The Origins of Aggregate Fluctuations in a Credit Network Economy

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Abstract

I build a network model of the economy in which intermediate goods are financed partly by supplier credit. The credit linkages between firms propagate financial shocks and amplify their aggregate effects. Combining firm- and industry-level data, I estimate aggregate and idiosyncratic shocks to industries in the US and find that financial shocks are a prominent driver of cyclical fluctuations, accounting for two-thirds of the drop in industrial production during the Great Recession. Furthermore, idiosyncratic financial shocks to a few key industries can explain a considerable portion of these effects. In contrast, productivity shocks had a negligible impact during the recession.

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1. INTRODUCTION

The recent financial crisis and ensuing recession have underscored the importance of external finance for the real economy. Generally, firms borrow extensively from their suppliers in the form of trade credit, or delayed payment terms provided by suppliers to their customers. Indeed, trade credit is the single most important source of short-term external financing for firms in the US, yet it has been largely absent from the business cycle literature. In this paper, I show that trade credit plays an important role in business cycle fluctuations.

To this end, I introduce trade credit into a network model of the economy and show that the credit interlinkages between firms can generate large fluctuations from small financial disturbances. I then use the framework to empirically shed light on the sources of observed fluctuations in the US. Accounting for the effects of the interlinkages between firms turns out to be crucial for identifying the sources of aggregate fluctuations in the US. In particular, I find financial shocks to be a key driver of cyclical fluctuations, particularly during the Great Recession. In contrast, productivity shocks play only a minor role.

The credit linkages I consider take the form of trade credit relationships between non-financial firms, in which a firm purchases intermediate goods on account and pays its supplier at a later date. Trade credit accounts for more than half of firms' short-term liabilities and more than one-third of their total liabilities in most OECD countries. In the US, trade credit was three times as large as bank loans and fifteen times as large as commercial paper outstanding on the aggregate balance sheet of non-financial corporations in 2012.[1] These facts point to the presence of strong credit linkages between non-financial firms.

An important feature of trade credit is that it leaves suppliers exposed to the financial distress of their customers.[2] A number of studies - including Jacobson and von

[1] See the Federal Reserve Board Flow of Funds.

[2] For example, the government bailout of the US automotive industry in 2008 was precipitated by an acute shortage of liquidity, which came about largely due to extended delays in payment for goods

Schedvin (2015), Boissay and Gropp (2012), Raddatz (2010), and Kalemli-Ozcan et al. (2014) - have found that firm- and industry-level trade credit linkages are an important channel through which financial shocks are transmitted upstream from firms to their suppliers. Yet the macroeconomic implications of trade credit have been largely overlooked in the literature.

I consider an economy similar to that of Bigio and La'O (2017), in which firms are organized in a production network and trade intermediate goods with one another. A moral hazard problem requires firms to make cash-in-advance payments to their suppliers before production takes place. As a result, firms face cash-in-advance constraints on their production. However, I assume that firms can delay part of these payments by borrowing from their suppliers. In particular, the moral hazard problem places a borrowing constraint on the amount of trade credit a firm can obtain from its suppliers, which depends on the firm's total cash flow.[3] Whereas in Bigio and La'O (2017) the tightness of financial constraints is fixed exogenously, trade credit in this framework implies that the tightness of constraints fluctuates endogenously with the cash-flow of downstream firms. As a result, credit linkages generate rich network effects by which financial shocks propagate through the economy.

When one firm is hit with an adverse shock to its cash on hand, there are two channels by which other firms in the economy are affected. First is the standard input-output channel: the shocked firm cuts back on production, reducing the supply of its good to its customers.[4] Second is a new credit linkage channel which tightens the financial constraints of upstream firms. That is, the shocked firm reduces the up-front payments it makes to its suppliers. Being more cash-constrained, these suppliers may be forced to cut back on their own production, and reduce the up-front payments to their own suppliers, etc. In this way, credit interlinkages propagate the firm-level financial shock across the economy. This additional upstream propagation turns out to be a powerful mechanism by which the financial conditions in the economy are

already delivered.

[3] I show how the market structure and the contracting environment pin down trade credit in equilibrium.

[4] This channel has been the focus of studies such as Acemoglu et al. (2012) and Bigio and La'O (2016).

tightened endogenously.

This mechanism is consistent with empirical evidence that, in response to tighter financial conditions, firms use trade credit more intensively, as their suppliers extend more trade credit to their distressed customers either through financing a higher proportion of purchased goods, or by extending the agreed maturity of the loans.[5] It is also consistent with the evidence that trade credit linkages transmit financial distress upstream from firms to their suppliers.[6]

Next, I evaluate the quantitative relevance of the mechanism. In order to overcome the paucity of data on trade credit, I first construct a proxy of inter-industry trade credit flows by combining firm-level balance sheet data from Compustat with industry-level input-output data from the Bureau of Economic Analysis (BEA). I thus produce a map of the credit network of the US economy at the three-digit NAICS level of detail, with which I calibrate the model.

Counterfactual exercises reveal the amplification mechanism to be quantitatively significant, amplifying financial shocks by 30-40 percent. Furthermore, the aggregate impact of an idiosyncratic (industry-level) financial shock depends jointly on the underlying structures of the credit and input-output networks of the economy. Based on this analysis, certain industries emerge as systemically important to the US economy, such as auto manufacturing and petroleum and coal manufacturing. Moreover, the systemic importance of an industry is closely related to the intensity of trade credit use by its largest trading partners. Thus, credit interlinkages play a significant role in exacerbating the effects of financial shocks and amplifying their aggregate effects.

In the empirical part of the paper, I use this theoretical framework to investigate which shocks drive cyclical fluctuations once we account for the network effects created by credit interlinkages. Accounting for these effects turns out to be crucial for identifying the sources of business cycle fluctuations in the US. My framework is rich enough to permit an empirical exploration of the sources of these fluctuations along two separate dimensions: the importance of productivity versus financial shocks, and that of aggregate versus idiosyncratic shocks. To address these issues, I use two

[5] Evidence suggests that, in this manner, suppliers use trade credit to insure their customers against liquidity shocks in order to preserve the relationship with customer. See Cunat (2007), Calomiris et al. (1995), and Nilsen (2002).

[6] See Jacobson and von Schedvin (2015), Boissay and Gropp (2012), Raddatz (2010), Jorion and Zhang (2009), and Carbo-Valverde et al. (2016).

methodological approaches.

My first approach involves identifying financial and productivity shocks without imposing the structure of my model on the data. To do this, I first construct quarterly measures of bank lending based on data from Call Reports collected by the FFIEC. I then augment an identified VAR of macro and monetary variables with this measure of bank lending, and with the excess bond premium of Gilchrist and Zakrajsek (2012), which reflects the risk-bearing capacity of the financial sector. I construct financial shocks as changes in bank lending which arise from orthogonalized innovations to the excess bond premium. [7] For productivity shocks, I use the quarterly, utilization-adjusted changes in total factor productivity (TFP) estimated by Fernald (2012).

Feeding these estimated shocks into the model, I find that, before 2007, productivity and financial shocks played a roughly equal role in generating cyclical fluctuations, together accounting for half of observed aggregate volatility in US industrial production. However, during the Great Recession, productivity shocks had virtually no adverse effects on industrial production - in fact, they actually *mitigated* the downturn. On the other hand, two-thirds of the peak-to-trough drop in aggregate industrial production during the recession can be accounted for by financial shocks, with the remainder unaccounted for by either shock. By propagating financial shocks across firms and exacerbating the financial conditions in the economy, trade credit linkages thus amplified the drop in aggregate industrial production during the recession.

With my second methodological approach, I empirically assess the relative contribution of aggregate versus idiosyncratic, industry-specific shocks in generating cyclical fluctuations. This involves estimating the model using a structural factor approach similar to that of Foerster et al. (2011), using data on the output and employment growth of US industrial production industries. I first use a log-linear approximation of the model to back-out the productivity and financial shocks to each industry required for the model to match the fluctuations in the output and employment data. Then, I use standard factor methods to decompose each of these shocks into an aggregate

[7] I construct the measure of bank lending in such a way that changes in the demand for bank lending are largely netted out. Therefore, changes in my measure of bank lending mostly reflect supply-side changes.

component and an idiosyncratic component.

Through variance decomposition I show that, while the idiosyncratic component of productivity shocks can account for a fraction of aggregate volatility before 2007, it played virtually no role during the Great Recession. Rather, nearly three-quarters of the drop in industrial production during the recession can be accounted for by aggregate financial shocks. In addition, the remainder can be accounted for by idiosyncratic financial shocks to a few systemically important industrial production industries - namely the oil and coal, chemical, and auto manufacturing industries. Furthermore, the credit and input-output linkages between industries played a significant role in propagating these industry-level shocks across the economy.

The broad picture which emerges from these two empirical analyses is that financial shocks have been a key driver of aggregate output dynamics in the US, particularly during the Great Recession.[8] Thus, when we account for the amplification mechanism of trade credit and input-output interlinkages, financial shocks seem to displace aggregate productivity shocks as a prominent driver of the US business cycle.

Related literature

This paper contributes to several strands of the literature. A growing literature examines the importance of network effects in macroeconomics, beginning with the seminal work of Long and Plosser (1983), and continuing with Dupor (1999), Horvath (2000), Shea (2002), Acemoglu et al. (2012), Acemoglu et al. (2015), Carvalho and Gabaix (2013), Baqaee (2016), and Liu (2017). These abstract away from financial frictions. The seminal work of Acemoglu et al. (2012) show that the network structure of an economy can generate aggregate fluctuations from idiosyncratic, firm-level shocks, using a frictionless input-output model of the economy.

The notable work of Bigio and La'O (2017) explores the interaction between financial frictions and the input-output structure of an economy by introducing financial constraints into a version of the Acemoglu et al. (2012) economy. However, they do not explicitly model any credit relationships between firms. As a result, the financial

[8] While shocks to aggregate TFP have long been relied upon as a principal source of cyclical fluctuations, the lack of direct evidence for such shocks has raised questions about their empirical viability.

constraints that firms face are fixed exogenously, and do not become tighter in response to shocks. Luo (2016) embeds an input-output structure in the framework of Gertler and Karadi (2011), with a role for trade credit. However, trade credit linkages do not propagate shocks across the economy *per se*.[9] Kiyotaki and Moore (1997) study theoretically how a shock to a firm in a credit chain can cause a cascade of defaults in a partial equilibrium framework. Gabaix (2011), Foerster et al. (2011), Stella (2014), and Atalay (2017) evaluate the contribution of idiosyncratic shocks to aggregate fluctuations. Jermann and Quadrini (2012) evaluate the importance of financial shocks by explicitly modeling the tradeoff between debt and equity financing. Ramirez (2017) uses an input-output model to explain certain empirical features of asset prices.

The rest of the paper is organized as follows. I first give a brief review of the empirical evidence on the role that trade credit plays in transmitting shocks across firms. In section I, I introduce the stylized model and derive the analytical results. In sections II-IV, I generalize the production network structure, discuss the construction of my proxy for credit flows and calibration, and summarize the quantitative results. In section V, I perform the empirical analyses.

Empirical evidence on the role of trade credit in business cycles

Here, I present a brief review of the empirical evidence on the role of trade credit in the business cycle. There are two notable features of this evidence. First, in response to a liquidity shock, distressed firms typically obtain more trade credit financing from their suppliers. Second, trade credit linkages transmit financial distress upstream from firms to their suppliers. The model will capture these two features of the data.

Cunat (2007), Molina and Preve (2012), and Love et al. (2007) all find evidence that suppliers increase trade credit financing when their customers experience tighter financial conditions. When a customer experiences a liquidity shock, suppliers typically either finance a higher proportion of their customer's purchased goods, or they extend the agreed maturity of the trade credit loans. Suppliers may accomodate their distressed customers in this way to preserve the relationships with their customers,

[9] In that paper, credit linkages only affect the interest rate that the bank charges firms. As such, all network effects are due to input-output linkages, as in Bigio and La'O (2017).

which are costly to reproduce. For example, Cunat (2007) finds evidence that trade credit is a mechanism for suppliers to insure their customers against liquidity shocks, when suppliers have an advantage over banks in enforcing the repayment of noncollateralized debt. Calomiris et al. (1995) show that, more generally, inter-firm financing via trade credit is countercyclical. This evidence goes back as early as Meltzer (1960) and Seiden (1964) and has been supported more recently by Atanasova (2007), Mateut et al. (2006), and Nilsen (2002), all of whom make similar findings.

Second, empirical evidence strongly suggests that trade credit linkages transmit financial distress upstream from firms to their suppliers. Jacobson and von Schedvin (2015) provide systematic evidence that trade credit chains propagate financial distress upstream from firms to their suppliers, using an exhaustive firm-level dataset on trade credit among firms in Sweden. Similar findings using firm-level data come from Boissay and Gropp (2012), Jorion and Zhang (2009), and Carbo-Valverde et al. (2016). Raddatz (2010) uses industry-level data to provide strong evidence for the upstream transmission of liquidity shocks through trade credit chains.

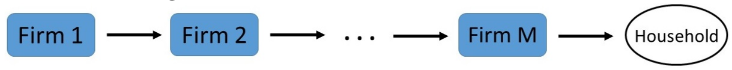
2. STYLIZED MODEL: VERTICAL PRODUCTION STRUCTURE

In this section, I build intuition with a simple model. The stylized nature of the production structure of the economy permits closed-form expressions for certain equilibrium variables. I will later generalize both the production structure and preferences.

There is one time period, consisting of two parts. At the beginning of the period, contracts are signed. At the end of the period, production takes place and contracts are settled. There are three types of agents: a representative household, firms, and a bank. There are M goods, each produced by a continuum of competitive firms with constant returns-to-scale in production. We can therefore consider each good as being produced by a representative, price-taking firm. I discuss the market structure in further detail below. Each good can be consumed by the household or used in the production of other goods.

The representative household supplies labor competitively to firms and consumes a final consumption good. It has preferences over consumption C and labor N given by $U(C,N)$, and a standard budget constraint, where w denotes the competitive wage earned from working, and π_i the profit earned by firm i.

Figure 1: Vertical Production Chain



$$U(C,N) = log C - N \qquad C = wN + \sum_{i=1}^{M} \pi_i \tag{1}$$

The household's optimality condition is given by $w = C$.

There are M price-taking firms who each produce a different good, for now arranged in a supply chain, where each firm produces an intermediate good for one other firm. The last firm in the chain produces the consumption good, which it sells to the household. Firms are indexed by their order in the supply chain, with $i = M$ denoting the producer of the final consumption good.

The production technology of firm i is Cobb-Douglas over labor and intermediate goods, where x_i denotes firm i's output, n_i its labor use, and x_{i-1} its use of good $i-1$, z_i denotes firm i's total factor productivity, η_i the share of labor in its production, and $\omega_{i,i-1}$ the share of good $i-1$ in firm i's total intermediate good use.

$$x_i = z_i n_i^{\eta_i} x_{i-1}^{\omega_{i,i-1}(1-\eta_i)} \tag{2}$$

Note that since in this supply chain economy each firm has one supplier and one customer, $\omega_{i,i-1} = 1$ for all i, and $\eta_1 = 1$. Let p_s denote the price of good s, where the final consumption good M is the numeraire and I normalize $p_M = 1$.

Limited enforcement problems between firms create a need for *ex ante* liquidity to finance working capital. The household cannot force any debt repayment. Therefore, firm i must pay the full value of wage bill, wn_i, up front to the household before production takes place. In addition, each firm i must pay for some portion of its intermediate goods purchases $p_{i-1}x_{i-1}$ up front to its supplier. Thus, firms are required to have some funds at the beginning of the period before any revenue is realized.

Firm i can delay payment to its supplier by borrowing some amount τ_{i-1} from its supplier, representing the trade credit loan given from $i-1$ to i. In addition, I assume each firm has some other exogenous source of funds b_i, which I interpret as a cash loan from an outside bank, for ease of exposition. The net payment that firm $i-1$ receives from its customer at the beginning of the period is therefore $p_{i-1}x_{i-1} - \tau_{i-1}$. Firm i's cash-in-advance constraint takes the form

$$\underbrace{wn_i}_{wage\ bill} + \underbrace{p_{i-1}x_{i-1} - \tau_{i-1}}_{CIA\ payment\ to\ supplier} \leq \underbrace{b_i}_{bank\ loan} + \underbrace{p_i x_i - \tau_i}_{CIA\ from\ customer} . \tag{3}$$

Thus, the cash that firm i is required to have in order to employ n_i units of labor and purchase x_{i-1} units of intermediate good $i-1$, is bounded by the amount of cash that firm i can collect at the beginning of the period. Note that trade credit appears on both sides of the constraint.

Contracting environment Firms face borrowing constraints on the size of loans they can obtain from their suppliers. Namely, the trade credit that firm i can obtain from its supplier $i-1$ is bounded from above by a fraction θ_i of its end-of-period revenue.

$$\tau_{i-1} \leq \theta_i p_i x_i \tag{4}$$

Underlying this borrowing constraint is a moral hazard problem described in detail in online appendix O.A2 in which firms have access to a diversion technology specific to each intermediate good. (Although this resembles a collateral constraint, suppliers cannot recover any of their customer's funds in the event of default.)[10]

Each firm i has a similar diversion technology for funds lent by the bank, with B_i parameterizing the severity of the moral hazard problem. In addition, firm i can credibly pledge a fraction $\alpha \,\varepsilon\, (0, 1]$ of its accounts receivable τ_i as collateral to borrow from the bank.[11]

[10] The parameter θ_i reflects the threshold above which the private benefit to diversion exceeds the cost. Alternatively, one could consider a collateral constraint in which firm i's accounts receivables τ_i serves as collateral for the trade credit loan, so that the constraint would be of the form $\tau_{i-1} \leq \theta_i \tau_i$. It is easy to show that this would yield qualitatively similar results, as trade credit linkages would still transmit shocks upstream from firms to their suppliers.

[11] The collateralizing of accounts receivables for borrowing, sometimes referred to as factoring, is prevalent in the data. See Mian and Smith Jr. (1992) and Omiccioli (2005), and see Ahn, Amiti, and Weinstein (2011) for a discussion of this in the context of international trade.

$$b_i \leq B_i p_i x_i + \alpha \tau_i. \tag{5}$$

The role of α is simply to introduce some degree of substitutability between cash and bank credit, whereby a firm can partially offset a fall in the up-front payment it collects from its customers by borrowing more from the bank. This will dampen the quantitative importance of trade credit in transmitting shocks, as discussed in section 2.2.

Market structure and equilibrium contracts How do firms choose how much to lend to their customers and borrow from their suppliers? Recall that representative firm i is actually comprised of a continuum of competitive firms with constant returns to scale production. These atomistic suppliers compete with one another along two margins by posting contracts which specify both a price of output and the trade credit to offer. In online appendix O.A3, I setup the game between suppliers and characterize the contracts which emerge in the Nash equilibrium.[12] In short, perfect competition amongst these atomistic suppliers forces them to offer their customers the maximum amount of trade credit permitted by the borrowing constraint (4). This result holds even when these suppliers are cash-constrained in equilibrium.[13]

Representative firm's problem We can re-write firm i's cash-in-advance constraint as

$$wn_i + p_{i-1}x_{i-1} \leq \underbrace{\chi_i p_i x_i}_{liquid\ funds} \tag{6}$$

where

$$\chi_i \equiv \underbrace{\frac{b_i}{p_i x_i} + \frac{\tau_{i-1}}{p_i x_i}}_{debt/revenue\ ratio} + \underbrace{1 - \frac{\tau_i}{p_i x_i}}_{cash/revenue\ ratio}. \tag{7}$$

Therefore, a firm's expenditure on inputs is bounded by the amount of funds it has at the beginning of the period. The variable χ_i describes the tightness of firm i's cash-

[12] In addition, atomistic firms do not internalize that their demand may have an effect on the price offered by their suppliers in equilibrium. As a result, their demand for trade credit is unbounded.

[13] These results are supported by micro-level evidence on trade credit: competition amongst suppliers is often sufficiently high that they are forced to offer their customers extended payment terms, even when they are cash-constrained. See, for instance, Barrot (2015).

in-advance constraint, and will play a key role in the mechanism of the model. The tightness of a firm's cash-in-advance constraint is comprised of the firm's debt-to-revenue ratio and its cash-to-revenue. These describe how much of the firm's revenue is financed by debt, and how much of its revenue is collected as a cash-in-advance payment, respectively. Notice that χ_i is decreasing in $\frac{\tau_i}{p_i x_i}$, the amount of i's output sold on credit: the more credit that i gives its customer, the less cash it collects at the beginning of the period.

Firm i chooses its input purchases n_i and x_{i-1}, and how much trade credit to borrow τ_{i-1}, to maximize its profits subject to its cash-in-advance constraint. Because competition pins down prices p_i and trade credit supply τ_i, the firm takes these as given.

$$\max_{n_i, x_{i-1}, \tau_{i-1}} \; p_i x_i - w n_i - p_{i-1} x_{i-1}$$

$$s.t. \; w n_i + p_{i-1} x_{i-1} \leq \chi_i(\tau_{i-1}) p_i x_i \tag{8}$$

$$\tau_{i-1} \leq \theta_i p_i x_i \tag{9}$$

Recall that contracts are such that firms borrow the maximum trade credit in equilibrium, implying the borrowing constraints (9) are always binding. I will show in section 2.2 on the credit linkage channel this is not crucial for the qualitative results.

Notice that a firm's production is constrained by how much liquid funds it has at the beginning of the period, given by the tightness of its cash-in-advance constraint (8). Henceforth, I refer to a firm whose cash-in-advance constraint is binding in equilibrium as a "constrained" firm. If firms are constrained in equilibrium, we can re-write the tightness χ_i of a firm's constraint using firms' binding borrowing constraints to replace τ_i and b_i.

$$\chi_i = \underbrace{B_i + \theta_i}_{debt/revenue\ ratio} + \underbrace{1 - (1-\alpha)\theta_{i+1}\frac{p_{i+1}x_{i+1}}{p_i x_i}}_{cash/revenue\ ratio} \tag{10}$$

Crucially, equation (4) shows that χ_i is an *equilibrium object* - it is an endogenous variable which depends on the firm's forward credit linkage θ_{i+1} and the revenue of its customer.[14] Hence, changes in the price of its customer's good affect the tightness of firm i's cash-in-advance constraint. [15] Here, the endogeneity of χ_i will be a critical determinant of how the economy responds to shocks.

Firm i's optimality conditions equate the ratio of expenditure on each type of input with the ratio of their share of production. I show in section 2.1 that firm i's cash-in-advance constraint (3) binds in equilibrium if and only if $\chi_i < 1$. Combining the first order conditions with the cash-in-advance constraint yields the optimality conditions below.[16]

$$w = \phi_i \eta_i \frac{p_i x_i}{n_i}\,, \qquad p_{i-1} = \phi_i \omega_{i,i-1}(1-\eta_i)\frac{p_i x_i}{x_{i-1}} \tag{11}$$

Here, $\phi_i \equiv \min\{1, \chi_i\}$ describes firm i's shadow value of funds.[17] ϕ_i is strictly less than one if and only if firm i's cash-in-advance is binding in equilibrium. Equations (11) says that, if binding, the cash-in-advance constraint inserts a wedge $\phi_i < 1$ between the marginal cost and marginal benefit of each input, representing the distortion in the firm's input use created by the constraint. A tighter cash-in-advance (lower χ_i) corresponds to a greater distortion, and lower output. Through χ_i , ϕ_i *endogenously* depends on shadow value funds of downstream firms ϕ_{i+1}, reflecting that firms' constraints are interdependent due to trade credit.

Note that there are two types of interlinkages between firms: input-output linkages, represented by input shares $\omega_{i,i-1}$ in production; and credit linkages, represented by the borrowing limits θ_i between firms. Each of these interlinkages will play a different role in generating network effects from shocks.

[14] Notice that the firm's debt-to-revenue ratio is fixed, because firms collateralize their end-of-period revenue for borrowing.

[15]This a key difference with Bigio and La'O (2017), in which the tightness of each firm's cash-in-advance is an exogenous parameter because there is no inter-firm lending.

[16]Since τ_{i-1} is important only insofar as it affects the tightnesses of firms' constraints, it shows up in firm i's first order conditions only through ϕ_i.

[17]More precisely, the shadow value of funds of firm i is given by $\frac{1}{\phi_i} - 1$.

2.1. Equilibrium

I close the model by imposing labor and goods market clearing conditions $N = \sum_{i=1}^{M} n_i$ and $C = Y \equiv x_M$. An equilibrium is a set of prices $\{p_{i_{i\varepsilon I}}, w\}$, and quantities x_i, n_i, $\tau_{i_{i\varepsilon I}}$ that (i) maximize the representative household's utility, subject to its budget constraint; (ii) maximize each firm's profits subject to its cash-in-advance, bank borrowing, and supplier borrowing constraints; and (iii) clear goods markets and the labor market.

Solving for the equilibrium The concavity in the household utility function pins down the equilibrium uniquely. This concavity means that, in general equilibrium, prices respond to firm production decisions. (This can be seen from equation (14) below). That prices enter cash-in-advance constraint means that firm production decisions affect the 'tightness' of the constraint. Solving for the equilibrium amounts to solving this fixed point problem. I now outline the closed-form solution for the equilibrium.

Let the final consumption good be the numeraire, and normalize its price to $p_M = 1$. First note from equation (10) that, since the final firm in the supply chain collects all revenue from the household up front, the tightness of its cash-in advance constraint is given by $\chi_M = B_M + \theta_M$, which pins down its equilibrium wedge $\phi_M = \min\{1, \chi_M\}$. Then, using the expression (10) and the optimality conditions (11) for each firm's intermediate goods, we can recursively solve for the tightness χ_i of each firm's constraint and wedges $\phi_i = \min\{1, \chi_i\}$, where

$$\chi_i = B_i + \theta_i + 1 - (1-\alpha)\theta_{i+1} \frac{1}{\phi_{i+1}(1-\eta_{i+1})}. \tag{12}$$

Here, I have imposed that the share $\omega_{i,i-1}$ of good $i-1$ in the intermediate good use of firm i is unity, reflecting the vertical structure of the supply chain.

We can also recursively use the optimality conditions for labor (11) to obtain an expression for each firm's labor expenditure as function of final output. For each firm i

$$wn_i = x_M \phi_i \eta_i \left(\prod_{j=i+1}^{M} \phi_j \left(1-\eta_j\right) \right). \tag{13}$$

Recall that the household's optimality condition, derived from (1), is $w = C$. Combining this condition with the market clearing condition $C = x_M$ for the consumption good yields

$$w = x_M. \tag{14}$$

From (14) we can see that the concavity in the household utility functions implies that prices are increasing in final output in equilibrium. Combining (14) with the expressions (11) for each firm's labor expenditure yields expressions for each firm's labor use.

$$n_i = \phi_i \eta_i \prod_{j=i+1}^{M} \phi_j \left(1 - \eta_j\right) \tag{15}$$

Using the inter-linked production functions (2) of each firm, we can produce an expression for equilibrium aggregate output as function of each firm's wedge ϕ_i and labor use (15)

$$Y \equiv x_M = \prod_{i=0}^{M-1} \left(z_{M-i} n_{M-i}^{\eta_{M-i}}\right)^{\delta_{M-i}} \tag{16}$$

where $\delta_{M-i} \equiv \prod_{j=0}^{i-1} \left(1 - \eta_{M-j}\right)$.

Thus, equilibrium aggregate output in the economy is determined by each firm's production function and financial constraint. To see this, let $\bar{Y}$ denote the aggregate output that would prevail in a frictionless input-output economy (à la Acemoglu et al. (2012)), given by $\bar{Y} \equiv \prod_{i=1}^{M} \tilde{\eta}_i^{\tilde{\eta}_i} z_i^{\tilde{\omega}_i}$.[18] Define aggregate liquidity in the economy as $\bar{\Phi} \equiv \prod_{i=1}^{M} \phi_i^{\sum_{j=1}^{i} \tilde{\eta}_j}$, an aggregation of all firm's shadow value of funds. And since the production structure of the economy is simply a supply chain, the share of firms $i-1$'s good in firm i's production is $\omega_{i,i-1} = 1$ for all i. Then we can write the analytical expression (16) for equilibrium aggregate output as an expression which is log-linear in $\bar{Y}$ and the aggregate liquidity in the economy.

$$Y = \bar{Y}\bar{\Phi} \tag{17}$$

[18] Here, $\tilde{\omega}_i \equiv \prod_{j=i+1}^{M} \omega_{j,j-1}$ denotes firm i's share in total intermediate good use, and $\tilde{\eta}_i \equiv \eta_i \tilde{\omega}_i$ denotes firm i's share of labor in aggregate output.

Intuitively, (17) says that equilibrium aggregate output is constrained by aggregate liquidity - the funds available to all firms to finance working capital at the beginning of the period. Note that if all firms are unconstrained, then $\bar{\Phi} = 1$ and $Y = \bar{Y}$. If one firm i is constrained, aggregate output depends on how its constraint affects the supply of intermediate good i for all downstream firms, given by $\sum_{j=1}^{i} \tilde{\eta}_j$.[19] To summarize, firms' financial constraints distort production in a way which depends on the underlying structures of the credit and input-output networks of the economy.

2.2. Aggregate Impact of Firm-Level Shocks

I now examine how the economy responds to firm-level financial shocks and productivity shocks. I model a *financial shock* to firm i by a change in B_i, the fraction of firm i's revenue that the bank will accept as collateral for the bank loan. This is a reduced-form way to capture a reduction in the supply of bank credit to firm i, and represents an exogenous tightening in firm i's financial constraint.[20]

If firm i is unconstrained in equilibrium, a marginal financial shock dB_i has no effect on its production - the firm has deep pockets and can absorb the shock. However, if the firm is constrained, then it is forced to reduce production as it can no longer finance as many inputs with up front payments. In addition to this direct effect, there are two types of network effects by which the shock affects other firms in the economy: input-output channel and the credit linkage channel.

Network effects: Standard input-output channel Through the first channel, which I call the *standard input-output channel*, the shock propagates through input-output interlinkages, increasing firms' input costs. This is the standard channel analyzed in the input-output literature, including Acemoglu et al. (2012) and Bigio and La'O (2017). The reduction in firm i's output increases the price p_i of good i. This acts as a supply shock to the customer downstream (firm $i+1$), who is now faced with a higher unit cost of its intermediate good. In response, firm $i+1$ cuts back on production, which causes the p_{i+1} to increase, etc. Thus, as a result of the shock to firm i, all

[19] Note that the credit network of the economy - i.e. the set $\{\theta_i\}_{\forall i \varepsilon I}$ - shows up implicitly in (12) through each ϕ_i.

[20] In the general network model in the following section, each firm sells some portion of its output directly to the household. In this setting, one could alternatively interpret the fall in B_i as a failed payment by final consumer. In either case, these are idiosyncratic shocks to the firm's liquid funds such that $\frac{d\chi_i}{dB_i} > 0$, and are not well-represented by a change in its productivity or technology.

firms downstream experience a supply shock to their intermediate goods, and cut back on production. This amplifies the shock because as firms reduce production, they cut back on employment which, in turn, reduces the wage and household consumption.[21] In addition, the shock travels upstream as suppliers adjust their output to respond to the fall in demand for their intermediate goods.

Network effects: Credit linkage channel There is also a new, additional channel of transmission - which I call the *credit linkage channel* - which describes how the financial constraints of upstream firms are tightened endogenously in response to the shock.

Recall that when firm i cuts back on production, the price p_i of its good rises. This increases the collateral value of its future cash flow, allowing it to delay payment for a larger fraction of its purchase from supplier $i-1$.[22] As a result, supplier $i-1$'s cash/revenue ratio falls, meaning the fraction of its revenue collected as up front payment falls. This tightens its cash-in-advance constraint - i.e. χ_{i-1} falls.[23]

$$\chi_{i-1} \downarrow \equiv \underbrace{B_{i-1} + \theta_{i-1}}_{debt/revenue\ ratio} + \underbrace{1 - \frac{\tau_{i-1}}{p_{i-1}x_{i-1}}}_{cash/revenue\ ratio \downarrow} \tag{18}$$

Thus, with less cash on-hand, the supplier $i-1$ is now faced with a tighter financial constraint itself. The supplier may therefore be forced to reduce production further, and thereby pass the shock to its own suppliers and customers. (This continues up the chain of firms). In this manner, the initial effect of the shock is amplified as upstream

[21] This channel is ultimately driven by the input specificity in each firm's production technology, as each downstream firm is unable to offset the supply shock by substituting away from using good i in their production, and each upstream firm is unable to offset the demand shock by finding other customers for its good.

[22] This is true even though the volume of trade credit τ_{i-1} may actually fall in response to the shock.

[23] More precisely, there are three effects of the shock $d\,B_i$ on χ_{i-1}. Recall from (10) that firm $i-1$'s cash/revenue ratio depends inversely on $\frac{p_i x_i}{p_{i-1}x_{i-1}}$. First, the shock increases p_i, as discussed above. Second, the fall in firm i's output increases the ratio $\frac{x_i}{x_{i-1}}$ due to the decreasing returns to x_{i-1}. And third, the fall in i's demand reduces the price p_{i-1} of good $i-1$. All of these effects unambiguously reduce χ_{i-1}.

firms experience tighter financial conditions.

But why doesn't firm $i-1$ reduce the trade credit loans it makes in order to increase its cash holdings and relax its own constraint? Recall that representative firm $i-1$ consists of a continuum of firms, and that perfect competition forces them to offer the maximum trade credit, even when they are themselves constrained. Note also that α mitigates the transmission, allowing firm $i-1$ to partially offset the lost up-front cash payments with a larger bank loan. Thus, α parameterizes the substitutability between cash and bank credit.

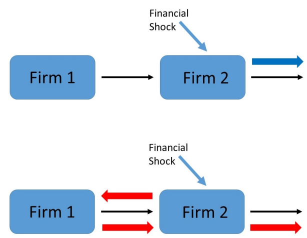
Importantly, this mechanism is consistent with two well-documented stylized facts on trade credit. First, in response to a liquidity shock, firms generally use trade credit more intensively to finance their intermediate goods: when a customer experiences a financial shock, suppliers extend more trade credit to their distressed customer either through financing a higher proportion of purchased goods, or by extending the agreed maturity of the trade credit loans.[24] (See Cunat (2007), Calomiris et al. (1995), and Nilsen (1994; 2002) for evidence.) This is captured in my stylized model: in response to a financial shock, firm i increases $\frac{\tau_{i-1}}{p_{i-1}x_{i-1}}$, the fraction of intermediate goods that firm i finances with trade credit.

Second, empirical evidence strongly suggests that trade credit linkages transmit financial distress upstream from firms to their suppliers. This is consistent across studies which use firm-level and industry level data on trade credit, such as Jacobson and von Schedvin (2015), Boissay and Gropp (2012), and Raddatz (2010), Jorion and Zhang (2009), and Carbo-Valverde et al. (2016).[25] In the model, the credit linkage channel implies that trade credit transmits a financial shock upstream from firms to their suppliers, consistent with the evidence.

[24]There is evidence that suppliers do this in order to preserve customer-supplier relationships, which are costly to reproduce. For example, Cunat (2007) finds evidence that trade credit is a mechanism for suppliers to insure their customers against liquidity shocks, when suppliers have an advantage over banks in enforcing the repayment of noncollateralized debt.

[25]Some observors have noted that aggregate accounts receivable fell during the Great Recession, and interpret this as a fall in the amount of trade credit extended by suppliers. However, this fall in aggregate trade credit undoubtedly reflects in part the fall in trade of intermediate goods which occurred during the recession (trade credit is generally proportional to trade in intermediate goods). Aggregate quantitites of trade credit are therefore not informative about the fraction of intermediate goods for which suppliers are willing to accept late payment. On the other hand, key to my model's mechanism is the intensity of trade credit use, given by the *fraction* of intermediate goods financed by trade credit. For this, the firm-level studies mentioned above provide better insight than aggregate quantities of trade credit.

Figure 2: Feedback Effect



It should be noted that it is not crucial for the qualitative results that the borrowing constraints (9) are binding in equilibrium. Even if equilibrium trade credit were defined by an interior optimum in which the borrowing constraints (9) do not bind, there would still be this upstream transmission of shocks as long as trade credit does not declined as a fraction of suppliers' revenue in response to a financial shock to industry i. In fact, this upstream transmission may be stronger if borrowing constraints are not binding in equilibrium: in response to a financial shock, firm i can borrow more from its supplier, further tightening its supplier's financial constraint.

Feedback effect created by transmission channels Importantly, the two transmission channels produce a feedback effect which amplifies the shock, as illustrated in Figure 2. Suppose that firm 2 is hit with an adverse financial shock, causing its cash-in-advance constraint to become tighter, and forcing it to cut back on production. The standard input-output channel, represented by the blue arrow, transmits the shock downstream in the form of a higher intermediate good price.

In addition, the credit linkage channel tightens the constraints of upstream firms, as firm 2 reduces the cash-in-advance payments it makes to its supplier. With a tighter financial constraint the supplier is forced to reduce production, which feeds back to firm 2 again in the form of higher price for the intermediate good. Thus, firm 2 is hit not only with a tighter financial constraint, but also endogenously higher input costs, (which it passes on to its customer, and so on). In this manner, the two channels interact to create a feedback loop represented by the red arrows, which exacerbates the initial shock.[26]

2.3. Impact of Firm-Level Shock on Aggregate Output

In light of these mechanisms, I now derive analytical expressions for how a firm-level financial shock affects aggregate output, and show that the credit network effects amplify the shock in a manner which depends on the structure of the credit linkages.

From (17), I decompose the change in aggregate output due to a financial shock to firm i into components reflecting the standard input-output channel and the credit linkage channel.

$$\frac{d\log Y}{dB_i} = \sum_{j=1}^{M} \bar{v}_j \frac{d\log \phi_j}{dB_i} \tag{19}$$

Here, the terms $\frac{d\log \phi_j}{dB_i}$ capture the credit linkage channel, and reflect how the financial shock to firm i affects the shadow value of funds of every other firm j in the network. The terms $\bar{v}_j$ capture the standard input-output channel, and map these changes in each ϕ_j into aggregate output. ($\bar{v}_j \equiv \sum_{k=1}^{j} \tilde{\eta}_k$ depends on the share of labor in aggregate output of each firm.) This decomposition will allow me to quantify the aggregate effects of each channel later on.

In an economy without the credit linkage channel, such Bigio and La'O (2017), each ϕ_j is fixed so that $\frac{d\log \phi_j}{dB_i} = 0$ for all $j \neq i$. In words, financial constraints would not respond endogenously to a shock. Therefore, (19) would reduce to $\frac{d\log Y}{dB_i} = \bar{v}_i$.

However, credit network effects *amplify* the effects of the firm-level financial shock on aggregate output. This is because $\frac{d\log \phi_j}{dB_i} \geq 0$ and therefore $\frac{d\log Y}{dB_i} = \sum_{j=1}^{M} \bar{v}_j \frac{d\log \phi_j}{dB_i} > \bar{v}_i$ (proved in online appendix O.A1). In addition, the credit network effects $\frac{d\log \phi_j}{dB_i}$ are weakly increasing in θ_{jk} for all firms i,j, and k. Thus, the aggregate impact of the financial shock depends on the location of firm i within the networks, and the strength of input-output and credit linkages between firms.

[26] A firm-level financial shock in my model therefore resembles an *aggregate* financial shock to all firms in a model with fixed constraints, e.g. Bigio and La'O (2017).

2.4. Impact of Firm-Level Productivity Shock on Aggregate Output

Now consider a productivity shock to firm i, represented by a fall in i's total factor productivity (TFP) z_i. It turns out that, due to Cobb-Douglas production, each firm's cash/revenue ratio, and therefore the tightness of their constraint ϕ_j, is independent of the productivity of firms z_i.[27] As a result, while the standard input-output channel amplifies the productivity shock just as in Acemoglu et al. (2012), the credit linkage channel does not.

Summary of theoretical results To summarize, the credit linkages between firms create a multiplier effect which amplifies the aggregate effects of firm-level shocks. The aggregate impact of these shocks depends on structure of the credit network, i.e. how firms borrow from and lend to one another.

3. GENERAL MODEL

To capture more features of the economy, I now allow for an arbitrary network structure so that each firm may trade with and borrow from or lend to any other firm in the economy.

I assume that each of the M goods can be consumed by the representative household or used in the production of other goods. The household's total consumption C is Cobb-Douglas over the M goods, and it has Greenwood-Hercowitz-Huffman preferences.[28]

$$U(C,N) = \frac{1}{1-\gamma}\left(C - \frac{1}{1+\varepsilon}N^{1+\varepsilon}\right)^{1-\gamma}, \qquad C \equiv \prod_{i=1}^{M} c_i^{\beta_i} \tag{20}$$

Here, ε and γ respectively denote the Frisch and income elasticity of labor supply. The household maximizes its utility subject to its budget constraint (1). This yields

[27] Acemoglu, Akcigit, and Kerr (2015) argue that Cobb-Douglas is a good approximation for production at the industry level.

[28] Quantitatively similar results hold for preferences which are additively separable in aggregate consumption C and labor N.

optimality conditions which equate the ratio of expenditure on each good with the ratio of their marginal utilities, and the competitive wage with the marginal rate of substitution between aggregate consumption and labor.

$$\frac{p_i c_i}{p_j c_j} = \frac{\beta_i}{\beta_j}, \qquad N^{1+\varepsilon} = C \tag{21}$$

Each firm can trade with all other firms. Firm i's production function is again Cobb-Douglas over labor and intermediate goods.

$$x_i = z_i^{\eta_i} n_i^{\eta_i} \left(\prod_{j=1}^{m} x_{ij}^{\omega_{ij}} \right)^{1-\eta_i} \tag{22}$$

Here, x_i denotes firm i's output and x_{ij} denotes firm i's use of good j. Since ω_{ij} denotes the share of j in i's total intermediate good use, I assume $\sum_{j=1}^{M} \omega_{ij} = 1$ so that each firm has constant returns to scale. The input-output structure of the economy can be summarized by the matrix Ω of intermediate good shares ω_{ij}.[29]

$$\Omega \equiv \begin{bmatrix} \omega_{11} & \omega_{12} & \cdots & \omega_{1M} \\ \omega_{21} & \omega_{22} & & \\ \vdots & & \ddots & \\ \omega_{M1} & & & \omega_{MM} \end{bmatrix}$$

Note that the production network is defined only by technology parameters. As we will see, the presence of financial frictions will distort inter-firm trade in equilibrium. Hence, Ω describes how firms would trade with each other in the absence of frictions.

Each firm's cash-in-advance constraint takes the same form as in the stylized model, with the exception that each firm has M suppliers and M customers instead of just one of each. τ_{is} denotes the trade credit loan that firm i receives from each of its suppliers s.

$$wn_i + \underbrace{\sum_{s=1}^{M} (p_s x_{is} - \tau_{is})}_{net\ CIA\ payment\ to\ suppliers} \leq b_i + \underbrace{p_i x_i - \sum_{c=1}^{M} \tau_{ci}}_{net\ CIA\ received\ from\ customers} \tag{23}$$

[29] This is simply a generalization of the input-output structure in the stylized model. In that case, the Ω would be given by a matrix of zeros, with one sub-diagonal of ones, reflecting the vertical production structure and the constant returns to scale technology of firms.

Firm i faces borrowing constraints with each of its suppliers, in which the loan from each supplier s is bounded above by a fraction θ_{is} of its total cash flow. Each firm can also borrow b_i from the bank up to a fraction B_i of its cash flow, and by pledging a fraction α of its accounts receivable $\sum_{c=1}^{M} \tau_{ci}$ as collateral.

$$\tau_{is} \leq \theta_{is} p_i x_i \qquad b_i \leq B_i p_i x_i + \alpha \sum_{c=1}^{M} \tau_{ci} \tag{24}$$

Underlying these constraints is the same moral hazard problem described in the stylized model. As in the stylized model, competition amongst suppliers in industry s forces them to offer the maximum trade credit permitted by the limited enforcement problem, so that the trade credit borrowing constraint always binds when industries are cash-constrained in equilibrium. The structure of the credit network between firms can be summarized by the matrix of θ_{ij}'s.

$$\Theta \equiv \begin{bmatrix} \theta_{11} & \theta_{12} & \cdots & \theta_{1M} \\ \theta_{21} & \theta_{22} & & \\ \vdots & & \ddots & \\ \theta_{M1} & & & \theta_{MM} \end{bmatrix}$$

Plugging the binding borrowing constraints into (23) yields a constraint on i's total input purchases, where χ_i describes the tightness of i's cash-in-advance constraint.

$$wn_i + \sum_{s=1}^{M} p_s x_{is} \leq \chi_i p_i x_i \tag{25}$$

Just as in the stylized version, χ_i is an an equilibrium object, where firm i's cash/revenue ratio depends on the prices p_c of its customer's goods and its forward credit linkages θ_{ci}.

$$\chi_i = \underbrace{B_i + \sum_{s=1}^{M} \theta_{is}}_{debt/revenue\ ratio} + \underbrace{1 - (1-\alpha) \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i}}_{cash/revenue\ ratio} \tag{26}$$

Firms choose labor and intermediate goods to maximize profits subject to their cash-in-advance constraint. Again, firm i's constraint inserts a wedge ϕ_i between the marginal cost and marginal revenue product of each input

$$n_i = \phi_i \eta_i \frac{p_i}{w} x_i \qquad x_{ij} = \phi_i (1 - \eta_i) \omega_{ij} \frac{p_i}{p_j} x_i \tag{27}$$

where the wedge $\phi_i = min\{1, \chi_i\}$ is determined by the firm's shadow value of funds. Market clearing conditions for labor and each intermediate good are given by

$$N = \sum_{i=1}^{M} n_i \qquad x_i = c_i + \sum_{c=1}^{M} x_{ci}. \tag{28}$$

The equilibrium conditions of this generalized model take the same form as in the stylized model, and the economy will behave in qualitatively the same way in response to shocks as in the stylized model. When taking this model to industry-level data, the calibration of the model will allow industries to differ in how financially constrained they are.

Relationship between firm influence and size A well-known critique of frictionless input-output models such as Acemoglu et al. (2012) is that the size of a firm, as measured by its share s_i of aggregate sales, is sufficient to determine the aggregate impact of a shock to sector i, and one does not need to know anything about the underlying input-output structure of the economy. Bigio and La'O (2017), however, show that this result breaks down when the economy has financial frictions. My model shows that when credit linkages between firms propagate shocks across the economy, the aggregate impact of an idiosyncratic shock depends also on the underlying structure of the credit network of the economy, summarized by the matrix Θ.

4. QUANTITATIVE ANALYSIS

Having established analytically that the credit network of the economy can amplify firm-level shocks, I now ask whether this mechanism is quantitatively significant for the US, and examine more carefully the role that the structure of the credit network plays. But before these questions can be addressed, I need disaggregated data on trade credit flows in order to calibrate the credit network of the US economy.

Figure 3: Constructing Proxy for Trade Credit Flows

4.1. Mapping the US Credit Network

Calibration of the trade credit parameters θ_{ij} requires data on credit flows between industry pairs; but data on credit flows at any level of detail is scarce. To overcome this paucity of data, I construct a proxy for trade credit flows τ_{ij} between industry pairs using industry-level input-output data and firm-level balance sheet data. I use input-output tables from the Bureau of Economic Analysis (BEA) and Compustat North America over the period 1997-2013. The BEA publishes annual input-output data at the three-digit NAICS level, at which there are 58 industries, excluding the financial sector. From this data, I observe annual trade flows between each industry-pair, which corresponds to $p_j x_{ij}$ in my model for every industry pair $\{i, j\}$. Compustat collects balance-sheet information annually from all publicly-listed firms in the US. The available data includes each firm's total accounts payable, accounts receivable, cost of goods sold, and sales in each year of the sample.

My strategy for constructing the proxy is illustrated in Figure 3. From the payables and receivables data, I observe how much, on average, firms in each industry have borrowed from all of their suppliers collectively, and lent to all of their customers collectively.[30] However, I do not observe how an industry's stock of trade credit and debt breaks down across each of its suppliers and customers. Therefore, I combine the input-output data with the payables and receivables data to approximate the fraction of sales from firms in industry j to firms in industry i made on credit, on average, yielding a proxy for trade credit flows τ_{ij} between each industry pair.

Many studies have found that large firms on average use trade credit less intensively than their smaller counterparts, presumably because they have greater access to other forms of financing.[31] Since the publicly-traded firms in the Compustat database tend to be large, my use of this data likely biases downward the extent of trade credit linkages between firms, and therefore potentially underestimates their quantitative importance

[30] The vast majority of accounts receivables and payables of US corporations consists of trade credit.
[31] See, for instance, Petersen and Rajan (1997).

in amplifying business cycles.

4.2. Calibration

With the proxy for trade credit flows at hand, I calibrate the general model to match US data. I calibrate technology parameters η_i and ω_{ij} to match the BEA input-output tables of the median year in my sample, 2005. The firm optimality conditions and CRS technology imply

$$\phi_i = \frac{wn_i + \sum_{j=1}^{M} p_j x_{ij}}{p_i x_i}. \tag{29}$$

The right-hand side of (29) is directly observable from the BEA's Direct Requirements table.

Looking through the lens of the model, the observed input-output tables reflect both technology parameters and distortions created by the financial constraints. My calibration strategy respects this feature. In particular, I calibrate technology parameters using firm i's optimality conditions for each input and my calibrated ϕ_i's.

$$\eta_i = \frac{wn_i}{\phi_i p_i x_i} \qquad \omega_{ij} = \frac{p_j x_{ij}}{(1-\eta_i)\phi_i p_i x_i} \tag{30}$$

Again the ratios $\frac{wn_i}{p_i x_i}$ and $\frac{p_j x_{ij}}{p_i x_i}$ are directly observable from the Direct Requirements tables for every industry i and j.

I calibrate the parameters θ_{ij}, representing the credit linkages between industries j and i, to match my proxy of inter-industry trade credit flows $\hat{\tau}_{ij}$ using industry i's binding borrowing constraint.

$$\theta_{ij} = \frac{\hat{\tau}_{ij}}{p_i x_i} \tag{31}$$

Industry i's total revenue $p_i x_i$ is directly observable from the Uses by Commodity tables. (Recall that I use the input-output tables for year 2005).

To calibrate B_i, the parameters reflecting the agency problem between firm i and the bank, recall the definition of ϕ_i given by (11), which depends on the technology parameters (calibrated as described above) and the tightness χ_i of each industry's cash-in-advance, where

$$\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - (1-\alpha) \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i}. \tag{32}$$

The total revenue of each industry $p_i x_i$ is observable from the Uses by Commodity tables, and ϕ_i and θ_{is} for all s were calibrated as described above. I therefore use (29) and (31) to back out B_i for each industry. Thus, the calibration of B_i ensures that $\phi_i < 1$, so that all industries are constrained to some degree in equilibrium.[32]

I follow the standard literature and set $\varepsilon = 1$ and $\gamma = 2$, which represent the Frisch and income elasticity, respectively. I set $\alpha = 0.2$ in my baseline calibration, but check the sensitivity of the quantitative results to varying α.[33]

5. A QUANTITATIVE EXPLORATION OF THE MODEL

With my model calibrated to match the US economy, I am in a position to examine the quantitative response of the economy to industry-level and aggregate productivity and financial shocks.

In this more general setting, the presence of higher-order linkages means there are now additional spillover effects. To illustrate, consider Figure 4. The petroleum and coal manufacturing industry and the utilities industry are linked by a common supplier, the oil and gas extraction industry. Suppose that firms in petroleum and coal manufacturing experience tighter financial constraints, forcing some to reduce production, and raising the price of petroleum. This corresponds to the standard input-output channel represented by the blue arrow.[34] In the absence of the credit linkage channel of transmission, firms in the utilities industry will remain largely unaffected by the shock.

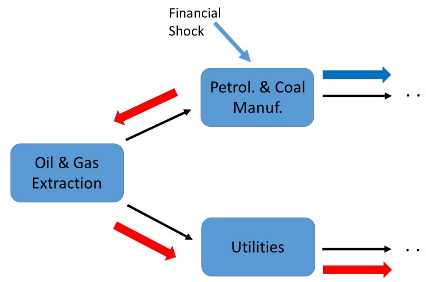
However, the shock causes petroleum and coal manufacturers to reduce the up front

[32]Recall that for a credit supply shock to have any effect on industry i, a necessary condition is that the industry be constrained in equilibrium.

[33] Recall that α is the fraction of receivables that industries can collateralize to borrow from the bank. Omiccioli (2005) finds that the median Italian firm in a sample collateralizes 20 percent of its accounts receivable for bank borrowing.

[34]In addition, the suppliers in the oil and gas industry will face lower demand from their customers, and reduce production accordingly.

Figure 4: Transmission Mechanism in the General Model



payments they make to their oil and gas suppliers. With tighter financial constraints, these suppliers reduce production, raising the price of oil and gas. As a result utilities firms pass these higher input costs downstream in the form of higher energy prices. These additional credit network effects further amplify the effects of the shock.

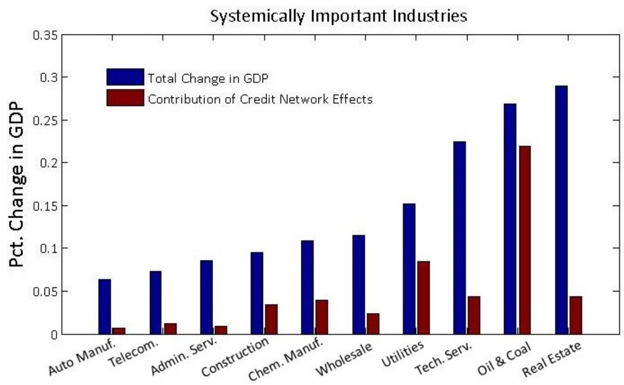
How large are these credit network effects likely to be? To answer this, I hit the US economy with an aggregate financial shock, and industry-level financial shocks, and measure the response in aggregate output to a log-linear approximation.

5.1. Response to an Aggregate Financial Shock

Suppose that the economy is hit with a one percent aggregate financial shock: each industry i's cash-in-advance constraint is tightened by one percent. [35] Under my conservative, baseline calibration, I find that US GDP falls by 2.92 percent - a large drop. Shutting off the credit linkage channel, I find that GDP falls by only 2.28 percent in response to the same aggregate shock. Thus, the credit network effects amplify the fall in GDP by about 30 percent. This is a conservative estimate of the quantitative relevance of the mechanism, given that the calibration uses data on only large, publicly-traded firms who use trade credit less intensively than other. Table 3 in the appendix reports the sensitivity of these results to the specification of $\alpha = 0.2$, the parameter controlling the substitutability of cash and bank credit.

[35] More specifically $dB_i = 0.01$ for all industries i. This can be interpreted as a one percent fall in the aggregate supply of credit.

Figure 5:



Notes: This chart shows the ten most systemically important US industries based on the counterfactual drop in GDP in response to a 1 percent industry-level financial shock. The contribution of credit network effects is computed numerically. This exercise is done excluding financial industries.

5.2. Response to Industry-Level Financial Shocks

Next, I ask which industries are likely to be systemically important to the US economy, in light of these network effects. I measure the systemic importance of industry i by the how much GDP falls in response to a 1 percent financial shock to industry i. This industry-specific shock should be interpreted as an exogenous tightening of the financial constraints of at least some firms in the industry.

Figure 5 shows a bar graph of the ten most systemically important industries in the US, based on this exercise. For each industry i, the blue bars show the elasticity of GDP with respect to B_i, or the percentage change in GDP in response to a 1 percent financial shock to industry i.

The model implies that an industry-level financial shock can have a strong impact on US GDP. For example, although the technical services industry accounts for only 0.069 percent of US GDP, a one percent financial shock this industry causes a fall in GDP of 0.19 percent - a multiplier of 2.75. The red bars indicate the magnitude of the credit network effects of the shock.[36] These credit network effects contribute substantially to this amplification, accounting for between one-fifth to half of the fall

[36] This is computed by subtracting the drop in GDP that occurs with credit linkage channel shut off, from the total drop in GDP. I shut off the credit linkage channel by imposing that financial constraints do not respond endogenously to financial shocks, i.e. $\frac{d\log\phi_j}{dB_i} = 0$ for all $j \neq i$.

in GDP in response to an industry-level shock, depending on the industry.

5.3. Mapping the Model to the Data

In order to map the model to the data, I extend the static model to be a repeated cross-section. Let X_t, N_t, B_t, and z_t denote the M-by-1 vectors of output growth, employment growth, financial shocks, and productivity for each industry respectively, in quarter t. The log-linearized model yields closed-form expressions for how the output and employment of each industry respond to financial and productivity shocks.

$$X_t = G_X B_t + H_X z_t \qquad N_t = G_N B_t + H_N z_t \tag{33}$$

The M-by-M matrices G_X and H_X (G_N and H_N) map industry-level financial and productivity shocks, respectively, into output growth (employment growth), and capture the effects of input-output and credit interlinkages in propagating shocks across industries. The elements of these matrices depend only on the model parameters, and therefore take their values from my calibration.

I construct the observed, quarterly cyclical fluctuations in the output $\hat{X}_t$ and employment $\hat{N}_t$ of US industrial production industries using data from the Federal Reserve Board's Industrial Production Indexes, which includes data on the output growth of these industries, and the Bureau of Labor Statistics' Quarterly Census of Employment and Wages, from which I observe the number of workers employed by each of these industries. At the three-digit NAICS level there are 23 such industries.[37] For each dataset, I take 1997 Q1 through 2013 Q4 as my sample period, and seasonally-adjust and de-trend each series. In the empirical analysis to follow, I use this data and the expressions (33) to decompose observed cyclical fluctuations into various components.

[37]Hours worked is not directly available at this level of industry detail and this frequency.

6. EMPIRICAL ANALYSES

In the empirical part of the paper, I use my theoretical framework to investigate which shocks drive observed cyclical fluctuations in the US, once we account for the network effects created by credit and input-output linkages between industries. The framework is rich enough to permit an empirical exploration of the sources of these fluctuations along two separate dimensions: the importance of productivity versus financial shocks, and that of aggregate versus idiosyncratic shocks.

To this end, I use two methodological approaches to identifying shocks. In the first, I identify shocks without imposing the structure of my model on the data. This permits a cleaner identification of financial and productivity shocks, and estimates a residual component of fluctuations which are not explained by either of these shocks. In the second approach, I identify shocks using a structural estimation of the model. While this attributes all fluctuations to financial and productivity shocks only, it allows for a decomposition between aggregate versus industry-level shocks.

6.1. First Method: Estimating Shocks without the Model

My first approach involves identifying financial and productivity shocks without imposing the structure of my model on the data - the identifying assumptions are completely independent of the model. An added advantage of this method is that it permits the estimation of a residual component of observed fluctuations - a component which is not explained by either shock. However, the shocks estimated using this method are assumed to be common to all industries.

6.1.1. Estimating financial shocks To identify credit supply shocks to the US economy, I estimate an identified VAR using a similar approach as Gilchrist and Zakrajsek (2011). To do this requires first constructing a measure of bank-intermediated business lending.

I construct a measure of aggregate business lending by US financial intermediaries using quarterly Call Report data collected by the FFIEC. To capture lending to the business sector, I use commercial and industrial loans outstanding and unused loan commitments - a cyclically-sensitive component of bank lending.[38] I thus construct a measure called the *business lending capacity* of the financial sector, as the sum of unused commitments and commercial and industrial loans outstanding in each quarter.[39]

To empirically identify credit supply shocks, I augment a standard VAR of macroeconomic and financial variables with the measure of business lending capacity, and the excess bond premium of Gilchrist and Zakrajsek (2012) - a component of corporate credit spreads designed to capture changes in the risk-bearing capacity of financial intermediaries.[40] The endogenous variables included in the VAR, ordered recursively, are: (i) the log-difference of real business fixed investment; (ii) the log-difference of real GDP; (iii) inflation as measured by the log-difference of the GDP price deflator; (iv) the quarterly average of the excess bond premium; (v) the log difference business lending capacity (vi) the quarterly (value-weighted) excess stock market return from CRSP; (vii) the ten-year (nominal) Treasury yield; and (viii) the effective (nominal) federal funds rate. The identifying assumption implied by this ordering is that stock prices, the risk-free rate, and bank lending can react contemporaneously to shocks to the excess bond premium, while real economic activity and inflation respond with a lag. I estimate the VAR using two lags of each endogenous variable.

To map the orthogonalized innovations in the excess bond premium into the financial shocks $\tilde{B}_t$ of my model, I make use of the impulse response function of business lending capacity, and construct financial shocks as changes in the supply of bank lending which arise due to orthogonalized innovations in the risk-bearing capacity of the financial sector. Figure 6 plots the time series of this shock.

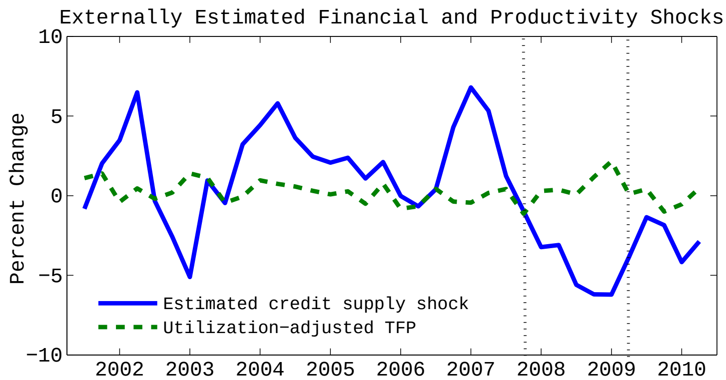
To allow for credit supply shocks to affect industries differentially depending on their dependence on external finance, I also load the financial shocks onto each industry

[38]Gilchrist and Zakrajsek (2011) show that the contraction in unused loan commitments was concomitant with onset of the financial crisis in 2007, while business loans outstanding contracted only with a lag of about four quarters.

[39]Changes in business lending capacity mostly reflect supply-side changes. To see why, consider the following example. Suppose that a business draws down an existing line of credit it has with its bank. This is recorded as a fall in unused commitments, but reflects an increase in demand for credit rather than a contraction in the supply of credit. However, the loan is now recorded as an on-balance sheet commercial or industrial loan. Therefore, the fall in unused commitments is exactly offset by the increase in commercial and industrial loans outstanding, leaving bank lending capacity unchanged. So this measure of business lending capacity is largely unresponsive to firms drawing down their lines of credit.

[40]I thank Simon Gilchrist for kindly sharing the excess bond premium data.

Figure 6:



Notes: This figure shows the series of quarter-to-quarter growth in utilization-adjusted TFP measure of Fernald (2012) and the credit supply shocks, estimated as changes in the business lending capacity of the financial sector which are due to orthogonalized innovations to the excess bond premium. Financial shocks were estimated using an identified VAR.. TFP data was obtained from the San Francisco Fed database.

based on a measure of the the industry's external finance dependence, constructed according to Rajan and Zingales (1998).[41] However, the results reported hereafter are for financial shocks $\hat{B}_t$ which load equally onto all industries.

6.1.2. Estimating productivity shocks The Federal Reserve Bank of San Francisco produces a quarterly series on TFP for the US business sector, adjusted for variations in factor utilization, according to Fernald (2012). As such, this series is readily mapped into my model as an aggregate productivity shock $\tilde{z}_t$. Figure 6 plots time series for this productivity shock. Let $\hat{z}_t \equiv \tilde{z}_t \vec{1}$ denote the M-by-1 vector of these shocks.

6.1.3. Decomposing observed fluctuations in industrial production With the estimated shocks at hand, I use log-linearized expression (33) to decompose observed cyclical fluctuations in industrial production into components coming from the financial shocks, productivity shocks, and a residual.

$$\hat{X}_t = G_X \hat{B}_t + H_X \hat{z}_t + \varepsilon_t \tag{34}$$

[41] In this manner, I obtain a time-varying, industry-specific financial shock $\tilde{B}_{it}$ which can be fed into the model.Although they varies across industries in any given quarter, these shocks to each industry are perfectly correlated across time, and so should not be interpreted as idiosyncratic shocks.

Table 1: Variance Decomposition of IP: 2001Q4:2007Q3

	Share of Aggregate Volatility
Productivity Shocks	0.205
Financial Shocks	0.279
Residual	0.516

Notes: This table reports the results of the variance decomposition of the quarterly time series of aggregate industrial production over the pre-recessionary period 2001 Q4 - 2007 Q3. Aggregate volatility is computed as the sample variance of observed aggregate industrial production. Financial shocks were estimated using an identified VAR, and capture quarterly credit supply shocks to the productive sector. Productivity shocks are estimated by Fernald (2012) as quarter-to-quarter, utilization-adjusted changes in TFP in the US, obtained from the San Francisco Fed database. The residual is the component of aggregate industrial production which is unexplained after these shocks are fed through the log-linearized model.

The residual ε_t is the component of these fluctuations which is unexplained by either of these shocks. I then feed these shocks into the model and perform a variance decomposition of aggregate industrial production.

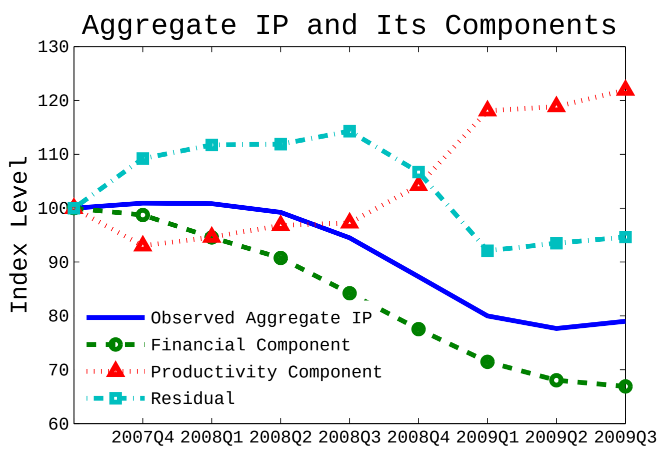
The variance decomposition of output before 2007 is given in Table 1. In the period 2001 - 2007, productivity and financial shocks played a roughly equal role in generating cyclical fluctuations, together accounting for half of observed aggregate volatility in US industrial production. The remaining half is unaccounted for by either type of shock.

However, the story is different for the Great Recession. Figure 7 plots the time series of aggregate industrial production during the Great Recession, as well as a simulation for each of its components.[42] These counterfactual series are constructed by feeding each of the estimated components through the model one at a time, and thus represents how aggregate industrial production would have evolved in the absence of other shocks, beginning in 2007 Q3.

During the recession, productivity shocks had virtually no adverse effects on industrial production - in fact, they actually *mitigated* the downturn. Rather, financial shocks are the main culprit, accounting for two-thirds of the peak-to-trough drop in aggregate industrial production during the recession. The remaining one-third is not accounted for by either shock. Furthermore, the credit network of these industries played a quantitatively significant role during this period, amplifying the effects of the financial shocks by about 15% (i.e. adding 3.98 percentage points to the peak-to-

[42] The time series for observed aggregate IP is constructed from the cyclical component of IP growth. It is constructed as an aggregate index of the observed industry-level growth rates.

Figure 7:



Notes: This figure shows the time series of aggregate industrial production and its components. Observed aggregate industrial production is an index constructed from the de-trended, seasonally-adjusted industry-level quarter-to-quarter growth rates in the output of the 23 industrial production industries at the three-digit NAICS level, obtained from FRB IP Indexes. Each of the other series depict counterfactual indexes constructed from the respective components of the observed series, beginning in 2007 Q3, and represent how aggregate IP would have evolved in the absence of other shocks. Financial shocks were estimated using an identified VAR Productivity shocks are estimated by Fernald (2012) as quarter-to-quarter, utilization-adjusted changes in TFP in the US, obtained from the San Francisco Fed database.

trough drop in the financial component of aggregate industrial production).

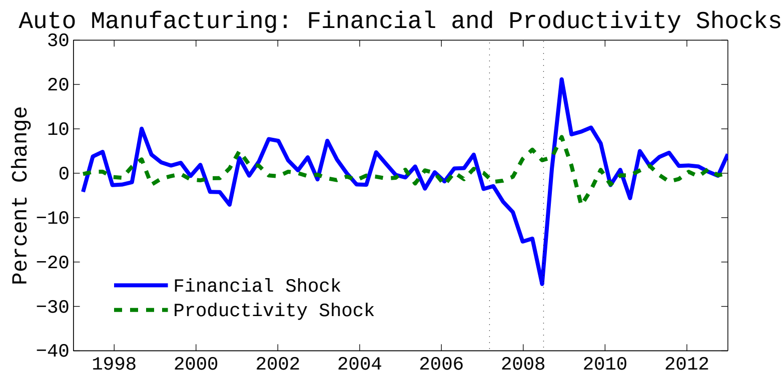
6.2. Second Method: Structural Factor Analysis

With my second methodological approach, I empirically assess the relative contribution of aggregate versus idiosyncratic shocks in generating cyclical fluctuations. This involves estimating the model using a structural factor approach similar to that of Foerster et al. (2011)[43], using data on the output and employment growth of US IP industries. The procedure involves two steps. I first use a log-linear approximation of the model to back-out the productivity and financial shocks to each industry required for the model to match the fluctuations in the output and employment data. Then, I use dynamic factor methods to decompose each of these shocks into an aggregate component and an idiosyncratic, industry-specific component.[44]

[43] Foerster et al. (2011) allow only for productivity shocks in driving observed fluctuations.

[44] As in Foerster et al. (2011), these latter shocks are specific to each industry, but idiosyncratic in the sense that they are uncorrelated across industries.

Figure 8:



Notes: This figure shows the quarterly time series of the productivity and financial shocks to the auto manufacturing industry over the sample period. Financial shocks are captured by percent changes in parameters B_i in the model, and thus represent exogenous tightnening in the cash-in-advance constraint of an industry. Productivity shocks are changes in TFP. These shocks were estimated using the log-linearized model, and quarterly data on the employment and output growth of IP industries, obtain from the BLS Quarterly Census of Employment and Wages and the FRB IP Indexes, respectively.

6.2.1. Step 1: Structural estimation of shocks

I first use a log-linear approximation of the model to back-out the productivity and financial shocks to each industry required for the model to match the fluctuations in the output and employment data. To do this, recall that from equations (33) I have an exactly identified system of equations. Given the observations $\hat{X}_t$ and $\hat{N}_t$, I then invert the system to back-out industry-level each quarter over my sample period 1997 Q1 to 2013 Q4. Denote by $\check{B}_t$ and $\check{z}_t$ the M-by-1 vectors of financial and productivity shocks estimated with this procedure in quarter t. And let $Q \equiv H_X - G_X G_N^{-1} H_N$.

$$\check{B}_t = G_N^{-1}\left(\hat{N}_t - H_N \check{z}_t\right) \qquad \check{z}_t = Q^{-1}\hat{X}_t - Q^{-1} G_X G_N^{-1} \hat{N}_t \tag{35}$$

Thus, I construct industry-level shocks as the observed fluctuations, filtered for the the network effects created by interlinkages. The model is able to separately identify these shocks because each type of shock has quantitatively differential effects on an industry's output and employment.[45]

Figure 8 shows the time series of the estimated financial and productivity shocks which hit the US auto manufacturing industry each quarter over the sample period.

Between 2007 and 2009, the output and employment of industrial production in-

[45]Namely, productivity shocks affect an industry's output relative to its employment through Cobb-Douglas production functions. On the other hand, financial shocks do not affect production functions, but tightens the cash-in-advance constraints.

dustries took a sharp drop for a number of quarters. As illustrated in the figure, this contraction shows up in the model as an acute tightening in the financial constraints of these firms, reaching up to a 25 percent decline in a single quarter.[46]

6.2.2. Step 2: Dynamic factor analysis

Next, I use factor methods to decompose the financial and productivity shocks, $\check{B}_t$ and $\check{z}_t$, into aggregate and idiosyncratic components.

$$\check{B}_t = \Lambda_B F_t^B + u_t \ , \qquad \check{z}_t = \Lambda_z F_t^z + v_t \tag{36}$$

Here, F_t^B and F_t^z are scalars denoting the common factors affecting the output and employment growth of each industry at quarter t, and are assumed to follow an AR(1) process; the residual components, u_t and v_t, are the idiosyncratic shocks. Hence, I estimate two dynamic factor models; one for the financial shocks $\check{B}_t$ and one for the productivity shocks $\check{z}_t$.[47]

To gauge the external validity of the structural factor analysis, I compare the aggregate financial shocks to the excess bond premium. The large aggregate financial shocks estimated by the structural factor analysis is broadly reflective of the severe credit crunch that occurred during this period.

6.2.3. Decomposing observed fluctuations in industrial production

To perform a variance decomposition of observed industrial production from 1997 Q1 to 2013 Q4, I follow the procedure described in appendix A2. For the full sample period, aggregate volatility is about 0.19%.[48] The results are summarized in Table 2.

[46]These features broadly hold across most industries in industrial production.

[47] I use standard methods to estimate the model. To predict the factors, I use both a one-step prediction method and Kalman smoother. The Kalman smoother yields factors which explain more of the data. Since it utilizes more information in predicting the factors, I use this method as my baseline. All subsequent reported results used the factors predicted using a Kalman smoother.

[48]This is roughly in line with the findings of Foerster et al. (2011). If I compute growth rates and aggregate volatility using the same scaling conventions as they, I find aggregate volatility to be about 9.35 compared to their 8.8 for 1972-1983 and 3.6 for 1984-2007. The higher volatility that I get comes

Table 2: Pre-Recession Composition of Agg. Vol.: 1997Q1:2006Q4

	Fraction of Agg. Vol. Explained
Productivity Shocks	0.365
Agg. Component	0.133
Idios. Component	0.232
Financial Shocks	0.635
Agg. Component	0.45
Idios. Component	0.185

Notes: This table reports the results of the variance decomposition of the quarterly time series of aggregate industrial production over the period 1997 Q1 - 2006 Q4. Aggregate volatility is computed as the sample variance of observed aggregate industrial production. Shocks to industrial production industries were estimated using the structural factor analysis of these industries' quarterly output and employment growth, obtained from the BLS Quarterly Census of Employment and Wages and the FRB IP Indexes, respectively. The aggregate and idiosycnratic components were estimated by dynamic factor analysis of the industry-level financial shocks, where the common components are assumed to follow an AR(1) process.

Before the Great Recession, aggregate volatility was driven primarily by aggregate financial shocks and idiosyncratic productivity shocks; aggregate financial shocks account for nearly a half of aggregate volatility. Nevertheless, idiosyncratic productivity shocks account for a quarter of aggregate volatility.

Furthermore, the credit network of industrial production industries amplified these shocks, accounting for nearly one-fifth of observed aggregate volatility.

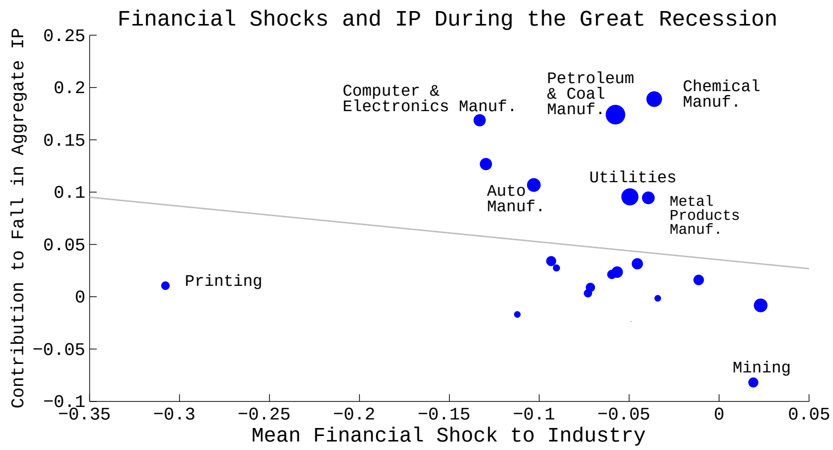
Aggregate financial shocks were the primary driver of the Great Recession. I perform an accounting exercise to evaluate how much of the peak-to-trough drop in aggregate industrial production over 2007Q4: 2009Q2 can be explained by each type of shock. I find that changes in productivity did not contribute to the decline in aggregate industrial production during the recession. In contrast, 73 percent of the drop in aggregate industrial production is due to an aggregate financial shock, and a sizable fraction of the the remainder can be accounted for by idiosyncratic financial shocks to the three most systemically important industries

Figure 9 depicts the relationship between industry-level financial shocks, an industry's contribution to aggregate output, and the systemic importance of an industry for industrial production industries during the Great Recession.

Large financial shocks to a few systemically important industries can explain the bulk of the decline in aggregate industrial production during the Great Recession. In fact, idiosyncratic shocks to the oil and coal products manufacturing, chemical prod-

from including the Great Recession in my sample period.

Figure 9:



Notes: This figure shows a scatter plot of industrial production industries by the mean quarterly financial shock to each industry during the recessionary period 2007 Q4 - 2009 Q2, and by each industry's contribution to the peak-to-trough fall in aggregate industrial production observed over this period. Each industry's spot is weighted by a measure of the industry's systemic importance to the US economy, computed using numerical simulations. The figure includes a least-squares line. Shocks were estimated using a structural factor approach and quarterly data on the employment and output growth of industrial production industries, obtained from the BLS Quarterly Census of Employment and Wages and the FRB Industrial Production Indexes, respectively. The shocks consist of both the aggregate and idiosyncratic components. An industry's contribution to the peak-to-trough drop in aggregate industrial production is computed by simulating the path of an index of each indsutry-level component of aggregate industrial production (i.e. how aggregate industrial production would evolve with shocks to exactly one industry) and computing the peak-to-trough change between 2007 Q4 and 2009 Q2.

ucts manufacturing, and auto manufacturing industries account for about 9 percent of the decline (or one-third of the decline unaccounted for by aggregate shocks), despite comprising only about 25 percent of aggregate industrial production. This suggests that idiosyncratic financial shocks to a few systemically important industries played a quantitatively significant role during the Great Recession.

In contrast, both the aggregate and idiosyncratic components of productivity shocks were slightly positive during this period on average. As such, changes in productivity did not contribute to the decline in aggregate industrial production during the recession.

6.3. Take-Aways from the Two Empirical Analyses

The broad picture which emerges from these empirical analyses is that financial shocks have been a key driver of aggregate output dynamics in the US, particularly during the Great Recession. While much of the previous literature has relied on shocks to aggregate TFP drive the business cycle, the dearth of direct evidence for such shocks has raised concerns about their empirical viability. I have argued that the credit and input-output interlinkages of firms can create a powerful mechanism by which a shock to one firm's financial constraint propagates across the economy. The confluence of my empirical results suggest that once we account for these interlinkages, financial shocks seem to displace aggregate productivity shocks as a prominent driver of the US business cycle.

7. CONCLUSION

In this paper, I showed that inter-firm lending plays an important role in business cycle fluctuations. First, I introduced supplier credit into a network model of the economy and show that trade credit interlinkages can create a powerful amplification mechanism. To evaluate the model quantitatively, I constructed a proxy of the credit linkages between US industries by combining firm-level balance sheet data and industry-level input-output data.

Finally, I used the model to investigate which shocks drive the US business cycle when we account for the linkages between industries. To do so, I identified shocks both structurally and without the use of my model. Feeding these shocks though the model showed financial shocks to be a key driver of aggregate fluctuations, particularly during the Great Recession, and productivity shocks to play only a minor role. Thus, accounting for the role that credit and input-output interlinkages play helps to capture the empirical importance of financial shocks in US business cycle fluctuations.

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Appendix

A1. Sensitivity Analysis

In the quantitative analysis, I computed the change in GDP to a counterfactual one percent aggregate financial shock. Table 3 reports these results for different values of α. While the multiplier effect of the credit network indeed falls as α approaches 1, credit network effects are quantitatively significant for reasonable values of α.

Table 3:

α	$P(\hat{\alpha} \leq \alpha)$	% Change in GDP	Credit Network Amplification
0	0.18	4.04%	77.2%
0.1	0.32	3.26%	43.0%
0.2	**0.5**	**2.92%**	**28.1%**
0.4	0.66	2.59%	13.6%
0.5	0.75	2.50%	9.6%
1	0.97	2.28%	0%

Notes: This table reports the results of the sensitivity analysis. Recall that α is the fraction of accounts receivable that banks can collateralize to borrow from the bank, and controls the substitutability of cash and bank credit for firms in the model. The first column indicates the value of α used. The second column yields the fraction of Italian firms which collateralizes less than α of their receivables to borrow from banks, as estimated by Omiccioli (2005). The third column lists the total percentage change in GDP in response to a 1 percent financial shock to all US industries. The fourth column lists by how much the credit network effects amplify the drop in GDP in response to the shock. The bold row indicates the baseline calibration.

A2. Structural Factor Analysis: Aggregate Volatility

Assume the shocks B_t and z_t in (33) are composed of an aggregate and idiosyncratic components.

$$B_t = \Lambda_B F_t^B + u_t \quad F_t^B = \gamma_B F_{t-1}^B + \iota_t^B \tag{37}$$

$$z_t = \Lambda_z F_t^z + v_t \quad F_t^z = \gamma_z F_{t-1}^z + \iota_t^z \tag{38}$$

Then letting Σ_{XX} denote the variance-covariance matrix of X_t, and $\bar{s}$ a vector of industry shares of aggregate output, aggregate volatility (of output) is approximately

$$\sigma^2 \equiv \bar{s}'\Sigma_{XX}\bar{s} = \bar{s}'G_X\Sigma_{BB}G_X'\bar{s} + \bar{s}'H_X\Sigma_{zz}H_X'\bar{s}\,. \tag{39}$$

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O.A1. PROOF OF AMPLIFICATION

From the definitions of χ_i and ϕ_i, we have

$$\phi_i = min\left\{1,\ \frac{1}{r_i}\left(B_i + \theta_{i,i-1} - \theta_{i+1,i}\frac{1}{\phi_{i+1}\omega_{i+1,i}(1-\eta_{i+1})}\right)\right\}.$$

Here, $r_i = 1$ denotes firm i's returns-to-scale. It follows that

$$\frac{d\,\phi_{i-1}}{d\,B_i} = \begin{cases} \frac{1}{r_i}\frac{\alpha\theta_{i,i-1}}{\phi_i\omega_{i,i-1}(1-\eta_i)} > 0 & if\ \phi_{i-1} < 1 \\ 0 & otherwise \end{cases}$$

$$\frac{d\,\phi_j}{d\,B_i} = 0\,\forall\, j > i\ \ and\ \ \frac{d\,\phi_j}{d\,B_i} = \frac{1}{r_i} > 0\ for\ j = i\,.$$

Putting these cases together, we can write $\frac{d\,log\,\phi_j}{d\,B_i}$ for any j.

$$\frac{d\,log\,\phi_j}{d\,B_i} = \begin{cases} \frac{1}{r_i} > 0 & if\ j = i \\ \frac{1}{\phi_j}\frac{1}{r_j}\frac{\theta_{kj}}{\phi_k\omega_{kj}(1-\eta_k)}\frac{d\,\phi_k}{d\,B_i} \geq 0\,\forall k & if\ j < i \\ 0 & otherwise \end{cases}$$

It follows that $\frac{d\,log\,\phi_j}{d\,B_i} \geq 0$ and $\frac{d}{d\,\theta_{ij}}\left(\frac{d\,log\,\phi_j}{d\,B_i}\right) \geq 0$.

O.A2. CONTRACTING ENVIRONMENT

Each firm has access to some diversion technology, whereby instead of investing a loaned input into production, it can invest it into another technology which yields a private, non-verifiable benefit to the firm. While suppliers know the characteristics of their customers' diversion technology, they do not observe how much of these inputs sold are used in production or diverted. For simplicity, I assume that firms cannot divert non-loaned inputs.[49]None of the value of a diverted input can be recovered by the supplier. Therefore, the supplier will never find it profitable to incentivize diversion.

I assume that the diversion technology is input-specific, in that each representative firm has access to a different diversion technology for each input. I also assume that atomistic suppliers observe how much they each lend to their customers.[50] These two assumptions imply that

[49]More specifically, I assume that once goods are paid for, registered publicly with an outside accountant, and so any diversion is publicly observed. This means only goods bought on credit can be privately diverted. While this simplifies the exposition, it is not important for qualitative results.

[50]If, contrary to this assumption, the diversion technology were different for each of a firm's atomistic supplier, then the firm could get around its borrowing constraint by borrowing a small amount from many suppliers, never approaching borrowing constraint with any single supplier, thereby rendering the borrowing constraint moot. Then no firm would ever be constrained. I make these two assumptions to preclude this situation.

an atomistic firm's incentive to divert input j will depend on how much of that input it is purchasing on credit from all of the atomistic suppliers of good j, but it will not depend on how much of a different input k it is purchasing on credit. Ultimately, this will imply each representative firm faces exactly one borrowing constraint for each good.

To operate the diversion technology for input j, representative firm i faces a constant marginal cost of diversion given by $f_{ij} > 0$. The marginal private benefit from diversion $\delta_{ij}(\frac{\tau_{ij}}{p_i x_i})$, which is a function monotonically increasing in the share of revenue financed by the trade credit loan, to capture in a reduced form way the notion that the severity of the moral hazard problem scales with the size of the firm. Suppose that the marginal cost and benefit intersect at θ_{ij} - i.e. θ_{ij} solves $f_{ij} = \delta'_{ij}(d)$. (For a given f_{ij} and δ_{ij}, θ_{ij} is unique because f_{ij} is constant and δ_{ij} is monotonic.) Then at the margin, a representative firm i will not divert a marginal input loaned from j if and only if $\frac{\tau_{ij}}{p_i x_i} \leq \theta_{ij}$, for that is the condition under which the marginal cost of diversion is less than the marginal benefit. Hence, we can treat θ_{ij} as a parameter, specific to each firm-pair, which governs the severity of the moral hazard problem between representative firms i and j. We can rewrite this as

$$\tau_{ij} \leq \theta_{ij} p_i x_i.$$

Note this environment produces a borrowing constraint which resembles a collateral constraint, in which some fraction of total revenue (receivables) can be credibly plegded to supplier to repay the trade credit loan from representative supplier j.

O.A3. MARKET STRUCTURE

This section shows how competition amongst suppliers pins down the amount of trade credit at the maximum allowed by the borrowing constraint.

Consider industry (i.e. representative firm) j. Recall that industry j is comprised of a continuum of perfectly competitive firms with CRS production, indexed by superscript i. (For example, y^i_j denotes the output of firm i in industry j. In what follows, I supress the subscript indexing each industry.) Entry into industry j is costless - upon entry, each firm draws a cash endownment b^i from a distribution G with support $[0, \bar{b}]$. (This is akin to b_i in the body of the text).

At the beginning of each period, each atomistic supplier in industry j posts a contract, which specifies a price of the good and the terms of trade credit. Atomistic firms in industry i choose which contracts to accept, and given these contracts, how many inputs to buy and borrow from each supplier. Each atomistic firm may have multiple suppliers or customers in other industries.

Suppliers compete with each other along two margins: the price of their good and the amount of trade credit. At the beginning of each period, each supplier i posts a contract consisting of a price p^i for its output and a fraction φ^i the payment which can be postponed until the end of the period. Hence, a contract offered by atomistic supplier i consists of (p^i, φ^i). Competition takes place amongst suppliers based on these contracts. (Given a contract, a buyer and seller also agree on an amount y^{ij} to of good i to be sold to atomistic firm j (in a downstream industry). Then the total trade credit can be written as $\tau^i = \varphi^i p^i y^{ij}$).

O.A3.1 Supply of Trade Credit

Here, I show how competition jointly determines the prices of intermediate goods and supply of credit. To do so, I setup the game between atomistic suppliers in an industry, and solve for the the Nash equilibrium of the game.

Consider the problem of firm i in industry j. Without loss of generality, suppose each firm's production takes only one input, given by $y^i = F(n^i)$. Let c denote firms' marginal cost (common to all firms in industry j), τ^i the trade credit offered by firm i to its customers in other industries, and τ^i_{j-1} the trade credit received by firm i by its suppliers in industry $j-1$. Then the firm's problem is to choose how much to produce y^i, what price p^i and payment terms τ^i to offer its customers, and how much to borrow its suppliers τ^i_{j-1} to maximize its profits subject to its cash-in-advance constraint. (Here, I express the trade credit loan τ^i as a function of the contract terms (p^i, φ^i)).

$$\max_{p^i,\varphi^i,n^i,\tau^i_{j-1}} \quad \pi^i = p^i y^i - cn^i$$

$$s.t.\ cn^i \leq p^i y^i - \tau^i(p^i,\varphi^i) + \tau^i_{j-1} + b^i \tag{40}$$

Let $D^i(p^i, \varphi^i)$ denote the demand that firm i faces from its customers in other industries, given the price p^i and trade credit φ^i it offers them. (For now, take this function as given - I will specify it below). Since the firm never finds it optimal to produce more or less than the demand it faces, we can set $y^i = D^i(p^i, \varphi^i)$. In addition, I assume borrowing is costless, so that it chooses to borrow as much as possible from the bank and its suppliers. Then we can rewrite the firm's problem as

$$\max_{p^i,\varphi^i} \ \pi^i = p^i D^i(p^i,\varphi^i) - cn^i$$

$$s.t.\ cn^i \leq p^i D^i(p^i,\varphi^i) - \tau^i(p^i,\varphi^i) + \tau^i_{j-1} + b^i \tag{41}$$

Letting λ^i denote the Lagrange multiplier, the firm's first order conditions are

$$\left(1+\lambda^i\right)\left(D^i(p^i,\varphi^i) + p^i \frac{dD^i}{dp^i}\right) = c\frac{dF^{-1}}{dy^i}\frac{dD^i}{dp^i}$$

$$\left(1+\lambda^i\right) p^i \frac{dD^i}{d\varphi^i} = c\frac{dF^{-1}}{dy^i}\frac{dD^i}{d\varphi^i} + \lambda^i$$

Before specifying D^i, i.e. how the demand that firm i faces depends on its choices of p^i and φ^i, I introduce the agency problem between the firm and its customers, which puts an upper bound on the trade credit firm i offers its customers.

There is a agency problem between firm i and its customers in other industries, discussed in the section on the contracting environment, which produces a borrowing limit, call itM_{j+1}, such that the loans to any atomistic firm j never exceed M_{j+1}. (Note that, in the body of the text, $M_{j+1} = \theta_{j+1}\frac{p_{j+1}y_{j+1}}{p_j y_j}$.)

I now specify $D^i(p^i, \varphi^i)$ - the demand that firm i faces as a function of its choices of φ^i and p^i. Firm i must compete with other firms in industry j for customers in industry $j+1$. Because of this competition, the demand that firm i faces depends on the price p^i it posts relative to that posted by its competitors p^k and the trade credit φ^i it offers relative to that offered by its competitors φ^k, for all $k \neq i$. Let $\underline{p} \equiv \min\{p^s\}_{\forall s}$ denote the lowest price offered by any firm within industry j, and let $\bar{\varphi} \equiv \max\{\varphi^s\}_{\forall s}$ denote the highest trade credit loan offered by any firm within industry j. Let K denote he number of firms in industry j who have set $\varphi^k = \bar{\varphi}$ and $p^k = \underline{p}$. The demand faced by firm i takes the following form. (Because the above agency problem precludes $\varphi^i > M_{j+1}$, I ignore this case.) Let $D_{j+1}(\underline{p}, \bar{\varphi})$ denote the total demand of customers in industry $j+1$, given the most competitive input prices $\underline{p}$ and trade credit $\bar{\varphi}$.

$$D^i\left(p^i, \varphi^i, \{p^k, \varphi^k\}_{\forall k \neq i}\right) = \begin{cases} D_{j+1}(\underline{p}, \bar{\varphi}) & if\ \varphi^i \leq M_{j+1}\ \&\ \varphi^i > \varphi^k\ \&\ p^i < p^k\ for\ all\ k \\ \frac{1}{K} D_{j+1}(\underline{p}, \bar{\varphi}) & if\ \varphi^i \leq M_{j+1}\ \&\ \varphi^i = \bar{\varphi}\ \&\ p^i = \underline{p} \\ \nu^i D_{j+1}(\underline{p}, \bar{\varphi}) & if\ \varphi^i \leq M_{j+1}\ \&\ [(\varphi^i = \bar{\varphi}\ \&\ p^i > p^k)\ or\ (\varphi^i < \bar{\varphi}\ \&\ p^i = \underline{p})]\ for\ some\ k \\ 0 & otherwise \end{cases}$$

The first case says that, if firm i out-competes all other suppliers in industry j by offering both the lowest price and highest loan, then firm i gets all the demand from customers in industry $j+1$. The second case says that if firm i is not unique in offering the lowest price and highest loan, then it shares the total demand with the other suppliers who offer the same terms. The third case states that if firm i offers the lowest competitive price but not the highest loan, or offers the highest loan but not the lowest price, then it faces some fraction $\nu^i\, \varepsilon\, [0, 1]$ of the total demand of customers. (In general, ν^i will depend on production function and constraint of customers in industry $j+1$. However, as I will show below, this case will never prevail in equilibrium, so leave ν^i unspecified.) The final case states that if firm i offers neither the lowest price nor the highest loan, then it is out-competed and faces none of the demand from customers.

Best Response Functions

Given the demand function D^i above and the first order conditions, we can now characterize firm i's best response function. I divide the function into 2 broad cases: when firm i is unconstrained in equilibrium (so that its CIA constraint does not bind), denoted with a subscript U; and when firm i is constrained in equilibrium, denoted with subscript C. Each of these cases can further be divided into four cases, described below.

First suppose that firm i is unconstrained in equilibrium. The firm's best response to the competition will be to incrementally increase its trade credit offer to capture all the demand costlessly. If he cannot do this because he's reached the maximum trade credit loan allowed by the agency problem, then he incrementally decreases the price to capture as much demand as possible. This is formalized below.

Case U1: If $\bar{\varphi} < M_{j+1}$

$$BR^i_{U,1}\left(\{p^k, \varphi^k\}_{k \neq i}\right) = \begin{bmatrix} \varphi^i = \bar{\varphi} + \varepsilon \\ p^i = \underline{p} \end{bmatrix}$$

In this case, the highest trade credit loan offered by any other competitor is still less than the limit required to prevent customers from diverting the loan. Since firm i is unconstrained, it can costlessly increase its trade credit offer by an arbitrarily small amount ε and get all the demand.

Case U2: If $\bar{\varphi} \geq M_{j+1}$ and $p_U^L < \underline{p} < p_U^H$. Here, p_U^L and p_U^H denote the lowest and highest prices, respectively, that firm i would be willing to offer in this case. (When $\bar{\varphi} > M_{j+1}$, then p_U^L solves $\pi^i = 0$ and p_U^H solves the FOC, when we impose $D^i = \nu^i D_{j+1}(p^i, \bar{\varphi})$. When $\bar{\varphi} = M_{j+1}$, p_U^L solves $\pi^i = 0$ when we impose $D^i = \frac{1}{K} D_{j+1}(p^i, \bar{\varphi})$, and p_U^H solves the FOC when we impose $D^i = D_{j+1}(p^i, \bar{\varphi})$).

$$BR_{U,2}^i\left(\{p^k, \varphi^k\}_{k\neq i}\right) = \begin{bmatrix} \varphi^i = \min\{\bar{\varphi},\, M_{j+1}\} \\ p^i = \underline{p} - \varepsilon \end{bmatrix}$$

In this case, the lowest price offered by firm i's competitors exceeds the lowest price firm i would be willing to offer, and is less than the maximum price firm i would be willing to offer. Firm i's best response is then to set its trade credit competitively (not high enough to incentivize diversion of the loan, and no lower than the lowest set by competitors). Also, it will want to undercut the price of its nearest competitor by an arbitrarily small amount, to get all the demand at minimum cost.

Case U3: If $\bar{\varphi} \geq M_{j+1}$ and $\underline{p} \geq p_U^H$

$$BR_{U,3}^i\left(\{p^k, \varphi^k\}_{k\neq i}\right) = \begin{bmatrix} \varphi^i = \min\{\bar{\varphi},\, M_{j+1}\} \\ p^i = p_U^H \end{bmatrix}$$

In this case, the lowest price offered by competitors exceeds what firm i is willing to offer. Then the only difference from the previous case is that firm i would set its price to p_U^H, capturing all demand. Any higher or lower price would reduce its profits.

Case U4: If $\bar{\varphi} \geq M_{j+1}$ and $\underline{p} \leq p_U^L$

$$BR_{U,4}^i\left(\{p^k, \varphi^k\}_{k\neq i}\right) = \begin{bmatrix} \varphi^i\, \varepsilon\, [\bar{\varphi}, M_{j+1}] \\ p^i = p_U^L \end{bmatrix}$$

In this case, the lowest price offered by competitors is less than what firm i is willing to offer. Then firm i will choose to set price to p_U^L (any lower and it would loose money on each unit). In addition, it will set its trade credit offer no lower than the competition and no higher than the borrowing limit. (An intermediate level might increase the demand it faces, depending on the customers' tradeoff between high trade credit and a high price).

Now suppose now that firm i is constrained in equilibrium.

Case C1: When $\bar{\varphi} < M_{j+1}$, it is not obvious whether firm i would prefer to reduce its price or increase its trade credit offered (since its now constrained in equilibrium), thereby capturing all the demand in the process. This depends on the tradeoff between the two, which depends on its production function and its constraint. But in general, its best response function in this case will be to pick the action which comes at minimal cost:

$$BR_{C,1}^i\left(\{p_j, \varphi_j\}_{j\neq i}\right) = \max\left\{\pi^i\left(\begin{bmatrix} \varphi_i = \bar{\varphi} + \varepsilon \\ p_i = \bar{p} \end{bmatrix}\right),\ \pi^i\left(\begin{bmatrix} \varphi_i = \bar{\varphi} \\ p_i = \bar{p} - \varepsilon \end{bmatrix}\right)\right\}$$

For Cases C2-C4, the best response functions will look the same as in Cases U2-U4, but for

p_C^H and p_C^L defined analogously to p_U^H and p_U^L (i.e. when firm i is constrained).

Case C5: For the constrained case, there is an additional case for $\bar{\varphi} < M_{j+1}$. If $\pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi} + \varepsilon) < \pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi})$ and $\pi^i(p^i = \underline{p} - \varepsilon,\ \varphi^i = \bar{\varphi}) < \pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi})$, then

$$BR^i_{C,5}\left(\{p_j, \varphi_j\}_{j \neq i}\right) = \begin{bmatrix} \varphi^i = \bar{\varphi} \\ p^i = \max\{\underline{p},\ p_C^L\} \end{bmatrix}$$

This states that if the demand that the firm faces at $p^i = \underline{\bar{p}}, \varphi^i = \bar{\varphi}$ exceeds the amount that it would be able to service (when constrained) by increasing φ^i or decreasing p^i by $\varepsilon > 0$, then it is not worth it to out-compete others by doing either. If firm i is unconstrained in equilibrium, this case never applies, because when $\bar{\varphi} < M_{j+1}$, then firm i can always strictly increase profits by setting $\varphi^i = \bar{\varphi} + \varepsilon$. However, when firm i is constrained in equilibrium, it is already producing at capacity (given the constraint). So increasing trade credit reduces its capacity by tightening its constraint. (Note however, that increasing trade credit increases the demand faced by the firm, which may allow it charge a higher price. This higher price can offset the reduced capacity. Therefore Case C5 is not the only possible case when the firm is constrained in equilibrium.)

$BR^i_{C,5}$ highlights that there is a tradeoff to increasing trade credit when the firm is constrained in equilibrium: on the downside, the firm can get all of the demand from customers. This can allow the firm to increase the price offered to customers, increase profits per unit sold. On the other hand, increasing trade credit tightens its constraint, reducing its capacity to produce. Given that the firm is already constrained to the extent that it cannot service any more demand, then increasing trade credit can make the firm worse off at the margin (the increased demand it faces is of no use if the firm has no capacity to service it). Given this tradeoff, it is not obvious, that the firm finds it optimal to marginally increase trade credit when it is already constrained. For this reason, one may be tempted to conclude that we could have an equilibrium in which all firms who produce in the industry are sufficiently constrained that $\bar{\varphi} < M_{j+1}$. However, as discussed below, this cannot be an equilibrium due to free entry.

As I will discuss below in the section on demand for trade credit, in equilibrium, each firm $j+1$ will demand the maximum trade credit M_{j+1} allowed by the borrowing constraint.

Sectoral Nash Equilibrium

Given the best response functions of all suppliers, I now characterize the equilibrium trade credit φ_i offered to customers. First suppose that all firms in industry j are unconstrained in equilibrium. The best response $BR^i_{U,1}$ ensures that each firm offers φ^i up to the maximium M_{j+1} allowed by the agency problem (because marginally increasing φ^i is costless for an unconstrained firm). Since all firms in industry j face the same marginal cost of production, they all produce at $p^i = p_U^L$ and share the demand.

Now suppose that some firms in industry j are constrained in equilibrium. I claim that free entry and the best response functions above imply that, in equilibrium, we have $\varphi^i = M_{j+1}$ in all cases, for all firms i. To see this, note first that the best response Cases C1-C4 push $\bar{\varphi}$ up to its limit M_{j+1}.

From $BR^i_{C,5}$, we could have all firms be sufficiently constrained that $\bar{\varphi} < M_{j+1}$ and none find it optimal to marginally increase φ^i - this could happen if their capacity to produce is less than or equal to the demand they currently face $\frac{1}{K}D_{j+1}(\underline{p},\bar{\varphi})$. (If that's the case, then their opportunity cost of increasing φ^i by $\varepsilon > 0$, $\underline{p}\frac{1}{K}D_{j+1}(\underline{p},\bar{\varphi})$, exceeds the benefit, $\underline{p}y^i$).

However, free entry precludes this as an equilibrium. To see this, note that as long as there are profits to be made, new firms would enter and set $\varphi^i = \bar{\varphi} < M_{j+1}$, increasing the measure K of firms who share the demand. Once K becomes sufficiently high, the opportunity cost of increasing φ^i by ε, $\underline{p}\frac{1}{K}D_{j+1}(\underline{p},\bar{\varphi})$, would be exceeded by the benefit $\underline{p}y^i > 0$. At this moment, all firms would find it optimal to increase φ^i up to the point that $\frac{1}{K}D_{j+1}(\underline{p},\bar{\varphi}) = y^i$ (i.e. their capacity is sufficient to meet the demand they face). Thus, as long as $\bar{\varphi} < M_{j+1}$, new entrants will always find it worthwhile to enter and set $\varphi^i > \bar{\varphi}$. This process of entry would push $\bar{\varphi}$ up to its limit until $\bar{\varphi} = M_{j+1}$. (And equilibrium price and profits are determined by how constrained firms are in equilibrium.)

Proposition: In all cases $\varphi^k = M_{j+1}$ for all firms k in industry j. In words, the best response functions above and free entry imply that the trade credit borrowing constraints always bind in equilibrium.

Proof: First, assume for the sake of contradiction that, in equilibrium, all firms k offer $\varphi^k = \bar{\varphi} > M_{j+1}$. By the agency problem outlined above, customers will find it optimal to divert the entire trade credit loans φ^k. Firm i knows this, and it knows it could increase profits by setting $\varphi^i = M_{j+1}$. Therefore, this cannot be an equilibrium

Now, assume for the sake of contradiction that all firms k offer $\varphi^k = \bar{\varphi} < M_{j+1}$ for all k. Consider firm i.

First, suppose that firm i is unconstrained in equilibrium. Then firm i can increase its profits by setting $\varphi^i = \bar{\varphi} + \varepsilon$. This is because setting \tau^i marginally higher comes at no cost (since the firm is unconstrained), and the firm can out-compete all competitors by doing so. Then this cannot be an equilibrium.

Now suppose that firm i is constrained in equilibrium. There are two cases to consider: either $\pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi} + \varepsilon) > \pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi})$ or $\pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi} + \varepsilon) \leq \pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi})$.

First suppose that $\pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi} + \varepsilon) > \pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi})$. Then, by definition firm i can increase profits by increasing \tau, even though this leaves it more constrained. The loss in profits due to reduced capacity to produce is more than offset by increase in profits due to higher price it can charge due to the higher demand it faces. Thus, this cannot be an equilibrium.

Now suppose that that $\pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi} + \varepsilon) \leq \pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi})$. Then the firm is sufficiently constrained that it cannot increase its profits by increasing the trade credit offers customers (the loss in profits due to a lower capacity to produce is larger than the gain from charging a higher price). However, free entry precludes this as an equilibrium.

To see this, note that the profits of firm i are an increasing function of the demand that it faces, which, in this case, is $\frac{1}{K}D_{j+1}(\underline{p},\bar{\varphi})$ (where K is the number of firms in industry j who offer $p^i = \underline{p}$ and $\varphi^i = \bar{\varphi}$). Suppose that K is finite, so that firm i is earning positive profits. Recall that there is an infinite pool of potential entrants, and entry ino the industry is costless. Given that there are positive profits to be made, new firms will enter, offering $\underline{p}$ and $\bar{\varphi}$. This free entry will mean K grows arbitrarily large, until the demand faced by each firm (and therefore the profits of each firm) becomes arbitrarily small. At some point, one of two things will happen. The demand faced by firm i will become sufficiently small that, it is is no longer constrained in equilibrium (a contradiction to the initial assumption). Or its profits will be become so small that the marginal increase in profits from raising trade credit (and therefore capturing all the demand and raising the price) will exceed the opportunity cost to doing so (0 current profits) - i.e. $\pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi} + \varepsilon) > \pi^i(p^i = \underline{p},\ \varphi^i = \bar{\varphi}) \approx 0$ (also a contradiction). Then, firm i will no longer find it optimal to offer $\bar{\tau}$ - it can now increase its profit by offering $\bar{\varphi} + \varepsilon$. Thus, this cannot be an equilibrium.

This result implies that due to perfect competition, all suppliers always offer the maximum trade credit allowed by the agency problem, i.e. M_{j+1}. In addition, because customers always take the maximum trade credit offered to them, regardless of whether they're constrained in equilibrium or not, the trade credit borrowing constraints always bind in equilibrium. Thus, $\varphi^i = M_{j+1}$ for all firms i in industry j. Q.E.D.

O.A3.2 Demand for Trade Credit

How much trade credit do firms demand? Two features of the model govern firms' demand for trade credit. First, trade credit loans are assumed to be costless for borrowers. So all else equal, when selecting a contract, an atomistic customer wants the maximum trade credit and lowest price. Second, atomistic customers do not internalize that collectively, their demand for trade credit has an effect on the price that suppliers can offer in equilibrium. As a result, each atomistic firm in industry i demands the maximum trade credit allowed by the borrowing constraint (4) from each of its atomistic suppliers in industry j.

To see this, recall that each representative firm consists of a continuum of atomisitic, competitive firms. Each atomistic supplier in industry j can have multiple atomistic customers in industry i, and each atomistic customer in industry i can have multiple atomistic suppliers in industry j. The price offered by any supplier in equilibrium depends on the tightness of that suppliers constraint, which in turn depends on interactions with all of that supplier's customers. Each customer chooses which contracts to accept, and separately, how many inputs to buy and borrow from each supplier, given those contracts.

When selecting a contract, an atomistic cutomer wants the maximum trade credit and lowest price, as discussed in the section on the demand for trade credit. Trade credit loans are assumed to be costless for borrowers (interest rate set to 0). Since firms may be constrained in equilibrium, having more trade credit weakly dominates having less. If a firm is unconstrained in equilibrium (in that its cash-in-advance constraint does not bind), it is indifferent between taking marginally more and marginally less trade credit from its suppliers. I assume for simplicity that when indifferent, firms elect to take more trade credit. These assumptions imply that firms would like to take as much trade credit as is offered to them by their suppliers. In any event, we will be concerned the case in which all representative firms are constrained in equilibrium.

While in general, there may be a tradeoff between the trade credit and price a supplier can offer due to suppliers' financial constraints, this tradeoff is not internalized by the atomistic customers. More specifically, they do not internalize that the amount of trade credit demanded may affect the price suppliers can offer in equilibrium. Why is this the case? Each customer knows that, if a given supplier is constrained, increasing its demand for trade credit result in the supplier offering a higher price (that, or the supplier will simply be unable to offer a competitive contract). To minimize the marginal price impact of his own demand for trade credit, a customer prefers to buy an infinitesimally small amount from many (all) suppliers. Since all atomistic suppliers within an industry are identical, they will all offer the same contract in equilibrium. This implies that in equilibrium, all suppliers will buy and lend an infinitesimal amount with all atomistic customers (i.e. the full continuum). Therefore, in equilibrium, the marginal price impact of any single customer's demand from trade credit is zero.

O.A3.3 Equilibrium Contracts

Given the contracts offered and demanded specified above, I now discuss which contracts are offered and accepted in equilibrium.

Each atomistic customer takes other customers' demand for trade credit as given. Then each firm demands the maximum trade credit from each supplier. But in equilibrium, the collective demand for trade credit from all customers affects the prices offered in equilibrium through competition and the constraints faced by suppliers. But no individual customer internalizes these price effects. As a result, equilibrium contracts are characterized by an offer of the maximum trade credit allowed by the borrowing constraint and (so that the constraint is binding) and the lowest price feasible given the supplier's constraint. In other words, atomistic suppliers take both prices and the fraction of output to be sold on credit as given in equilibrium.

Key to this result is that, because each atomistic supplier can have multiple atomistic customers, the price offered by each supplier is determined in equilibrium from each supplier's constraint (an aggregate outcome which depends on the demand for trade credit of all the supplier's customers), while the trade credit demanded by each customer not aggregate but individual. So in equilibrium, the amount of trade credit offered in each supplier's contract is pinned down by the borrowing constraint, and the price is the lowest possible given these constraints.