

Models of money based on imperfect monitoring and pairwise meetings:  
policy implications

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Why do we want models in monetary economics?

- address policy questions (normative economics)
- *explain* observations that seem puzzling or paradoxical (positive (?) economics)

Today's talk is mainly about policy questions

## Two pillars of modern monetary economics

- imperfect monitoring (necessary for money to be essential)
  - asymmetric information in the form of private histories
  - costly record-keeping (why use poker chips?)
- pairwise meetings (not necessary for money to be essential)
  - have to be justified in other ways

## Three roles of pairwise meetings

- have always appeared in descriptions of double-coincidence problems

“Since occasions where two persons can just satisfy each other’s desires are rarely met, a material was chosen to serve as a general medium of exchange.” (Paulus, a 2nd century Roman jurist)

- helps rationalize the asymmetric-information foundation for imperfect monitoring
- deals with puzzles that models of centralized trade seem unable to address

## Study good policies in three kinds of settings

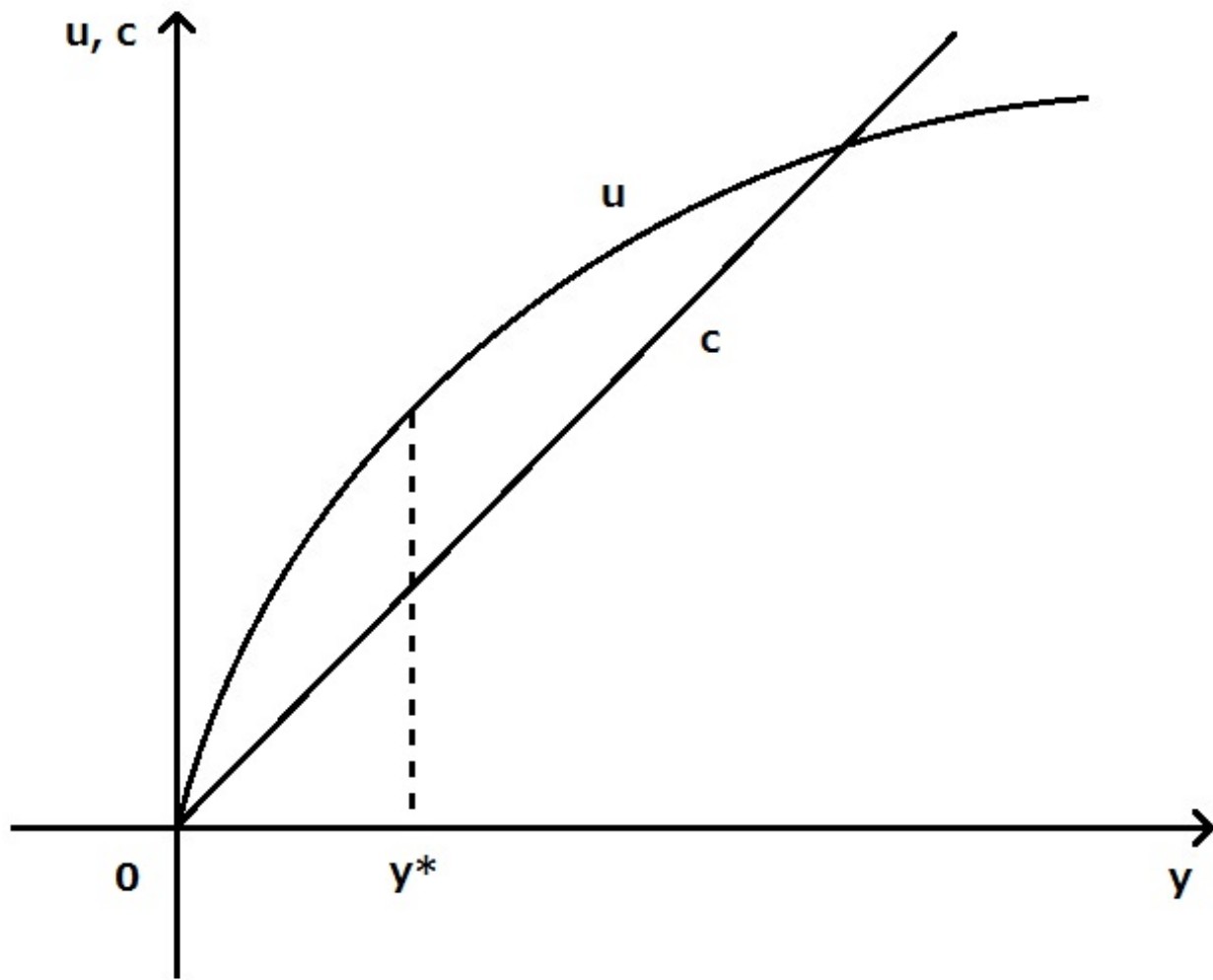
- no-monitoring and currency is the only asset
- no-monitoring, but with currency and higher return assets
- some monitoring and currency is the only asset

## Shi 1995 and Trejos-Wright 1995

- discrete time with nonatomic measure of people
- each maximizes expected discounted utility with discount factor  $\beta$
- pairwise meetings at random: a person is a consumer with prob  $1/K$  and is a producer with prob  $1/K$ ; integer  $K \geq 2$
- period utility: consumer  $u(y)$ ; producer  $-c(y)$ ;
- no commitment: let  $\hat{y}$  be maximum production under *perfect monitoring* and let  $y^* = \arg \max[u(y) - c(y)] > 0$ ; perfect-monitoring optimum is  $\min\{\hat{y}, y^*\}$  in every single-coincidence meeting
- no-monitoring

## Extensions of Shi 1995 and Trejos-Wright 1995

- they studied the model with money holdings in  $\{0, 1\}$ 
  - the distribution of money is unaffected by trades
  - there is no scope for transfers of money
- first two applications: abandon their limitation on asset holdings, but assume that asset holdings in meetings are common knowledge
- third application: keeps the  $\{0, 1\}$  restriction on asset holdings, but generalize the no-monitoring assumption
  - some exogenous fraction of people are perfectly monitored and the rest not at all (see Cavalcanti-Wallace 1999)





## Ex ante optima from among implementable allocations

For each model, find the allocation that maximizes *ex ante representative-agent discounted utility* subject to two main restrictions:

- the allocation is a steady state
- the allocation is in the pairwise core for each meeting

Ex ante means before initial assets holdings are assigned and before type (monitored or not-monitored) is determined

Wataru Nozawa and Hoonsik Yang (unpublished, part of Yang's 2016 Ph.D. dissertation)

Consider Trejos-Wright 1995 with individual money holdings in  $\{0, 1, 2, \dots, B\}$  and no explicit taxation

Sequence of actions:

- state of the economy is a distribution over  $\{0, 1, 2, \dots, B\}$ , denoted  $\pi$
- then pairwise meetings at random and trade with lotteries
- then (probabilistic) transfers: non-negative and weakly increasing in a person's money holding
- then inflation modeled as probabilistic *iid* loss  $\delta$  (disintegration) of each unit of money held

## Maximization problem

For given parameters, choose  $\pi$ ;  $y_{ij}$  and a lottery over money surrendered by the consumer; the transfers; and  $\delta$  to maximize

$$\frac{1}{K(1 - \beta)} \sum_{i=0}^B \sum_{j=0}^B \pi(i)\pi(j)[u(y_{ij}) - c(y_{ij})]$$

subject to

- steady-state condition
- each trade is feasible and in the pairwise core for the meeting

Parameters:  $B = 3$ ,  $K = 3$ ,  $u(y) = 1 - e^{-\kappa y}$ , and  $c(y) = y$

Table 1. Optimal prob (%) of a unit transfer: the top number is for those with 0 or 1, the bottom number is for those with 2

$\kappa \setminus \beta$	.15	.2	.25	.3	.35	.4	.5	.6	.7	.8
20	0 0	0 72	0 73	0 74	0 0	3.5 3.5	3.1 3.1	13.0 13.0	11.4 11.4	0 0
15	-	0 0	0 65	0 67	0 67	0 0	0 0	5.3 5.3	2.8 2.8	0 0
12	-	-	0 0	0 59	0 61	0 0	0 0	2.1 2.1	0 0	0 0
10	-	-	-	0 0	0 55	0 56	0 0	0 0	0 0	0 0
8	-	-	-	-	0 0	0 47	0 0	0 0	0 0	0 0
6	-	-	-	-	-	0 0	0 38	0 0	0 0	0 0
5	-	-	-	-	-	-	0 0	0 0	0 0	0 0
4	-	-	-	-	-	-	-	0 0.6	0 0	0 0
3	-	-	-	-	-	-	-	-	0 0.9	0 0

## Lesson

Optimal policy is not simple

- even for this very simple model, the best policy is very dependent on the parameters

The best policy ranges from paying substantial interest on large holdings of money to giving lump-sum transfers—very different policies

- the former spreads out the distribution of money holdings, while the latter compresses it

## Coexistence of money and higher return assets

Hicks (1935): coexistence is the main challenge for monetary theory

- For Hicks, assuming a demand for money or putting money in the utility function are *nonsenses*
- Hicks proposed transaction costs (which we ought to regard as another *nonsense*)

Cash-in-advance (CIA) did not exist when Hicks wrote

- Whether it gives rise to coexistence depends on your equilibrium notion

Individual defection or defection by the pair in each meeting

With individual defection, CIA is implementable

- each person choose from  $\{yes, no\}$  as response to a planner suggested trade

With defection by pairs, it, generally, is not

Tao Zhu and I (JET, 2007) took as our challenge getting coexistence while allowing cooperative defection of the pair in each meeting

The partial equilibrium setting (strictly increasing and concave continuation value of nominal wealth)

Stage 1: portfolio choice

- government offers one-period discount bonds at a given price
- people with only money choose a portfolio
- (in the general equilibrium, interest is financed by money-creation)

Stage 2. Trejos-Wright (1995) with general portfolios of money and bonds



## Stage-2 pairwise meetings

Bonds and money are perfect substitutes in terms of their payoffs at the start of the next date

Therefore, pairwise-core outcomes are defined as a set of pairs: output in a meeting and the amount of *wealth* transferred

There are many pairwise-core outcomes: they range from giving all the gains-from-trade to the consumer to giving all to the producer

That allows us to reward buyers with a lot of money and, thereby, gives people an incentive to leave stage-1 with some money

## Comments

Role of pairwise trade (nondegenerate pairwise core)

Multiplicity (the favored assets could be money, bonds, or neither)

Welfare: can it be beneficial to have coexistence?

- Hu and Rocheteau (JET 2013)
  - uses a version of Lagos-Wright (2005) with capital and money
  - helps avoid over-accumulation of capital
- Hoonsik Yang (unpublished, part of his 2016 Ph.D. dissertation)
  - works with examples in the Zhu-Wallace setting and the following allowable portfolios:  $(0, 0)$ ,  $(1, 0)$ ,  $(2, 0)$  and  $(0, 1)$ ,  $(1, 1)$ ,  $(0, 2)$

## Cavalcanti and Wallace 1999

Maintain all of Trejos-Wright 1995 including  $\{0, 1\}$  money holdings, but assume

- some exogenous fraction of people are perfectly monitored,  $m$ -people; the rest,  $n$ -people, not monitored at all
  - the endogenous state: the distribution of money between the two types
- the model was designed to compare inside (private) money and outside money as alternative monetary systems

Here: two numerical examples, from joint work with Alexei Deviatov, that uses an outside-money version to explore optimal policy

## Monitoring and punishment

- ex ante identical people, but
  - fraction  $\alpha$  become permanently monitored ( $m$ -people)
  - rest are permanently nonmonitored ( $n$ -people)
- for  $m$ -people, histories (and money holdings) are common knowledge
- for  $n$ -people, they are private
- monitored status and consumer/producer status are common knowledge
- only punishment is individual  $m \rightarrow n$

## Implementable stationary allocations

- state of economy:  $(\theta_m, \theta_n) \in [0, \alpha] \times [0, 1 - \alpha]$ , fractions with money
- state of meeting:  $(s, s') \in S \times S$ , where  $S = \{m, n\} \times \{0, 1\}$
- a stationary allocation is  $(\theta_m, \theta_n)$ , trades (including lotteries) in meetings, and transfers consistent with a steady state, a constant  $(\theta_m, \theta_n)$
- a stationary allocation is implementable if
  - trades are in the *pairwise core* and *IC* for  $n$  people
  - transfers are *IR* and *IC* for  $n$  people

## The planner

Choose a stationary and implementable allocation to maximize ex ante expected utility before initial  $s$  is realized for each person

- ex ante expected utility is proportional to

$$\sum_{s \in S} \sum_{s' \in S} \pi_s \pi_{s'} [u(y_{ss'}) - c(y_{ss'})]$$

where  $y_{ss'}$  is output in the  $(s, s')$  meeting and

$$(\pi_{m1}, \pi_{m0}, \pi_{n1}, \pi_{n0}) = (\theta_m, \alpha - \theta_m, \theta_n, 1 - \alpha - \theta_n)$$

- first-best is proportional to  $u(y^*) - c(y^*)$ , where  $y^* = \arg \max [u(y) - c(y)]$

Example 1: Optimal inflation (Deviatov and Wallace 2010; working paper)

- inflation: a person who ends trade with money loses it with some probability
- parameters:  $u(y) = 1 - e^{-10y}$ ,  $c(y) = y$ ,  $K = 3$ ,  $\beta = .59$ ,  $\alpha = 1/4$ ;
  - arbitrary except for  $\beta$ ; high enough so that if  $\alpha = 1$ , then  $\min\{\hat{y}, y^*\} = y^*$ ; low enough so that if  $\alpha = 0$ , then optimal  $(y, \theta_n) \ll (y^*, 1/2)$ , where  $y$  is output in *trade* meetings

## The optimum

- welfare equal to 34% of first best
- all  $m$ -people start each date with money ( $\theta_m = \alpha$ )
- 24% of  $n$ -people start with money ( $\theta_n = .24(1 - \alpha)$ )
- 16% inflation rate
- transfers to  $m$  people, none to  $n$  people



Table 1. Optimal trades	
(prod)(con)	(output/ $y^*$ )/(money from consumer)
$(n0)(n1)^*$	0.57/(1)
$(n0)(m1)^*$	0.57/(1)
$(m1)(n0)$	0.11/(0)
$(m1)(n1)^*$	0.38/(1)
$(m1)(m1)^*$	0.38/(0)

More row-2 meetings than row-4 meetings ( $\pi_{n1} < \pi_{n0}$ ). Therefore,

- net inflow of money into holdings of  $n$ -people (more row-2 meetings than row-4 meetings)
- inflation and transfers to  $m$ -people (surprise?)

Similar inside-money result is in Deviatov and Wallace (*RED* 2014)

Inside money is different because money is issuer-specific

An  $m$  person defects without useful money

Transfers not needed, but the optimum has spending by  $m$  people that exceeds earnings at each date, which produces inflation (surprise?)

Example 2. Optimal seasonal policy (Deviatov and Wallace *JME* 2009)

Same model except: a seasonal and zero average inflation is imposed

Parameters:  $\alpha = 1/4$ ,  $K = 3$ ,  $u(y) = 2y^{1/2}$ ,  $\beta = .95$ , and

$$c_t(y) = \begin{cases} y/(0.80) & \text{if } t \text{ is odd (winter)} \\ y/(1.25) & \text{if } t \text{ is even (summer)} \end{cases}$$

First date is odd (winter)

Implementable allocations: same except allowed to be two-date periodic

- If  $\alpha = 1$ , then the first-best is implementable
- If  $\alpha = 0$ , then the optimum is  $(y_t, \theta_{nt}) = (y_t^*, 1/2)$  and welfare equal to  $1/4$  of first-best welfare

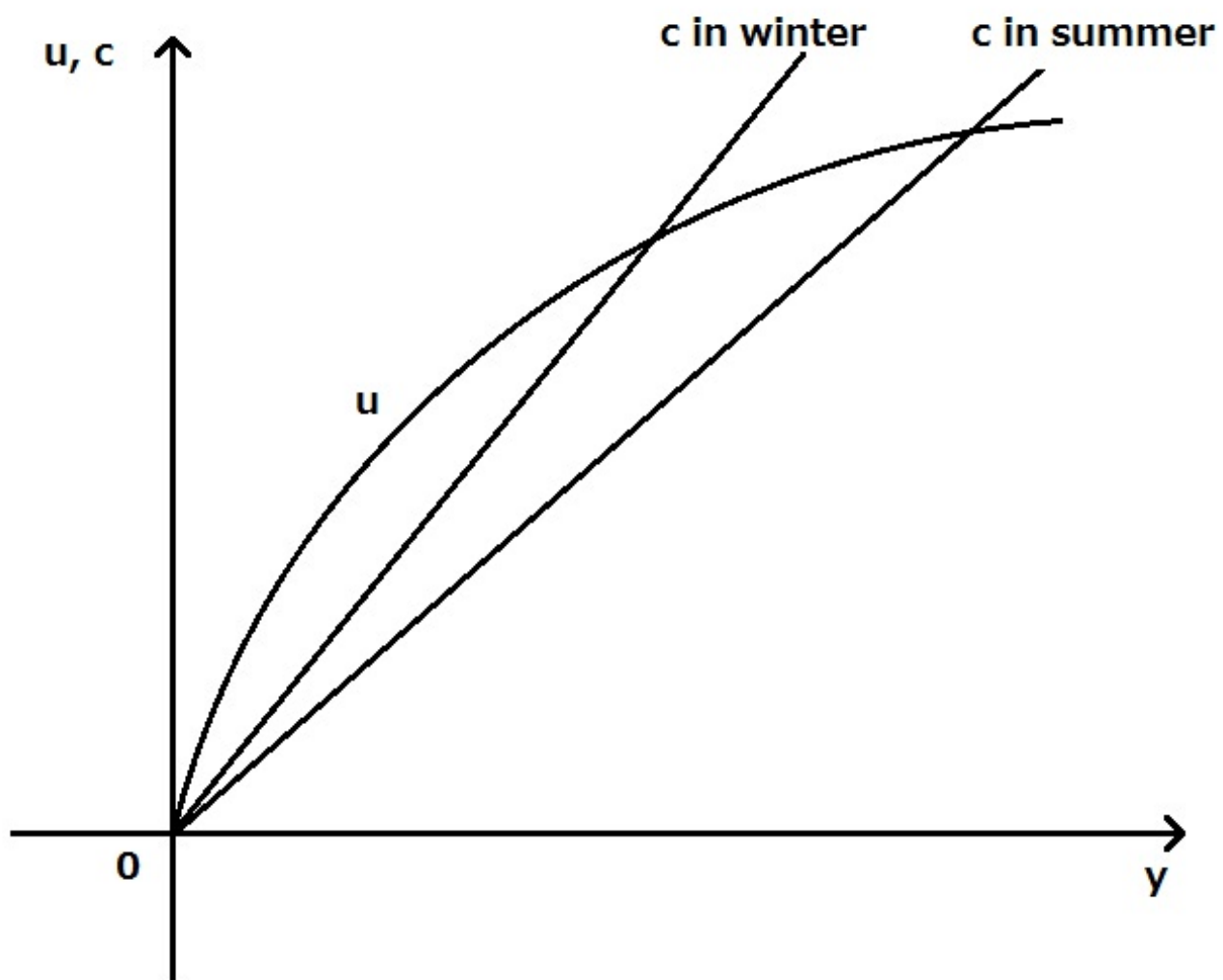


Table 2. Optimal quantity of money and welfare		
	beginning of winter	beginning of summer
$\theta_m$	$\alpha$	$\alpha$
$\theta_n$	0.312	0.309
welfare/first-best welfare	.4558	

there are no transfers to  $n$ -people

a higher quantity of money at beginning of winter

comparison to a no-intervention (constant money-supply) optimum

- no-intervention optimum also has  $\theta_m = \alpha$  at each date and, therefore, a constant  $\theta_n$  and zero net flows between types at each date
- gain from intervention is about 1/2% in terms of consumption

Table 3. Optimal trades		
meeting	(output/ $y_t^*$ )/(money transferred)	
(prod)(con)	winter	summer
( $n0$ )( $n1$ )	0.95/(1.00)	0.95/(1.00)
( $n0$ )( $m1$ )	0.85*/(0.51)	0.78*/(0.78)
( $m1$ )( $n0$ )	0.16/(0)	0.17/(0)
( $m1$ )( $n1$ )	1.18 <sup>†</sup> /(0.81)	0.84* <sup>†</sup> /(1.00)
( $m1$ )( $m1$ )	1.00/(0)	0.84*/(0)

- net outflow from holdings of  $n$  people in winter, matching net inflow in summer
- $m$ -people surrender money at beginning of summer and receive an exactly offsetting transfer at beginning of winter
  - interpretation as planner loans: zero-interest loans to  $m$ -people at beginning of winter with repayment at beginning of summer (surprise and explanation?)

What can we learn from a few numerical examples?

They are consistent with the following related conclusions:

- if you know the model, then intervention is optimal
- even the *qualitative* nature of optimal intervention is not obvious
- optimal intervention depends on all the details

May need more examples to make those conclusions convincing

## Concluding remarks

Somewhat standard view: judge a model not by the observations that inspired it, but by its other implications that were not known when the model was formulated. In that sense, the above applications are a tribute to the Shi and Trejos-Wright models.

Possible extensions:

- what if people in a meeting can hide assets
- nonstationary allocations and time consistent policy
- a large finite number of agents rather than a continuum