend of the log real GDP from 1980:Q1 to 1991:Q1 and the cubic trend from 1980:Q1 to 2019:Q4. Comparing the linear and cubic trends, we observe that Japan experienced a significant decline in output growth.

[Figure 1 about here.]

In this paper, we develop a model with wealth preferences and perform an assessment of the model in accounting for the slowdown in output growth. The literature on wealth preferences has found that strong wealth preferences could lead to an inherently stagnant economy in the steady state.[3] In the standard model without wealth preferences, households receive market interest from saving. In the model with wealth preferences, however, households receive additional benefits of saving, namely holding wealth. Under our preference assumptions, the additional benefits incentivize households to give up more consumption to enjoy holding more wealth. In turn, the permanent shortage in aggregate demand or the strong desire for saving leads to permanently low real interest rate and low inflation, which well characterize the secular stagnation observed in advanced

[1]In the figure, we normalize the level of the estimated output trends to the actual real GDP in 1990:Q1.

[2]We confirmed a similar pattern in the potential GDP series by the Congressional Budget Office.

[3]The earliest examples of these studies are Ono (1994) and Ono (2001), among others. In a recent paper, Michau (2018) also proposes a model with wealth preferences which leads to secular stagnation in the steady state.

economies.

We incorporate two components into the standard monetary growth model. As discussed, the first component is wealth preferences. Following the literature (e.g., Michau (2018), Michaillat and Saez (2021), Hashimoto, Ono, and Schlegl (2021)), we introduce wealth preferences with a strictly positive marginal utility in equilibrium. This preference assumption leads to a strong desire for saving compared with the case without wealth preferences. The second component is downward nominal wage rigidity (DNWR), which is widely discussed in recent studies.[4] Together with wealth preferences, this DNWR plays an important role for generating secular stagnation in the monetary growth model.

We demonstrate that our model endogenously generates a slowdown in output growth in the transition path to the stagnation steady state. In particular, we theoretically show that output initially equals potential output that is represented by productivity in our model, but output later falls below potential output. In our model, the household with wealth preferences accumulates wealth rather than consuming enough to reach the potential output. The aggregate demand weakened by wealth preferences leads to disinflation while the DNWR is not binding. However, once the DNWR binds, inflation no longer decreases, the aggregate demand determines output, and output growth is no longer affected by productivity. Then, the weakened aggregate demand falls short of the potential output, the aggregate demand shortage occurs, and the growth rate of output is lower than that of productivity.

In our numerical simulations, we focus the slowdown in output growth in the US after the Great Recession and that in Japan in the 1990s. In the US data, our model explains 93 percent of the slowdown in output growth in 2017. Our numerical simulation for Japan also performs well. The model can explain 72 percent of the slowdown in output growth in 2001. While our model is extremely simplified, the simulated output explains differences between the realized output trend and the output trend that was expected before the Great Recession remarkably well.

Our model can also predict the permanent decline in the real interest rate, which is closely linked to the secular stagnation hypothesis.[5] In the standard consumption Euler equation, the

[4]See Barattieri, Basu, and Gottschalk (2014), Sigurdsson and Sigurdardottir (2016), Schmitt-Grohé and Uribe (2016), Fallick, Lettau, and Wascher (2016), Hazell and Taska (2020), and Grigsby, Hurst, and Yildirmaz (2021), among others.

[5]Characterizations of secular stagnation appear in Baldwin and Teulings (2014) and Krugman (2014).

real interest rate is determined by the household's subjective discount rate and the growth rate of consumption. However, as pointed out by Michaillat and Saez (2021), wealth preferences in the consumption Euler equation create discounting in the real interest rate. This discounting leads to a persistently low real interest rate, consistent with the data under secular stagnation.

In our model, inflation also declines persistently until the output trend deviates from the productivity trend. This observation is roughly consistent with inflation in the US before and after the Great Recession. Hall (2011) points out that US inflation declined during the 1990s, but became stable even in the presence of long-lasting slack in the economy from the Great Recession. Our model interprets the missing deflation as being a consequence of the binding DNWR, where inflation stops declining even if aggregate demand falls short of aggregate supply.

Previous studies have fallen into one of four groups in explaining secular stagnation. The first focuses on the productivity slowdown (e.g., Fernald (2015), Gordon (2015), Takahashi and Takayama (2022)). While this group considers the decline in productivity as the main driver of secular stagnation, our paper assumes a constant productivity growth rate to highlight the degree to which wealth preferences alone can explain the observed slowdown in output growth. The second group emphasizes the impact of demographic changes on saving in explaining the declining real interest rate. (e.g., Carvalho, Ferrero, and Nechio (2016), Gagnon, Johannsen, and Lopez-Salido (2021), Jones (2022)). We exclude this potentially important factor from our model because we focus on the mechanism behind the impact of wealth preferences on saving. The third group relies on debt deleveraging (e.g., Hall (2011), Eggertsson and Krugman (2012), Mian and Sufi (2014), Guerrieri and Lorenzoni (2017), Eggertsson, Mehrotra, and Robbins (2019)). Among these studies, Eggertsson, Mehrotra, and Robbins (2019) introduce debt deleveraging into the model which declines in productivity and changes in demography are built in. They numerically evaluate their model of secular stagnation and discuss policy evaluation in their model.[6]

Our paper is categorized into the fourth group, which introduces wealth preferences into a standard macroeconomic model. This group assumes a strong desire for liquidity or wealth (e.g., Michau (2018), Illing, Ono, and Schlegl (2018)).[7] The study closest to ours is Michau (2018).

[6]Ikeda and Kurozumi (2019) discuss monetary policy rules to prevent secular stagnation in a model with financial frictions and endogenous total factor productivity growth.

[7]In recent studies, models with wealth preferences are analyzed using the New Keynesian framework (Michaillat and Saez (2021)) and search models (Michaillat and Saez (2022)).

He incorporates wealth preferences and the DNWR into the standard neoclassical growth model and shows the existence of both the neoclassical and stagnation steady states in his model. By contrast, our model has a unique steady state, and endogenously generates a regime change from efficient allocation in the neoclassical economy to inefficient allocation in the stagnant economy.[8] In contrast to Michau (2018), we implement a quantitative assessment whether the model with wealth preferences can explain the slowdown in output growth together with permanent declines in the real interest rate and inflation.

The paper is organized as follows. Section 2 presents our simple growth model. Section 3 studies the model dynamics and presents the main analytical results. In Section 4, we simulate the model and show that its predictions are consistent with the data. Section 5 concludes.

2 The model

2.1 Setup

The representative household solves the following maximization problem:

$$\max \quad \int_0^\infty e^{-\rho t}\left[u(c_t) + v(m_t) + \beta(a_t)\right] dt, \tag{1}$$

$$\text{s.t.} \quad \dot{a}_t = r_t(a_t - m_t) - \pi_t m_t + w_t n_t - c_t + \tau_t, \tag{2}$$

$$n_t \leq 1, \tag{3}$$

where a_0 is given. The notation is standard: c_t is consumption, m_t is real money balances ($m_t = M_t/P_t$, where M_t is nominal money balances and P_t is the price level), and a_t is total real asset holdings. In the budget constraint, r_t is the real interest rate ($a_t - m_t = b_t$ represents the illiquid asset holdings of the household), π_t is inflation or the opportunity cost of holding money, w_t is real wages, n_t is labor supply, and τ_t are lump-sum transfers from government. The parameter $\rho > 0$ is the household's subjective discount rate. The budget constraint (2) suggests that the sources of consumption and saving ($\dot{a}_t$) equal income from asset holdings ($r_t(a_t - m_t) - \pi_t m_t$),

[8] In this sense, our analysis also differs from Benigno and Fornaro (2018), who develop an endogenous growth model with downward nominal wage rigidities. They show that weak growth depresses aggregate demand and that the resulting aggregate demand shortage may lead to the stagnation steady state. In contrast to our study, stagnation arises as a self-fulfilling equilibrium.

labor income ($w_t n_t$), and the lump-sum transfers from government (τ_t). In this maximization problem, we simplify the labor supply and assume that $n_t \leq 1$. We also assume no capital in the economy, so $a_t = m_t$ holds for all t (i.e., $b_t = 0$ in equilibrium).

Our preference assumptions on the utility from wealth are critical. We assume that the household has an insatiable desire for wealth. The utility from wealth satisfies $\beta'(a_t) > 0$, $\beta''(a_t) \leq 0$, and $\beta'(a_t)$ is strictly positive and constant in equilibrium. Except for $\beta(a_t)$, the utility functions are standard and take a constant-relative-risk-aversion form. The utility from consumption satisfies $u'(c_t) > 0$, $u''(c_t) < 0$, and $\lim_{c_t \to \infty} u'(c_t) = 0$, and the utility from real money balances satisfies $v'(m_t) > 0$, $v''(m_t) < 0$, and $\lim_{m_t \to \infty} v'(m_t) = 0$.

The simplest specification that satisfies the above conditions for $\beta(a_t)$ is a linear function $\beta(a_t) = \beta \times a_t$, where β is a positive constant. The linearity assumption follows Michau (2018) and extends Ono (1994) and Ono (2001), in which the household has an insatiable desire for liquidity. We assume the linearity of $\beta(a_t)$ for simplicity, not for the necessity of our main results.

A necessary condition for our main results is that the marginal utility from wealth is strictly positive and constant in the stagnation steady state. As argued by Michau (2018) and Michaillat and Saez (2021), there are a variety of alternative specifications for the utility from wealth that generates positive constant marginal utility. In these studies, while the concavity of the utility function is ensured, the marginal utility from the wealth is constant in equilibrium.[9] Because these specifications lead to the same results, we employ linear utility for simplicity.[10]

The first-order conditions are

$$
\begin{aligned}
u'(c_t) &= \lambda_t, \\
v'(m_t) &= (r_t + \pi_t)\lambda_t, \\
\dot{\lambda}_t &= (\rho - r_t)\lambda_t - \beta'(a_t),
\end{aligned}
$$

[9] Michau (2018) and Hashimoto, Ono, and Schlegl (2021) consider the preferences for wealth excluding money, $\beta(a_t - m_t^s)$ where m_t^s is the real money supply and the household takes it as given. They assume that $\beta'(a_t - m_t^s) > 0$, $\beta''(a_t - m_t^s) < 0$, and $\lim_{a_t \to \infty} \beta'(a_t - m_t^s) = 0$, but $\beta'(0) > 0$. Thus, the marginal utility from the wealth is constant in equilibrium, where $b_t = a_t - m_t^s = 0$. Michaillat and Saez (2021) allow for the utility from relative wealth $a_t(i) - \tilde{a}_t$. Here, $a_t(i)$ denotes the wealth at the individual household level and $\tilde{a}_t$ is the average wealth in the economy, and the household takes $\tilde{a}_t$ as given. They assume that $\beta'(a_t(i) - \tilde{a}_t) > 0$, $\beta''(a_t(i) - \tilde{a}_t) < 0$ and $\lim_{a_t(i) \to \infty} \beta'(a_t(i) - \tilde{a}_t) = 0$, but again $\beta'(0) > 0$ where $a_t(i) = \tilde{a}_t$.

[10] The constant marginal utility results in equilibrium money holdings beyond the amount the consumers use for their transactions. Nevertheless, constant marginal utility is not necessarily inconsistent with the neuroscientific evidence. Based on lab experiments, Camerer, Loewenstein, and Prelec (2005) argue that "people value money without carefully computing what they plan to buy with it." (p.35)

where λ is the Lagrange multiplier for the budget constraint. The transversality condition is given by $\lim_{t\to\infty} e^{-\rho t}\lambda_t a_t = 0$.

We rewrite the above equations as

$$\frac{v'(m_t)}{u'(c_t)} = r_t + \pi_t, \tag{4}$$

$$\sigma\frac{\dot{c}_t}{c_t} = r_t - \rho + \frac{\beta'(a_t)}{u'(c_t)}. \tag{5}$$

In (4), the household pays the opportunity cost of holding money, $r_t + \pi_t$, to receive the marginal benefits $v'(m_t)$ (or $v'(m_t)/u'(c_t)$ when measured by the unit of consumption goods). In (5), the household pays the marginal cost of saving, $\rho + \sigma\dot{c}_t/c_t$, where σ denotes the degree of relative risk aversion. This is the household's consumption discount rate that allows for the household's risk aversion.[11] Regarding the marginal benefits of saving, the household receives market returns on illiquid assets r_t and the marginal utility from wealth $\beta'(a_t)$ (or $\beta'(a_t)/u'(c_t)$ when measured by the unit of consumption goods). When the wealth preferences are absent, (5) reduces to the standard Euler equation $\dot{c}_t/c_t = \sigma^{-1}(r_t - \rho)$ and saving only yields market returns r_t. When the wealth preferences are present, however, saving generates additional benefits of $\beta'(a_t)/u'(c_t)$. Thus, the household would give up more consumption and accept a lower interest to enjoy holding more wealth.

Eliminating r_t from (4) and (5) and allowing for $b_t = 0$ in equilibrium yield the condition for the substitution between consumption and liquidity:

$$\Omega(m_t, c_t) = \rho + \sigma\frac{\dot{c}_t}{c_t} + \pi_t, \tag{6}$$

$$\text{where} \qquad \Omega(m_t, c_t) \equiv \frac{v'(m_t) + \beta'(m_t)}{u'(c_t)}.$$

Here $\Omega(m_t, c_t)$ denotes the sum of the marginal benefits of holding real money balances (measured by the unit of consumption goods). In $\Omega(m_t, c_t)$, $v'(m_t)$ is benefits from increasing money and $\beta'(m_t)$ is an additional benefit of increasing wealth as a whole. The right-hand side of (6) represents the opportunity cost of holding money. To hold an additional unit of the liquid asset, the household

[11] The consumption discount rate is, in general, represented by the sum of the steady-state discount rate and the growth rate of marginal utility. In equation, it is given by $\rho - [du'(c_t)/dt][1/u'(c_t)] = \rho + \sigma\dot{c}_t/c_t$.

has to give up consumption goods by an amount equal to the household's consumption discount rate $(\rho + \sigma \dot{c}/c_t)$ and the inflation rate (π_t).

There is a representative firm in the competitive market in the economy. In our model, the firm's technology is linear:

$$y_t = \theta_t n_t, \tag{7}$$

where y_t is output and θ_t is productivity. With this production function, the firm's labor demand condition is

$$w_t = \theta_t. \tag{8}$$

Critical assumptions in our model are that nominal wage inflation $\dot{W}_t/W_t$ cannot be lower than the lower bound γ (i.e., the DNWR), and that θ_t increases at an exogenous rate of $g > 0$.[12] Real wage inflation is g from (8) and thus nominal wage inflation is $\pi_t + g$. As a result, the DNWR is expressed as $\dot{W}_t/W_t = \pi_t + g \geq \gamma$, or $\pi_t \geq \gamma - g$, where $\gamma - g$ is the lowest level of price inflation.

The government has a budget constraint $\tau_t = \mu m_t^s$, where $m_t^s = M_t^s/P_t$ and M_t^s is the nominal money supply. Throughout the paper, we assume that the money growth rate is strictly positive $(\mu > 0)$ and sufficiently high:

$$\mu > \gamma - g, \tag{9}$$

which means that the money growth rate always exceeds the lowest level of inflation.

We rewrite the assumption for the DNWR as the complementarity slackness condition in the labor market:

$$(\pi_t + g - \gamma)(1 - n_t) = 0. \tag{10}$$

The economy has two regimes depending on the complementary slackness condition. If $n_t = 1$, the economy is in the state of full employment and $\pi_t > \gamma - g$. We refer to this regime as the high-inflation regime. Alternatively, if $n_t < 1$, there is unemployment in the labor market with the binding DNWR and $\pi_t = \gamma - g$. We refer to this regime as the low-inflation regime.

The market-clearing conditions are

[12] Under the assumption that θ_t grows at g, labor demand decreases with productivity and will eventually disappear. We can circumvent this problem by changing the assumptions on labor productivity. In particular, the representative household endogenously chooses its effort to increase labor income but pays a quadratic cost of effort $[\phi_t/2]e_t^2$ in which ϕ_t declines at an exogenous rate of g. In the appendix, we describe the model with endogenous effort and explain how we can address this problem.

1. Goods market $c_t = y_t = \theta_t n_t$,

2. Labor market $(\pi_t + g - \gamma)(1 - n_t) \geq 0$,

3. Money market $m_t = m_t^s$,

4. Bond market $b_t = 0$.

A competitive equilibrium of the model is the set of allocations $\{c_t, y_t, n_t, m_t, a_t\}$ and prices $\{w_t, r_t, \pi_t\}$ that satisfy the following: i) The representative household maximizes (1) subject to (2) and (3); ii) The representative firm maximizes profits; iii) The government's transfers and money supply are specified as above; and iv) All markets clear except for labor market. The labor market-clearing condition depends on the complementary slackness condition (10).

2.2 High-inflation regime

The high-inflation regime is characterized by full employment $n_t = 1$ and high inflation $\pi_t > \gamma - g$ where the DNWR is not binding. We summarize the equilibrium conditions as follows:

$$\begin{aligned} \frac{\dot{y}_t}{y_t} &= g, && (11)\\ \frac{\dot{m}_t}{m_t} &= \mu - \Omega(m_t, y_t) + \rho + \sigma g, && (12)\\ \pi_t &= \Omega(m_t, y_t) - (\rho + \sigma g), && (13)\\ r_t &= \rho + \sigma g - \frac{\beta'(m_t)}{u'(y_t)}. && (14) \end{aligned}$$

To derive (11), recall that the production function (7) and goods market-clearing condition imply $c_t = y_t = \theta_t$. Here, $y_t = \theta_t$ means that the growth rate of output is g. Next, (5) leads to (14), where we use $\dot{y}_t/y_t = g$ and $a_t = m_t$ in equilibrium. It is straightforward to show (13) from (6). By assumption, inflation is higher than $\gamma - g$ in this regime. Finally, (12) is obtained from the definition of $m_t = M_t/P_t$, $\dot{m}_t/m_t = \mu - \pi_t$. Given the initial value θ_0, we obtain the output level $y_t = \theta_t$. Moreover, we can numerically solve (12) for m_t given m_0.

It is convenient to define the threshold value of $\Omega(m_t, y_t)$ in which output grows at the rate of g but inflation is as low as $\gamma - g$. Substituting $\dot{y}_t/y_t = g$ and $\pi_t = \gamma - g$ into (6) gives the threshold value of $\Omega(m_t, y_t)$:

$$\Omega^* = \rho + \gamma + (\sigma - 1)g. \tag{15}$$

We use the threshold Ω^* to evaluate the allocation in the low-inflation regime.

2.3 Low-inflation regime

The low-inflation regime is characterized by unemployment $n_t < 1$ and the lower bound of inflation $\pi_t = \gamma - g$ where the DNWR is binding. We summarize the equilibrium conditions as follows:

$$\begin{aligned}
\frac{\dot{y}_t}{y_t} &= g - \sigma^{-1}\left[\rho + \gamma + (\sigma - 1)g - \Omega(m_t, y_t)\right], && (16)\\
\frac{\dot{m}_t}{m_t} &= \mu - (\gamma - g), && (17)\\
\pi_t &= \gamma - g, && (18)\\
r_t &= \frac{v'(m_t)}{u'(y_t)} - (\gamma - g). && (19)
\end{aligned}$$

Our assumption corresponds to (18). Equation (17) immediately follows from (18) because $\dot{m}_t/m_t = \mu - \pi_t$. To obtain the expression for output growth (16), we use (6) and the equilibrium condition $c_t = y_t$. The real interest rate r_t can be obtained from (4) together with $\pi_t = \gamma - g$.

To compare the system of equations between the high- and low-inflation regimes, we rewrite (16) – (19) as

$$\begin{aligned}
\frac{\dot{y}_t}{y_t} &= g - \sigma^{-1}\left[\Omega^* - \Omega(m_t, y_t)\right], && (20)\\
\frac{\dot{m}_t}{m_t} &= \mu - \Omega^* + \rho + \sigma g, && (21)\\
\pi_t &= \Omega^* - (\rho + \sigma g), && (22)\\
r_t &= \rho + \sigma g - \frac{\beta'(m_t)}{u'(y_t)} - \left[\Omega^* - \Omega(m_t, y_t)\right], && (23)
\end{aligned}$$

where we use the definitions of $\Omega(m_t, y_t)$ and Ω^* shown in (6) and (15), respectively.

Comparisons between the two regimes reveal that there is additional downward pressure on output growth and the real interest rate when the DNWR binds. Equation (20) indicates that output growth can be lower than g depending on the sign of $\Omega^* - \Omega(m_t, y_t)$. In particular, it indicates that output growth is lower than or equal to g if $\Omega(m_t, y_t) \leq \Omega^*$. Similar interpretations can apply to the real interest rate from a comparison between (14) and (23).

3 The model dynamics

In this section, we specify the functional forms of $u(c_t)$, $v(m_t)$, and $\beta(a_t)$ and investigate the model dynamics. We first show that the stagnation steady state exists in the presence of the wealth preferences. We then demonstrate that output initially grows at the same rate as productivity but slows in the transition path to this stagnation steady state. This slowdown in output growth endogenously occurs even if productivity continues to grow at a steady rate. Finally, we also show that the model without wealth preferences fails to generate the slowdown in output growth.

In what follows, we assume that the functions $u(c_t)$, $v(m_t)$, and $\beta(a_t)$ are given by

$$u(c_t) = \ln c_t \tag{24}$$

$$v(m_t) = v\frac{m_t^{1-\eta}}{1-\eta}, \qquad v > 0, \quad \eta > 0, \tag{25}$$

$$\beta(a_t) = \beta a_t, \qquad \beta \geq 0, \tag{26}$$

where, with a slight abuse of notations, $v > 0$ and $\beta \geq 0$ represent a parameter for the functions $v(m_t)$ and $\beta(a_t)$.

These utility functions simplifies the condition for the substitution between consumption and liquidity. The market-clearing conditions $c_t = y_t$ in equilibrium and $\sigma = 1$ under (24) imply

$$\Omega(m_t, y_t) = \rho + \frac{\dot{y}_t}{y_t} + \pi_t, \tag{27}$$

which is the key equation for understanding the transition dynamics toward the stagnation steady state.

3.1 The stagnation steady state

We characterize the steady state as a preparatory step for analyzing the model dynamics. The steady state in our model is the "stagnation" steady state in which the DNWR binds and output converges to a constant value. We show that the stagnation steady state can arise under our preference assumptions.

Suppose that nominal wage inflation hits the lower bound γ at $t = t^*$. Equation (17) implies that m_t goes to ∞ as $t \to \infty$ because of (9). In particular,

$$m_t = m_{t^*} \exp[(\mu - \gamma + g)(t - t^*)] \;\Rightarrow\; \lim_{t\to\infty} m_t = \infty, \tag{28}$$

where m_{t^*} is the real money balances in the period from which the DNWR starts binding. Then, in the stagnation steady state where y_t converges to a constant value, the limit of the left-hand side of (27) becomes

$$\lim_{t\to\infty} \Omega(m_t, y_t) = \lim_{t\to\infty} [v'(m_t) + \beta'(m_t)]y_t = \beta y^{ss},$$

where a superscript ss on a variable denotes the stagnation steady-state value.

To prove the existence of the stagnation steady state, we use the transversality condition: $\lim_{t\to\infty} \lambda_t a_t \exp(-\rho t) = \lim_{t\to\infty} \lambda_t m_t \exp(-\rho t) = 0$. Taking the time derivative of this condition translates the condition into $\dot{\lambda}_t/\lambda_t + \dot{m}_t/m_t - \rho < 0$. In the stagnation steady state, $\dot{\lambda}_t/\lambda_t = -\dot{c}_t/c_t = 0$ and $\dot{m}_t/m_t = \mu - (\gamma - g)$. Thus, $\mu - (\gamma - g) < \rho$ ensures the transversality condition. Note that as long as the assumption of $\mu > 0$ holds, the transversality condition implies that $g < \rho + \gamma$.

Now, the allocations in the steady state are characterized as follows. Regarding output, (27) becomes $\beta y^{ss} = \rho + \gamma - g$ because $\lim_{t\to\infty} \Omega(m_t, y_t) = \beta y^{ss}$, $\dot{y}_t/y_t = 0$, and $\pi_t = \gamma - g$. The steady-state output is given by

$$y^{ss} = \frac{\rho + \gamma - g}{\beta}, \tag{29}$$

which is strictly positive because $g < \rho + \gamma$. Because of the DNWR, we have $\pi^{ss} = \gamma - g$. The nominal interest rate in this steady state is zero because of (4): $\lim_{t\to\infty}(r_t + \pi_t) = \lim_{t\to\infty}[v'(m_t)y_t] = 0$. From this equation, the real interest rate is given by $r^{ss} = -\pi^{ss} = g - \gamma$. Clearly, $\dot{y}_t/y_t = 0$ and $\dot{m}_t/m_t = \mu - (\gamma - g) > 0$ from (9).

3.2 Transition dynamics to the stagnation steady state

We are now ready to discuss the transition path to the stagnation steady state. The following proposition summarizes our main results.

Proposition 1. *Suppose that productivity grows at a strictly positive growth rate ($g > 0$) and that* *the money growth rate exceeds the lower bound of inflation ($\mu > \gamma - g$) and is strictly positive ($\mu > 0$). Then, under the preference assumptions specified by (24)–(26) with a strictly positive β, we have a unique dynamic path of output toward the stagnation steady state. Output growth is endogenously determined. In particular, let t^* be the period of time from which the DNWR binds. Then,*

- *For $0 < t < t^*$, the growth rate of output is equal to that of productivity.*
- *For $t = t^*$, the growth rate of output starts declining.*
- *For $t > t^*$, the growth rate of output is lower than that of productivity.*
- *As $t \to \infty$, the growth rate of output converges to zero.*

Here, we assume that the initial productivity θ_0 is such that $\pi_0 = \Omega(m_0, \theta_0) - (\rho + g) > \Omega^* - (\rho + g)$. To ensure the transversality condition, we assume that $\mu - (\gamma - g) < \rho$.

Proof. See Appendix A.1. □

The model endogenously generates a slowdown in output growth. This slowdown results from a regime change from the high- to the low-inflation regime. When the economy is in the high-inflation regime, $n_t = 1$ holds.[13] Thus, output equals the potential output or equivalently productivity (i.e., $y_t = \theta_t$). Thus, output grows at the same rate as productivity. The steady growth of output persists as long as the DNWR is not binding (i.e., $\pi_t > \gamma - g$). However, once the DNWR binds at $t = t^*$, the growth rate becomes lower than g for $t \geq t^*$. Output growth then declines over time and converges to zero in the stagnation steady state.

The prediction in Proposition 1 is consistent with the slowdown in output growth observed in the US after the Great Recession and in Japan after the early 1990s. In the debate on secular stagnation, output growth was high before the Great Recession but declined after the Great Recession. In the context of our model, output growth is as high as g when $t < t^*$, but it becomes lower than g when $t > t^*$. Note that this endogenous slowdown in output growth occurs even if productivity continues to grow at the same rate. In other words, the model explains the slowdown in output

[13] For the economy to be in the high-inflation regime at the initial period, productivity in the initial period θ_0 must be sufficiently low. The sufficiently low θ_0 leads to high inflation that satisfies the condition of $\pi_0 = \Omega(m_0, \theta_0) - (rho + g) > \Omega^* - (\rho + g)$ in Proposition prop:1.

growth without relying on the productivity slowdown. Moreover, the slowdown in output growth is permanent. Output growth declines over time as $t \to \infty$ and converges to zero when $t = \infty$.

To understand the model dynamics, we focus on $\Omega(m_t, y_t)$ given by (27). Under our specification, $\Omega(m_t, y_t) = (\beta + v m_t^{-\eta}) y_t$ is decreasing in m_t and increasing in y_t. That is, there are two offsetting effects on $\Omega(m_t, y_t)$. In the growing economy, both m_t and y_t grow over time, so the dynamics of $\Omega(m_t, y_t)$ are not clear analytically. Numerically, however, the marginal benefits of holding real money balances $\Omega(m_t, y_t)$ decreases over time because the effect of m_t on $\Omega(m_t, y_t)$ overwhelms the effect of y_t on $\Omega(m_t, y_t)$. Intuitively, this is because strong preferences for wealth make real money balances grow faster than consumption.

Now, given the declining marginal benefits of holding money, the marginal cost represented by the right-hand side of (27) must decline in equilibrium. As (27) shows, given a constant ρ, either output growth ($\dot{y}_t / y_t$) or inflation (π_t) must decline. If the economy is in the high-inflation regime, output growth can be kept at $\dot{y}_t / y_t = g$. This is because decreased inflation can lower the marginal cost of holding money. By contrast, inflation can no longer decrease if the economy is in a low-inflation regime. Only through the slowdown in output growth can the marginal cost of holding money decrease.

We emphasize that the aggregate demand shortage drives this slowdown in output growth. In our model, the household's wealth preferences weaken the aggregate demand growth by substituting wealth for consumption goods. Initially, the weakened aggregate demand leads to disinflation since the DNWR is not binding. Because prices are flexible, the economy can achieve the efficient level of output. However, when the DNWR makes price adjustment rigid, the weakened aggregate demand determines equilibrium output. As a result, the growth rate of output is g for $t < t^*$ and becomes lower than g for $t \geq t^*$. Moreover, the equilibrium output for $t \geq t^*$ is inefficient in terms of welfare.

Some remarks on the transition path are in order. First, all variables including $\dot{y}_t / y_t$ and $\dot{m}_t / m_t$ smoothly switch from a path described from (11)–(14) to a path specified by (20)–(23) without a jump. We can confirm this by evaluating $\Omega(m_t, y_t)$ at Ω^*. When the marginal benefit of holding money is at the threshold level, (20)–(23) at $t = t^*$ all reduce to (11)–(14), suggesting smooth transitions from the high- to the low-inflation regime. Second, output growth also exhibits a smooth transition to the stagnation steady state. Since $\Omega(m_t, y_t)$ in (16) smoothly converges to its limit of βy^{ss}, output growth smoothly converges to zero as $t \to \infty$:

$$\lim_{t\to\infty} \frac{\dot{y}_t}{y_t} = \lim_{t\to\infty} [g + \Omega(m_t, y_t) - (\rho+\gamma)] = g + \beta y^{ss} - \rho - \gamma = 0 \tag{30}$$

where the first equality is from $\sigma = 1$ in (16) and the last equality comes from (29).

3.3 The role of wealth preferences

The results of Proposition 1 critically depend on the household's wealth preferences with a strictly positive β. In this subsection, we show that if there are no wealth preferences (i.e., $\beta = 0$), the model fails to generate the endogenous slowdown in output growth, in contrast to Proposition 1. The following proposition summarizes our results under $\beta = 0$.

Proposition 2. *Suppose that productivity grows at a strictly positive growth rate ($g > 0$) and that the money growth rate exceeds the lower bound of inflation ($\mu > \gamma - g$). Then, under the preference assumptions specified by (24)–(26) but with $\beta = 0$, we have a unique dynamic path of output growth such that*

- *For any $t > 0$, the growth rate of output is constant.*
- *The growth rate of output is equal to $\min\{g, \eta[\mu - (\gamma - g)]\}$.*

The growth rate of output is g when the DNWR does not bind or $\eta[\mu - (\gamma - g)]$ when the DNWR binds. The transversality condition is given by $(1-\eta)\min\{g, \eta[\mu - (\gamma - g)]\} < \rho$.

Proof. See the Appendix A.2 □

The proposition suggests that the model without wealth preferences ($\beta = 0$) fails to generate an endogenous slowdown in output growth. Instead, parameters in the model predetermine the regime in equilibrium. For example, if the economy is initially in a high-inflation regime, the growth rate moves in tandem with productivity. Output always equals its potential level and the allocation is efficient. If the economy instead starts from a low-inflation regime, the economy experiences low economic growth given the rigid wage adjustment arising from the DNWR. In this case, output is always below its potential level and the allocation is inefficient. The aggregate demand shortage make economic growth slower than g. In either case, however, we do not observe a slowdown in output growth, in contrast to the case of $\beta > 0$. Moreover, the growth rate of output with the binding DNWR does not converge to zero. Even if the DNWR is binding, there is no stagnation steady state under $\beta = 0$. Therefore, the essential ingredient for generating slowdown in output growth and stagnation steady state is a strictly positive β in the wealth preferences.[14]

While the model with wealth preferences is consistent with the observed slowdown in output growth, it is not necessarily clear whether the model numerically explains the data. It is also worth assessing macroeconomic variables other than output growth. The following section assesses the model with wealth preferences based on output growth, the real interest rate, and inflation.

4 Simulating the model

This section simulates the model for the US and Japan. We will explore whether our model can quantitatively explain the observed slowdown in output growth as well as the declining real interest rate and inflation.

4.1 Calibrations

Before simulating the model, we calibrate parameters for simulations. We first discuss the timing of the regime change from the high-inflation regime to the low-inflation regime. Comparing the linear trend and the cubic trend shown in Figure 1, we assume that t^* is the period in which the cubic trend (the dashed line in the figure) starts falling below the linear trend (the dot-dashed line). A closer look at Figure 1 leads to t^* at 2001:Q3 for the US and at 1989:Q1 for Japan.

Some deep parameters are assumed to be common between the US and Japan. The subjective discount factor ρ is 0.04. The degree of relative risk aversion for c_t is one ($\sigma = 1$) and that for m_t is four ($\eta = 4$).

Other deep parameters differ between the US and Japan, and we use the data to parameterize them. We first calibrate the parameters for the US economy. Recall that we estimated the linear trend (the dot-dashed line in the upper panel of Figure 1) from log real GDP over 1990:Q1 and 2007:Q1. The mean growth rate over the sample period (or the slope of the linear trend) is 0.022 at

[14] A natural question is what happens if nominal wages are fully flexible, but β is strictly positive. As (27) suggests, π_t may continue to decline without the slowdown in output growth. However, it can be shown that there is no monetary equilibrium with a strictly positive β. For details, see Ono (2001).

the annual rate. We take this value as the growth rate of productivity, $g = 0.022$ for simulation. The money growth rate μ is 0.043, using the mean growth rate of M2 stock (per capita) over 1990:Q1–2019:Q4. To parameterize γ, we utilize $\pi_t = \gamma - g$ given by (18). The value of $\gamma = \pi_t + g$ is 0.039 where we use the mean of inflation over 2001:Q3–2019:Q1 as π_t in $\gamma = \pi_t + g$.[15] Here, inflation is the year-on-year inflation calculated from the Core PCEPI (the Personal Consumption Expenditure Price Index Excluding Food and Energy). To pin down v and β in the utility functions $v(m_t)$ and $\beta(a_t)$, we target the 2007:Q1 values of the cubic trend for the real interest rate and the velocity of money. For the real interest rate, we use the 1 year real Treasury yields from the database of the Cleveland Federal Reserve Bank and estimated by Haubrich, Pennacchi, and Ritchken (2012).[16] The real interest rate in 2007:Q1 is 1.72 percent. For the velocity of money, we employ the velocity of M2. The velocity of money in 2007:Q1 is 2.04. We calibrate v and β to hit these target values. The resulting values of v and β are $v = 0.055$ and $\beta = 0.014$, respectively.

We next calibrate the parameters for the Japanese economy. We set $g = 0.039$ from the growth rate of real GDP per capita between 1980:Q1 and 1991:Q1. We take the year 1991 for the end of the sample period in calculating g because the business cycle peaked in 1991 before output growth declined for decades. We parameterize $\mu = 0.041$ from the per capita M2 growth averaged over 1980:Q2 –2019:Q4. As before, we calibrate γ at 0.042 using (18). Here, we calculate the mean inflation from the Core CPI (the Consumer Price Index, All items less fresh food).[17] To obtain v and β in $v(m_t)$ and $\beta(a_t)$ for Japan, we target the real interest rate and the velocity in 1991:Q1. We construct the real interest rate using the nominal interest rate and the actual inflation from 1986:Q3 to 2019:Q4.[18] We use the velocity of M2 as in the calibration for the US. We calibrate

[15] The period over 2001:Q3–2019:Q1 corresponds to time satisfying $t > t^*$. Given that the mean of inflation is 0.017 and $g = 0.022$, the resulting value of γ is 0.039, which seems too high for the lower bound on the nominal wage inflation rate. However, we employ $\gamma = 0.039$ for three reasons. First, the main purpose of our analysis is to explore whether the model replicates the observed slowdown in output growth. In our simulation, we confirmed that the value of γ does not largely affect the transition dynamics of output growth to the steady state. Second, we are also interested in whether the model can generate declining inflation for $t < t^*$. If we set γ at a lower value, the level of inflation would be lower than the data. Finally, the parameterized value ensures our assumptions that $\mu > \gamma - g$ and $\rho > \mu - (\gamma - g)$. We require these assumptions for equilibrium to exist.

[16] The most recent data are available at `https://www.clevelandfed.org/our-research/indicators-and-data/inflation-expectations/background-and-resources.aspx#research`.

[17] The consumer prices are adjusted for consumption tax hikes in 1997 and 2014.

[18] To compute the 1-year real interest rate, we use the 12-month Japanese Yen London Interbank Offered Rate (JPY LIBOR) as an interest rate benchmark over 1986:Q3–2019:Q4. We combine the core inflation from 1986:Q3 to 1989:Q4 and the tax-adjusted core inflation from 1990:Q1 to 2019:Q4 because the consumption-tax-adjusted core inflation is only available from 1990. Note that inflation is not adjusted for the introduction of the consumption tax in 1989:Q2 and the increase in 2019:Q4.

v and β to hit the target of the 1991:Q1 values of the real interest rate (3.67 percent in cubic trend) and the velocity of money (1.06 in cubic trend). We obtained $v = 0.0658$ and $\beta = 0.027$, respectively.

We solve the model by a method similar to the shooting algorithm. We calculate the transition path forward from t^* to $\bar{t}$, where $\bar{t}$ is a sufficiently large number to approximate $t = \infty$. In this transition path, the economy moves toward the stagnation steady state. We guess the output in the period of a regime change (denoted by y_{t^*}) and compute the real money balances in the same period (denoted by m_{t^*}).[19] Together with y_{t^*} and m_{t^*}, we use (16) and (17) to obtain future variables $y_{t+\Delta}$ and $m_{t+\Delta}$, where Δ is a small increment of time. We iterate this calculation forward until we have $y_t \simeq y_{t+\Delta}$. Define $y_{\bar{t}}$ as the output for which $y_t \simeq y_{t+\Delta}$ is satisfied. We compare $y_{\bar{t}}$ with the steady state output y^{ss}. If $y_{\bar{t}} \simeq y^{ss}$, we conclude that the transition path to the stagnation steady state is obtained. If not, we update the guess of the output y_{t^*} and iterate computations until we have $y_{\bar{t}} \simeq y^{ss}$. Regarding the transition path for $t < t^*$, output growth always equals productivity growth g. We compute the transition path backward from t^* to $t = t_0$, where t_0 denotes the first period of the sample.

4.2 Simulation results

Figures 2 and 3 report our simulation results for the US and Japan, respectively. The upper panel presents the log real GDP per capita, the middle panel shows the real interest rate, and the right panel is inflation. Each panel has the simulated data (the solid line), the estimated cubic trend (the dashed line), and the actual data (the dotted line). Our model has no stochastic shocks. Therefore, our model aims to explain the estimated cubic trend rather than the actual data.

4.2.1 The US

The upper panel of Figure 2 presents the US output. In this panel, the simulated output at t^* is equalized to the linear trend in the same period. We will see how closely the simulated output after t^* tracks the cubic trend. Because the simulated output grows at g when $t < t^*$, the simulated output matches perfectly with the linear trend, which grows at the same rate of g.

[19] Here we use $\Omega(m_{t^*}, y_{t^*}) = \Omega^*$ to obtain m_{t^*}. The threshold value Ω^* is given by (15).

[Figure 2 about here.]

Comparing the solid and dashed lines in the upper panel of Figure 2 reveals that the model can account for a substantial fraction of the slowdown in output growth relative to the linear trend (the dotted line). For example, the model can explain 68.9 percent of the slowdown in output growth in 2012 and 93.0 percent of the slowdown in output growth in 2017.[20] Thus, the model can almost entirely explain the slowdown in output growth in 2017.

The middle panel shows the simulated real interest rate. The model successfully replicates the real interest rate that has decreased since the 1990s. The decline in the cubic trend (dashed line) is 3.13 percentage points lower, from 2.43 percent in 1994:Q3 to -0.70 percent in 2014:Q1. The simulated real interest rate exhibits a similar pattern to the data. The simulated real interest rate was 2.74 percent in 1990:Q1 and -1.00 percent in 2019:Q1. The magnitude of the decline in the simulated real interest rate is 3.74 percentage points, which is close to the 3.13 percentage point decline in the real interest rate. As discussed in the previous section, there is additional downward pressure on the real interest rate represented by $\Omega^* - \Omega(m_t, y_t)$ when the DNWR binds (See (23)). This downward pressure arises from the household's insatiable strong desire for wealth. The lower panel of Figure 2 plots inflation.[21] Overall, the simulated inflation replicates the estimated cubic trend quite well, especially in replicating the decline in the 1990s. The figure indicates that trend inflation (the dashed line) decreased steadily by 2003:Q2 and became stable at around 1.5–2 percent. In simulation, inflation also decreases steadily by 2001:Q3 and becomes constant at $\gamma - g$ (See (22)). The decline in simulated inflation results from the weak aggregate demand for consumption goods.

4.2.2 Japan

Figure 3 repeats the same exercise using the Japanese data. Overall, the model performs well in the case of Japan. Recall that we set the timing of the regime change at 1989:Q1. Thus, changes in the model dynamics become significant in the early 1990s. Once again, the model can explain

[20] In the case of 2012, the linear trend of output is 43.6 percent higher in 2012:Q1 than in 1990:Q1. The cubic trend is 33.6 percent higher in 2012:Q1 than in 1990:Q1. Thus, the slowdown in output growth is $10.3(= 43.6 - 33.6)$ percentage points between the linear and cubic trends. Next, the simulated output trend is 36.5 percent higher in 2012:Q1 than in 1990:Q1, so output declines $7.1(= 43.6 - 36.5)$ percentage points. Taking the ratio of these numbers yields $0.689 = 0.071/0.103$, so the model can explain 68.9 percent of the slowdown in output growth. We can implement the same calculation to obtain the number for 2017.

[21] We use year-on-year inflation to remove noises in inflation.

a substantial fraction of the slowdown in output growth relative to the linear trend. Shown in the upper panel of Figure 3, the model explains 53.6 percent of the slowdown in output growth as of 1996 and 72.4 percent of the slowdown in output growth in 2001. The simulated real interest rate closely keeps track of the cubic trend of the real interest rate that declined from about 5 percent to a slightly negative value of around -0.2 percent.[22] Finally, the simulated inflation also explains the observed decreases in the early 80s.

[Figure 3 about here.]

5 Conclusion

Output growth in advanced economies was persistently low after the Great Recession. In this paper, we explained this slowdown in output growth by introducing wealth preferences into a standard monetary growth model. We theoretically showed that our model generates a slowdown in output growth in the transition path to the stagnation steady state. Using numerical simulations, we found that our model explains 93 percent of the slowdown in output growth in 2017 for the US and 72 percent of the slowdown in output growth in 2001 for Japan. In addition to output growth, the model successfully reproduces the magnitude of the decreases in the real interest rate and inflation.

It is quite surprising that a simple model can account for the long-run patterns in the data. Further work would enrich our understanding of secular stagnation and its policy prescriptions. Many questions are left open. What are the implications of growth-enhancing policy on the model dynamics? What happens to output growth and the real interest rate if we prompt nominal wage adjustment by removing institutional frictions in the labor market? What are the impacts of the forward guidance on output growth? Exploring these directions would be important for future research.

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[22] Note that we plot the real interest rates from 1986:Q2 because of the data availability of the nominal interest rate.

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A Proofs of Propositions

A.1 Proof of Proposition 1

To prove Proposition 1, the phase diagram in the (m_t, y_t) plane is convenient. We derive the loci in each inflation regime in Figure 4.

[Figure 4 about here.]

We first consider the high-inflation regime. Equation (11) implies that $\dot{y}_t > 0$ for any $t < t^*$. We thus focus on the $\dot{m}_t = 0$ locus. The $\dot{m}_t = 0$ locus is obtained from (12) and given by $\mu - \Omega(m_t, y_t) + \rho + g = 0$. Under our preference assumptions, $\Omega(m_t, y_t) = (\beta + v m_t^{-\eta}) y_t$. Therefore, the $\dot{m}_t = 0$ locus can be rewritten as

$$\begin{aligned} y_t &= \frac{\rho + g + \mu}{\beta + v m_t^{-\eta}} \\ &= \frac{\rho + \gamma + [\mu - (\gamma - g)]}{\beta + v m_t^{-\eta}} \\ &= f_H(m_t), \end{aligned} \tag{31}$$

where we define the $\dot{m}_t = 0$ locus by $y_t = f_H(m_t)$. In Figure 4, the locus is drawn as the red solid line. Here m_t increases with time whenever (m_t, y_t) lies to the right of the $\dot{m}_t = 0$ locus. Together with $\dot{y}_t > 0$ under the high-inflation regime, the directions of the changes are indicated by red arrows in the figure.

Next, consider the low-inflation regime. Under the assumption of (9), (17) implies that $\dot{m}_t > 0$ for any $t > t^*$. We thus focus on the $\dot{y}_t = 0$ locus. The $\dot{y}_t = 0$ locus is obtained from (16) and given by $g - (\rho + \gamma) + \Omega(m_t, y_t) = 0$. Again, noting that $\Omega(m_t, y_t) = (\beta + vm_t^{-\eta})y_t$, the $\dot{y}_t = 0$ locus can be rewritten as

$$\begin{aligned} y_t &= \frac{\rho + \gamma - g}{\beta + vm_t^{-\eta}} \\ &= f_L(m_t), \end{aligned} \tag{32}$$

where we define the $\dot{y}_t = 0$ locus by $y_t = f_L(m_t)$. Note that $f_L(m_t) < f_H(m_t)$ holds because $\mu - (\gamma - g) > 0$ from (9) and $g > 0$. In Figure 4, the locus is drawn as the blue solid line. Here y_t increases with time whenever (m_t, y_t) lies to the left of the $\dot{y}_t = 0$ locus. Together with $\dot{m}_t > 0$ under the low-inflation regime, the directions of the changes are indicated by blue arrows in the figure.

Let us introduce another locus that determines the regime change. This locus is defined as a set of (m_t, y_t) in which output growth is g and inflation is $\gamma - g$. Substitute $\dot{y}_t/y_t = g$ and $\pi_t = \gamma - g$ into (27) to get $\Omega(m_t, y_t) = \rho + \gamma$. Again, using $\Omega(m_t, y_t) = (\beta + vm_t^{-\eta})y_t$, we rewrite this condition at the threshold as

$$\begin{aligned} y_t &= \frac{\rho + \gamma}{\beta + vm_t^{-\eta}} \\ &= f_T(m_t), \end{aligned} \tag{33}$$

where we define the locus as $y_t = f_T(m_t)$. As shown in Figure 4, we have $f_L(m_t) < f_T(m_t) < f_H(m_t)$ given m_t because of $\mu > \gamma - g$ and $g > 0$. It is easy to show that the economy is in the high-inflation regime when (m_t, y_t) lies above the locus.[23] When (m_t, y_t) lies below the locus, the

[23] We can prove this claim by contradiction. Suppose that $\pi_t = \gamma - g$ and $\dot{y}_t/y_t < g$. We do not need to consider $\pi_t < \gamma - g$ and $\dot{y}_t/y_t > g$ because they are infeasible. Then, (27) implies that $\Omega(m_t, y_t) = \rho + \dot{y}_t/y_t + \pi_t < \rho + g + \gamma - g = \rho + \gamma = \Omega^*$. That is, $\Omega(m_t, y_t) < \Omega^*$. It immediately follows from $\Omega(m_t, y_t) = (\beta + vm_t^{-\eta})y_t$ that $y_t < f_T(m_t)$, which contradicts the supposition.

DNWR is binding and the economy is in the low-inflation regime.

Figure 4 also draws the optimal time path (the curve with arrows) starting from the initial state of the economy. By assumption, at the initial state, $\pi_0 > \gamma - g$. Thus, the economy is in the high-inflation regime, and (m_0, y_0) is located above the locus of $f_T(m_t)$. Once (m_t, y_t) goes to the right of the $f_T(m_t)$ locus, the economy moves to the low-inflation regime. Recall that (20) and (21) indicate that (m_t, y_t) switches from the high-inflation regime to the low-inflation regime without a jump. Also, as $t \to \infty$, $m_t \to \infty$ from (28) and $\dot{y}_t/y_t \to 0$ from (30). Thus, (m_t, y_t) asymptotically converges to the dotted line located at the bottom in which $y_t = y^{ss}$. The optimal time path never exceeds the dotted line because $\dot{y}_t/y_t$ is always positive as long as (m_t, y_t) is located above the blue solid line (See Figure 4).

To complete the proof, we confirm that the time path shown in the figure is saddle-path stable. Let us redefine $z_t = 1/m_t$ to obtain the system of the equation under the low-inflation regime:

$$
\begin{aligned}
\frac{\dot{y}_t}{y_t} &= g - \left[(\rho + \gamma) - \Omega(1/z_t, y_t)\right], \\
\frac{\dot{z}_t}{z_t} &= -(\mu - \gamma + g)
\end{aligned}
$$

By linearizing the above two equations around the stagnation steady state $(z_t, y_t) = (0, y^{ss})$, we have the eigenvalues ζ such that

$$
\begin{vmatrix} \rho + \gamma - g - \zeta & 0 \\ 0 & -(\mu - \gamma + g) - \zeta \end{vmatrix} = 0,
$$

where we use (29) to replace $\beta y^{ss} = \rho + \gamma - g$. As mentioned in the main text, combining the condition $\mu - (\gamma - g) < \rho$ derived from the transversality condition with the condition $\mu > 0$ implies that $g < \rho + \gamma$. The condition $g < \rho + \gamma$ then ensures that one eigenvalue is positive. In addition, the assumption $\mu - (\gamma - g) > 0$ in (9) implies that the other eigenvalue is negative. Because the system of equations under the low-inflation regime includes one jump variable y_t and one predetermined variable z_t, the time path is saddle-path stable and unique.

Along any path located above the saddle path m_t eventually becomes negative so that it is infeasible. Along any path located below the saddle path c_t eventually becomes zero so that the transversality condition does not hold. Therefore, the saddle path is the unique equilibrium path.

A.2 Proof of Proposition 2

The proposition discusses the case of $\beta = 0$ to clarify the role of wealth preferences. To prove the proposition, we focus on the elasticity of $\Omega(m_t, y_t)$ with respect to m_t and study the dynamics of $\Omega(m_t, y_t) = [v'(m_t) + \beta'(m_t)]y_t$. We then discuss the equilibrium under the high- and low-inflation regimes.

Let ε_m be the elasticity of $\Omega(m_t, y_t)$ with respect to m_t.

$$\varepsilon_m(m_t) = \frac{\partial \Omega(m_t, y_t)}{\partial m_t} \frac{m_t}{\Omega_t} = -\eta \frac{v m_t^{-\eta}}{\beta + v m_t^{-\eta}}. \tag{34}$$

The growth rate of $\Omega_t = \Omega(m_t, y_t)$ is

$$\frac{\dot{\Omega}_t}{\Omega_t} = \frac{\dot{y}_t}{y_t} + \varepsilon_m(m_t)\frac{\dot{m}_t}{m_t}.$$

In general, the dynamics of Ω_t are described by a nonlinear differential equation because the elasticity of Ω_t depends on m_t. However, when $\beta = 0$, the above equation simplifies to

$$\frac{\dot{\Omega}_t}{\Omega_t} = \frac{\dot{y}_t}{y_t} - \eta\frac{\dot{m}_t}{m_t}. \tag{35}$$

Next, we study the dynamics of Ω_t under the high- and low-inflation regimes. We will show below that Ω_t is constant in equilibrium when $\beta = 0$. This constancy of $\Omega(m_t, y_t)$ along with (35) implies that $\dot{y}_t/y_t = \eta\dot{m}_t/m_t$ holds in equilibrium.

Output growth under the high-inflation regime Substituting (11) and (17) into (35) yields the differential equation for Ω_t under the high-inflation regime:

$$\begin{aligned}\frac{\dot{\Omega}_t}{\Omega_t} &= g - \eta(\mu - \Omega_t + \rho + g) \\ &= g - \eta[\mu - (\gamma - g)] + \eta(\Omega_t - \Omega^*),\end{aligned} \tag{36}$$

where we use (15) for the second equality. Equation (36) is the differential equation for Ω_t with a positive coefficient on Ω_t. Thus, if there is a deviation of Ω_t from its steady-state value, Ω_t would explode to ∞. Therefore, when $\beta = 0$, only $\dot{\Omega}_t = 0$ is feasible in equilibrium. Imposing $\dot{\Omega}_t = 0$ on (36), we have the steady-state value of Ω_t under the high-inflation regime with $\beta = 0$.

$$\Omega_H^{ss} = \Omega^* + [\mu - (\gamma - g)] - \frac{g}{\eta}. \tag{37}$$

The high-inflation regime under $\beta = 0$ is feasible only when $g < \eta[\mu - (\gamma - g)]$. To see this, suppose that $g \geq \eta[\mu - (\gamma - g)]$ under the high-inflation regime. In this case, (37) implies that $\Omega_H^{ss} \leq \Omega^*$. Given $\dot{y}_t/y_t = g$ in the high-inflation regime, (15) and (27) imply that $\Omega_H^{ss} \leq \Omega^*$ is rewritten as $\Omega_H^{ss} = \rho + \dot{y}_t/y_t + \pi_t = \rho + g + \pi_t < \Omega^* = \rho + \gamma$ and thus $\pi_t < \gamma - g$. However, it violates the assumption of the DNWR, $\pi > \gamma - g$.

Output growth under the low-inflation regime Substituting (17) and (20) into (35) yields

$$\frac{\dot{\Omega}_t}{\Omega_t} = g - \Omega^* + \Omega_t - \eta[\mu - (\gamma - g)]. \tag{38}$$

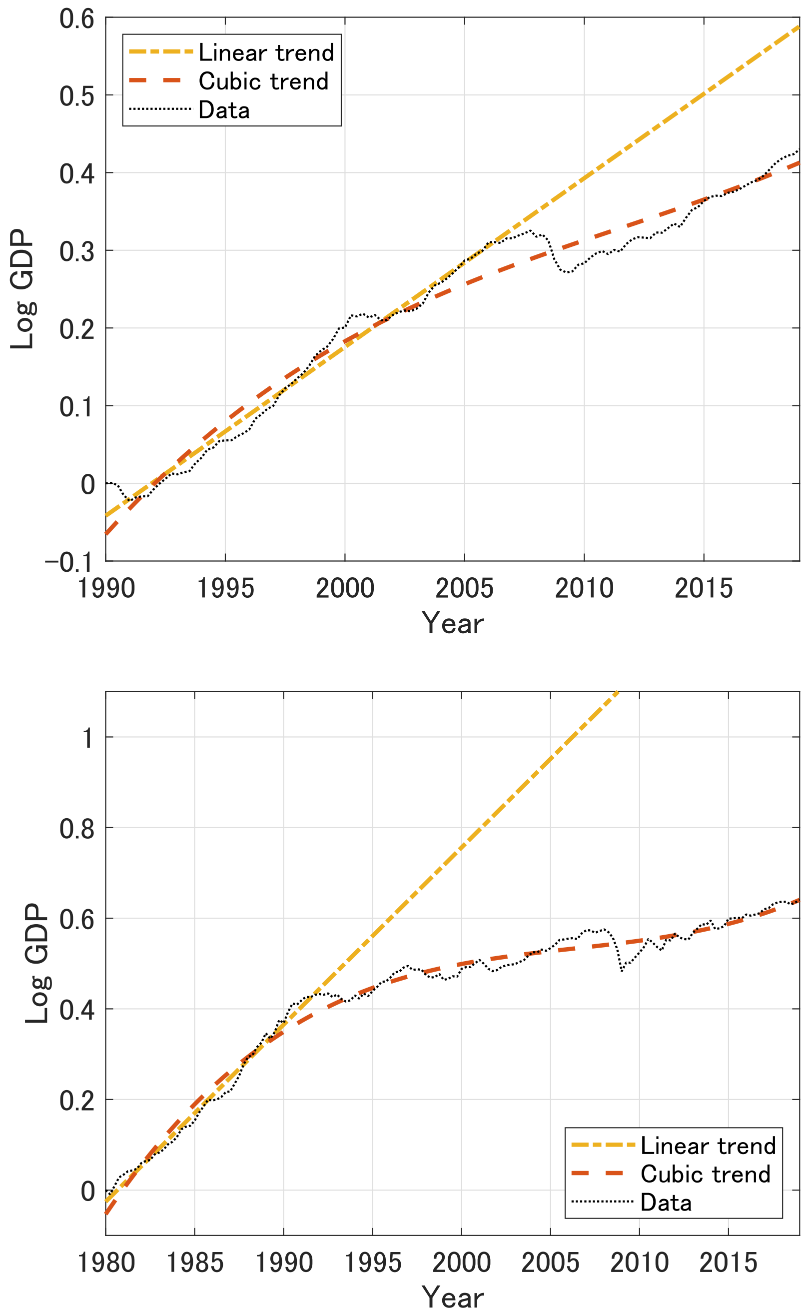
As in (36), (38) is the differential equation for Ω_t with a positive coefficient on Ω_t. Once again, only $\dot{\Omega}_t = 0$ is feasible in equilibrium. Imposing $\dot{\Omega}_t = 0$ on (38), we have the steady-state value under the low-inflation regime with $\beta = 0$:

$$\Omega_L^{ss} = \Omega^* + \eta[\mu - (\gamma - g)] - g \tag{39}$$

The low-inflation regime under $\beta = 0$ is feasible only when $g \geq \eta[\mu - (\gamma - g)]$. To prove this, suppose that $g < \eta[\mu - (\gamma - g)]$ in the low-inflation regime. In this case, (39) implies that $\Omega_L^{ss} > \Omega^*$. Given $\pi_t = \gamma - g$ in the low-inflation regime, (15) and (27) imply that the condition $\Omega_L^{ss} > \Omega^*$ is rewritten as $\Omega_L^{ss} = \rho + \dot{y}_t/y_t + \pi_t = \rho + \dot{y}_t/y_t + \gamma - g > \Omega^* = \rho + \gamma$ and thus $\dot{y}_t/y_t > g$. However, it violates the feasibility because $y_t = \theta_t n_t$ where $n_t \leq 1$.

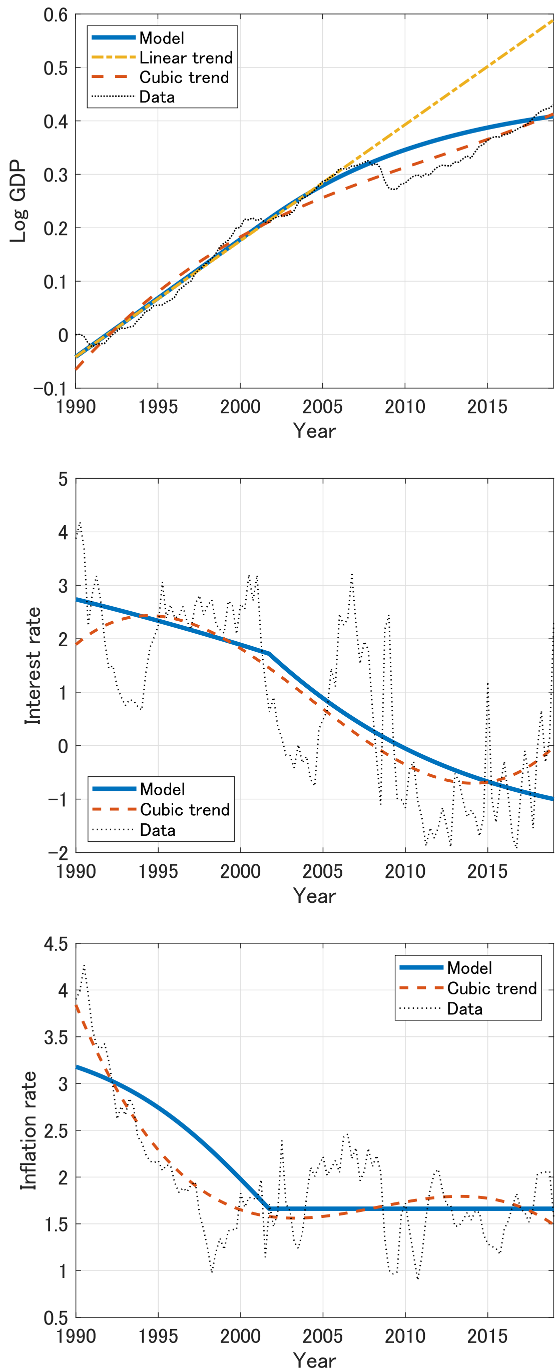
To summarize, there is no endogenous slowdown in output growth when $\beta = 0$. The parameters in the model fully determine the regime. If $g < \eta[\mu - (\gamma - g)]$, the growth rate of output is g and the economy remains in the high-inflation regime. Alternatively, if $g \geq \eta[\mu - (\gamma - g)]$, the growth rate of output is $\eta[\mu - (\gamma - g)]$ and the economy is always in the low-inflation regime from the initial period.

Figure 1: Real GDP per capita and trend



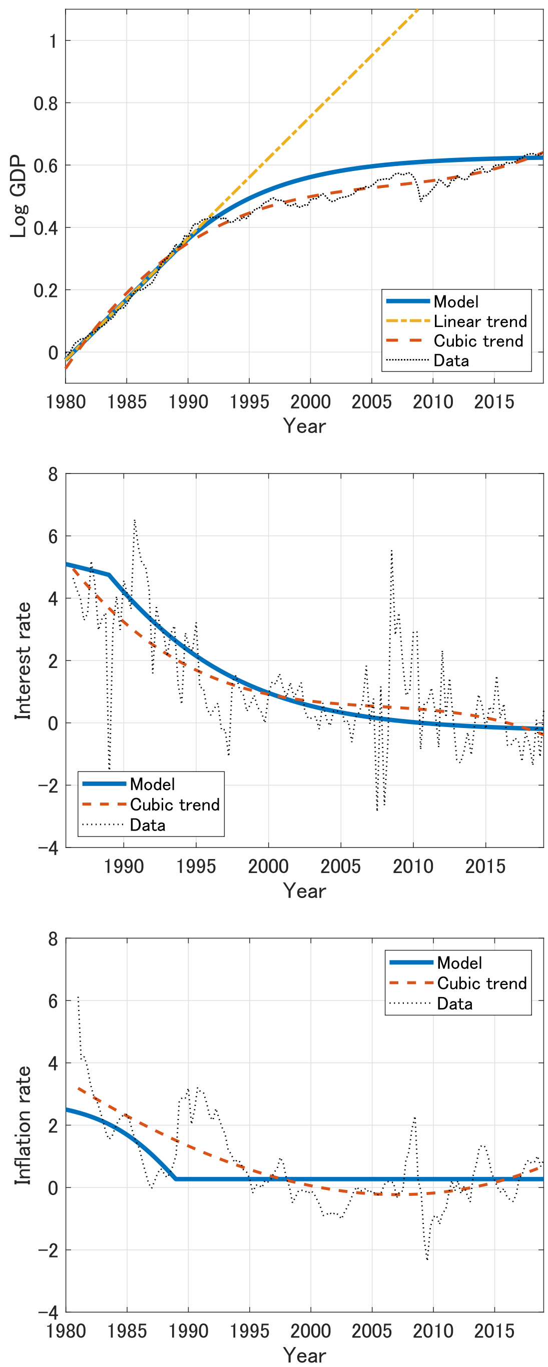
Notes: Each panel of the figure plots the estimated output trend for the log real GDP per capita, along with the actual data. The upper panel plots the US data over 1990:Q1–2019:Q4 while the lower panel shows the Japanese data over 1980:Q1–2019:Q4. In each panel, the dotted line represents actual GDP. The dashed line is the cubic trend of output. The dot-dashed line is the linear trend estimated from the subsample. We use the data over 1990:Q1–2007:Q1 for the US and the data over 1980:Q1–1991:Q1 for Japan. The linear trend after the last period of the subsample, the projected values are reported. In the figure, the estimated trends and the actual GDP are normalized to its initial value (1990:Q1 in the US and 1980:Q1 in Japan).

Figure 2: Simulation results for the US: Output, the real interest rate, and inflation



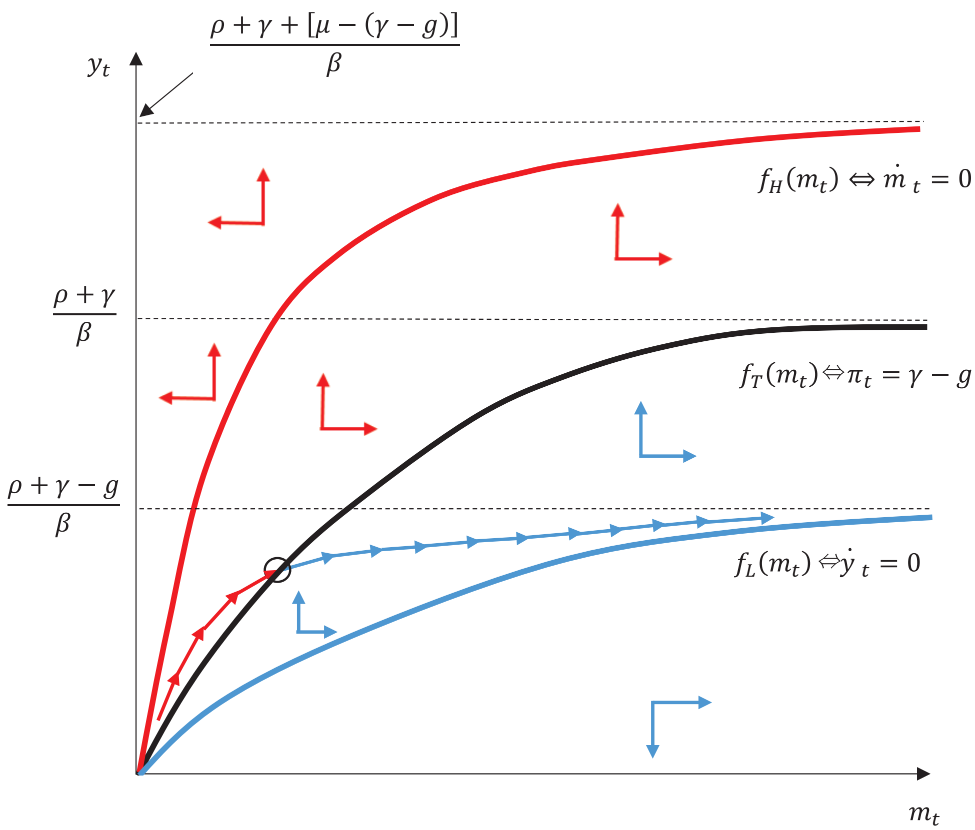
Notes: Each panel of the figure compares the simulated data, the estimated cubic trend, and the actual data in the US. The solid line represents the simulated data, the dashed line represents the estimated cubic trend, and the dotted line represents the actual data. The upper panel is the log real GDP, the middle panel is the real interest rate, and the lower panel is inflation. The dot-dashed line in the upper panel is the linear trend estimated from the data over 1990:Q1–2007:Q1. For the details of the data, see the main text.

Figure 3: Simulation results for Japan: Output, the real interest rate, and inflation



Notes: Each panel of the figure compares the simulated data, the estimated cubic trend, and the actual data in Japan. The dot-dashed line in the upper panel is the linear trend estimated from the data over 1980:Q1–1991:Q1. For the other details, see the notes in Figure 2.

Figure 4: Phase diagram



Notes: The red solid line denoted by $y_t = f_H(m_t)$ represents the locus that achieves $\dot{m}_t = 0$ when the economy is in the high-inflation regime. This $\dot{m}_t = 0$ locus determines the direction of change in m_t in the high-inflation regime. In this regime, $\dot{y}_t > 0$ always holds. The blue solid line denoted by $y_t = f_L(m_t)$ is the locus that achieves $\dot{y}_t = 0$ when the economy is in the low-inflation regime. This $\dot{y}_t = 0$ locus determines the direction of change in y_t in the low-inflation regime. In this regime, $\dot{m}_t > 0$ always holds. The black solid line denoted by $y_t = f_T(m_t)$ points to the locus that achieves $\dot{y}_t/y_t = g$ at the lowest level of inflation $\pi_t = \gamma - g$. If (m_t, y_t) is located above (below) the locus, the economy is in the high- (low-)inflation regime.