Does State-Dependent Wage Setting Generate Multiple Equilibria?*

Shuhei Takahashi†

August 27, 2018

Abstract

Does wage setting exhibit strategic complementarity and produce multiple equilibria? This study constructs a discrete-time New Keynesian model in which the timing of individual wage adjustments is endogenous. I explore steady-state equilibrium of the state-dependent wage-setting model both analytically and numerically. For reasonable parameter values, complementarity in wage setting is weak and multiple equilibria are unlikely to exist at the steady state. The uniqueness of equilibrium is robust to imperfect consumption insurance and deflation.

JEL classification: E24; E31; E32; E52

Keywords: State-dependent wage setting; New Keynesian model; Multiple equilibria; Strategic complementarity; Incomplete markets; Deflation.

*I would like to thank seminar and conference participants at the AEI Joint Workshop, Kobe University, China Meeting of the Econometric Society, International Conference on Computing in Economics and Finance, Australasian Meeting of the Econometric Society, Tohoku University, and European Meeting of the Econometric Society for their comments. I gratefully acknowledge financial support from Grant-in-Aid for Young Researchers (B) 17K13700, Grant-in-Aid for Scientific Research (B) 16H03626, Grant-in-Aid for Scientific Research (A) 16H02026, and the Zengin Foundation for Studies on Economics and Finance. Any errors are my own.

†Institute of Economic Research, Kyoto University, Yoshida-Honmachi, Sakyo-ku, Kyoto 606-8501, Japan. Tel: +81-75-753-7153. Email: takahashi@kier.kyoto-u.ac.jp

1 Introduction

Nominal wages change in a staggered way: infrequently and with the timing of adjustments not completely synchronized. The resulting variation in relative wages is considered a major welfare cost of nominal wage stickiness in the New Keynesian literature (e.g., Erceg, Henderson, and Levin (2000)). Nonetheless, individual wage adjustments have not been comprehensively analyzed. While empirical studies provide evidence that macroeconomic conditions affect the frequency of wage changes and thereby suggest state dependency in wage setting, existing New Keynesian models typically assume time-dependent wage setting (e.g., Taylor (1980); Calvo (1983)) and fix the timing of wage adjustments exogenously.[1]

With the aim of better understanding wage adjustments and their consequences, the present study constructs a New Keynesian model in which the timing of wage adjustments is endogenous. The multiplicity of equilibria is a natural concern under such state-dependent wage setting. For price setting, the uniqueness of equilibrium in New Keynesian models is known to depend on a time horizon. Specifically, for an essentially static environment similar to Blanchard and Kiyotaki (1987), Ball and Romer (1991) argue that price setting is characterized by strategic complementarity and multiple equilibria often exist. By contrast, for the seminal dynamic state-dependent pricing model by Dotsey, King, and Wolman (1999), John and Wolman (2004, 2008) show that multiple equilibria do not arise under empirically plausible parameterization.[2] However, these results for price setting may not carry over to wage setting. As Huang and Liu (2002) point out in a time-dependent setting model, households' incentive to stabilize their relative wage is typically stronger than firms' incentive to stabilize their relative price.[3] Hence, the nonuniqueness of equilibrium might be a more

[1] Taylor (1999) reviews studies for several countries and concludes that the frequency of wage adjustments increases with the rate of inflation. According to Daly, Hobijn, and Lucking (2012) and Daly and Hobijn (2014), nominal wage stickiness rises in recessions in the United States.

[2] The Dotsey, King, and Wolman (1999) framework has been used for various analyses, such as the New Keynesian Phillips curve (Bakhshi, Kahn, and Rudolf (2007)), optimal monetary policy (Nakov and Thomas (2014)), short-run monetary transmission (Dotsey and King (2005, 2006)), US inflation (Klenow and Kryvtsov (2008)), and exchange rate dynamics (Landry (2009, 2010)).

[3] This result provides an explanation for the common finding (e.g., Huang and Liu (2002); Christiano, Eichenbaum, and Evans (2005)) that in a New Keynesian model with time-dependent setting, nominal wage

serious problem for wage setting than for price setting and analysis of state-dependent wage setting is of interest.

I investigate the possibility of multiple equilibria in a dynamic New Keynesian model with state-dependent wage setting. The wage-setting side of the model is based on Takahashi (2017). As in Blanchard and Kiyotaki (1987) and Erceg, Henderson, and Levin (2000), households supply a differentiated labor service and set the wage rate for their labor. Unlike under time-dependent wage setting, in which the timing of wage changes is exogenous, households endogenously determine when to adjust their wage subject to a fixed wage-setting cost. Like firms' price-setting costs in the Dotsey, King, and Wolman (1999) model, the wage-setting costs are stochastic and heterogeneous across households, leading to staggered wage adjustments. Thus, the only difference compared to a standard time-dependent wage-setting model is that the timing of wage changes is endogenous. Furthermore, to make the impact of state dependency in wage setting as transparent as possible, the present study assumes perfect competition and flexible prices in the goods market.

Using analytical and numerical methods developed by John and Wolman (2004, 2008), the present study examines the uniqueness of steady-state equilibrium in the state-dependent wage-setting model. I first analytically explore steady-state equilibrium under some restricted but empirically relevant parameterization. I find that for a high discount factor close to one, which is empirically relevant, wage setting is characterized by weak complementarity and multiple sticky-wage equilibria are unlikely to exist. This result is the same as that for price setting shown by John and Wolman (2004, 2008). Numerical analysis then finds that the uniqueness result holds under more general parameterization. Furthermore, the uniqueness of steady-state equilibrium is robust to several extensions of the baseline model.

While Blanchard and Kiyotaki (1987) analyze both price setting and wage setting, their model is not fully dynamic. Dynamic state-dependent wage-setting models are scarce, com-

stickiness generates larger short-run money nonneutrality than nominal price stickiness does.

pared to dynamic state-dependent pricing models.[4] Takahashi (2017) is the first study to analyze state-dependent wage setting in a dynamic New Keynesian model. The model includes various features and aggregate shocks similar to those in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). Costain, Nakov, and Zarzalejos (2017) develop a state-dependent wage-setting model with idiosyncratic shocks. Their model describes micro-level wage adjustments more accurately than the Takahashi (2017) model does. The main focus of these prior studies is the short-run implications of state-dependent wage setting, and neither analyzes the uniqueness of steady-state equilibrium.[5]

The analysis herein closely follows the work by John and Wolman (2004, 2008) on the Dotsey, King, and Wolman (1999) model of state-dependent pricing. Relevant equations and analytical results are similar between price setting and wage setting, although numerical results differ. There are also important differences between the present work and the prior studies. First, a constant marginal disutility of labor is assumed in the analytical investigation of John and Wolman (2004, 2008), whereas in what follows, the marginal disutility is allowed to increase with labor hours and individual labor hours influence wage-setting decisions. For that, I generalize the analysis by John and Wolman (2004, 2008), including their case as a special one.

Second, in addition to the baseline case of perfect consumption insurance, the present study analyzes a case with incomplete markets. By contrast, John and Wolman (2004, 2008) assume a representative household and do not explore the role of financial markets. Exploring the market incompleteness is important because the assumption of complete markets might be too strong. Indeed, an increasing number of studies introduce imperfect risk sharing in a New Keynesian framework (e.g., Braun and Nakajima (2012); Gornemann, Kuester, and Nakajima (2016); Kaplan, Moll, and Violante (2018)). Under incomplete markets, relative

[4] Examples of state-dependent pricing models, other than the Dotsey, King, and Wolman (1999) model, include those by Caplin and Spulber (1987), Caplin and Leahy (1991), Devereux and Siu (2007), Golosov and Lucas (2007), Gertler and Leahy (2008), Nakamura and Steinsson (2010), Costain and Nakov (2011a,b), and Midrigan (2011).

[5] These prior studies examine the interaction between price stickiness and wage stickiness, whereas the present study assumes flexible prices and focuses on wage setting.

wage variation leads to fluctuations not only in labor hours, but also in consumption. This gives additional motive for smoothing relative wages. However, I numerically show that imperfect consumption insurance does not affect the possibility of multiple steady-state equilibria and that the uniqueness of equilibrium holds in the dynamic state-dependent wage-setting model with incomplete markets.

Third, while John and Wolman (2004, 2008) focus on inflation, the present study also considers deflation. The Japanese experience motivates this analysis. The Japanese economy experienced mild, but persistent deflation from the mid-1990s to the mid-2010s and price behavior has been analyzed in various studies (e.g., Weinstein and Broda (2008); Ueda, Watanabe, and Watanabe (2018)).[6] I analytically show that a condition for the uniqueness of steady-state equilibrium is milder under deflation than under inflation. Specifically, multiple sticky-wage equilibria are ruled out for any discount factor under deflation, whereas under inflation, they are completely ruled out only in a limiting case where the discount factor approaches one. Furthermore, numerical analysis finds no evidence for multiple equilibria under deflation.

The rest of the present paper is organized as follows. Section 2 describes the benchmark model. Section 3 analytically explores the uniqueness of steady-state equilibrium in the model under particular parameter assumptions. Section 4 then analyzes this issue numerically under less restrictive assumptions. Section 5 considers several extensions of the benchmark model. Section 6 concludes.

2 Model

As in Takahashi (2017), I introduce fixed costs for wage adjustments in an otherwise standard discrete-time New Keynesian model. Fixed wage-setting costs differ across households and evolve independently over time. Therefore, the timing of wage adjustments, which is endoge-

[6] Other studies include Hirose (2014), Sudo, Ueda, Watanabe, and Watanabe (2018), and Watanabe and Watanabe (2018).

nous, differs across households. The present study makes two departures from Takahashi (2017). First, to focus on wage setting, perfect competition and flexible prices in the goods market are assumed. Second, instead of labor costs as in Takahashi (2017), wage-setting costs are included as utility costs.[7]

2.1 Central Bank

The central bank maintains a constant growth rate of money supply:

$$\frac{M_{t+1}^s}{M_t^s} = \mu, \tag{1}$$

where M_t^s is the money supply and $\mu > 1$.[8]

2.2 Labor Aggregator

As in Erceg, Henderson, and Levin (2000), a representative labor aggregator combines differentiated labor services, $n_t(h)$, indexed by $h \in [0,1]$, and all firms hire composite labor from the aggregator. The composite labor supplied by the aggregator is given by

$$N_t^s = \left(\int_0^1 n_t(h)^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where $\epsilon > 1$. Cost minimization by the labor aggregator implies that the demand for each labor service is

$$n_t^d(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\epsilon} N_t^s, \tag{3}$$

where $W_t(h)$ is the nominal wage rate for type-h labor service and W_t is the aggregate wage index, which is defined by

[7] The specification of wage-setting costs is required for the analytical approach taken in Section 3. It is straightforward to include labor costs for wage adjustments in the numerical analysis in Section 4. See Section 5.3 for the detail.

[8] Section 5.2 considers deflation, $\mu \in (0,1)$.

$$W_t = \left(\int_0^1 W_t(h)^{1-\epsilon} dh \right)^{\frac{1}{1-\epsilon}}. \tag{4}$$

2.3 Firms

A representative firm (or perfectly competitive firms) produces a single good using labor. The production function is

$$Y_t = N_t^d, \tag{5}$$

where Y_t is output and N_t^d is labor input. The firm maximizes its static profit. Prices are flexible and the aggregate nominal price P_t equals W_t, which is the nominal marginal cost.

2.4 Households

There is a continuum of households (measure one). Each household, indexed by $h \in [0, 1]$, supplies a differentiated labor service $n_t(h)$. Each household also sets the wage rate for their labor as $W_t(h)$. Wage changes incur a fixed utility cost $\varpi_t(h)$, which is drawn from a time-invariant continuous distribution $G(\varpi)$ with support $[0, \bar{\varpi}]$, $\bar{\varpi} < \infty$. These costs are independently and identically distributed over time and across households.

A household's preference is represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t(h)^{1-\sigma}}{1-\sigma} - \chi n_t^s(h)^\zeta - \varpi_t(h) I_t(h) \right), \tag{6}$$

where $\beta \in [0, 1), \sigma > 0, \chi > 0, \zeta \geq 1$, $c_t(h)$ is consumption, and $n_t^s(h)$ is hours worked.[9] The function $I_t(h)$ takes the value 1 if the household resets its wage in the period and takes the value 0 otherwise.

As in a standard New Keynesian model, households have identical initial wealth and access to perfect insurance for consumption. Thus, consumption is the same for all households, that is, $c_t(h) = C_t$ for all h and for all t, where C_t is aggregate consumption. Furthermore,

[9] A log consumption utility function is assumed for $\sigma = 1$.

money demand is given by

$$\ln \frac{M_t^d}{P_t} = \ln C_t, \tag{7}$$

where M_t^d is the quantity of money demanded by households.[10]

Let $x_t(h) \equiv W_t(h)/M_t$ be the wage rate prevailing in the current period relative to the current period's money stock ($M_t = M_t^s = M_t^d$ in equilibrium). Households supply labor hours demanded as in (3), that is, $n_t^s(h) = n_t^d(h)$. Let λ_t be the marginal utility of consumption, which is the same for all households. Given (3), current utility relating to wage-setting decisions is given by

$$\begin{aligned} \pi(x_t(h)) &= \lambda_t \frac{W_t(h)}{P_t}\left(\frac{W_t(h)}{W_t}\right)^{-\varepsilon} N_t^s - \chi\left[\left(\frac{W_t(h)}{W_t}\right)^{-\varepsilon} N_t^s\right]^{\zeta} \\ &= \lambda_t (x_t(h)C_t)^{1-\varepsilon} N_t^s - \chi\left[(x_t(h)C_t)^{-\varepsilon} N_t^s\right]^{\zeta}. \end{aligned} \tag{8}$$

Households' wage-setting problem is described recursively as follows. Let $V(x_{t-1}(h), \varpi_t(h))$ be the value function of households whose wage relative to the money stock in the last period is $x_{t-1}(h) = W_{t-1}(h)/M_{t-1}$ and whose adjustment cost in the current period is $\varpi_t(h)$. Note that

$$V(x_{t-1}(h), \varpi_t(h)) = \max\left\{V^A(\varpi_t(h)), V^{NA}(x_{t-1}(h))\right\}. \tag{9}$$

First, $V^A(\varpi_t(h))$ is the value function of households when they adjust their wage in the current period and satisfies

$$V^A(\varpi_t(h)) = -\varpi_t(h) + \max_{x_t}\left\{\pi(x_t) + \beta E\left[V(x_t, \varpi_{t+1}(h))\right]\right\}. \tag{10}$$

Households pay a fixed wage-setting cost $\varpi_t(h)$ and set their wage to maximize the sum of current utility and discounted expected utility. The optimal wage x_t^* is common to all adjusting households, as per the standard time-dependent wage setting. Hence, the value of adjusting households is independent of the wage set in the previous period $x_{t-1}(h)$ and it

[10] Interest-elastic money demand is analyzed in Section 5.4.

depends only on the current adjustment cost $\varpi_t(h)$.

Second, $V^{NA}(x_{t-1}(h))$ is the value function of households when they keep their wage unchanged from the last period and satisfies

$$V^{NA}(x_{t-1}(h)) = \left\{ \pi \left(\frac{x_{t-1}(h)}{\mu} \right) + \beta E \left[V \left(\frac{x_{t-1}(h)}{\mu}, \varpi_{t+1}(h) \right) \right] \right\}. \tag{11}$$

Households keep their nominal wage unchanged from the last period, that is, $W_t(h) = W_{t-1}(h)$. Hence, their wage decreases relative to money stock. In particular, $x_t(h) = x_{t-1}(h)/\mu$. Households obtain current utility and expected discounted utility based on the decreased wage. The value of non-adjusting households is independent of the current adjustment cost $\varpi_t(h)$ and it depends only on the wage carried over from the last period $x_{t-1}(h)$.

An important property of the present model is that since adjusting households set the same wage, at the start of any given period, a fraction $\omega_{t,q}$ of households charge $x^*_{t-q}, q = 1, ..., Q_t$. For each wage vintage, there is a cutoff wage-setting cost and households whose cost is below the cutoff choose to reset their wage in the current period. Hence, for each wage vintage, a fraction $\alpha_{t,q}$ of households change their wage. The number of wage vintages Q_t and the wage distribution are endogenous. Since the inflation rate is positive and wage-setting costs are bounded, households eventually increase their wage and hence Q_t is finite.

2.5 Equilibrium

I analyze the model's steady-state equilibrium in which real variables are constant with a constant inflation rate that equals the money growth rate of μ. Hereafter, time subscripts are dropped for expository purposes. A steady-state competitive equilibrium satisfies the following conditions.

1. Households' optimization:

   $V(x_{-1}(h), \varpi(h)), V^A(\varpi(h))$, and $V^{NA}(x_{-1}(h))$ satisfy (9), (10), and (11), respectively, while x^* is the associated optimal wage. Furthermore, (7) holds.

2. Firms' optimization:

   The representative firm maximizes its profit under the technology described in (5).

3. Labor aggregator's optimization:

   The representative labor aggregator chooses $n_t^d(h)$ as in (2) and (3).

4. Goods market clearing: $C = Y$.

5. Money market clearing: $M = M^s = M^d$.

6. Labor market clearing: $N = N^s = N^d$ and $n(h) = n^s(h) = n^d(h)$ for all $h \in [0, 1]$.

7. Monetary policy:

   The central bank conducts monetary policy as described in (1).

8. Wage distribution:

   The evolution of the wage distribution is consistent with households' wage-setting decisions. Further, the distribution of wages (relative to money stock) is unchanged over time.

3 Analytical Approach

This section analytically examines the uniqueness of steady-state equilibrium of the state-dependent wage-setting model described in Section 2. The analysis here closely follows the work of John and Wolman (2004, 2008) with respect to the Dotsey, King, and Wolman (1999) state-dependent price-setting model.

Throughout the present paper, a focus is on a situation in which wages are fixed for no more than two periods ($Q = 2$).[11] In such a case, households' wage setting is characterized by

[11] John and Wolman (2004, 2008) focus on a case in which prices are fixed for two periods at most.

two variables: α and x^*. First, α is the (ex-ante) probability of wage changes in the current period before the current wage-setting cost is drawn, when households adjusted their wage in the previous period. Note that under the assumption that wages are fixed for two periods at most, households certainly adjust their wage in the current period if they did not do so in the previous period. Second, x^* is the wage rate (relative to the current money stock) chosen by adjusting households. Recall that adjusting households choose the same wage.

Let $v(\alpha; s)$ be the value of an adjusting household that has a constant adjustment probability α and sets the associated optimal wage $x^*(\alpha; s)$, under the aggregate state s. The value $v(\alpha; s)$ is gross of the current fixed cost of wage adjustments and it satisfies

$$\begin{aligned} v(\alpha; s) &= \pi(x^*(\alpha; s); s) + \beta\alpha \left\{ v(\alpha; s) - E\left[\varpi | \varpi < G^{-1}(\alpha)\right] \right\} \\ &\quad + \beta(1-\alpha) \left\{ \pi\left(\frac{x^*(\alpha; s)}{\mu}; s\right) + \beta\left[v(\alpha; s) - E(\varpi)\right] \right\}. \end{aligned} \tag{12}$$

The first term is current utility. The other terms pertain to expected utility. With probability α, the household will adjust its wage in the next period again. In that case, the household receives $v(\alpha; s)$, while the expected utility cost is $E\left[\varpi | \varpi < G^{-1}(\alpha)\right]$. With probability $(1 - \alpha)$, the household will not adjust its wage in the next period. In that case, the household receives $\pi\left(x^*(\alpha; s)/\mu\right)$ in the next period. Furthermore, the household will certainly adjust its wage in the following period, which gives $\beta\left[v(\alpha; s) - E(\varpi)\right]$.

Rearranging (12) leads to

$$v(\alpha; s) = \frac{\pi(x^*(\alpha; s); s) + \beta(1-\alpha)\pi\left(\frac{x^*(\alpha; s)}{\mu}; s\right) - \beta\alpha E\left[\varpi | \varpi < G^{-1}(\alpha)\right] - \beta^2(1-\alpha)E(\varpi)}{(1-\beta)[1+\beta(1-\alpha)]}. \tag{13}$$

The present section assumes that $\zeta + \sigma - 2 + \varepsilon(1 - \zeta) = 0$.[12] Under the condition, the optimal wage x^* becomes independent of the aggregate state s, that is, $x^*(\alpha) = x^*(\alpha; s)$ for all s.

Lemma 1 *Suppose that* $\zeta + \sigma - 2 + \varepsilon(1 - \zeta) = 0$. *Given* α, *the optimal wage of an adjusting*

[12]Note that $\sigma = \zeta = 1$, which is John and Wolman (2004, 2008)'s specification, satisfies the condition. The condition allows $\zeta > 1$.

household is

$$x^*(\alpha) = \left[\frac{\varepsilon\chi\zeta}{\varepsilon-1}\frac{1+\beta(1-\alpha)\mu^{\varepsilon\zeta}}{1+\beta(1-\alpha)\mu^{\varepsilon-1}}\right]^{\frac{1}{\varepsilon(\zeta-1)+1}} = \left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}g(\alpha,\beta)\right)^{\frac{1}{\varepsilon(\zeta-1)+1}}, \tag{14}$$

where

$$g(\alpha,\beta) \equiv \frac{1+\beta(1-\alpha)\mu^{\varepsilon\zeta}}{1+\beta(1-\alpha)\mu^{\varepsilon-1}}. \tag{15}$$

Proof. *See the Appendix.* ■

If households do not care about the future ($\beta = 0$) or if they will certainly adjust their wage in the next period ($\alpha = 1$), then $g(\alpha,\beta)^{1/[\varepsilon(\zeta-1)+1]} = 1$, that is, each period households set the static optimal wage for the period $W^* = x^*M = [\varepsilon\chi\zeta/(\varepsilon-1)]^{1/[\varepsilon(\zeta-1)+1]}M$. When $\alpha \in [0,1)$ and $\beta \in (0,1)$,

$$1 < g(\alpha,\beta)^{\frac{1}{\varepsilon(\zeta-1)+1}} < \left(\frac{1+\mu^{\varepsilon\zeta}}{1+\mu^{\varepsilon-1}}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}} < \mu, \tag{16}$$

and adjusting households charge a wage that is between the static optimal wage for the current period W^* and that for the next period $W^{*\prime} = x^*M' = [\varepsilon\chi\zeta/(\varepsilon-1)]^{1/[\varepsilon(\zeta-1)+1]}\mu M$. Note that $g(\alpha,\beta)$ decreases with α and increases with β. As α increases, households will more likely to adjust their wage in the next period. Thus, the reset wage decreases and becomes closer to the current static optimal wage. In contrast, as β increases, households put a larger weight on the next period. Therefore, the reset wage increases and becomes closer to the static optimal wage for the next period.[13]

Given the adjustment probability α, the optimal wage $x^*(\alpha)$ is unique, but the optimal α might not be unique. Hence, households might randomize their adjusting strategies. Furthermore, an asymmetric equilibrium in which different households pursue different strategies could exist. For simplicity, I focus on a pure-strategy symmetric steady-state equilibrium, in which all households choose the same constant adjusting probability and thereby the same

[13] As $\mu \to \infty$, $\left[(1+\mu^{\varepsilon\zeta})/(1+\mu^{\varepsilon-1})\right]^{1/[\varepsilon(\zeta-1)+1]} \to \infty$.

constant reset wage.

In such a pure-strategy symmetric steady-state equilibrium, the aggregate state s is represented by the aggregate adjustment probability $\bar{\alpha}$. As shown in the Appendix, aggregate consumption is

$$C(\bar{\alpha}) = \left(\frac{\varepsilon - 1}{\varepsilon\chi\zeta}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}} \frac{r(\bar{\alpha}, 1)^{\frac{1}{\varepsilon-1}}}{g(\bar{\alpha}, \beta)^{\frac{1}{\varepsilon(\zeta-1)+1}}}, \tag{17}$$

where

$$r(\alpha, \beta) = \frac{1 + \beta(1-\alpha)\mu^{\varepsilon-1}}{1 + \beta(1-\alpha)}. \tag{18}$$

Note that $N(\bar{\alpha}) = C(\bar{\alpha})$ and $\lambda(\bar{\alpha}) = C(\bar{\alpha})^{-\sigma}$. Hence, (8) can be written as

$$\begin{aligned} \pi(x^*(\alpha); s(\bar{\alpha})) &= \lambda(\bar{\alpha})(x^*(\alpha)C(\bar{\alpha}))^{1-\varepsilon} N(\bar{\alpha}) - \chi\left[(x^*(\alpha)C(\bar{\alpha}))^{-\varepsilon} N(\bar{\alpha})\right]^{\zeta} \\ &= C(\bar{\alpha})^{(1-\varepsilon)\zeta} x^*(\alpha)^{-\varepsilon\zeta}\left(x^*(\alpha)^{1-\varepsilon+\varepsilon\zeta} - \chi\right), \end{aligned} \tag{19}$$

where $\zeta + \sigma - 2 + \varepsilon(1-\zeta) = 0$ is imposed. Since $(1-\varepsilon)\zeta < 0$ and $x^*(\alpha)^{1-\varepsilon+\varepsilon\zeta} > \chi$, $\pi(x^*(\alpha); s(\bar{\alpha}))$ decreases with $C(\bar{\alpha})$. This feature is important for the following analysis.

Consider the best response of an individual household's adjustment probability α to the aggregate adjustment probability $\bar{\alpha}$:

$$\alpha(\bar{\alpha}) = \arg\max v(\alpha; s(\bar{\alpha})). \tag{20}$$

A pure-strategy symmetric steady-state equilibrium is a fixed point of the best-response correspondence, and any fixed point of the best-response correspondence is a pure-strategy symmetric steady-state equilibrium.

From the result of Lemma 1, (13) is rewritten as

$$v(\alpha; s(\bar{\alpha})) = \frac{\Pi_{SUM}(\alpha, \bar{\alpha}) - C_{SUM}(\alpha)}{1-\beta}, \tag{21}$$

where

$$\Pi_{SUM}(\alpha,\bar{\alpha}) = \frac{\pi(x^*(\alpha);s(\bar{\alpha}))+\beta(1-\alpha)\pi(\frac{x^*(\alpha)}{\mu};s(\bar{\alpha}))}{1+\beta(1-\alpha)}$$
$$= \left(\frac{\varepsilon\zeta-\varepsilon+1}{\varepsilon\zeta}\right)\left(\frac{D(\alpha,\beta)}{D(\bar{\alpha},\beta)^{\zeta}}\right)^{\varepsilon-1}\left(\frac{r(\alpha,\beta)}{r(\alpha,1)}\right), \quad (22)$$

$$C_{SUM}(\alpha) = \frac{\beta\alpha E\left[\varpi|\varpi<G^{-1}(\alpha)\right]+\beta^2(1-\alpha)E(\varpi)}{1+\beta(1-\alpha)}, \quad (23)$$

and

$$D(\alpha,\beta) = \left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}} \frac{r(\alpha,1)^{\frac{1}{\varepsilon-1}}}{g(\alpha,\beta)^{\frac{1}{\varepsilon(\zeta-1)+1}}}. \quad (24)$$

See the Appendix for the derivation of (22).

The following lemma characterizes $D(\alpha,\beta)$, which determines aggregate consumption $C(\bar{\alpha})$ when $\alpha=\bar{\alpha}$, as shown in (17).

Lemma 2 (Lemma 2 of John and Wolman (2008)) *For $\alpha\in[0,1]$, (i) when β is sufficiently small, $\partial D(\alpha,\beta)/\partial\alpha<0$; (ii) when β is sufficiently large, there exists $\tilde{\alpha}$ in $(0,1)$ such that $\partial D(\alpha,\beta)/\partial\alpha<0$ for $\alpha<\tilde{\alpha}$ and $\partial D(\alpha,\beta)/\partial\alpha>0$ for $\alpha>\tilde{\alpha}$; (iii) when β is sufficiently large, $D(\alpha,\beta)$ attains its maximum on $[0,1]$ at $\alpha=1$.*

Proof. *See the Appendix.* ■

On the one hand, a higher α means more adjusting households. Since under a positive inflation rate, adjusting households increase their wage, an increase in α tends to increase the wage (price) index, which lowers aggregate consumption. This is reflected in $\partial r(\alpha,1)/\partial\alpha<0$. On the other hand, as discussed above, an increase in α lowers the reset wage, which is reflected in $\partial g(\alpha,\beta)/\partial\alpha<0$. This works to lower the wage (price) index, which increases aggregate consumption. When β is low, the first effect dominates the second because households discount the future highly and α does not substantially affect the reset wage. When β is higher, the sign of $\partial D(\alpha,\beta)/\partial\alpha$ depends on the relative strengths of the two effects. For sufficiently large β, the contribution of the second effect increases as α increases. Hence, the sign of $\partial D(\alpha,\beta)/\partial\alpha$ switches from negative to positive as α increases.[14]

Next, the best-response correspondence (20) is analyzed. Given $\bar{\alpha}$, there could be multiple local maxima for $v(\alpha; s(\bar{\alpha}))$. Hence, determining the optimal α requires comparison between local maxima. First, there could be one or multiple local maxima for $\alpha \in (0,1)$. As in John and Wolman (2004, 2008), such local maxima are called the interior arm of the best-response correspondence and they are defined for $\alpha \in (0,1)$ as

$$\alpha^{int}(\bar{\alpha}) = \left\{\alpha : \frac{\partial v(\alpha; s(\bar{\alpha}))}{\partial \alpha} = 0 \text{ and } \frac{\partial^2 v(\alpha; s(\bar{\alpha}))}{\partial \alpha^2} < 0\right\}. \tag{25}$$

Second, there could be a local maximum at $\alpha = 1$, which occurs when $\partial v(\alpha; s(\bar{\alpha}))/\partial\alpha > 0$ at $\alpha = 1$. As in John and Wolman (2004, 2008), the local maximum is called the flexible arm of the best-response correspondence. Note that setting $\alpha = 0$ or not adjusting a wage under any realization of adjustment costs cannot be a global maximum. Hence, the case is not considered below.

The following analysis considers a case whereby if $\alpha^{int}(\bar{\alpha})$ exists, $\alpha^{int}(\bar{\alpha})$ is unique for $\bar{\alpha} \in [0,1]$. Numerical analysis suggests that this is always the case.[15] In such a case, the best-response correspondence is defined as $\alpha(\bar{\alpha}) = \alpha^{int}(\bar{\alpha})$ if $v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha})) \geq v(1; s(\bar{\alpha}))$ and $\alpha(\bar{\alpha}) = 1$ if $v(1; s(\bar{\alpha})) \geq v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))$. I start with analyzing the interior arm of the best-response correspondence in the next lemma.

Lemma 3 (Lemma 3 of John and Wolman (2008)) *(i) For small β, the interior arm of the best-response correspondence exhibits complementarity everywhere; (ii) As $\beta \to 1$, the interior arm of the best-response correspondence does not exhibit complementarity at any fixed point; (iii) As $\beta \to 1$, the interior arm of the best-response correspondence has a unique fixed point α^*.*

[14] For intermediate β, the sign of $\partial D(\alpha,\beta)/\partial\alpha$ could switch from positive to negative as α increases.

[15] John and Wolman (2004, 2008) numerically show the same for the Dotsey, King, and Wolman (1999) state-dependent pricing model. However, the finding in the present study is not wholly obvious because the relevant equations are different for price setting and wage setting. It could be fruitful for future research to examine conditions under which a unique interior local maximum exists for $v(\alpha; s(\bar{\alpha}))$.

Proof. *See the Appendix.* ■

Complementarity means that an increase in the aggregate adjustment probability $\bar{\alpha}$ leads to an increase in the individual adjustment probability α, which requires that the marginal utility of increasing α, $\partial v(\alpha; s(\bar{\alpha}))/\partial\alpha$, must increase with $\bar{\alpha}$. Recall that the term relating to wage-setting costs in (21), $C_{SUM}(\alpha)$, is independent of $\bar{\alpha}$. Hence, complementarity requires that

$$\frac{\partial \Pi_{SUM}(\alpha, \bar{\alpha})}{\partial \alpha} = \frac{\beta \left[\pi(x^*(\alpha); s(\bar{\alpha})) - \pi(\frac{x^*(\alpha)}{\mu}; s(\bar{\alpha}))\right]}{[1+\beta(1-\alpha)]^2} \tag{26}$$

increases with $\bar{\alpha}$ or $\partial^2 \Pi_{SUM}(\alpha, \bar{\alpha})/\partial\alpha\partial\bar{\alpha} > 0$.

Consider sufficiently small β. As Lemma 2 (i) shows, an increase in $\bar{\alpha}$ decreases aggregate consumption. Hence, as shown in (19), static utility increases in both the current and next periods proportionally. Moreover, for small β, the reset wage is closer to the static optimal wage for the current period, meaning that the numerator of (26) is positive. Thus, $\partial \Pi_{SUM}(\alpha, \bar{\alpha})/\partial\alpha$ is positive and increases with $\bar{\alpha}$, which implies that $\partial^2 \Pi_{SUM}(\alpha, \bar{\alpha})/\partial\alpha\partial\bar{\alpha} > 0$.

Now consider sufficiently large β close to 1. As Lemma 2 (ii) shows, for $\bar{\alpha} < \tilde{\alpha}$, an increase in $\bar{\alpha}$ decreases aggregate consumption and increases static utility. Further, at a fixed point, α is relatively low, too. Hence, the reset wage is closer to the static optimal wage for the next period, meaning that the numerator of (26) is likely to be negative. Accordingly, $\partial \Pi_{SUM}(\alpha, \bar{\alpha})/\partial\alpha$ is negative and decreases with $\bar{\alpha}$. That is, increasing $\bar{\alpha}$ makes $\partial \Pi_{SUM}(\alpha, \bar{\alpha})/\partial\alpha$ more negative. By contrast, for $\bar{\alpha} > \tilde{\alpha}$, an increase in $\bar{\alpha}$ increases aggregate consumption and decreases static utility. Further, α is relatively high at a fixed point and the reset wage is closer to the current static optimal wage, meaning that the numerator of (26) is likely to be positive. Thus, $\partial \Pi_{SUM}(\alpha, \bar{\alpha})/\partial\alpha$ is positive and decreases with $\bar{\alpha}$. In summary, for β close to 1, it is likely that $\partial^2 \Pi_{SUM}(\alpha, \bar{\alpha})/\partial\alpha\partial\bar{\alpha} < 0$ at a fixed point and that the interior arm of the best-response correspondence does not show complementarity. Although it is possible that $\partial^2 \Pi_{SUM}(\alpha, \bar{\alpha})/\partial\alpha\partial\bar{\alpha} > 0$, such a possibility disappears as $\beta \to 1$.

The discussion thus far shows that for sufficiently large β, multiple equilibria with sticky wages ($\alpha < 1$) are unlikely to exist. However, there could be two equilibria, one with sticky wages and the other with flexible wages ($\alpha = 1$). The next proposition gives the necessary conditions for such multiple equilibria and the sufficient conditions for ruling them out.

Proposition 4 (Proposition 4 of John and Wolman (2008)) *Let β be sufficiently large such that the interior arm has a unique fixed point, denoted by α^*. Let $\hat{\alpha}$ be as defined in (60) in the Appendix. (i) As $\beta \to 1$, the necessary conditions for multiple equilibria are $\alpha^* < \hat{\alpha}$ and $v(\alpha^{int}(\hat{\alpha}); s(\hat{\alpha})) < v(1; s(\hat{\alpha}))$; (ii) As $\beta \to 1$, multiple symmetric steady-state equilibria are ruled out if*

$$\frac{\varepsilon\zeta - \varepsilon + 1}{\varepsilon\zeta}\left(\frac{\varepsilon - 1}{\varepsilon\chi\zeta}\right)^{\frac{(\varepsilon-1)(1-\zeta)}{\varepsilon(\zeta-1)+1}}\left\{\left[\frac{\left(\frac{1+\mu^{\varepsilon-1}}{2}\right)^{-\zeta}}{\left(\frac{1+\mu^{\varepsilon\zeta}}{1+\mu^{\varepsilon-1}}\right)^{-\frac{\zeta(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}\right] - \left[\frac{\left(\frac{1+\mu^{\varepsilon-1}}{2}\right)^{-(\zeta-1)}}{\left(\frac{1+\mu^{\varepsilon\zeta}}{1+\mu^{\varepsilon-1}}\right)^{-\frac{(\zeta-1)(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}\right]\right\} > E(\varpi) \tag{27}$$

or

$$E(\varpi) - C_{SUM}(\hat{\alpha}) >$$
$$\frac{\varepsilon\zeta - \varepsilon + 1}{\varepsilon\zeta}\left(\frac{\varepsilon - 1}{\varepsilon\chi\zeta}\right)^{\frac{(\varepsilon-1)(1-\zeta)}{\varepsilon(\zeta-1)+1}}\left\{\frac{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon\zeta}}{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}\right]^{\frac{\zeta(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}{1+(1-\hat{\alpha})}\right]^{\zeta}} - \frac{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon\zeta}}{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}\right]^{\frac{(\zeta-1)(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}{1+(1-\hat{\alpha})}\right]^{\zeta-1}}\right\}. \tag{28}$$

Proof. *See the Appendix.* ■

To obtain two equilibria, one with sticky wages and the other with flexible wages, the best-response correspondence is the interior arm first, has a fixed point $\alpha^* < 1$, and thereafter moves up to the flexible arm. As shown in the Appendix, as $\beta \to 1$, such an upward jump of the best-response correspondence is not possible when $\bar{\alpha} \geq \hat{\alpha}$. Thus, the best-response correspondence moves up at $\bar{\alpha} < \hat{\alpha}$ and α^* must be smaller than $\hat{\alpha}$. Note also that $v(\alpha^*; s(\alpha^*)) \geq v(1; s(\alpha^*))$ because α^* is the optimum when $\bar{\alpha} = \alpha^*$. To obtain an equilibrium with flexible wages, $v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha})) < v(1; s(\bar{\alpha}))$ must hold for some $\bar{\alpha} \in [\alpha^*, \hat{\alpha})$. Since, as shown in the Appendix, $v(1; s(\bar{\alpha}))$ increases with $\bar{\alpha}$ more rapidly than $v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))$ does for $\bar{\alpha} \in [\alpha^*, \hat{\alpha})$, a necessary condition for $v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha})) < v(1; s(\bar{\alpha}))$ for $\bar{\alpha} \in [\alpha^*, \hat{\alpha})$ is $v(\alpha^*(\hat{\alpha}); s(\hat{\alpha})) < v(1; s(\hat{\alpha}))$.

The second part of the proposition gives conditions for ruling out multiple equilibria. The first condition (27) implies that when adjustment costs are small, sticky wages cannot be an equilibrium: The best response at $\bar{\alpha} = \alpha^*$ is $\alpha = 1$ and thus α^* is not an equilibrium. The second condition (28) suggests that when adjustment costs are large, flexible wages cannot be an equilibrium.

These two conditions rule out multiple equilibria for most long-run inflation rates. For the benchmark parameterization considered in Section 4, for example, multiple equilibria are ruled out except when the annual inflation rate is 1.19–1.67%. Note that these conditions are sufficient but not necessary for ruling out multiple equilibria, one with sticky wages and the other with flexible wages.

4 Numerical Approach

This section numerically analyzes the uniqueness of steady-state equilibrium in the state-dependent wage-setting model.

Benchmark parameter values are standard and determined as follows. One period in the model is one quarter. The Frisch labor supply elasticity is 1: $\zeta = 2$. The coefficient of relative risk aversion σ is 2. The assumption made in Section 3 (i.e., $\zeta + \sigma - 2 + \varepsilon(1 - \zeta) = 0$) then implies that the elasticity of substitution for differentiated labor services ε is 2, which lies in the range considered by Huang and Liu (2002). The disutility parameter χ is 6.75, so that when wage-setting costs are eliminated from the present model, households use one-third of their time endowment, which is normalized to 1, for working.

As in Dotsey, King, and Wolman (1999), the inverse of the distribution of wage-setting

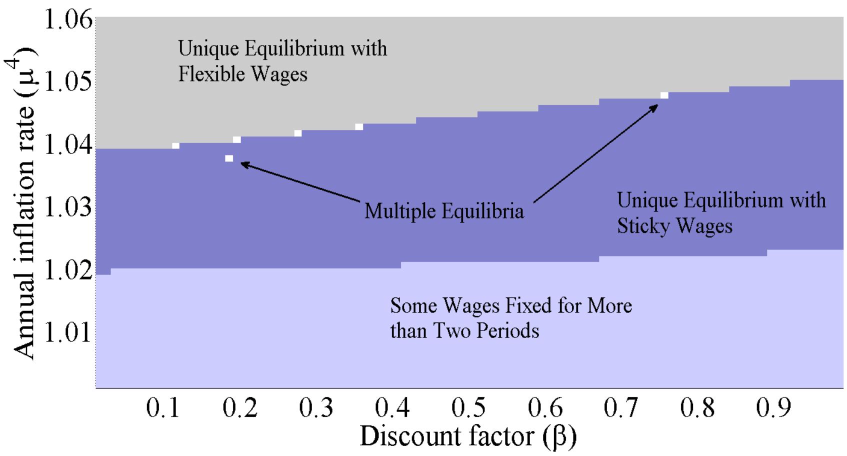

Figure 1: Benchmark: Discount factor.

cost is, for $z \in [0, 1]$,

$$G^{-1}(z) = \bar{\varpi} \frac{\text{atan}(bz - d\pi) + \text{atan}(d\pi)}{\text{atan}(b - d\pi) + \text{atan}(d\pi)}, \tag{29}$$

where $b = 16$ and $d = 1$ (see Figure 2 of Dotsey, King, and Wolman (1999)). The maximum wage-setting cost $\bar{\varpi}$ is 0.0004, so that as shown below, some wages are fixed for exactly two periods when the annual inflation rate is around 2–4%, which is in line with recent experiences in developed countries. The maximum wage-setting cost is relatively small: It is equivalent to 0.013% of the equilibrium consumption when wage-setting costs are removed from the present model.

I start by analyzing how the number of steady-state equilibria depends on the discount factor β and the inflation rate μ. For β, 99 values linearly spaced between 0.01 and 0.99 are considered. The typical calibration is $\beta = 0.99$, which implies that the real annual interest rate is 4%. For μ, 60 annual inflation rates (μ^4) linearly spaced from 0.1% to 6% are considered. In the United States, the annual CPI inflation rate has not exceeded 6% in the last 20 years.

The result is presented in Figure 1. As is consistent with the analytical result in Section 3, multiple sticky-wage equilibria arise only when the discount factor is low. Specifically, this occurs in only one case: there are two sticky-wage equilibria when the discount factor β is

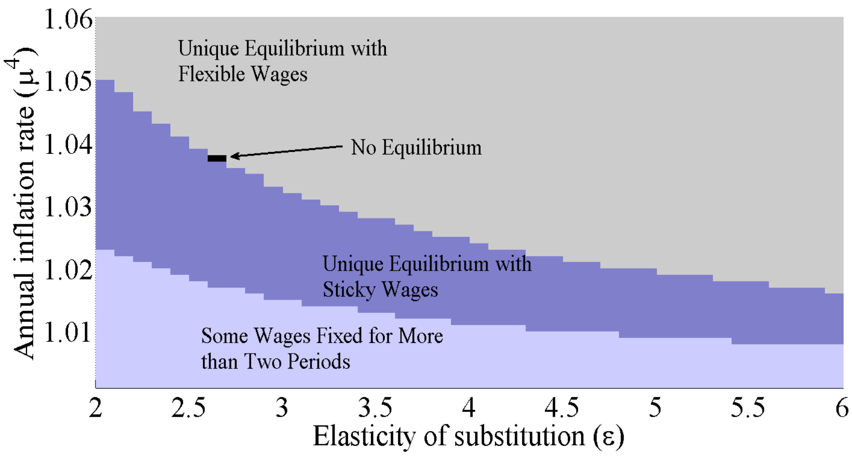

Figure 2: Benchmark: Elasticity of substitution.

0.18 and the annual inflation rate is 3.7% ($\mu^4 = 1.037$). Note that strategic complementarity in wage setting is a necessary condition for such multiple equilibria, but not a sufficient condition. For relatively low β, several cases lead to two equilibria, one with sticky wages and the other with flexible wages, in the region between the unique sticky-wage equilibrium and the unique flexible-wage equilibrium. However, such multiple equilibria disappear for $\beta > 0.75$. Thus, steady-state equilibrium is unique when the discount factor takes a standard value close to 1.

I next vary the elasticity of substitution for differentiated labor services ε and the inflation rate μ. For ε, 41 values linearly spaced between 2 and 6, which is the range considered by Huang and Liu (2002), are investigated. I keep other parameters unchanged from the baseline case and $\beta = 0.99$.[16] As shown in Figure 2, multiple equilibria do not exist for any case.

However, there is a case in which a pure-strategy symmetric steady-state equilibrium does not exist, as found by John and Wolman (2004, 2008) for the Dotsey, King, and Wolman (1999) model. The non-existence case occurs when the best-response correspondence jumps down from the flexible arm to the interior arm: In Figure 2, it occurs when the elasticity of substitution for differentiated labor services ε is 2.6 and the annual inflation rate is 3.7%

[16]The Appendix explains how the optimal wage and aggregate consumption are computed for a general case in which the optimal wage depends on the aggregate adjustment probability.

($\mu^4 = 1.037$).

Note also that wages become more flexible as the elasticity of substitution for different labor types ε increases. As ε increases, individual labor hours change with the relative wage more elastically. Hence, households choose to adjust their wage more frequently to smooth their labor hours.[17] The effect is strong. For example, when $\varepsilon = 21$, which is the value used by Christiano, Eichenbaum, and Evans (2005), the maximum wage-setting cost $\bar{\varpi}$ needs to be 0.039 or 1.28% of the equilibrium consumption of a case without wage-setting costs, if some wages are to be fixed for more than three periods (9 months) for an annual inflation rate of 2%.[18] Hence, the cost must be more than 100 times larger than the benchmark case.

The distribution of wage-setting costs could also be important. Here I report the result for a uniform distribution between 0 and $\bar{\varpi} = 0.0004$. Other parameters inherit their original value with $\beta = 0.99$. A uniform distribution implies a higher share of households drawing an intermediate wage-setting cost compared to the benchmark distribution, and the impact of state dependency in wage setting is expected to become stronger. However, as shown in Figure 3, the number of steady-state equilibria is reasonably similar to the benchmark case and multiple equilibria do not arise when the discount factor is close to 1. The uniqueness of steady-state equilibrium also holds under other distributions for wage-setting costs.[19]

Lastly, I examine a higher inflation rate than that considered so far. Specifically, I consider a situation where some wages are fixed for exactly two periods when the annual inflation rate is around 10%. For that, the maximum wage-setting cost $\bar{\varpi}$ is increased 10-fold to 0.004, which is 0.13% of the equilibrium consumption of a case without wage-setting costs. Other parameters are fixed at their benchmark value. Figure 4 shows the result of changing the discount factor β when the elasticity of substitution for differentiated labor service ε is

[17] In a state-dependent wage-setting model similar to that herein, Takahashi (2017) finds that an increase in the elasticity of substitution for differentiated labor services could reduce aggregate wage stickiness in response to monetary shocks and thereby money nonneutrality. By contrast, in time-dependent wage-setting models, a higher elasticity typically increases aggregate wage stickiness and short-run money nonneutrality (e.g., Huang and Liu (2002)).

[18] I find that the main result of the present study, the uniqueness of steady-state equilibrium, holds even under higher elasticity of substitution for differentiated labor services than the range considered here.

[19] These results are available upon request.

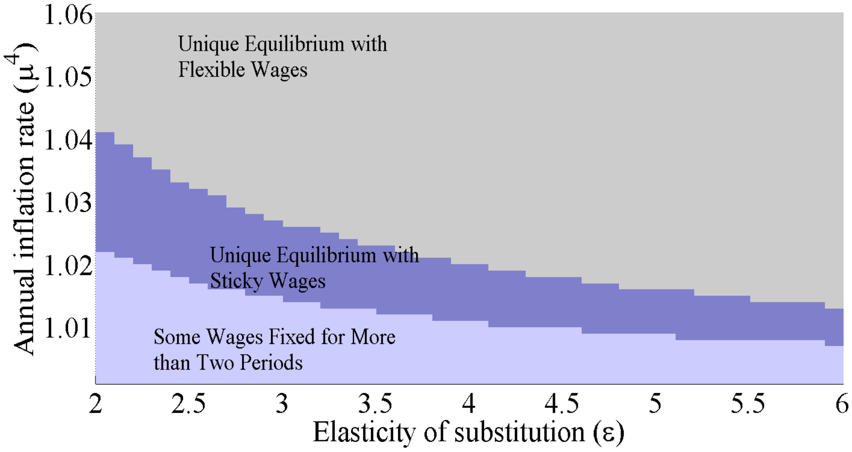

Figure 3: Uniform distribution: Elasticity of substitution.

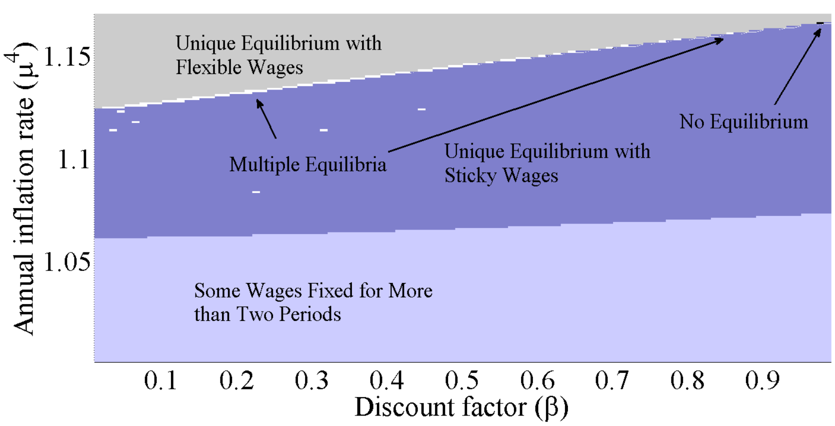

Figure 4: High inflation: Discount factor.

2. Figure 5 presents the result of changing ε when β is 0.99. As shown, multiple equilibria occur more frequently than in the benchmark case. However, the main result of the present study is robust: Multiple equilibria do not arise when the discount factor β is high and close to 1. For example, when $\varepsilon = 2$, multiple equilibria disappear for $\beta \geq 0.85$.

In summary, the substantial numerical analyses presented in this section support the analytical results reported in Section 3. Multiple steady-state equilibria do not exist under typical and empirically plausible parameter values in the dynamic state-dependent wage-setting model.

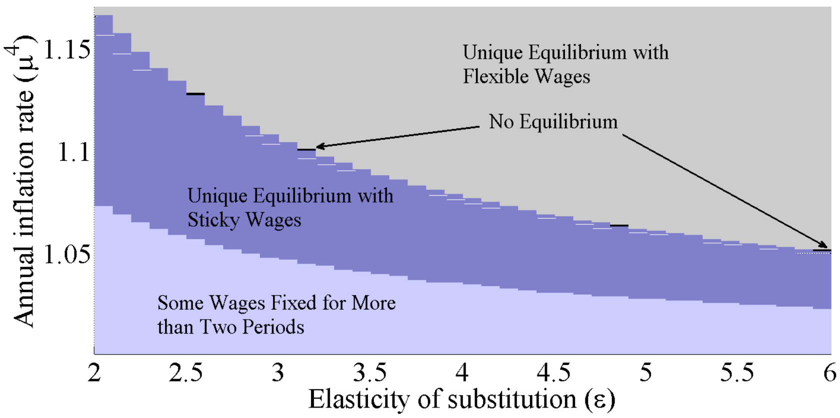

Figure 5: High inflation: Elasticity of substitution.

5 Extensions

This section considers several extensions of the benchmark model and confirms that the uniqueness of steady-state equilibrium in the state-dependent wage-setting model is robust.

5.1 Imperfect Consumption Insurance

Following most prior studies in the New Keynesian literature, the benchmark model assumes perfect insurance for consumption. However, the presence of complete asset markets might be too strong an assumption. Indeed, recent studies relax this assumption and analyze a New Keynesian model with imperfect consumption insurance. As such, this subsection considers imperfect risk sharing in the present state-dependent wage-setting model. Specifically, an extreme situation is considered in which all households live hand to mouth and consume their labor income in each period. Hence, both insurance markets and savings vehicles are excluded.[20]

Since households consume their labor income each period, $Pc(h) = W(h)n(h)$. Current

[20] The benchmark model implicitly assumes that seigniorage revenue is returned to households in a lump-sum way. The present case with incomplete markets assumes that the economy is cashless and that as in Nakamura and Steinsson (2010), the central bank executes monetary policy by keeping the growth rate of nominal GDP constant, interpreting M_t as nominal GDP. This avoids discussions on the redistribution of seigniorage revenue.

utility is then given by

$$\begin{aligned}\pi^{IM}(x(h);s) &= \frac{c(h)^{1-\sigma}}{1-\sigma} - \chi n^s(h)^\zeta \\ &= \frac{\left[\left(\frac{W(h)}{W}\right)^{1-\varepsilon} N\right]^{1-\sigma}}{1-\sigma} - \chi\left[\left(\frac{W(h)}{W}\right)^{-\varepsilon} N\right]^\zeta \\ &= \frac{\left[(x(h)C)^{1-\varepsilon} N\right]^{1-\sigma}}{1-\sigma} - \chi\left[(x(h)C)^{-\varepsilon} N\right]^\zeta . \end{aligned} \tag{30}$$

Let $v^{IM}(\alpha;s)$ be the value of an adjusting household that has a constant adjustment probability α and sets the associated optimal wage $x^*(\alpha;s)$, under the aggregate state s. The value is gross of the current adjustment cost and it is written as

$$v^{IM}(\alpha;s) = \frac{\pi^{IM}(x^*(\alpha;s);s) + \beta(1-\alpha)\pi^{IM}\left(\frac{x^*(\alpha;s)}{\mu};s\right) - \beta\alpha E\left[\varpi|\varpi < G^{-1}(\alpha)\right] - \beta^2(1-\alpha)E(\varpi)}{(1-\beta)[1+\beta(1-\alpha)]}. \tag{31}$$

A numerical method is used to analyze the incomplete markets model because analytical investigation is not possible.[21] Utility $\pi^{IM}(x(h);s)$ in (30) and the value $v^{IM}(\alpha;s)$ in (31) are computed using the optimal wage and aggregate consumption, both of which are derived in the Appendix. Parameter values are as per the baseline case and $\beta = 0.99$.

Figure 6 shows the number of steady-state equilibria for the annual inflation rate μ^4 and the elasticity of substitution for differentiated labor services ε. The main conclusion of the present study is robust: Multiple steady-state equilibria do not exist when the discount factor is close to 1.

Note also that wages become more flexible under incomplete markets than under complete markets. Under incomplete markets, not only do non-adjusting households work longer than adjusting households, they also consume more. Meanwhile, because of the curvature of

[21] Like Lemma 1, it is possible to derive a condition that renders the optimal wage independent of aggregate consumption. However, even when the discount factor is close to 1, the sign of the partial derivative of aggregate consumption with respect to the aggregate adjustment probability depends on the inflation rate and parameters of the utility function.

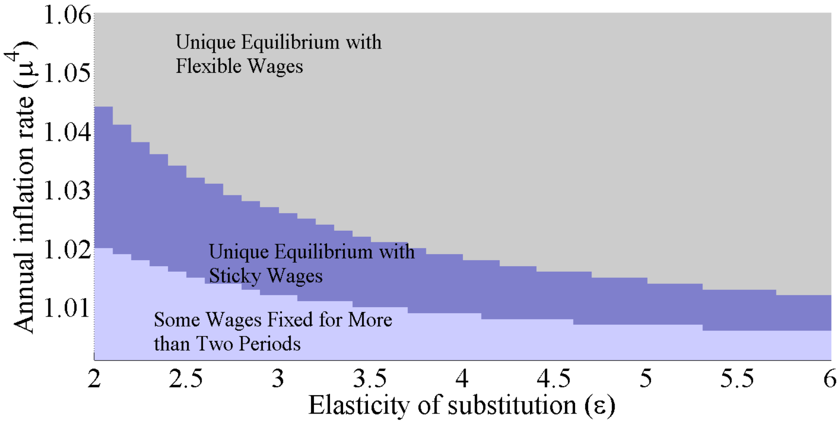

Figure 6: Incomplete markets: Elasticity of substitution.

the utility function, households prefer to smooth consumption. Hence, households choose to adjust their wage more frequently. As a result, imperfect consumption insurance increases wage flexibility and lowers the threshold inflation rate for the flexible-wage equilibrium relative to the benchmark model with complete asset markets.

5.2 Deflation

The analysis thus far has assumed a positive inflation rate, $\mu > 1$. This is empirically justifiable because the average inflation rate is positive in most countries, even though deflation occurs temporarily. One exception is Japan, where mild deflation started in the mid-1990s and continued until the mid-2010s. The CPI inflation rate has become slightly positive in recent years, but concern remains that deflation could soon again follow. Motivated by the Japanese experience, this subsection examines deflation, $\mu \in (0, 1)$.

The analytical derivation for the inflation case is carried over to the deflation case and the optimal wage is characterized as in Lemma 1. However, for $\mu \in (0, 1)$, households decrease their wage over time. In particular, for $\alpha \in [0, 1)$ and $\beta \in (0, 1)$,

$$\mu < \left(\frac{1 + \mu^{\varepsilon\zeta}}{1 + \mu^{\varepsilon-1}} \right)^{\frac{1}{\varepsilon(\zeta-1)+1}} < g(\alpha, \beta)^{\frac{1}{\varepsilon(\zeta-1)+1}} < 1. \tag{32}$$

As in the case of inflation, adjusting households charge a wage that is between the static optimal wage for the current period W^* and that for the next period $W^{*'} = x^* M' = \mu W^*$. However, the static optimal wage is lower for the next period than for the current period. Note also that $g(\alpha, \beta)$ increases with α and decreases with β. As α increases, households will more likely to decrease their wage in the next period. Thus, the reset wage increases and becomes closer to the current static optimal wage. In contrast, as β increases, households put a larger weight on the next period. Hence, the reset wage decreases and becomes closer to the static optimal wage for the next period.

The deflation case differs from the inflation case in terms of how increasing the aggregate adjustment probability affects aggregate consumption. Specifically, Lemma 2 is modified as follows.

Lemma 5 *Suppose that* $\mu \in (0, 1)$. *For* $\alpha \in [0, 1]$ *and* $\beta \in [0, 1)$, $\partial D(\alpha, \beta)/\partial\alpha > 0$ *and* $D(\alpha, \beta)$ *attains its maximum on* $[0, 1]$ *at* $\alpha = 1$.

Proof. *See the Appendix.* ■

On the one hand, a higher α means more adjusting households. Since under $\mu \in (0, 1)$, adjusting households decrease their wage, an increase in α tends to decrease the wage (price) index, which increases aggregate consumption. This is reflected in $\partial r(\alpha, 1)/\partial\alpha > 0$. On the other hand, as discussed above, an increase in α raises the reset wage, which is reflected in $\partial g(\alpha, \beta)/\partial\alpha > 0$ for $\mu \in (0, 1)$. This tends to raise the wage (price) index, which decreases aggregate consumption. Unlike the case of $\mu > 1$, the first effect always dominates the second for $\mu \in (0, 1)$. Therefore, $\partial D(\alpha, \beta)/\partial\alpha > 0$.

The property of the interior arm of the best-response correspondence is also altered. In particular, Lemma 3 is modified as follows.

Lemma 6 *Suppose that* $\mu \in (0, 1)$. *For* $\beta \in [0, 1)$, *the interior arm of the best-response correspondence does not exhibit complementarity anywhere and the interior arm of the best-response correspondence has a unique fixed point* α^*.

Proof. *See the Appendix.* ■

While the Appendix provides the formal proof, (26) gives the intuition. As shown in Lemma 5, $\partial D(\alpha,\beta)/\partial\alpha > 0$ for $\beta \in [0,1)$ and $\alpha \in [0,1]$. Hence, an increase in $\bar{\alpha}$ decreases static utility in the current and next periods proportionally. Further, unlike the case with $\mu > 1$, the numerator of (26) is always positive: $\pi(x^*) - \pi(x^*/\mu) > 0$.[22] That is, the reset wage is closer to the static optimal wage in the current period than to the static optimal wage in the next period. For $\mu \in (0,1)$, households' wage (relative to money stock) is higher and thus their labor hours are lower in the future period than in the current period, making households put a smaller weight on the future period. In sum, $\partial^2\Pi_{SUM}(\alpha,\bar{\alpha})/\partial\alpha\partial\bar{\alpha} < 0$ and the interior arm of the best-response correspondence does not exhibit complementarity for any $\beta \in [0,1)$. As a result, there is a unique fixed point.

The analysis so far shows that under deflation, $\mu \in (0,1)$, multiple sticky-wage equilibria do not exist for any discount factor $\beta \in [0,1)$. However, two equilibria, one with sticky wages and the other with flexible wages, might exist. The next proposition, which is a modification of Proposition 4, rules out such multiple equilibria as $\beta \to 1$.

Proposition 7 *Suppose that $\mu \in (0,1)$. Multiple equilibria are ruled out as $\beta \to 1$.*

Proof. *See the Appendix.* ■

To obtain two equilibria, one with sticky wages and the other with flexible wages, the best-response correspondence is the interior arm first, has a fixed point $\alpha^* < 1$, and thereafter moves up to the flexible arm. As shown in the Appendix, such an upward jump of the best-response correspondence is not possible as $\beta \to 1$. Note that while Lemmas 5 and 6 hold for $\beta \in [0,1)$, Proposition 7 requires $\beta \to 1$.

[22]Note that $\pi(x^*) - \pi(x^*/\mu)$ increases with x^* and with $g(\alpha,\beta) = (1+\mu^{\varepsilon\zeta})/(1+\mu^{\varepsilon-1})$,

$$sgn[\pi(x^*) - \pi(x^*/\mu)] = sgn[\frac{\varepsilon\zeta}{\varepsilon-1}\frac{1+\mu^{\varepsilon\zeta}}{1+\mu^{\varepsilon-1}}(1-\mu^{\varepsilon-1}) - (1-\mu^{\varepsilon\zeta})].$$

It is straightforward to show that the right-hand side is positive for $\mu \in (0,1)$.

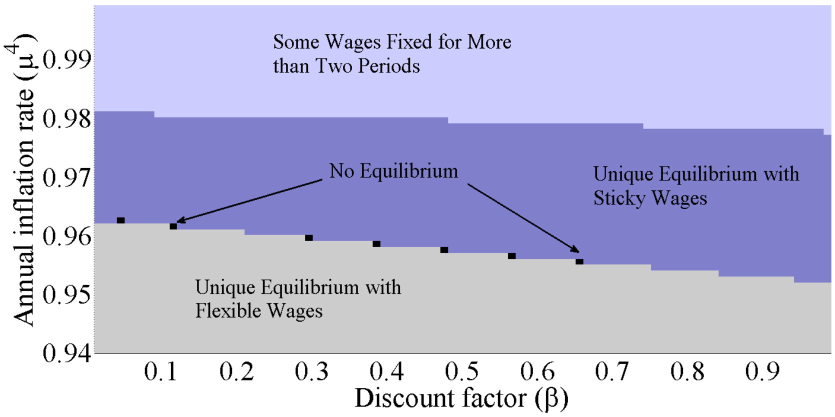

Figure 7: Deflation: Discount factor.

I move on to numerical analysis. Parameter values are as per the benchmark case. For the inflation rate μ, 60 annual inflation rates from –6% to –0.1% are examined. Figure 7 shows the result of varying the discount factor β and the inflation rate μ when the elasticity of substitution for differentiated labor services ε is 2. Consistent with the analytical result, multiple sticky-wage equilibria never arise. In addition, multiple equilibria, one with sticky wages and the other with flexible wages, also do not exist for $\beta \in [0, 1)$, even though it is proven analytically only for $\beta \to 1$. Hence, the unique equilibrium exists in most cases, although a non-existence case also arises. As shown in Figure 8, the uniqueness of equilibrium when β is close to 1 ($\beta = 0.99$) is robust to changing the value of ε.

5.3 Labor Costs for Wage Adjustments

This subsection considers labor costs for wage adjustments, instead of utility costs as in the benchmark model. Specifically, as in Takahashi (2017), each household uses composite labor to adjust their wage. This leads to two changes in the benchmark model. First, wage-setting costs need to be evaluated, multiplying the marginal utility of consumption and the real wage (constant to 1). Hence, the cost term in (21) is no longer independent of the aggregate adjustment probability. Second, some labor is used for wage changes. Thus, at the aggregate

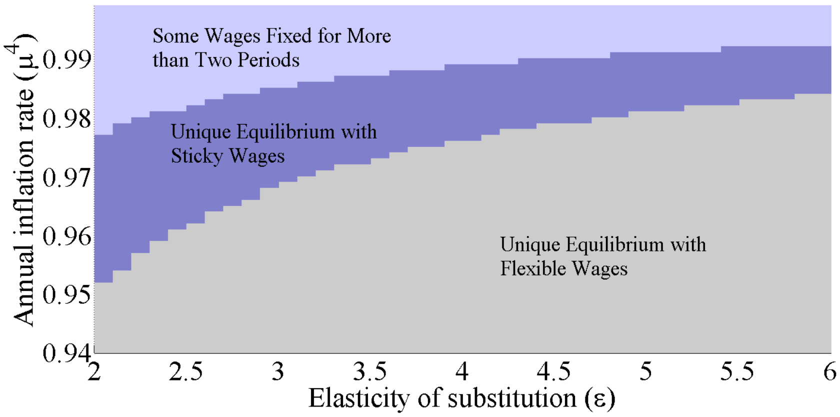

Figure 8: Deflation: Elasticity of substitution.

level, consumption is equal to total labor minus labor used for wage adjustments. Because of the two modifications, an analytical approach cannot be used for analyzing the model with labor costs for wage adjustments.

Accordingly, the extended model is solved numerically. Parameter values are as per the baseline model and $\beta = 0.99$. The maximum wage-setting cost $\bar{\varpi}$ is reset so that as in the baseline model, it is equal to 0.013% of the equilibrium consumption when wage-setting costs are removed from the present model.

Figure 9 shows how the number of steady-state equilibria varies with the inflation rate μ and the elasticity of substitution for differentiated labor services ε. The result is reasonably similar to that for the baseline model and multiple equilibria do not exist when the discount factor β is close to 1.

5.4 Interest-Elastic Money Demand

The benchmark model assumes that money demand is independent of a nominal interest rate. This subsection relaxes that assumption and instead assumes a positive interest elasticity of

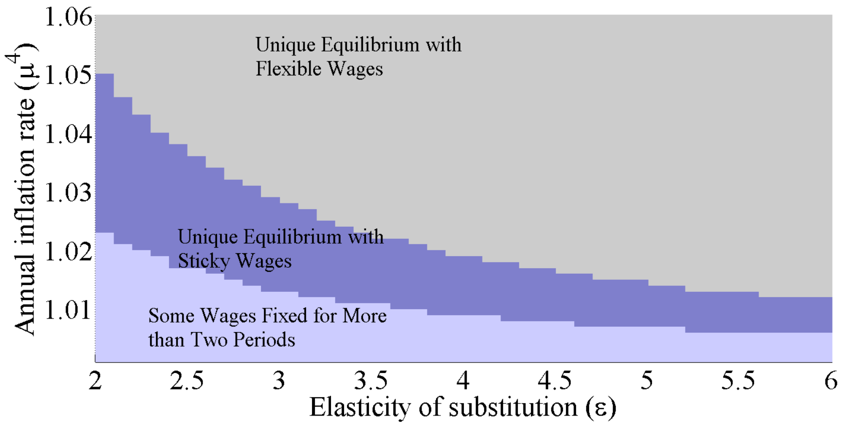

Figure 9: Labor costs for wage changes: Elasticity of substitution.

money demand. Specifically, money demand is given by

$$\ln \frac{M_t^d}{P_t} = \ln C_t - \eta R_t, \tag{33}$$

where $\eta > 0$ is the interest semi-elasticity of money demand and R_t is the net nominal interest rate. At steady-state equilibrium, $1 + R_t = \mu/\beta$ and thus

$$x_t(h) \equiv \frac{W_t(h)}{M_t} = \frac{W_t(h)e^{\eta\left(\frac{\mu}{\beta}-1\right)}}{W_t C_t}. \tag{34}$$

Let $v^{PE}(\alpha; s)$ be the value of an adjusting household that has a constant adjustment probability α and sets the associated optimal wage $x^*(\alpha; s)$, under the aggregate state s. Note that

$$v^{PE}(\alpha; s) = \frac{\pi^{PE}(x^*(\alpha; s); s) + \beta(1-\alpha)\pi^{PE}\left(\frac{x^*(\alpha;s)}{\mu}; s\right) - \beta\alpha E\left[\varpi | \varpi < G^{-1}(\alpha)\right] - \beta^2(1-\alpha)E(\varpi)}{(1-\beta)[1+\beta(1-\alpha)]}, \tag{35}$$

with

$$\pi^{PE}(x_t(h); s) = \lambda_t \left[e^{-\eta\left(\frac{\mu}{\beta}-1\right)} x_t(h) C_t\right]^{1-\varepsilon} N_t - \chi \left\{\left[e^{-\eta\left(\frac{\mu}{\beta}-1\right)} x_t(h) C_t\right]^{-\varepsilon} N_t\right\}^{\zeta}. \tag{36}$$

Under the same assumption as the benchmark model (i.e., $\zeta + \sigma - 2 + \varepsilon(1-\zeta) = 0$), the optimal wage becomes independent of the aggregate state s. Specifically, as shown in the Appendix, the optimal wage is given by

$$x^{PE*}(\alpha) = e^{\eta\left(\frac{\mu}{\beta}-1\right)} \left[\frac{\varepsilon\chi\zeta}{\varepsilon-1}\frac{1+\beta(1-\alpha)\mu^{\varepsilon\zeta}}{1+\beta(1-\alpha)\mu^{\varepsilon-1}}\right]^{\frac{1}{\varepsilon(\zeta-1)+1}} = e^{\eta\left(\frac{\mu}{\beta}-1\right)} \left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}g(\alpha,\beta)\right)^{\frac{1}{\varepsilon(\zeta-1)+1}}. \tag{37}$$

Recall that in the case of zero interest elasticity of money demand, the static optimal wage is $W^* = x^*M = [\varepsilon\chi\zeta/(\varepsilon-1)]^{1/[\varepsilon(\zeta-1)+1]}M$. With positive interest elasticity of money demand, M is effectively multiplied by $e^{\eta(\mu/\beta-1)}$, as are the static optimal wages for the current and next periods. Hence, the reset wage is also multiplied by $e^{\eta(\mu/\beta-1)}$.

As in the benchmark case, (35) is rewritten as

$$v^{PE}(\alpha; s(\bar{\alpha})) = \frac{\Pi^{PE}_{SUM}(\alpha,\bar{\alpha}) - C^{PE}_{SUM}(\alpha)}{1-\beta}, \tag{38}$$

where

$$\Pi^{PE}_{SUM}(\alpha,\bar{\alpha}) = e^{\eta\left(\frac{\mu}{\beta}-1\right)(\varepsilon-1)} \left(\frac{\varepsilon\zeta-\varepsilon+1}{\varepsilon\zeta}\right) \left(\frac{D^{PE}(\alpha,\beta)}{D^{PE}(\bar{\alpha},\beta)^{\zeta}}\right)^{\varepsilon-1} \left(\frac{r(\alpha,\beta)}{r(\alpha,1)}\right), \tag{39}$$

$$C^{PE}_{SUM}(\alpha) = C_{SUM}(\alpha), \tag{40}$$

and

$$D^{PE}(\alpha,\beta) = e^{-\eta\left(\frac{\mu}{\beta}-1\right)} \left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}} \frac{r(\alpha,1)^{\frac{1}{\varepsilon-1}}}{g(\alpha,\beta)^{\frac{1}{\varepsilon(\zeta-1)+1}}}. \tag{41}$$

See the Appendix for the derivation of (39).

It is straightforward to show that the arguments for the benchmark model can be applied to the present case. Specifically, Lemma 2 and Lemma 3 hold without any modifications. Hence, when the discount factor β is close to 1, it is unlikely that the interior arm of the best-response correspondence shows complementarity at a fixed point and multiple sticky-wage equilibria are unlikely to exist.

There is no change in the necessary conditions for multiple equilibria, one with sticky wages and the other with flexible wages. However, the sufficient conditions for ruling out such multiple equilibria are modified. Specifically, those conditions become

$$e^{\eta(\mu-1)\zeta(\varepsilon-1)}\left(\frac{\varepsilon\zeta-\varepsilon+1}{\varepsilon\zeta}\right)\left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{(\varepsilon-1)(1-\zeta)}{\varepsilon(\zeta-1)+1}}\left\{\left[\frac{\left(\frac{1+\mu^{\varepsilon-1}}{2}\right)^{-\zeta}}{\left(\frac{1+\mu^{\varepsilon\zeta}}{1+\mu^{\varepsilon-1}}\right)^{-\frac{\zeta(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}\right]-\left[\frac{\left(\frac{1+\mu^{\varepsilon-1}}{2}\right)^{-(\zeta-1)}}{\left(\frac{1+\mu^{\varepsilon\zeta}}{1+\mu^{\varepsilon-1}}\right)^{-\frac{(\zeta-1)(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}\right]\right\}>E(\varpi) \tag{42}$$

or

$$E(\varpi)-C_{SUM}(\hat{\alpha})>$$
$$e^{\eta(\mu-1)\zeta(\varepsilon-1)}\left(\frac{\varepsilon\zeta-\varepsilon+1}{\varepsilon\zeta}\right)\left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{(\varepsilon-1)(1-\zeta)}{\varepsilon(\zeta-1)+1}}\left\{\frac{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon\zeta}}{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}\right]^{\frac{\zeta(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}{1+(1-\hat{\alpha})}\right]^{\zeta}}-\frac{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon\zeta}}{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}\right]^{\frac{(\zeta-1)(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}{1+(1-\hat{\alpha})}\right]^{\zeta-1}}\right\}. \tag{43}$$

The positive interest elasticity of money demand renders (42) easier to satisfy than in the benchmark case, whereas (43) becomes more difficult to satisfy.

Next, a numerical method is used to analyze the uniqueness of steady-state equilibrium. I set $\eta = 4$, so that as the annualized nominal interest rate increases by 1 percentage point, real money demand decreases by 1% (Christiano, Eichenbaum, and Evans (2005)). Other parameters are fixed at their benchmark value.

For $\eta = 4$, as $\beta \to 1$, multiple equilibria, one with sticky wages and the other with flexible wages, are ruled out except when the annual inflation rate is 1.14–1.57%. For $\eta = 17.65$, which is the value used by Dotsey, King, and Wolman (1999), such multiple equilibria are ruled out except when the annual inflation rate is 1.00%–1.32%. Hence, multiple equilibria are ruled out for most long-run inflation rates. Recall that these conditions are sufficient, not necessary, for eliminating multiple equilibria.

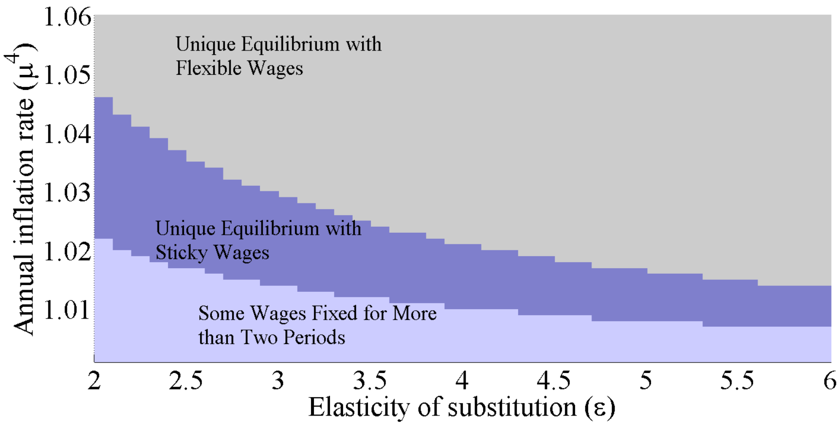

Figure 10: Interest-elastic money demand: Elasticity of substitution.

Figure 10 shows how the number of steady-state equilibria depends on the elasticity of substitution for differentiated labor services ε and the long-run inflation rate μ under $\eta = 4$ and $\beta = 0.99$. Multiple equilibria do not exist for any case. Thus, the main conclusion of the present study is unchanged. More generally, the result is quite similar to that in Figure 2. Consistent with recent experiences in most developed countries, μ is relatively low. Furthermore, β is close to 1, in line with the typical calibration for New Keynesian models. Hence, μ/β is close to 1 and setting $\eta > 0$ does not change the model's steady-state equilibrium substantially.

6 Conclusion

Nominal wage stickiness is an important issue in macroeconomics. Indeed, New Keynesian models, a modern framework for policy analysis, highlight welfare losses arising from staggered wage adjustments. It is thus important to analyze how the timing of individual wage adjustments is determined. However, most prior studies fix the timing of wage adjustments exogenously. Toward addressing this gap, the present study constructs a New Keynesian model with fixed costs for wage adjustments. The presence of fixed costs leads to infrequent and endogenous individual wage adjustments. I then analyze and explore whether such state-dependent wage setting generates multiple equilibria in the long run. Using analytical and numerical approaches, I find that multiple steady-state equilibria are unlikely to exist in a reasonably calibrated dynamic New Keynesian model with state dependency in wage setting.

For future research, it would be interesting to conduct welfare analyses and to compare the results with those under time-dependent wage setting. Furthermore, it is an open question whether and how state dependency in wage setting influences equilibrium determinacy under various short-run monetary policy rules. I leave these questions for future research.

References

Bakhshi, H., H. Kahn, and B. Rudolf (2007): "The Phillips Curve under State-Dependent Pricing," *Journal of Monetary Economics*, 54(8), 2321–2345.

Ball, L., and D. Romer (1991): "Sticky Prices as Coordination Failure," *American Economic Review*, 81(3), 539–552.

Blanchard, O. J., and N. Kiyotaki (1987): "Monopolistic Competition and the Effects of Aggregate Demand," *American Economic Review*, 77(4), 647–666.

Braun, A. R., and T. Nakajima (2012): "Uninsured Countercyclical Risk: An Aggregation Result and Application to Optimal Monetary Policy," *Journal of the European Economic Association*, 10(6), 1450–1474.

Calvo, G. A. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12(3), 383–398.

Caplin, A., and J. Leahy (1991): "State-Dependent Pricing and the Dynamics of Money and Output," *Quarterly Journal of Economics*, 106(3), 683–708.

Caplin, A. S., and D. F. Spulber (1987): "Menu Costs and the Neutrality of Money," *Quarterly Journal of Economics*, 102(4), 703–725.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Costain, J., and A. Nakov (2011a): "Distributional Dynamics Under Smoothly State-Dependent Pricing," *Journal of Monetary Economics*, 58(6-8), 646–665.

——— (2011b): "Price Adjustments in a General Model of State-Dependent Pricing," *Journal of Money, Credit and Banking*, 43(2-3), 385–406.

Costain, J., A. Nakov, and B. P. Zarzalejos (2017): "Monetary Policy Implications of State-Dependent Prices and Wages," mimeo.

Daly, M., and B. Hobijn (2014): "Downward Nominal Wage Rigidities Bend the Phillips Curve," *Federal Reserve Bank of San Francisco*, 2013-08.

Daly, M., B. Hobijn, and B. Lucking (2012): "Why Has Wage Growth Stayed Strong?," *FRBSF Economic Letter*, 2012-11.

Devereux, M. B., and H. E. Siu (2007): "State Dependent Pricing and Business Cycle Asymmetries," *International Economic Review*, 48(1), 281–310.

Dotsey, M., and R. G. King (2005): "Implications of State-Dependent Pricing for Dynamic Macroeconomic Models," *Journal of Monetary Economics*, 52(1), 213–242.

——— (2006): "Pricing, Production, and Persistence," *Journal of the European Economic Association*, 4(5), 893–928.

Dotsey, M., R. G. King, and A. L. Wolman (1999): "State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output," *Quarterly Journal of Economics*, 114(2), 655–690.

ERCEG, C. J., D. W. HENDERSON, AND A. T. LEVIN (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46(2), 281–313.

GERTLER, M., AND J. LEAHY (2008): "A Phillips Curve with an Ss Foundation," *Journal of Political Economy*, 116(3), 533–572.

GOLOSOV, M., AND R. E. LUCAS, JR. (2007): "Menu Costs and Phillips Curves," *Journal of Political Economy*, 115(2), 171–199.

GORNEMANN, N., K. KUESTER, AND M. NAKAJIMA (2016): "Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy," *International Finance Discussion Papers, Board of Governors of the Federal Reserve System.*

HIROSE, Y. (2014): "An Estimated DSGE Model with a Deflation Steady State," *CAMA Working Paper*, 52.

HUANG, K. X. D., AND Z. LIU (2002): "Staggered Price-Setting, Staggered Wage-Setting, and Business Cycle Persistence," *Journal of Monetary Economics*, 49(2), 405–433.

JOHN, A. A., AND A. L. WOLMAN (2004): "An Inquiry into the Existence and Uniqueness of Equilibrium with State-Dependent Pricing," *Federal Reserve Bank of Richmond Working Paper*, 4(4).

——— (2008): "Steady-State Equilibrium with State-Dependent Pricing," *Journal of Monetary Economics*, 55(2), 383–405.

KAPLAN, G., B. MOLL, AND G. VIOLANTE (2018): "Monetary Policy According to HANK," *American Economic Review*, 108(3), 697–743.

KLENOW, P. J., AND O. KRYVTSOV (2008): "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?," *Quarterly Journal of Economics*, 123(3), 863–904.

LANDRY, A. (2009): "Expectations and Exchange Rate Dynamics: A State-Dependent Pricing Approach," *Journal of International Economics*, 78(1), 60–71.

——— (2010): "State-Dependent Pricing, Local-Currency Pricing, and Exchange Rate Pass-Through," *Journal of Economic Dynamics and Control*, 34(10), 1859–1871.

MIDRIGAN, V. (2011): "Menu Costs, Multiproduct Firms, and Aggregate Fluctuations," *Econometrica*, 79(4), 1139–1180.

NAKAMURA, E., AND J. STEINSSON (2010): "Monetary Non-Neutrality in a Multi-Sector Menu Cost Model," *Quarterly Journal of Economics*, 125(3), 961–1013.

NAKOV, A., AND C. THOMAS (2014): "Optimal Monetary Policy with State-Dependent Pricing," *International Journal of Central Banking*, 10(3), 49–94.

SMETS, F., AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

SUDO, N., K. UEDA, K. WATANABE, AND T. WATANABE (2018): "Working Less and Bargain Hunting More: Macroimplications of Sales During Japan's Lost Decade," *Journal of Money, Credit and Banking*, 50(2-3), 449–478.

TAKAHASHI, S. (2017): "State Dependency in Price and Wage Setting," *International Journal of Central Banking*, 13(1), 151–189.

TAYLOR, J. B. (1980): "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy*, 88(1), 1–23.

——— (1999): "Staggered Price and Wage Setting in Macroeconomics," in *Handbook of Macroeconomics, Volume 1*, ed. by J. B. Taylor, and M. Woodford, pp. 1009–1050. Elsevier Science B.V.

Ueda, K., K. Watanabe, and T. Watanabe (2018): "Product Turnover and the Cost of Living Index: Quality vs. Fashion Effects," *American Economic Journal: Macroeconomics*, forthcoming.

Watanabe, K., and T. Watanabe (2018): "Why Has Japan Failed to Escape from Deflation?," *Asian Economic Policy Review*, 13, 23–41.

Weinstein, D., and C. Broda (2008): "Exporting Deflation? Chinese Exports and Japanese Prices," *NBER Working Paper Series*, 13942.

Appendix

Proof of Lemma 1

From (8),

$$\frac{\partial \pi(x)}{\partial x} = (1-\varepsilon)\lambda x^{-\varepsilon} C^{1-\varepsilon} N + \chi\varepsilon\zeta x^{-\varepsilon\zeta-1} C^{-\varepsilon\zeta} N^{\zeta}. \tag{44}$$

Since adjusting households set x to maximize $\pi(x) + \beta(1-\alpha)\pi(x/\mu)$, the optimal wage x^* satisfies

$$\begin{aligned} &(1-\varepsilon)\lambda x^{*-\varepsilon} C^{1-\varepsilon} N + \chi\varepsilon\zeta x^{*-\varepsilon\zeta-1} C^{-\varepsilon\zeta} N^{\zeta} \\ &+\beta(1-\alpha)[(1-\varepsilon)\lambda x^{*-\varepsilon} C^{1-\varepsilon} N \mu^{\varepsilon-1} + \chi\varepsilon\zeta x^{*-\varepsilon\zeta-1} C^{-\varepsilon\zeta} N^{\zeta} \mu^{\varepsilon\zeta}] = 0. \end{aligned} \tag{45}$$

Rearranging (45),

$$x^{*\varepsilon(\zeta-1)+1} = \frac{\varepsilon\chi\zeta}{\varepsilon-1} \frac{1+\beta(1-\alpha)\mu^{\varepsilon\zeta}}{1+\beta(1-\alpha)\mu^{\varepsilon-1}} \frac{C^{-\varepsilon\zeta} N^{\zeta}}{\lambda C^{1-\varepsilon} N}. \tag{46}$$

Note that $\lambda = C^{-\sigma}$ and $N = C$. Then, (46) is written as

$$x^{*\varepsilon(\zeta-1)+1} = \frac{\varepsilon\chi\zeta}{\varepsilon-1} \frac{1+\beta(1-\alpha)\mu^{\varepsilon\zeta}}{1+\beta(1-\alpha)\mu^{\varepsilon-1}} C^{\zeta+\sigma-2+\varepsilon(1-\zeta)}. \tag{47}$$

Letting $\zeta + \sigma - 2 + \varepsilon(1-\zeta) = 0$ leads to (14).

Baseline Model

Consider a pure-strategy symmetric steady-state equilibrium. Let $\bar{\alpha}$ be the probability of adjusting wages in the current period when households adjusted their wage in the last period. Let $\bar{\omega}$ be the fraction of adjusting households in the current period. Since households certainly adjust their wage in the current period when they did not do so in the last period,

$$(1-\bar{\omega})+\bar{\alpha}\bar{\omega}=\bar{\omega} \text{ or } \bar{\omega}=\frac{1}{2-\bar{\alpha}}. \tag{48}$$

From (4),

$$W=\left[\bar{\omega}W^{*1-\varepsilon}+(1-\bar{\omega})\left(\frac{W^*}{\mu}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{49}$$

From (7), $M=M^s=M^d$, and $P=W$, (49) can be written as

$$\left[\bar{\omega}\left(\frac{W^*}{M}\right)^{1-\varepsilon}+(1-\bar{\omega})\left(\frac{W^*}{M\mu}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}C=1. \tag{50}$$

By the definition of x^* and (48),

$$\left[\frac{1}{2-\bar{\alpha}}\left(x^*(\bar{\alpha})\right)^{1-\varepsilon}+\frac{1-\bar{\alpha}}{2-\bar{\alpha}}\left(\frac{x^*(\bar{\alpha})}{\mu}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}C=1. \tag{51}$$

Using (47),

$$\left\{\begin{array}{c}\frac{1}{2-\bar{\alpha}}\left[\frac{\varepsilon\chi\zeta}{\varepsilon-1}g(\bar{\alpha},\beta)C^{\zeta+\sigma-2+\varepsilon(1-\zeta)}\right]^{\frac{1-\varepsilon}{\varepsilon(\zeta-1)+1}}\\ +\frac{1-\bar{\alpha}}{2-\bar{\alpha}}\left[\frac{\varepsilon\chi\zeta}{\varepsilon-1}g(\bar{\alpha},\beta)C^{\zeta+\sigma-2+\varepsilon(1-\zeta)}\right]^{\frac{1-\varepsilon}{\varepsilon(\zeta-1)+1}}\mu^{\varepsilon-1}\end{array}\right\}^{\frac{1}{1-\varepsilon}}C=1. \tag{52}$$

Rearranging (52),

$$C^{\frac{\xi+\sigma-1}{\varepsilon\xi-\varepsilon+1}}=\left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}}\frac{\left[\frac{1+(1-\bar{\alpha})\mu^{\varepsilon-1}}{1+(1-\bar{\alpha})}\right]^{\frac{1}{\varepsilon-1}}}{g(\bar{\alpha},\beta)^{\frac{1}{\varepsilon(\zeta-1)+1}}}. \tag{53}$$

Setting $\zeta+\sigma-2+\varepsilon(1-\zeta)=0$ leads to (17).

By (14), (17), and (19),

$$
\begin{aligned}
\Pi_{SUM}(\alpha,\bar{\alpha}) &= x^*(\alpha)^{-\varepsilon\zeta} C(s(\bar{\alpha}))^{(1-\varepsilon)\zeta} \frac{\left(x^*(\alpha)^{1-\varepsilon+\varepsilon\zeta} - \chi\right) + \beta(1-\alpha)\mu^{\varepsilon\zeta}\left(x^*(\alpha)^{1-\varepsilon+\varepsilon\zeta}\mu^{\varepsilon-1-\varepsilon\zeta} - \chi\right)}{1+\beta(1-\alpha)} \\
&= \left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}\right)^{-\frac{\varepsilon\zeta}{\varepsilon(\zeta-1)+1}} g(\alpha,\beta)^{-\frac{\varepsilon\zeta}{\varepsilon(\zeta-1)+1}} r(\bar{\alpha},1)^{-\zeta} \left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}\right)^{-\frac{(1-\varepsilon)\zeta}{\varepsilon(\zeta-1)+1}} g(\bar{\alpha},\beta)^{-\frac{(1-\varepsilon)\zeta}{\varepsilon(\zeta-1)+1}} \\
&\quad \frac{\left(\frac{\varepsilon\chi\zeta}{\varepsilon-1} g(\alpha,\beta) - \chi\right) + \beta(1-\alpha)\mu^{\varepsilon\zeta}\left(\frac{\varepsilon\chi\zeta}{\varepsilon-1} g(\alpha,\beta)\mu^{\varepsilon-1-\varepsilon\zeta} - \chi\right)}{1+\beta(1-\alpha)} \\
&= \left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}\right)^{-\frac{\zeta}{\varepsilon(\zeta-1)+1}} \frac{1}{r(\bar{\alpha},1)^{\zeta} g(\alpha,\beta)^{\frac{\varepsilon\zeta}{\varepsilon(\zeta-1)+1}} g(\bar{\alpha},\beta)^{\frac{(1-\varepsilon)\zeta}{\varepsilon(\zeta-1)+1}}} \\
&\quad \left\{\frac{\varepsilon\chi\zeta}{\varepsilon-1} \frac{g(\alpha,\beta)[1+\beta(1-\alpha)\mu^{\varepsilon-1}]}{1+\beta(1-\alpha)} - \frac{\chi[1+\beta(1-\alpha)\mu^{\varepsilon\zeta}]}{1+\beta(1-\alpha)}\right\} \\
&= \frac{\chi(\varepsilon\zeta-\varepsilon+1)}{\varepsilon-1} \left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}\right)^{-\frac{\zeta}{\varepsilon(\zeta-1)+1}} \frac{g(\alpha,\beta) r(\alpha,\beta)}{r(\bar{\alpha},1)^{\zeta} g(\alpha,\beta)^{\frac{\varepsilon\zeta}{\varepsilon(\zeta-1)+1}} g(\bar{\alpha},\beta)^{\frac{(1-\varepsilon)\zeta}{\varepsilon(\zeta-1)+1}}} \\
&= \frac{\chi(\varepsilon\zeta-\varepsilon+1)}{\varepsilon-1} \left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}\right)^{-\frac{\zeta}{\varepsilon(\zeta-1)+1}} \frac{g(\alpha,\beta)^{\frac{1-\varepsilon}{\varepsilon(\zeta-1)+1}} r(\alpha,\beta)}{g(\bar{\alpha},\beta)^{\frac{(1-\varepsilon)\zeta}{\varepsilon(\zeta-1)+1}} r(\bar{\alpha},1)^{\zeta}}. \qquad (54)
\end{aligned}
$$

Rearranging (54) with (24) leads to (22).

Proof of Lemma 2

Note that

$$
\frac{\frac{\partial D(\alpha,\beta)}{\partial\alpha}}{D(\alpha,\beta)} = \left(\frac{1}{\varepsilon-1}\right)\left\{\left[\frac{\frac{\partial r(\alpha,\beta)}{\partial\alpha}}{r(\alpha,\beta)} - \frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}\frac{\frac{\partial g(\alpha,\beta)}{\partial\alpha}}{g(\alpha,\beta)}\right] - \left(\frac{\frac{\partial r(\alpha,\beta)}{\partial\alpha}}{r(\alpha,\beta)} - \frac{\frac{\partial r(\alpha,1)}{\partial\alpha}}{r(\alpha,1)}\right)\right\}, \qquad (55)
$$

$$
\frac{\frac{\partial r(\alpha,\beta)}{\partial\alpha}}{r(\alpha,\beta)} = -\frac{\beta(\mu^{\varepsilon-1}-1)}{[1+\beta(1-\alpha)][1+\beta(1-\alpha)\mu^{\varepsilon-1}]}, \qquad (56)
$$

$$
\frac{\frac{\partial r(\alpha,1)}{\partial\alpha}}{r(\alpha,1)} = -\frac{(\mu^{\varepsilon-1}-1)}{[1+(1-\alpha)][1+(1-\alpha)\mu^{\varepsilon-1}]}, \qquad (57)
$$

and

$$
\frac{\frac{\partial g(\alpha,\beta)}{\partial\alpha}}{g(\alpha,\beta)} = -\frac{\beta(\mu^{\varepsilon\zeta}-\mu^{\varepsilon-1})}{[1+\beta(1-\alpha)\mu^{\varepsilon-1}][1+\beta(1-\alpha)\mu^{\varepsilon\zeta}]}. \qquad (58)
$$

With (56) and (58),

$$\begin{aligned}
&\frac{\frac{\partial r(\alpha,\beta)}{\partial\alpha}}{r(\alpha,\beta)} - \frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}\frac{\frac{\partial g(\alpha,\beta)}{\partial\alpha}}{g(\alpha,\beta)} \\
= & \frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}\frac{\beta(\mu^{\varepsilon\zeta}-\mu^{\varepsilon-1})}{[1+\beta(1-\alpha)\mu^{\varepsilon-1}][1+\beta(1-\alpha)\mu^{\varepsilon\zeta}]} - \frac{\beta(\mu^{\varepsilon-1}-1)}{[1+\beta(1-\alpha)][1+\beta(1-\alpha)\mu^{\varepsilon-1}]} \\
= & \beta\frac{\frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}(\mu^{\varepsilon\zeta}-\mu^{\varepsilon-1})[1+\beta(1-\alpha)] - (\mu^{\varepsilon-1}-1)[1+\beta(1-\alpha)\mu^{\varepsilon\zeta}]}{[1+\beta(1-\alpha)][1+\beta(1-\alpha)\mu^{\varepsilon-1}][1+\beta(1-\alpha)\mu^{\varepsilon\zeta}]} \\
= & \beta\frac{\beta(1-\alpha)\left[\frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}(\mu^{\varepsilon\zeta}-\mu^{\varepsilon-1}) - (\mu^{\varepsilon-1}-1)\mu^{\varepsilon\zeta}\right] + \frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}(\mu^{\varepsilon\zeta}-\mu^{\varepsilon-1}) - (\mu^{\varepsilon-1}-1)}{[1+\beta(1-\alpha)][1+\beta(1-\alpha)\mu^{\varepsilon-1}][1+\beta(1-\alpha)\mu^{\varepsilon\zeta}]} \\
= & \frac{\beta\left[(\mu^{\varepsilon-1}-1)\mu^{\varepsilon\zeta} - \frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}(\mu^{\varepsilon\zeta}-\mu^{\varepsilon-1})\right]}{[1+\beta(1-\alpha)][1+\beta(1-\alpha)\mu^{\varepsilon-1}][1+\beta(1-\alpha)\mu^{\varepsilon\zeta}]} \\
& \left[\frac{\frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}(\mu^{\varepsilon\zeta}-\mu^{\varepsilon-1}) - (\mu^{\varepsilon-1}-1)}{(\mu^{\varepsilon-1}-1)\mu^{\varepsilon\zeta} - \frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}(\mu^{\varepsilon\zeta}-\mu^{\varepsilon-1})} - \beta(1-\alpha)\right] \\
= & \frac{\beta\left[(\mu^{\varepsilon-1}-1)\mu^{\varepsilon\zeta} - \frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}(\mu^{\varepsilon\zeta}-\mu^{\varepsilon-1})\right]}{[1+\beta(1-\alpha)][1+\beta(1-\alpha)\mu^{\varepsilon-1}][1+\beta(1-\alpha)\mu^{\varepsilon\zeta}]}[(1-\hat{\alpha}) - \beta(1-\alpha)], \qquad (59)
\end{aligned}$$

where

$$\hat{\alpha} = 1 - \frac{\frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}(\mu^{\varepsilon\zeta}-\mu^{\varepsilon-1}) - (\mu^{\varepsilon-1}-1)}{(\mu^{\varepsilon-1}-1)\mu^{\varepsilon\zeta} - \frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}(\mu^{\varepsilon\zeta}-\mu^{\varepsilon-1})}. \qquad (60)$$

By contrast, with (56) and (57),

$$\frac{\frac{\partial r(\alpha,\beta)}{\partial\alpha}}{r(\alpha,\beta)} - \frac{\frac{\partial r(\alpha,1)}{\partial\alpha}}{r(\alpha,1)} = \frac{(1-\beta)(\mu^{\varepsilon-1}-1)[1-\beta(1-\alpha)^2\mu^{\varepsilon-1}]}{[1+\beta(1-\alpha)][1+\beta(1-\alpha)\mu^{\varepsilon-1}][1+(1-\alpha)][1+(1-\alpha)\mu^{\varepsilon-1}]}. \qquad (61)$$

Substituting (59) and (61) into (55) leads to

$$\begin{aligned}
\frac{\frac{\partial D(\alpha,\beta)}{\partial\alpha}}{D(\alpha,\beta)} = & \left(\frac{1}{\varepsilon-1}\right)\frac{\beta\mu^{\varepsilon-1}\left[(\mu^{\varepsilon\zeta}-\mu^{\varepsilon\zeta-\varepsilon+1}) - \frac{\varepsilon-1}{\varepsilon\zeta-\varepsilon+1}(\mu^{\varepsilon\zeta-\varepsilon+1}-1)\right]}{[1+\beta(1-\alpha)][1+\beta(1-\alpha)\mu^{\varepsilon-1}][1+\beta(1-\alpha)\mu^{\varepsilon\zeta}]}[(1-\hat{\alpha})-\beta(1-\alpha)] \\
& -\left(\frac{1}{\varepsilon-1}\right)\frac{(1-\beta)(\mu^{\varepsilon-1}-1)[1-\beta(1-\alpha)^2\mu^{\varepsilon-1}]}{[1+\beta(1-\alpha)][1+\beta(1-\alpha)\mu^{\varepsilon-1}][1+(1-\alpha)][1+(1-\alpha)\mu^{\varepsilon-1}]}. \qquad (62)
\end{aligned}$$

(i) As $\beta \to 0$, the first term of (62) goes to 0. The second term goes to $-1/(\varepsilon-1) \cdot (\mu^{\varepsilon-1}-1)/\{[1+(1-\alpha)][1+(1-\alpha)\mu^{\varepsilon-1}]\}$. For $\mu > 1$, $\mu^{\varepsilon-1}-1 > 0$ and $\partial D(\alpha,\beta)/\partial\alpha < 0$

for all $\alpha \in [0,1]$.

(ii) Consider $\beta = 1$. The second term of (62) is 0. Note that $(\mu^{\varepsilon\zeta} - \mu^{\varepsilon\zeta-\varepsilon+1}) - (\varepsilon-1)/[\varepsilon(\zeta-1)+1] \cdot (\mu^{\varepsilon\zeta-\varepsilon+1} - 1) > 0$ for all $\mu > 0$ but 1.[23] Hence, $\partial D(\alpha,\beta)/\partial\alpha < 0$ for $\alpha < \hat{\alpha}$ and $\partial D(\alpha,\beta)/\partial\alpha > 0$ for $\alpha > \hat{\alpha}$. Furthermore, the first and second terms of (62) are continuous in β. Hence, when β is sufficiently close to 1, there exists $\tilde{\alpha}$ such that $\partial D(\alpha,\beta)/\partial\alpha < 0$ for $\alpha < \tilde{\alpha}$ and $\partial D(\alpha,\beta)/\partial\alpha > 0$ for $\alpha > \tilde{\alpha}$.

Note that $\hat{\alpha} \in (0,1)$. Let $\hat{\alpha} \equiv 1 - g(\mu)/f(\mu)$, where $g(\mu) \equiv (\varepsilon-1)/(\varepsilon\zeta-\varepsilon+1)\cdot(\mu^{\varepsilon\zeta-\varepsilon+1} - 1) - (1 - 1/\mu^{\varepsilon-1})$ and $f(\mu) \equiv \mu^{\varepsilon\zeta} - \mu^{\varepsilon\zeta-\varepsilon+1} - (\varepsilon-1)/(\varepsilon\zeta-\varepsilon+1)\cdot(\mu^{\varepsilon\zeta-\varepsilon+1} - 1)$. Note that $g(1) = f(1) = 0$. Furthermore, $f'(\mu) = \varepsilon\zeta(\mu^{\varepsilon\zeta-1} - \mu^{\varepsilon\zeta-\varepsilon})$ and $g'(\mu) = (\varepsilon-1)(\mu^{\varepsilon\zeta-\varepsilon} - \mu^{-\varepsilon})$. Hence, $f'(\mu) > 0$ and $g'(\mu) > 0$ for $\mu > 1$ and $f'(\mu) < 0$ and $g'(\mu) < 0$ for $\mu \in (0,1)$. These results ensure that $f(\mu) > 0$ and $g(\mu) > 0$ for all $\mu > 0$ and $\hat{\alpha} \in (0,1)$ means that $f(\mu) > g(\mu)$. Let $h(\mu) \equiv f(\mu) - g(\mu)$ and $h(1) = h'(1) = 0$. Furthermore, $h''(u) = f''(u) - g''(u) = \varepsilon\zeta(\varepsilon\zeta-1)(\mu^{\varepsilon\zeta-2} - \mu^{\varepsilon\zeta-\varepsilon-1}) + \varepsilon(\varepsilon-1)(\mu^{\varepsilon\zeta-\varepsilon-1} - \mu^{-\varepsilon-1})$. Hence, for $\mu > 1$, $h''(\mu) > 0$, which implies that $h'(\mu) > 0$ and $h(\mu) > 0$. Thus, $\hat{\alpha} \in (0,1)$ and $\tilde{\alpha} \in (0,1)$ for $\mu > 1$.

(iii) It is sufficient to show that $D(1,\beta) > D(0,\beta)$ because, as given in (ii), for β sufficiently close to 1, there exists $\tilde{\alpha} \in (0,1)$ such that $\partial D(\alpha,\beta)/\partial\alpha < 0$ for $\alpha < \tilde{\alpha}$ and $\partial D(\alpha,\beta)/\partial\alpha > 0$ for $\alpha > \tilde{\alpha}$. Note that

$$D(1,\beta) = \left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}} \frac{r(1,1)^{\frac{1}{\varepsilon-1}}}{g(1,\beta)^{\frac{1}{\varepsilon(\zeta-1)+1}}} = \left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}} \tag{63}$$

and

$$D(0,\beta) = \left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}} \frac{r(0,1)^{\frac{1}{\varepsilon-1}}}{g(0,\beta)^{\frac{1}{\varepsilon(\zeta-1)+1}}} = \left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}} \frac{\left(\frac{1+\mu^{\varepsilon-1}}{2}\right)^{\frac{1}{\varepsilon-1}}}{\left(\frac{1+\beta\mu^{\varepsilon\zeta}}{1+\beta\mu^{\varepsilon-1}}\right)^{\frac{1}{\varepsilon\zeta-\varepsilon+1}}}. \tag{64}$$

[23] Let $z(\mu) \equiv (\mu^{\varepsilon\zeta} - \mu^{\varepsilon\zeta-\varepsilon+1}) - (\varepsilon-1)/(\varepsilon\zeta-\varepsilon+1)\cdot(\mu^{\varepsilon\zeta-\varepsilon+1} - 1) = (\mu^{\varepsilon\zeta} - 1) - \varepsilon\zeta/(\varepsilon\zeta-\varepsilon+1)\cdot(\mu^{\varepsilon\zeta-\varepsilon+1} - 1)$. Note that $z(1) = 0$ and $z'(\mu) = \varepsilon\zeta\mu^{\varepsilon\zeta-1} - \varepsilon\zeta\mu^{\varepsilon\zeta-\varepsilon} = \varepsilon\zeta(\mu^{\varepsilon\zeta-1} - \mu^{\varepsilon\zeta-\varepsilon})$. Hence, $z'(\mu) < 0$ for $\mu \in (0,1)$ and $z'(\mu) > 0$ for $\mu > 1$. Thus, $z(\mu) > 0$ for $\mu > 0$.

Since $D(\alpha, \beta)$ is continuous in β, it is sufficient to show that $D(1,1) > D(0,1)$, which is equivalent to the following condition:

$$\ln(1+\mu^{\varepsilon\zeta}) > -\frac{\varepsilon\zeta - \varepsilon + 1}{\varepsilon - 1}\ln 2 + \frac{\varepsilon\zeta}{\varepsilon - 1}\ln(1+\mu^{\varepsilon-1}). \tag{65}$$

Let $m(\mu) \equiv \ln(1+\mu^{\varepsilon\zeta}) + (\varepsilon\zeta - \varepsilon + 1)/(\varepsilon - 1) \cdot \ln 2 - \varepsilon\zeta/(\varepsilon - 1) \cdot \ln(1+\mu^{\varepsilon-1})$. Note that $m(1) = 0$. Furthermore,

$$\begin{aligned} m'(\mu) &= \frac{\varepsilon\zeta\mu^{\varepsilon\zeta-1} + \varepsilon\zeta\mu^{\varepsilon\zeta+\varepsilon-2} - \varepsilon\zeta\mu^{\varepsilon-2} - \varepsilon\zeta\mu^{\varepsilon\zeta+\varepsilon-2}}{(1+\mu^{\varepsilon\zeta})(1+\mu^{\varepsilon-1})} \\ &= \frac{\varepsilon\zeta(\mu^{\varepsilon\zeta-1} - \mu^{\varepsilon-2})}{(1+\mu^{\varepsilon\zeta})(1+\mu^{\varepsilon-1})}. \end{aligned} \tag{66}$$

Since $\varepsilon\zeta - 1 > \varepsilon - 2$, $m'(\mu) > 0$ for $\mu > 1$. Hence, $m(\mu) > 0$ for $\mu > 1$, which implies that for sufficiently large β, $D(1, \beta) > D(0, \beta)$ and $D(\alpha, \beta)$ attains its maximum at $\alpha = 1$.

Proof of Lemma 3

Since (25) implies that $\partial v(\alpha^{int}; s(\bar{\alpha}))/\partial\alpha = 0$, by the implicit function theorem,

$$\frac{\partial \alpha^{int}}{\partial \bar{\alpha}} = -\frac{\frac{\partial^2 v(\alpha^{int}; s(\bar{\alpha}))}{\partial\alpha\partial\bar{\alpha}}}{\frac{\partial^2 v(\alpha^{int}; s(\bar{\alpha}))}{\partial\alpha^2}}. \tag{67}$$

By (25), the denominator of the right-hand side of (67) is negative. Hence,

$$\begin{aligned} sgn\left(\frac{\partial \alpha^{int}}{\partial \bar{\alpha}}\right) &= sgn\left(\frac{\partial^2 v(\alpha^{int}; s(\bar{\alpha}))}{\partial\alpha\partial\bar{\alpha}}\right) \\ &= sgn\left(\frac{\partial^2 \Pi_{SUM}(\alpha^{int}, \bar{\alpha})}{\partial\alpha\partial\bar{\alpha}}\right), \end{aligned} \tag{68}$$

where (21) is used. Note also that by (22),

$$\frac{\partial \Pi_{SUM}(\alpha, \bar{\alpha})}{\partial\alpha} = \Pi_{SUM}(\alpha, \bar{\alpha})\left[(\varepsilon - 1)\frac{\frac{\partial D(\alpha,\beta)}{\partial\alpha}}{D(\alpha, \beta)} + \frac{\frac{\partial r(\alpha,\beta)}{\partial\alpha}}{r(\alpha, \beta)} - \frac{\frac{\partial r(\alpha,1)}{\partial\alpha}}{r(\alpha, 1)}\right]. \tag{69}$$

(i) By (55) and (59), (69) can be written as

$$\begin{aligned}\frac{\partial \Pi_{SUM}(\alpha,\bar{\alpha})}{\partial \alpha} &= \Pi_{SUM}(\alpha,\bar{\alpha})\left[\frac{\frac{\partial r(\alpha,\beta)}{\partial \alpha}}{r(\alpha,\beta)} - \frac{\varepsilon-1}{\varepsilon(\zeta-1)+1}\frac{\frac{\partial g(\alpha,\beta)}{\partial \alpha}}{g(\alpha,\beta)}\right] \\ &= \Pi_{SUM}(\alpha,\bar{\alpha})\frac{\beta\left[(\mu^{\varepsilon-1}-1)\mu^{\varepsilon\zeta} - \frac{(\varepsilon-1)}{\varepsilon(\zeta-1)+1}(\mu^{\varepsilon\zeta}-\mu^{\varepsilon-1})\right]}{[1+\beta(1-\alpha)][1+\beta(1-\alpha)\mu^{\varepsilon-1}][1+\beta(1-\alpha)\mu^{\varepsilon\zeta}]} \\ &\quad \cdot[(1-\hat{\alpha})-\beta(1-\alpha)]. \qquad (70)\end{aligned}$$

When β is sufficiently small, $(1-\hat{\alpha})-\beta(1-\alpha)>0$ and $\partial\Pi_{SUM}(\alpha,\bar{\alpha})/\partial\alpha>0$. Note that $\bar{\alpha}$ appears only in $\Pi_{SUM}(\alpha,\bar{\alpha})$. As shown in Lemma 2 (i), when β is sufficiently small, $\partial D(\bar{\alpha},\beta)/\partial\alpha<0$. Furthermore, a decrease in $D(\bar{\alpha},\beta)$ or in aggregate consumption $C(\bar{\alpha})$ increases $\Pi_{SUM}(\alpha,\bar{\alpha})$. Thus, $\partial^2\Pi_{SUM}(\alpha,\bar{\alpha})/\partial\alpha\partial\bar{\alpha}>0$, and for sufficiently small β, $\partial\alpha^{int}/\partial\bar{\alpha}>0$ and the interior arm of the best-response correspondence shows complementarity for $\bar{\alpha}\in[0,1]$.

(ii)(iii) When $\beta=1$, (69) implies that

$$\frac{\partial^2\Pi_{SUM}(\alpha,\bar{\alpha})}{\partial\alpha\partial\bar{\alpha}} = (\varepsilon-1)\frac{\partial\Pi_{SUM}(\alpha,\bar{\alpha})}{\partial\bar{\alpha}}\frac{\frac{\partial D(\alpha,1)}{\partial\alpha}}{D(\alpha,1)}. \qquad (71)$$

Note that by (22)

$$\frac{\partial\Pi_{SUM}(\alpha,\bar{\alpha})}{\partial\bar{\alpha}} = -\frac{\zeta(\varepsilon-1)}{D(\bar{\alpha},\beta)}\frac{\partial D(\bar{\alpha},\beta)}{\partial\alpha}\Pi_{SUM}(\alpha,\bar{\alpha}). \qquad (72)$$

Combining (71) and (72) leads to

$$\frac{\partial^2\Pi_{SUM}(\alpha,\bar{\alpha})}{\partial\alpha\partial\bar{\alpha}} = -\frac{\zeta(\varepsilon-1)^2\Pi_{SUM}(\alpha,\bar{\alpha})}{D(\bar{\alpha},1)D(\alpha,1)}\left[\frac{\partial D(\bar{\alpha},1)}{\partial\alpha}\frac{\partial D(\alpha,1)}{\partial\alpha}\right]. \qquad (73)$$

At a fixed point, $\alpha^{int}=\bar{\alpha}$. Hence, $\partial^2\Pi_{SUM}(\alpha,\bar{\alpha})/\partial\alpha\partial\bar{\alpha}<0$ at any fixed point. Hence, $\partial\alpha^{int}/\partial\bar{\alpha}<0$ and the interior arm of the best-response correspondence does not show complementarity at any fixed point.

Next, consider β sufficiently close to 1. From (68), (70), and (72),

$$\begin{aligned} sgn\left(\frac{\partial\alpha^{int}}{\partial\bar{\alpha}}\right) &= sgn\left\{\frac{\partial\Pi_{SUM}(\alpha^{int},\bar{\alpha})}{\partial\bar{\alpha}}[(1-\hat{\alpha})-\beta(1-\alpha^{int})]\right\} \\ &= sgn\left[\frac{\partial D(\bar{\alpha},\beta)}{\partial\alpha}(\breve{\alpha}-\alpha^{int})\right], \end{aligned} \tag{74}$$

where $\breve{\alpha} \equiv [\hat{\alpha} - (1-\beta)]/\beta$.

When β is sufficiently close to 1, according to Lemma 2 (ii), $\partial D(\bar{\alpha},\beta)/\partial\alpha < 0$ for $\bar{\alpha} < \tilde{\alpha}$ and $\partial D(\bar{\alpha},\beta)/\partial\alpha > 0$ for $\bar{\alpha} > \tilde{\alpha}$. Hence, $\partial\alpha^{int}/\partial\bar{\alpha} > 0$ if $\bar{\alpha} < \tilde{\alpha}$ and $\alpha^{int} < \breve{\alpha}$ or $\bar{\alpha} > \tilde{\alpha}$ and $\alpha^{int} > \breve{\alpha}$. Define $\Lambda \equiv [\min(\breve{\alpha},\tilde{\alpha}),\max(\breve{\alpha},\tilde{\alpha})]$. Note that as $\beta \to 1$, $\Lambda \to \{\emptyset\}$ because $\tilde{\alpha} \to \hat{\alpha}$ and $\breve{\alpha} \to \hat{\alpha}$. Let a fixed point be $\alpha^* = \alpha^{int} = \bar{\alpha}$. For almost all cost distributions $G(\xi)$, $\alpha^* \notin \Lambda$ because as $\beta \to 1$, $\Lambda \to \{\emptyset\}$. Hence, at a fixed point, $\partial\alpha^{int}/\partial\bar{\alpha} < 0$. Since a necessary condition for multiple equilibria is $\partial\alpha^{int}/\partial\bar{\alpha} > 0$ at a fixed point, there should be a unique fixed point.

Proof of Proposition 4

(i) I start by showing that a necessary condition for multiple equilibria is $\alpha^* < \hat{\alpha}$. Suppose that $\alpha^* \geq \hat{\alpha}$. To obtain multiple equilibria, the best-response correspondence must jump up from the interior arm to the flexible arm at $\bar{\alpha} \in [\alpha^*, 1]$. However, this is not possible. First, $v(1; s(\alpha^*)) \leq v(\alpha^*; s(\alpha^*))$ because the best response is α^* for $\bar{\alpha} = \alpha^*$. Second, for $\bar{\alpha} \in [\alpha^*, 1]$, $v(1; s(\bar{\alpha}))$ decreases with $\bar{\alpha}$ more rapidly than $v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))$ and therefore $v(1; s(\bar{\alpha})) < v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))$ for $\bar{\alpha} \in [\alpha^*, 1]$, which implies that the best-response correspondence cannot jump up. Hence, a flexible-wage equilibrium does not exist and multiple equilibria cannot exist. The second point is shown as follows. For $\bar{\alpha} \in [\alpha^*, 1]$,

$$\begin{aligned} \frac{\partial v(1; s(\bar{\alpha}))}{\partial\bar{\alpha}} &= \frac{1}{1-\beta}\frac{\partial\Pi_{SUM}(1,\bar{\alpha})}{\partial\bar{\alpha}} \\ = -\frac{\zeta(\varepsilon-1)}{1-\beta}\left(\frac{D(1,\beta)}{D(\bar{\alpha},\beta)^{\zeta}}\right)^{\varepsilon-1} &\frac{r(1,\beta)}{r(1,1)}\frac{\frac{\partial D(\bar{\alpha},\beta)}{\partial\alpha}}{D(\bar{\alpha},\beta)} < 0, \end{aligned} \tag{75}$$

whereas

$$\frac{\partial v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))}{\partial \bar{\alpha}} = \frac{1}{1-\beta} \frac{\partial \Pi_{SUM}(\alpha^{int}(\bar{\alpha}), \bar{\alpha})}{\partial \bar{\alpha}}$$
$$= -\frac{\zeta(\varepsilon - 1)}{1-\beta} \left(\frac{D(\alpha^{int}(\bar{\alpha}), \beta)}{D(\bar{\alpha}, \beta)^{\zeta}} \right)^{\varepsilon - 1} \frac{r(\alpha^{int}(\bar{\alpha}), \beta)}{r(\alpha^{int}(\bar{\alpha}), 1)} \frac{\frac{\partial D(\bar{\alpha}, \beta)}{\partial \alpha}}{D(\bar{\alpha}, \beta)} < 0. \quad (76)$$

Since β is sufficiently large, from Lemma 2 (iii),

$$D(\alpha^{int}(\bar{\alpha}), \beta) < D(1, \beta). \quad (77)$$

Further, since $r(\alpha, \beta) < r(\alpha, 1)$ for $\alpha \in [0, 1)$,

$$\frac{r(\alpha^{int}(\bar{\alpha}), \beta)}{r(\alpha^{int}(\bar{\alpha}), 1)} < \frac{r(1, \beta)}{r(1, 1)} = 1. \quad (78)$$

By (77) and (78),

$$\left| \frac{\partial v(1; s(\bar{\alpha}))}{\partial \bar{\alpha}} \right| > \left| \frac{\partial v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))}{\partial \bar{\alpha}} \right|. \quad (79)$$

Thus, $v(1; s(\bar{\alpha})) < v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))$ for $\bar{\alpha} \in [\alpha^*, 1]$.

By contrast, suppose that $\alpha^* < \hat{\alpha}$. In this case, the best-response correspondence is initially the interior arm and then moves up to the flexible arm at $\bar{\alpha} \in [\alpha^*, \hat{\alpha})$. The reason is that as shown above, when a discontinuity of the best-response correspondence occurs at $\bar{\alpha} \geq \hat{\alpha}$, the best-response correspondence must move down. A necessary condition for an upward jump of the best-response correspondence at $\bar{\alpha} \in [\alpha^{**}, \hat{\alpha})$ is $v(1; s(\bar{\alpha})) > v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))$ at $\bar{\alpha} \in [\alpha^*, \hat{\alpha})$. At $\bar{\alpha} = \alpha^*$, $v(1; s(\alpha^*)) \leq v(\alpha^{int}(\alpha^*); s(\alpha^*))$. Both $v(1; s(\bar{\alpha}))$ and $v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))$ increase with $\bar{\alpha}$ for $\bar{\alpha} \in [\alpha^*, \hat{\alpha})$ because $\partial D(\bar{\alpha}, \beta)/\partial \alpha < 0$ for $\bar{\alpha} < \hat{\alpha}$ and as $\beta \to 1$ (Lemma 2 (ii)). Further, because of (77) and (78), $v(1; s(\bar{\alpha}))$ increases more rapidly than $v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))$ does. In other words, for $\bar{\alpha} \in [0, \hat{\alpha})$,

$$\frac{\partial v(1; s(\bar{\alpha}))}{\partial \bar{\alpha}} > \frac{\partial v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))}{\partial \bar{\alpha}}. \quad (80)$$

Hence, a necessary condition for $v(1;s(\bar{\alpha})) > v(\alpha^{int}(\bar{\alpha});s(\bar{\alpha}))$ for $\bar{\alpha} \in [\alpha^*, \hat{\alpha})$ is $v(1;s(\hat{\alpha})) > v(\alpha^{int}(\hat{\alpha});s(\hat{\alpha}))$.

(ii) The first condition implies that

$$\begin{aligned}
&\frac{\varepsilon\zeta - \varepsilon + 1}{\varepsilon\zeta}\left(\frac{\varepsilon - 1}{\varepsilon\chi\zeta}\right)^{\frac{(\varepsilon-1)(1-\zeta)}{\varepsilon(\zeta-1)+1}}\left\{\left[\frac{\left(\frac{1+\mu^{\varepsilon-1}}{2}\right)^{-\zeta}}{\left(\frac{1+\mu^{\varepsilon\zeta}}{1+\mu^{\varepsilon-1}}\right)^{-\frac{\zeta(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}\right] - \left[\frac{\left(\frac{1+\mu^{\varepsilon-1}}{2}\right)^{-(\zeta-1)}}{\left(\frac{1+\mu^{\varepsilon\zeta}}{1+\mu^{\varepsilon-1}}\right)^{-\frac{(\zeta-1)(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}\right]\right\} > E(\varpi) \\
&\Longrightarrow \frac{\varepsilon\zeta - \varepsilon + 1}{\varepsilon\zeta}\left[\left(\frac{D(1,1)}{D(0,1)^\zeta}\right)^{\varepsilon-1} - \left(\frac{D(0,1)}{D(0,1)^\zeta}\right)^{\varepsilon-1}\right] > E(\varpi) \\
&\Longrightarrow \frac{\varepsilon\zeta - \varepsilon + 1}{\varepsilon\zeta}\left[\left(\frac{D(1,1)}{D(0,1)^\zeta}\right)^{\varepsilon-1} - \left(\frac{D(\alpha^{int}(0),1)}{D(0,1)^\zeta}\right)^{\varepsilon-1}\right] \\
&\qquad > C_{SUM}(1) - C_{SUM}(\alpha^{int}(0)), \qquad (81)
\end{aligned}$$

where $D(\alpha^{int}(0),1) < D(0,1)$, $E(\zeta) = C_{SUM}(1)$, and $C_{SUM}(\alpha^{int}(0)) > 0$ are used from the second to third lines. As $\beta \to 1$, $r(\alpha,\beta)/r(\alpha,1) \to 1$. Hence, (81) implies that $v(1;s(0)) > v(\alpha^{int}(0);s(0))$. Since, as shown in (i), $v(1;s(\bar{\alpha}))$ increases more rapidly in $\bar{\alpha}$ than $v(\alpha^{int}(\bar{\alpha});s(\bar{\alpha}))$ does for $\bar{\alpha} \in [0,\hat{\alpha}]$, $v(1;s(\alpha^*)) > v(\alpha^*;s(\alpha^*))$, which implies that $\bar{\alpha} = \alpha^*$ is not the equilibrium. Hence, multiple equilibria are not possible.

The second condition implies that

$$\begin{aligned}
&E(\varpi) - C_{SUM}(\hat{\alpha}) > \frac{\varepsilon\zeta - \varepsilon + 1}{\varepsilon\zeta}\left(\frac{\varepsilon - 1}{\varepsilon\chi\zeta}\right)^{\frac{(\varepsilon-1)(1-\zeta)}{\varepsilon(\zeta-1)+1}} \\
&\cdot\left\{\frac{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon\zeta}}{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}\right]^{\frac{\zeta(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}{1+(1-\hat{\alpha})}\right]^{\zeta}} - \frac{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon\zeta}}{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}\right]^{\frac{(\zeta-1)(\varepsilon-1)}{\varepsilon(\zeta-1)+1}}}{\left[\frac{1+(1-\hat{\alpha})\mu^{\varepsilon-1}}{1+(1-\hat{\alpha})}\right]^{\zeta-1}}\right\} \\
&\Longrightarrow E(\varpi) - C_{SUM}(\hat{\alpha}) > \frac{\varepsilon\zeta - \varepsilon + 1}{\varepsilon\zeta}\left[\left(\frac{D(1,1)}{D(\hat{\alpha},1)^\zeta}\right)^{\varepsilon-1} - \left(\frac{D(\hat{\alpha},1)}{D(\hat{\alpha},1)^\zeta}\right)^{\varepsilon-1}\right] \\
&\Longrightarrow C_{SUM}(1) - C_{SUM}(\hat{\alpha}) > \Pi_{SUM}(1,\hat{\alpha}) - \Pi_{SUM}(\hat{\alpha},\hat{\alpha}) \\
&\Longrightarrow v(\hat{\alpha};s(\hat{\alpha})) > v(1;s(\hat{\alpha})) \\
&\Longrightarrow v(\alpha^{int}(\hat{\alpha});s(\hat{\alpha})) > v(1;s(\hat{\alpha})), \qquad (82)
\end{aligned}$$

where the last condition holds because $v(\alpha^{int}(\hat{\alpha}); s(\hat{\alpha})) > v(\hat{\alpha}; s(\hat{\alpha}))$. This violates a necessary condition for multiple equilibria.

Imperfect Consumption Insurance

From (30),

$$\frac{\partial \pi^{IM}(x)}{\partial x} = (1-\varepsilon)x^{(\varepsilon-1)\sigma-\varepsilon}C^{(\varepsilon-1)\sigma-\varepsilon+1}N^{1-\sigma} + \zeta\chi\varepsilon x^{-\varepsilon(\zeta-1)-\varepsilon-1}C^{-\varepsilon(\zeta-1)-\varepsilon}N^{\zeta}. \tag{83}$$

Since adjusting households set x to maximize $\pi^{IM}(x) + \beta(1-\alpha)\pi^{IM}(x/\mu)$, the optimal wage x^* satisfies

$$\begin{aligned} &(1-\varepsilon)x^{*(\varepsilon-1)\sigma-\varepsilon}C^{(\varepsilon-1)\sigma-\varepsilon+1}N^{1-\sigma} + \zeta\chi\varepsilon x^{*-\varepsilon(\zeta-1)-\varepsilon-1}C^{-\varepsilon(\zeta-1)-\varepsilon}N^{\zeta} \\ &+\beta(1-\alpha)\left[(1-\varepsilon)x^{*(\varepsilon-1)\sigma-\varepsilon}C^{(\varepsilon-1)\sigma-\varepsilon+1}N^{1-\sigma}\mu^{(1-\varepsilon)(\sigma-1)} + \zeta\chi\varepsilon x^{*-\varepsilon(\zeta-1)-\varepsilon-1}C^{-\varepsilon(\zeta-1)-\varepsilon}N^{\zeta}\mu^{\varepsilon\zeta}\right] = 0. \end{aligned} \tag{84}$$

Rearranging (84) with $N = C$ leads to

$$x^{IM*\varepsilon\zeta+1+\sigma(\varepsilon-1)-\varepsilon} = \frac{\varepsilon\chi\zeta}{\varepsilon-1}g^{IM}(\alpha,\beta)C^{\zeta(1-\varepsilon)-(\sigma-1)(\varepsilon-2)}, \tag{85}$$

where

$$g^{IM}(\alpha,\beta) \equiv \frac{1+\beta(1-\alpha)\mu^{\varepsilon\zeta}}{1+\beta(1-\alpha)\mu^{(1-\varepsilon)(\sigma-1)}}. \tag{86}$$

Putting (85) with $\alpha = \bar{\alpha}$ into (51) leads to

$$\left\{ \begin{array}{c} \frac{1}{2-\bar{\alpha}}\left[\left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}g^{IM}(\bar{\alpha},\beta)C^{\zeta(1-\varepsilon)-(\sigma-1)(\varepsilon-2)}\right)^{\frac{1}{\varepsilon(\zeta-1)+1+\sigma(\varepsilon-1)}}\right]^{1-\varepsilon} \\ +\frac{1-\bar{\alpha}}{2-\bar{\alpha}}\left[\frac{1}{\mu}\left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}g^{IM}(\bar{\alpha},\beta)C^{\zeta(1-\varepsilon)-(\sigma-1)(\varepsilon-2)}\right)^{\frac{1}{\varepsilon(\zeta-1)+1+\sigma(\varepsilon-1)}}\right]^{1-\varepsilon} \end{array} \right\}^{\frac{1}{1-\varepsilon}} C = 1. \tag{87}$$

Rearranging (87),

$$C^{IM\frac{\xi+\sigma-1}{\varepsilon\xi+1+\sigma(\varepsilon-1)-\varepsilon}} = \left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{1}{\varepsilon(\zeta-1)+1+\sigma(\varepsilon-1)}} \frac{\left[\frac{1+\beta(1-\bar{\alpha})\mu^{\varepsilon-1}}{1+\beta(1-\bar{\alpha})}\right]^{\frac{1}{\varepsilon-1}}}{g^{IM}(\bar{\alpha},\beta)^{\frac{1}{\varepsilon(\zeta-1)+1+\sigma(\varepsilon-1)}}}. \quad (88)$$

Proof of Lemma 5

Consider (62). According to the proof for Lemma 2, for $\mu \in (0,1)$, $(\mu^{\varepsilon\zeta} - \mu^{\varepsilon\zeta-\varepsilon+1}) - (\varepsilon - 1)/[\varepsilon(\zeta-1)+1] \cdot (\mu^{\varepsilon\zeta-\varepsilon+1} - 1) > 0$ and $\hat{\alpha} < 0$. Hence, the first term of (62) is positive. The second term is negative for $\mu \in (0,1)$. Thus, for $\alpha \in [0,1]$ and $\beta \in [0,1)$, $\partial D(\alpha,\beta)/\partial\alpha > 0$ and $D(\alpha,\beta)$ attains its maximum on $[0,1]$ at $\alpha = 1$.

Proof of Lemma 6

Consider (70) for $\beta \in [0,1)$. Since for $\mu \in (0,1)$, $(\mu^{\varepsilon-1}-1)\mu^{\varepsilon\zeta} - (\varepsilon-1)/[\varepsilon(\zeta-1)+1] \cdot (\mu^{\varepsilon\zeta} - \mu^{\varepsilon-1}) > 0$ and $\hat{\alpha} < 0$, $\partial\Pi_{SUM}(\alpha,\bar{\alpha})/\partial\alpha > 0$. Further, as shown in Lemma 5, $\partial D(\bar{\alpha},\beta)/\partial\alpha > 0$. Recall that an increase in $D(\bar{\alpha},\beta)$ or in aggregate consumption $C(\bar{\alpha})$ reduces $\Pi_{SUM}(\alpha,\bar{\alpha})$. Thus, for $\beta \in [0,1)$, $\partial^2\Pi_{SUM}(\alpha,\bar{\alpha})/\partial\alpha\partial\bar{\alpha} < 0$ and $\partial\alpha^{int}/\partial\bar{\alpha} < 0$, which implies that the interior arm of the best-response correspondence does not show complementarity for all $\bar{\alpha} \in [0,1]$ and it has a unique fixed point α^*.

Proof of Proposition 7

To obtain multiple equilibria, the best-response correspondence needs to move up from the interior arm to the flexible arm at $\bar{\alpha} \in [\alpha^*, 1]$. Note that $v(1; s(\alpha^*)) \leq v(\alpha^*; s(\alpha^*))$ because the best response is α^* at $\bar{\alpha} = \alpha^*$. Meanwhile, $D(\alpha^{int}(\bar{\alpha}),\beta) < D(1,\beta)$ by Lemma 5 and as $\beta \to 1$, $r(\alpha^{int}(\bar{\alpha}),\beta)/r(\alpha^{int}(\bar{\alpha}),1) \to 1$. Hence, as $\beta \to 1$, (79) holds, which implies that $v(1; s(\bar{\alpha}))$ decreases with $\bar{\alpha}$ more rapidly than $v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))$. Thus, as $\beta \to 1$, $v(1; s(\bar{\alpha})) < v(\alpha^{int}(\bar{\alpha}); s(\bar{\alpha}))$ for $\bar{\alpha} \in [\alpha^*, 1]$ and the flexible-wage equilibrium does not exist.

Interest-Elastic Money Demand

From (36),

$$\frac{\partial \pi^{PE}(x)}{\partial x} = (1-\varepsilon)e^{-\eta\left(\frac{\mu}{\beta}-1\right)(1-\varepsilon)}\lambda x^{-\varepsilon}C^{1-\varepsilon}N + \chi\varepsilon\zeta e^{\eta\left(\frac{\mu}{\beta}-1\right)\varepsilon\zeta}x^{-\varepsilon\zeta-1}C^{-\varepsilon\zeta}N^{\zeta}. \tag{89}$$

Since adjusting households set x to maximize $\pi^{PE}(x)+\beta(1-\alpha)\pi^{PE}(x/\mu)$, the optimal wage x^* satisfies

$$\begin{aligned} &(1-\varepsilon)e^{-\eta\left(\frac{\mu}{\beta}-1\right)(1-\varepsilon)}\lambda x^{*-\varepsilon}C^{1-\varepsilon}N + \chi\varepsilon\zeta e^{\eta\left(\frac{\mu}{\beta}-1\right)\varepsilon\zeta}x^{*-\varepsilon\zeta-1}C^{-\varepsilon\zeta}N^{\zeta} \\ &+\beta(1-\alpha)\left[(1-\varepsilon)e^{-\eta\left(\frac{\mu}{\beta}-1\right)(1-\varepsilon)}\lambda x^{*-\varepsilon}C^{1-\varepsilon}N\mu^{\varepsilon-1} + \chi\varepsilon\zeta e^{\eta\left(\frac{\mu}{\beta}-1\right)\varepsilon\zeta}x^{*-\varepsilon\zeta-1}C^{-\varepsilon\zeta}N^{\zeta}\mu^{\varepsilon\zeta}\right] = 0. \end{aligned} \tag{90}$$

Rearranging (90),

$$x^{PE*\varepsilon(\zeta-1)+1} = \frac{\varepsilon\chi\zeta}{\varepsilon-1}\frac{1+\beta(1-\alpha)\mu^{\varepsilon\zeta}}{1+\beta(1-\alpha)\mu^{\varepsilon-1}}e^{\eta\left(\frac{\mu}{\beta}-1\right)[\varepsilon(\zeta-1)+1]}\frac{C^{-\varepsilon\zeta}N^{\zeta}}{\lambda C^{1-\varepsilon}N}. \tag{91}$$

Note that $\lambda = C^{-\sigma}$ and $C = N$. Hence, (91) can be written as

$$x^{PE*\varepsilon(\zeta-1)+1} = \frac{\varepsilon\chi\zeta}{\varepsilon-1}\frac{1+\beta(1-\alpha)\mu^{\varepsilon\zeta}}{1+\beta(1-\alpha)\mu^{\varepsilon-1}}e^{\eta\left(\frac{\mu}{\beta}-1\right)[\varepsilon(\zeta-1)+1]}C^{\zeta+\sigma-2+\varepsilon(1-\zeta)}. \tag{92}$$

Letting $\zeta+\sigma-2+\varepsilon(1-\zeta)=0$ leads to (37).

Putting (92) with $\alpha=\bar{\alpha}$ into (51) leads to

$$\left\{ \begin{array}{c} \frac{1}{2-\bar{\alpha}}\left[e^{\eta\left(\frac{\mu}{\beta}-1\right)}\left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}g(\bar{\alpha},\beta)C^{\zeta+\sigma-2+\varepsilon(1-\zeta)}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}}\right]^{1-\varepsilon} \\ +\frac{1-\bar{\alpha}}{2-\bar{\alpha}}\left[\frac{1}{\mu}e^{\eta\left(\frac{\mu}{\beta}-1\right)}\left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}g(\bar{\alpha},\beta)C^{\zeta+\sigma-2+\varepsilon(1-\zeta)}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}}\right]^{1-\varepsilon} \end{array} \right\}^{\frac{1}{1-\varepsilon}} C = 1. \tag{93}$$

Rearranging (93),

$$C^{PE\frac{\xi+\sigma-1}{\varepsilon\xi-\varepsilon+1}} = e^{-\eta\left(\frac{\mu}{\beta}-1\right)} \left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{1}{\varepsilon(\zeta-1)+1}} \frac{\left[\frac{1+\beta(1-\bar{\alpha})\mu^{\varepsilon-1}}{1+\beta(1-\bar{\alpha})}\right]^{\frac{1}{\varepsilon-1}}}{g(\bar{\alpha},\beta)^{\frac{1}{\varepsilon(\zeta-1)+1}}}. \tag{94}$$

Given (37) and (94),

$$\begin{aligned}
\Pi_{SUM}^{PE}(\alpha,\bar{\alpha}) &= \frac{\pi^{PE}(x^*(\alpha);s(\bar{\alpha}))+\beta(1-\alpha)\pi^{PE}(\frac{x^*(\alpha)}{\mu};s(\bar{\alpha}))}{1+\beta(1-\alpha)} \\
&= x^{PE*}(\alpha)^{-\varepsilon\zeta}C^{PE}(\bar{\alpha})^{(1-\varepsilon)\zeta}\left(\frac{\beta}{\mu}\right)^{-\eta\varepsilon\zeta} \\
&\quad \frac{\left\{\begin{array}{c}\left[e^{-\eta\left(\frac{\mu}{\beta}-1\right)[\varepsilon(\zeta-1)+1]}x^{PE*}(\alpha)^{1-\varepsilon+\varepsilon\zeta}-\chi\right] \\ +\beta(1-\alpha)\mu^{\varepsilon\zeta}\left[e^{-\eta\left(\frac{\mu}{\beta}-1\right)[\varepsilon(\zeta-1)+1]}x^{PE*}(\alpha)^{1-\varepsilon+\varepsilon\zeta}\mu^{\varepsilon-1-\varepsilon\zeta}-\chi\right]\end{array}\right\}}{1+\beta(1-\alpha)} \\
&= e^{-\eta\left(\frac{\mu}{\beta}-1\right)(1-\varepsilon)\zeta}\left(\frac{\varepsilon-1}{\varepsilon\chi\zeta}\right)^{\frac{\zeta}{\varepsilon(\zeta-1)+1}} g(\alpha,\beta)^{-\frac{\varepsilon\zeta}{\varepsilon(\zeta-1)+1}} \frac{r(\bar{\alpha},1)^{\frac{(1-\varepsilon)\zeta}{\varepsilon-1}}}{g(\bar{\alpha},\beta)^{\frac{(1-\varepsilon)\zeta}{\varepsilon(\zeta-1)+1}}} \\
&\quad \frac{\left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}g(\alpha,\beta)-\chi\right)+\beta(1-\alpha)\mu^{\varepsilon\zeta}\left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}g(\alpha,\beta)\mu^{\varepsilon-1-\varepsilon\zeta}-\chi\right)}{1+\beta(1-\alpha)} \\
&= e^{-\eta\left(\frac{\mu}{\beta}-1\right)(1-\varepsilon)\zeta}\left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}\right)^{-\frac{\zeta}{\varepsilon(\zeta-1)+1}} \frac{1}{r(\bar{\alpha},1)^{\zeta}g(\alpha,\beta)^{\frac{\varepsilon\zeta}{\varepsilon(\zeta-1)+1}}g(\bar{\alpha},\beta)^{\frac{(1-\varepsilon)\zeta}{\varepsilon(\zeta-1)+1}}} \\
&\quad \left\{\frac{\varepsilon\chi\zeta}{\varepsilon-1}\frac{g(\alpha,\beta)[1+\beta(1-\alpha)\mu^{\varepsilon-1}]}{1+\beta(1-\alpha)}-\frac{\chi[1+\beta(1-\alpha)\mu^{\varepsilon\zeta}]}{1+\beta(1-\alpha)}\right\} \\
&= e^{-\eta\left(\frac{\mu}{\beta}-1\right)(1-\varepsilon)\zeta}\frac{\chi(\varepsilon\zeta-\varepsilon+1)}{\varepsilon-1}\left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}\right)^{-\frac{\zeta}{\varepsilon(\zeta-1)+1}} \frac{g(\alpha,\beta)r(\alpha,\beta)}{r(\bar{\alpha},1)^{\zeta}g(\alpha,\beta)^{\frac{\varepsilon\zeta}{\varepsilon(\zeta-1)+1}}g(\bar{\alpha},\beta)^{\frac{(1-\varepsilon)\zeta}{\varepsilon(\zeta-1)+1}}} \\
&= e^{-\eta\left(\frac{\mu}{\beta}-1\right)(1-\varepsilon)\zeta}\frac{\chi(\varepsilon\zeta-\varepsilon+1)}{\varepsilon-1}\left(\frac{\varepsilon\chi\zeta}{\varepsilon-1}\right)^{-\frac{\zeta}{\varepsilon(\zeta-1)+1}} \frac{g(\alpha,\beta)^{\frac{1-\varepsilon}{\varepsilon(\zeta-1)+1}}r(\alpha,\beta)}{g(\bar{\alpha},\beta)^{\frac{(1-\varepsilon)\zeta}{\varepsilon(\zeta-1)+1}}r(\bar{\alpha},1)^{\zeta}}. \qquad (95)
\end{aligned}$$

Rearranging (95) with (24) leads to (39).