# A Technology-Gap Model of Premature Deindustrialization

Ippei Fujiwara
Faculty of Economics, Keio University
Crawford School of Public Policy, ANU

Kiminori Matsuyama
Department of Economics
Northwestern University

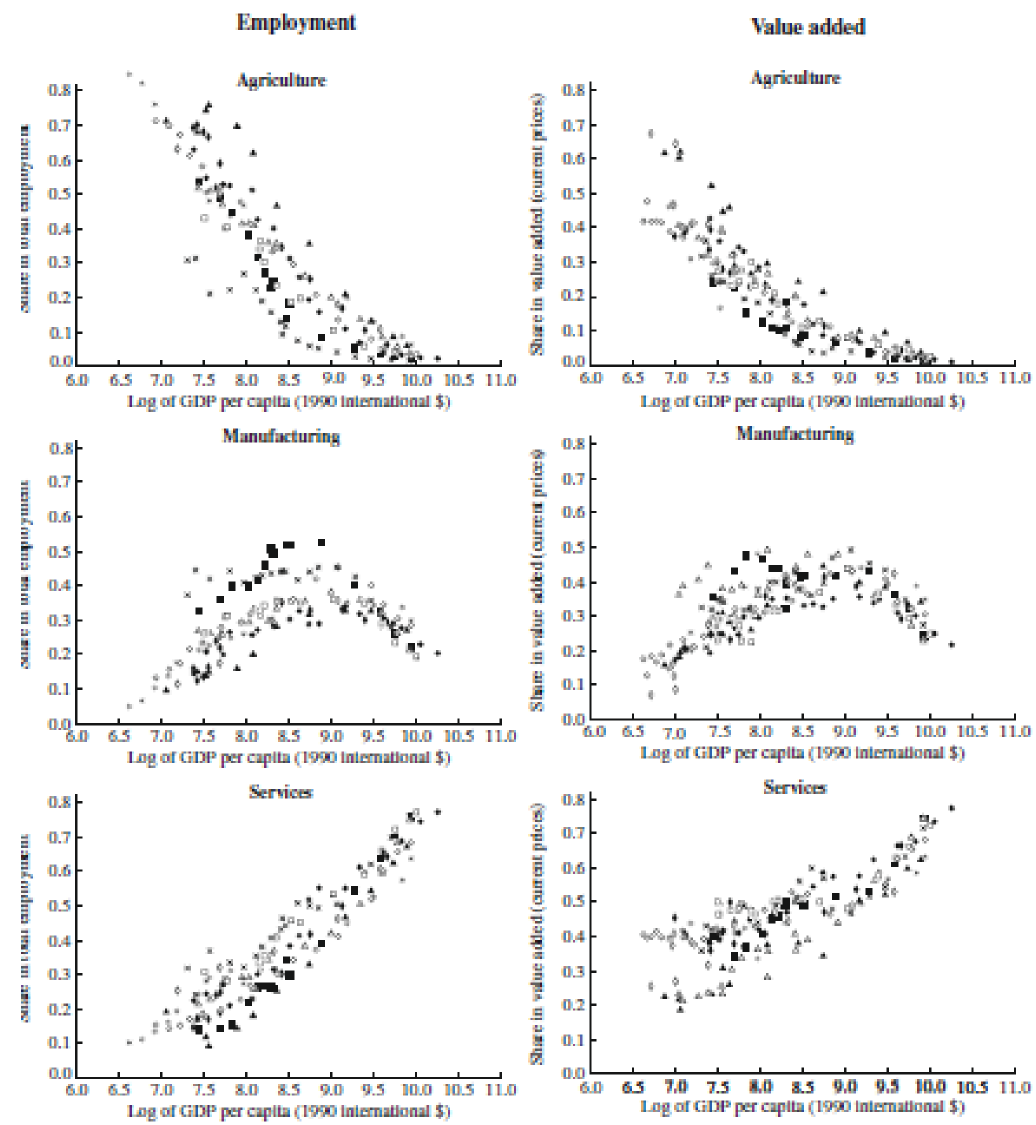
Virtual Seminar in Macroeconomics
University of Tokyo
October 28, 2021

# Introduction

## Structural Change

As per capita income rises, employment or value-added shares

- fall in Agriculture
- rise in Services
- rise and fall in Manufacturing




**From Herrendorf-Rogerson-Valentinyi (2014)**

*Evidence from Long Time Series for the Currently Rich Countries* (Belgium, Finland, France, Japan, Korea, Netherlands, Spain, Sweden, United Kingdom, and United States) 1800-2000

## Premature Deindustrialization (Rodrik, J. Econ Growth, 2016)

Late industrializers reach their M-peak and start deindustrializing

- *later* in time
- at *lower* per capita income levels
- with the *lower* peak M-sector shares,

compared to early industrializers.

By "premature" no welfare connotations intended.

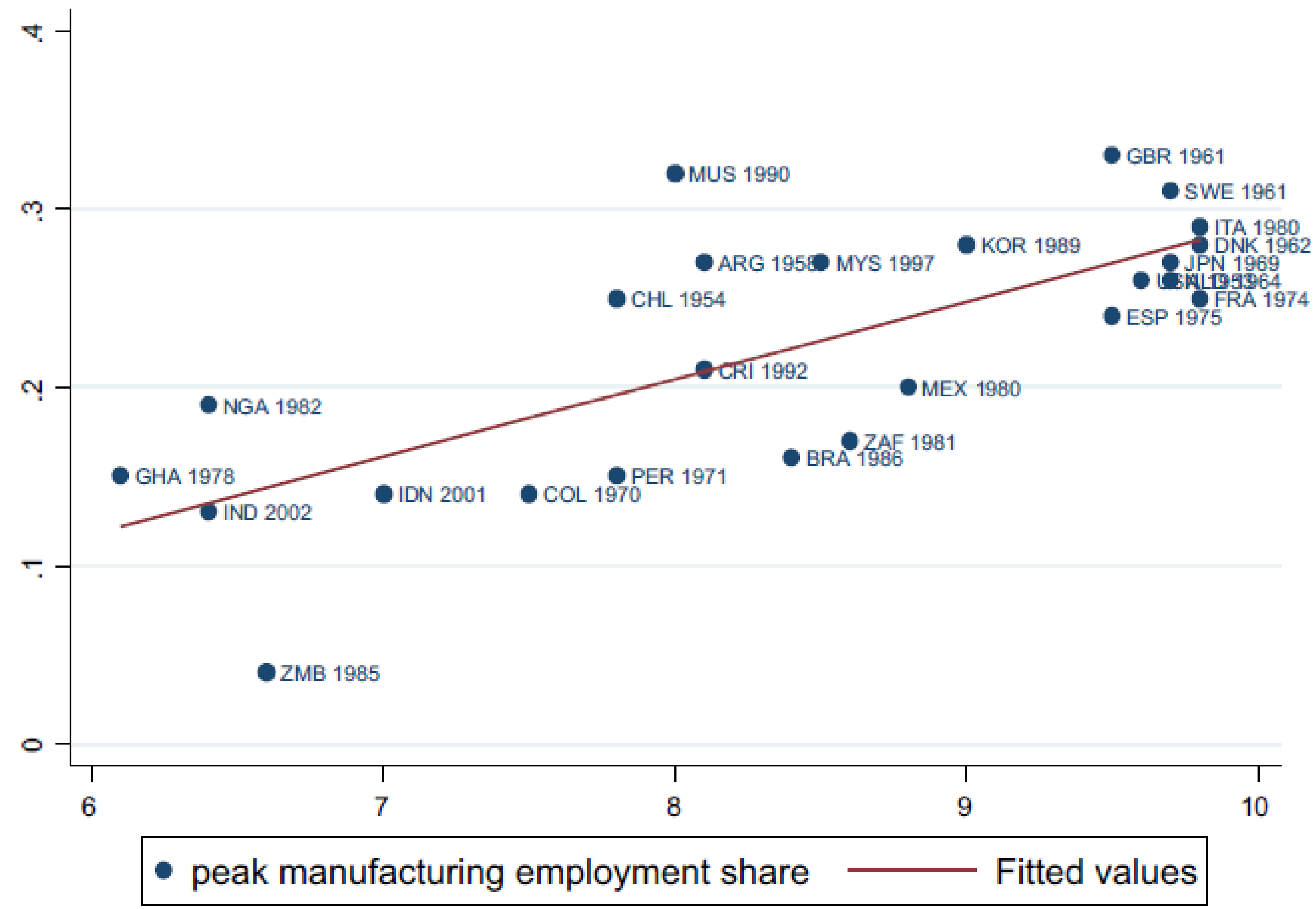



Fig. 5 Income at which manufacturing employment peaks (logs)

## This Paper: A Simple Model of Premature Deindustrialization (PD)

### Key Ingredients

**3 Goods/Sectors**, 1 = (**A**)griculture, 2 = (**M**)anufacturing, 3 = (**S**)ervices, *homothetic CES with gross complements*.

**Frontier Technology**: $\bar{A}_j(t) = \bar{A}_j(0)e^{g_j t}$, with $g_1 > g_2 > g_3 > 0$ ⇒ a decline of A, a rise of S, and a hump-shaped of M in each country through **the Baumol (relative price) effect**, as in Ngai-Pissarides (2007)

**Actual Technology Used**: $\tilde{A}_j(t) = \bar{A}_j(t - \lambda_j)$ due to **Adoption Lags,** $(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda$
$\lambda \geq 0$: **Technology Gap:** country-specific, as in Krugman (1985)
$\theta_j > 0$: sector-specific, unlike Krugman (1985), common across countries

- Countries differ only in one dimension, $\lambda \geq 0$, in their ability to adopt the frontier technologies.
- $\theta_j > 0$ controls how much the technology gap affects the adoption lag and hence productivity in each sector.

$$\tilde{A}_j(t) = \bar{A}_j(t - \lambda_j) = \bar{A}_j(0)e^{-\lambda_j g_j}e^{g_j t} = \bar{A}_j(0)e^{-g_j \theta_j \lambda}e^{g_j t} \implies \frac{\partial}{\partial \lambda}\ln\left(\frac{\tilde{A}_j(t)}{\tilde{A}_k(t)}\right) = -(\theta_j g_j - \theta_k g_k)$$

$\lambda$ has **no "growth" effect, but negative "level" effects** *proportional* to $\theta_j g_j$ in sector-*j*

**Key Mechanisms:**

- $\theta_j$ magnifies the impact of the technology gap on the adoption lag: $\frac{\partial}{\partial \theta_j}\left(\frac{\partial \lambda_j}{\partial \lambda}\right) > 0$ (*supermodularity*)
- $g_j$ magnifies the (negative) impact of the adoption lag on productivity: $\frac{\partial}{\partial g_j}\left(\frac{\partial}{\partial \lambda_j} \ln e^{-\lambda_j g_j}\right) < 0$ *(log-submodularity)*

**Main Results:** PD occurs (i.e., A high-$\lambda$ country reaches its peak later, with lower peak M-share at lower peak time per capita income) under the conditions:

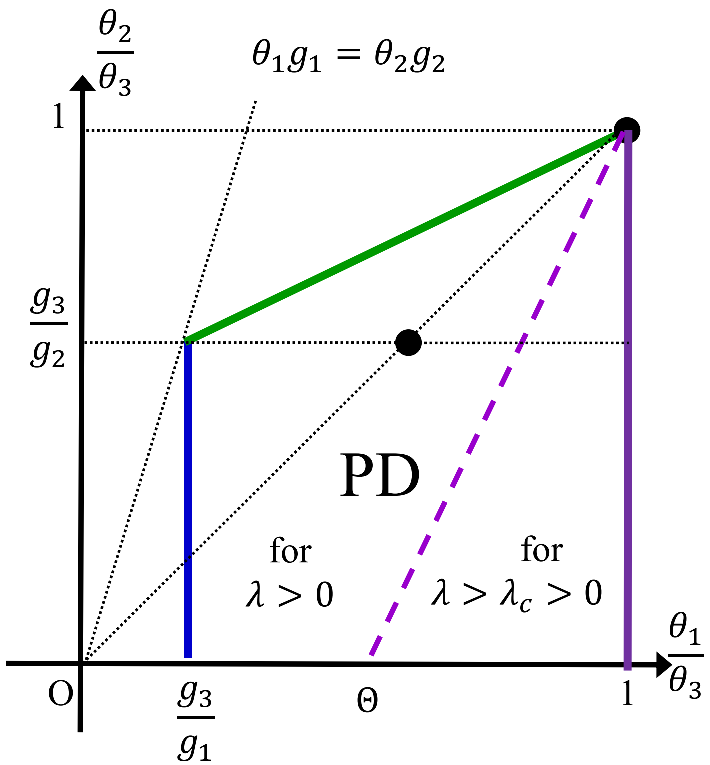
i) $\theta_1 g_1 > \theta_3 g_3$: cross-country productivity difference larger in A than in S.

ii) $\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} > \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}$: technology adoption takes not too long in M.

iii) $\theta_1 < \theta_3$: Technology adoption takes longer in S than in A.

i) & ii) $\Rightarrow \theta_1 g_1 > \theta_2 g_2, \theta_3 g_3$: cross-country productivity difference the largest in A.

ii) & iii) $\Rightarrow \theta_1, \theta_2 < \theta_3$: Technology adoption takes longest in S.




**Two Extensions:** We show these results are robust against introducing

- **The Engel (income) effect** (through nonhomothetic CES)

But, the Engel effect alone could not generate PD without counterfactual implications.

- **Catching-up** (with an exponential decay in $\lambda$), unless the catching up speed is too large.

**Literature Review.** Herrendorf-Rogerson-Valentinyi (14) for a broad survey on structural change

**Related to The Baseline Model**
*Premature Deindustrialization,* **Rodrik (16)**
*The Baumol Effect:* Baumol (67), **Ngai-Pissarides (07),** Nordhaus (08)
*Sectoral implications of cross-country heterogeneity in technology development*

- *Log-supermodularity:* **Krugman** (85), Matsuyama (05), Costinot (09), Costinot-Vogel (15)
- *Productivity difference across countries the largest in A:* Caselli (05), Gollin et.al. (14, AERP&P)
- *Small adoption lags in M;* Rodrik (2013)

**Related to Two Extensions**
*The Engel Effect (Nonhomotheticity);* Murphy et.al. (89), Matsuyama (92,02), Kongsamut et.al. (01), Foellmi-Zweimueller (08), Buera-Kaboski (09,12), Boppart (14), **Comin-Lashkari-Mestieri (21),** Matsuyama (19), Lewis et.al. (21), Bohr-Mestieri-Yavuz (21)
*Catching-Up/Technology Diffusion:* Acemoglu (08), Comin-Mestieri (18)

**The Issues We Abstract From**
*Sector-level productivity growth rate differences across countries:* Huneeus-Rogerson (20)
*Open economy implications:* Matsuyama (92,09), Uy-Yi-Zhang (13), Sposi-Yi-Zhang (19), Fujiwara-Matsuyama (WinP)
*Endogenous growth, externalities,* Matsuyama(92),
*Sectoral wedges/misallocation:* Caselli(05), Gollin et.al. (14 QJE) and many others
*Nominal vs. Real expenditure; Employment vs. Value Added shares; Compatibility with aggregate balance growth, investment vs consumption, sector-specific factor intensities, skill premium, home production, productivity slowdown, etc.*

# Structural Change, the Baumol Effect, and Adoption Lags

# Three Complementary Goods/Competitive Sectors, $j = 1, 2, 3$

Sector-1 = **(A)**griculture, Sector-2 = **(M)**anufacturing, Sector-3 = **(S)**ervices.

**Demand System:** $L$ Identical HH, each supplies 1 unit of mobile labor at $w$; $\kappa_j$ units of factor specific to $j$ at $\rho_j$.

**Budget Constraint:** $$\sum_{j=1}^{3} p_j c_j \leq E \equiv w + \sum_{j=1}^{3} \rho_j \kappa_j$$

**CES Preferences:** $$U(\mathbf{c}) = \left[\sum_{j=1}^{3} (\beta_j)^{\frac{1}{\sigma}} (c_j)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

with $\beta_j > 0$ and $0 < \sigma < 1$ (gross complementarity)

**Expenditure Shares:** $$m_j \equiv \frac{p_j c_j}{E} = \frac{\beta_j (p_j)^{1-\sigma}}{\sum_{k=1}^{3} \beta_k (p_k)^{1-\sigma}} = \beta_j \left(\frac{E/p_j}{U}\right)^{\sigma-1}$$

# Three Competitive Sectors: Production

**Cobb-Douglas**

$$Y_j = A_j\left(\kappa_j L\right)^{\alpha}\left(L_j\right)^{1-\alpha}$$

$A_j > 0$: the TFP of sector-$j$; $\alpha \in [0,1)$ the share of specific factor.

**Employment Share**

$$s_j \equiv \frac{L_j}{L}; \qquad \sum_{j=1}^{3} s_j = 1$$

**Output per worker**
**Output per capita**

$$\frac{Y_j}{L_j} = \tilde{A}_j\left(s_j\right)^{-\alpha}; \qquad \frac{Y_j}{L} = \tilde{A}_j\left(s_j\right)^{1-\alpha}$$

where $\tilde{A}_j \equiv A_j\left(\kappa_j\right)^{\alpha}$.

With Cobb-Douglas, $wL_j = (1-\alpha)p_jY_j$, implying the employment shares equal to

**Value-Added Shares**

$$\frac{p_jY_j}{EL} = \frac{p_jY_j}{\sum_{k=1}^{3} p_kY_k} = s_j = \frac{L_j}{L}$$

**Equilibrium:** The expenditure shares are equal to the employment and value-added shares.

$$m_j = \frac{p_j Y_j}{EL} = s_j$$

which lead to

**Equilibrium Shares**

$$s_j = \frac{\left[\beta_j^{\frac{1}{\sigma-1}} \tilde{A}_j\right]^{-a}}{\sum_{k=1}^{3} \left[\beta_k^{\frac{1}{\sigma-1}} \tilde{A}_k\right]^{-a}}$$

**Per Capita Income**

$$U = \left\{\sum_{k=1}^{3} \left[\beta_k^{\frac{1}{\sigma-1}} \tilde{A}_k\right]^{-a}\right\}^{-\frac{1}{a}}$$

where

$$a \equiv \frac{1-\sigma}{1-\alpha(1-\sigma)} = -\frac{\partial \log\left(s_j/s_k\right)}{\partial \log\left(\tilde{A}_j/\tilde{A}_k\right)} > 0.$$

This captures how much relatively high productivity in a sector contributes to its relatively low equilibrium share.

**Productivity Growth:** $\{\tilde{A}_j(t)\}_{j=1}^{3}$ change according to:

$$\tilde{A}_j(t) = \bar{A}_j(t-\lambda_j) = \bar{A}_j(0)e^{g_j(t-\lambda_j)} = \bar{A}_j(0)e^{-\lambda_j g_j}e^{g_j t}$$

$\bar{A}_j(t) = \bar{A}_j(0)e^{g_j t}$: **Frontier Technology** in $j$, with a constant **growth rate** $g_j > 0$.
$\tilde{A}_j(t) = \bar{A}_j(t-\lambda_j)$; $\lambda_j$ = **Adoption Lag** in $j$.

- $g_j$ and $\lambda_j$ are sector-specific.
- $\lambda_j$ has **no "growth" effect.**
- $\lambda_j$ has **the "level" effect,** $e^{-\lambda_j g_j}$, which is decreasing in $\lambda_j$ and the effect is proportional to $g_j$

Key: Log-submodularity, $\frac{\partial}{\partial g_j}\left(\frac{\partial}{\partial \lambda_j}\ln e^{-\lambda_j g_j}\right) < 0$: $g_j$ magnifies the negative effect of the adoption lag on productivity

A large adoption lag would not matter much in a sector with slow productivity growth.
Even a small adoption lag would matter a lot in a sector with fast productivity growth.

$$U(t) = \left\{\sum_{k=1}^{3}\left[\beta_k^{\frac{1}{\sigma-1}}\tilde{A}_k\right]^{-a}\right\}^{-\frac{1}{a}} = \left\{\sum_{k=1}^{3}\tilde{\beta}_k e^{-a g_k(t-\lambda_k)}\right\}^{-\frac{1}{a}}, \quad where\ \tilde{\beta}_k \equiv \left(\beta_k^{\frac{1}{\sigma-1}}\bar{A}_k(0)\right)^{-a} = \left(\frac{\beta_k^{\frac{1}{1-\sigma}}}{\bar{A}_k(0)}\right)^{a} > 0.$$

Longer adoption lags would shift down the time path of $U(t)$.

**Relative Prices:**

$$\left(\frac{p_j(t)}{p_k(t)}\right)^{1-\alpha(1-\sigma)} = \left(\frac{\beta_j}{\beta_k}\right)^{\alpha} \frac{\bar{A}_k(0)}{\bar{A}_j(0)} e^{(\lambda_j g_j - \lambda_k g_k)} e^{(g_k - g_j)t}$$

**Relative Growth Effect:** $p_j(t)/p_k(t)$ is de(in)creasing over time if $g_j > (<) g_k$.
**Relative Level Effect:** A higher $\lambda_j g_j - \lambda_k g_k$ raises $p_j(t)/p_k(t)$ at any point in time.
*Note*: For a fixed $\lambda_j$, a higher $g_j$ makes the relative price of *j* higher (though declining faster).

**Relative Sector Shares:**

$$\frac{s_j(t)}{s_k(t)} = \left[\frac{\beta_j^{\frac{1}{\sigma-1}} \bar{A}_j(t-\lambda_j)}{\beta_k^{\frac{1}{\sigma-1}} \bar{A}_k(t-\lambda_k)}\right]^{-a} = \frac{\tilde{\beta}_j}{\tilde{\beta}_k} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t}$$

**Relative Growth Effect:** $s_j(t)/s_k(t)$ is de(in)creasing over time if $g_j > (<) g_k$.
Shift from faster growing sectors to slower growing sectors over time.
**Relative Level Effect:** A higher $\lambda_j g_j - \lambda_k g_k$ raises $s_j(t)/s_k(t)$ at any point in time.
*Note*: For a fixed $\lambda_j$, a higher $g_j$ makes the relative share of *j* higher (though declining faster).

**Structural Change with the Baumol (Relative Price) Effect:** Let $g_1 > g_2 > g_3 > 0$

**Decline of Agriculture:** $s_1(t)$ is decreasing in $t$, because

$$\frac{1}{s_1(t)} - 1 = \frac{s_2(t)}{s_1(t)} + \frac{s_3(t)}{s_1(t)} = \left[\frac{\tilde{\beta}_2}{\tilde{\beta}_1} e^{a(\lambda_2 g_2 - \lambda_1 g_1)}\right] e^{a(g_1 - g_2)t} + \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1} e^{a(\lambda_3 g_3 - \lambda_1 g_1)}\right] e^{a(g_1 - g_3)t}$$

**Rise of Services:** $s_3(t)$ is increasing in $t$, because

$$\frac{1}{s_3(t)} - 1 = \frac{s_1(t)}{s_3(t)} + \frac{s_2(t)}{s_3(t)} = \left[\frac{\tilde{\beta}_1}{\tilde{\beta}_3} e^{a(\lambda_1 g_1 - \lambda_3 g_3)}\right] e^{-a(g_1 - g_3)t} + \left[\frac{\tilde{\beta}_2}{\tilde{\beta}_3} e^{a(\lambda_2 g_2 - \lambda_3 g_3)}\right] e^{-a(g_2 - g_3)t}$$

**Rise and Fall of Manufacturing:** $s_2(t)$ is hump-shaped in $t$, because

$$\frac{1}{s_2(t)} - 1 = \frac{s_1(t)}{s_2(t)} + \frac{s_3(t)}{s_2(t)} = \left[\frac{\tilde{\beta}_1}{\tilde{\beta}_2} e^{a(\lambda_1 g_1 - \lambda_2 g_2)}\right] e^{-a(g_1 - g_2)t} + \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_2} e^{a(\lambda_3 g_3 - \lambda_2 g_2)}\right] e^{a(g_2 - g_3)t},$$

where $\tilde{\beta}_k \equiv \left(\beta_k^{\frac{1}{\sigma-1}} \bar{A}_k(0)\right)^{-a} = \left(\frac{\beta_k^{\frac{1}{1-\sigma}}}{\bar{A}_k(0)}\right)^{a} > 0.$

$g_1 > g_2$ pushes labor out of A to M; $g_2 > g_3$ pulls labor out of M to S.

**Manufacturing Peak:** "^" indicates the peak. From $s_2'(\hat{t}) = 0$,

$$\hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3} + \hat{t}_0, \qquad \text{where } \frac{\bar{A}_1(\hat{t}_0)}{\bar{A}_3(\hat{t}_0)} = \frac{\bar{A}_1(0)}{\bar{A}_3(0)} e^{(g_1 - g_3)\hat{t}_0} \equiv \left(\frac{\beta_1}{\beta_3}\right)^{\frac{1}{1-\sigma}} \left(\frac{g_1 - g_2}{g_2 - g_3}\right)^{\frac{1}{a}}$$

**Two Normalizations:**

$$\frac{\tilde{\beta}_1}{\tilde{\beta}_3} \equiv \left\{\left(\frac{\beta_1}{\beta_3}\right)^{\frac{1}{\sigma-1}} \frac{\bar{A}_1(0)}{\bar{A}_3(0)}\right\}^{-a} = \frac{g_2 - g_3}{g_1 - g_2} \Leftrightarrow \hat{t}_0 = 0$$

The calendar time is reset so that the country with $\lambda_j = 0$ reaches its M-peak at $\hat{t} = 0$.

$$\tilde{\beta}_1 + \tilde{\beta}_2 + \tilde{\beta}_3 = 1$$

The country with $\lambda_j = 0$ reaches its M-peak at $U(\hat{t}) = 1$.

Then,

**Peak Time**

$$\hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3}.$$

**Peak M-Share**

$$\frac{1}{s_2(\hat{t})} = 1 + \left(\frac{1}{\tilde{\beta}_2} - 1\right) e^{\frac{a(g_1 - g_2)(g_2 - g_3)}{(g_1 - g_3)}\left(\frac{\lambda_1 g_1 - \lambda_2 g_2}{g_1 - g_2} - \frac{\lambda_2 g_2 - \lambda_3 g_3}{g_2 - g_3}\right)}$$

**Peak Time Per Capita Income**

$$U(\hat{t}) = \left\{(1 - \tilde{\beta}_2) e^{-a g_1 g_3\left(\frac{\lambda_1 - \lambda_3}{g_1 - g_3}\right)} + \tilde{\beta}_2 e^{-a g_2\left(\frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3} - \lambda_2\right)}\right\}^{-\frac{1}{a}}$$

# Technology Gaps and Premature Deindustrialization

Consider the world with many countries with

$$(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda$$

$\lambda \geq 0$: **Technology Gap,** Country-specific

$\theta_j > 0$: Sector-specific, capturing the inherent difficulty of technology adoption, common across countries

- **Countries differ only in one dimension**, $\lambda$, in their ability to adopt the frontier technologies.
- $\theta_j > 0$ determines how the technology gap affects the adoption lag in that sector.

$$\frac{\tilde{A}_j(t)}{\tilde{A}_k(t)} = \frac{\bar{A}_j(0)}{\bar{A}_k(0)} e^{-(\theta_j g_j - \theta_k g_k)\lambda} e^{(g_j - g_k)t} \Rightarrow \frac{\partial}{\partial \lambda} \ln\left(\frac{\tilde{A}_j(t)}{\tilde{A}_k(t)}\right) = -(\theta_j g_j - \theta_k g_k)$$

Cross-country productivity difference is larger in sector-$j$ than in sector-$k$ if $\theta_j g_j > \theta_k g_k$.

**Peak Time**

$$\hat{t} = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3}\lambda.$$

**Peak M-Share**

$$\frac{1}{s_2(\hat{t})} = 1 + \left(\frac{1}{\tilde{\beta}_2} - 1\right) e^{\frac{(g_1 - g_2)(g_2 - g_3)}{(g_1 - g_3)}\left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}\right)a\lambda}$$

**Peak Time Per Capita Income**

$$U(\hat{t}) = \left\{(1 - \tilde{\beta}_2) e^{-g_1 g_3\left(\frac{\theta_1 - \theta_3}{g_1 - g_3}\right)a\lambda} + \tilde{\beta}_2 e^{-g_2\left(\frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} - \theta_2\right)a\lambda}\right\}^{-\frac{1}{a}}$$

**Figure 1: Conditions for Premature Deindustrialization (PD) only with the Baumol (Relative Price) Effect**

$$\frac{\partial \hat{t}}{\partial \lambda} > 0 \text{ for all } \lambda > 0 \Leftrightarrow \theta_1 g_1 > \theta_3 g_3.$$

With $\theta_1 g_1 > \theta_3 g_3$, the price of A is high and the price of S is low relative to M in a high-$\lambda$ country, which delays structural change.

$$\frac{\partial s_2(\hat{t})}{\partial \lambda} < 0 \text{ for all } \lambda > 0 \Leftrightarrow \frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} > \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}$$

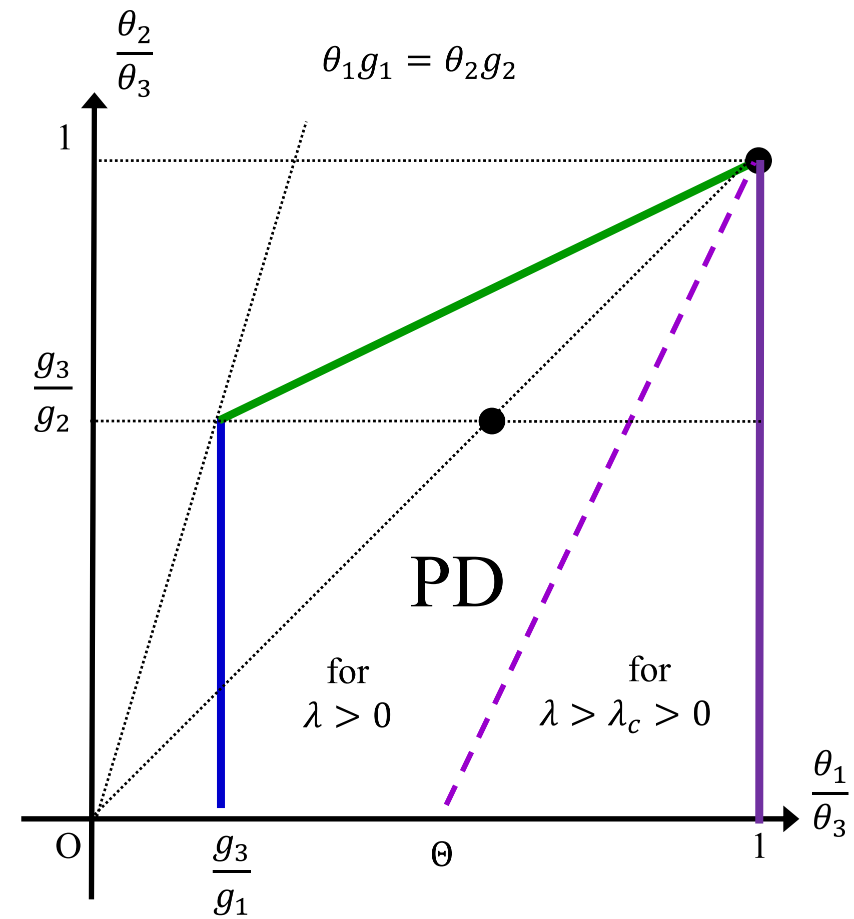
With a low $\theta_2$, which has no effect on $\hat{t}$, the price of M is low relative to both A & S in a high-$\lambda$ country, which keeps the M-share low.

Under the above condition,

$$\frac{\partial U(\hat{t})}{\partial \lambda} < 0 \text{ for a sufficiently large } \lambda \Leftrightarrow \theta_1 < \theta_3$$

$$\frac{\partial U(\hat{t})}{\partial \lambda} < 0 \text{ for all } \lambda > 0 \Leftrightarrow 0 < (1 - \Theta)\left(1 - \frac{\theta_2}{\theta_3}\right) < \left(1 - \frac{\theta_1}{\theta_3}\right),$$

where $g_3/g_1 < \Theta < 1$.




These conditions jointly imply $\theta_1 g_1 > \theta_2 g_2, \theta_3 g_3$ (productivity differences the largest in A) and $\theta_1, \theta_2 < \theta_3$ (adoption lag the longest in S).

## Some Examples

## Example 1: No Premature Deindustrialization (PD)

Uniform Adoption Lags, as in Krugman (1985)

$$\theta_1 = \theta_2 = \theta_3 = 1 \iff \lambda_1 = \lambda_2 = \lambda_3 = \lambda > 0$$

$$\Longrightarrow \hat{t} = \lambda; \quad s_2(\hat{t}) = \tilde{\beta}_2; \quad U(\hat{t}) = 1$$

- The country's technology gap causes a delay in the peak time, $\hat{t}$, by $\lambda > 0$.
- The peak M-share & per capita income at the peak time unaffected.

Each country follows exactly the same development path of early industrializers *with a delay*. No PD!!

## Example 2: Premature Deindustrialization (PD)

$$\frac{g_3}{g_1} < \frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} \equiv \theta < 1$$

1st Inequality $\Longrightarrow \theta_1 g_1 > \theta_3 g_3$ Equality: $\Longrightarrow \theta_1 g_1 > \theta_2 g_2$
Cross-country productivity differences the largest in A. e.g., Caselli (2005), Gollin et.al. (2014, AERP&P)

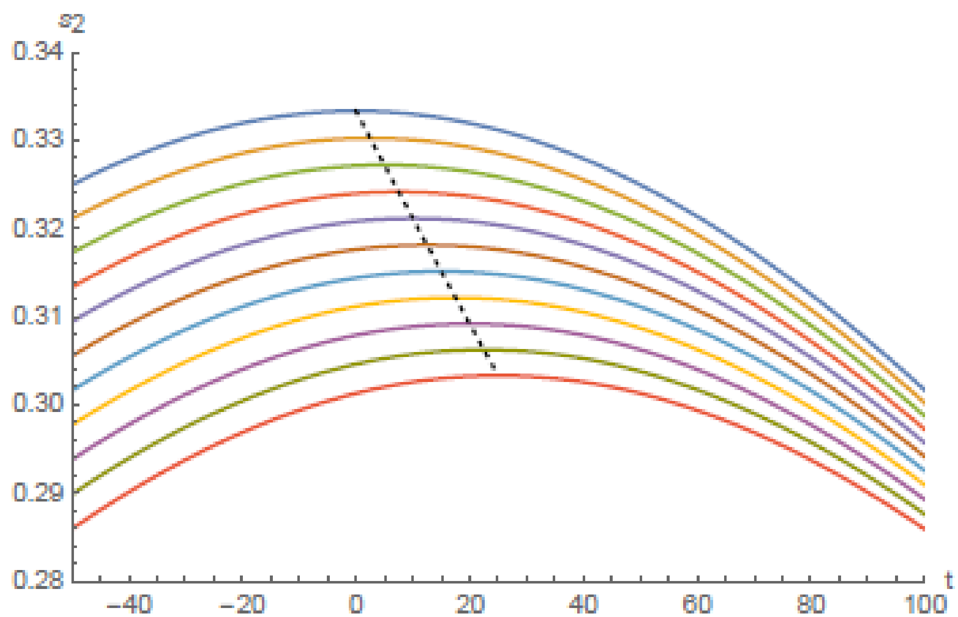
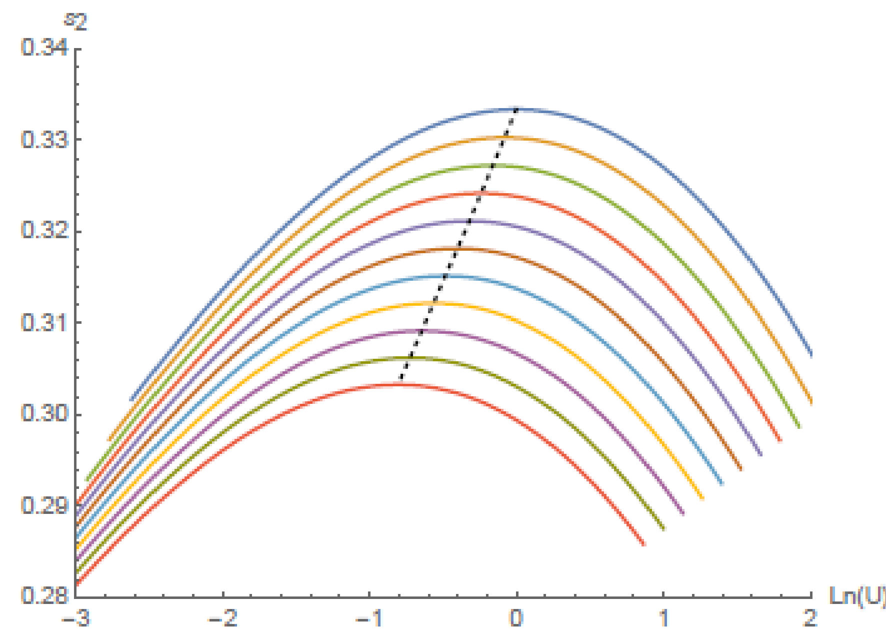
2nd Inequality $\Longrightarrow \theta_1, \theta_2 < \theta_3$
Technology adoption the hardest in S (due to its intangible nature of technology).

**Peak Time** $$\hat{t} = \left(\frac{\theta g_1 - g_3}{g_1 - g_3}\right)\theta_3\lambda \quad \Longrightarrow \quad \frac{\partial \hat{t}}{\partial \lambda} > 0$$

**Peak M-Share** $$\frac{1}{s_2(\hat{t})} = 1 + \left(\frac{1}{\tilde{\beta}_2} - 1\right) e^{\frac{a(g_1 - g_2)g_3}{g_1 - g_3}(1-\theta)\theta_3\lambda} \quad \Longrightarrow \quad \frac{\partial s_2(\hat{t})}{\partial \lambda} < 0$$

**Peak Time Per Capita Income** $$U(\hat{t}) = \left\{(1 - \tilde{\beta}_2) e^{\frac{g_1 g_3}{g_1 - g_3}(1-\theta)\theta_3 a\lambda} + \tilde{\beta}_2 e^{\frac{g_2 g_3}{g_1 - g_3}(1-\theta)\theta_3 a\lambda}\right\}^{-\frac{1}{a}} \quad \Longrightarrow \quad \frac{\partial U(\hat{t})}{\partial \lambda} < 0$$

**Example 2 Continued:** Numerical Illustrations. In all cases, we use
$g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$; $\alpha = 1/3$, and $\sigma = 0.6$ (hence $a = 6/13$).
$\tilde{\beta}_j = 1/3$ for $j = 1,2,3 \Rightarrow s_2(\hat{t}) = \tilde{\beta}_2 = 1/3$; $\hat{U}(\hat{t}) = 1$; $\hat{t} = 0$ for $\lambda = 0$.

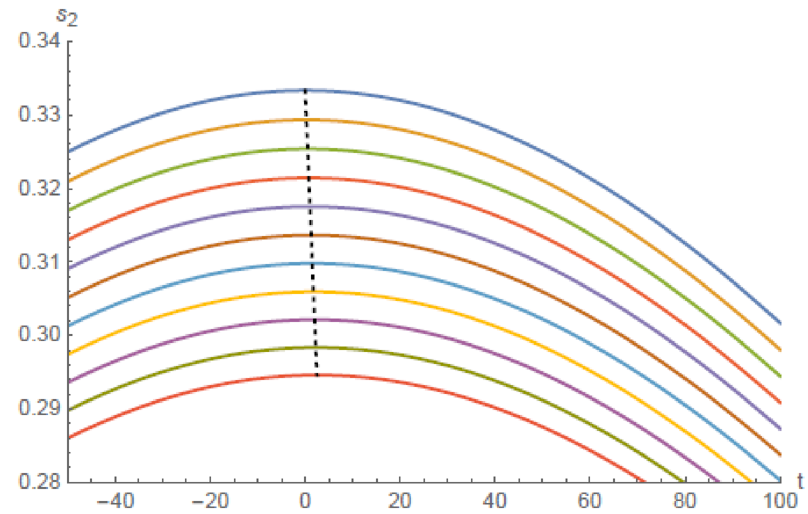
<table>
<tr><td></td><td>$\left(t, s_2(t)\right)$</td><td>$\left(\ln U(t), s_2(t)\right)$</td></tr>
<tr><td>$$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.5$$<br>$$\Longrightarrow \theta_1 g_1 > \theta_2 g_2 = \theta_3 g_3$$<br>Cross-country productivity differences are the same in M & in S in this case.</td><td>
</td><td>
</td></tr>
<tr><td colspan="3">λ=0 λ=10 λ=20 λ=30 λ=40 λ=50 λ=60 λ=70 λ=80 λ=90 λ=100</td></tr>
</table>

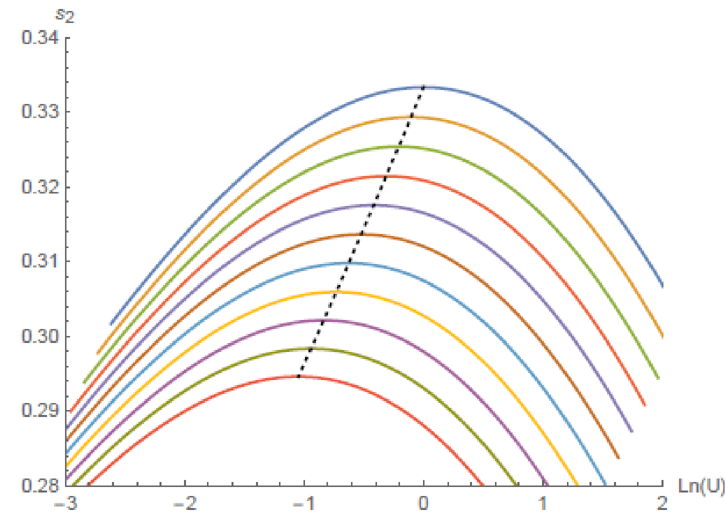
## **Example 2 Continued:** Numerical Illustrations

$$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.35$$

$$\Longrightarrow \theta_1 g_1 > \theta_3 g_3 > \theta_2 g_2$$

Cross-country productivity differences the smallest in M.

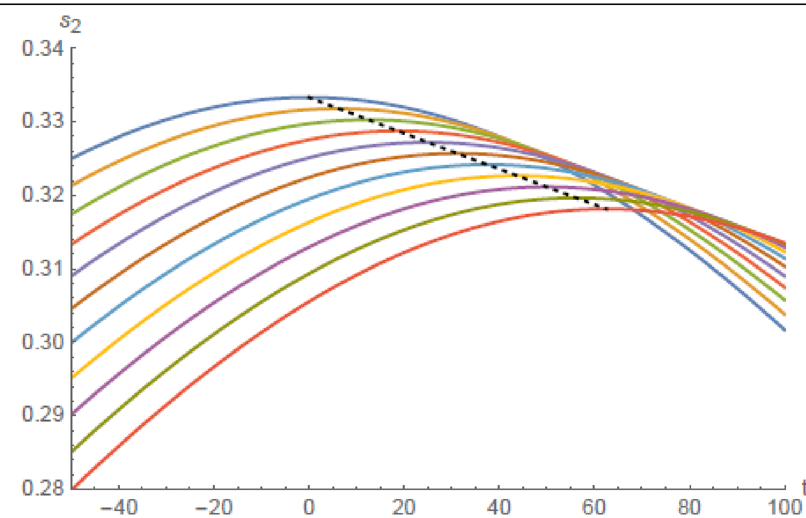


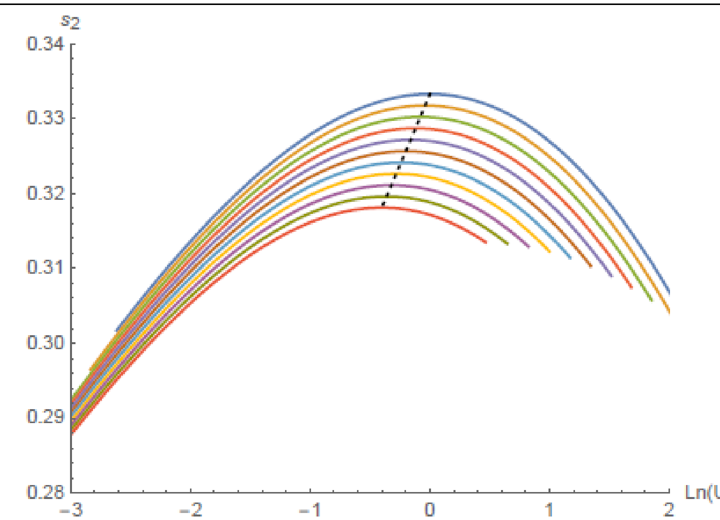




$$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.75$$

$$\Longrightarrow \theta_1 g_1 > \theta_2 g_2 > \theta_3 g_3$$

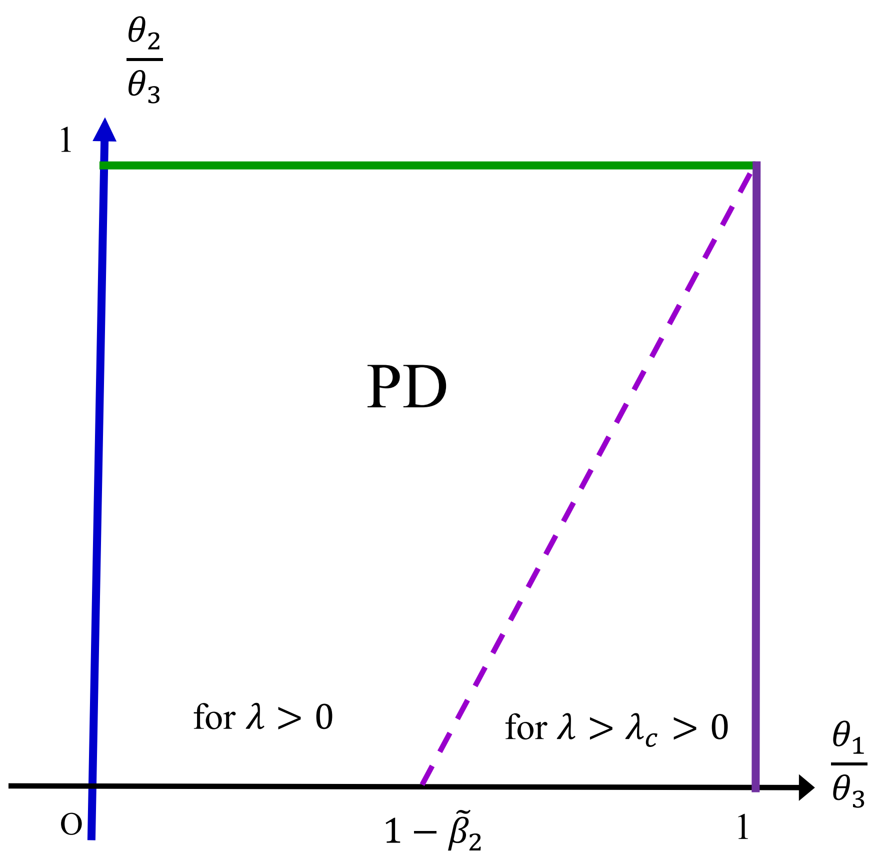
Cross-country productivity differences the smallest in S.



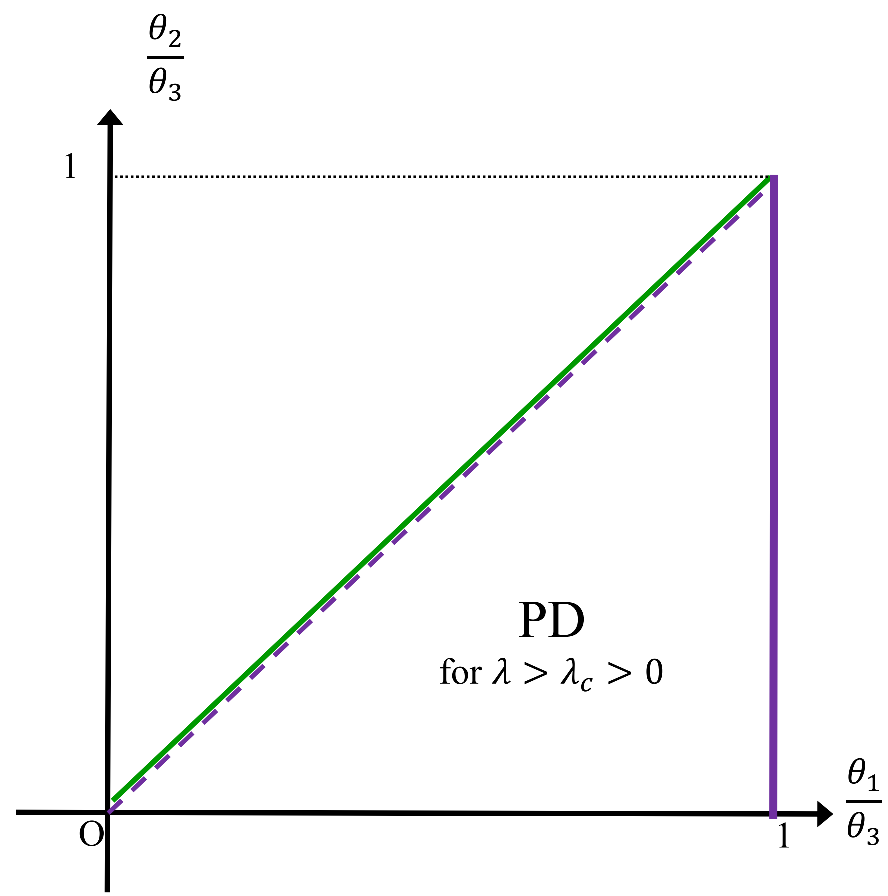



## Some Limit Cases

For $g_3/g_1 \to 0; g_3/g_2 \to 1 \Longrightarrow \Theta \to 1 - \tilde{\beta}_2$




For $g_3/g_1 \to 0;\ g_3/g_2 \to 0 \Longrightarrow \Theta \to 0$



# Introducing the Engel Effect

**The Engel Law through Isoelastic Nonhomothetic CES;** Comin-Lashkari-Mestieri (2021), Matsuyama (2019)

$$\left[\sum_{j=1}^{3}(\beta_j)^{\frac{1}{\sigma}}\left(\frac{c_j}{U^{\varepsilon_j}}\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \equiv 1$$

Normalize $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3$; with $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$, we go back to the standard homothetic CES.
With $\sigma < 1$, $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3$ ⇒ **the income elasticity the lowest in A and the highest in S.**

By maximizing $U$ subject to $\sum_{j=1}^{3} p_j c_j \le E$,

**Expendíture Shares** $$m_j \equiv \frac{p_j c_j}{E} = \frac{\beta_j\left(U^{\varepsilon_j}p_j\right)^{1-\sigma}}{\sum_{k=1}^{3}\beta_k(U^{\varepsilon_k}p_k)^{1-\sigma}} = \beta_j\left(\frac{U^{\varepsilon_j}p_j}{E}\right)^{1-\sigma} \Longrightarrow \frac{m_j}{m_k} = \frac{\beta_j}{\beta_k}\left(\frac{p_j}{p_k}U^{\varepsilon_j-\varepsilon_k}\right)^{1-\sigma}$$

**Indirect Utility Function:**

$$\left[\sum_{j=1}^{3}\beta_j\left(\frac{U^{\varepsilon_j}p_j}{E}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \equiv 1$$

**Cost-of-Living Index:**

$$\left[\sum_{j=1}^{3}\beta_j\left(\frac{U^{\varepsilon_j-1}p_j}{P}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \equiv 1 \Leftrightarrow U \equiv \frac{E}{P}$$

**Income Elasticity:**

$$\eta_j \equiv \frac{\partial \ln c_j}{\partial \ln(U)} = 1 + \frac{\partial \ln m_j}{\partial \ln(E/P)} = 1 + (1-\sigma)\left\{\varepsilon_j - \sum_{k=1}^{3} m_k\varepsilon_k\right\}$$

**Structural Change with the Engel (Income) Effect:** Let $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$.
Then, even with constant relative prices,

**Decline of Agriculture:** $s_1(t) = m_1(t)$ is decreasing in $U(t)$, because

$$\frac{1}{s_1(t)} - 1 = \frac{m_2(t)}{m_1(t)} + \frac{m_3(t)}{m_1(t)} = \frac{\beta_2}{\beta_1}\left(\frac{p_2}{p_1}U(t)^{\varepsilon_2-\varepsilon_1}\right)^{1-\sigma} + \frac{\beta_3}{\beta_1}\left(\frac{p_3}{p_1}U(t)^{\varepsilon_3-\varepsilon_1}\right)^{1-\sigma}$$

**Rise of Services:** $s_3(t) = m_3(t)$ is increasing in $U(t)$, because

$$\frac{1}{s_3(t)} - 1 = \frac{m_1(t)}{m_3(t)} + \frac{m_2(t)}{m_3(t)} = \frac{\beta_1}{\beta_3}\left(\frac{p_1}{p_3}U(t)^{\varepsilon_1-\varepsilon_3}\right)^{1-\sigma} + \frac{\beta_2}{\beta_3}\left(\frac{p_2}{p_3}U(t)^{\varepsilon_2-\varepsilon_3}\right)^{1-\sigma}$$

**Rise and Fall of Manufacturing:** $s_2(t) = m_2(t)$is hump-shaped in $U(t)$, because

$$\frac{1}{s_2(t)} - 1 = \frac{m_1(t)}{m_2(t)} + \frac{m_3(t)}{m_2(t)} = \frac{\beta_1}{\beta_2}\left(\frac{p_1}{p_2}U(t)^{\varepsilon_1-\varepsilon_2}\right)^{1-\sigma} + \frac{\beta_3}{\beta_2}\left(\frac{p_3}{p_2}U(t)^{\varepsilon_3-\varepsilon_2}\right)^{1-\sigma}.$$

$\varepsilon_1 < \varepsilon_2$ pushes labor out of A to M; $\varepsilon_2 < \varepsilon_3$ pulls labor out of M to S.

The production side is the same as before. By following the same step, we obtain

**Equilibrium Shares**

$$s_j = \frac{\left[\beta_j^{\frac{1}{\sigma-1}} \tilde{A}_j\right]^{-a}}{[U^{\varepsilon_j}]^{-a}}, \qquad \text{where } \sum_{k=1}^{3} \frac{\left[\beta_k^{\frac{1}{\sigma-1}} \tilde{A}_k\right]^{-a}}{[U^{\varepsilon_k}]^{-a}} \equiv 1$$

With $\tilde{A}_j(t) = \bar{A}_j(t-\lambda_j) = \bar{A}_j(0)e^{g_j(t-\theta_j\lambda)}$,

$s_2(t)$:

$$\frac{1}{s_2(t)} = U(t)^{a(\varepsilon_1-\varepsilon_2)}\left[\frac{\tilde{\beta}_1}{\tilde{\beta}_2} e^{a(\theta_1 g_1-\theta_2 g_2)\lambda}\right] e^{-a(g_1-g_2)t} + 1 + U(t)^{a(\varepsilon_3-\varepsilon_2)}\left[\frac{\tilde{\beta}_3}{\tilde{\beta}_2} e^{a(\theta_3 g_3-\theta_2 g_2)\lambda}\right] e^{a(g_2-g_3)t}$$

$U(t)$:

$$U(t)^{a\varepsilon_1}\tilde{\beta}_1 e^{-ag_1(t-\theta_1\lambda)} + U(t)^{a\varepsilon_2}\tilde{\beta}_2 e^{-ag_2(t-\theta_2\lambda)} + U(t)^{a\varepsilon_3}\tilde{\beta}_3 e^{-ag_3(t-\theta_3\lambda)} \equiv 1$$

$s_2'(t) = 0$:

$$(g_1-g_2) = (g_2-g_3)U^{a(\varepsilon_3-\varepsilon_2)}\left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1}\right] e^{a(\theta_3 g_3-\theta_1 g_1)\lambda} e^{a(g_1-g_3)t}$$
$$+ \frac{\left\{(\varepsilon_1-\varepsilon_2) + (\varepsilon_3-\varepsilon_2)U^{a(\varepsilon_3-\varepsilon_1)}\left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1}\right] e^{a(\theta_3 g_3-\theta_1 g_1)\lambda} e^{a(g_1-g_3)t}\right\}\left\{g_1 U^{a(\varepsilon_1-\varepsilon_2)}\tilde{\beta}_1 e^{-ag_1(t-\theta_1\lambda)} + g_2\tilde{\beta}_2 e^{-ag_2(t-\theta_2\lambda)} + g_3 U^{a(\varepsilon_3-\varepsilon_2)}\tilde{\beta}_3 e^{-ag_3(t-\theta_3\lambda)}\right\}}{\varepsilon_1 U^{a(\varepsilon_1-\varepsilon_2)}\tilde{\beta}_1 e^{-ag_1(t-\theta_1\lambda)} + \varepsilon_2\tilde{\beta}_2 e^{-ag_2(t-\theta_2\lambda)} + \varepsilon_3 U^{a(\varepsilon_3-\varepsilon_2)}\tilde{\beta}_3 e^{-ag_3(t-\theta_3\lambda)}}.$$

$\hat{t}$ and $\hat{U}$ solve the equation for $U(t)$ and the equation for $s_2'(t) = 0$, simultaneously.
Then, $\hat{s}_2$ can be obtained by plugging $\hat{t}$ and $\hat{U}$ into the equation for $s_2(t)$

**(Analytically Solvable) "Unbiased" Case**

$$0 < \mu \equiv \frac{\varepsilon_2 - \varepsilon_1}{g_1 - g_2} = \frac{\varepsilon_3 - \varepsilon_2}{g_2 - g_3} < \frac{1}{g_1 - \bar{g}}, \qquad \text{where} \qquad \bar{g} \equiv \frac{g_1 + g_2 + g_3}{3}$$

**Peak Time**

$$\hat{t} = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3}\lambda - \ln\left\{(1 - \tilde{\beta}_2)e^{-g_1 g_3\left(\frac{\theta_1 - \theta_3}{g_1 - g_3}\right)a\lambda} + \tilde{\beta}_2 e^{-g_2\left(\frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} - \theta_2\right)a\lambda}\right\}^{-\frac{1}{a}\left(\frac{\mu}{1+\mu\bar{g}}\right)}$$

**Peak M-Share**

$$\frac{1}{s_2(\hat{t})} = 1 + \left(\frac{1}{\tilde{\beta}_2} - 1\right)e^{\frac{(g_1 - g_2)(g_2 - g_3)}{(g_1 - g_3)}\left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}\right)a\lambda}$$

**Peak Time Per Capita Income**

$$U(\hat{t}) = \left\{(1 - \tilde{\beta}_2)e^{-g_1 g_3\left(\frac{\theta_1 - \theta_3}{g_1 - g_3}\right)a\lambda} + \tilde{\beta}_2 e^{-g_2\left(\frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} - \theta_2\right)a\lambda}\right\}^{-\frac{1}{a}\left(\frac{1}{1+\mu\bar{g}}\right)}$$

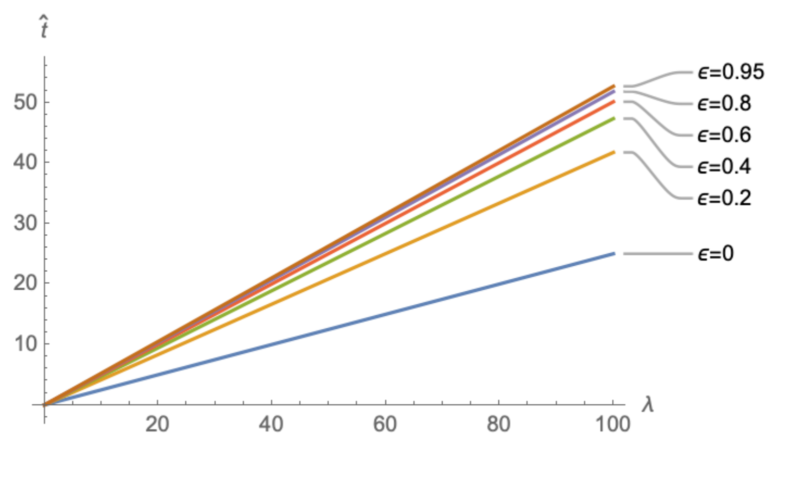
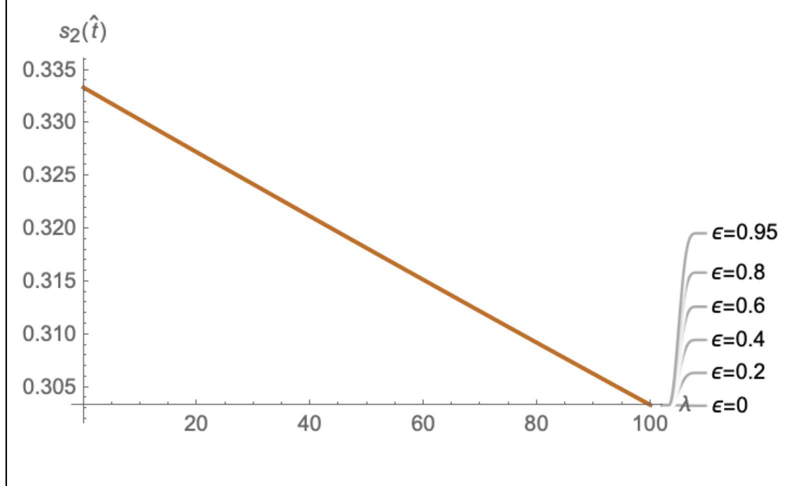
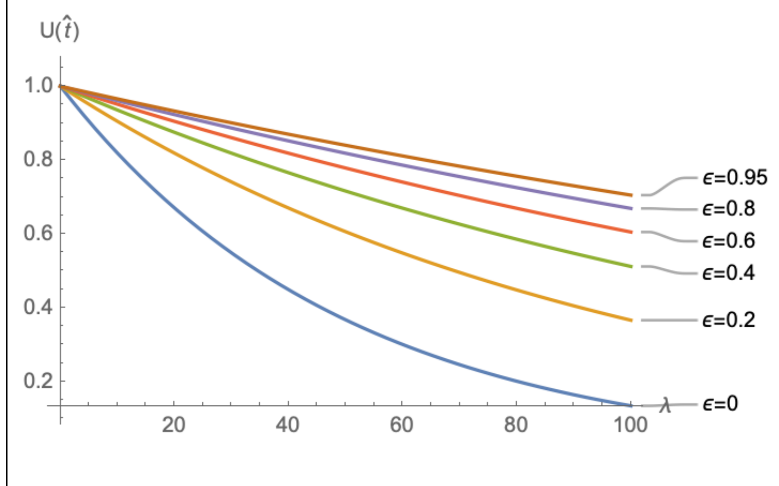
$\frac{\partial s_2(\hat{t})}{\partial \lambda} < 0$; $\frac{\partial U(\hat{t})}{\partial \lambda} < 0$ under the same condition; $\frac{\partial \hat{t}}{\partial \lambda} > 0$ under a weaker condition. With $g_1, g_2, g_3$ fixed, a higher $\mu$ has

- **No effect** on $\hat{t}, s_2(\hat{t}), U(\hat{t})$ for the country with $\lambda = 0$.
- A further delay in $\hat{t}$ for every country with $\lambda > 0$.
- **No effect on** $s_2(\hat{t})$ for every country with $\lambda > 0$.
- A smaller decline in $U(\hat{t})$ for each country with $\lambda > 0$.

## (Analytically Solvable) "Unbiased" Case: A Numerical Illustration

$g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$, $\theta = 0.5$, $a = 6/13$; $\tilde{\beta}_j = 1/3$ for $j = 1,2,3$.

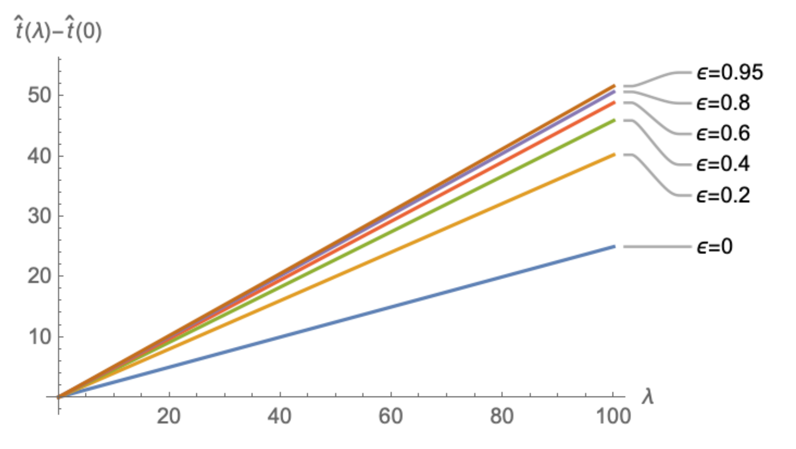
In this case, $g_1 - g_2 = g_2 - g_3 = \bar{g} = 1.2\% > 0 \Longrightarrow \varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 < \varepsilon_3 = 1 + \epsilon$ for $0 < \epsilon = (1.2\%)\mu < 1$

| Peak Time | Peak M-Share | Peak Time Per Capita Income |
| --- | --- | --- |
|   |   |   |

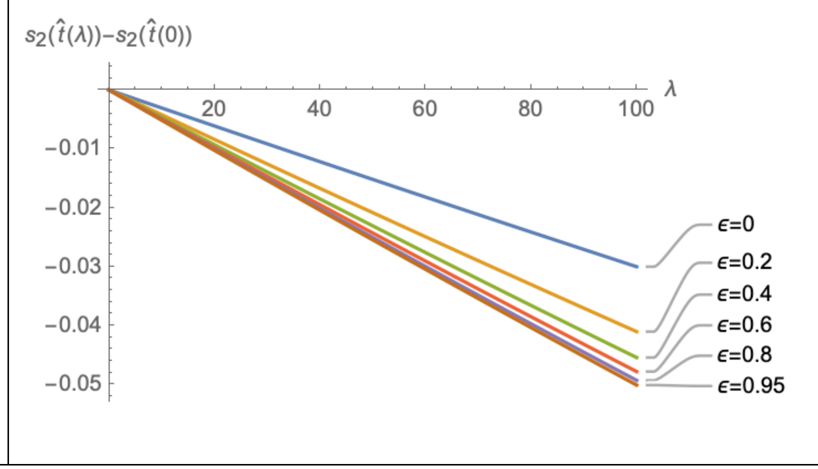
**(Empirically More Plausible) Biased Case**:

$\varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 + \frac{\epsilon}{3} < \varepsilon_3 = 1 + \frac{2\epsilon}{3}$ for $0 < \epsilon < 1 \Rightarrow \frac{g_1 - g_2}{g_2 - g_3} = 1 < \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2} = 4$, as in CLM (2021).

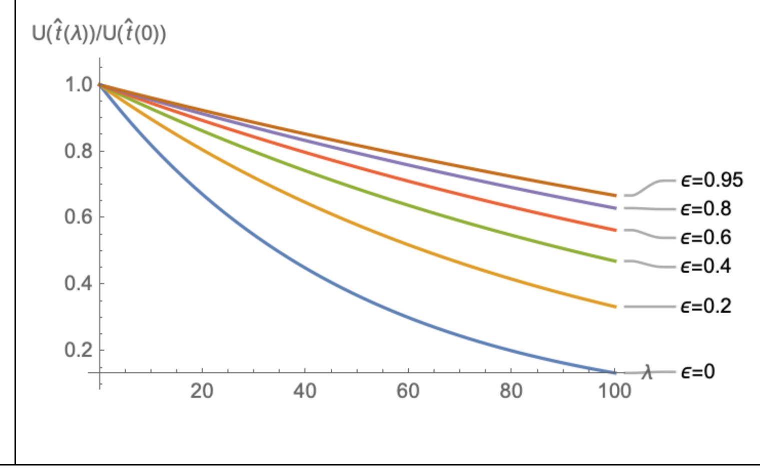
**Peak Time**




**Peak M-Share**



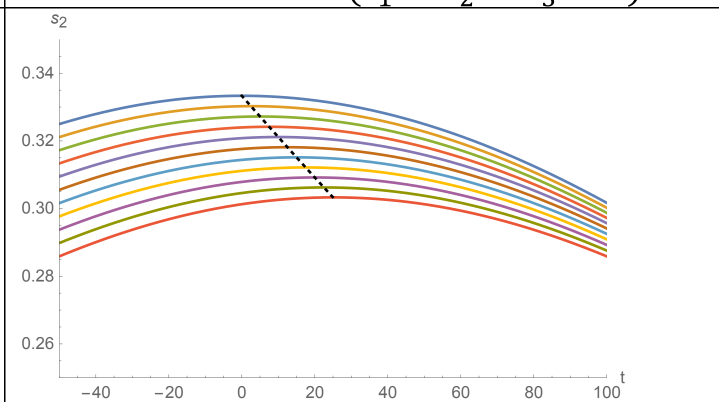
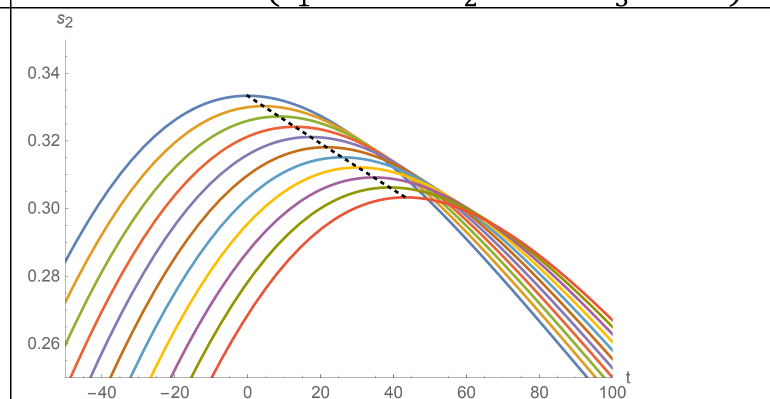
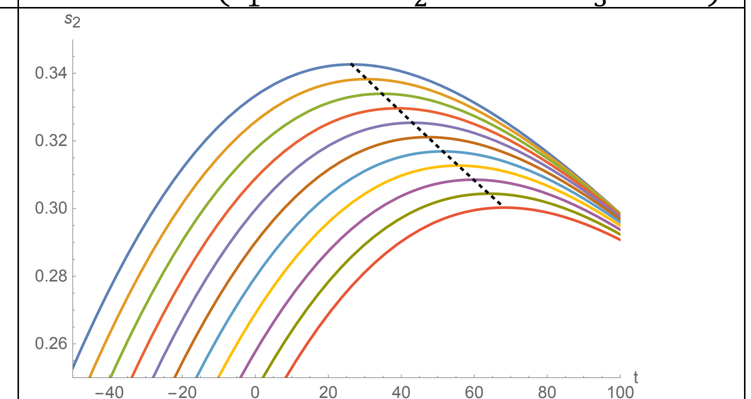
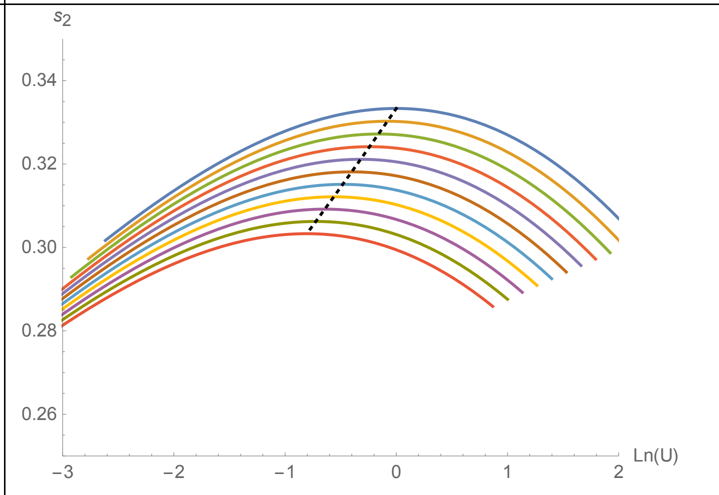
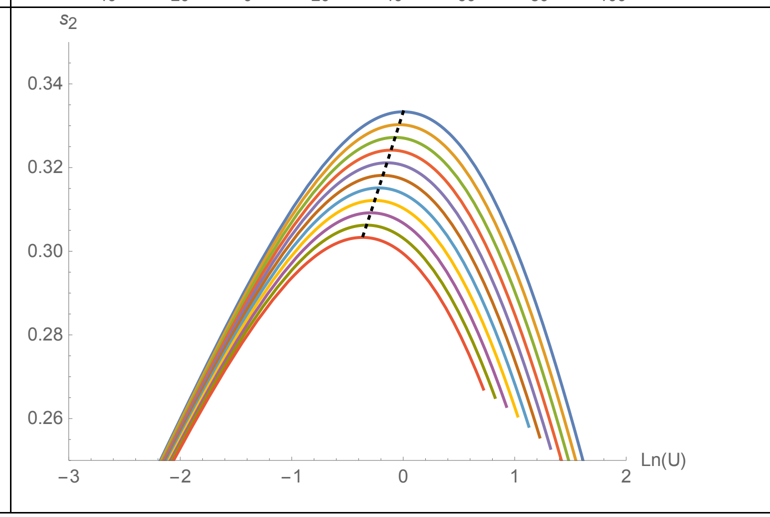
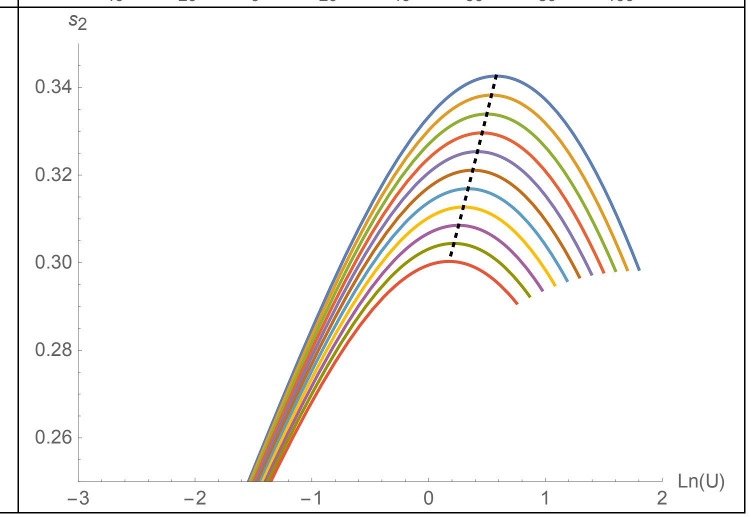

**Peak Time Per Capita Income**




PD ($\frac{\partial \hat{t}}{\partial \lambda} > 0, \frac{\partial s_2(\hat{t})}{\partial \lambda} < 0, \frac{\partial U(\hat{t})}{\partial \lambda} < 0$). Relative to the frontier country, a higher $\epsilon$ causes a high-$\lambda$ country to have

- A further delay in $\hat{t}$
- A *larger* decline in $s_2(\hat{t})$.
- A smaller decline in $U(\hat{t})$.

# Stronger nonhomotheticity changes the shape of the time paths significantly.
# It does not change the implications on PD, i.e., how technology gaps affect $\hat{t}, s_2(\hat{t})$, and $U(\hat{t})$.

| | $\epsilon = 0$ | $\epsilon = 0.6$ | |
|---|---|---|---|
| | Homothetic case ($\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$) | Unbiased case($\varepsilon_1 = .4 < \varepsilon_2 = 1 < \varepsilon_3 = 1.6$) | Biased case($\varepsilon_1 = .4 < \varepsilon_2 = 1.2 < \varepsilon_3 = 1.4$) |
| $(t, s_2(t))$ |   |   |   |
| $(\ln U(t), s_2(t))$ |   |   |   |

## Premature Deindustrialization (PD) through the Engel (Income) Effect Only

What happens if we rely *entirely* on the Engel effect, by removing the Baumol effect with $g_1 = g_2 = g_3 = \bar{g} > 0$, while keeping $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$?

**Peak Time**

$$\hat{t} = \frac{1}{a\bar{g}} \ln\left\{(1 - \tilde{\beta}_2)e^{\frac{(\varepsilon_3\theta_1 - \varepsilon_1\theta_3)}{(\varepsilon_3 - \varepsilon_1)}a\bar{g}\lambda} + \tilde{\beta}_2 e^{\left(\theta_2 + \frac{(\theta_1 - \theta_3)}{(\varepsilon_3 - \varepsilon_1)}\varepsilon_2\right)a\bar{g}\lambda}\right\}$$

**Peak M-Share**

$$\frac{1}{s_2(\hat{t})} - 1 = \left(\frac{1}{\tilde{\beta}_2} - 1\right) e^{\frac{\varepsilon_3\theta_3}{(\varepsilon_3 - \varepsilon_1)}\left[\left(1 - \frac{\varepsilon_1}{\varepsilon_3}\right)\left(1 - \frac{\theta_2}{\theta_3}\right) - \left(1 - \frac{\varepsilon_2}{\varepsilon_3}\right)\left(1 - \frac{\theta_1}{\theta_3}\right)\right]a\bar{g}\lambda}$$

**Peak Time Per Capita Income**

$$\ln U(\hat{t}) = \frac{\theta_1 - \theta_3}{\varepsilon_3 - \varepsilon_1}\bar{g}\lambda$$

with the two normalizations

$$\left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2}\right)\frac{\tilde{\beta}_1}{\tilde{\beta}_3} = 1; \;\; \tilde{\beta}_1 + \tilde{\beta}_2 + \tilde{\beta}_3 = 1$$

which ensures $U(\hat{t}) = 1$ and $\hat{t} = 0$ for $\lambda = 0$.

**Conditions for Premature Deindustrialization (PD) only with the Engel Effect**

$$\frac{\partial U(\hat{t})}{\partial \lambda} < 0 \text{ for all } \lambda > 0 \Leftrightarrow 0 < \frac{\theta_1}{\theta_3} < 1$$

With a low $\theta_1$ and a high $\theta_3$, the price of the income elastic S is high relative to the income inelastic A in a high-$\lambda$ country, which make it necessary to reallocate labor to S at earlier stage of development.

$$\frac{\partial s_2(\hat{t})}{\partial \lambda} < 0 \text{ for all } \lambda > 0 \Leftrightarrow \frac{\theta_1 - \theta_2}{\varepsilon_2 - \varepsilon_1} > \frac{\theta_2 - \theta_3}{\varepsilon_3 - \varepsilon_2}$$

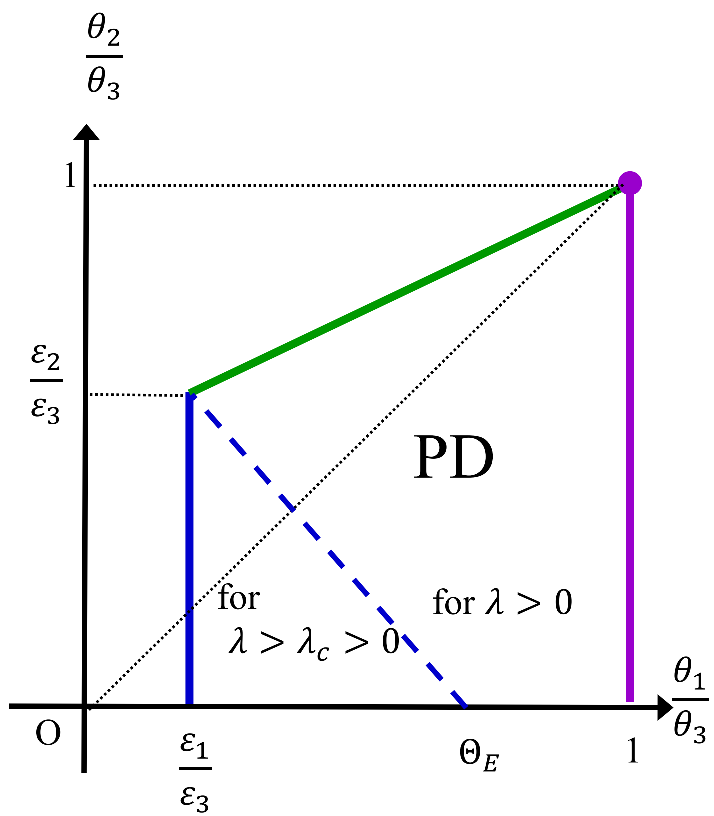
With a low $\theta_2$, which has no effect on $U(\hat{t})$, the price of M is low relative to both A & S in a high-$\lambda$ country, which keeps the M-share low.

Under the above condition,

$$\frac{\partial \hat{t}}{\partial \lambda} > 0 \ \text{ for a sufficiently large } \lambda \Leftrightarrow \frac{\theta_1}{\theta_3} > \frac{\varepsilon_1}{\varepsilon_3}$$

$$\frac{\partial \hat{t}}{\partial \lambda} > 0 \ \text{ for all } \lambda > 0 \Leftrightarrow \left(\Theta_E - \frac{\varepsilon_1}{\varepsilon_3}\right)\left[1 - \left(\frac{\varepsilon_3}{\varepsilon_2}\right)\frac{\theta_2}{\theta_3}\right] < \frac{\theta_1}{\theta_3} - \frac{\varepsilon_1}{\varepsilon_3} < 1 - \frac{\varepsilon_1}{\varepsilon_3}$$

where $\varepsilon_1/\varepsilon_3 < \Theta_E < 1$.




With $g_1 = g_2 = g_3 = \bar{g}$, PD occurs only if $\theta_1\bar{g}, \theta_2\bar{g} < \theta_3\bar{g}$, that is, when cross-country productivity difference is *the largest in S.*

# Introducing Catching Up

## Narrowing a Technology Gap

We assumed that $\lambda$ is time-invariant. This implies

The sectoral productivity growth rate is constant over time & identical across countries.
[In contrast, the aggregate growth rate, $g_U(t) \equiv U'(t)/U(t) = \sum_{k=1}^{3} g_k s_k(t)$, declines over time, $g_U'(t) = g_1 s_1'(t) + g_2 s_2'(t) + g_3 s_3'(t) = (g_1 - g_2)s_1'(t) + (g_3 - g_2)s_3'(t) < 0$, the so-called Baumol's cost disease.]

*What if technological laggards can* ***narrow a technology gap****, and hence achieve a higher productivity growth in each sector?*

**Countries differ only in the *initial* value of lambda, $\boldsymbol{\lambda_0}$**, converging exponentially over time at **the same rate,**

$$\tilde{A}_j(t) = \bar{A}_j(0)e^{g_j(t-\theta_j\lambda_t)}, \quad \text{where } \lambda_t = \lambda_0 e^{-g_\lambda t}, \quad g_\lambda > 0.$$

$$\Rightarrow \frac{1}{s_2(t)} = \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a[(\theta_1 g_1 - \theta_2 g_2)\lambda_t - (g_1 - g_2)t]} + 1 + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a[(\theta_3 g_3 - \theta_2 g_2)\lambda_t + (g_2 - g_3)t]}$$

Again, by setting the calendar time such that $\hat{t}_0 = 0$ for the frontier country with $\lambda_0 = 0$,

**Peak Time**

$$\hat{t} = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3}\lambda_{\hat{t}} + D(g_\lambda \lambda_{\hat{t}})$$

**Peak Share**

$$\frac{1}{s_2(\hat{t})} = 1 + \left(\frac{\tilde{\beta}_1 + \tilde{\beta}_3}{\tilde{\beta}_2}\right)\left[\frac{(g_2 - g_3)e^{a(g_2-g_1)D(g_\lambda\lambda_{\hat{t}})} + (g_1 - g_2)e^{a(g_2-g_3)D(g_\lambda\lambda_{\hat{t}})}}{g_1 - g_3}\right]\left[e^{\frac{a(g_1-g_2)(g_2-g_3)}{(g_1-g_3)}}\right]^{\left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} + \frac{\theta_3 g_3 - \theta_2 g_2}{g_2 - g_3}\right)\lambda_{\hat{t}}}$$

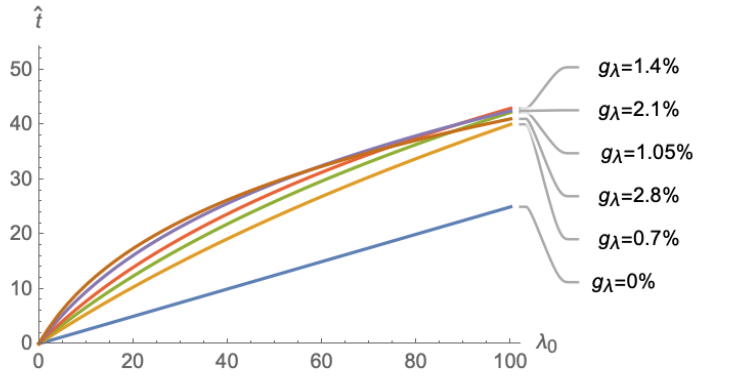
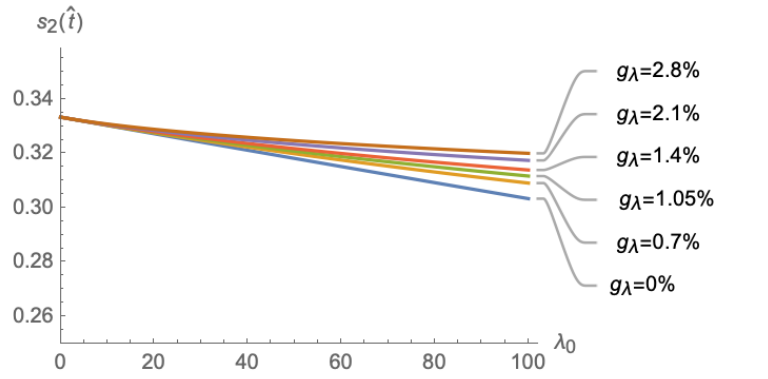
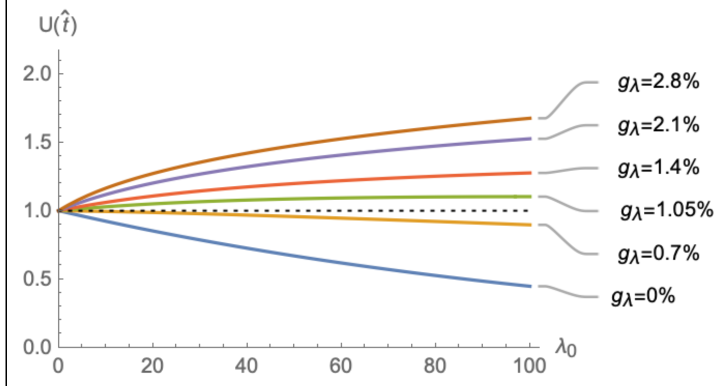
**Peak Time Per Capita Income**

$$U(\hat{t}) = \left\{\left(\tilde{\beta}_1 e^{-a g_1 D(g_\lambda\lambda_{\hat{t}})} + \tilde{\beta}_3 e^{-a g_3 D(g_\lambda\lambda_{\hat{t}})}\right)e^{-a\frac{(\theta_1-\theta_3)g_1 g_3}{g_1-g_3}\lambda_{\hat{t}}} + \left(\tilde{\beta}_2 e^{-a g_2 D(g_\lambda\lambda_{\hat{t}})}\right)e^{-a\frac{(\theta_1-\theta_2)g_1 g_2 + (\theta_2-\theta_3)g_2 g_3}{g_1-g_3}\lambda_{\hat{t}}}\right\}^{-\frac{1}{a}}$$

where

$$D(g_\lambda\lambda_{\hat{t}}) = \frac{1}{a(g_1 - g_3)}\ln\left[\left(\frac{g_1 - g_2 + (\theta_1 g_1 - \theta_2 g_2)g_\lambda\lambda_{\hat{t}}}{g_2 - g_3 - (\theta_3 g_3 - \theta_2 g_2)g_\lambda\lambda_{\hat{t}}}\right)\left(\frac{g_2 - g_3}{g_1 - g_2}\right)\right].$$

For $g_\lambda = 0, D(g_\lambda\lambda_{\hat{t}}) = D(0) = 0$, and all the parts in red disappear, and we go back to the baseline model.

| **Peak Time** | **Peak M-Share** | **Peak time Per Capita Income** |
|---|---|---|
|  |  |  |







Technological laggards

- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, unless $g_\lambda$ is too large: Comin-Mestieri (2018)

# Concluding Remarks

A simple model of Rodrik's (2016) PD based on

- **Differential productivity growth rates across complementary sectors**, as in Baumol (67), Ngai-Pissarides (07).
- **Countries heterogeneous only in their technology gaps**, as in Krugman (1985).
- Sectors differ in the extent to which technology gap affects their adoption lags, unlike in Krugman (1985)

We find that PD occurs for

- cross-country productivity difference larger in A than in S.
- technology adoption takes not too long in M.
- Technology adoption takes longer in S than in A.

which implies that cross-country productivity difference the largest in A; that technology adoption the longest in S.

The baseline model assumes **homothetic CES (**to focus on the Baumol effect) and **no catching up** (to isolate the level effect from the growth effect).

In two extensions, we showed that the results are ***robust*** against introducing

- **The Engel effect** with income-elastic S & income-inelastic A, using nonhomothetic CES: CLM(21), Matsuyama(19)

The Engel effect changes the shape of the time paths, but not the implications on technology gaps on PD

The Engel effect *alone* could not generate PD w/o counterfactual implications on cross-country productivity differences

- **Narrowing a technology gap** to allow technological laggards to catch up

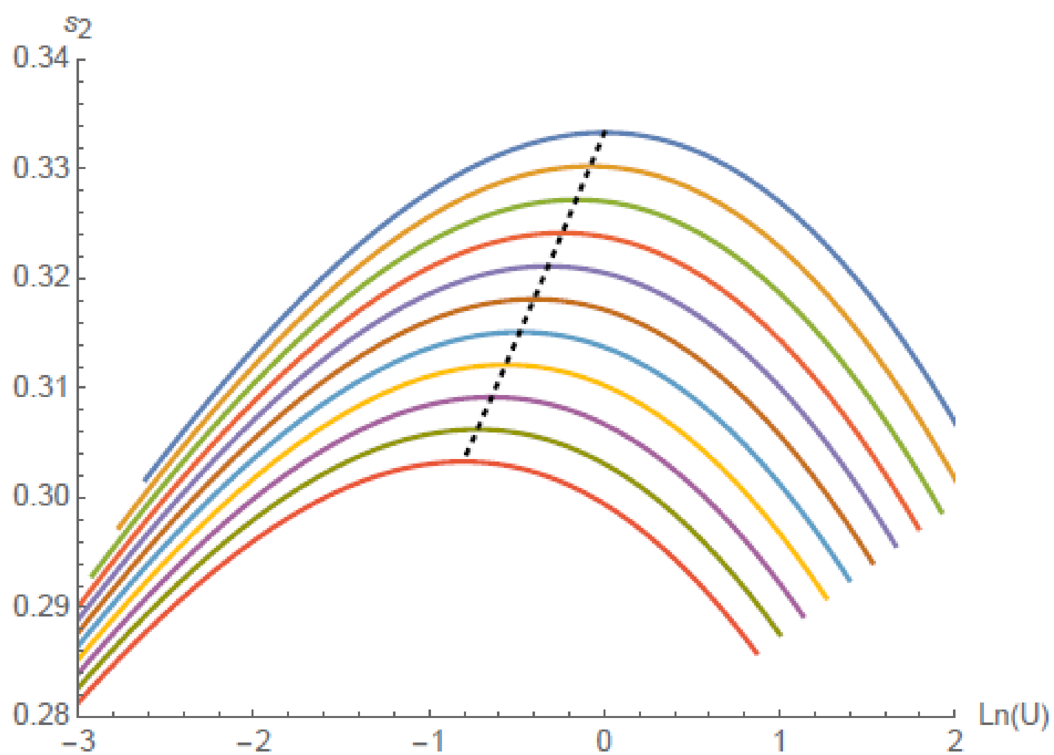
unless the catching-up speed is too large.

# Appendix

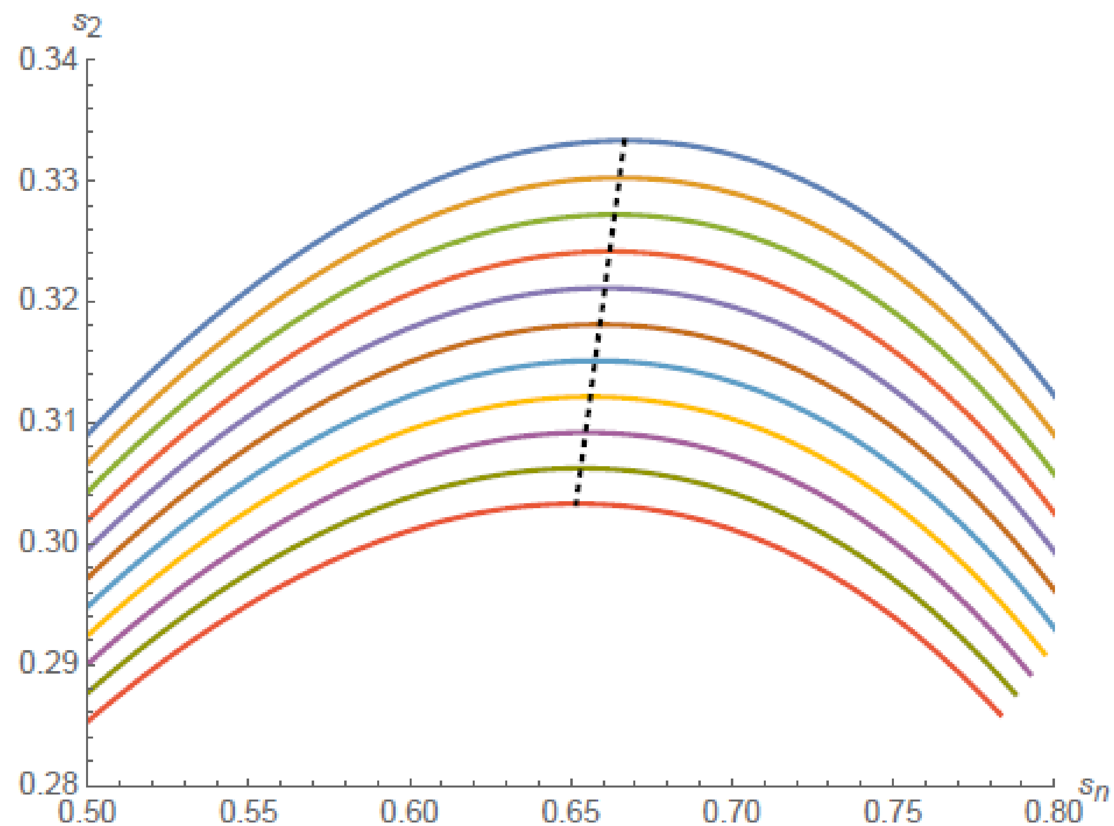
**Appendix: Non-agricultural share as another measure of development**, $1 - s_1(\hat{t}) = s_2(\hat{t}) + s_3(\hat{t}) \equiv s_n(\hat{t})$

Baseline Homothetic Case:

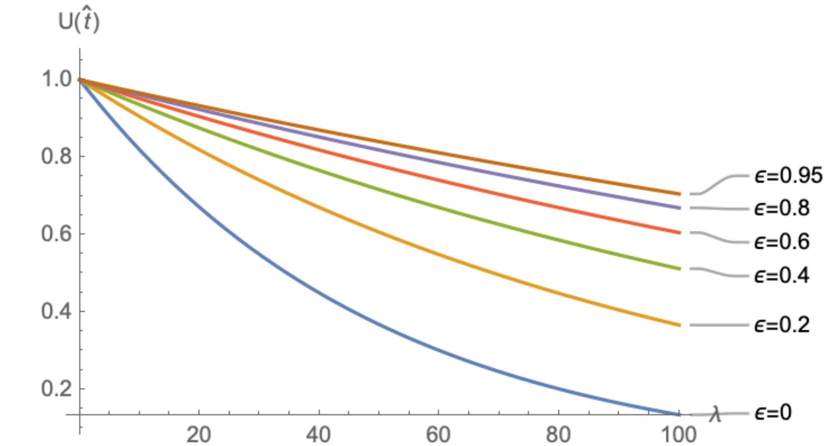
$(\ln U(t), s_2(t))$

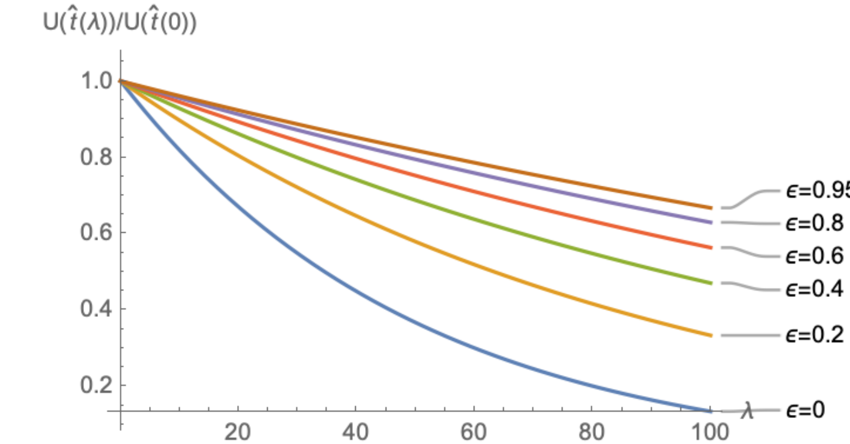


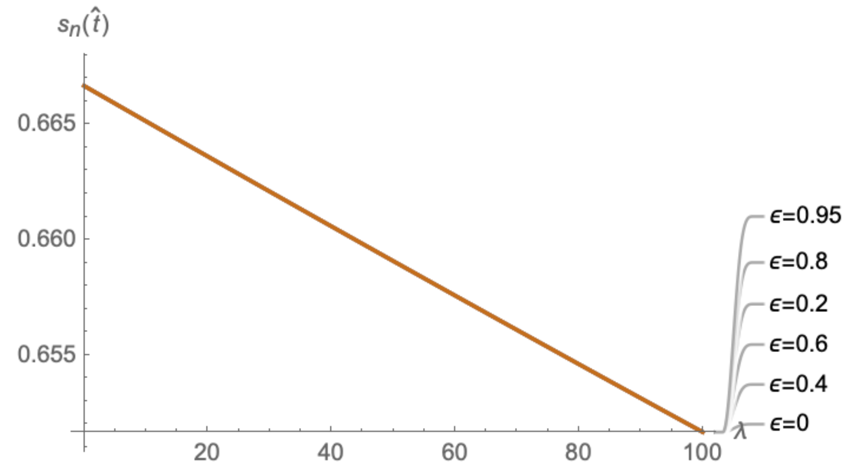

$(s_n(t), s_2(t))$

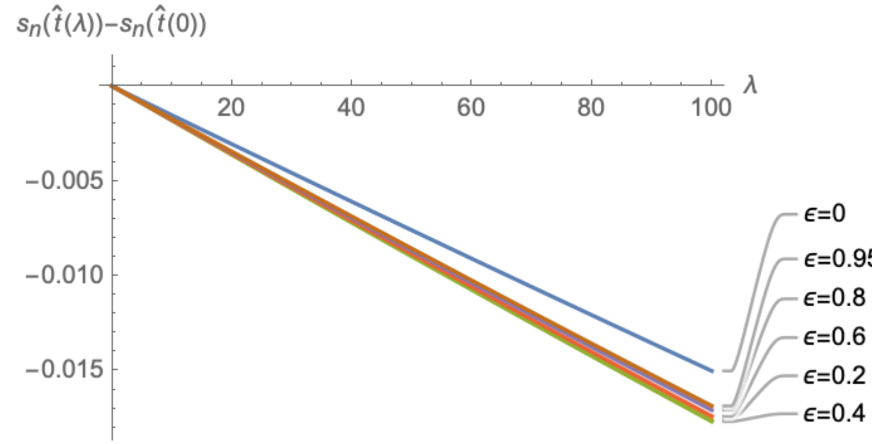


Nonhomothetic Cases:

<table>
<tr><td></td><td>Unbiased: $\varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 < \varepsilon_3 = 1 + \epsilon$</td><td>Biased: $\varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 + \epsilon/3 < \varepsilon_3 = 1 + 2\epsilon/3$</td></tr>
<tr><td>$\ln U(t)$</td><td>


</td><td>


</td></tr>
<tr><td>$s_n(t)$</td><td>


</td><td>


</td></tr>
</table>