# Stagnation Traps

Gianluca Benigno and Luca Fornaro*

November 2015

**Abstract**

We provide a Keynesian growth theory in which pessimistic expectations can lead to very persistent, or even permanent, slumps characterized by unemployment and weak growth. We refer to these episodes as *stagnation traps*, because they consist in the joint occurrence of a liquidity and a growth trap. In a stagnation trap, the central bank is unable to restore full employment because weak growth pushes the interest rate against the zero lower bound, while growth is weak because low aggregate demand results in low profits, limiting firms' investment in innovation. Policies aiming at restoring growth can successfully lead the economy out of a stagnation trap, thus rationalizing the notion of job creating growth.

*JEL Codes:* E32, E43, E52, O42.

*Keywords:* Secular Stagnation, Liquidity Traps, Growth Traps, Endogenous Growth, Multiple Equilibria.

*Benigno: London School of Economics, CEPR, and Centre for Macroeconomics; G.Benigno@lse.ac.uk. Fornaro: CREI, Universitat Pompeu Fabra, Barcelona GSE and CEPR; LFornaro@crei.cat. We would like to thank Philippe Aghion, Pierpaolo Benigno, Javier Bianchi, Francisco Buera, Roger Farmer, Andrea Ferrero, Jordi Gali, Jang-Ting Guo, Bob Hall, Chad Jones, Tommaso Monacelli, Alwyn Young and seminar participants at the HECER/Bank of Finland, University of St. Andrews, Federal Reserve Board, IMF, CREI, Kent University, Queen Mary University, ECB-CFS-Bundesbank, Federal Reserve Bank of Minneapolis, LSE and participants at the RIDGE workshop on Financial Crises, Federal Reserve Bank of San Francisco conference on the New Normal for Monetary Policy, ESSIM, NBER Conference on Multiple Equilibria and Financial Crises, Riksbank conference on Deflation - Causes, Consequences and Policies, SED Meeting 2015, Barcelona GSE Summer Forum 2015, Sciences Po Summer Workshop and EUI-CEPR-IMF conference on Secular Stagnation for useful comments. We thank Julia Faltermeier, Andresa Lagerborg and Martin Wolf for excellent research assistance. This research has been supported by ESRC grant ES/I024174/1, the Spanish Ministry of Science and Innovation (grant ECO2011-23192) and the Fondation Banque de France Research Grant. The contribution by Luca Fornaro to this paper has been prepared under the Lamfalussy Fellowship Program sponsored by the European Central Bank. Any views expressed are only those of the authors and do not necessarily represent the views of the ECB or the Eurosystem.

## 1 Introduction

Can insufficient aggregate demand lead to economic stagnation, i.e. a protracted period of low growth and high unemployment? Economists have been concerned with this question at least since the Great Depression, but recently interest in this topic has reemerged motivated by the two decades-long slump affecting Japan since the early 1990s, as well as by the slow recoveries characterizing the US and the Euro area in the aftermath of the 2008 financial crisis. Indeed, all these episodes have been characterized by long-lasting slumps in the context of policy rates at, or close to, their zero lower bound, leaving little room for conventional monetary policy to stimulate demand. Moreover, during these episodes potential output growth has been weak, resulting in large deviations of output from pre-slump trends.[1]

In this paper we present a theory in which very persistent, or even permanent, slumps characterized by unemployment and weak growth are possible. Our idea is that the connection between depressed demand, low interest rates and weak growth, far from being casual, might be the result of a two-way interaction. On the one hand, unemployment and weak aggregate demand might have a negative impact on firms' investment in innovation, and result in low growth. On the other hand, low growth might depress the real interest rates and push nominal rates close to their zero lower bound, thus undermining the central bank's ability to maintain full employment by cutting policy rates.

To formalize this insight, and explore its policy implications, we propose a *Keynesian growth* framework that sheds lights on the interactions between endogenous growth and liquidity traps. The backbone of our framework is a standard model of vertical innovation, in the spirit of Aghion and Howitt (1992) and Grossman and Helpman (1991). We modify this classic endogenous growth framework in two directions. First, we introduce nominal wage rigidities, which create the possibility of involuntary unemployment, and give rise to a channel through which monetary policy can affect the real economy.[2] Second, we take into account the zero lower bound on the nominal interest rate, which limits the central bank's ability to stabilize the economy with conventional monetary policy. Our theory thus combines the Keynesian insight that unemployment might arise due to weak aggregate demand, with the notion, developed by the endogenous growth literature,

[1] Ball (2014) estimates the long-run consequences of the 2008 global financial crisis in several countries and documents significant losses in terms of potential output. Christiano et al. (2015) find that the US Great Recession has been characterized by a very persistent fall in total factor productivity below its pre-recession trend. Cerra and Saxena (2008) analyse the long-run impact of deep crises, and find, using a large sample of countries, that crises are often followed by permanent negative deviations from pre-crisis trends. A similar conclusion is reached by Blanchard et al. (2015), who also find that recessions are in many cases followed by a slowdown in the growth rate of the economy.

[2] A growing body of evidence emphasizes how nominal wage rigidities represent an important transmission channel through which monetary policy affects the real economy. For instance, this conclusion is reached by Christiano et al. (2005) using an estimated medium-scale DSGE model of the US economy, and by Olivei and Tenreyro (2007), who show that monetary policy shocks in the US have a bigger impact on output in the aftermath of the season in which wages are adjusted. Eichengreen and Sachs (1985) and Bernanke and Carey (1996) describe the role of nominal wage rigidities in exacerbating the downturn during the Great Depression. Similarly, Schmitt-Grohé and Uribe (2011) document the importance of nominal wage rigidities for the 2001 Argentine crisis and for the 2008-2009 recession in the Eurozone periphery. Micro-level evidence on the importance of nominal wage rigidities is provided by Fehr and Goette (2005), Gottschalk (2005), Barattieri et al. (2010) and Fabiani et al. (2010).

that productivity growth is the result of investment in innovation by profit-maximizing agents. We show that the interaction between these two forces can give rise to prolonged periods of low growth and high unemployment. We refer to these episodes as *stagnation traps*, because they consist in the joint occurrence of a liquidity and a growth trap.

In our economy there are two types of agents: firms and households. Firms' investment in innovation determines endogenously the growth rate of productivity and potential output of our economy. As in the standard models of vertical innovation, firms invest in innovation to gain a monopoly position, and so their investment in innovation is positively related to profits. Through this channel, a slowdown in aggregate demand that leads to a fall in profits, also reduces investment in innovation and the growth rate of the economy. Households supply labor and consume, and their intertemporal consumption pattern is characterized by the traditional Euler equation. The key aspect is that households' current demand for consumption is affected by the growth rate of potential output, because productivity growth is one of the determinants of households' future income. Hence, a low growth rate of potential output is associated with lower future income and a reduction in current aggregate demand.

This two-way interaction between productivity growth and aggregate demand results in two steady states. First, there is a full employment steady state, in which the economy operates at potential and productivity growth is robust. However, our economy can also find itself in an unemployment steady state. In the unemployment steady state aggregate demand and firms' profits are low, resulting in low investment in innovation and weak productivity growth. Moreover, monetary policy is not able to bring the economy at full employment, because the low growth of potential output pushes the interest rate against its zero lower bound. Hence, the unemployment steady state can be thought of as a stagnation trap.

Expectations, or animal spirits, are crucial in determining which equilibrium will be selected. For instance, when agents expect growth to be low, expectations of low future income reduce aggregate demand, lowering firms' profits and their investment, thus validating the low growth expectations. Through this mechanism, pessimistic expectations can generate a permanent liquidity trap with involuntary unemployment and stagnation. We also show that, aside from permanent liquidity traps, pessimistic expectations can give rise to liquidity traps of finite, but arbitrarily long, expected duration.

We then examine the policy implications of our framework. First we study optimal interest rate policy. We show that a central bank operating under commitment can design interest rate rules that eliminate the possibility of stagnation traps. However, we also show that if the central bank lacks the ability to commit to its future actions stagnation traps are possible even when interest rates are set optimally. We then turn to policies aiming at sustaining the growth rate of potential output, by subsidizing investment in productivity enhancing activities. While these policies have been studied extensively in the context of the endogenous growth literature, here we show that they operate not only through the supply side of the economy, but also by stimulating aggregate demand during a liquidity trap. In fact, we show that an appropriately designed subsidy to innovation can push the economy out of a stagnation trap and restore full employment, thus capturing the notion of job creating growth. However, our framework suggests that, in order to be effective, the subsidy to innovation has to be sufficiently aggressive, so as to provide a "big push" to the economy.

This paper is related to several strands of the literature. First, the paper is related to Hansen's secular stagnation hypothesis (Hansen, 1939), that is the idea that a drop in the real natural interest rate might push the economy in a long-lasting liquidity trap, characterized by the absence of any self-correcting force to restore full employment. Hansen formulated this concept inspired by the US Great Depression, but recently some researchers, most notably Summers (2013) and Krugman (2013), have revived the idea of secular stagnation to rationalize the long duration of the Japanese liquidity trap and the slow recoveries characterizing the US and the Euro area after the 2008 financial crisis. To the best of our knowledge, the only existing framework in which permanent liquidity traps are possible due to a fall in the real natural interest rate has been provided by Eggertsson and Mehrotra (2014).[3] However, the source of their liquidity trap is very different from ours. In their framework, liquidity traps are generated by shocks that alter households' lifecycle saving decisions. Instead, in our framework the drop in the real natural interest rate that generates a permanent liquidity trap originates from an endogenous drop in investment in innovation and productivity growth.

Second, our paper is related to the literature on poverty and growth traps. This literature discusses several mechanisms through which a country can find itself permanently stuck with inefficiently low growth. Examples of this literature are Murphy et al. (1989), Matsuyama (1991), Galor and Zeira (1993) and Azariadis (1996).[4] Different from these contributions, we show that a liquidity trap can be the driver of a growth trap. Indeed, the intimate connection between the two traps lead us to put forward the notion of stagnation traps.

As in the seminal frameworks presented by Aghion and Howitt (1992), Grossman and Helpman (1991) and Romer (1990), long-run growth in our model is the result of investment in innovation by profit-maximizing agents. A small, but growing, literature has considered the interactions between short-run fluctuation and long run growth in this class of models (Fatas, 2000; Barlevy, 2004; Comin and Gertler, 2006; Aghion et al., 2010; Nuño, 2011; Queraltó, 2013), as well as some of the implications for fiscal or monetary policy (Aghion et al., 2009, 2014; Chu and Cozzi, 2014). However, to the best of our knowledge, we are the first ones to study monetary policy in an endogenous growth model featuring a zero lower bound constraint on the policy rate, and to show that the interaction between endogenous growth and monetary policy creates the possibility of

[3]The literature studying liquidity traps in micro-founded models has traditionally focused on slumps generated by ad-hoc preference shocks, as in Krugman (1998), Eggertsson and Woodford (2003), Eggertsson (2008) and Werning (2011), or by financial shocks leading to tighter access to credit, as in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011). In all these frameworks liquidity traps are driven by a temporary fall in the natural interest rate, and permanent liquidity traps are not possible. Benhabib et al. (2001) show that, when monetary policy is conducted through a Taylor rule, changes in inflation expectations can give rise to permanent liquidity traps. However, in their framework the real natural interest rate during a permanent liquidity trap is equal to the one prevailing in the full employment equilibrium.

[4]See Azariadis and Stachurski (2005) for an excellent survey of this literature.

long periods of stagnation.

Finally, our paper is linked to the literature on fluctuations driven by confidence shocks and sunspots. Some examples of this vast literature are Kiyotaki (1988), Benhabib and Farmer (1994, 1996), Francois and Lloyd-Ellis (2003), Farmer (2012) and Bacchetta and Van Wincoop (2013). We contribute to this literature by describing a new channel through which pessimistic expectations can give rise to economic stagnation.

The rest of the paper is composed of four sections. Section 2 describes the baseline model. Section 3 shows that pessimistic expectations can generate arbitrarily long lasting stagnation traps. Section 4 extends the baseline model in several directions. Section 4 discusses some policy implications . Section 5 concludes.

# 2 Baseline Model

Consider an infinite-horizon closed economy. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. The economy is inhabited by households, firms, and by a central bank that sets monetary policy.

## 2.1 Households

There is a continuum of measure one of identical households deriving utility from consumption of a homogeneous "final" good. The lifetime utility of the representative household is:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - 1}{1 - \sigma} \right) \right], \tag{1}$$

where $C_t$ denotes consumption, $0 < \beta < 1$ is the subjective discount factor, $\sigma$ is the inverse of the elasticity of intertemporal substitution, and $E_t[\cdot]$ is the expectation operator conditional on information available at time $t$.

Each household is endowed with one unit of labor and there is no disutility from working. However, due to the presence of nominal wage rigidities to be described below, a household might be able to sell only $L_t < 1$ units of labor on the market. Hence, when $L_t = 1$ the economy operates at full employment, while when $L_t < 1$ there is involuntary unemployment, and the economy operates below capacity.

Households can trade in one-period, non-state contingent bonds $b_t$. Bonds are denominated in units of currency and pay the nominal interest rate $i_t$. Moreover, households own all the firms and each period they receive dividends $d_t$ from them.

The intertemporal problem of the representative household consists in choosing $C_t$ and $b_{t+1}$ to maximize expected utility, subject to a no-Ponzi constraint and the budget constraint:

$$P_t C_t + \frac{b_{t+1}}{1 + i_t} = W_t L_t + b_t + d_t,$$

where $P_t$ is the nominal price of the final good, $b_{t+1}$ is the stock of bonds purchased by the household in period $t$, and $b_t$ is the payment received from its past investment in bonds. $W_t$ denotes the nominal wage, so that $W_t L_t$ is the household's labor income.

The optimality conditions are:

$$\lambda_t = \frac{C_t^{-\sigma}}{P_t} \tag{2}$$

$$\lambda_t = \beta(1+i_t)E_t\left[\lambda_{t+1}\right], \tag{3}$$

where $\lambda_t$ denotes the Lagrange multiplier on the budget constraint, and the transversality condition $\lim_{s\to\infty} E_t\left[\frac{b_{t+s}}{(1+i_t)...(1+i_{t+s})}\right] = 0$.

## 2.2 Final good production

The final good is produced by competitive firms using labor and a continuum of measure one of intermediate inputs $x_j$, indexed by $j \in [0,1]$. Denoting by $Y_t$ the output of final good, the production function is:

$$Y_t = L_t^{1-\alpha} \int_0^1 A_{jt}^{1-\alpha} x_{jt}^{\alpha} dj, \tag{4}$$

where $0 < \alpha < 1$, and $A_{jt}$ is the productivity, or quality, of input $j$.[5]

Profit maximization implies the demand functions:

$$P_t(1-\alpha)L_t^{-\alpha} \int_0^1 A_{jt}^{1-\alpha} x_{jt}^{\alpha} di = W_t \tag{5}$$

$$P_t \alpha L_t^{1-\alpha} A_{jt}^{1-\alpha} x_{jt}^{\alpha-1} = P_{jt}, \tag{6}$$

where $P_{jt}$ is the nominal price of intermediate input $j$. Due to perfect competition, firms in the final good sector do not make any profit in equilibrium.

## 2.3 Intermediate goods production and profits

In every industry $j$ producers compete as price-setting oligopolists. One unit of final output is needed to manufacture one unit of intermediate good, regardless of quality, and hence every producer faces the same marginal cost $P_t$. Our assumptions about the innovation process will ensure that in every industry there is a single leader able to produce good $j$ of quality $A_{jt}$, and a fringe of competitors which are able to produce a version of good $j$ of quality $A_{jt}/\gamma$. The parameter $\gamma > 1$ captures the distance in quality between the leader and the followers. Given this market structure, it is optimal for the leader to capture the whole market for good $j$ by charging the price:[6]

$$P_{jt} = \xi P_t \quad \text{where} \quad \xi \equiv \min\left(\gamma^{\frac{\alpha}{1-\alpha}}, \frac{1}{\alpha}\right) > 1. \tag{7}$$

[5] More precisely, for every good $j$, $A_{jt}$ represents the highest quality available. In principle, firms could produce using a lower quality of good $j$. However, as in Aghion and Howitt (1992) and Grossman and Helpman (1991), the structure of the economy is such that in equilibrium only the highest quality version of each good is used in production.

[6] For a detailed derivation see, for instance, the appendix to chapter 7 of Barro and Sala-i Martin (2004).

This expression implies that the leader charges a constant markup $\xi$ over its marginal cost. Intuitively, $1/\alpha$ is the markup that the leader would choose in absence of the threat of entry from the fringe of competitors. Instead, $\gamma^{\alpha/(1-\alpha)}$ is the highest markup that the leader can charge without losing the market to its competitors. It follows that if $1/\alpha \leq \gamma^{\alpha/(1-\alpha)}$ then the leader will charge the unconstrained markup $1/\alpha$, otherwise it will set a markup equal to $\gamma^{\alpha/(1-\alpha)}$ to deter entry. In any case, the leader ends up satisfying all the demand for good $j$ from final good producers.

Expressions (6) and (7) imply that the quantity produced of a generic intermediate good $j$ is:

$$x_{jt} = \left(\frac{\alpha}{\xi}\right)^{\frac{1}{1-\alpha}} A_{jt} L_t. \tag{8}$$

Combining expressions (4) and (8) gives:

$$Y_t = \left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}} A_t L_t, \tag{9}$$

where $A_t \equiv \int_0^1 A_{jt} dj$ is an index of average productivity of the intermediate inputs. Hence, production of the final good is increasing in the average productivity of intermediate goods and in aggregate employment. Moreover, the profits earned by the leader in sector $j$ are given by:

$$P_{jt} x_{jt} - P_t x_{jt} = P_t \varpi A_{jt} L_t$$

$$\text{where} \quad \varpi \equiv (\xi - 1) \left(\frac{\alpha}{\xi}\right)^{\frac{1}{1-\alpha}}.$$

According to this expression, a leader's profits are increasing in the productivity of its intermediate input and on aggregate employment. The dependence of profits from aggregate employment is due to the presence of a market size effect. Intuitively, high employment is associated with high production of the final good and high demand for intermediate inputs, leading to high profits in the intermediate sector.

### 2.4 Research and innovation

There is a large number of entrepreneurs that can attempt to innovate upon the existing products. A successful entrepreneur researching in sector $j$ discovers a new version of good $j$ of quality $\gamma$ times greater than the best existing version, and becomes the leader in the production of good $j$.[7]

Entrepreneurs can freely target their research efforts at any of the continuum of intermediate goods. An entrepreneur that invests $I_{jt}$ units of the final good to discover an improved version of product $j$ innovates with probability:

$$\mu_{jt} = \min\left(\frac{\chi I_{jt}}{A_{jt}}, 1\right),$$

[7] As in Aghion and Howitt (1992) and Grossman and Helpman (1991), all the research activities are conducted by entrants. Incumbents do not perform any research because the value of improving over their own product is smaller than the profits that they would earn from developing a leadership position in a second market.

where the parameter $\chi > 0$ determines the productivity of research.[8] The presence of the term $A_{jt}$ captures the idea that innovating upon more advanced and complex products requires a higher investment, and ensures stationarity in the growth process. We consider time periods small enough so that the probability that two or more entrepreneurs discover contemporaneously an improved version of the same product is negligible. This implies that the probability that a product is improved is the sum of the success probabilities of all the entrepreneurs targeting that product.[9] With a slight abuse of notation, we then denote by $\mu_{jt}$ the probability that an improved version of good $j$ is discovered at time $t$.

We now turn to the reward from research. A successful entrepreneur obtains a patent and becomes the monopolist during the following period. For simplicity, in our baseline model we assume that the monopoly position of an innovator lasts a single period, after which the patent is allocated randomly to another entrepreneur.[10] The value $V_t(\gamma A_{jt})$ of becoming a leader in sector $j$ and attaining productivity $\gamma A_{jt}$ is given by:

$$V_t(\gamma A_{jt}) = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} P_{t+1} \varpi \gamma A_{jt} L_{t+1} \right]. \tag{10}$$

$V_t(\gamma A_{jt})$ is equal to the expected profits to be gained in period $t+1$, $P_{t+1}\varpi\gamma A_{jt}L_{t+1}$, discounted using the households' discount factor $\beta\lambda_{t+1}/\lambda_t$. Profits are discounted using the households' discount factor because entrepreneurs finance their investment in innovation by selling equity claims on their future profits to the households. Competition for households' funds leads entrepreneurs to maximize the value to the households of their expected profits.

Free entry into research implies that expected profits from researching cannot be positive, so that for every good $j$:[11]

$$P_t \geq \frac{\chi}{A_{jt}} V_t(\gamma A_{jt}),$$

holding with equality if some research is conducted aiming at improving product $j$.[12] Combining

[8] Our formulation of the innovation process follows closely chapter 7 of Barro and Sala-i Martin (2004) and Howitt and Aghion (1998). An alternative is to assume, as in Grossman and Helpman (1991), that labor is used as input into research. This alternative assumption would lead to identical results, since ultimately output in our model is fully determined by the stock of knowledge and aggregate labor.

[9] Alternatively, we could have assumed that every period only a single entrepreneur has the chance to invest in research in a given sector.

[10] This assumption, which is drawn from Aghion and Howitt (2009) and Acemoglu et al. (2012), simplifies considerably the analysis. In section 4.3 we show that our results extend to a setting in which, more conventionally, the innovator's monopoly position lasts until a new version of the product is discovered.

[11] To derive this condition, consider that an entrepreneur that invests $I_{jt}$ in research has a probability $\chi I_{jt}/A_{jt}$ of becoming a leader which carries value $V_t(\gamma A_{jt})$. Hence, the expected return from this investment is $\chi I_{jt} V_t(\gamma A_{jt})/A_{jt}$. Since the investment costs $P_t I_{jt}$, the free entry condition in the research sector implies:

$$P_t I_{jt} \geq \frac{\chi I_{jt}}{A_{jt}} V_t(\gamma A_{jt}).$$

Simplifying we obtain the expression in the main text.

[12] It is customary in the endogenous growth literature to restrict attention to equilibria in which in every period a positive amount of research is targeted toward every intermediate good. We take a slightly more general approach, and allow for cases in which expected profits from research are too low to induce entrepreneurs to invest in innovation. This degree of generality will prove important when we will discuss the policy implications of the framework.

this condition with expression (38) gives:

$$\frac{P_t}{\chi} \geq \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} P_{t+1} \gamma \varpi L_{t+1} \right].$$

Notice that this condition does not depend on any variable specific to sector $j$, because the higher profits associated with more advanced sectors are exactly offset by the higher research costs. As is standard in the literature, we then focus on symmetric equilibria in which the probability of innovation is the same in every sector, so that $\mu_{jt} = \chi I_{jt}/A_{jt} = \mu_t$ for every $j$. We can then summarize the equilibrium in the research sector with the complementary slackness condition:

$$\mu_t \left( \frac{P_t}{\chi} - \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} P_{t+1} \gamma \varpi L_{t+1} \right] \right) = 0. \tag{11}$$

Intuitively, either some research is conducted, so that $\mu_t > 0$, and free entry drives expected profits in the research sector to zero, or the expected profits from researching are negative and no research is conducted, so that $\mu_t = 0$.

### 2.5 Aggregation and market clearing

Market clearing for the final good implies:[13]

$$Y_t - \int_0^1 x_{jt} dj = C_t + \int_0^1 I_{jt} dj, \tag{12}$$

where the left-hand side of this expression is the GDP of the economy, while the right-hand side captures the fact that all the GDP has to be consumed or invested in research. Using equations (8) and (9) we can write GDP as:

$$Y_t - \int_0^1 x_{jt} dj = \left( \frac{\alpha}{\xi} \right)^{\frac{\alpha}{1-\alpha}} \left( 1 - \frac{\alpha}{\xi} \right) A_t L_t. \tag{13}$$

The assumption of a unitary labor endowment implies that $L_t \leq 1$. Since labor is supplied inelastically by the households, $1 - L_t$ can be interpreted as the unemployment rate. For future reference, when $L_t = 1$ we say that the economy is operating at potential, while when $L_t < 1$ the economy operates below capacity and there is a negative output gap.

Long run growth in this economy takes place through increases in the quality of the intermediate goods, captured by increases in the productivity index $A_t$. By the law of large numbers, a fraction

[13] The goods market clearing condition can be derived combining the households' budget constraint, with the expression for firms' profits:

$$d_t = \underbrace{P_t Y_t - W_t L_t - \xi P_t \int_0^1 x_{jt} dj}_{\text{profits from final sector}} + \underbrace{(\xi - 1) P_t \int_0^1 x_{jt} dj - P_t \int_0^1 I_{jt} dj}_{\text{profits from intermediate sector}},$$

where profits are net of research expenditure, and the equilibrium condition $b_{t+1} = 0$, deriving from the assumption of identical households.

$\mu_t$ of intermediate products is improved every period. Hence $A_t$ evolves according to:

$$A_{t+1} = \mu_t \gamma A_t + (1 - \mu_t) A_t,$$

while the (gross) rate of productivity growth is:

$$g_{t+1} \equiv \frac{A_{t+1}}{A_t} = \mu_t (\gamma - 1) + 1. \tag{14}$$

Recalling that $\mu_t = \chi I_{jt}/A_{jt}$, this expression implies that higher investment in research in period $t$ is associated with faster productivity growth between periods $t$ and $t+1$.

### 2.6 Wages, prices and monetary policy

Following a tradition that goes back at least to Keynes' *General Theory*, we consider frictions in the adjustment of nominal wages. The presence of nominal wage rigidities plays two roles in our analysis. First, it creates the possibility of involuntary unemployment, by ensuring that nominal wages are positive even in presence of unemployment.[14] Second, it opens the door to a stabilization role for monetary policy. Indeed, as we will see, prices inherit part of wage stickiness, so that the central bank can affect the real interest rate of the economy through movements in the nominal interest rate.

In our baseline model, we consider the simplest possible form of nominal wage rigidities and assume that wages evolve according to:

$$W_t = \bar{\pi}^w W_{t-1}. \tag{15}$$

This expression implies that nominal wage inflation is constant and equal to $\bar{\pi}^w$, and could be derived from the presence of large menu costs from deviating from the constant wage inflation path.

To be clear, our results do not rely at all on this extreme form of wage stickiness. Indeed, in section 4.2 we generalize our results to an economy in which wages are allowed to respond to fluctuations in employment, giving rise to a wage Phillips curve. However, considering an economy with constant wage inflation simplifies considerably the analysis, and allows us to characterize transparently the key economic forces at the heart of the model.

Turning to prices, combining equations (5) and (8) gives:

$$P_t = \frac{1}{1-\alpha} \left( \frac{\xi}{\alpha} \right)^{\frac{\alpha}{1-\alpha}} \frac{W_t}{A_t}.$$

Intuitively, prices are increasing in the marginal cost of firms producing the final good. An increase in wages puts upward pressure on marginal costs and leads to a rise in prices, while a rise in

[14] In fact, since households do not experience disutility from working, if nominal wages were fully flexible any wages would fall to zero if $L_t < 1$.

productivity reduces marginal costs and prices. This expression, combined with the law of motion for wages, can be used to derive an equation for price inflation:

$$\pi_t \equiv \frac{P_t}{P_{t-1}} = \frac{\bar{\pi}^w}{g_t}, \tag{16}$$

which implies that price inflation is increasing in wage inflation and decreasing in productivity growth.

The central bank implements its monetary policy stance by setting the nominal interest rate according to the truncated Taylor rule:

$$1 + i_t = \max\left((1+\bar{i})\, L_t^{\phi}, 1\right),$$

where $\bar{i} \geq 0$ and $\phi > 0$. Under this rule the central bank tries to stabilize output around its potential level by cutting the interest rate in response to falls in employment. The nominal interest rate is subject to a zero lower bound constraint, which, as we show in appendix *B*, can be derived from standard arbitrage between money and bonds.

We make the following assumptions about monetary policy and wage inflation to guarantee the existence and local determinacy of a steady state in which the economy operates at full employment.

**Assumption 1** *The parameters $\bar{i}$, $\bar{\pi}^w$ and $\phi$ satisfy:*

$$\bar{i} = \bar{\pi}^w \beta^{-\frac{1}{\sigma}} (\chi\gamma\varpi)^{\frac{\sigma-1}{\sigma}} - 1 \tag{17}$$

$$\bar{\pi}^w > \beta^{\frac{1}{\sigma}} (\chi\gamma\varpi)^{\frac{1-\sigma}{\sigma}} \tag{18}$$

$$\phi > 1. \tag{19}$$

Condition (17) ensures that the intercept of the interest rate rule is consistent with existence of a full employment steady state, while condition (18) implies that inflation and trend growth in the full employment steady state are sufficiently high so that the zero lower bound constraint on the nominal interest rate is not binding. Instead, assumption (19), which requires the central bank to respond sufficiently strongly to fluctuations in employment, fulfills two roles. First, in the spirit of the Taylor principle of New Keynesian models, it ensures that the full employment steady state is locally determinate. Second, as we show below, this assumption implies that in absence of the zero lower bound there are no steady states other than the full employment steady state.[15]

## 2.7 Equilibrium

The equilibrium of our economy can be described by four simple equations. The first one is the *Euler* equation, which captures households' consumption decisions. Combining households'

[15] All these results are derived in the proofs to propositions 1 and 2.

optimality conditions (2) and (3) gives:

$$C_t^{-\sigma} = \beta(1+i_t)E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right].$$

According to this standard Euler equation, demand for consumption is increasing in expected future consumption and decreasing in the real interest rate, $(1+i_t)/\pi_{t+1}$.

To understand how productivity growth relates to demand for consumption, it is useful to combine the previous expression with $A_{t+1}/A_t = g_{t+1}$ and $\pi_{t+1} = \bar{\pi}^w/g_{t+1}$ to obtain:

$$c_t^{\sigma} = \frac{g_{t+1}^{\sigma-1}\bar{\pi}^w}{\beta(1+i_t)E_t\left[c_{t+1}^{-\sigma}\right]}, \tag{20}$$

where we have defined $c_t \equiv C_t/A_t$ as consumption normalized by the productivity index. This equation shows that the impact of productivity growth on present demand for consumption depends on the elasticity of intertemporal substitution, $1/\sigma$. In fact, there are two contrasting effects. On the one hand, faster productivity growth is associated with higher future consumption. This income effect leads households to increase their demand for current consumption in response to a rise in productivity growth. On the other hand, faster growth is associated with a fall in expected inflation. Keeping everything else constant, lower expected inflation increases the real interest rate inducing households to postpone consumption. This substitution effect points toward a negative relationship between growth and current demand for consumption. For low levels of intertemporal substitution, i.e. for $\sigma > 1$, the income effect dominates and the relationship between growth and demand for consumption is positive. Instead, for high levels of intertemporal substitution, i.e. for $\sigma < 1$, the substitution effect dominates and the relationship between growth and demand for consumption is negative. Finally, for the special case of log utility, $\sigma = 1$, the two effects cancel out and growth does not affect present demand for consumption.

Empirical estimates based on aggregate consumption data point toward a low elasticity of intertemporal substitution (Hall, 1988).[16] Hence, in the main text we will focus attention on the case $\sigma > 1$, while we provide an analysis of the cases $\sigma < 1$ and $\sigma = 1$ in the appendix.

**Assumption 2** *The parameter $\sigma$ satisfies:*

$$\sigma > 1.$$

Under this assumption, the Euler equation implies a positive relationship between the pace of innovation and demand for present consumption.

The second key relationship in our model is the *growth* equation, which is obtained by combining

[16] Similar results are reached by Ogaki and Reinhart (1998) and Basu and Kimball (2002). Using estimates based on micro data, Vissing-Jørgensen (2002) finds higher values of the elasticity of intertemporal substitution, but they still tend to be lower than 1.

equation (2) with the optimality condition for investment in research (11):

$$(g_{t+1}-1)\left(1-\beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\sigma} g_{t+1}^{-\sigma}\chi\gamma\varpi L_{t+1}\right]\right)=0. \tag{21}$$

This equation captures the optimal investment in research by entrepreneurs. For values of profits sufficiently high so that some research is conducted in equilibrium and $g_{t+1}>1$, this equation implies a positive relationship between growth and expected future employment. Intuitively, a rise in employment, and consequently in output gap and aggregate demand, is associated with higher monopoly profits. In turn, higher expected profits induce entrepreneurs to invest more in research, leading to a positive impact on the growth rate of the economy. This is the classic market size effect emphasized by the endogenous growth literature.

The third equation combines the goods market clearing condition (12), the GDP equation (13) and the fact that $\int_0^1 I_{jt}dj = A_t(g_{t+1}-1)/(\chi(\gamma-1))$:[17]

$$c_t=\left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}}\left(1-\frac{\alpha}{\xi}\right)L_t-\frac{g_{t+1}-1}{\chi(\gamma-1)} \tag{22}$$

Keeping output constant, this equation implies a negative relationship between productivity-adjusted consumption and growth, because to generate faster growth the economy has to devote a larger fraction of output to innovation activities, reducing the resources available for consumption.

Finally, the fourth equation is the monetary policy rule:

$$1+i_t=\max\left((1+\bar{i})\,L_t^{\phi},1\right). \tag{23}$$

We are now ready to define an equilibrium as a set of processes $\{g_{t+1},L_t,c_t,i_t\}_{t=0}^{+\infty}$ satisfying equations (20) − (23).

# 3 Stagnation traps

In this section we show that the interaction between aggregate demand and productivity growth can give rise to prolonged periods of low growth, low interest rates and high unemployment, which we call stagnation traps. We start by considering non-stochastic steady states, and we derive conditions on the parameters under which two steady states coexist, one of which is a stagnation trap. We then show that stagnation traps of finite expected duration are also possible.

[17] To derive this condition, consider that:

$$\int_0^1 I_{jt}dj=\int_0^1 A_{jt}I_{jt}/A_{jt}dj=A_t\int_0^1 I_{jt}/A_{jt}dj=A_tI_{jt}/A_{jt}=A_t\mu_t/\chi=A_t(g_{t+1}-1)/(\chi(\gamma-1)).$$

### 3.1 Non-stochastic steady states

Non-stochastic steady state equilibria are characterized by constant values for productivity growth $g$, employment $L$, normalized consumption $c$ and the nominal interest rate $i$ satisfying:

$$g^{\sigma-1} = \frac{\beta(1+i)}{\bar{\pi}^w} \tag{24}$$

$$g^{\sigma} = \max\left(\beta\chi\gamma\varpi L, 1\right) \tag{25}$$

$$c = \left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}}\left(1-\frac{\alpha}{\xi}\right)L - \frac{g-1}{\chi(\gamma-1)} \tag{26}$$

$$1+i = \max\left(\left(1+\bar{i}\right)L^{\phi}, 1\right), \tag{27}$$

where the absence of a time subscript denotes the value of a variable in a non-stochastic steady state. We now show that two steady state equilibria can coexist: one characterized by full employment, and one by involuntary unemployment.

**Full employment steady state.** Let us start by describing the full employment steady state, which we denote by the subscripts $f$. In the full employment steady state the economy operates at full capacity, and hence $L_f = 1$. The growth rate associated with the full employment steady state $g_f$ can then be found by setting $L = 1$ in equation (25), while the nominal interest rate that supports this steady state can be obtained by setting $g = g_f$ in equation (24).

The following proposition characterizes the full employment steady state.

**Proposition 1** *Suppose assumptions 1 and 2 hold and that*

$$1 < \left(\beta\chi\gamma\varpi\right)^{\frac{1}{\sigma}} < 1 + (\gamma-1)\min\left(1, \chi\left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}}\left(1-\frac{\alpha}{\xi}\right)\right) \tag{28}$$

*Then, there exists a unique full employment steady state with $L_f = 1$. The full employment steady state is characterized by positive growth ($g^f > 1$) and by a positive nominal interest rate ($i_f > 0$). Moreover, the full employment steady state is locally determinate.*[18]

Condition (28) ensures that in the full employment steady state investment in innovation is sufficiently small so that steady state consumption is positive. Moreover, condition (28) also implies that in the full employment steady state the innovation probability lies between zero and one ($0 < \mu_f < 1$), an assumption often made in the endogenous growth literature.

As anticipated in section 2.6, our assumptions about trend inflation and monetary policy guarantee that the nominal interest rate in the full employment steady state is positive, and that the full employment steady state is locally determinate.

**Unemployment steady state.** Aside from the full employment steady state, the economy can find itself in a permanent liquidity trap with low growth and involuntary unemployment. We denote this unemployment steady state with subscripts $u$. To derive the unemployment steady

[18] All the proofs can be found in appendix *A*.

state, consider that with $i = 0$ equation (24) implies:

$$g_u = \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}.$$

Since $\bar{i} > 0$ it follows immediately from equation (24) that $g_u < g_f$. Moreover, notice that equation (24) can be written as $(1+i)/\pi = g^\sigma/\beta$. Hence, $g_u < g_f$ implies that the real interest rate $(1+i)/\pi$ in the unemployment steady state is lower than in the full employment steady state. To see that the liquidity trap steady state is characterized by unemployment, consider that by equation (25) $g_u^\sigma = \max(\beta\chi\gamma\varpi L_u, 1)$. Now use $\beta\chi\gamma\varpi = g_f$ to rewrite expression (25) as:

$$L_u \leq \left(\frac{g_u}{g_f}\right)^\sigma < 1,$$

where the second inequality derives from $g_u < g_f$.

Since productivity growth cannot be negative, in order for an unemployment steady state to exist it must be that $g_u \geq 1$.[19] If this is the case, assumption (19) guarantees that when employment is equal to $L_u$ the central bank sets the nominal interest rate to zero.

Finally, the unemployment steady state is locally indeterminate, so that animal spirits and sunspots can generate fluctuations around it. This result is not surprising, given that in the unemployment steady state the central bank is constrained by the zero lower bound, and hence monetary policy cannot respond to changes in aggregate demand driven by self-fulfilling expectations.

The following proposition summarizes our results about the unemployment steady state.

**Proposition 2** *Suppose assumptions 1 and 2 hold and that*

$$1 < \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} < 1 + \frac{\frac{\xi}{\alpha} - 1}{\xi - 1}\left(\frac{\beta}{(\bar{\pi}^w)^\sigma}\right)^{\frac{1}{\sigma-1}}\frac{\gamma - 1}{\gamma}. \tag{29}$$

*Then, there exists a unique unemployment steady state. At the unemployment steady state the economy is in a liquidity trap ($i_u = 0$), there is involuntary unemployment ($L_u < 1$), and both growth and the real interest rate are lower than in the full employment steady state ($g_u < g_f$ and $1/\pi_u < (1 + i_f)/\pi_f$). Moreover, the unemployment steady state is locally indeterminate.*

The role of assumption (29) is to ensure existence and uniqueness of the unemployment steady. In fact, this assumption guarantees that $g_u > 1$ , so that by equation (25) there exists a unique value of $L$ consistent with $g = g_u$.[20] Moreover, the second inequality of assumption (29) makes sure that $c_u > 0$.

[19] Since $\beta < 1$, in our baseline model an unemployment steady state exists only if $\bar{\pi}^w < 1$, that is if wage inflation is negative and the real interest rate positive. However, as we show in section 4.2, this is not a strict implication of our framework, and it is not hard to modify the model to allow for positive wage inflation in the unemployment steady state, for instance by introducing precautionary savings due to idiosyncratic shocks.

[20] Notice that this assumption rules out the case $g_u = 1$. Under this knife-edged case an unemployment steady state might exist, but it will not be unique, since by equation (25) multiple values of $L$ are consistent with $g = 1$.

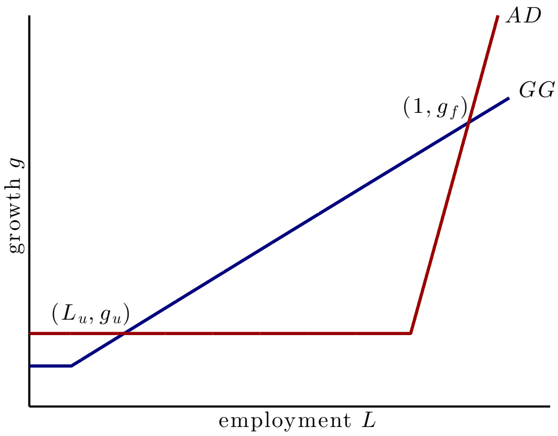


**Figure 1:** Non-stochastic steady states.

We think of this second steady state as a *stagnation trap*, that is the combination of a liquidity and a growth trap. In a liquidity trap the economy operates below capacity because the central bank is constrained by the zero lower bound on the nominal interest rate. In a growth trap, lack of demand for firms' products depresses investment in innovation and prevents the economy from developing its full growth potential. In a stagnation trap these two events are tightly connected. We illustrate this point with the help of a diagram.

Figure 1 depicts the two key relationships that characterize the steady states of our model in the $L - g$ space. The first one is the growth equation (25), which corresponds to the $GG$ schedule. For sufficiently high $L$, the $GG$ schedule is upward sloped. The positive relationship between $L$ and $G$ can be explained with the fact that, for $L$ high enough, an increase in employment and production is associated with a rise in firms' profits, while higher profits generate an increase in investment in innovation and productivity growth. Instead, for low values of $L$ the $GG$ schedule is horizontal. These are the values of employment for which investing in research is not profitable, and hence they are associated with zero growth.

The second key relationship combines the Euler equation (24) and the policy rule (27):

$$g^{\sigma-1} = \frac{\beta}{\bar{\pi}^w} \max\left((1+\bar{i})L^{\phi}, 1\right).$$

Graphically, this relationship is captured by the $AD$, i.e. aggregate demand, curve. The upward-sloped portion of the $AD$ curve corresponds to cases in which the zero lower bound constraint on the nominal interest rate is not binding.[21] In this part of the state space, the central bank responds to a rise in employment by increasing the nominal rate. In turn, to be consistent with households' Euler equation, a higher interest rate must be coupled with faster productivity growth.[22] Hence, when monetary policy is active the $AD$ curve generates a positive relationship between $L$ and $g$. Instead, the horizontal portion of the $AD$ curve corresponds to values of $L$ for which the zero lower bound constraint binds. In this case, the central bank sets $i = 0$ and steady state growth is

[21] Precisely, the zero lower bound constraint does not bind when $L \geq (1+\bar{i})^{-1/\phi}$.
[22] Recall that we are focusing on the case $\sigma > 1$.

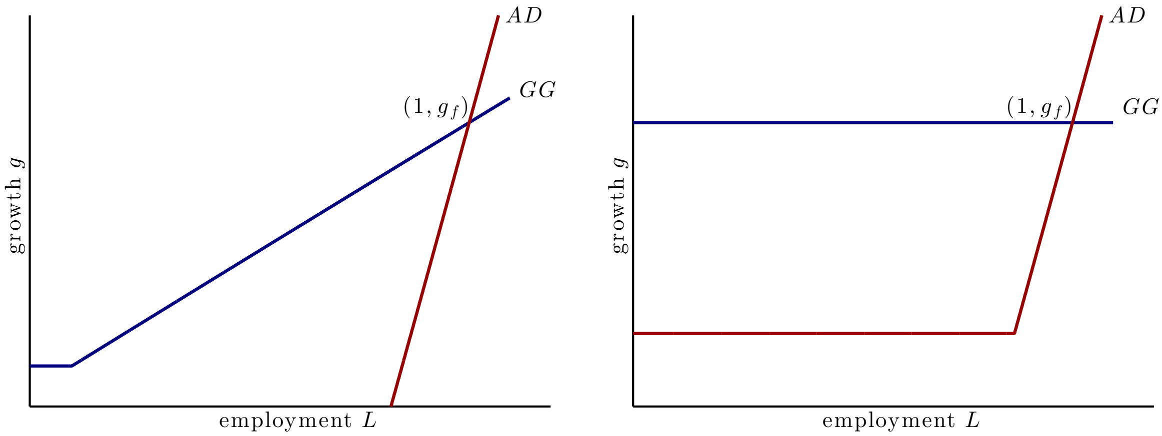


**Figure 2:** Understanding stagnation traps. Left panel: economy without zero lower bound. Right panel: economy with exogenous growth.

independent of $L$ and equal to $(\beta/\bar{\pi}^w)^{1/(\sigma-1)}$. As long as the conditions specified in propositions 1 and 2 hold, the two curves cross twice and two steady states are possible.

Importantly, both the presence of the zero lower bound and the procyclicality of investment in innovation are needed to generate steady state multiplicity. Suppose that the central bank is not constrained by the zero lower bound, and hence that liquidity traps are not possible. As illustrated by the left panel of figure 2, in this case the $AD$ curve reduces to an upward sloped curve, steeper than the $GG$ curve, and the unemployment steady state disappears. Indeed, assumption (19) ensures that, in absence of the zero lower bound, the central bank's reaction to unemployment is always sufficiently strong to ensure that the only possible steady state is the full employment one.

Now suppose instead that productivity growth is constant and equal to $g_f$. In this case the $GG$ curve reduces to a horizontal line at $g = g_f$, and again the full employment steady state is the only possible one. Intuitively, if growth is not affected by variations in employment, then aggregate demand is always sufficiently strong so that in steady state the zero lower bound constraint on the nominal interest rate does not bind, ensuring that the economy operates at full employment. We refer to the unemployment steady state as a stagnation trap to capture the tight link between liquidity and growth traps suggested by our model.

We are left with determining what makes the economy settle in one of the two steady states. This role is fulfilled by expectations. Suppose that agents expect that the economy will permanently fluctuate around the full employment steady state. Then, their expectations of high future growth sustain aggregate demand, so that a positive nominal interest rate is consistent with full employment. In turn, if the economy operates at full employment then firms' profits are high, inducing high investment in innovation and productivity growth. Conversely, suppose that agents expect that the economy will permanently remain in a liquidity trap. In this case, low expectations about growth and future income depress aggregate demand, making it impossible for the central bank to sustain full employment due to the zero lower bound constraint on the interest rate. As a result the economy operates below capacity and firms' profits are low, so that investment in innovation is also low, justifying the initial expectations of weak growth. Hence, in our model expectations can be self-fulfilling, and sunspots, that is confidence shocks unrelated to fundamentals, can determine real outcomes.

Before moving on, it is useful to compare our notion of stagnation traps with the permanent liquidity traps that can arise in New-Keynesian models. In standard New-Keynesian models productivity growth is exogenous, and there is a unique real interest rate consistent with a steady state. As shown by Benhabib et al. (2001), permanent liquidity traps can occur in these frameworks if agents coordinate their expectations on an inflation rate equal to the inverse of the steady state real interest rate. Because of this, New-Keynesian models typically feature two steady states, one of which is a permanent liquidity trap. These two steady states are characterized by the same real interest rate, but by different inflation and nominal interest rates, with the liquidity trap steady state being associated with inflation below the central bank's target.

In contrast, in our framework endogenous growth is key in opening the door to steady state multiplicity and permanent liquidity traps. Crucially, in our model the two steady states feature different growth and real interest rates, with the liquidity trap steady state being associated with low growth and low real interest rate. Instead, inflation expectations do not play a major role. In fact, once a wage Phillips curve is introduced in the model, it might very well be the case that inflation in the unemployment steady state is the same, or even higher, than in the full employment one. We will go back to this point in section 4.2.

Summarizing, the combination of growth driven by investment in innovation from profit-maximizing firms and the zero lower bound constraint on monetary policy can produce stagnation traps, that is permanent, or very long lasting, liquidity traps characterized by unemployment and low growth. All it takes is a sunspot that coordinates agents' expectations on the unemployment steady state.

### 3.2 Sunspots and temporary stagnation traps

Though our model can allow for economies which are permanently in a liquidity trap, it is not difficult to construct equilibria in which the expected duration of a trap is finite.

To construct an equilibrium featuring a temporary liquidity trap we have to put some structure on the sunspot process. Let us start by denoting a sunspot by $\xi_t$. In a sunspot equilibrium agents form their expectations about the future after observing $\xi$, so that the sunspot acts as a coordination device for agents' expectations. To be concrete, let us consider a two-state discrete Markov process, $\xi_t \in (\xi_o, \xi_p)$, with transition probabilities $Pr\left(\xi_{t+1} = \xi_o | \xi_t = \xi_o\right) = 1$ and $Pr\left(\xi_{t+1} = \xi_p | \xi_t = \xi_p\right) = q_p < 1$. The first state is an absorbing optimistic equilibrium, in which agents expect to remain forever around the full employment steady state. Hence, once $\xi_t = \xi_o$ the economy settles on the full employment steady state, characterized by $L = 1$ and $g = g_f$. The second state $\xi_p$ is a pessimistic equilibrium with finite expected duration $1/(1 - q_p)$. In this state the economy is in a liquidity trap with unemployment. We consider an economy that starts in the pessimistic equilibrium.

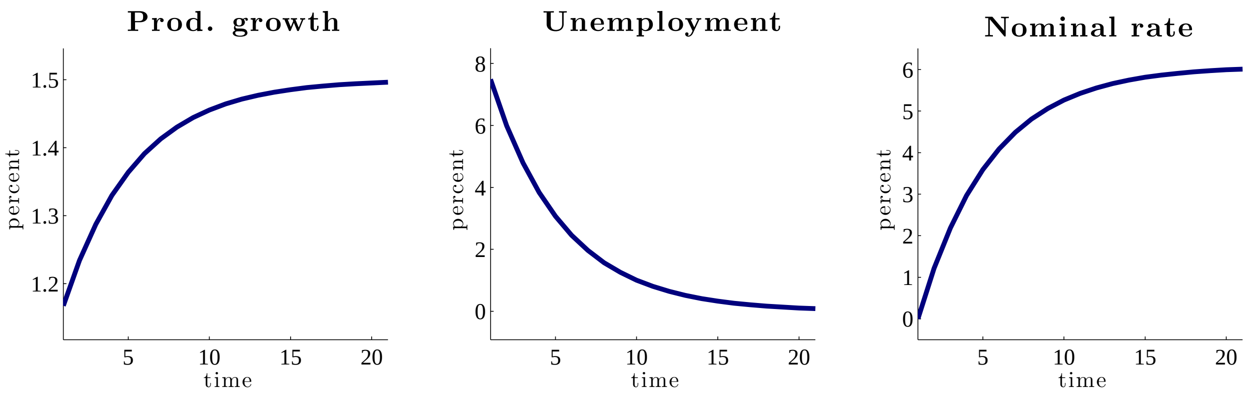


**Figure 3:** Temporary liquidity traps: expected dynamics.

Under these assumptions, as long as the pessimistic sunspot shock persists the equilibrium is described by equations (20), (21) and (39), which, using the fact that in the pessimistic state $i = 0$, can be written as:

$$g_p^{\sigma-1} = \frac{\beta}{\bar{\pi}^w} \left( q_p + (1 - q_p) \left( \frac{c_p}{c_f} \right)^\sigma \right) \tag{30}$$

$$(g_p - 1) \left( g_p^\sigma - \beta \chi \gamma \varpi \left( q_p L_p + (1 - q_p) \left( \frac{c_p}{c_f} \right)^\sigma \right) \right) = 0. \tag{31}$$

$$c_p = \left( \frac{\alpha}{\xi} \right)^{\frac{\alpha}{1-\alpha}} \left( 1 - \frac{\alpha}{\xi} \right) L_p - \frac{g_p - 1}{\chi(\gamma - 1)}, \tag{32}$$

where the subscripts $p$ denote the equilibrium while pessimistic expectations prevail. Similar to the case of the unemployment steady state, in the pessimistic equilibrium the zero lower bound constraint on the interest rate binds, there is involuntary unemployment and growth is lower than in the optimistic state.

Characterizing analytically the equilibrium described by equations (30) – (32) is challenging, but some results can be obtained by writing equation (30) as:

$$g_p^{\sigma-1} = g_u^{\sigma-1} \left( q_p + (1 - q_p) \left( \frac{c_p}{c_f} \right)^\sigma \right).$$

It can be shown that $c_p/c_f$ is smaller than one, i.e. switching to the optimistic steady state entails an increase in productivity-adjusted consumption. Hence, the equation above implies that temporary liquidity traps feature slower growth compared to permanent ones. Indeed, for reasonable parameterizations, growth and employment are both increasing in the expected duration of the trap, so that traps of shorter expected duration are characterized by sharper contractions.

Figure 3 displays the expected path of productivity growth, unemployment and the nominal interest rate during a temporary liquidity trap.[23] The economy starts in the pessimistic equilibrium, characterized by low growth, high unemployment and a nominal interest rate equal to zero. From the second period on, each period agents expect that the economy will leave the trap and go back

[23] The value of the parameters are described in section 4.3.

to the full employment steady state with a constant probability. Hence, the probability that the economy remains in the trap decreases with time, explaining the upward path for expected productivity growth, employment and the nominal interest rate. However, even though the economy eventually goes back to the full employment steady state, the post-trap increase in the growth rate is not sufficiently strong to make up for the low growth during the trap, so that the trap generates a permanent loss in output.

This example shows that pessimistic expectations can plunge the economy into a temporary liquidity trap with unemployment and low growth. Eventually the economy will recover, but the liquidity trap lasts as long as pessimistic beliefs persist. Hence, long lasting liquidity trap driven by pessimistic expectations can coexist with the possibility of a future recovery.

# 4 Extended Model and Numerical Exercise

In this section we extend the model in three directions. We first show that the introduction of precautionary savings can give rise to stagnation traps characterized by positive inflation and a negative real interest rate. We then show that our key results do not rely on the assumption of a constant wage inflation rate. Lastly, we perform a simple calibration exercise to examine a setting in which, consistent with standard models of vertical innovation, monopoly rents of innovators can last longer than a single period.

## 4.1 Precautionary savings and negative real rates

In our baseline framework positive growth and positive inflation cannot coexist during a permanent liquidity trap. Intuitively, if the economy is at the zero lower bound with positive inflation, then the real interest rate must be negative. But then, to satisfy households' Euler equation, the steady state growth rate of the economy must also be negative. Conversely, to be consistent with positive steady state growth the real interest rate must be positive, and when the nominal interest rate is equal to zero this requires deflation.

However, it is not hard to think about mechanisms that could make positive growth and positive steady state inflation coexist in an unemployment steady state. One possibility is to introduce precautionary savings. In appendix $C$, we lay down a simple model in which every period a household faces a probability $p$ of becoming unemployed. An unemployed household receives an unemployment benefit, such that its income is equal to a fraction $b < 1$ of the income of an employed household. Unemployment benefits are financed with taxes on the employed households. We also assume that unemployed households cannot borrow and that trade in firms' share is not possible.

As showed in the appendix, under these assumptions the equilibrium is described by the same equations of the baseline model except for the Euler equation (20), which is replaced by:

$$c_t^{\sigma} = \frac{\bar{\pi}^w g_{t+1}^{\sigma-1}}{\beta(1+i_t)\rho E_t\left[c_{t+1}^{-\sigma}\right]},$$

where:

$$\rho \equiv 1 - p + p/b^{\sigma} > 1.$$

The unemployment steady state is now characterized by:

$$g_u = \left(\frac{\rho\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}.$$

Since $\rho > 1$, an unemployment steady state in which both inflation and growth are positive is now possible.

The key intuition behind this result is that the presence of uninsurable idiosyncratic risk depresses the natural interest rate.[24] Indeed, the presence of uninsurable idiosyncratic risk drives up the demand for precautionary savings. Since the supply of saving instruments is fixed, higher demand for precautionary savings leads to a lower equilibrium interest rate. This is the reason why an economy with uninsurable unemployment risk can reconcile positive steady state growth with a negative real interest rate. Hence, once the possibility of uninsurable unemployment risk is taken into account, it is not hard to imagine a permanent liquidity trap with positive growth, positive inflation and negative real interest rate.

## 4.2 Introducing a wage Phillips curve

Our basic model features a constant wage inflation rate. Here we introduce a wage Phillips curve, and discuss the implications of our model for inflation and the role of wage flexibility.

To make things simple, let us assume that nominal wages are downwardly rigid:

$$W_t \geq \psi\left(L_t\right) W_{t-1},$$

with $\psi' > 0$, $\psi(1) = \bar{\pi}^w$. This formulation, in the spirit of Schmitt-Grohé and Uribe (2012), allows wages to fall at a rate which depends on unemployment. Capturing some nonmonetary costs from adjusting wages downward, here wages are more downwardly flexible the more employment is below potential. This formulation gives rise to a nonlinear wage Phillips curve. For levels of wage inflation greater than $\bar{\pi}^w$ output is at potential. Instead, if wage inflation is less than $\bar{\pi}^w$ there is a positive relationship between inflation and the output gap.

Similar to the baseline model, monetary policy follows a truncated Taylor rule in which the nominal interest rate responds to deviations of wage inflation from a target:

$$1 + i_t = \max\left((1+\bar{i})\left(\frac{\pi_t^w}{\bar{\pi}^w}\right)^{\phi}, 1\right). \tag{33}$$

For symmetry with the baseline model, we have assumed that the central bank's target for wage inflation is $\bar{\pi}^w$. Moreover, we assume that $\phi$ is such that $\psi(1)L^{\phi-1}/\psi(L) < 1$ for any $0 \leq L \leq 1$. This assumption, similar to assumption (19) of the baseline model, ensures local determinacy of

[24]See Bewley (1977), Huggett (1993) and Aiyagari (1994).

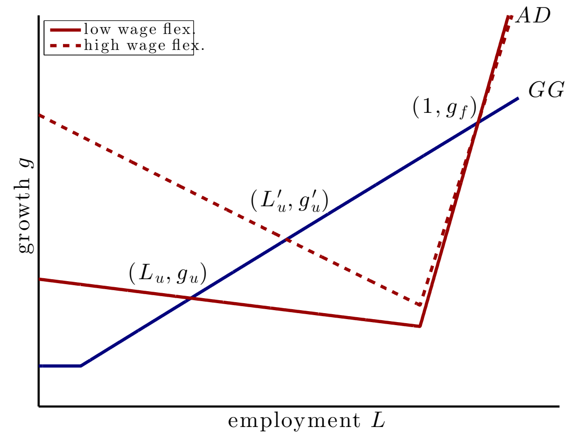


**Figure 4:** Steady states with variable inflation.

the full employment steady state and that, in the absence of the zero lower bound, there are no other steady states than the full employment one.

A steady state of the economy is now described by (25), (26), (33) and:

$$g^{\sigma-1} = \frac{\beta(1+i)}{\pi^w} \tag{34}$$

$$\pi^w \geq \psi(L). \tag{35}$$

It is easy to check that the presence of the wage Phillips curve does not have any impact on the full employment steady state, which is characterized by $L = 1$, $g = g_f$, $i = i_f$ and $\pi^w = \bar{\pi}^w$.

Let us now turn to the unemployment steady state. Combining equations (33) − (35) and using $i = 0$, gives:

$$g_u = \left(\frac{\beta}{\psi(L_u)}\right)^{\frac{1}{\sigma-1}}. \tag{36}$$

This expression implies a negative relationship between growth and employment. To understand this relationship, consider that in a liquidity trap the real interest rate is just the inverse of expected inflation. Due to the wage Phillips curve, as employment increases wage inflation rises generating higher price inflation. Hence, in a liquidity trap a higher employment is associated with a lower real interest rate. The consequence is that during a permanent liquidity trap a rise in employment must be associated with lower productivity growth, to be consistent with the lower real interest rate. As illustrated by figure 4, graphically this is captured by the fact that the $AD$ curve, obtained by combining equations (33) − (35), is downward sloped for values of $L$ low enough so that the zero lower bound constraint binds.

To solve for the equilibrium unemployment steady state, combine equations (25) and (36) to obtain:

$$L_u = \frac{1}{\beta\chi\gamma\varpi}\left(\frac{\beta}{\psi(L_u)}\right)^{\frac{\sigma}{\sigma-1}}. \tag{37}$$

Since the left-hand side of this expression is increasing in $L_u$, while the right hand-side is decreasing in $L_u$, there is a unique $L_u$ that characterizes the unemployment steady state. Moreover, since $L_u < 1$, the presence of a Phillips curve implies that the unemployment steady state is now characterized by lower wage inflation than the full employment steady state. In sum, the presence of a wage Phillips curve does not alter the key properties of the unemployment steady state, while adding the realistic feature that in the unemployment steady state the central bank undershoots its wage inflation target.

Turning to price inflation, recalling that $\pi_t = \pi_t^w / g_{t+1}$, we have that:

$$\frac{\pi_u}{\pi_f} = \frac{\pi_u^w}{\pi_f^w} \frac{g_f}{g_u}.$$

Since $\pi_u^w < \pi_f^w$ and $g_f > g_u$, depending on parameter values price inflation in the unemployment steady state can be above, below or even equal to price inflation in the full employment steady state. This result is due to the fact that in the unemployment steady state the depressive impact on price inflation originating from low wage inflation is counteracted by the upward pressure exerted by low productivity growth.

To conclude this section, we note that higher wage flexibility, captured by a steeper wage Phillips curve, is associated with better outcomes in the unemployment steady state. For instance, this result can be seen by considering that expression (37) implies, since $\psi(L_u) < \bar{\pi}^w$, that the endogenous fall in wage inflation sustains employment in the unemployment steady state. Figure 4 illustrates graphically the impact of higher wage flexibility on the determination of the unemployment steady state. The figure shows that higher wage flexibility steepens the downward portion of the $AD$ curve, leading to higher growth and employment in the unemployment steady state.

### 4.3 Numerical exercise

In this section, we explore further the properties of the model by performing a simple calibration exercise. To be clear, the objective of this exercise is not to provide a careful quantitative evaluation of the framework or to replicate any particular historical event. In fact, both of these tasks would require a much richer model. Rather, our aim is to show that the magnitudes implied by the model are quantitatively relevant and reasonable.

For our numerical exercise we enrich the baseline model in two dimensions. First, along the lines of section 4.2, we consider an economy with endogenous wage inflation. Second, we relax the assumption of one period monopoly rents for innovators in favor of a, more conventional, setting in which the duration of innovators' rents is endogenous.

**Endogenous duration of rents from innovation.** In our baseline model we assume that the rents from innovation last a single period, after which the innovator's patent expires. Here we consider a setting in which every period the innovator retains its patent with probability $1 - \eta$.[25]

[25] Standard models of vertical innovation typically assume that $\eta = 0$. We follow Acemoglu and Akcigit (2012) and consider the case $\eta > 0$ to be able to match a realistic value for the spending in R&D-to-GDP ratio.

Under these assumptions, the value of becoming a leader in sector $j$ is:

$$V_t(\gamma A_{jt}) = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( P_{t+1} \varpi \gamma A_{jt} L_{t+1} + (1 - \mu_{jt+1} - \eta) V_{t+1}(\gamma A_{jt}) \right) \right], \tag{38}$$

The first term inside the parenthesis, also present in the baseline model, captures the expected profits to be gained in period $t+1$. In addition, the value of a successful innovation includes the value of being a leader in period $t+1$, $V_{t+1}(\gamma A_{jt})$, times the probability that the entrepreneur remains the leader in period $t+1$, $1 - \mu_{jt+1} - \eta$. Notice that the probability of maintaining the leadership is decreasing in $\mu_{jt+1}$, capturing the fact that the discovery of a better version of product $j$ terminates the monopoly rents for the incumbent. As in the baseline model, future payoffs are discounted using the households' discount factor $\beta \lambda_{t+1}/\lambda_t$.

For simplicity, in this section we restrict attention to equilibria in which in every period a positive amount of research is targeted toward every intermediate good. In this case, free entry into research implies that expected profits from researching are zero for every product. This zero profit condition implies that:[26]

$$P_t = \frac{\chi}{A_{jt}} V_t(\gamma A_{jt}).$$

for every good $j$. Moreover, we focus on symmetric equilibria in which the probability of innovation is the same in every sector, so that $\mu_{jt} = \chi I_{jt}/A_{jt} = \mu_t$ for every $j$. In this case, $V_t(\gamma A_{jt}) = \bar{V}_t \gamma A_{jt}$ for every $j$, while free entry into the research sector in period $t+1$ implies $\bar{V}_{t+1} = P_{t+1}/(\gamma\chi)$. Combining these conditions with expression (38) gives:

$$\frac{P_t}{\chi} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( P_{t+1} \varpi \gamma L_{t+1} + (1 - \mu_{t+1} - \eta) \frac{P_{t+1}}{\chi} \right) \right].$$

This expression summarizes the equilibrium in the research sector. Combining this expression with equation (2) and using $\mu_t = (g_{t+1} - 1)/(\chi(\gamma - 1))$ gives:

$$1 = \beta E_t \left[ \left( \frac{c_t}{c_{t+1}} \right)^{\sigma} \left( g_{t+1}^{-\sigma} \chi \gamma \varpi L_{t+1} + 1 - \frac{g_{t+2} - 1}{\chi(\gamma - 1)} - \eta \right) \right] = 0,$$

which replaces equation (21) of the baseline model.

**Parameters.** We choose the length of a period to correspond to a year. For many parameters we follow Schmitt-Grohé and Uribe (2012). Hence, the discount factor is set to $\beta = 0.99$ to target a real interest rate in the full employment steady state of 4 percent, and the inverse of the elasticity of intertemporal substitution is set to $\sigma = 2$, a standard value in the business-cycle literature. Moreover, we choose the value of $\chi$, the parameter determining the productivity of research, so

[26] To derive this condition, consider that an entrepreneur that invests $I_{jt}$ in research has a probability $\chi I_{jt}/A_{jt}$ of becoming a leader which carries value $V_t(\gamma A_{jt})$. Hence, the expected return from this investment is $\chi I_{jt} V_t(\gamma A_{jt})/A_{jt}$. Since the investment costs $P_t I_{jt}$, the zero expected profits condition in the research sector implies:

$$P_t I_{jt} = \frac{\chi I_{jt}}{A_{jt}} V_t(\gamma A_{jt}).$$

Simplifying we obtain the expression in the main text.

**Table 1: Parameters**

| | Value | Source/Target |
|---|---|---|
| Risk aversion | $\sigma = 2$ | Standard value |
| Discount factor | $\beta = 0.99$ | $(1 + i_f)/\pi_f = 1.04$ |
| Wage inflation at full emp. | $\bar{\pi}^w = 1.035$ | $\pi_f = 1.02$ |
| Slope of wage Phillips curve | $\tilde{\psi} = 1.07$ | $\bar{\pi}^w 0.95^{\tilde{\psi}} = 0.98$ |
| Productivity of research | $\chi = 42.08$ | $g_f = 1.015$ |
| Innovation step | $\gamma = 1.05$ | Acemoglu and Akcigit (2012) |
| Share of labor in gross output | $1 - \alpha = 0.512$ | Profits/GDP $= 5\%$ |
| Prob. patent expires | $\eta = 0.19$ | $I_f/GDP_f = 3\%$ |

that growth in the full employment steady state is equal to 1.5 percent, the average growth rate of per capita output in the postwar United States.

The step size of innovations is set to $\gamma = 1.05$, as in Acemoglu and Akcigit (2012). We set the labor share in gross output to $\alpha = 0.512$ to target a share of profits in GDP of 5 percent, and the probability that a patent expires to $\eta = 0.19$ to match a ratio of spending in R&D-to-GDP of 3 percent. These targets correspond to the profit share and spending in R&D-to-GDP ratio implied by the benchmark calibration of the model in Acemoglu and Akcigit (2012).

We specify a functional form for the wage Phillips curve similar to the one in Schmitt-Grohé and Uribe (2012). We thus assume that:

$$\psi(L) = \bar{\pi}^w L^{\tilde{\psi}}.$$

We set $\bar{\pi}^w = 3.5\%$ so that price inflation in the full employment steady state is equal to 2 percent. We calibrate the parameter $\tilde{\psi}$, which governs the elasticity of wage inflation to employment, as in Schmitt-Grohé and Uribe (2012). In particular, we set $\tilde{\psi}$ so that at an unemployment rate of 5 percent nominal wages fall by 2 percent per year, that is we impose the restriction $0.98 = \bar{\pi}^w 0.95^{\tilde{\psi}}$.

**Results.** Table 2 displays several statistics from this calibrated version of the model. The first column refers to the full employment steady state. As targeted in the calibration, productivity growth is 1.5 percent, while, by definition of the full employment steady state, unemployment is equal to zero. The real interest rate is equal to its calibration target of 4 percent, which, coupled with the 2 percent price inflation, implies a nominal interest rate of about 6 percent. Wage inflation is 3.53 percent, approximately equal to the sum of price inflation and productivity growth. Finally, about 3 percent of GDP is spent on research, as targeted in the calibration.

Column two shows the statistics for the unemployment steady state. Productivity growth is 0.13 points lower than in the full employment steady state. This difference might seem small, but, as we will see, it generates sizable welfare losses. Moreover, this number is in line with the response of productivity growth to policy changes typically found in Schumpeterian growth models (Acemoglu and Akcigit, 2012; Aghion et al., 2013). Unemployment is equal to 5.25 percent, while the real interest rate is 3.74 percent, so 26 basis point lower than in the full employment steady state. Moreover, the unemployment steady state is characterized by deflation and falling nominal

**Table 2: Calibrated examples**

| | Full employment steady state | Unemployment steady state | | Temporary liquidity trap | |
|---|---|---|---|---|---|
| | | Benchmark | Ex. growth | Benchmark | Ex. growth |
| Productivity growth | 1.50 | 1.37 | 1.50 | 1.17 | 1.50 |
| Unemployment rate | 0.00 | 5.25 | 5.37 | 7.50 | 8.13 |
| Nominal interest rate | 6.08 | 0.00 | 0.00 | 0.00 | 0.00 |
| Real interest rate | 4.00 | 3.74 | 4.00 | 6.20 | 7.35 |
| Price inflation | 2.00 | −3.61 | −3.85 | −5.84 | −6.84 |
| Wage inflation | 3.53 | −2.28 | −2.40 | −4.74 | −5.45 |
| R&D/GDP | 2.96 | 2.86 | *n/a* | 2.49 | *n/a* |
| Consumption equivalent | 0.00 | 11.06 | 5.67 | 2.32 | 0.97 |

Note: All the values are expressed in percentage points. *Ex. growth* stands for model with productivity growth exogenous and equal to $g_f$. *n/a* stands for not applicable.

wages. Interestingly, the ratio of spending in R&D-to-GDP in the unemployment steady state is 2.86 percent, so only slightly lower than in the full employment steady state. This happens because in the unemployment steady state both spending in R&D and GDP are lower than in the full employment one.

To get a sense of the welfare losses entailed by the unemployment steady state, we computed the consumption equivalent with respect to the full employment steady state. This is defined as the proportional permanent increase in consumption that households living in an economy stuck in the unemployment steady state must receive in order to be indifferent with respect to switching to the full employment steady state.[27] As shown in the last row of table 2, the welfare losses associated with the unemployment steady state are equivalent to an 11.06 percent permanent increase in consumption. This is a large welfare loss compared to the gains from stabilization policies usually obtained in business-cycle models. We will see shortly that about half of these welfare losses are a direct cause of the endogenous drop in productivity growth.

In the third column, we report statistics referring to the unemployment steady state of a counterfactual economy featuring exogenous productivity growth set equal to $g_f$.[28] This counterfactual is useful, because it isolates the role played by the endogenous productivity growth in the benchmark economy. The unemployment steady state of the economy with exogenous growth features a higher unemployment rate and lower inflation compared to the benchmark economy. The difference in inflation is partly due to the fact that the endogenous drop in productivity growth featured

[27]More formally, for any generic expected consumption stream $E_0\{C_t\}_{t=0}^{\infty}$ we compute the consumption equivalent $\epsilon$ with respect to the full employment steady state as:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{((1+\epsilon)C_t)^{1-\sigma}-1}{1-\sigma}\right)\right]=\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{(C_{f,t})^{1-\sigma}-1}{1-\sigma}\right)\right],$$

where $\{C_{f,t}\}_{t=0}^{\infty}$ is the consumption stream in the full employment steady state. When performing these computations we consider economies that start with the same initial level of productivity $A_0$.

[28]The economy with exogenous productivity growth closely resembles the one described by Schmitt-Grohé and Uribe (2012). For the economy with exogenous productivity growth we maintained the same parameters of our benchmark economy, and the statistics relative to the full employment steady state reported in column 1 also apply to the economy with exogenous productivity growth. The only exception is that, since in the counterfactual economy productivity growth is fully exogenous, investment in research is set equal to zero.

in the benchmark economy raises firms' marginal cost, inducing firms to charge higher prices and mitigating the fall in inflation,[29] and partly due to the fact that wage inflation is lower in the exogenous growth economy. The full employment and unemployment steady state of the exogenous economy feature the same real interest rate. This highlights the role of the endogenous drop in productivity growth in depressing the real interest rate in the unemployment steady state of the benchmark economy. Turning to welfare, the unemployment steady state of the exogenous growth economy is characterized by large welfare losses, equal to a 5.67 percent permanent decrease in consumption. However, these welfare losses are about half of the ones characterizing the benchmark economy, meaning that the endogenous drop in productivity growth has a large negative impact on welfare.

Columns 4 and 5 show the statistics for a temporary stagnation trap of expected duration of five years, respectively for the benchmark economy and for the economy with exogenous growth. Starting from the benchmark economy, qualitatively a temporary liquidity trap resembles the unemployment steady state, but the difference with respect to the full employment steady state are quantitatively larger. For instance, during the temporary liquidity trap productivity growth is 0.33 percent lower and unemployment is 7.5 percent higher than in the full employment steady state. The only exception is represented by the real interest rate, which during the temporary liquidity trap rises above its value in the full employment steady state. This can be explained with the large drop in price inflation occurring during the trap. The welfare losses linked to the temporary liquidity trap are smaller than the ones generated by the unemployment steady state, but still quantitatively significant since they are equal to a 2.32 percent permanent drop in consumption. Comparing column 4 and 5 gives broadly the same picture of the unemployment steady state. In particular, when productivity growth is exogenous a temporary liquidity trap generates a sharper rise in unemployment, and a temporary liquidity trap generates welfare losses that are about half the size of the ones of the benchmark economy.

## 5 Policy Implications

We now turn to the policy implications of our model. We start by considering optimal interest rate policy, both under commitment and discretion. We then turn to growth policies aiming at sustaining investment in productivity enhancing activities. For simplicity, we discuss the role of these policies in the context of the baseline model described in section 3.

### 5.1 Interest rate policy

In section 3 we have shown that stagnation traps can arise if the central bank follows a Taylor-type rule, in which the interest rate responds monotonically to employment or wage inflation. In this section we examine the robustness of this result to optimal interest rate policy. The key lesson

[29] Christiano et al. (2015) provide evidence on the role of the slowdown in productivity growth during the US Great Recession in sustaining inflation.

that we derive is that the ability to commit by the central bank is crucial. As we will see, under full commitment a central bank can implement interest rate policies that rule out stagnation traps. Instead, if the central bank operates under discretion stagnation traps are possible, even if interest rates are set optimally.

**Commitment.** Let us start by examining a central bank that operates under commitment. Consider a central bank that adopts an interest rate peg, by committing to set $i_t = i_f$ in any date and state. Clearly, this policy rules out the unemployment steady state, because the only steady state consistent with $i = i_f$ is the full employment one.[30] Moreover, this policy rules out the persistent liquidity traps described in section 3.2, because they would require the nominal interest rate to be equal to zero. More broadly, pegging the interest rate to $i_f$ rules out stagnation traps, because for these to occur agents should anticipate a protracted period of zero nominal interest rate. However, as it is well known from the New-Keynesian literature, pegging the interest rate opens the door to sunspot fluctuations around the full employment steady state.[31] In fact, this is precisely one of the reasons why adopting interest rate rules that respond to employment or inflation might be desirable. Hence, ruling out stagnation traps by pegging the interest rate comes at the risk of self-fulfilling fluctuations around the full employment steady state.

Another option for a central bank under commitment is to adopt a non-linear interest rate rule, in the spirit of the one proposed by Schmitt-Grohé and Uribe (2012). This approach combines a standard Taylor rule, that operates in "normal" times, with an interest rate peg, adopted by the central bank when expectations turn pessimistic. To see how this approach works, define $s_t$ as a binary value that follows:

$$s_t = \begin{cases} 1 & \text{if } i_{t-1} = 0 \\ 0 & \text{if } g_t \geq g_f \\ s_{t-1} & \text{otherwise,} \end{cases}$$

for $t \geq 0$ with $s_{-1} = 0$. Now consider a central bank that follows the rule:

$$i_t = \begin{cases} \max\left( \left(1+\bar{i}\right) L_t^{\phi} - 1, 0 \right), & \text{if } s_t = 0 \\ i_f & \text{otherwise.} \end{cases}$$

Under this rule, the central bank switches to an interest rate peg the period after the nominal interest rate hits the zero lower bound. The peg is maintained for one period, after which the central bank returns to the interest rate rule considered in section 3.

This policy eliminates the unemployment steady state and the persistent stagnation traps of section 3.2. In fact, for stagnation traps to occur agents should coordinate their expectations on a protracted period of low interest rates, a possibility ruled out by this non-linear Taylor rule. At the same time, this policy rule eliminates sunspot fluctuations around the full employment steady state. Hence, a central bank under commitment can rule out the stagnation traps discussed in

[30] See the proof to proposition 1.

[31] Again, see the proof to proposition 1.

section 3, while still preserving determinacy of the full employment steady state, by adopting a nonlinear interest rate rule.

**Discretion.** The picture changes dramatically if the central bank does not have the ability to commit to its future actions. The following proposition characterizes the behavior of a benevolent central bank that operates under discretion.

**Proposition 3** *Consider a central bank that operates under discretion and maximizes households' expected utility, subject to (20), (21), (39), $L_t \leq 1$ and $i_t \geq 0$. The solution to this problem satisfies:*

$$i_t (L_t - 1) = 0.$$

The intuition behind this result is straightforward. The discretionary central bank seeks to maximize current employment.[32] From the goods market clearing condition, employment is increasing in consumption and investment in research (both normalized by productivity):

$$L_t = \frac{1}{\Psi} \left( c_t + \frac{I_t}{A_t} \right), \tag{39}$$

where $\Psi \equiv (\alpha/\xi)^{\frac{\alpha}{1-\alpha}} (1 - \alpha/\xi)$. In turn, equations (20) and (21) imply that, holding expectations about the future as given, both consumption and investment in research are decreasing in the nominal interest rate. In fact, when the nominal interest rate falls also the real interest rate decreases, inducing households to frontload their consumption and entrepreneurs to increase investment in research, thus stimulating output and employment. It follows that, as long as the zero lower bound constraint does not bind, the central bank is able to set the nominal interest rate low enough so that the economy operates at full employment and $L_t = 1$. However, if a negative nominal interest rate is needed to reach full employment then the best that the discretionary central bank can do is to set $i_t = 0$. Hence, the economy can be in one of two regimes. Either the economy operates at full employment and the zero lower bound constraint on the interest rate does not bind, or the economy is in a liquidity trap with unemployment.[33]

We now show that under a discretionary central bank the economy can experience the same kind of stagnation traps described in section 3. Let us take the perspective of a discretionary central bank operating in period $t = 0$. Consider a case in which expectations coordinate on the unemployment steady state, so that $E_0[i_t] = 0$, $E_0[L_{t+1}] = L_u$ and $E_0[c_{t+1}] = c_u$ for every future date $t > 0$ and state. From section 3 we know that if the central bank sets $i_0 = 0$, then $L_0 = L_u$ so the economy will experience unemployment. Can the central bank do better by setting a positive nominal interest rate? The answer is no, because by raising the nominal interest rate above zero

[32] Notice that the economy is subject to three sources of inefficiency. First, part of the labor endowment might not be used in production. Second, due to monopolistic competition production of intermediate goods is inefficiently low. Third, investment in research is subject to the intertemporal spillover and business stealing effects, studied by Aghion and Howitt (1992) and Grossman and Helpman (1991). However, interest rate policy can only seek to correct the first distortion, since a change in the interest rate moves consumption and investment in innovation in the same direction.

[33] In fact, the optimal policy under discretion is equivalent to the truncated Taylor rule considered in the baseline model with $\phi \to +\infty$.

the central bank would further depress demand for consumption and investment, thus pushing employment below $L_u$. Hence, if expectations coordinate on the unemployment steady state the best response of a central bank under discretion is to set $i_0 = 0$, implying that $L_0 = L_u$, $g_1 = g_u$ and $c_0 = c_u$. A similar reasoning holds in any date $t \geq 0$, meaning that the central bank's actions validate agents' expectations and push the economy in the unemployment steady state. Hence, under discretionary monetary policy the same steady states analyzed in section 2 are possible.[34] Moreover, one can show that under discretion temporary stagnations traps of the kind described in section 3.2 are also possible.

These results highlight the key role that the ability to commit plays in avoiding stagnation traps through interest rate policy. Under commitment, the central bank can design interest rate policies that make expectations of a prolonged liquidity trap inconsistent with equilibrium, thus ruling out the possibility of long periods of stagnation. Instead, under discretion the central bank inability to commit to its future actions leaves the door open to stagnation episodes.

### 5.2 Growth Policy

One of the root causes of a stagnation trap is the weak growth performance of the economy, which is in turn due to entrepreneurs' limited incentives to innovate due to weak demand for their products. This suggests that subsidies to investment in innovation or to firms' profits might be a helpful tool in the management of stagnation traps. In fact, these policies have been extensively studied in the context of endogenous growth models as a tool to overcome inefficiencies in the innovation process. However, here we show how policies that foster productivity growth can also play a role in stimulating aggregate demand and employment during a liquidity trap.

The most promising form of growth policies to exit a stagnation trap are those that loosen the link between profits and investment in innovation. For instance, suppose that the government provides a subsidy to innovation, in the form of a lump-sum transfer $s_{jt}$ given to entrepreneurs in sector $j$ to finance investment in innovation.[35] The subsidy can be state contingent and sector specific, and it is financed with lump-sum taxes on households. Under these assumptions, the zero profit condition for research in sector $j$ becomes:[36]

$$V_t(\gamma A_{jt}) = \frac{P_t A_{jt}}{\chi}\left(1 - \frac{s_{jt}}{I_{jt}}\right),$$

where $V_t(\gamma A_{jt})$ is defined as in (38). The presence of the term $s_{jt}/I_{jt}$ is due to the fact that

[34] This result can also be derived using the graphical approach of figure 1. In fact, the only difference is that in the case of a discretionary central bank the upward portion of the $AD$ curve becomes a vertical line at $L = 1$.

[35] More precisely, we assume that in sector $j$ the government devotes an aggregate amount of resources $s_t A_j t$ to sustain innovation. These resources are equally divided among all the entrepreneurs operating in innovation in that sector.

[36] With the subsidy, the cost of investing $I_{jt}$ in research is $P_t(I_{jt} - s_{jt})$, which gives an expected gain of $\chi I_{jt} V_{jt}(\gamma A_{jt})/A_{jt}$. The zero expected profits condition for research in sector $j$ then implies:

$$P_t\left(I_{jt} - s_{jt}\right) = \frac{\chi I_{jt}}{A_{jt}} V_t(\gamma A_{jt}).$$

Rearranging this expression we obtain the expression in the main text.

entrepreneurs have to finance only a fraction $1 - s_{jt}/I_{jt}$ of the investment in research, while the rest is financed by the government. This expression implies that entrepreneurs are willing to invest in innovation even when the value of becoming a leader is zero, since if $V_t(\gamma A_{jt}) = 0$ then $I_{jt} = s_{jt}$. Hence, assuming that the government can ensure that entrepreneurs cannot divert the subsidy away from innovation activities, investment in innovation will be always at least equal to the subsidy $s_{jt}$, so $I_{jt} \geq s_{jt}$.

Let us now consider the macroeconomic implications of the subsidy. For simplicity, we keep on focusing on symmetric equilibria in which every sector $j$ has the same innovation probability, and hence we consider subsidies of the form $s_{jt} = s_t A_{jt}$. Assuming a positive subsidy $s_t > 0$, the growth equation (21) is replaced by:

$$\left(1 - \frac{s_t \chi(\gamma - 1)}{g_{t+1} - 1}\right) = \beta E_t \left[ \left(\frac{c_t}{c_{t+1}}\right)^{\sigma} g_{t+1}^{-\sigma} \chi \gamma \varpi L_{t+1} \right], \tag{40}$$

where to derive this expression we have followed the same steps taken in section 3 and used $I_{jt}/A_{jt} = \mu_t/\chi = (g_{t+1} - 1)/(\gamma - 1)$. Notice that the expression above implies that $g_{t+1} > 1$, since with the subsidy in place investment in innovation is always positive.

We start by showing that an appropriately chosen subsidy can eliminate the unemployment steady state. To gain intuition, consider a non-state contingent subsidy $s_t = s$. In this case, in steady state the growth equation becomes:

$$g^{\sigma} \left(1 - \frac{s\chi(\gamma - 1)}{g - 1}\right) = \beta \chi \gamma \varpi L. \tag{41}$$

Notice that the term in round brackets on the left-hand side of expression (41) is smaller than one, because $sA_{jt} < I_{jt}$. Hence, for given $L$, steady state growth is increasing in $s$.

Provided that the central bank sets $\bar{i}$ appropriately, the economy features a unique full employment steady state, and a higher subsidy is associated with faster growth in the full employment equilibrium. Now turn to the unemployment steady state. In the unemployment steady state productivity growth must be equal to $g_u = (\beta/\bar{\pi}^w)^{(1/(\sigma-1))}$, to satisfy households' Euler equation. As discussed in section 3, this condition implies that an unemployment steady state exists only if there exists a nonnegative value for employment such that productivity growth is equal to $(\beta/\bar{\pi}^w)^{(1/(\sigma-1))}$. However, a sufficiently high subsidy can guarantee that the growth rate of the economy will always be higher than $(\beta/\bar{\pi}^w)^{(1/(\sigma-1))}$. It follows that by setting a sufficiently high subsidy the government can rule out the possibility that the economy might fall in a permanent stagnation trap.

**Proposition 4** *Suppose that there is a subsidy to innovation satisfying:*

$$s > \left( \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma - 1}} - 1 \right) \frac{1}{\chi(\gamma - 1)}, \tag{42}$$

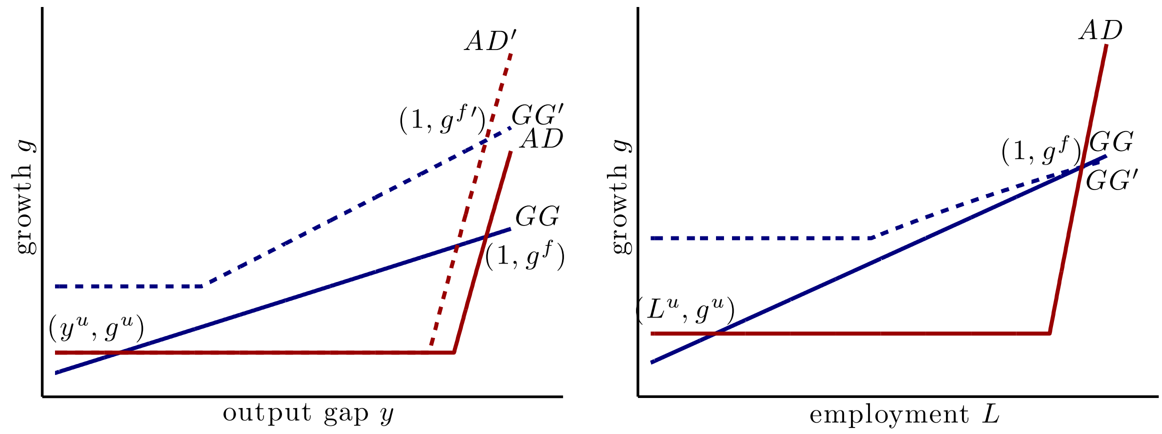


**Figure 5:** Subsidy to innovation.

*and that the central bank sets $\bar{i}$ according to:*

$$\bar{i} = \frac{\bar{\pi}^w g_f^{\sigma-1}}{\beta} - 1 \geq 1,$$

*where $g_f$ solves equation (41) evaluated at $L = 1$, and $\phi \geq 1$. Then there exists a unique steady state characterized by full employment.*

Intuitively, the subsidy to innovation guarantees that even if firms' profits were to fall substantially, investment in innovation would still be relatively high. In turn, a high investment in innovation stimulates growth and aggregate demand, since a high future income is associated with a high present demand for consumption. By implementing a sufficiently high subsidy, the government can eliminate the possibility that aggregate demand will be low enough to make the zero lower bound constraint on the nominal interest rate bind. It is in this sense that growth policies can be thought as a tool to manage aggregate demand in our framework. Importantly, to be effective a subsidy to innovation has to be large enough, otherwise it might not have any positive impact on the economy.

The left panel of figure 5 illustrates these points graphically. The solid lines correspond to the benchmark economy, while the dashed lines represent an economy with a subsidy to investment in innovation. The subsidy makes the $GG$ curve shift up, because for a given level of output gap the subsidy increases the growth rate of the economy.[37] Moreover, the $GG$ curve has a horizontal portion corresponding to the growth rate attained when only the subsidy is invested in research. If the subsidy is sufficiently high, as it is the case in the figure, the unemployment steady state disappears and the only possible steady state is the full employment one.

One potential issue is that ruling out stagnation traps with a constant subsidy to innovation could come at the cost of an inefficiently high investment in innovation in the full employment steady state. However, it is not difficult to design a subsidy scheme that rules out the unemployment steady state, while maintaining the full employment steady state unchanged. For example, consider

[37] The subsidy also induces a shift left of the $AD$ curve, because it has an impact on $\bar{i}$.

a countercyclical subsidy to innovation such that $s_t = s(1 - L_t)$.

Under this policy, the subsidy is decreasing in the output gap. In particular, the subsidy is equal to zero when the economy operates at full employment, so that the full employment steady state is not affected by the subsidy. However, if the subsidy is sufficiently large so that condition (42) holds, this countercyclical policy rules out the unemployment steady state.

The impact of a countercyclical subsidy is illustrated by the right panel of figure 5. The subsidy makes the $GG$ curve rotate up. If $s$ is sufficiently high, the movement of the curve is large enough to eliminate the unemployment steady state, while still leaving the full employment steady state unchanged. Hence, it is possible to design subsidy schemes that rule out the unemployment steady state without distorting the full employment one.

Summarizing, there is a role for subsidies to growth-enhancing investment, a typical supply side policy, in stimulating aggregate demand so as to rule out liquidity traps driven by expectations of weak future growth. In turn, the stimulus to aggregate demand has a positive impact on employment. In this sense, our model helps rationalizing the notion of job creating growth.

## 6 Conclusion

We develop a Keynesian growth model in which endogenous growth interacts with the possibility of involuntary unemployment due to weak aggregate demand. The model features two steady states. One is a stagnation trap, that is a permanent liquidity trap characterized by unemployment and weak growth. All it takes for the economy to fall into a stagnation trap is a shift toward pessimistic expectations about future growth. Aside form permanent liquidity traps, the model can also generate liquidity traps of arbitrarily long expected duration. We show that large policy interventions to support growth can lead the economy out of a stagnation trap, thus shedding light on the role of growth policies in stimulating aggregate demand and employment.

# Appendix

## A Proofs

### A.1 Proof of proposition 1 (existence, uniqueness and local determinacy of full employment steady state)

**Proof.** We start by proving existence. A steady state is described by the system (24)-(27). Setting $L = 1$ and using the first inequality in condition (28), equation (25) implies:

$$g_f = (\beta\chi\gamma\varpi)^{\frac{1}{\sigma}} \tag{A.1}$$

Then equation (24) implies:

$$1+i_f=\frac{\bar{\pi}^w g_f^{\sigma-1}}{\beta}. \tag{A.2}$$

Assumptions (17) and (18) guarantee that $i_f>0$, and that $\bar{i}=i_f$, so that the interest rate rule (27) is compatible with the existence of a full employment steady state. Moreover, combining equations (25) and (26) evaluated at $L=1$ and $g=g_f$, one can check that the second inequality in condition (28) ensures that $c_f>0$. Hence, a full employment steady state exists.

To prove uniqueness, consider that equation (A.1) implies that there is only one value of $g$ consistent with the full employment steady state, while equation (A.2) establishes that there is a unique value of $i$ consistent with $g=g_f$. Hence, the full employment steady state is unique.

We now show that, under the assumption $\phi>1$, the full employment steady state is locally determinate. A loglinear approximation of equations (20)−(23) around the full employment steady state gives:

$$(\sigma-1)\hat{g}_{t+1}=\hat{i}_t+\sigma(\hat{c}_t-E_t[\hat{c}_{t+1}]) \tag{A.3}$$

$$\hat{L}_t=\frac{c_f}{y^f}\hat{c}_t+\left(1-\frac{c_f}{y^f}\right)\frac{g_f}{g_f-1}\hat{g}_{t+1} \tag{A.4}$$

$$\sigma\hat{g}_{t+1}=\sigma(\hat{c}_t-E_t[\hat{c}_{t+1}])+E_t\left[\hat{L}_{t+1}\right] \tag{A.5}$$

$$\hat{i}_t=\phi\hat{L}_t, \tag{A.6}$$

where $\hat{x}\equiv\log(x_t)-\log(x^f)$ for every variable $x$, except for $\hat{g}_t\equiv g_t-g_f$ and $\hat{i}\equiv i_t-\bar{i}$, while $y^f\equiv(\alpha/\xi)^{\alpha/(1-\alpha)}(1-\alpha/\xi)$ is GDP normalized by the productivity index. This system can be written as:

$$\hat{L}_t=\xi_1E_t[\hat{L}_{t+1}]+\xi_2E_t[g_{t+2}] \tag{A.7}$$

$$\hat{g}_{t+1}=\xi_3E_t[\hat{L}_{t+1}]+\xi_4E_t[g_{t+2}], \tag{A.8}$$

where

$$\xi_1\equiv\frac{\frac{y^f}{c_f}-\frac{1}{\sigma}+\left(1+\frac{y^f-c_f}{c_f}\frac{g_f}{g_f-1}\right)}{\frac{y^f}{c_f}+\phi\left(1+\frac{y^f-c_f}{c_f}\frac{g_f}{g_f-1}\right)}$$

$$\xi_2\equiv-\frac{\frac{y^f-c_f}{c_f}\frac{g_f}{g_f-1}}{\frac{y^f}{c_f}+\phi\left(1+\frac{y^f-c_f}{c_f}\frac{g_f}{g_f-1}\right)}$$

$$\xi_3\equiv1-\phi\xi_1$$

$$\xi_4\equiv-\phi\xi_2.$$

The system is determinate if and only if:

$$|\xi_1\xi_4-\xi_2\xi_3|<1 \tag{A.9}$$

$$|\xi_1+\xi_4|<1+\xi_1\xi_4-\xi_2\xi_3. \tag{A.10}$$

Condition ($A.9$) holds if:

$$\phi > \frac{\frac{y^f - c_f}{y^f}\frac{g_f}{g_f - 1} - 1}{\frac{c_f}{y^f} + \frac{y^f - c_f}{y^f}\frac{g_f}{g_f - 1}},$$

while condition ($A.10$) holds if:

$$\phi > 1 - \frac{1}{\sigma}.$$

Since $c_f < y_f$, $\phi > 1$ is sufficient for both conditions to hold. ■

## A.2 Proof of proposition 2 (existence and uniqueness of unemployment steady state)

**Proof.** We start by showing that it is not possible to have an unemployment steady state with a positive nominal interest rate. Suppose that this is not the case, and that there is a steady state with $1 + i = (1 + \bar{i})L^{\phi}$ and $0 \leq L < 1$. Then combining equations (24) and (25), and using $\beta\chi\gamma\varpi = g_f$ and $1 + \bar{i} = g_f^{\sigma-1}\bar{\pi}^w/\beta$ gives:

$$g_f L^{\frac{\phi}{\sigma-1}} = \left(\max\left(g_f^{\sigma} L, 1\right)\right)^{\frac{1}{\sigma}}. \tag{A.11}$$

Assumption (19) implies that the left-hand side of this expression is smaller than the right-hand side for any $0 \leq L < 1$. Hence, we have found a contradiction and an unemployment steady state with $i > 0$ is not possible.

We now prove that an unemployment steady state with $i = 0$ exists and is unique. Setting $i = 0$, equation (24) implies that there is a unique value of $g = (\beta/\bar{\pi}^w)^{1/(\sigma-1)} = g_u$ consistent with the unemployment steady state. Moreover, since $i_f > 0$ equation (24) also implies that $g_u < g_f$, while the first inequality in condition (29) implies $g_u > 1$. Evaluating equation (25) at $g = g_u$ we have:

$$L_u = \frac{g_u^{\sigma}}{\beta\chi\gamma\varpi},$$

ensuring that there is a unique value of $L = L_u > 0$ consistent with $g = g_u$. Moreover, using $g_u < g_f$ and equation (25) gives $L_u < 1$. Combining equations (25) and (26) evaluated at $L = L_u$ and $g = g_u$, one can check that the second inequality in condition (29) ensures that $c_u > 0$. Hence, the unemployment steady state exists and is unique. Finally, using $i_f > 0$ and $g_u < g_f$ one can see that $1/\pi_u = g_u/\bar{\pi}^w < (1 + i_f)g_f/\bar{\pi}^w = (1 + i_f)/\pi_f$, so that the real interest rate in the unemployment steady state is lower than the one in the full employment steady state.

To prove local indeterminacy one can follow the steps of the proof to proposition one. Since in the neighborhood of the unemployment steady state $\hat{i}_t = 0$, it is easy to show that condition ($A.10$) cannot be satisfied. ■

## A.3 Proof to proposition 3 (optimal discretionary monetary policy)

**Proof.** Under discretion, every period the central bank maximizes the representative household expected utility subject to (20), (21), (39), $L_t \leq 1$ and $i_t \geq 0$, taking future variables as given. The central bank's problem can be written as:

$$\max_{L_t, c_t, g_{t+1}, i_t} E_t\left[\sum_{\tau=t}^{\infty} \beta^{\tau}\left(\frac{(C_{\tau}^{1-\sigma}-1}{1-\sigma}\right)\right] = E_t\left[\beta^t A_t^{1-\sigma}\left(\frac{c_t^{1-\sigma}}{1-\sigma} + g_{t+1}\nu_t^1\right)\right] - \frac{1}{(1-\beta)(1-\sigma)},$$

subject to:

$$L_t = \frac{1}{\Psi}\left(c_t + \frac{g_{t+1}-1}{\chi(\gamma-1)}\right)$$

$$c_t = \left(\frac{g_{t+1}^{\sigma-1}}{1+i_t}\right)^{\frac{1}{\sigma}} \nu_t^2$$

$$g_{t+1} = \max\left(1, \frac{\nu_t^3}{1+i_t}\right)$$

$$L_t \leq 1$$

$$i_t \geq 0,$$

where the third constraint is obtained by combining (20) and (21), and:

$$\Psi \equiv \left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}}\left(1-\frac{\alpha}{\xi}\right) \tag{A.12}$$

$$\nu_t^1 = E_t\left[\sum_{\tau=t+1}^{\infty} \beta^{\tau}\frac{\left(c_{\tau}\Pi_{\hat{\tau}=t+2}^{\tau} g_{\hat{\tau}}\right)^{1-\sigma}}{1-\sigma}\right)]$$

$$\nu_t^2 = \left(\frac{\bar{\pi}^w}{\beta E_t\left[c_{t+1}^{-\sigma}\right]}\right)^{\frac{1}{\sigma}}$$

$$\nu_t^3 = \bar{\pi}^w \chi\gamma\varpi\frac{E_t[L_{t+1}c_{t+1}^{-\sigma}]}{E_t[c_{t+1}^{-\sigma}]}.$$

$\nu_t^1$, $\nu_t^2$ and $\nu_t^3$ are taken as given by the central bank, because they are function of parameters and expectations about future variables only.

Notice that the objective function is strictly increasing in $c_t$ and $g_{t+1}$. Also notice that from the second and third constraints we can write $c_t = c(i_t)$ with $c'(i_t) < 0$ and $g_{t+1} = g(i_t)$ with $g'(i_t) \leq 0$. We can then rewrite the problem of a central bank under discretion as

$$\min i_t,$$

subject to:

$$L_t = \frac{1}{\Psi}\left(c(i_t) + \frac{g(i_t)-1}{\chi(\gamma-1)}\right)$$

$$L_t \leq 1$$

$$i_t \geq 0.$$

The solution to this problem can be expressed by the complementary slackness condition:

$$i_t \left(L_t - 1\right) = 0.$$

■

### A.4 Proof of proposition 4 (steady state with subsidy to innovation)

The proof that a full employment steady state exists and is unique follows the steps of the proof to proposition 1.

We now prove that there is no steady state with unemployment. Following the proof to proposition 2, one can check that if another steady state exists, it must be characterized by $i = 0$. Equation (20) implies that in this steady state growth must be equal to $(\beta/\bar{\pi}^w)^{(1/(\sigma-1))}$. But with the subsidy in place the lowest growth rate possible is obtained by setting $L = 0$ in equation (41) and is equal to $1 + s\chi(\gamma - 1)$. Then, since condition (42) implies $1 + s\chi(\gamma - 1) > (\beta/\bar{\pi}^w)^{(1/(\sigma-1))}$, an unemployment steady state is not possible. ■

## B Model with money

In this appendix we explicitly introduce money in the model. The presence of money rationalizes the zero lower bound constraint on the nominal interest rate, but does not alter the equilibrium conditions of the model.

Following Krugman (1998) and Eggertsson (2008) we assume that households need to hold at least a fraction $\nu > 0$ of production in money balances $M$:

$$M_t \geq \nu P_t Y_t. \tag{B.1}$$

The household's budget constraint is now:

$$P_t C_t + \frac{b_{t+1}}{1 + i_t} + M_t = W_t L_t + b_t + M_{t-1} + d_t - T_t, \tag{B.2}$$

where $M_t$ denotes money holdings at time $t$ to be carried over at time $t + 1$, and $T$ are lump-sum taxes paid to the government.

The optimality condition with respect to $M_t$ is:

$$\lambda_t = \beta E_t \left[\lambda_{t+1}\right] + \mu_t, \tag{B.3}$$

where $\mu_t > 0$ is the Lagrange multiplier on constraint ($B.1$). Combining optimality conditions (3) and ($B.3$) it is easy to see that the presence of money implies $i_t \geq 0$. Intuitively, households will always prefer to hold money, rather than an asset which pays a non-contingent negative nominal return. It is also easy to see that constraint ($B$.1) binds if $i_t > 0$, but it is slack if $i_t = 0$. Hence, households' money demand is captured by the complementary slackness condition:

$$i_t \left( M_t - \nu P_t \left( \frac{\alpha}{\xi} \right)^{\frac{\alpha}{1-\alpha}} A_t L_t \right) = 0, \tag{B.4}$$

with $i_t \geq 0$ and $M_t \geq \nu P_t \left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}} A_t L_t$, where we have substituted $Y_t$ using equation (9).

To close the model, we assume that the government runs a balanced budget:

$$T_t = M_t - M_{t-1},$$

so that seignorage revenue is rebated to households via lump sum taxes.

We can now define an equilibrium as a set of processes $\{L_t, c_t, g_{t+1}, i_t, M_t, P_t\}_{t=0}^{+\infty}$ satisfying equations (20) – (23), ($B$.4) and $P_t = \bar{\pi}^w P_{t-1}/g_t$, given initial values $P_{-1}$, $A_0$.[38]

Notice that to solve for the path of $L_t$, $c_t$, $g_{t+1}$ and $i_t$ only equations (20) – (23) are needed. Given values for $L_t$, $A_t$ and $P_t$, the only use of the money demand equation ($B$.4) is to define the money supply $M_t$ consistent with the central bank's interest rate rule. Specifically, when $i_t > 0$ the equilibrium money supply is $M_t = \nu P_t (\alpha/\xi)^{\frac{\alpha}{1-\alpha}} A_t L_t$, while when $i_t = 0$ any money supply $M_t \geq \nu P_t (\alpha/\xi)^{\frac{\alpha}{1-\alpha}} A_t L_t$ is consistent with equilibrium.

These results do not rest on the specific source of money demand assumed. For instance, similar results can be derived in a setting in which households derive utility from holding real money balances, as long as a cashless limit is considered (Eggertsson and Woodford, 2003).

# C Model with unemployment risk

In this appendix, we lay down the model with idiosyncratic unemployment risk described in section 4.2. In this model, each household faces in every period a constant probability $p$ of being unemployed. The employment status is revealed to the household at the start of the period. An unemployed household receives an unemployment benefit, such that its income is equal to a fraction $b < 1$ of the income received by employed households. Unemployment benefits are financed with taxes on employed households.

The budget constraint of a household now becomes:

$$P_t C_t + \frac{b_{t+1}}{1 + i_t} = \nu_t W_t L_t + b_t + d_t + T_t.$$

The only change with respect to the benchmark model is the presence of the variables $\nu$ and $T$, which summarize the impact of the employment status on a household's budget. $\nu$ is an indicator variable that takes value 1 if the household is employed, and 0 if the household is unemployed. $T$

[38]To derive the law of motion for $P_t$ we have used the equilibrium condition $\pi_t = \pi_t^w / g_t$ and the law of motion for $W_t$.

represents a lump-sum transfer for unemployed households, and a tax for employed households. $T$ is set such that the income of an unemployed household is equal to a fraction $b$ of the income of an employed household.[39]

Moreover, here we assume that households cannot borrow:

$$b_{t+1} \geq 0,$$

and that trade in firms' shares is not possible, so that every household receives the same dividends $d$.

The Euler equation is now:

$$c_t^{-\sigma} = \frac{\beta(1+i_t)E_t\left[c_{t+1}^{-\sigma}\right]}{g_{t+1}^{\sigma-1}\bar{\pi}^w} + \mu_t,$$

where $\mu_t$ is the Lagrange multiplier on the borrowing constraint and, as in the main text, $c_t \equiv C_t/A_t$.

We start by showing that the borrowing constraint binds only for unemployed households. Since neither households nor firms can borrow, in equilibrium every period every household consumes her entire income. Denoting, by $c^e$ and $c^u$ the consumption of respectively an employed and an unemployed household, we have $c_t^u = bc_t^e < c_t^e$. Moreover, due to the assumption of i.i.d. idiosyncratic shocks, $E_t\left[c_{t+1}^{-\sigma}\right]$ is independent of the employment status. Hence, from the Euler equation it follows that $\mu_t > 0$ only for the unemployed, and so the borrowing constraint does not bind for employed households.

The Euler equation of the employed households is:

$$(c_t^e)^\sigma = \frac{\bar{\pi}^w g_{t+1}^{\sigma-1}}{\beta(1+i_t)\rho E_t\left[\left(c_{t+1}^e\right)^{-\sigma}\right]},$$

where $\rho \equiv 1-p+p/b^\sigma > 1$, and we have used the fact that the probability of becoming unemployed is independent of aggregate shocks. Moreover, using $c_t = pc_t^u + (1-p)c_t^e = c_t^e(bp+1-p)$, we obtain:

$$c_t^\sigma = \frac{\bar{\pi} g_{t+1}^{\sigma-1}}{\beta(1+i_t)\rho E_t\left[c_{t+1}^{-\sigma}\right]}.$$

This equation, which is the equivalent of equation (20) in the baseline model, determines the demand for consumption in the model with idiosyncratic unemployment risk.

[39] More precisely, an unemployed household receives a transfer:

$$T = \frac{bW_tL_t + (b-1)d_t}{1+\frac{bp}{1-p}},$$

while an employed household pays a tax

$$T = -\frac{p}{1-p}\frac{bW_tL_t + (b-1)d_t}{1+\frac{bp}{1-p}}.$$

# D The cases of $\sigma = 1$ and $\sigma < 1$

In the main text we have focused attention on the empirically relevant case of low elasticity of intertemporal substitution, by assuming that $\sigma > 1$. In this appendix we consider the alternative cases $\sigma = 1$ and $\sigma < 1$. The key result is that under these cases the steady state is unique.

We start by analyzing the case of $\sigma = 1$. In steady state, equation (24) can be written as:

$$1 = \frac{\beta(1+i)}{\bar{\pi}^w}.$$

Intuitively, under this case changes in the growth rate of the economy have no impact on the equilibrium nominal interest rate. Hence, if there exists a full employment equilibrium featuring a positive nominal interest rate, it is easy to check that no unemployment equilibrium can exist.

We now turn to the case $\sigma < 1$. Under this case, equation (24) implies a negative relationship between growth and the nominal interest rate. Supposing that a full employment equilibrium featuring a positive nominal interest rate exists, if a liquidity trap equilibrium exists, it must feature a higher growth rate than the full employment one, i.e. $g_u > g_f$. Since $L_f = 1$, it must be the case that $L_u \leq L_f$. But equation (25) implies a nonnegative steady state relationship between $g$ and $L$. Then we cannot have a steady state in which $g_u > g_f$ and $L_u \leq L_f$, so that, if the economy features a full employment steady state, an unemployment steady state cannot exist.

## References


Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous (2012) "The Environment and Directed Technical Change," *The American Economic Review*, Vol. 102, No. 1, pp. 131–166. 10

Acemoglu, Daron and Ufuk Akcigit (2012) "Intellectual property rights policy, competition and innovation," *Journal of the European Economic Association*, Vol. 10, No. 1, pp. 1–42. 25, **??**, 4.3

Aghion, P and P Howitt (2009) "The economics of growth." 10

Aghion, Philippe, Ufuk Akcigit, and Jesús Fernández-Villaverde (2013) "Optimal capital versus labor taxation with innovation-led growth," NBER Working Paper No. 19086. 4.3

Aghion, Philippe, George-Marios Angeletos, Abhijit Banerjee, and Kalina Manova (2010) "Volatility and growth: Credit constraints and the composition of investment," *Journal of Monetary Economics*, Vol. 57, No. 3, pp. 246–265. 1

Aghion, Philippe, Philippe Bacchetta, Romain Ranciere, and Kenneth Rogoff (2009) "Exchange rate volatility and productivity growth: The role of financial development," *Journal of Monetary Economics*, Vol. 56, No. 4, pp. 494–513. 1

Aghion, Philippe, David Hemous, and Enisse Kharroubi (2014) "Cyclical fiscal policy, credit constraints, and industry growth," *Journal of Monetary Economics*, Vol. 62, pp. 41–58. 1

Aghion, Philippe and Peter Howitt (1992) "A Model of Growth Through Creative Destruction," *Econometrica*, Vol. 60, No. 2, pp. 323–351. 1, 5, 7, 32

Aiyagari, S.R. (1994) "Uninsured idiosyncratic risk and aggregate saving," *The Quarterly Journal of Economics*, Vol. 109, No. 3, pp. 659–684. 24

Azariadis, Costas (1996) " The Economics of Poverty Traps: Part One: Complete Markets," *Journal of Economic Growth*, Vol. 1, No. 4, pp. 449–96. 1

Azariadis, Costas and John Stachurski (2005) "Poverty Traps," in Philippe Aghion and Steven Durlauf eds. *Handbook of Economic Growth*, Vol. 1 of Handbook of Economic Growth: Elsevier, Chap. 5. 4

Bacchetta, Philippe and Eric Van Wincoop (2013) "The great recession: a self-fulfilling global panic," NBER Working Paper No. 19062. 1

Ball, Laurence M (2014) "Long-Term Damage from the Great Recession in OECD Countries," NBER Working Paper No. 20185. 1

Barattieri, A., S. Basu, and P. Gottschalk (2010) "Some evidence on the importance of sticky wages," NBER Working Paper No. 16130. 2

Barlevy, Gadi (2004) "The Cost of Business Cycles Under Endogenous Growth," *The American Economic Review*, Vol. 94, No. 4, pp. 964–990. 1

Barro, Robert J. and Xavier Sala-i Martin (2004) *Economic Growth*: MIT Press, 2nd edition. 6, 8

Basu, Susanto and Miles Kimball (2002) "Long-run labor supply and the elasticity of intertemporal substitution for consumption," manuscript, University of Michigan. 16

Benhabib, Jess and Roger EA Farmer (1994) "Indeterminacy and increasing returns," *Journal of Economic Theory*, Vol. 63, No. 1, pp. 19–41. 1

——— (1996) "Indeterminacy and sector-specific externalities," *Journal of Monetary Economics*, Vol. 37, No. 3, pp. 421–443. 1

Benhabib, Jess, Stephanie Schmitt-Grohé, and Martın Uribe (2001) "The perils of Taylor rules," *Journal of Economic Theory*, Vol. 96, No. 1, pp. 40–69. 3, 3.1

Bernanke, B.S. and K. Carey (1996) "Nominal wage stickiness and aggregate supply in the Great Depression," *The Quarterly Journal of Economics*, Vol. 111, No. 3, pp. 853–883. 2

Bewley, T. (1977) "The Permanent Income Hypothesis and Welfare Economics," *Journal of Economics Theory*, Vol. 16, No. 2, pp. 252–292. 24

Blanchard, O., E. Cerutti, and L. Summers (2015) "Inflation and activity - two explorations, and their monetary policy Implications," Manuscript. 1

Cerra, V. and S.C. Saxena (2008) "Growth dynamics: the myth of economic recovery," *The American Economic Review*, Vol. 98, No. 1, pp. 439–457. 1

Christiano, Lawrence J, Martin S Eichenbaum, and Mathias Trabandt (2015) "Understanding the Great Recession," *American Economic Journal: Macroeconomics*, Vol. 7, No. 1, pp. 110–167. 1, 29

Christiano, L.J., M. Eichenbaum, and C.L. Evans (2005) "Nominal rigidities and the dynamic effects of a shock to monetary policy," *Journal of political Economy*, Vol. 113, No. 1, pp. 1–45. 2

Chu, Angus C and Guido Cozzi (2014) "R&D and Economic Growth in a Cash-In-Advance Economy," *International Economic Review*, Vol. 55, No. 2, pp. 507–524. 1

Comin, Diego and Mark Gertler (2006) "Medium-Term Business Cycles," *The American Economic Review*, pp. 523–551. 1

Eggertsson, G. and P. Krugman (2012) "Debt, deleveraging, and the liquidity trap: a Fisher-Minsky-Koo approach," *The Quarterly Journal of Economics*, Vol. 127, No. 3, pp. 1469–1513. 3

Eggertsson, G. and M. Woodford (2003) "The zero bound on interest rates and optimal monetary policy," *Brookings Papers on Economic Activity*, Vol. 2003, No. 1, pp. 139–211. 3, B

Eggertsson, Gauti B (2008) "Great Expectations and the End of the Depression," *The American Economic Review*, Vol. 98, No. 4, pp. 1476–1516. 3, B

Eggertsson, Gauti and Neil Mehrotra (2014) "A Model of Secular Stagnation," Mimeo, Brown University. 1

Eichengreen, B. and J. Sachs (1985) "Exchange Rates and Economic Recovery in the 1930s," *The Journal of Economic History*, Vol. 45, No. 4, pp. 925–946. 2

Fabiani, S., C. Kwapil, T. Rõõm, K. Galuscak, and A. Lamo (2010) "Wage rigidities and labor market adjustment in Europe," *Journal of the European Economic Association*, Vol. 8, No. 2-3, pp. 497–505. 2

Farmer, Roger EA (2012) "Confidence, Crashes and Animal Spirits," *The Economic Journal*, Vol. 122, No. 559, pp. 155–172. 1

Fatas, Antonio (2000) "Do business cycles cast long shadows? Short-run persistence and economic growth," *Journal of Economic Growth*, Vol. 5, No. 2, pp. 147–162. 1

Fehr, E. and L. Goette (2005) "Robustness and real consequences of nominal wage rigidity," *Journal of Monetary Economics*, Vol. 52, No. 4, pp. 779–804. 2

Francois, Patrick and Huw Lloyd-Ellis (2003) "Animal spirits through creative destruction," *American Economic Review*, pp. 530–550. 1

Galor, Oded and Joseph Zeira (1993) "Income Distribution and Macroeconomics," *Review of Economic Studies*, Vol. 60, No. 1, pp. 35–52. 1

Gottschalk, P. (2005) "Downward nominal-wage flexibility: real or measurement error?" *Review of Economics and Statistics*, Vol. 87, No. 3, pp. 556–568. 2

Grossman, Gene M and Elhanan Helpman (1991) "Quality ladders in the theory of growth," *The Review of Economic Studies*, Vol. 58, No. 1, pp. 43–61. 1, 5, 7, 8, 32

Guerrieri, V. and G. Lorenzoni (2011) "Credit crises, precautionary savings, and the liquidity trap," NBER Working Paper No. 17583. 3

Hall, Robert E (1988) "Intertemporal Substitution in Consumption," *The Journal of Political Economy*, Vol. 96, No. 2, pp. 339–357. 2.7

Hansen, Alvin H (1939) "Economic progress and declining population growth," *The American Economic Review*, pp. 1–15. 1

Howitt, Peter and Philippe Aghion (1998) "Capital accumulation and innovation as complementary factors in long-run growth," *Journal of Economic Growth*, Vol. 3, No. 2, pp. 111–130. 8

Huggett, M. (1993) "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of economic Dynamics and Control*, Vol. 17, No. 5-6, pp. 953–969. 24

Kiyotaki, Nobuhiro (1988) "Multiple expectational equilibria under monopolistic competition," *The Quarterly Journal of Economics*, pp. 695–713. 1

Krugman, P. (1998) "It's baaack: Japan's slump and the return of the liquidity trap," *Brookings Papers on Economic Activity*, Vol. 1998, No. 2, pp. 137–205. 3, B

Krugman, Paul (2013) "Secular Stagnation, Coalmines, Bubbles, and Larry Summers," *New York Times,(Novemeber 16)*. 1

Matsuyama, Kiminori (1991) "Increasing Returns, Industrialization, and Indeterminacy of Equilibrium," *The Quarterly Journal of Economics*, Vol. 106, No. 2, pp. 617–50. 1

Murphy, Kevin M, Andrei Shleifer, and Robert W Vishny (1989) "Industrialization and the Big Push," *Journal of Political Economy*, Vol. 97, No. 5, pp. 1003–26. 1

Nuño, Galo (2011) "Optimal research and development and the cost of business cycles," *Journal of Economic Growth*, Vol. 16, No. 3, pp. 257–283. 1

Ogaki, Masao and Carmen M Reinhart (1998) "Measuring intertemporal substitution: The role of durable goods," *Journal of Political Economy*, Vol. 106, No. 5, pp. 1078–1098. 16

Olivei, G. and S. Tenreyro (2007) "The timing of monetary policy shocks," *The American Economic Review*, pp. 636–663. 2

Queraltó, Albert (2013) "Financial Market Frictions, Productivity Growth and Crises in Emerging Economies," Mimeo, Federal Reserve Board. 1

Romer, Paul M (1990) "Endogenous technological change," *Journal of political Economy*, pp. S71–S102. 1

Schmitt-Grohé, S. and M. Uribe (2011) "Pegs and Pain," NBER Working Paper No. 16847. 2

Schmitt-Grohé, Stephanie and Martín Uribe (2012) "The making of a great contraction with a liquidity trap and a jobless recovery," NBER Working Paper. 4.2, 4.3, 28, 5.1

Summers, Lawrence (2013) "Why stagnation might prove to be the new normal," *The Financial Times*. 1

Vissing-Jørgensen, Annette (2002) "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economy*, Vol. 110, No. 4. 16

Werning, Ivan (2011) "Managing a Liquidity Trap: Monetary and Fiscal Policy," NBER Working Paper No. 17344. 3