

Can the Laffer curve for consumption tax be hump-shaped?*

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Abstract

This paper characterizes the shape of the Laffer curve for consumption tax analytically. The Laffer curve for consumption tax can be hump-shaped if the utility function is an additively separable one in consumption and labor supply. Conversely, it cannot be hump-shaped if the utility function is the one employed by previous researchers. The difference in the utility functions has quantitatively significant effects on the peak tax rates of the Laffer curves for labor and capital income taxes.

Keywords: Laffer curve; tax revenue; consumption tax

JEL classification: E62; H20; H30

1 Introduction

The main objective of this paper is to investigate the Laffer curve for consumption tax. As in Waninski (1978), Arthur B. Laffer's conjecture is that the Laffer curve is hump-shaped.¹ This is because an increase in a tax rate would have two opposing effects on the tax revenue. In the first effect, the tax revenue would increase as a direct consequence of raising the tax rate. In the second effect, the tax revenue reduces because a high tax rate discourages economic activities of labor supply, capital accumulation, consumption, and output. Contrary to Laffer's conjecture, Trabandt and Uhlig (2011, 2013) recently show that the Laffer curve for consumption tax is monotonically increasing, whereas the Laffer curves for labor and capital income taxes are hump-shaped. It is also found that the monotonically increasing Laffer curve for consumption tax is robust to some variations of the models. Their finding has a big impact on many kind of fiscal issues like fiscal limits and fiscal sustainability because those are affected by the maximum size of government tax revenue. However, most of results of Trabandt and Uhlig (2011, 2013) are based on numerical analyses, and it is not clear whether the Laffer curve for consumption tax is generally monotonically increasing or not.

This paper characterizes the shape of the Laffer curve for consumption tax both in a simple static general equilibrium model and a standard neoclassical growth model. In a simple static model, output is produced by linear technology of labor, no capital stock, no government consumption, and the tax revenue is used only for the lump-sum transfer. In a neoclassical growth model, capital stock, investment expenditure, government debt, and net imports are introduced to the dynamic setting à la Trabandt and Uhlig (2011). Both of the consumption tax revenue curve and the total tax revenue curve, including labor and capital tax revenues, are considered as the Laffer curves.

¹There are two definitions of the Laffer curve. In this paper, the Laffer curve is defined as the tax revenue curve following Trabandt and Uhlig (2011). On the other hand, in the textbooks of public finance, like Gruber (2015), it is defined as the hump-shaped tax revenue curve.

The Laffer curve for consumption tax can be hump-shaped if the utility function is an additively separable one in consumption and labor supply, whereas this is not so if the utility function is the one employed by Trabandt and Uhlig (2011). A necessary and sufficient condition for the hump-shaped consumption tax revenue curve is also derived under general functional form of the utility. The key parameters for the hump-shaped Laffer curve are the relative risk aversion (RRA) and the labor supply elasticity in the utility function. For the hump-shaped Laffer curve, RRA and the inverse of labor supply elasticity should be sufficiently small. An increase in the consumption tax rate has a negative effect on the tax revenue in that it reduces aggregate labor supply and aggregate consumption. Thus, the parameter of labor supply elasticity in the utility function is important. The aggregate labor supply and aggregate consumption elasticities can be greater than one under sufficiently small values of RRA and the inverse of labor supply elasticity parameters in the case of an additively separable utility function, whereas this cannot be the case for the Trabandt-Uhlig utility. The difference in the functional form of the utility has quantitatively significant effects on the peak tax rates of the Laffer curves for labor and capital income taxes. The quantitative impacts of the difference in the utility function on the peak tax rates of the Laffer curves for labor and capital income taxes are about 10% when the Laffer curve for consumption tax is not hump-shaped. They exceed 30% when the Laffer curve is hump-shaped.

Both additively separable and Trabandt-Uhlig utility functions are often employed in macroeconomics. For examples, Gali (2008) employs the additively separable utility, whereas King and Rebelo (1999) employed Trabandt-Uhlig utility. It is rare to focus on the effect of the difference in utility functions. However, this paper illustrates an example where the difference in the utility functions has a significant effect on the Laffer curves.

Note that the models in this paper are plain vanilla frictionless neoclassical ones. Some frictions, like home production and tax evasions, would be apparently sources of hump-shaped Laffer curve for consumption tax. However, this paper focuses on the

economies without such frictions, and investigates the shape of Laffer curve for consumption tax in standard neoclassical model.

The Laffer curve has been investigated by various researchers. Ireland (1994) find that the hump-shaped Laffer curve for capital income tax using an AK model. Schmitt-Grohè and Uribe (1997) derivate the hump-shaped Laffer curve for labor income tax in a neoclassical growth model. Trabandt and Uhlig (2011, 2013) estimate the Laffer curves for consumption, labor, and capital taxes for the U.S. and EU14 using a neoclassical growth model. Nutahara (2015) applies the model of Trabandt and Uhlig (2011) to the Japanese economy. Fève, Matheron, and Sahuc (2013) investigate the Laffer curves for consumption, labor, and capital taxes in an incomplete-market economy. Holter, Krueger, and Stepanchuk (2014) focus on the effect of households' heterogeneity and progressive tax scheme on the peak tax rate of the Laffer curve for labor income tax using an overlapping generations model.

This paper is closely related to the papers by Trabandt and Uhlig (2011, 2013) and Nutahara (2015), who estimate the Laffer curve for consumption tax. They employ a utility function with constant labor supply elasticity and use numerical analyses to show that the Laffer curve for consumption tax is monotonically increasing. Kobayashi (2014) investigates whether the consumption tax revenue is bounded using a neoclassical growth model with the log utility function. He finds that although the fixed supply of production factor affects the boundedness of the consumption tax revenue, the Laffer curve continues to be monotonically increasing in his model. In the paper by Fève, Matheron, and Sahuc (2013), the Laffer curve for consumption tax is not hump-shaped because they employ the log utility function. The main contribution of the present paper is to find that the Laffer curve for consumption tax can be hump-shaped if the utility is additively separable.

Our finding has implications on the literature of fiscal reform because the consumption tax is receiving a lot of attention as a useful tool to finance the government expen-

diture as in Braun and Joines (2015), Hansen and Imrohoroglu (2016) and Kitao (2016). In this literature, the consumption tax is highlighted because the welfare loss from the consumption tax is less than those from other distortionary taxes, and because the Laffer curves for other taxes, like labor income tax, are hump-shaped and the tax revenues are limited. According to our finding, the consumption tax might not be useful if the Laffer curve for consumption tax is hump-shaped.

The remainder of the paper is organized as follows. Section 2 introduces the simple static model and shows the main result. Section 3 extends the result of Section 2 to a dynamic setting à la Trabandt and Uhlig (2011). Section 4 discusses the results. Section 5 concludes.

2 Simple static economy

Assuming a simple static economy, the Laffer curve for consumption tax is characterized in this section.

2.1 Model

The representative households supply labor n to firms and earn wage rate w . They also receive government transfers s . Let τ_t^c denote consumption tax. The budget constraint of households is

$$(1 + \tau^c) c \leq wn + s, \tag{1}$$

where c denotes consumption.

The firms are perfectly competitive. Their production function is

$$y = n, \tag{2}$$

where y denotes output.

The government budget constraint is

$$s \leq T, \quad (3)$$

where total tax revenue T is defined by

$$T = \tau^c c. \quad (4)$$

Since there is no investment and government consumption, the resource constraint of this closed economy is

$$y = c. \quad (5)$$

Two types of utility functions are considered. The one is an additively separable one such that

$$U^{AS} = \frac{c^{1-\eta} - 1}{1-\eta} - \kappa n^{1+\lambda},$$

where η is the relative risk aversion (that is the inverse of the IES under a dynamic setting), and $1/\lambda$ is the labor supply elasticity.² This type of utility function is often employed in the literature on the new Keynesian business cycle (see Gali, 2008). The other is one, called “Trabandt-Uhlig utility function” in this paper, such that

$$U^{TU} = \frac{1}{1-\eta} \left\{ c^{1-\eta} \left[1 - \kappa(1-\eta)n^{1+\lambda} \right]^\eta - 1 \right\},$$

which is a static version employed by Trabandt and Uhlig (2011). If $\eta = 1$, these two utility functions are identical. Otherwise, these two specifications are different.

2.2 Laffer curve for consumption tax in a static economy

First, consider the consumption tax revenue curve as the Laffer curve. The key element here is the elasticity of aggregate consumption to the consumption tax rate. If it is

²In this paper, $1/\lambda$ is called “the labor supply elasticity,” but it is often interpreted as “Frisch elasticity” in the literature. A discussion on this topic appears in Section 4.

greater than one, an increase in the consumption tax rate increases the consumption tax revenue, and vice versa. In this model, consumption equals labor supply by the resource constraint and production function.

In the case of the additively separable utility function, the optimization condition for the consumption–labor choice is

$$\kappa(1 + \lambda)c^\eta n^\lambda = \frac{1}{1 + \tau^c} w. \quad (6)$$

Solving this condition yields

$$c = n = [\kappa(1 + \lambda)(1 + \tau^c)]^{-1/(\eta + \lambda)}, \quad (7)$$

and the elasticity of aggregate consumption to the consumption tax rate is

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c}{1 + \tau^c} \cdot \frac{1}{\eta + \lambda}. \quad (8)$$

It is easily shown that $\left| \frac{dc/c}{d\tau^c/\tau^c} \right|$ is increasing in τ^c , $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = 0$ if $\tau^c = 0$, and $\left| \frac{dc/c}{d\tau^c/\tau^c} \right|$ converges to $\frac{1}{\eta + \lambda}$ as τ^c approaches infinity. Therefore, the Laffer curve for consumption tax can be hump-shaped if $\frac{1}{\eta + \lambda}$ is greater than one.

The following is a formal statement of a necessary and sufficient condition for a hump-shaped consumption tax revenue curve for consumption tax.

Proposition 1. *Suppose that the utility function is additively separable; U^{AS} . The consumption tax revenue curve for consumption tax is hump-shaped if and only if $\eta + \lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\eta + \lambda}{1 - \eta - \lambda}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing.*

Proof. Note that

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| - 1 = \frac{1}{\eta + \lambda} \cdot \frac{\tau^c}{1 + \tau^c} [(1 - \eta - \lambda)\tau^c - (\eta + \lambda)]. \quad (9)$$

Suppose that $\eta + \lambda = 1$. In this case, $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| - 1 < 0$ and the consumption tax revenue is monotonically increasing.

Suppose $\eta + \lambda \neq 1$. In this case,

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| - 1 = \left(\frac{1 - \eta - \lambda}{\eta + \lambda} \right) \left(\frac{\tau^c}{1 + \tau^c} \right) \left(\tau^c - \frac{\eta + \lambda}{1 - \eta - \lambda} \right).$$

If $\eta + \lambda \geq 1$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| \leq 1$.

If $\eta + \lambda < 1$, then $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| \leq 1$ for $\tau^c \leq (\eta + \lambda)/(1 - \eta - \lambda)$, and $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| > 1$ for $\tau^c > (\eta + \lambda)/(1 - \eta - \lambda)$. \square

The parameters in the utility function, η and λ , should be small because the hump-shaped consumption tax revenue curve can be understood by the optimization condition for the consumption–labor choice (6). The consumption tax revenue curve can be hump-shaped if an increase in the consumption tax rate reduces the labor supply by a sufficient amount. The key parameter is the inverse of λ , that is, the labor supply elasticity to the effective after-tax wage rate $w/(1 + \tau^c)$, that is also interpreted as the relative price of leisure with respect to consumption. Then, a low value of λ implies a highly distorted increase in the consumption tax rate. In general equilibrium, consumption c is closely related to the labor supply n through the resource constraint and production function. In the current setting, $c = n$. Then, the parameter η (the inverse of the IES) works as the inverse of the aggregate labor supply elasticity. As a result, the inverse of $\eta + \lambda$ is the elasticity of the aggregate labor supply in general equilibrium as in (7). Then, the inverse of $\eta + \lambda$ is the maximum of the elasticity of consumption since $c = n$.

In the case of the Trabandt-Uhlig utility function, the optimization condition for the consumption–labor choice is

$$\eta(1 + \lambda) \left(\frac{\kappa c n^\lambda}{1 - \kappa(1 - \eta)n^{1+\lambda}} \right) = \frac{1}{1 + \tau^c} w. \quad (10)$$

Solving this condition yields

$$c = n = [\tau^c \eta \kappa (1 + \lambda) + \kappa(\eta \lambda + 1)]^{-1/(1+\lambda)}, \quad (11)$$

and the elasticity of consumption to the consumption tax rate is

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c \eta \kappa}{\tau^c \eta \kappa (1 + \lambda) + \kappa(\eta \lambda + 1)}. \quad (12)$$

Contrary to Proposition 1, the Trabandt-Uhlig utility function cannot generate a hump-shaped Laffer curve for consumption tax as in Proposition 2, since $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| < 1$ for $\tau^c \geq 0$.

Proposition 2. *Suppose that the utility function is Trabandt-Uhlig; U^{TU} . The consumption tax revenue curve for consumption tax is monotonically increasing.*

Proof. It is obvious that

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c \eta \kappa}{\tau^c \eta \kappa (1 + \lambda) + \kappa (\eta \lambda + 1)} < 1.$$

□

So far, the consumption tax revenue curve is considered to be a Laffer curve. By introducing labor income tax, the Laffer curve refers to the total tax revenue. In this case, the budget constraint of a household becomes

$$(1 + \tau^c) c \leq (1 - \tau^n) wn + s, \quad (13)$$

and the total tax revenue is

$$T_t = \tau^c c + \tau^n wn. \quad (14)$$

Propositions 3 and 4 are analogues of Propositions 1 and 2.

Proposition 3. *Suppose that the utility function is additively separable; U^{AS} . The total tax revenue curve for consumption tax is hump-shaped if and only if $\tau^n < \eta + \lambda < 1$ and the revenue is maximized at $\tau^c = \frac{\eta + \lambda - \tau^n}{1 - \eta - \lambda}$. If $\eta + \lambda \leq \tau^n < 1$, the total tax revenue curve for consumption tax is monotonically decreasing. Otherwise, the total tax revenue curve for consumption tax is monotonically increasing.*

Proof. See Appendix A. □

Proposition 4. *Suppose that the utility function is Trabandt-Uhlig; U^{TU} . The total tax revenue curve for consumption tax is monotonically increasing.*

Proof. See Appendix B. □

As in the consumption tax revenue curve, the condition $\eta + \lambda < 1$ is necessary for the hump-shaped total tax revenue curve for consumption tax in the case of the additively separable utility function U^{AS} , and in the case of Trabandt-Uhlig utility function, the total tax revenue curve for consumption tax is monotonically increasing. Note that the consumption tax revenue curve might be monotonically decreasing if labor income tax rate is sufficiently high ($\eta + \lambda \leq \tau^n$). This is interpreted as the case where the peak consumption tax rate that maximizes the total tax revenue ($\tau^c = \frac{\eta + \lambda - \tau^n}{1 - \eta - \lambda}$) of the hump-shaped total tax revenue curve is negative.

2.3 Case of general form of the utility

So far, the functional form of the utility is specified. In this subsection, the consumption tax revenue curve under general form of the utility is investigated.

The utility function is $U(c, n)$ with standard assumption $U_c > 0$, $U_{cc} < 0$, $U_n < 0$, and $U_{nn} \leq 0$.

The consumption tax revenue curve is hump-shaped iff (1) $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| < 1$ if $\tau^c = 0$, (2) $\left| \frac{dc/c}{d\tau^c/\tau^c} \right|$ is increasing in τ^c , and (3) $\left| \frac{dc/c}{d\tau^c/\tau^c} \right| > 1$ if $\tau^c \rightarrow \infty$.

The elasticity of consumption is given by

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c}{1 + \tau^c} \times \left[-\frac{cU_{cc}}{U_c} + \frac{nU_{nn}}{U_n} + \frac{2 + \tau^c}{1 + \tau^c} \cdot \frac{nU_{cn}}{U_n} \right]^{-1}. \quad (15)$$

This form tell us that the elasticity of consumption consists of three parts. The first is the relative risk aversion $-cU_{cc}/U_c$. The second is about the disutility from labor supply nU_{nn}/U_n . The last is about the cross term of consumption and labor supply nU_{cn}/U_n .

In the case of additively separable utility U^{AS} ,

$$-\frac{cU_{cc}}{U_c} = \eta, \quad \frac{nU_{nn}}{U_n} = \lambda, \quad \text{and} \quad \frac{nU_{cn}}{U_n} = 0. \quad (16)$$

and then,

$$\left| \frac{dc/c}{d\tau^c/\tau^c} \right| = \frac{\tau^c}{1 + \tau^c} \times \frac{1}{\eta + \lambda}. \quad (17)$$

In the case of Trabandt-Uhlig utility U^{TU} ,

$$-\frac{cU_{cc}}{U_c} = \eta, \quad \frac{nU_{nn}}{U_n} = \lambda + \frac{(1 - \eta)^2 \kappa (1 + \lambda) n^{1+\lambda}}{1 - \kappa (1 - \eta) n^{1+\lambda}}, \quad \text{and} \quad \frac{nU_{cn}}{U_n} = 1 - \eta. \quad (18)$$

and then, The elasticity of consumption goes to $1/(1 + \lambda)$ as $\tau^c \rightarrow \infty$. Note that the relative risk aversion disappears as the limit of $\tau^c \rightarrow \infty$ in the case of Trabandt-Uhlig utility.

3 Dynamic economy à la Trabandt and Uhlig (2011)

In this section, the result of Section 2 is extended to a neoclassical growth model à la Trabandt and Uhlig (2011).

3.1 Model

The representative households hold capital stock k_{t-1} and debt b_{t-1} as assets at the beginning of the period. They supply labor n_t and capital stock k_{t-1} to firms, and earn the wage rate w_t , rental rate of capital d_t , and interest rate on debt R_t^b . They also receive government transfers s_t and transfers from abroad m_t . The latter can be interpreted as net imports as discussed by Trabandt and Uhlig (2011). Let τ_t^c , τ_t^n , and τ_t^k denote the consumption tax, labor tax, and capital tax rates, respectively. The budget constraint of households is

$$(1 + \tau_t^c)c_t + x_t + b_t \leq (1 - \tau_t^n)w_t n_t + (1 - \tau_t^k)(d_t - \delta)k_{t-1} + \delta k_{t-1} + R_t^b b_t + s_t + m_t, \quad (19)$$

where c_t denotes consumption, δ denotes the depreciation rate of capital, and x_t is investment. The capital stock evolves according to the following equation.

$$k_t = (1 - \delta)k_{t-1} + x_t. \quad (20)$$

The firms are perfectly competitive. Their production function is

$$y_t = \xi^t k_{t-1}^\theta n_t^{1-\theta}, \quad (21)$$

where ξ denotes the technology growth rate, and θ denotes the capital share in production. The profit maximization problem implies

$$w_t = (1 - \theta) \frac{y_t}{n_t} \quad \text{and} \quad (22)$$

$$d_t = \theta \frac{y_t}{k_{t-1}}. \quad (23)$$

The government budget constraint is

$$g_t + s_t + R_t^b b_{t-1} \leq b_t + T_t, \quad (24)$$

where g_t denotes the government consumption, and the total tax revenue T_t is defined by

$$T_t = \tau_t^c c_t + \tau_t^n w_t n_t + \tau_t^k (d_t - \delta) k_{t-1}. \quad (25)$$

The resource constraint of this economy is

$$y_t = c_t + x_t + g_t - m_t. \quad (26)$$

The additively separable utility function for this dynamic economy is

$$\mathbb{U}^{AS} = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\eta} - 1}{1-\eta} - \kappa \psi^{t(1-\eta)} n_t^{1+\lambda} + v(g_t) \right],$$

where $\psi^{t(1-\eta)}$ guarantees the existence of a balanced growth path, and $v(\cdot)$ is an increasing function. The Trabandt-Uhlig utility function is

$$\mathbb{U}^{TU} = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\eta} \left\{ c_t^{1-\eta} \left[1 - \kappa(1-\eta) n_t^{1+\lambda} \right]^\eta - 1 \right\} + v(g_t) \right].$$

Following Trabandt and Uhlig (2011), the Laffer curve for consumption tax is given by the relationship between the tax revenue and tax rate on the balanced growth path.

Let the growth rate on the balanced growth path be $\psi = \xi^{1/(1-\theta)}$. It is assumed that government debt b_t is on the balanced growth path; $b_{t-1} = \psi^t \bar{b}$. It is also assumed that $g_t = \phi_g y_t$ and $m_t = \phi_m y_t$.³ The equilibrium system at the balanced growth path is described in Appendix C.

3.2 Laffer curve for consumption tax in the dynamic economy

Propositions 5 and 6 refer to the consumption tax revenue curve in the dynamic economy.

Proposition 5. *Suppose that the utility function is additively separable; \mathbb{U}^{AS} . The consumption tax revenue curve for consumption tax is hump-shaped if and only if $\eta + \lambda < 1$, and the revenue is maximized at $\tau^c = \frac{\eta + \lambda}{1 - \eta - \lambda}$. Otherwise, the consumption tax revenue curve for consumption tax is monotonically increasing.*

Proof. See Appendix D. □

Proposition 6. *Suppose that the utility function is Trabandt-Uhlig; \mathbb{U}^{TU} . The consumption tax revenue curve for consumption tax is monotonically increasing.*

Proof. See Appendix E. □

³Trabandt and Uhlig (2011) employ alternative assumptions: $g_t = \psi^t \bar{g}$ and $m_t = \psi^t \bar{m}$. The constant steady-state ratio of government consumption to GDP is interpreted as the government controls g_t/y_t as in Hayashi and Prescott (2002). The constant steady-state ratio of net imports to GDP would be interpreted as net imports being closely related to the total income of the home country. These assumptions of constant steady-state ratios are used to prove Propositions 5–8. Under these assumptions, an increase in the consumption tax rate decreases both output and government consumption. This decrease in government consumption implies a positive wealth effect and then consumption increases. Therefore, the Laffer curve for consumption tax is more unlikely to be hump-shaped than those under the assumptions employed by Trabandt and Uhlig (2011).

Note that these two propositions are the same as Propositions 1 and 2, while the dynamic economy has far richer structure (capital, investment, debt evolution, etc.) than the static economy in Section 2.

Propositions 7 and 8 refer to the total tax revenue curve for consumption tax.

Proposition 7. *Suppose that the utility function is additively separable; \mathbb{U}^{AS} . The total tax revenue curve for consumption tax is hump-shaped if and only if*

$$\eta + \lambda < 1 \quad \text{and} \quad \left(\frac{y}{c}\right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{k}{y}\right) \right] < \eta + \lambda,$$

where

$$\begin{aligned} d &= \frac{1}{1 - \tau^k} \left[\frac{\psi^n}{\beta} - 1 + \delta \right], \\ \frac{k}{y} &= \frac{\theta}{d}, \\ \frac{c}{y} &= 1 - [\psi - (1 - \delta)] \frac{\theta}{d} - \phi_g + \phi_m, \end{aligned}$$

and the revenue is maximized at $\tau^c = \frac{1}{1 - \eta - \lambda} \left\{ (\eta + \lambda) - \left(\frac{y}{c}\right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{k}{y}\right) \right] \right\}$.

Otherwise, the total tax revenue curve for consumption tax is

- *U-shaped if $\eta + \lambda > 1$ and $\left(\frac{y}{c}\right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{k}{y}\right) \right] > \eta + \lambda$.*
- *monotonically increasing if $\eta + \lambda > 1$ and $\left(\frac{y}{c}\right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{k}{y}\right) \right] \leq \eta + \lambda$.*
- *monotonically increasing if $\eta + \lambda = 1$ and $\left(\frac{y}{c}\right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{k}{y}\right) \right] < \eta + \lambda$.*
- *flat if $\eta + \lambda = 1$ and $\left(\frac{y}{c}\right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{k}{y}\right) \right] = \eta + \lambda$.*
- *monotonically decreasing if $\eta + \lambda = 1$ and $\left(\frac{y}{c}\right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{k}{y}\right) \right] > \eta + \lambda$.*
- *monotonically decreasing if $\eta + \lambda < 1$ and $\left(\frac{y}{c}\right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{k}{y}\right) \right] \geq \eta + \lambda$.*

Proof. See Appendix F. □

Proposition 8. *Suppose that the utility function is Trabandt-Uhlig; \mathbb{U}^{TU} . The total tax revenue curve for consumption tax is monotonically increasing if and only if*

$$\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{k}{y} \right) \leq \frac{1 - \eta}{\eta} (1 - \theta)(1 - \tau^n) + (1 + \lambda) \left(\frac{c}{y} \right),$$

where

$$\begin{aligned} d &= \frac{1}{1 - \tau^k} \left[\frac{\psi^n}{\beta} - 1 + \delta \right]. \\ \frac{k}{y} &= \frac{\theta}{d}, \\ \frac{c}{y} &= 1 - [\psi - (1 - \delta)] \frac{\theta}{d} - \phi_g + \phi_m. \end{aligned}$$

Otherwise, the total tax revenue curve for consumption tax is U-shaped.

Proof. See Appendix G. □

Propositions 7 and 8 imply that there is a possibility that the total tax revenue curve might be U-shaped under some parameter values. Under this situation, the total tax revenue is decreasing and increasing if the consumption tax rate is low and sufficiently high, respectively. The U-shaped total tax revenue curve for consumption tax is generated when the labor and capital income tax rate are high. The decreases in these tax revenues associated with an increase in the consumption tax rate dominate the increase in consumption tax revenue if the consumption tax rate is low.

4 Discussion

4.1 Likelihood of a hump-shaped Laffer curve for consumption tax

According to Propositions 1, 3, 5, and 7, it is necessary for $\eta + \lambda < 1$ to generate a hump-shaped Laffer curve for consumption tax. For this condition, both η and λ should be less than one. The likelihood of this condition is discussed in this subsection.

The condition $\eta < 1$ might be supported by the empirical findings of Chetty (2006). He estimates RRA using 33 existing estimates of wage and income elasticities. The average of his estimated RRAs is 0.71, and the range of estimated RRAs is from 0.15 to 1.78 in the additive utility case.

The parameter λ should not be not restricted by evidence on the Frisch elasticity as claimed by Christiano, Trabandt, and Walentin (2010), although it is often interpreted as the inverse of Frisch elasticity, and the values are set depending on the estimations using micro data analyses. Empirical evidence from micro data implies that the Frisch elasticity is very small. However, as in the seminal works by Hansen (1985) and Rogerson (1988), even if the individual elasticity of labor supply is zero, the aggregate labor supply can be sensitive to the changes in the real wage rate. Recent papers by Keane and Rogerson (2011, 2012) claim that small micro and large macro elasticities of labor supply are consistent once human capital accumulation and the intensive and extensive margins are controlled. Christiano, Trabandt, and Walentin (2010) estimate this parameter for the U.S. economy by using Bayesian impulse response matching, and find that λ is around 0.1.

Therefore, some recent empirical evidence supports small values of η and λ . It would imply that a hump-shaped Laffer curve for consumption tax is possible.

4.2 Numerical results of the Laffer curve for consumption tax

Sections 2 and 3 characterize the shape of the Laffer curve for consumption tax and show that the Laffer curve can be hump-shaped in the case of the additively separable utility. This subsection presents some numerical results.

The parameter values are the same as those employed by Trabandt and Uhlig (2011) for the U.S. economy. The capital share in the production function θ is 0.35. The depreciation rate of capital δ is 0.083. The steady-state ratio of debt to output b/y is 0.63. The steady-state ratio of government expenditure to output g/y is 0.08. The steady-state

ratio of transfer from abroad to output m/y is 0.04. The balanced growth parameter ψ is 1.02. The steady-state real interest rate R is 1.04. The steady-state labor supply h is 0.2. The steady-state capital income tax rate is 0.36, labor income tax is 0.28, and consumption tax rate is 0.05.

Figure 1 summarizes the shape of the tax revenue curve for consumption tax in the dynamic model. The horizontal axis is η , and the vertical axis is λ . “I” denotes the region of the monotonically increasing total tax revenue curve, “D,” the region of the monotonically decreasing curve, “H,” the region of the hump-shaped curve, and “U,” the region of the U-shaped curve. At the point “F,” that is the intersection of the two lines, the total tax revenue is flat. The panels on the left and right are the cases of the additively separable utility \mathbb{U}^{AS} and the Trabandt-Uhlig utility \mathbb{U}^{TU} , respectively. The upper panels are the benchmark case with $\tau^n = 0.36$. The middle and lower panels are the cases of $\tau^n = 0.7$ and $\tau^n = 0.9$, respectively.

[Insert Figure 1]

Figure 2 shows a numerical example of the total tax revenue curves for consumption tax and components (consumption tax revenue, labor income tax revenue, and capital income tax revenue) in the dynamic model. The procedure to calculate the tax revenue curves is described in Appendix H. The horizontal axes are consumption tax rates. The vertical axes are normalized tax revenue. (The tax revenues are normalized such that the total tax revenues in the case of the baseline consumption tax rate $\tau^c = 0.05$ are 100.) The circles denote the peak tax rates that maximize the total tax revenues. The vertical dotted lines show the baseline consumption tax rate of 5%. The utility function parameters are set such that $\eta = 0.71$ and $\lambda = 0.1$. The value of η is consistent with Chetty (2006), and that of λ is consistent with the value estimated by Christiano, Trabandt, and Walentin (2010). As already shown in Sections 2 and 3, the total tax revenue curve for consumption tax is hump-shaped in the case of the additively separable utility function \mathbb{U}^{AS} , and it is monotonically increasing for the Trabandt-Uhlig utility function

\mathbb{U}^{TU} . The peak tax rate that maximizes the total tax revenue of the additively separable utility is 210.7%, whereas the consumption tax revenue is maximized at 426.32%, that is $(\eta + \lambda)/(1 - \eta - \lambda)$. The maximized total tax revenue is 143.4.

The important finding from this figure is the maximized government revenue is not so large. The government can increase their revenue by only 43% from the case of the baseline consumption tax rate.

[Insert Figure 2]

4.3 Interpretation of the finding by Kobayashi (2014)

Kobayashi (2014) find that there is a ceiling of consumption tax revenue in a neoclassical growth model with the log period utility

$$U = \log(c_t) + \gamma \log(1 - n_t). \quad (27)$$

while the consumption tax revenue curve is still monotonically increasing.

Our result can highlight his finding. In his utility, RRA is one and the labor supply elasticity is $(1 - n)/n$. Since an increase in τ^c reduces labor supply, $(1 - n)/n \rightarrow \infty$ as $\tau^c \rightarrow \infty$. Then, his log utility can be interpreted as the additively separable utility with $\eta = 1$ and $\lambda = 0$. $\eta + \lambda = 1$ implies that the elasticity of consumption goes to one as $\tau^c \rightarrow \infty$, and it is consistent with a ceiling of consumption tax revenue.

4.4 Quantitative significance of the difference in utility functions on the Laffer curves for labor and capital income taxes

Propositions 1–8 show that the difference in the functional form of the utility has significant effects on the shape of the Laffer curve for consumption tax. In this subsection, the quantitative effects of this difference on the total tax revenue curves for labor and capital income taxes are examined.

Figure 3 shows the total tax revenue curves for labor income tax in the cases of the additively separable and Trabandt-Uhlig utility functions in the dynamic model. The left-hand panel shows the case of $\eta = 2$ and $\lambda = 1$, employed by Trabandt and Uhlig (2011), and the right, of $\eta = 0.71$ and $\lambda = 0.1$, which generate the hump-shaped total tax revenue curves for consumption tax. The real lines are the total tax revenue curves in the case of the additively separable utility \mathbb{U}^{AS} . The dotted lines are the total tax revenue curves in the case of the Trabandt-Uhlig utility \mathbb{U}^{TU} . The total tax revenues are normalized such that those are 100 at the baseline labor income tax rate $\tau^n = 0.28$. The other parameter values are the same as in the previous subsection. Figure 3 tells that the difference in the utility functions has significant effects on the peak tax rates of the total tax revenue curves for labor income tax. These peak tax rates are 71.5% (additively separable utility) and 59.26% (Trabandt-Uhlig utility) for $\eta = 2$ and $\lambda = 1$. Notably, the difference in the peak tax rates is more than 10% even for Trabandt and Uhlig's (2011) parameter values. This impact is much strengthened for $\eta = 0.71$ and $\lambda = 0.1$: the peak tax rates are 39.98% (additively separable utility) and 53.33% (Trabandt-Uhlig utility).

[Insert Figure 3]

It is noticed that the peak tax rate of the total tax revenue curve for labor income tax of the additively separable utility is greater than that of the Trabandt-Uhlig utility if $\eta = 2$ and $\lambda = 1$. On the other hand, that of the additively separable utility is less than that of the Trabandt-Uhlig utility if $\eta = 0.71$ and $\lambda = 0.1$. This difference is accounted for by the difference in the equilibrium elasticity of labor supply with respect to labor income tax rate.

Remark 1. *Let the equilibrium elasticity of labor supply with respect to labor income tax rate in the case of the additively separable utility \mathbb{U}^{AS} be ε_n^{AS} , and that in the case of*

the Trabandt-Uhlig utility \mathbb{U}^{TU} be ε_n^{TU} .

$$\begin{aligned}\varepsilon_n^{AS} &< \varepsilon_n^{TU} && \text{if } \eta > 1, \\ \varepsilon_n^{AS} &\geq \varepsilon_n^{TU} && \text{if } \eta \leq 1.\end{aligned}$$

Proof. See Appendix I. □

Figure 4 shows the capital income tax analogue of Figure 3. The total tax revenues are normalized such that those are 100 at the baseline capital income tax rate $\tau^k = 0.36$. The peak tax rates in the case of $\eta = 2$ and $\lambda = 1$ are 71.11% (additively separable utility) and 59.32% (Trabandt-Uhlig utility), and the difference is more than 10% as well. This impact is much strengthened for $\eta = 0.71$ and $\lambda = 0.1$: the peak tax rates are 46.70% (additively separable utility) and 69.49% (Trabandt-Uhlig utility).

[Insert Figure 4]

Finally, Figures 3 and 4 show that the difference in the utility functions has quantitatively significant effects on the peak tax rates of the Laffer curves for labor and capital income taxes.

5 Concluding remarks

This paper has characterized the shape of the Laffer curve for consumption tax. The Laffer curve for consumption tax can be hump-shaped if the utility function is an additively separable in consumption and labor supply. On the other hand, it cannot be hump-shaped if the utility function is the one employed by Trabandt and Uhlig (2011). This is because the aggregate labor supply and consumption elasticities with respect to the consumption tax rate can be greater than one under sufficiently high parameter values of the IES and labor supply elasticity if the utility is additively separable, whereas the opposite stands when the utility is Trabandt-Uhlig.

This paper has also shown that the Laffer curve for consumption tax can be hump-shaped under empirically relevant parameter values. At the same time, the difference in the functional form of the utility has quantitatively significant effects on the peak tax rates of the Laffer curves for labor and capital income taxes.

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Appendix

A Proof of Proposition 3

Proof. The optimization condition for the consumption–labor choice,

$$\kappa(1 + \lambda)c^\eta n^\lambda = \frac{1 - \tau^n}{1 + \tau^c} w,$$

indicates that

$$c = \left[\frac{\kappa(1 + \lambda)}{1 - \tau^n} (1 + \tau^c) \right]^{-\frac{1}{\eta+\lambda}}.$$

Since the total tax revenue is

$$\begin{aligned} T &= \tau^c c + \tau^n w n \\ &= (\tau^c + \tau^n) \left[\frac{\kappa}{1 - \tau^n} (1 + \tau^c) \right]^{-\frac{1}{\eta+\lambda}}, \end{aligned}$$

then

$$\frac{dT}{d\tau^c} = \left[\frac{\kappa(1+\lambda)}{1-\tau^n} (1+\tau^c) \right]^{-\frac{1}{\eta+\lambda}-1} \left(\frac{\kappa(1+\lambda)}{1-\tau^n} \right) \left[\tau^c \left(\frac{\eta+\lambda-1}{\eta+\lambda} \right) + \frac{\eta+\lambda-\tau^n}{\eta+\lambda} \right].$$

Suppose $\eta + \lambda = 1$, then $\frac{dT}{d\tau^c} > 0$.

Suppose $\eta + \lambda \neq 1$, then

$$\frac{dT}{d\tau^c} = \left[\frac{\kappa(1+\lambda)}{1-\tau^n} (1+\tau^c) \right]^{-\frac{1}{\eta+\lambda}-1} \left(\frac{\kappa(1+\lambda)}{1-\tau^n} \right) \left(\frac{\eta+\lambda-1}{\eta+\lambda} \right) \left[\tau^c - \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda} \right].$$

If $\eta + \lambda > 1$, then $\frac{dT}{d\tau^c} > 0$.

If $\eta + \lambda < 1$, then $\frac{dT}{d\tau^c} > 0$ for $\tau^c < \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$, and $\frac{dT}{d\tau^c} < 0$ for $\tau^c > \frac{\eta+\lambda-\tau^n}{1-\eta-\lambda}$. \square

B Proof of Proposition 4

Proof. By the optimization condition for the consumption–labor choice,

$$\eta(1+\lambda) \left(\frac{\kappa c n^\lambda}{1-\kappa(1-\eta)n^{1+\lambda}} \right) = \frac{1-\tau^n}{1+\tau^c} w,$$

it follows that

$$c = (1-\tau^n)^{1/(1+\lambda)} \left[\tau^c \eta \kappa (1+\lambda) + \kappa(\eta\lambda+1) - \tau^n \kappa(1-\eta) \right]^{-1/(1+\lambda)}.$$

The total tax revenue is

$$\begin{aligned} T &= \tau^c c + \tau^n w n \\ &= (\tau^c + \tau^n) (1-\tau^n)^{1/(1+\lambda)} \left[\tau^c \eta \kappa (1+\lambda) + \kappa(\eta\lambda+1) - \tau^n \kappa(1-\eta) \right]^{-1/(1+\lambda)}. \end{aligned}$$

Then,

$$\begin{aligned} \frac{dT}{d\tau^c} &= (1-\tau^n)^{1/(1+\lambda)} \left[\tau^c \eta \kappa (1+\lambda) + \kappa(\eta\lambda+1) - \tau^n \kappa(1-\eta) \right]^{-1/(1+\lambda)-1} \\ &\quad \times \left[\tau^c \eta \kappa \lambda + \kappa(\eta\lambda+1-\tau^n) \right] > 0. \end{aligned}$$

\square

C Equilibrium system of the dynamic model

The equilibrium system of the dynamic model is

$$\begin{aligned}
(1 + \tau_t^c)\lambda_t &= u_1(c_t, n_t), \\
\lambda_t(1 - \tau_t^n)w_t &= -u_2(c_t, n_t), \\
\lambda_t &= \beta E_t \left\{ \lambda_{t+1} \left[(1 - \delta) + (1 - \tau_{t+1}^k)(d_{t+1} - \delta) + \delta \right] \right\}, \\
\lambda_t &= \beta E_t \left[\lambda_{t+1} R_{t+1}^b \right], \\
k_t &= (1 - \delta)k_{t-1} + x_t, \\
y_t &= \xi^t [k_{t-1}]^\theta n_t^{1-\theta}, \\
w_t &= (1 - \theta) \frac{y_t}{n_t}, \\
d_t &= \theta \frac{y_t}{k_{t-1}}, \\
y_t &= c_t + x_t + g_t - m_t, \\
T_t &= \tau_t^c c_t + \tau_t^n w_t n_t + \tau_t^k (d_t - \delta) k_{t-1},
\end{aligned}$$

where the marginal utilities are defined by

$$\begin{aligned}
u_1(c_t, n_t) &\equiv (c_t)^{-\eta}, \\
u_2(c_t, n_t) &\equiv -\kappa(1 + \lambda)\psi^{t(1-\gamma)}n_t^\lambda
\end{aligned}$$

if the utility function is the additively separable \mathbb{U}^{AS} , and by

$$\begin{aligned}
u_1(c_t, n_t) &\equiv (c_t)^{-\eta} \left[1 - \kappa(1 - \eta)n_t^{1+\lambda} \right]^\eta, \\
u_2(c_t, n_t) &\equiv -\eta(1 + \lambda) \left\{ (c_t)^{1-\eta} \left[1 - \kappa(1 - \eta)n_t^{1+\lambda} \right]^{\eta-1} \kappa n_t^\lambda \right\}
\end{aligned}$$

if the utility function is Trabandt-Uhlig \mathbb{U}^{TU} .

Detrend the equilibrium system by $\psi = \xi^{1/(1-\theta)}$, and let $a_t/\xi^t \equiv \tilde{a}_t$ (except for $\tilde{k}_{t-1} \equiv$

k_{t-1}/ξ^t and λ). The detrended equilibrium system is

$$\begin{aligned}
(1 + \tau_t^c)\tilde{\lambda}_t &= u_1(\tilde{c}_t, n_t), \\
\tilde{\lambda}_t(1 - \tau_t^n)\tilde{w}_t &= -u_2(\tilde{c}_t, n_t), \\
\tilde{\lambda}_t &= \beta\psi^{-\eta}E_t\left\{\tilde{\lambda}_{t+1}\left[(1 - \delta) + (1 - \tau_{t+1}^k)(d_{t+1} - \delta) + \delta\right]\right\}, \\
\tilde{\lambda}_t &= \beta\psi^{-\eta}E_t\left[\tilde{\lambda}_{t+1}R_{t+1}^b\right], \\
\psi\tilde{k}_t &= (1 - \delta)\tilde{k}_{t-1} + \tilde{x}_t, \\
\tilde{y}_t &= \left[\tilde{k}_{t-1}\right]^\theta n_t^{1-\theta}, \\
\tilde{w}_t &= (1 - \theta)\frac{\tilde{y}_t}{n_t}, \\
d_t &= \theta\frac{\tilde{y}_t}{\tilde{k}_{t-1}}, \\
\tilde{y}_t &= \tilde{c}_t + \tilde{x}_t + \tilde{g}_t - \tilde{m}_t, \\
\tilde{T}_t &= \tau_t^c\tilde{c}_t + \tau_t^n\tilde{w}_t n_t + \tau_t^k(d_t - \delta)\tilde{k}_{t-1}.
\end{aligned}$$

On the balanced growth path, the system becomes

$$\begin{aligned}
(1 + \tau^c)\tilde{\lambda} &= u_1(\tilde{c}, n), \\
\tilde{\lambda}(1 - \tau^n)\tilde{w} &= -u_2(\tilde{c}, n), \\
1 &= \beta\psi^{-\eta}\left[(1 - \delta) + (1 - \tau^k)(d - \delta) + \delta\right], \\
1 &= \beta\psi^{-\eta}R^b, \\
\psi\tilde{k} &= (1 - \delta)\tilde{k} + \tilde{x}, \\
\tilde{y} &= \left[\tilde{k}\right]^\theta n^{1-\theta}, \\
\tilde{w} &= (1 - \theta)\frac{\tilde{y}}{n}, \\
d &= \theta\frac{\tilde{y}}{\tilde{k}}, \\
\tilde{y} &= \tilde{c} + \tilde{x} + \tilde{g} - \tilde{m}_t, \\
\tilde{T} &= \tau^c\tilde{c} + \tau^n\tilde{w}n + \tau^k(d - \delta)\tilde{k}.
\end{aligned}$$

The balanced growth path values are obtained by

$$\begin{aligned}
R^b &= \frac{\psi^\eta}{\beta}, \\
d &= \frac{1}{1 - \tau^k} [R^b - 1 + \delta], \\
\frac{\tilde{k}}{\tilde{y}} &= \frac{\theta}{d}, \\
\frac{\tilde{x}}{\tilde{y}} &= [\psi - (1 - \delta)] \frac{\tilde{k}}{\tilde{y}}, \\
\frac{\tilde{c}}{\tilde{y}} &= 1 - \frac{\tilde{x}}{\tilde{y}} - \frac{\tilde{g}}{\tilde{y}} + \frac{\tilde{m}}{\tilde{y}}, \\
\frac{n}{\tilde{y}} &= \left[\frac{\tilde{y}}{\tilde{k}} \right]^{\theta/(1-\theta)}, \\
\tilde{w} &= (1 - \theta) \frac{\tilde{y}}{\tilde{n}},
\end{aligned}$$

given $\tilde{g}/\tilde{y} = \phi_g$ and $\tilde{m}/\tilde{y} = \phi_m$. From this system, the following lemma is obtained from the balanced growth path equilibrium system.

Lemma 1. *On the balanced growth path, the dividend (d), capital–output ratio ($k/y = \tilde{k}/\tilde{y}$), investment–output ratio ($x/y = \tilde{x}/\tilde{y}$), consumption–output ratio ($c/y = \tilde{c}/\tilde{y}$), and labor–output ratio (n/\tilde{y}) are independent from the consumption tax rate (τ^c).*

D Proof of Proposition 5

Proof. By the optimization condition for the consumption–labor choice,

$$\kappa(1 + \lambda)\tilde{c}^\eta n^\lambda = \frac{1 - \tau^n}{1 + \tau^c} \tilde{w},$$

it follows that

$$\tilde{y} = (1 + \tau^c)^{-1/(\eta+\lambda)} \left[\frac{1 - \theta}{\kappa(1 + \lambda)} (1 - \tau^n) \left(\frac{\tilde{c}}{\tilde{y}} \right)^{-\eta} \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/(\eta+\lambda)}.$$

Since $\tilde{c}/\tilde{y} = c/y$ and h/\tilde{y} are independent of τ^c as in Lemma 1 of Appendix C, it follows that

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} = \frac{d\tilde{y}/\tilde{y}}{d\tau^c/\tau^c} = -\frac{1}{\eta + \lambda} \cdot \frac{\tau^c}{1 + \tau^c}.$$

Then,

$$\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| - 1 = \frac{1}{\eta + \lambda} \cdot \frac{\tau^c}{1 + \tau^c} \left\{ (1 - \eta - \lambda)\tau^c - (\eta + \lambda) \right\}.$$

Suppose $\eta + \lambda = 1$. In this case, $\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| - 1 < 0$.

Suppose $\eta + \lambda \neq 1$. In this case,

$$\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| - 1 = \frac{1 - \eta - \lambda}{\eta + \lambda} \cdot \frac{\tau^c}{1 + \tau^c} \left\{ \tau^c - \frac{\eta + \lambda}{1 - \eta - \lambda} \right\}.$$

If $\eta + \lambda \geq 1$, then $\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| \leq 1$ for $\tau^c \geq 0$.

If $\eta + \lambda < 1$, then $\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| \leq 1$ for $\tau^c \leq (\eta + \lambda)/(1 - \eta - \lambda)$, and $\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| > 1$ for $\tau^c > (\eta + \lambda)/(1 - \eta - \lambda)$. \square

E Proof of Proposition 6

Proof. The optimization condition for the consumption–labor choice,

$$\eta(1 + \lambda) \left\{ \frac{\kappa \tilde{c} n^\lambda}{1 - \kappa(1 - \eta)n^{1+\lambda}} \right\} = \frac{1 - \tau^n}{1 + \tau^c} (1 - \theta) \frac{\tilde{y}}{h},$$

yields that

$$\tilde{y} = \left(\frac{\tilde{y}}{n} \right) (\kappa)^{-1/(1+\lambda)} \left[(1 - \eta) + \frac{1}{1 - \theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta(1 + \lambda) \frac{1 + \tau^c}{1 - \tau^n} \right]^{-1/(1+\lambda)}.$$

Since $\tilde{y} > 0$ for $\tau^c \geq 0$,

$$(1 - \eta) + \frac{1}{1 - \theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta(1 + \lambda) \frac{1}{1 - \tau^n} > 0.$$

Since $\tilde{c}/\tilde{y} = c/y$ is independent of τ^c as in Lemma 1 of Appendix C, it follows that

$$\frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} = \frac{d\tilde{y}/\tilde{y}}{d\tau^c/\tau^c} = -\frac{\frac{1}{1-\theta} \left(\frac{\tilde{c}}{\tilde{y}}\right) \eta \frac{\tau^c}{1-\tau^n}}{(1-\eta) + \frac{1}{1-\theta} \left(\frac{\tilde{c}}{\tilde{y}}\right) \eta \left(\frac{1+\tau^c}{1-\tau^n}\right) (1+\lambda)}.$$

Letting

$$\Psi = (1-\eta) + \frac{1}{1-\theta} \left(\frac{\tilde{c}}{\tilde{y}}\right) \eta \left(\frac{1+\tau^c}{1-\tau^n}\right) (1+\lambda) > 0,$$

it follows that

$$\left| \frac{d\tilde{c}/\tilde{c}}{d\tau^c/\tau^c} \right| - 1 = -\frac{1}{\Psi} \left\{ (1-\eta) + \frac{1}{1-\theta} \left(\frac{\tilde{c}}{\tilde{y}}\right) \eta (1+\lambda) \frac{1}{1-\tau^n} + \frac{1}{1-\theta} \left(\frac{\tilde{c}}{\tilde{y}}\right) \eta \frac{\tau^c}{1-\tau^n} \lambda \right\} < 0.$$

□

F Proof of Proposition 7

Proof. The total tax revenue is

$$\begin{aligned} \tilde{T} &= \tau^c \tilde{c} + \tau^n \tilde{w}n + \tau^k (d - \delta) \tilde{k} \\ &= \left[\tau^c \left(\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}}\right) \right] \tilde{y}. \end{aligned}$$

Since $\tilde{c}/\tilde{y} = c/y$ and $\tilde{k}/\tilde{y} = k/y$ are independent of τ^c as in Lemma 1 of Appendix C, the first-order derivative is

$$\frac{d\tilde{T}}{d\tau^c} = \left(\frac{\tilde{c}}{\tilde{y}}\right) \tilde{y} + \left[\tau^c \left(\frac{\tilde{c}}{\tilde{y}}\right) + \tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}}\right) \right] \frac{d\tilde{y}}{d\tau^c}.$$

Since

$$\tilde{y} = (1 + \tau^c)^{-1/(\eta+\lambda)} \left[\frac{1-\theta}{\kappa(1+\lambda)} (1-\tau^n) \left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta} \left(\frac{n}{\tilde{y}}\right)^{-1-\lambda} \right]^{1/(\eta+\lambda)},$$

then

$$\begin{aligned} \frac{d\tilde{T}}{d\tau^c} &= (1 + \tau^c)^{-1/(\eta+\lambda)-1} \left[\frac{1-\theta}{\kappa(1+\lambda)} (1-\tau^n) \left(\frac{\tilde{c}}{\tilde{y}}\right)^{-\eta} \left(\frac{n}{\tilde{y}}\right)^{-1-\lambda} \right]^{1/(\eta+\lambda)} \times \\ &\quad \left(\frac{\tilde{c}}{\tilde{y}}\right) \left\{ 1 + \tau^c \left(\frac{\eta + \lambda - 1}{\eta + \lambda}\right) - \frac{1}{\eta + \lambda} \left(\frac{\tilde{y}}{\tilde{c}}\right) \left[\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}}\right) \right] \right\}. \end{aligned}$$

Suppose that $\eta + \lambda = 1$. Then,

$$\frac{d\tilde{T}}{d\tau^c} = (1 + \tau^c)^{-1/(\eta+\lambda)-1} \left[\frac{1 - \theta}{\kappa(1 + \lambda)} (1 - \tau^n) \left(\frac{\tilde{c}}{\tilde{y}} \right)^{-\eta} \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/(\eta+\lambda)} \times \\ \left(\frac{\tilde{c}}{\tilde{y}} \right) \frac{1}{\eta + \lambda} \left\{ \eta + \lambda - \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \right\}.$$

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] < \eta + \lambda$, then, $\frac{d\tilde{T}}{d\tau^c} > 0$.

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] > \eta + \lambda$, then $\frac{d\tilde{T}}{d\tau^c} < 0$.

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] = \eta + \lambda$, then $\frac{d\tilde{T}}{d\tau^c} = 0$.

Suppose that $\eta + \lambda \neq 1$. It follows that

$$\frac{d\tilde{T}}{d\tau^c} = (1 + \tau^c)^{-1/(\eta+\lambda)-1} \left[\frac{1 - \theta}{\kappa(1 + \lambda)} (1 - \tau^n) \left(\frac{\tilde{c}}{\tilde{y}} \right)^{-\eta} \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/(\eta+\lambda)} \left(\frac{\tilde{c}}{\tilde{y}} \right) \left(\frac{\eta + \lambda - 1}{\eta + \lambda} \right) \times \\ \left[\tau^c - \frac{1}{\eta + \lambda - 1} \left\{ \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] - (\eta + \lambda) \right\} \right].$$

Suppose that $\eta + \lambda > 1$.

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \leq \eta + \lambda$, then $\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c \geq 0$.

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] > \eta + \lambda$,

then $\frac{d\tilde{T}}{d\tau^c} < 0$ for $\tau^c < \frac{1}{\eta+\lambda-1} \left\{ \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] - (\eta + \lambda) \right\}$,

and $\frac{d\tilde{T}}{d\tau^c} \geq 0$ for $\tau^c > \frac{1}{\eta+\lambda-1} \left\{ \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] - (\eta + \lambda) \right\}$.

Suppose that $\eta + \lambda < 1$.

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \geq \eta + \lambda$, then $\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c \geq 0$.

If $\left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] < \eta + \lambda$,

then $\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c < \frac{1}{1-\eta-\lambda} \left\{ (\eta + \lambda) - \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \right\}$,

and $\frac{d\tilde{T}}{d\tau^c} < 0$ for $\tau^c > \frac{1}{1-\eta-\lambda} \left\{ (\eta + \lambda) - \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n(1 - \theta) + \tau^k(d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \right\}$. □

G Proof of Proposition 8

Proof. The total tax revenue is

$$\tilde{T} = \tau^c \tilde{c} + \tau^n \tilde{w}n + \tau^k (d - \delta) \tilde{k} \left[\tau^c \left(\frac{\tilde{c}}{\tilde{y}} \right) + \tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \tilde{y}.$$

Since $\tilde{c}/\tilde{y} = c/y$ and $\tilde{k}/\tilde{y} = k/y$ are independent of τ^c as in Lemma 1 of Appendix C, the first-order derivative is

$$\frac{d\tilde{T}}{d\tau^c} = \left(\frac{\tilde{c}}{\tilde{y}} \right) \tilde{y} + \left[\tau^c \left(\frac{\tilde{c}}{\tilde{y}} \right) + \tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \right] \frac{d\tilde{y}}{d\tau^c}.$$

Since

$$\tilde{y} = \left(\frac{\tilde{y}}{n} \right) \kappa^{-1/(1+\lambda)} \left[(1 - \eta) + \frac{1}{1 - \theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta (1 + \lambda) \left(\frac{1 + \tau^c}{1 - \tau^n} \right) \right]^{-1/(1+\lambda)},$$

it follows that

$$\frac{d\tilde{y}}{d\tau^c} = -\frac{1}{1 - \theta} \left(\frac{\tilde{c}}{n} \right) \left(\frac{1}{1 - \tau^n} \right) \eta \kappa^{-1/(1+\lambda)} \left[(1 - \eta) + \frac{1}{1 - \theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta (1 + \lambda) \left(\frac{1 + \tau^c}{1 - \tau^n} \right) \right]^{-1/(1+\lambda)-1}.$$

Then,

$$\begin{aligned} \frac{d\tilde{T}}{d\tau^c} = & \left(\frac{\tilde{c}}{n} \right) \kappa^{-1/(1+\lambda)} \left[(1 - \eta) + \frac{1}{1 - \theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta (1 + \lambda) (1 + \tau^c) \right]^{-1/(1+\lambda)-1} \frac{1}{1 - \theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \left(\frac{1}{1 - \tau^n} \right) \eta \times \\ & \left\{ \tau^c \lambda - \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) - \frac{1 - \eta}{\eta} (1 - \theta) (1 - \tau^n) - (1 + \lambda) \left(\frac{\tilde{c}}{\tilde{y}} \right) \right] \right\}. \end{aligned}$$

If $\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) \leq \frac{1 - \eta}{\eta} (1 - \theta) (1 - \tau^n) + (1 + \lambda) \left(\frac{\tilde{c}}{\tilde{y}} \right)$, then $\frac{d\tilde{T}}{d\tau^c} \geq 0$.

If $\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) > \frac{1 - \eta}{\eta} (1 - \theta) (1 - \tau^n) + (1 + \lambda) \left(\frac{\tilde{c}}{\tilde{y}} \right)$,

then $\frac{d\tilde{T}}{d\tau^c} < 0$ for $\tau^c < \frac{1}{\lambda} \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) - \frac{1 - \eta}{\eta} (1 - \theta) (1 - \tau^n) - (1 + \lambda) \left(\frac{\tilde{c}}{\tilde{y}} \right) \right]$,

and $\frac{d\tilde{T}}{d\tau^c} > 0$ for $\tau^c > \frac{1}{\lambda} \left(\frac{\tilde{y}}{\tilde{c}} \right) \left[\tau^n (1 - \theta) + \tau^k (d - \delta) \left(\frac{\tilde{k}}{\tilde{y}} \right) - \frac{1 - \eta}{\eta} (1 - \theta) (1 - \tau^n) - (1 + \lambda) \left(\frac{\tilde{c}}{\tilde{y}} \right) \right]$.

□

H Procedure for the numerical calculations

Given the steady-state labor supply $n = 0.2$, the parameter of disutility of labor, κ , is calibrated as follows. First, the steady-state values are calculated by

$$\begin{aligned} d &= \frac{1}{1 - \tau^k} \left[\frac{\psi^\eta}{\beta} - 1 \right] + \delta, \\ \frac{\tilde{k}}{\tilde{y}} &= \frac{\theta}{d}, \\ \frac{\tilde{x}}{\tilde{y}} &= [\psi - (1 - \delta)] \frac{\tilde{k}}{\tilde{y}}, \\ \frac{\tilde{c}}{\tilde{y}} &= 1 - \frac{\tilde{x}}{\tilde{y}} - \frac{\tilde{g}}{\tilde{y}} + \frac{\tilde{m}}{\tilde{y}}, \\ \frac{n}{\tilde{y}} &= \left[\frac{\tilde{k}}{\tilde{y}} \right]^{-\theta/(1-\theta)}, \\ \tilde{y} &= n \times \left(\frac{n}{\tilde{y}} \right)^{-1}. \end{aligned}$$

If the utility is additively separable \mathbb{U}^{AS} , κ is given by

$$\kappa = \frac{1 - \theta}{\tilde{y}^{\eta+\lambda}(1 + \lambda)} \frac{1 - \tau^n}{1 + \tau^c} \left(\frac{\tilde{c}}{\tilde{y}} \right)^{-\eta} \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda}.$$

If the utility is Trabandt-Uhlig \mathbb{U}^{TU} , κ is given by

$$\kappa = \tilde{y}^{-(1+\lambda)} \left(\frac{\tilde{y}}{n} \right)^{1+\lambda} \left[(1 - \eta) + \frac{1}{1 - \theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta (1 + \lambda) \frac{1 + \tau^c}{1 - \tau^n} \right]^{-1}.$$

Given the value of κ , the output is given by

$$\tilde{y} = (1 + \tau^c)^{-1/(\eta+\lambda)} \left[\frac{1 - \theta}{\kappa(1 + \lambda)} (1 - \tau^n) \left(\frac{\tilde{c}}{\tilde{y}} \right)^{-\eta} \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/(\eta+\lambda)},$$

if the utility is additively separable \mathbb{U}^{AS} . If the utility is Trabandt-Uhlig \mathbb{U}^{TU} , the output is given by

$$\tilde{y} = \left(\frac{\tilde{y}}{n} \right) \kappa^{-1/(1+\lambda)} \left[(1 - \eta) + \frac{1}{1 - \theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta (1 + \lambda) \frac{1 + \tau^c}{1 - \tau^n} \right]^{-1/(1+\lambda)}.$$

The associated capital stock and consumption are

$$\begin{aligned}\tilde{k} &= \frac{\tilde{k}}{\tilde{y}} \times \tilde{y}, \\ \tilde{c} &= \frac{\tilde{c}}{\tilde{y}} \times \tilde{y},\end{aligned}$$

respectively. Finally, the total tax revenue is given by

$$\begin{aligned}T &= \tau^c \tilde{c} + \tau^n \tilde{w}n + \tau^k (d - \delta) \tilde{k} \\ &= \tau^c \tilde{c} + \tau^n (1 - \theta) \tilde{y} + \tau^k (d - \delta) \tilde{k}.\end{aligned}$$

I Proof of Remark 1

Suppose that the additively separable utility \mathbb{U}^{AS} . In this case, as in Appendix D,

$$\tilde{y} = (1 + \tau^c)^{-1/(\eta+\lambda)} \left[\frac{1 - \theta}{\kappa(1 + \lambda)} (1 - \tau^n) \left(\frac{\tilde{c}}{\tilde{y}} \right)^{-\eta} \left(\frac{n}{\tilde{y}} \right)^{-1-\lambda} \right]^{1/(\eta+\lambda)}.$$

Since \tilde{y}/n is independent from τ^n as in Lemma 1, the equilibrium elasticity of labor supply with respect to labor income tax rate is

$$\varepsilon_n^{AS} = \left| \frac{dn/n}{d\tau^n/d\tau^n} \right| = \left| \frac{d\tilde{y}/\tilde{y}}{d\tau^n/d\tau^n} \right| = \frac{1}{\eta + \lambda} \cdot \frac{\tau^n}{1 - \tau^n}.$$

It is easily found that $\left| \frac{dn/n}{d\tau^n/d\tau^n} \right|$ is increasing in τ^n . $\left| \frac{dn/n}{d\tau^n/d\tau^n} \right| = 0$ if $\tau^n = 0$, and $\left| \frac{dn/n}{d\tau^n/d\tau^n} \right| \rightarrow \infty$ if $\tau^n \rightarrow 1$.

Suppose that the Traband-Uhlig utility \mathbb{U}^{TU} , In this case, the equilibrium output is given by

$$\tilde{y} = \left(\frac{\tilde{y}}{n} \right) (\kappa)^{-1/(1+\lambda)} \left[(1 - \eta) + \frac{1}{1 - \theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta (1 + \lambda) \frac{1 + \tau^c}{1 - \tau^n} \right]^{-1/(1+\lambda)},$$

as in Appendix E. Since \tilde{y}/n is independent from τ^n as in Lemma 1, the equilibrium elasticity of labor supply with respect to labor income tax rate is

$$\varepsilon_n^{TU} = \left| \frac{dn/n}{d\tau^n/d\tau^n} \right| = \left| \frac{d\tilde{y}/\tilde{y}}{d\tau^n/d\tau^n} \right| = \frac{1}{1 + \lambda} \cdot \frac{\tau^n}{1 - \tau^n} \cdot \frac{\frac{1}{1-\theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta (1 + \lambda) \frac{1+\tau^c}{1-\tau^n}}{(1 - \eta) + \frac{1}{1-\theta} \left(\frac{\tilde{c}}{\tilde{y}} \right) \eta (1 + \lambda) \frac{1+\tau^c}{1-\tau^n}}$$

Then, $\left| \frac{dn/n}{d\tau^n/\tau^n} \right|$ is increasing in τ^n . $\left| \frac{dn/n}{d\tau^n/\tau^n} \right| = 0$ if $\tau^n = 0$, and $\left| \frac{dn/n}{d\tau^n/\tau^n} \right| \rightarrow \infty$ if $\tau^n \rightarrow 1$.

Suppose that $\eta > 1$. In this case,

$$\frac{1}{\eta + \lambda} < \frac{1}{1 + \lambda} < \frac{1}{1 + \lambda} \cdot \frac{\frac{1}{1-\theta} \left(\frac{\varepsilon}{y}\right) \eta(1 + \lambda)^{\frac{1+\tau^c}{1-\tau^n}}}{(1 - \eta) + \frac{1}{1-\theta} \left(\frac{\varepsilon}{y}\right) \eta(1 + \lambda)^{\frac{1+\tau^c}{1-\tau^n}}}.$$

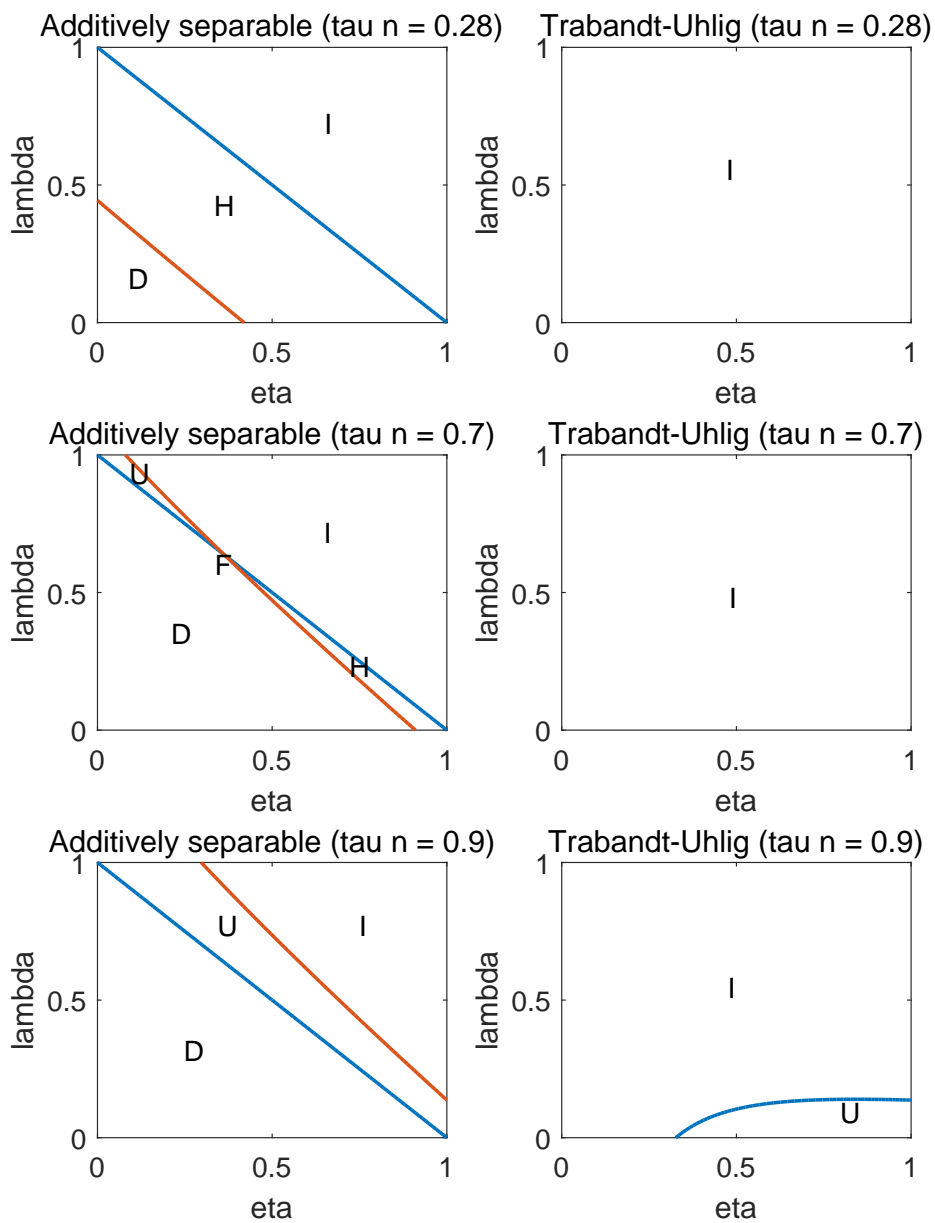
Then, $\varepsilon_n^{TU} > \varepsilon_n^{AS}$.

Suppose that $\eta \leq 1$. In this case,

$$\frac{1}{\eta + \lambda} \geq \frac{1}{1 + \lambda} \geq \frac{1}{1 + \lambda} \cdot \frac{\frac{1}{1-\theta} \left(\frac{\varepsilon}{y}\right) \eta(1 + \lambda)^{\frac{1+\tau^c}{1-\tau^n}}}{(1 - \eta) + \frac{1}{1-\theta} \left(\frac{\varepsilon}{y}\right) \eta(1 + \lambda)^{\frac{1+\tau^c}{1-\tau^n}}}.$$

Then, $\varepsilon_n^{TU} \leq \varepsilon_n^{AS}$.

Figure 1: Shape of the total tax revenue curve for consumption tax



Note - I: monotonically increasing, H: hump-shaped, D: monotonically decreasing, U: U-shaped, F: flat.

Figure 2: Total tax revenue curve for consumption tax: $\eta = 0.71$ and $\lambda = 1/10$

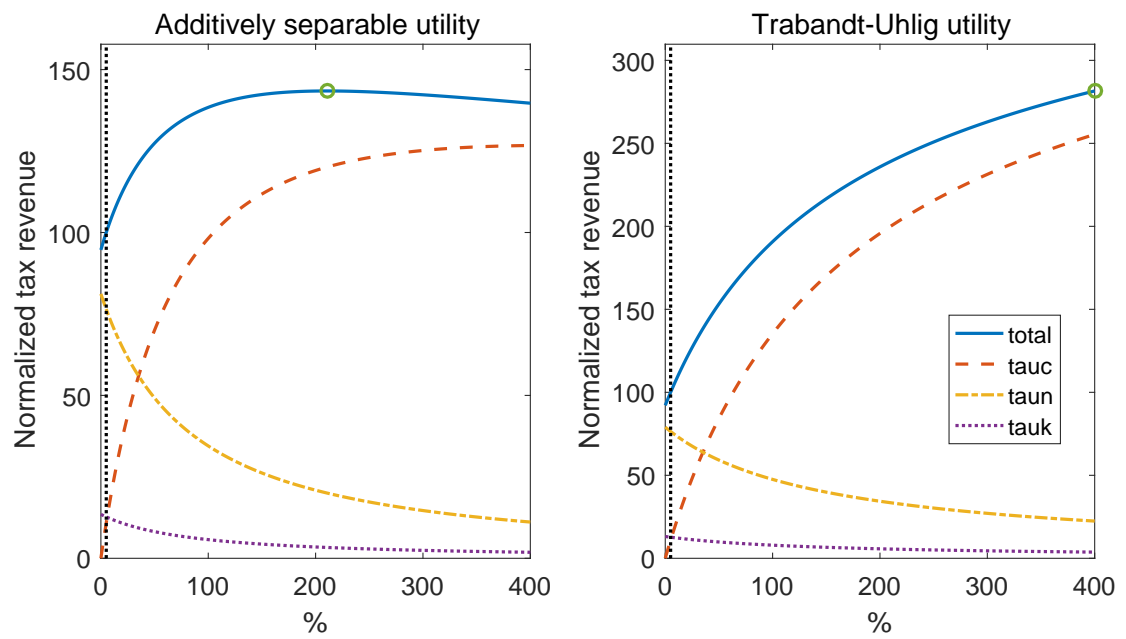


Figure 3: Total tax revenue curves for labor income tax

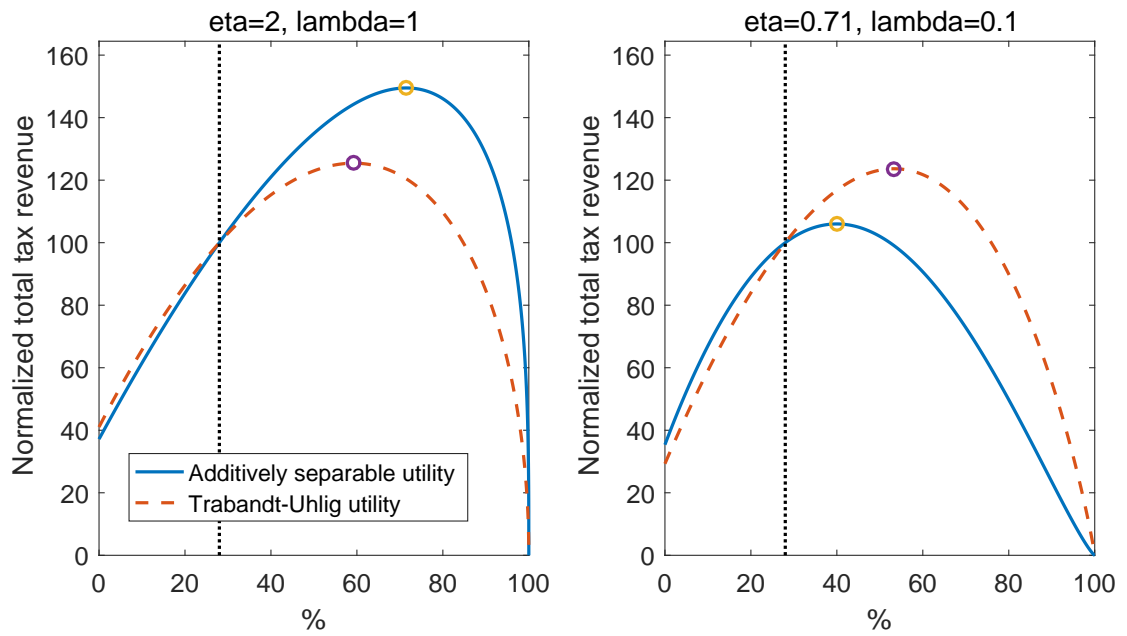


Figure 4: Total tax revenue curves for capital income tax

