Preliminary and Incomplete

Structural estimation of sovereign default models: The source of financial frictions*

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ABSTRACT

Emerging market business cycles feature a higher variability of consumption relative to output, a strongly countercyclical trade balance. Recent RBC studies regard financial frictions as the primary source of these features, rather than nonstationary productivity shocks. However, the source of financial frictions may be nonstationary productivity shocks, although they are not a direct cause of these stylized facts. Sovereign default models are suitable for addressing this issue since financial frictions are endogenous, and nonstationary productivity shocks may or may not become important. Therefore, we quantitatively evaluate the importance of nonstationary productivity shocks in sovereign default models using simulated tempering sequential Monte Carlo. Our main result indicates that nonstationary productivity shocks are the important source of financial frictions in terms of the random-walk component criterion. Our result bridges a gap between two strands of the literature.

*JEL classification:*E32, E62, F41, F44

Keywords: Sovereign default, Business cycles, Financial imperfections, Particle filter, Sequential Monte Carlo, Full nonlinear DSGE

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* The views expressed in this paper are those of the authors and do not necessarily reflect the views of Policy Research Institute, Ministry of Finance Japan.

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1. Introduction

The empirical regularities of the business cycles of emerging economies are excessive volatility in consumption, countercyclical current account balance, countercyclical interest rate, and frequent default at equilibrium (e.g. Neumeyer and Perri, 2005; Uribe and Yue, 2006; Aguiar and Gopinath, 2006, 2007; García-Cicco et al., 2010; Uribe and Schmitt-Grohé, 2017). There are two strands of literature that explain these stylized facts. The first strand claims that the most important source of these characteristics is a permanent productivity shocks. The second strand emphasizes financial frictions rather than nonstationary productivity shocks.

Aguiar and Gopinath (2007) introduce nonstationary productivity shocks into an open-economy real business cycle (RBC) model, and successfully replicate the characteristics of emerging economies. As the source of nonstationarity, Boz et al. (2011) suggest informational frictions, and Naoussi and Tripier (2013) suggest the level of income, quality of institutions, and the size of credit markets.

In contrast, García-Cicco et al. (2010) and Chang and Fernández (2013) emphasize stationary financial frictions rather than nonstationary productivity shocks. They add financial frictions to an RBC model, and report that it performs much better than frictionless RBC models with nonstationary productivity shocks. Especially, without financial frictions, RBC models tend to generate nearly random-walk trade balances, and fail to replicate excess volatility in consumption. Furthermore, under economies with financial frictions, the persistence and the variance of nonstationary productivity shocks are very small, thus play negligible role. Álvarez-Parra et al. (2013) introduce nondurable and durable goods, and also report only a minor effect for trend output shocks.

However, the source of financial frictions may be nonstationary productivity shocks, although their direct effect on business cycles is weak. It is possible that the source, nonstationary productivity shocks, seem to play a small role where the outcomes, financial frictions, are concurrently evaluated.

Sovereign default models are suitable for addressing this issue because financial frictions are endogenous, and nonstationary productivity shocks may or may not become the cause of them. Financial frictions (e.g. country-risk premium shocks, realistic debt-elasticity of the country premium) considered in RBC studies are microfounded endogenous mechanisms in sovereign default models. The drivers are productivity shocks which may be stationary or nonstationary or both. The model has repeatedly succeeded in replicating the stylized facts of the business cycles of emerging economies[1], but there is no structural estimation works, to the best of our knowledge.

Aguiar and Gopinath (2006) compare the sovereign default model with stochastic trend, nonstationary productivity shocks, and the model with stable trend. They report that the former performs better than the latter. Stochastic trend makes the decision to default relatively more sensitive to the particular realization of the productivity shocks and less sensitive to the amount of external debt. This generates a positive correlation between interest rate and the trade balance, which is consistent to data. Their results suggest nonstationary productivity shocks are the important source of financial frictions.

Therefore, we estimate a structural sovereign default model, and evaluate the effects of both stationary and nonstationary productivity shocks. Comparing two types of productivity shocks is meaningful for the literature. Stationary productivity shocks keep importance in the financial frictions in García-Cicco et al. (2010) and Chang and Fernández (2013), whereas nonstationary productivity shocks lose importance.

Our main result indicates that nonstationary productivity shocks are the important sources of financial frictions in terms of random-walk components, historical decompositions and parameter estimates. The result bridges a gap between two strands of the literature. As García-Cicco et al. (2010) and Chang and Fernández (2013) note, financial frictions have a substantial effect on the

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[1] See, e.g., Arellano (2008), Cuadra and Sapriza (2008), Alfaro and Kanczuk (2009), Hatchondo and Martinez (2009), Yue (2010), Boz (2011), Mendoza and Yue (2012), and Durdu et al. (2013).

business cycles of emerging economies. On the other hand, nonstationary productivity shocks are the important as the sources of financial frictions.

Another contribution of our study is that we are the first to provide parameter estimates of sovereign default models by structural estimation. Calibrations differ between studies (Table 1) although all the models examine the Argentine economy, since there is much less agreement on the magnitude of default costs[2]. Our result shows domestic costs (decreases in output) and the probability of regaining financial market access are relatively small. This is because our estimation method measures the length of default periods directly. Low probabilities of reentry are matched to the actual length of default in Argentina and Mexico. This makes domestic costs relatively small to keep defaults frequent.

Our estimation strategy is similar to Bayesian estimation on full-nonlinear DSGE models such as Gust et al. (2012) since almost all structural sovereign default models are solved by full-nonlinear methods[3]. Sovereign default models require particle filters to evaluate likelihood since they have no closed form solution.

We adopt simulated tempering sequential Monte Carlo (SMC) algorithm instead of random-walk Metropolis–Hastings algorithm (RWMH). Simulated tempering SMC is proposed by Herbst and Schorfheide (2014, 2015), and the algorithm overcomes some weaknesses of RWMH. In the RWMH algorithm, parameter draws may have severe autocorrelation, or may get stuck in local mode and fail

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[2] In a broad survey of sovereign defaults, Panizza et al. (2009) find that the main costs of sovereign defaults are exclusion from international capital markets or trade, interest rate spikes, and large reductions in output. Regarding exclusion from international trade, Rose (2005) explains that sovereign default decreases bilateral trade by around 8% for 15 years. On the other hand, Gelos et al. (2011) report the average exclusion is four years (in the 1980s) or 0–2 years (after 1980). Martinez and Sandleris (2011) find a decrease in trade of 3.2% for five years. As for spikes in interest rates, Flandreau and Zumer (2004) find that defaults increase spreads by around 90 basis points, whereas Borensztein and Panizza (2009) find that the effect is 250–400 basis points. The effects of domestic costs and large decreases in output are found to be 0.6% by Chuan and Sturzenegger (2005) and 0.6–2.5% by Borensztein and Panizza (2009), respectively.

[3] See, for example, Arellano (2008), Cuadra and Sapriza (2008), Alfaro and Kanczuk (2009), Hatchondo and Martinez (2009), Yue (2010), Boz (2011), Mendoza and Yue (2012), and Durdu et al. (2013).

to explore the posteriors entirely as Chib and Ramamurthy (2010) and Herbst and Schorfheide (2014), Hirose et al. (2017) mention. A simulated tempering SMC propagates particles of the parameter vectors to the whole prior space. This feature is very important for our study because there is much less agreement on the parameters of sovereign default models. To the best of our knowledge, our study is the first application of a simulated tempering SMC to a full nonlinear regime-switching DSGE model.

The remainder of this paper is organized as follows. In Section 2, we construct a sovereign default model. Section 3 proposes an estimation strategy tailored to sovereign default models. We present the estimation results in Section 4, and provide sensitivity analysis in Section 5. Section 6 provides concluding comments.

Table 1

Calibration of previous studies on structural sovereign default models

		Aguiar and Gopinath (2007)	Arellano (2008), Cuadra and Sapriza (2008)	Alfaro and Kanczuk (2009)	Hatchondo and Martinez (2009)	Yue (2010)	Boz (2011)	Mendoza and Yue (2012)	Durdu et al. (2013)
β	Discount factor	0.410	0.825	0.5	0.815	0.269	0.885	0.600	0.825
σ	Risk aversion	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
r	Risk-free interest rate	4.0%	6.8%	4.0%	4.0%	4.0%	4.0%	4.0%	6.8%
ρ_z	Persistence of transitory shocks	0.9	0.945	0.85	0.9	0.41	0.91	0.95	0.945
σ_z	SD of transitory shocks	0.034	0.025	0.044	0.027	0.0253	0.0192	0.017	0.015
θ	Probability of reentry	34.4%	73.4%	50.0%	—	—	29.3%	29.3%	73.4%
$(1-\lambda_\alpha)$	Asymmetric domestic cost	—	3.1%	—	—	—	—	—	3.1%
$(1-\lambda_\beta)$	Proportional domestic cost	2.0%	—	10%	10%, 20%, 50%	2.0%	—	—	–

Note: Discount factor, risk-free interest rate, and probability of reentry are annualized.

2. The model

2.1. The model economy

We consider a sovereign default model with both stationary and nonstationary shocks similar to that utilized by Aguiar and Gopinath (2006) and Arellano (2008). Unlike these two studies, however, our model has both proportional and asymmetric domestic costs. Aguiar and Gopinath (2006) have only proportional domestic costs and Arellano (2008) has only asymmetric ones. We discuss this in Section 2.2.

We assume that there is a single tradable good. The economy receives a stochastic endowment stream given by:

$$Y_t = e^{z_t}\Gamma_{\mathrm{t}}, \tag{1}$$

where Γ_{t} denotes the trend, and z_t is a transitory shock. Trend and transitory shocks are discussed in Section 2.4. Households are identical, and maximize their utility according to:

$$E_0 \sum_{t=0}^{x} \beta^t u(C_t), \tag{2}$$

where $0 < \beta < 1$ is the discount factor, C is consumption, and $u(\cdot)$ is an increasing and strictly concave utility function. The utility function is assumed to display a constant coefficient of relative risk aversion σ as follows:

$$u(C) = \frac{(C)^{1-\sigma}}{1-\sigma}, \tag{3}$$

A benevolent government maximizes the present expected discounted value of future utility flows of households in equation (2). The government utilizes international borrowing to smooth consumption and alters its time path. The government buys one-period discount bonds B' at price $q(B',Y)$, which is endogenously determined depending on the government's incentives to default, the total amount of sovereign debt and endowment. Positive values of B' indicate that the government purchases bonds, and negative values of B' indicate that the government issues bonds in international financial markets. Earnings on the government portfolio are distributed as lump sums to households. The resource constraint of the economy when the government chooses to repay the debts is:

$$C = Y + B - q(B',Y)B'. \tag{4}$$

The government is excluded from international financial markets when it chooses to default. The resource constraint in the default state is:

$$C = Y^{def}, \tag{5}$$

where Y^{def} is the endowment when in the default state. The definition of Y^{def} is discussed in Section 2.2.

Foreign investors are assumed to evaluate defaultable bonds in a risk-neutral manner. In every period, risk-neutral investors lend B' to maximize expected profits ϕ as follows:

$$\phi = qB' - \frac{1 - \delta(B',Y)}{1 + r}, \tag{6}$$

where $\delta(B', Y)$ is the default probability depending on the debt accumulation and aggregate shocks.

2.2. Domestic costs

Our model is equipped with both proportional and asymmetric domestic costs. Aguiar and Gopinath (2006) use only proportional costs, whereas Arellano (2008) adopts only asymmetric costs. Similarly, other structural models have proportional costs only or asymmetric costs only. However, adopting both costs is important because the effects on business cycles differ between them. Proportional costs are incurred immediately at default, while asymmetric costs are incurred only if output fluctuates above the mean level. This means that proportional costs always reduce the default incentive; however, asymmetric costs do not inhibit default incentives when output is much lower than the mean level.

Regarding the estimation of domestic costs, structural estimation has a significant advantage over reduced-form estimation. Panizza et al. (2009) note that regressions containing domestic costs cannot avoid two biases. First, defaults could be endogenous to decreases in output. Second, it is possible that output does not fall because of defaults. Structural estimation overcomes these problems. In structural estimation, the effects of productivity shocks and domestic costs are explicitly identified, and asymmetric costs are one of the determinants of the default decision of the government.

Asymmetric costs are adopted by Arellano (2008), Cuadra and Sapriza (2008), and Cuadra et al. (2010). The asymmetric costs are expressed as:

$$Y^{def} = \begin{cases} Y & if\ Y < (1-\lambda_\alpha)E[Y] \\ (1-\lambda_\alpha)E[Y] & if\ Y \geq (1-\lambda_\alpha)E[Y] \end{cases}. \tag{7}$$

Proportional costs are adopted by Aguiar and Gopinath (2006), Alfaro and Kanczuk (2009), Hatchondo and Martinez (2009), and Yue (2010). The proportional costs are given as:

$$Y^{def} = (1 - \lambda_\beta)Y. \tag{8}$$

In our model, two types of domestic costs are combined. Through estimation, we test which of these two costs is most appropriate for our model.

$$Y^{def} = \begin{cases} (1-\lambda_\beta)\mathrm{Y} & if\ (1-\lambda_\beta)\mathrm{Y} < (1-\lambda_\alpha)E[Y] \\ (1-\lambda_\alpha)E[Y] & if\ (1-\lambda_\beta)\mathrm{Y} \geq (1-\lambda_\alpha)E[Y] \end{cases}. \tag{9}$$

2.3. Recursive formulation

Let $V^o(B, \mathrm{Y})$ denote the government's value function before the default or repayment decision. Define $V^c(B, \mathrm{Y})$ as the value associated with not defaulting, and denote $V^d(Y)$ as the value associated with default. $V^o(B, \mathrm{Y})$ satisfies:

$$V^o(B, \mathrm{Y}) = \max_{\{c,d\}}\{V^c(\mathrm{B}, \mathrm{Y}), V^d(Y)\}. \tag{10}$$

The decision is featured by:

$$D(B, Y) = \begin{cases} 1 & if\ V^c(\mathrm{B}, \mathrm{Y}) < V^d(Y) \\ 0 & otherwise \end{cases}. \tag{11}$$

The economy becomes an autarky when the government chooses default. The value function is given by the equation:

$$V^{d}(Y) = u(Y^{def}) + \beta \int_{Y'} [\theta V^{o}(0, Y') + (1-\theta) V^{d}(Y')] f(Y', Y)\, dY', \tag{12}$$

where θ is the probability that the economy regains access to international financial markets. When the government decides to repay, the value function is given by:

$$V^{c}(B, Y) = \max_{(B')} \left\{ u(Y - q(B', Y)B' + B) + \beta \int_{y'} V^{o}(B', Y') f(Y', Y) dY' \right\}. \tag{13}$$

Therefore, the default probabilities $\delta(B', Y)$ are given by:

$$\delta(B', Y) = \int_{D(B')} f(Y', Y) dY'. \tag{14}$$

The bond price that satisfies the lender's zero-profit condition is:

$$q(B')B' = \frac{1 - F\left(Y^{*}(B')\right)}{1 + r} B'. \tag{15}$$

2.4. Stationary and nonstationary productivity shocks and detrending

In equation (1), the endowment stream is comprised of stationary and nonstationary productivity shocks. The transitory productivity shock is:

$$z_t = \rho z_{t-1} + \varepsilon_t^{z}. \tag{16}$$

The stochastic trend is formulated as:

$$\Gamma_t = g_t \Gamma_{t-1}, \tag{17}$$

$$\ln g_t = (1-\rho_g)\left(\ln \mu_g - \frac{1}{2}\frac{\sigma_g^2}{(1-\rho_g^2)}\right) + \rho_g \ln g_{t-1} + \varepsilon_t^g. \tag{18}$$

The state vector is unbounded because the endowment stream has a trend. We normalize the nonstationary element following Aguiar and Gopinath (2006). We normalize the variable X by $\mu_g \Gamma_{t-1}$ and denote $X/\mu_g \Gamma_{t-1}$ by x, whereas Aguiar et al. (2016) normalize their variables by Γ_t as in DSGE studies such as Smets and Wouters (2007) and Chang and Fernández (2013). Importantly, the method of Aguiar and Gopinath (2006) enables us to analyze the effect of trend shocks on cycles. The logged and detrended endowment streams of equation (1) are expressed as:

$$\ln Y_t - (ln\mu_g + ln\Gamma_{t-1}) = z_t + ln\Gamma_t - (ln\mu_g + ln\Gamma_{t-1}),$$

$$\ln y_t = z_t + ln \mathrm{g}_t - ln\mu_g. \tag{19}$$

Thus, the detrended endowment stream is composed of both stationary and nonstationary productivity shocks:

$$c = y + b - q(b', y)b'. \tag{20}$$

The detrended value functions become:

$$V^d(y) = u(y^{def}) + \beta \int_{y'} [\theta V^o(0, y') + (1-\theta)V^d(y')] f(y', y)\, dy', \tag{21}$$

$$V^c(b,y) = \max_{(b')}\left\{u(c) + \beta \int_{y'} V^o(b',y')f(y',y)dy'\right\}. \tag{22}$$

The model is solved using value function iterations based mainly on Aguiar and Gopinath (2007) and Arellano (2008) (see Algorithm 1).

3. Estimation strategy

3.1. Data

The target economies are Argentina and Mexico following the studies of Aguiar and Gopinath (2006), García-Cicco et al. (2010), and Chang and Fernández (2013). Furthermore, almost all structural sovereign default models investigate the Argentinean economy. We use data of real GDP per capita, external debt stock, interest rates, and default states. External debt stocks are deflated by dollar expected inflation rates, and divided by total population. We use deposit rate provided by World Bank as country specific interest rate since they have longer data than EMBI + spread. We offer the estimation result using EMBI + spread for the robustness check. Interest rates are also deflated by dollar expected inflation rates. The default states are defined by Standard & Poor's.

3.2. State space representation

The state transition equation is $s_t = \Phi(s_{t-1}, \epsilon_t ; \vartheta)$, where $\epsilon_t \sim F_\epsilon(\cdot ; \vartheta)$. The measurement equation is $\mathbb{y}_t = \Psi(s_t, t; \vartheta) + u_t$, where $u_t \sim F_u(\cdot ; \vartheta)$. After solving for the state transition equation, we map the variables computed in our model to the observables.

The measurement equations include the one-period lag of the growth of technology. We assume balanced growth as in most studies on DSGE models such as Smets and Wouters (2007) and Chang and Fernández (2013), although they add the current growth of technology to their measurement equations, not that of the one-period lag. The reason is we detrend the model using $\mu_g\Gamma_{t-1}$, whereas they detrend the model using present level of technology corresponding to Γ_t in this paper. Log-differenced series are decomposed as follows:

$$
\begin{aligned}
d\ln Y_t \quad &= \left(\ln Y_t - \ln\mu_g - \ln\Gamma_{t-1}\right) - \left(\ln Y_{t-1} - \ln\mu_g - \ln\Gamma_{t-2}\right) + \ln\mu_g + \ln\Gamma_{t-1} - \ln\mu_g - \ln\Gamma_{t-2} \\
&= \ln y_t - \ln y_{t-1} + \ln\Gamma_{t-1} - \ln\Gamma_{t-2} \\
&= \hat{y}_t - \hat{y}_{t-1} + \ln g_{t-1}. \qquad (23)
\end{aligned}
$$

Similarly, foreign assets B_t are detrended. We assume bond prices and default decisions have no trend as is assumed in other DSGE model studies. The measurement equations are:

$$
\begin{bmatrix} dlnY_t^{obs} \\ dlnB_t^{obs} \\ q_t^{obs} \\ def_t^{obs} \end{bmatrix} = \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{b}_t - \hat{b}_{t-1} \\ q_t \\ def_t \end{bmatrix} + \begin{bmatrix} lng_{t-1} \\ lng_{t-1} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} u_{y,t} \\ u_{b,t} \\ u_{q,t} \\ u_{def,t} \end{bmatrix}. \qquad (24)
$$

Our measurement equations have measurement errors. The main reason for adding measurement error is to avoid stochastic singularity, which arises when there are more observables than shocks in the model. The shocks in our model are two productivity shocks, but there are three observables in our model. According to Schmitt-Grohé and Uribe (2012), adding measurement error is a way to circumvent stochastic singularity of the model. García-Cicco et al. (2010) and Chang and Fernández (2013) also adopt measurement error for the same reason. Measurement errors are restricted to a maximum of 20% of the empirical standard deviation to avoid measurement errors absorbing variability as discussed by An and Schorfheide (2007) and García-Cicco et al. (2010).

We add a default decision to the measurement equations since the definition of sovereign default by Standard & Poor’s is very clear, and we can easily observe the state of the sovereign. Similarly, some studies applying a state space model add default decisions or crises to the measurement equations, such as Schwaab et al. (2016) and Rose and Spiegel (2010, 2011, and 2012). Standard & Poor's defines default as the failure to meet a principal or interest payment on the due date contained in the original terms of a debt issue.

3.3. Simulated tempering SMC–SMC algorithm

The simulated tempering SMC–SMC algorithm is an estimation strategy that is combined with particle filtering. We use the simulated tempering SMC algorithm proposed by Herbst and Schorfheide (2014, 2015) (see algorithms 2–4 in the Appendix), as well as their particle filter to evaluate likelihood, hence the label SMC–SMC. This paper is the first attempt to apply this estimation method to a full nonlinear regime-switching DSGE model.

The most important feature of the SMC algorithm for this paper is that it explores entire prior ranges propagating particles of parameter vectors. It prevents parameter draws from getting stuck in the local mode.

All priors are uniform to secure objectiveness such as in Fernández-Villaverde and Rubio-Ramírez (2005) and García-Cicco et al. (2010). The ranges cover all calibration values of the referenced studies of structural sovereign default models (see Tables 1 and 2).

Table 2

Priors.

Parameters		Distributions	Hyperparameters of uniform distribution
β	Discount factor	Uniform	$(0.0, 1.0)$
σ	Risk aversion	Uniform	$(0.0, 10.0)$
r	Risk-free interest rate	Uniform	$(0.0, 1.0)$
ρ_z	Persistence of stationary productivity shock	Uniform	$(0.0, 1.0)$
σ_z	SD of stationary productivity shock	Uniform	$(0.0, 10.0)$
θ	Probability of reentry	Uniform	$(0.0, 1.0)$
$(1-\lambda_\alpha)$	Asymmetric domestic costs	Uniform	$(0.0, 1.0)$
$(1-\lambda_\beta)$	Proportional domestic costs	Uniform	$(0.0, 1.0)$
μ_g	Gross mean growth	Uniform	$(1.0, 1.3)$
ρ_g	Persistence of nonstationary productivity shock	Uniform	$(0.0, 1.0)$
σ_g	SD of nonstationary productivity shock	Uniform	$(0.0, 10.0)$

The hyperparameters of the SMC algorithm are $\mathrm{N} = 2000, N_{\phi} = 100, \lambda = 2.1, N_{blocks} = 6, M = 1$ and $\alpha = 0.9$. These are the number of particles for the parameter vectors, the number of stages, the parameter for the tempering schedule, the number of blocks, the number of MH steps at each stage, and parameter controls for the weight of the proposals' mixture components. The number of particles for the likelihood evaluations, N_{filter}, is 20,000. The chain is initialized by priors. The tempering schedule $\{\phi_n\}_{n=1}^{N_\phi}$ is determined by the equation:

$$\phi_n = \left(\frac{n-1}{N_\phi - 1}\right)^{\lambda}. \tag{25}$$

As the number of stages increases, each stage requires additional likelihood evaluations. The scale parameter is adjusted by approximately 25–40% along with the tempering schedule. N_{filter} is large enough to obtain robust results efficiently according to Amisano and Tristani (2010) and Malik and Pitt (2011).

The total number of likelihood estimations in the SMC algorithm is equal to $N \times N_\phi \times N_{blocks} \times M = 3.6$ million, which is sufficiently large. In our experience, the draws in an MCMC estimation of a DSGE model are rarely more than one million.[4]

4. Results

4.1. The importance of nonstationary productivity shocks

The random-walk component suggests that nonstationary productivity shocks are the important sources of financial frictions (Table 3). A historical decomposition also indicates that nonstationary

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[4] García-Cicco et al. (2010) used two million iterations in their MCMC estimation, but their model has substantially more parameters than in this paper. Log-linearized DSGE model estimation studies that use a basic Kalman filter for likelihood evaluation often conduct 500,000 MCMC iterations. DSGE model studies that use a particle filter have fewer iterations than the Kalman filter case.

productivity shocks are as important as stationary ones (Fig. 1). The random-walk component is a criterion for the importance of trend shocks proposed by Aguiar and Gopinath (2007). The equation for the random-walk component is given by:

$$\frac{\sigma_{\Delta\tau}^2}{\sigma_{\Delta sr}^2} = \frac{\alpha^2\sigma_g^2/\left(1-\rho_g\right)^2}{[2/(1+\rho_z)]\sigma_z^2 + \left[\alpha^2\sigma_g^2/\left(1-\rho_g^2\right)\right]}, \tag{26}$$

where α is the labor exponent used in an RBC model; however, our model and most other structural sovereign default models do not use this parameter. Instead, we calibrate $\alpha = 0.32$ or $\alpha = 0.68$, which are the same values as used by Aguiar and Gopinath (2007), García-Cicco et al. (2010), and Chang and Fernández (2013).

Table 3

Random-walk component.

Argentina	Our model		García-Cicco et al. (2010)		
Capital income share	α = 0.32	α = 0.68	α = 0.32		
Random-walk component	1.25	3.46	0.07		
Mexico	**Our model**		**Aguiar and Gopinath (2007)**	**García-Cicco et al. (2010)**	**Chang and Fernández (2013)**
Capital income share	α = 0.32	α = 0.68	α = 0.68	α = 0.32	α = 0.68
Random-walk component	1.15	3.19	0.88–1.13	0.01	0.88

The reason for the high random-walk component is the high persistency of trend shock ρ_g (Table 4). Higher persistency of output means that positive shocks are expected to persist over time causing the economy to default less frequently in good states. The economy can borrow relatively more, resulting in relatively high levels of debt when booms. Similarly, negative income shocks are expected persist long time, thus the economy cannot support high debt. These properties are consistent to data of Argentina and Mexico, the external debt stock just prior to default is large, but defaults occur frequently[5].

The effect of higher persistency is similar between stationary and nonstationary productivity shocks. The difference is that nonstationary productivity shocks support or reduce external debt not only through business cycle fluctuations, but also through trend growth, whereas stationary productivity shocks drive debts only through the former.

On the other hand, historical decompositions show, at the start of default, harsh negative *stationary* productivity shocks reduce output in both Argentina and Mexico. This suggests that the particular realization of stationary productivity shock is the dominant factor of default.

Table 4

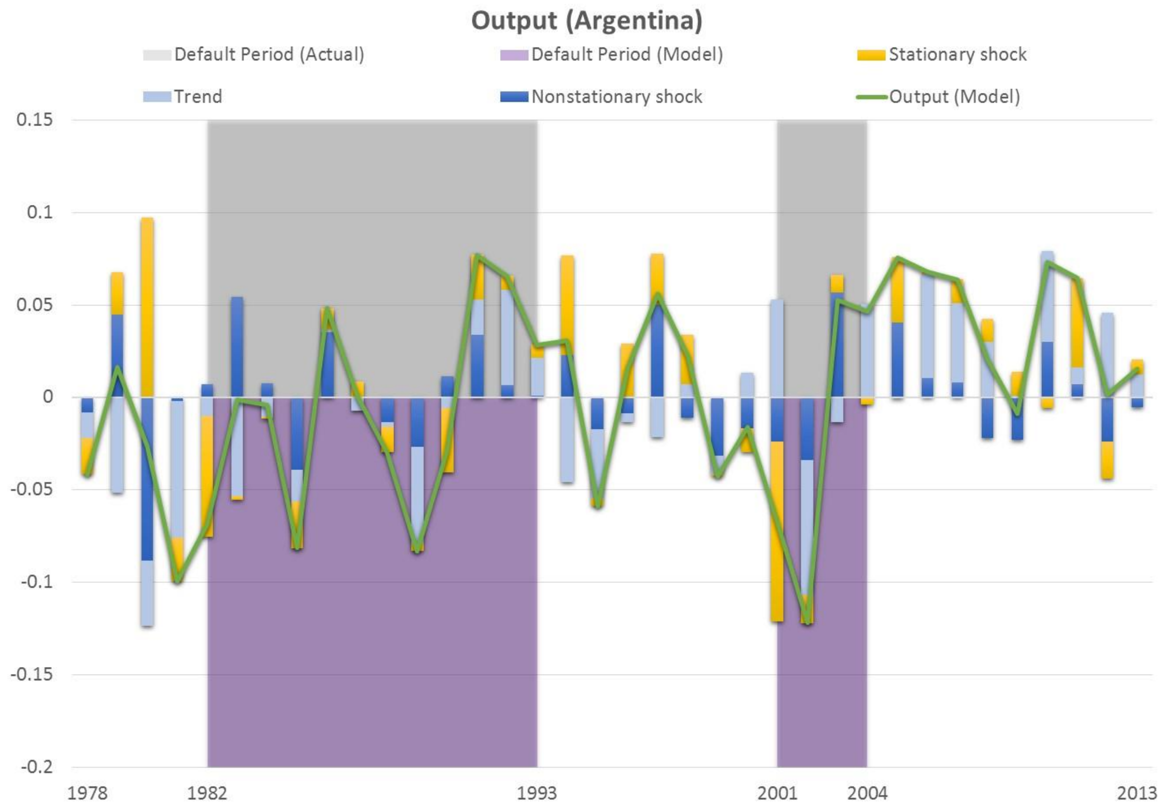
Posteriors (mean).

Argentina		Priors	Mean	Standard Deviation
β	Discount factor	Uniform	0.80	0.050
σ	Risk aversion	Uniform	1.88	0.12
r	Risk-free interest rate	Uniform	0.038	0.0023
ρ_z	Persistence of transitory shock	Uniform	0.85	0.051
σ_z	SD of transitory shock	Uniform	0.024	0.0014
θ	Probability of reentry	Uniform	0.21	0.013
$(1-\lambda_\alpha)$	Asymmetric domestic costs	Uniform	0.0068	0.0060
$(1-\lambda_\beta)$	Proportional domestic costs	Uniform	0.0033	0.0030
μ_g	Gross mean growth	Uniform	1.015	0.017
ρ_g	Persistence of trend shock	Uniform	0.75	0.044
σ_g	SD of trend shock	Uniform	0.024	0.0014
Marginal data density			-72.45	

Mexico		Priors	Mean	Standard Deviation
β	Discount factor	Uniform	0.81	0.051
σ	Risk aversion	Uniform	1.86	0.11
r	Risk-free interest rate	Uniform	0.037	0.0022

[5] The stylized facts are observed not only in our sample periods, but throughout a longer period of history. Uribe and Schmitt-Grohé (2017) show that the debt-to-GNP ratio in default years is approximately 14% higher than average, and in Argentina default occurred five times and in Mexico default occurred eight times from 1824 to 2014.

ρ_z	Persistence of transitory shock	Uniform	0.83	0.051
σ_z	SD of transitory shock	Uniform	0.022	0.0015
θ	Probability of reentry	Uniform	0.21	0.014
$(1-\lambda_\alpha)$	Asymmetric domestic costs	Uniform	0.010	0.0048
$(1-\lambda_\beta)$	Proportional domestic costs	Uniform	0.013	0.0066
μ_g	Gross mean growth	Uniform	1.015	0.020
ρ_g	Persistence of trend shock	Uniform	0.73	0.046
σ_g	SD of trend shock	Uniform	0.023	0.0015
Marginal data density			-317.55	



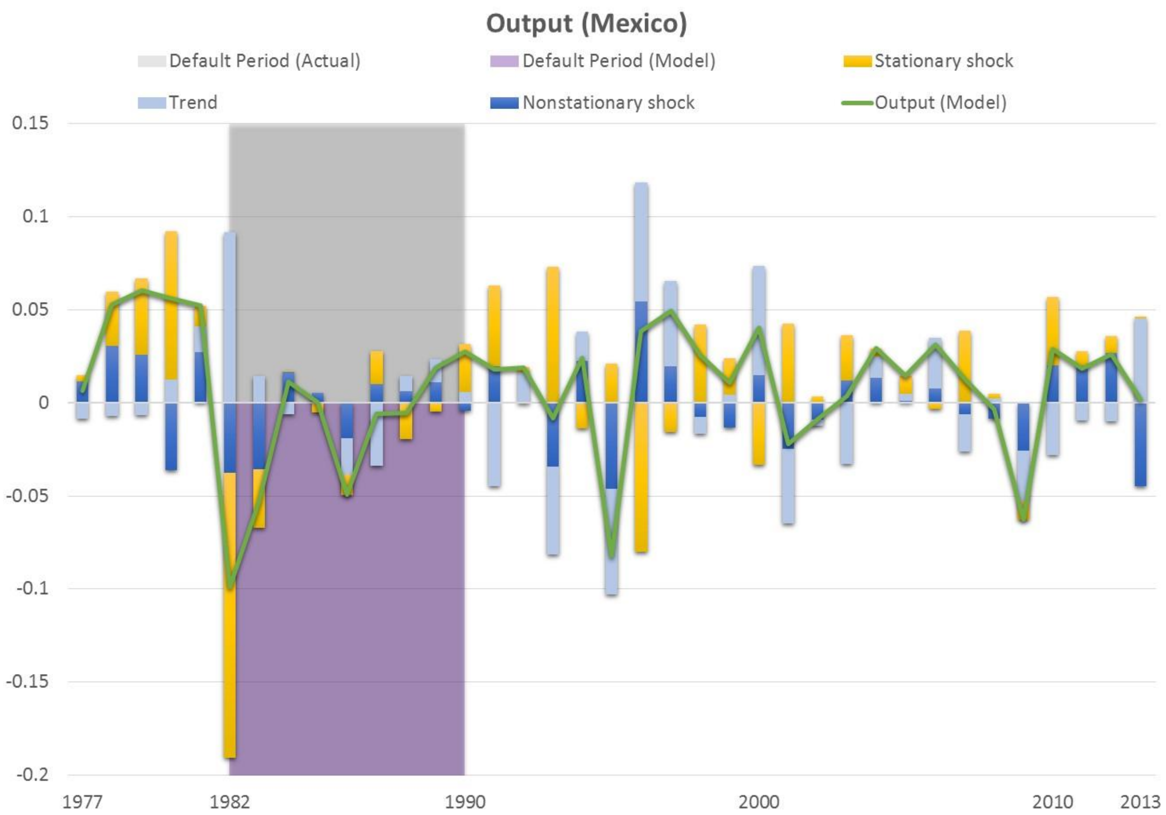


Fig. 1. Historical decompositions. Notes: Observables are log-differenced. Fluctuations in output can be fully decomposed, but external debt and bond prices cannot be decomposed because they are endogenously determined by numerical nonlinear policy functions.

4.2. Stylized facts of emerging economies

The second moments demonstrate that our model replicates well the empirical regularities of emerging economies (Table 5). Our model satisfactorily exhibits volatile output, excess volatility of consumption, countercyclical trade balance, and countercyclical interest rate (procyclical bond price). Autocorrelations of the trade balance–output ratio are low, and do not exhibit random-walk behavior, which García-Cicco et al. (2010) observe in a frictionless RBC model. Our models show negative correlations between the trade balance and interest rate, that is, a positive correlation between the trade balance and bond prices as in Aguiar and Gopinath (2006).

Table 5

Second moments.

Argentina	Data	Our model
$\sigma_{\Delta y}$	5.74	5.40
$\sigma_{\Delta c}/\sigma_{\Delta y}$	1.50	1.16
$\sigma_{tb/y}$	3.87	2.80
$Corr(\Delta y, tb/y)$	−0.17	−0.18
$Corr(\Delta y, q)$	0.50	0.34
$Corr(tb/y, q)$	−0.17	−0.051
$Serial\ corr(tb/y)$	0.67	0.13

Mexico	Data	Our model
$\sigma_{\Delta y}$	3.38	3.11
$\sigma_{\Delta c}/\sigma_{\Delta y}$	1.28	1.6
$\sigma_{tb/y}$	3.29	2.84
$Corr(\Delta y, tb/y)$	−0.45	−0.71
$Corr(\Delta y, q)$	0.32	0.41
$Corr(tb/y, q)$	−0.78	−0.3
$Serial\ corr(tb/y)$	0.75	0.17

Note: Standard deviations are reported in percentage points.

4.3. The costs of sovereign default in Argentina and Mexico

Our estimates of the probability of reentry and domestic costs are relatively low among preceding studies (Tables 1 and 4). The reason for these differences is that the default period is measured to be matched to observables in our Bayesian estimation, whereas previous studies matched the moments of the data. The observed long default period makes the probability of reentry low. This inhibits default; hence, domestic cost becomes relatively small to keep default frequent.

Domestic costs seem to be small, but they are amplified by the low probability of reentry when calculating the value in equation (12).

5. Sensitivity analysis

5.1. Another interest rate data: EMBI + spread and U.S. interest rate

We use deposit rate as the country-specific interest rate since longer data are available. However, many studies on emerging economies create interest rate series with EMBI + spread and U.S. interest rate. We present robustness check for our main result estimating the sovereign default model using EMBI + spread and U.S. interest rate.

As for Mexico, EMBI + spread during default period, 1982-1990, is not available. Thus, we estimate and analyze only Argentine economy. Regarding Argentina, EMBI + spread during default period, 2001-2004, is available. We construct the country-specific interest rate as the sum of the EMBI + spread for Argentina and the 90-day Treasury-Bill rate deflated by expected dollar inflation.

The result shows that parameter estimates are similar to main analysis. Random-walk components show that nonstationary productivity shocks are also important in this robustness check.

Table 6

Posteriors (mean).

Argentina		Priors	Mean	
β	Discount factor	Uniform	0.82	0.06
σ	Risk aversion	Uniform	1.92	0.14
r	Risk-free interest rate	Uniform	0.038	0.0026
ρ_z	Persistence of transitory shock	Uniform	0.88	0.06
σ_z	SD of transitory shock	Uniform	0.024	0.0017
θ	Probability of reentry	Uniform	0.21	0.015
$(1-\lambda_\alpha)$	Asymmetric domestic costs	Uniform	0.0050	0.00034
$(1-\lambda_\beta)$	Proportional domestic costs	Uniform	0.0025	0.00018
μ_g	Gross mean growth	Uniform	1.026	0.029
ρ_g	Persistence of trend shock	Uniform	0.77	0.055
σ_g	SD of trend shock	Uniform	0.024	0.0017
Marginal data density			-95.20	
Random-walk component			1.47 (α = 0.32)	3.97 (α = 0.68)

5.2. Another detrending method: HP-filter

Many studies on sovereign default models use an HP filter such as Aguiar and Gopinath (2006) and Mendoza and Yue (2012), whereas we use log-differenced series assuming balanced growth. Due to the difference of detrending methods, our parameter estimates may be different from conventional literature. Thus, we provide the estimation using HP-filter to detrend the data. We test whether parameter estimates in this case show similar tendency to our main results.

The measurement equation is different from our main analysis. In the literature on DSGE model estimation with particle filtering, Fernández-Villaverde and Rubio-Ramírez (2005, 2007) and Malik and Pitt (2011) also use an HP filter to detrend their data. The measurement equations of the HP-filter approach are as follows. The tilde denotes deviations from trend.

$$\begin{bmatrix} \tilde{y}_t^{obs} \\ \tilde{b}_t^{obs} \\ q_t^{obs} \\ def_t^{obs} \end{bmatrix} = \begin{bmatrix} \tilde{y}_t \\ \tilde{b}_t \\ q_t \\ def_t \end{bmatrix} + \begin{bmatrix} u_{y,t} \\ u_{B,t} \\ u_{q,t} \\ u_{def,t} \end{bmatrix}. \tag{27}$$

The result is provided by table 7. Probability of reentry and domestic costs are relatively small comparing to the preceding studies (Table 1, Table 7). Thus, we confirmed that the tendency of parameter estimates is similar to our main result (Table 4).

Table 7

Posteriors (mean).

Argentina		Priors	Mean	Standard Deviation
β	Discount factor	Uniform	0.68	0.11
σ	Risk aversion	Uniform	1.82	0.67
r	Risk-free interest rate	Uniform	0.038	0.014
ρ_z	Persistence of transitory shock	Uniform	0.74	0.11
σ_z	SD of transitory shock	Uniform	0.033	0.0071
θ	Probability of reentry	Uniform	0.12	0.049
$(1-\lambda_\alpha)$	Asymmetric domestic costs	Uniform	0.017	0.0072
$(1-\lambda_\beta)$	Proportional domestic costs	Uniform	0.00015	0.00
Marginal data density			-400.01	

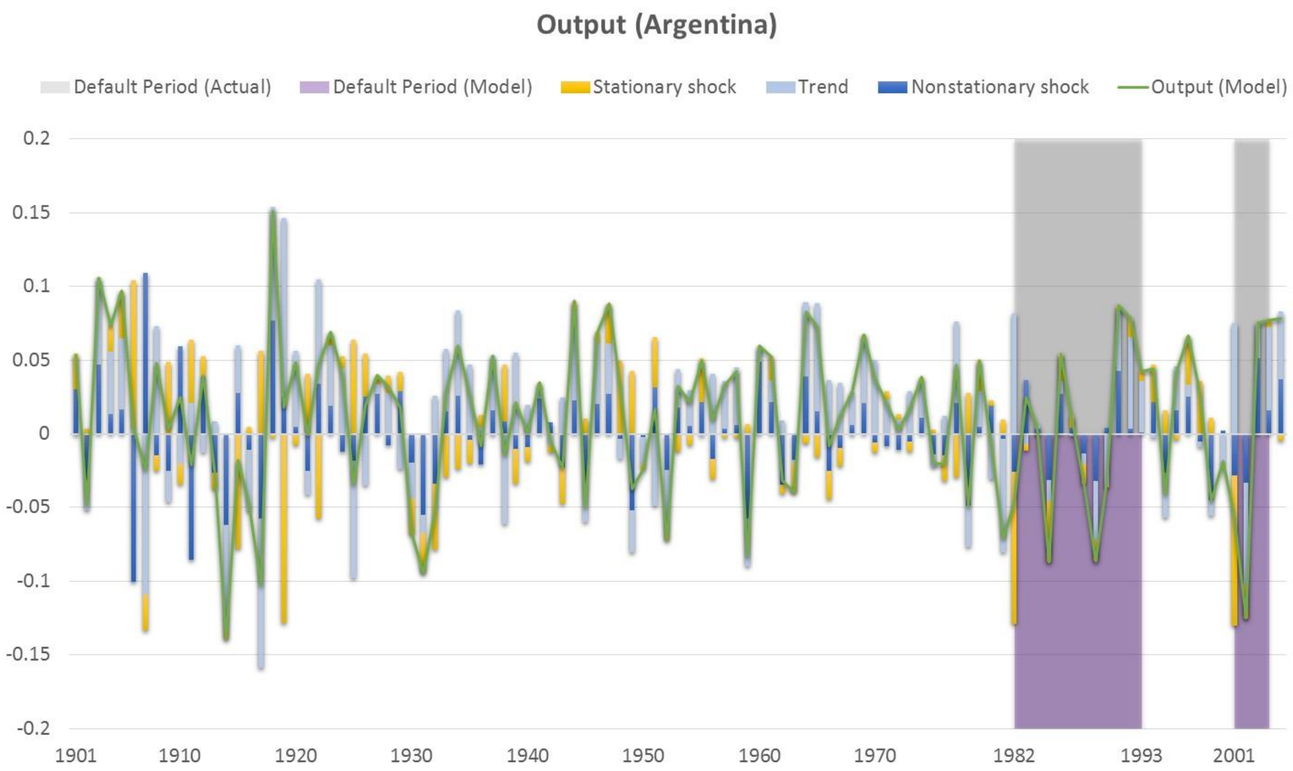
Mexico		Priors	Mean	Standard Deviation
β	Discount factor	Uniform	0.71	0.11
σ	Risk aversion	Uniform	1.51	0.36
r	Risk-free interest rate	Uniform	0.033	0.0070
ρ_z	Persistence of transitory shock	Uniform	0.67	0.11
σ_z	SD of transitory shock	Uniform	0.023	0.0043
θ	Probability of reentry	Uniform	0.11	0.024
$(1-\lambda_\alpha)$	Asymmetric domestic costs	Uniform	0.023	0.0051
$(1-\lambda_\beta)$	Proportional domestic costs	Uniform	0.0002	0.00
Marginal data density			-208.08	

5.3. Application to longer data

García-Cicco et al. (2010) point out that the period 1980–2005 only contains between one and a half and two cycles that are deviations from a cubic trend, and this may make nonstationary productivity shocks more important than actual. Using 100 years of data, we could not conduct the estimation same as main analysis because such long historical data for external debt and interest rates are unavailable. Instead, under the parameter estimates of our main analysis, we apply a particle filter to the data for the Argentinean and Mexican economies used in García-Cicco et al. (2010), in particular, output and trade balance data over the period 1900–2005. Then, we check whether nonstationary productivity shocks are important throughout this period. The measurement equations are as follows:

$$\begin{bmatrix} dlnY_t^{obs} \\ (tb/y)_t^{obs} \\ def_t^{obs} \end{bmatrix} = \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ (tb/y)_t \\ def_t \end{bmatrix} + \begin{bmatrix} lng_{t-1} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} u_{y,t} \\ u_{tb/y,t} \\ u_{def,t} \end{bmatrix}. \tag{28}$$

As a result, the historical decompositions show that nonstationary productivity shocks are as important as stationary productivity shocks over the period 1905–2005 (Fig. 2). The second moments of the model also replicate the excess volatility of consumption, countercyclical interest rate, and trade balance (Table 8). The model also predicts default periods accurately, although we use trade balance data instead of external debt data. Stationary shocks also play an important role at the start of default periods.



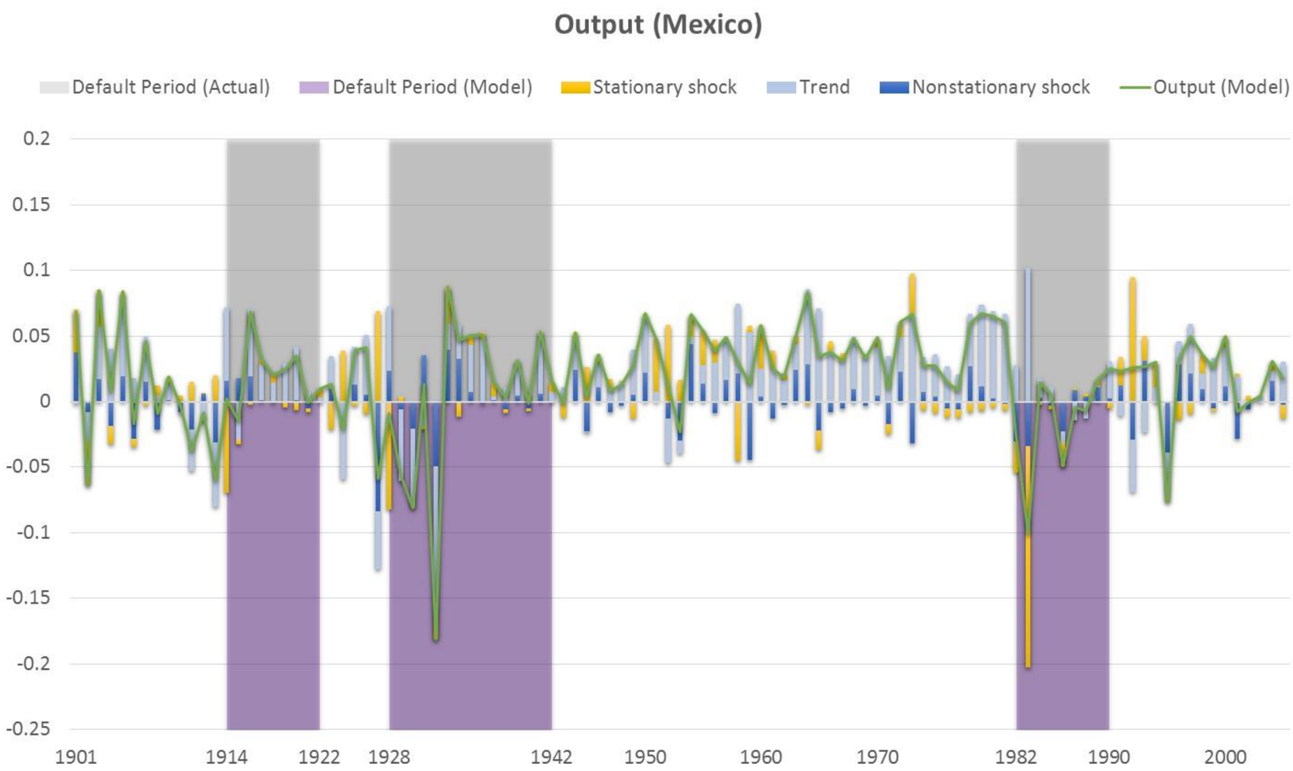


Fig. 2. Historical decomposition in the period 1905–2005.

Table 8

Second moments in the period 1905–2005.

Argentina	Data	Model
$\sigma_{\Delta y}$	5.37	4.79
$\sigma_{\Delta c}/\sigma_{\Delta y}$	1.41	1.38
$\sigma_{tb/y}$	5.14	2.70
$Corr\ (\Delta y, tb/y)$	−0.05	−0.44
$Corr\ (\Delta y, q)$	–	0.21
$Corr\ (tb/y, q)$	–	−0.16
$Serial\ corr\ (tb/y)$	0.58	0.38

Mexico	Data	Model
$\sigma_{\Delta y}$	4.26	4.20
$\sigma_{\Delta c}/\sigma_{\Delta y}$	1.45	1.36
$\sigma_{tb/y}$	4.22	2.25
$Corr\ (\Delta y, tb/y)$	−0.18	−0.39
$Corr\ (\Delta y, q)$	–	0.31
$Corr\ (tb/y, q)$	–	−0.15
$Serial\ corr (tb/y)$	0.72	0.23

Note: Standard deviations are reported in percentage points.

6. Conclusions

The major character of the business cycles of emerging economies are excessive volatility in consumption, countercyclical current account balance, countercyclical interest rate, and frequent default at equilibrium. There are two strands of literature that explain these stylized facts. The first strand claims that the most important source of these characteristics is a permanent productivity shocks. The second strand emphasizes financial frictions rather than nonstationary productivity shocks. However, the former may be the source of the latter. We estimate sovereign default models, and examine this hypothesis.

To obtain robust and accurate results efficiently, we adopt simulated tempering SMC–SMC as the estimation strategy. It explores parameter estimates using whole prior spaces. Furthermore, we adopt a uniform distribution as priors for all parameters to obtain objective results.

The results indicate that nonstationary productivity shocks are the important sources of financial frictions in terms of random-walk components, historical decompositions, second moments, and parameter estimates. Therefore, we bridge a gap between Aguiar and Gopinath (2007) and García-Cicco et al. (2010).

A natural extension to this paper is to add other important shocks in emerging economies such as interest rate shocks, terms of trade shocks and so forth. Uribe and Yue (2006) show that one of the major drivers of interest rate fluctuations in emerging economies is the monetary policy of the United States. Schmitt-Grohé and Uribe (2016) and Na et al. (2014) introduce downward nominal wage rigidity into a model, and replicate defaults with large currency devaluations. Our estimation framework is applicable to these important issues.

Acknowledgments

We are grateful for useful comments from Makoto Nirei, Junko Koeda, Kozo Ueda, Ryo Jinnai, Makoto Nakajima, Munechika Katayama, Masaru Inaba, Tomoaki Yamada, Kensuke Miyazawa, Jun-Hyung Ko, Kengo Nutahara, Kazufumi Yamana, Go Kotera, Ryo Ishida, and seminar participants at Waseda University, Kyoto University, the Ministry of Finance Japan, and participants of the DSGE Conference in Matsuyama.

References

Aguiar, M., Gopinath, G., 2006. Defaultable debt, interest rates and the current account. Journal of International Economics 69 (1), 64–83.

Aguiar, M., Gopinath, G., 2007. Emerging market business cycles: the cycle is the trend. Journal of Political Economy 115 (1), 69-102.

Aguiar, M., Chatterjee, S., Cole, H., Stangebye, Z., 2016. Quantitative models of sovereign debt crises. NBER Working Paper 22125.

Alfaro, L., Kanczuk, F., 2009. Optimal reserve management and sovereign debt. Journal of International Economics 77 (1), 23–36.

Álvarez-Parra, F., Brandao-Marques, L., Toledo, M., 2013. Durable goods, financial frictions, and business cycles in emerging economies. Journal of Monetary Economics 60 (6), 720–736.

Amisano, G., Tristani, O., 2010. Euro area inflation persistence in an estimated nonlinear DSGE model. Journal of Economic Dynamics and Control 34 (10), 1837–1858.

An, S., Schorfheide, F., 2007. Bayesian analysis of DSGE models. Econometric Reviews 26 (2–4), 113–172.

Arellano, C., 2008. Default risk and income fluctuations in emerging economies. American Economic Review 98 (3), 690–712.

Borensztein, E., Panizza, U., 2009. The costs of sovereign default. IMF Economic Review 56 (4), 683–741.

Boz, E., 2011. Sovereign default, private sector creditors, and IFIs. Journal of International Economics 83 (1), 72–82.

Boz, E., Daude, C., Durdu, C.B., (2011). Emerging market business cycles: learning about the trend. Journal of Monetary Economics 58 (6–8), 616–631.

Chang, R., Fernández, A., 2013. On the sources of aggregate fluctuations in emerging economies. International Economic Review 54 (4), 1265-1293.

Chib, S., Ramamurthy, S., 2010. Tailored randomized block MCMC methods with application to DSGE models. Journal of Econometrics 115 (1), 19–38.

Chuan, P., Sturzenegger, F., 2005. Default episodes in the 1980s and 1990s: what have we learned? In: Aizenman, J., Pinto, B. (Eds.), Managing Economic Volatility and Crisis, Cambridge University Press, Cambridge, MA, pp.

Cuadra, G., Sanchez, J.M., Sapriza, H., 2010. Fiscal policy and default risk in emerging markets. Review of Economic Dynamics 13 (2), 452–469.

Cuadra, G., Sapriza, H., 2008. Sovereign default, interest rates and political uncertainty in emerging markets. Journal of International Economics 76 (1), 78–88.

Durdu, C.B., Nunes, R., Sapriza, H., 2013. News and sovereign default risk in small open economies. Journal of International Economics 91 (1), 1–17.

Fernández-Villaverde, J., Rubio-Ramírez, J.F., 2005. Estimating dynamic equilibrium economies: linear versus nonlinear likelihood. Journal of Applied Econometrics 20 (7), 891–910.

Fernández-Villaverde, J., Rubio-Ramírez, J.F., 2007. Estimating macroeconomic models: a likelihood approach. Review of Economic Studies 74 (4), 1059–1087.

Flandreau, M., Zumer, F., 2004. The making of global finance 1880–1913. OECD Development Center Research Monograph.

García-Cicco, J.,Pancrazzi, R., Uribe, M., 2010. Real business cycles in emerging countries? American Economic Review 100 (5), 2510–2531.

Gelos, R.G., Sahay, R., Sandleris, G., 2011. Sovereign borrowing by developing countries: what determines market access? Journal of International Economics 83 (2), 243–254.

Gust, C., López-Salido, D., Smith, M.E., 2012. The empirical implications of the interest-rate lower bound. Finance and Economics Discussion Series 83, Board of Governors of the Federal Reserve System.

Hatchondo, J.C., Martinez, L., 2009. Long-duration bonds and sovereign defaults. Journal of International Economics 79 (1), 117–125.

Herbst, E., Schorfheide, F., 2014. Sequential Monte Carlo sampling for DSGE models. Journal of Applied Econometrics 29 (7), 1073–1098.

Herbst, E., Schorfheide, F., 2015. Bayesian Estimation of DSGE Models. Princeton University Press, Princeton.

Hirose, Y., Kurozumi, T., Zandweghe, W., V., 2017. Monetary Policy and Macroeconomic Stability Revisited. Research Working Papers, 2017, 17-01, Federal Reserve Bank of Kansas City.

Martinez, J.V., Sandleris, G., 2011. Is it punishment? Sovereign defaults and the decline in trade. Journal of International Money and Finance 30 (6), 909–930.

Malik, S., Pitt, M.K., 2011. Particle filters for continuous likelihood evaluation and maximization. Journal of Econometrics 165 (2), 190–209.

Mendoza, E., Yue, V.Z., 2012. A general equilibrium model of sovereign default and business cycles. The Quarterly Journal of Economics 127 (2), 889–946.

Na, S., Schmitt-Grohé, S., Uribe, M., Yue, V.Z., 2014. A model of the twin Ds: optimal default and devaluation. NBER Working Paper No. 20314, July 2014.

Naoussi, C.F., Tripier, F., 2013. Trend shocks and economic development. Journal of Development Economics 103, 29–42.

Neumeyer, P.A., Perri, F., 2005. Business cycles in emerging economies: the role of interest rates. Journal of Monetary Economics 52 (2), 345–380.

Panizza, U., Sturzenegger, F., Zettelmeyer, J., 2009. The economics and law of sovereign debt and default. Journal of Economic Literature 47 (3), 651–698.

Rose, A.K., 2005. One reason countries pay their debts: renegotiation and international trade. Journal of International Economics 77 (1), 189–206.

Rose, A.K., Spiegel, M.M., 2010. Cross-country causes and consequences of the 2008 crisis: international linkages and American exposure. Pacific Economic Review 15 (3), 340–363.

Rose, A.K., Spiegel, M.M., 2011. Cross-country causes and consequences of the crisis: an update. European Economic Review 55 (3), 309–324.

Rose, A.K., Spiegel, M.M., 2012. Cross-country causes and consequences of the 2008 crisis: early warning. Japan and the World Economy 24 (1), 1–16.

Schmitt-Grohé, S., Uribe, M., 2012. What's news in business cycles. Econometrica 80 (6), 2733–2764.

Schmitt-Grohé, S., Uribe, M., 2016. Downward nominal wage rigidity, currency and industry factors. Journal of Political Economy 124 (5), 1466–1514.

Schwaab, B., Koopman, S.J., Lucus, A., 2016. Global credit risk: world, country and industry factors. Journal of Applied Econometrics doi:10.1002/jae.2521.

Smets, F., Wouters, R., 2007. Shocks and frictions in US business cycles: a Bayesian DSGE approach. American Economic Review 97 (3), 586–606.

Uribe, M., Schmitt-Grohé, S., 2017. Open Economy Macroeconomics. Princeton University Press. Princeton.

Uribe, M., Yue, V.Z., 2006. Country spreads and emerging countries: who drives whom? Journal of International Economics 69 (1), 6–36.

Yue, V.Z., 2010. Sovereign default and debt renegotiation. Journal of International Economics 80 (2), 176–187.

Preliminary and Incomplete

Appendix: Algorithms in detail

Most of our algorithms follow Herbst and Schorfheide (2014, 2015).

Algorithm 0

Policy function iteration.

1. Guess a value function $V^o(b_t, y_t)$ and bond price $q(b_{t+1}, y_t)$
2. At each (b_{t+1}, y_t), update $v^d(y_t)$ and $v^c(b_t, y_t)$
3. Update $V^o(b_t, y_t)$, the default rule, the implied ex ante default probability, and the bond price function
4. Iterate step 2 and step 3 until convergence occurs

Algorithm 1

Simulated tempering SMC.

1. Initialization

Draw $\theta_1^i, \cdots \theta_1^N$ from the prior $\pi(\theta_1)$ and set $w_1^i = N^{-1}$, $\mathrm{i} = 1, \cdots \mathrm{N}$.

2. Recursion For $\mathrm{n} = 2, \cdots, N_\phi$

(a) Correction

Reweight the particles from stage $\mathrm{n} - 1$ by defining the incremental and normalized weights by calculating incremental and normalized weights $\tilde{w}_n^i = \left(p\left(\mathbb{Y}|\theta_{n-1}^i\right)\right)^{\phi_n - \phi_{n-1}}$ and $\widetilde{W}_n^i = \frac{\tilde{w}_n^i W_{n-1}^i}{\frac{1}{N}\sum_{i=1}^N \tilde{w}_n^i W_{n-1}^i}$.

$E_{\pi_n}[h(\theta)]$ is approximated to $\tilde{h}_{n,N} = \frac{1}{N}\sum_{i=1}^N h\left(\theta_{n-1}^i\right) W_n^i$

(b) Selection

Compute the effective sample size $ESS_n = \frac{N}{\left(\frac{1}{N}\sum_{i=1}^{N}(\widetilde{W}_i^n)^2\right)}$.

If $\hat{\rho}_n = 1$, where $\hat{\rho}_n = \mathcal{L}\{ESS_n < 0.5N\}$

Resample the particles by multinomial resampling, $\{\hat{\theta}\}_{i=1}^{N} = \{\theta_{n-1}^i, \widetilde{W}_n^i\}_{i=1}^{N}$. Let $W_n^i = 1$.

If $\hat{\rho}_n = 0$

Let $\hat{\theta}_n^i = \theta_{n-1}^i$ and $W_n^i = \widetilde{W}_n^i$. $E_{\pi_n}[h(\theta)]$ is approximated to

$\hat{h}_{n,N} = \frac{1}{N}\sum_{i=1}^{N} h(\hat{\theta}_n^i)\, W_n^i$.

(c) Mutation

Propagate the particles $\{\hat{\theta}_\mathrm{i}, W_n^i\}$ using the Metropolis–Hastings algorithm with transition density $\theta_n^i \sim K_n(\theta_n | \hat{\theta}_n^i; \xi_n)$ and stationary $\pi_n(\theta)$ (see Algorithm 2). $E_{\pi_n}[h(\theta)]$ is approximated to $\bar{h}_{n,N} = \frac{1}{N}\sum_{i=1}^{N} h(\theta_n^i)\, W_n^i$.

3. Final importance of sampling approximation of $E_\pi[h(\theta)]$

When $\mathrm{n} = N_\phi$, $\bar{h}_{N_\phi,N} = \sum_{i=1}^{N} h\left(\theta_{N_\phi}^i\right) W_{N_\phi}^i$

Algorithm 2

Particle mutation; Prior to executing Algorithm 1.

1. Generate blocks randomly

Generate a sequence of random partitions $\{B_n\}_{n=2}^{N_\phi}$ of θ_n into N_{blocks} equally sized blocks, denoted by $\theta_{n,b}$. Let $\theta_{n,b}^*$ and $\Sigma_{n,b}^*$ be the partitions of θ_n^* and Σ_n^* that correspond to the subvector $\theta^*_{n,b}$.

---

2. MH steps: For i = 1 to M:

For b = 1 to N_{blocks}:

(a) Proposal draw

Draw proposal ϑ_b from the mixture distribution

$$\vartheta_b|\left(\theta^i_{n,b,m-1}, \theta^i_{n,-b,m}, \theta^*_{n,b}, \Sigma^*_{n,b}\right) \sim \alpha N\left(\theta^i_{n,b,m-1}, c_n^2 \Sigma^*_{n,b}\right) + \frac{1-\alpha}{2} N\left(\theta^i_{n,b,m-1}, c_n^2 diag\left(\Sigma^*_{n,b}\right)\right) + \frac{1-\alpha}{2} N\left(\theta^*_{n,b}, c_n^2 \Sigma^*_{n,b}\right)$$

(b) Solve the model and evaluate the proposal

Solve the model (see section)

Evaluate the likelihood

(c) Accept or reject

Calculate

$$\alpha\left(\vartheta_b|\theta^i_{n,b,m-1}, \theta^i_{n,-b,m}, \theta^*_{n,b}, \Sigma^*_{n,b}\right)$$

$$= min\left\{1, \frac{p^{\phi_n}\left(\mathbb{Y}|\vartheta_b, \theta^i_{n,-b,m}\right) p\left(\vartheta_b, \theta^i_{n,-b,m}\right) / q\left(\vartheta_b|\theta^i_{n,b,m-1}, \theta^i_{n,-b,m}, \theta^*_{n,b}, \Sigma^*_{n,b}\right)}{p^{\phi_n}\left(\mathbb{Y}|\theta^i_{n,b,m-1}, \theta^i_{n,-b,m}\right) p\left(\theta^i_{n,b,m-1}, \theta^i_{n,-b,m}\right) / q\left(\theta^i_{n,b,m-1}|\vartheta_b, \theta^i_{n,-b,m}, \theta^*_{n,b}, \Sigma^*_{n,b}\right)}\right\}$$

and let

$$\theta^i_{n,b,m} = \begin{cases} \vartheta_b & with\ probability\ \alpha\left(\vartheta_b|\theta^i_{n,b,m-1}, \theta^i_{n,-b,m}, \theta^*_{n,b}, \Sigma^*_{n,b}\right) \\ \theta^i_{n,b,m-1} & otherwiese \end{cases}$$

---

3. Final step

Let $\theta^i_{n,b} = \theta^i_{n,b,M}$

Algorithm 3

Adaptive particle mutation; Prior to Step 1 of Algorithm 2.

1. Importance of sampling approximation

Calculate the importance of the sampling approximations $\tilde{\theta}_n$ and $\tilde{\Sigma}_n$ of $E_{\pi_n}[\theta]$ and $V_{\pi_n}[\theta]$ according to particles $\{\theta_{n-1}^i, \widetilde{W}_n^i\}_{i=1}^N$.

2. Adjusting the scaling factor

Calculate the average rejection rate $\hat{R}_{n-1}(\hat{\xi}_{n-1})$, based on the mutation step in iteration $n-1$, across N_{blocks}.

Adjust the scaling factor following $\hat{c}_2 = c^*$ and $\hat{c}_n = \hat{c}_{n-1} f\left(1 - \hat{R}_{n-1}(\hat{\xi}_{n-1})\right)$, where $f(x) = 0.95 + 0.10\frac{e^{16(x-0.25)}}{1+e^{16(x-0.25)}}$

3. Replacement

Replace ξ_n with $\hat{\xi}_n = \left[\hat{c}_n, \tilde{\theta}_n, vech(\tilde{\Sigma}_n)'\right]'$.

Algorithm 4

Particle filter.

1. Initialization

Draw the initial particles from $s_0^j \sim p(s_0|\theta)$ and set $W_0^j = 1$.

2. Recursion For $t = 1, \cdots, T$

(a) Forecasting s_t

Propagate the particles $\{s_{t-1}^j, W_{t-1}^j\}$ by simulating the state-transition equation:

$$\tilde{s}_t^j = \Phi(s_{t-1}^j, \epsilon_t^j; \theta), \qquad \epsilon_t^j \sim F_\epsilon(\cdot; \theta)$$

(b) Forecasting $\mathbb{y}_t$

Calculate the incremental weights:

$$\widetilde{w}_t^j = p\left(\mathbb{y}_t|\tilde{s}_t^j, \theta\right)$$

Approximate the predictive density $p(\mathbb{y}_t|\mathbb{Y}_{1:t-1}, \theta)$ as follows:

$$p(\mathbb{y}_t|\mathbb{Y}_{1:t-1}, \theta) = \frac{1}{N_{filter}} \sum_{j=1}^{N_{filter}} \widetilde{w}_t^j W_{t-1}^j$$

(c) Updating

Calculate the normalized weights

$$\widetilde{W}_t^j = \frac{\widetilde{w}_t^j W_{t-1}^j}{\frac{1}{N_{filter}} \sum_{j=1}^{N_{filter}} \widetilde{w}_t^j W_{t-1}^j}$$

(d) Selection

If $\hat{\rho}_t = 1$, where $\hat{\rho}_t = \mathcal{L}\{ESS_t < 0.5N\}$, where $ESS_t = M/\left(\frac{1}{N_{filter}} \sum_{j=1}^{N_{filter}} \left(W_t^j\right)^2\right)$

Resample the particles by multinomial resampling, $\{s_t^j\}_{j=1}^{N_{filter}} = \{\tilde{s}_t^j, \widetilde{W}_t^j\}_{j=1}^{N_{filter}}$. Let $W_t^j = 1$.

If $\hat{\rho}_t = 0$

Let $s_t^j = \tilde{s}_t^j$ and $W_t^j = \widetilde{W}_t^j$.

3. Likelihood approximation

The approximation of the likelihood function is given by:

$$\hat{p}(\mathbb{Y}_{1:T}|\theta) = \prod_{t=1}^{T} \left(\frac{1}{N_{filter}} \sum_{j=1}^{N_{filter}} \widetilde{w}_t^j W_{t-1}^j \right)$$