Designing a Simple Loss Function for the Fed: Does the Dual Mandate Make Sense?*

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Abstract

Yes, it makes a lot of sense. Using the Smets and Wouters (2007) model of the U.S. economy, we find that the role of the output gap should be equal to or even more important than that of inflation when designing a simple loss function to represent household welfare. Moreover, we document that a loss function with nominal wage inflation and the hours gap provides an even better approximation of the true welfare function than a standard objective based on inflation and the output gap. Our results hold up when we introduce interest rate smoothing in the simple mandate to capture the observed gradualism in policy behavior and to ensure that the probability of the federal funds rate hitting the zero lower bound is negligible.

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1 Introduction

Variable and high rates of price inflation in the 1970s and 1980s led many countries to delegate the conduct of monetary policy to "instrument-independent" central banks. Drawing on learned experiences, many societies gave their central banks a clear mandate to pursue price stability and instrument independence to achieve it.[1] Advances in academic research, notably the seminal work of Rogoff (1985) and Persson and Tabellini (1993), supported a strong focus on price stability as a means to enhance the independence and credibility of monetary policymakers. As discussed in further detail in Svensson (2010), an overwhelming majority of these central banks also adopted an explicit inflation target to further strengthen credibility and facilitate accountability. One exception to common central banking practice is the U.S. Federal Reserve, which since 1977 has been assigned the so-called "dual mandate" which requires it to "promote maximum employment in a context of price stability". Only as recently as January 2012, the Fed finally announced an explicit long-run inflation target, but also made clear its intention to keep a balanced approach between mitigating deviations of both inflation and employment from target.

Although the Fed has established credibility for the long-run inflation target, an important question is whether its heavy focus on resource utilization can be justified. Our reading of the academic literature up to date, perhaps most importantly the seminal work by Woodford (2003), is that resource utilization should be assigned a small weight relative to inflation under the reasonable assumption that the underlying objective of monetary policy is to maximize welfare of the households inhabiting the economy. Drawing on results in Rotemberg and Woodford (1998), Woodford (2003) showed that the objective function of households in a basic New Keynesian sticky-price model could be approximated as a (purely) quadratic function in inflation and the output gap, with the weights determined by the specific features of the economy. A large literature that followed used these insights to study various aspects of optimal monetary policy.[2] A potential drawback with the main body of this literature is that it focused on relatively simple calibrated (or partially estimated) models. Our goal in this paper is to revisit this issue within the context of an estimated

[1] The academic literature often distinguishes between *goal-* and *instrument-independent* central banks. Goal independence, i.e. the freedom of the central bank to set its own goals, is difficult to justify in a democratic society. However, instrument independence, i.e. the ability of the central bank to determine the appropriate settings of monetary policy to achieve a given mandate without political interference, is arguably less contentious if the central bank can be held accountable for its actions.

[2] As a prominent example, Erceg, Henderson and Levin (2000) showed that when both wages and prices are sticky, wage inflation enters into the quadratic approximation in addition to price inflation and the output gap. Within an open economy context, Benigno and Benigno (2008) studied how international monetary cooperative allocations could be implemented through inflation targeting aimed at minimizing a quadratic loss function consisting of only domestic variables such as GDP inflation and the output gap.

medium-scale model of the U.S. economy. Specifically, we use the workhorse Smets and Wouters (2007) model—SW henceforth—of the U.S. economy to examine how a simple objective for the central bank should be designed in order to approximate the welfare of households in the model economy as closely as possible. For instance, does the Federal Reserve's strong focus on resource utilization improve households' welfare relative to a simple mandate that focuses more heavily on inflation?

Even though it is optimal and ideal to implement the Ramsey policy directly, the overview of central banking mandates by Reis (2013) and Svensson (2010) shows that most advanced countries have not asked their central bank to implement such a policy for society. Instead, many central banks are mandated to follow a simple objective that involves only a small number of economic variables.[3] We believe there are several important reasons for assigning a simple mandate. First, it would be for all practical purposes infeasible to describe the utility-based welfare criterion for an empirically plausible model, as it would include too high a number of targets in terms of variances and covariances of different variables.[4] Instead, a simple objective facilitates communication of policy actions with the public and makes the conduct of monetary policy more transparent. Second, a simple mandate also enhances accountability of the central bank, which is of key importance. Third and finally, prominent scholars like Svensson (2010) argue that a simple mandate is more robust to model and parameter uncertainty than a complicated state-contingent Ramsey policy.[5]

Given the widespread discussion and adoption of simple mandates, we analyze how these perform relative to the Ramsey policy. In that sense, our exercise is similar in spirit to the literature designing simple interest rate rules (see for example Kim and Henderson, 2005, and Schmitt-Grohé and Uribe, 2007). As a final exercise, we complement our extensive analysis of simple mandates with a brief analysis of simple rules: we are interested in knowing how simple interest rate rules compare with simple mandates. Of key interest to us is also whether the widely used rules proposed by Taylor (1993, 1999) approximates Ramsey policy as well as a simple mandate.

We assume that the central bank operates under commitment when maximizing its simple objective.[6] We believe commitment is a good starting point for three reasons. First, the evidence provided by Bodenstein, Hebden and Nunes (2012), Debortoli, Maih and Nunes (2014), and De-

[3] The dual mandate was codified only in the Federal Reserve Reform Act of 1977. See Bernanke (2013) for a summary of Federal Reserve's one hundred years.

[4] For instance, the utility-based welfare criterion in the SW model contains more than 90 target variables. See also Edge (2003), who derives analytically the welfare criterion for a model with capital accumulation.

[5] As an alternative to simple mandates, Taylor and Williams (2009) argue in favor of simple and robust policy rules.

[6] By contrast, Rogoff (1985) assumes that the central bank operates under discretion.

bortoli and Lakdawala (2013) suggests that the Federal Reserve operates with a high degree of commitment. Second, the University of Michigan and the Survey of Professional Forecasters measures of long-term expected inflation rates have remained well anchored during the crisis. This indicates that the Federal Reserve was able to credibly commit to price stability, although it has communicated a strong emphasis on stabilizing the real economy. Third, since simple interest rate rules as well as Ramsey policy imply commitment, this assumption enables us to directly compare such frameworks with the simple objectives we consider.

As noted earlier, we adopt the SW model in our analysis. This model represents a prominent example of how the U.S. economy can be described by a system of dynamic equations consistent with optimizing behavior. As such, it should be less prone to the Lucas (1976) critique than other prominent studies on optimal monetary policy that are based on backward-looking models (see e.g. Rudebusch and Svensson, 1999, and Svensson, 1997).[7] Moreover, many of the existing papers which use models based on optimizing behavior have often relied on simple calibrated models without capital formation.[8] Even though policy recommendations are model consistent, their relevance may be questioned given the simplicity of these models and the fact that they have not been estimated. By conducting normative analysis with an empirically realistic model, this paper achieves the objective of providing theoretically coherent yet empirically relevant policy recommendations.

A conventional procedure for estimating such a model, following the seminal work of Smets and Wouters (2003), is to form the likelihood function for a first-order approximation of the dynamic equations and to use Bayesian methods to update the priors of the deep parameters. Doing so yields a posterior distribution for the parameters. In a normative analysis that involves an evaluation of a specific criterion function, it may be important to allow for both parameter and model uncertainty.[9] However, before doing such a fully fledged analysis, we believe it is instructive to start out by performing a normative exercise in the context of a specific model and specific parameter values. We assume that the parameters in the SW model are fixed at their posterior mode, and the optimal policy exercises take as constraints all the SW model equations except the estimated ad hoc monetary policy rule. Instead, the central bank pursues policy to best achieve the objective that it is mandated to accomplish.

[7] Consistent with this argument, several papers estimating dynamic general-equilibrium models that are closely related to the SW model have also found that the deep parameters are largely invariant to alternative assumptions about the conduct of monetary policy. For example, see Adolfson, Laséen, Lindé and Svensson (2011), Ilbas (2012), and Chen, Kirsanova and Leith (2013).

[8] See e.g. the classical paper by Clarida, Gali and Gertler (1999).

[9] See Walsh (2005) as an example.

Our main findings are as follows. First, we find that adding a term involving a measure of real activity in the objective function appears to be much more important than previously thought. A positive weight on any of the typical variables like the output gap, the level of output, and the growth rate of output improves welfare significantly. Moreover, among these standard activity measures, a suitably chosen weight on the model-consistent output gap delivers the lowest welfare loss. Specifically, we find that in a simple loss function—with the weight on annualized inflation normalized to unity—the optimized weight on the output gap is about 1. This is considerably higher than the reference value of 0.048 derived in Woodford (2003) and the value of 0.25 assumed by Yellen (2012).[10] In our model, the chosen weight for the output gap has important implications for inflation volatility, as the model features a prominent inflation-output gap trade-off along the efficient frontier as defined in the seminal work of Taylor (1979) and Clarida, Galí and Gertler (1999). Our basic finding that the central bank should respond vigorously to resource utilization is consistent with the arguments in Reifschneider, Wascher and Wilcox (2013) and English, López-Salido and Tetlow (2013).

At first glance, our results may appear to be contradictory to Justiniano, Primiceri and Tambalotti (2013), who argue that there is no important trade-off between stabilizing inflation and the output gap. However, the different findings can be reconciled by recognizing that the key drivers behind the trade-off in the SW model—the price- and wage-markup shocks—are absent in the baseline model analyzed by Justiniano et al. (2013).[11] While our reading of the literature is that considerable uncertainty remains about the role of these inefficient shocks as drivers of business cycle fluctuations, our results hold irrespectively. In particular, if inefficient shocks are irrelevant for business cycle fluctuations, then stabilizing inflation or output is approximately equivalent and attaching a high weight to output is still optimal. And as long as inefficient shocks do play some role as in SW, then the high weight on output stabilization becomes imperative. Furthermore, we demonstrate that our findings apply even when only one of the markup shocks is present or when the variance of both the inefficient price- and wage-markup shocks are reduced substantially, following for instance the recent evidence provided in Galí, Smets and Wouters (2011).

Our second important finding is that a loss function with nominal wage inflation and the hours gap provides an even better approximation to the household true welfare function than a simple standard inflation-output gap based objective. As is the case with the inflation-output gap based

[10] Yellen (2012) assumed a value of unity for the unemployment gap, which by the Okun's law translates into a value of 0.25 for the output gap.

[11] The alternative model of Justiniano et al. (2013) includes wage-markup shocks and is closer to the model in this paper.

simple objective, the hours gap—defined as the difference between actual and potential hours worked per capita—should be assigned a large weight in such a loss function. The reason why targeting labor market variables provides a better approximation of the Ramsey policy is that the labor market in the SW model features large nominal wage frictions and mark-up shocks, and these frictions in factor markets become even more important to correct than the distortions in the product markets (sticky prices and price mark-up shocks).

Third, we show that our basic result is robust to a number of important perturbations of the simple loss function; notably when imposing realistic limitations on the extent to which monetary policy makers change policy interest rates. Fourth and finally, we find that our simple mandates outperform the conventional Taylor-type interest rate rules, and that only more complicated rules—e.g. including terms like the level and the change of resource utilization measures—approximate Ramsey policy as well.

This paper proceeds as follows. We start by presenting the SW model and describe how to compute the Ramsey policy and to evaluate the alternative monetary policies. Section 3 reports the benchmark results. The robustness of our results along some key dimensions are subsequently discussed in Section 4, while the comparison with simple rules is discussed in Section 5. Finally, Section 6 provides some concluding remarks and suggestions for further research.

2 The Model and Our Exercise

The analysis is conducted with the model of Smets and Wouters (2007). The model includes monopolistic competition in the goods and labor market and nominal frictions in the form of sticky price and wage settings, while allowing for dynamic inflation indexation. It also features several real rigidities: habit formation in consumption, investment adjustment costs, variable capital utilization, and fixed costs in production. The model dynamics are driven by six structural shocks: the two inefficient shocks—a price-markup shock and a wage-markup shock—follow an ARMA(1,1) process, while the remaining four shocks (total factor productivity, risk premium, investment-specific technology, and government spending shocks) follow an AR(1) process. All the shocks are assumed to be uncorrelated, with the exception of a positive correlation between government spending and productivity shocks, i.e. $Corr(e_t^g, e_t^a) = \rho_{ag} > 0$. The only departure from the original SW model is that we explicitly consider the central bank's decision problem from an optimal perspective rather than including their (Taylor-type) interest rate rule and the associated monetary policy shock.

To that end, we first derive the utility-based welfare criterion. Rotemberg and Woodford (1998) showed that—under the assumption that the steady state satisfies certain efficiency conditions—the objective function of households can be transformed into a (purely) quadratic function using the first-order properties of the constraints. With this quadratic objective function, optimization subject to linearized constraints would be sufficient to obtain accurate results from a normative perspective. Some assumptions about efficiency were unpalatable as exemplified by the presence of positive subsidies that would make the steady state of the market equilibrium equivalent to that of the social planner.[12] Therefore, many researchers—including Benigno and Woodford (2012)—extended the LQ transformation to a general setting without the presence of such subsidies. Benigno and Woodford (2012) demonstrated that the objective function of the households could be approximated by a (purely) quadratic form:

$$\sum_{t=0}^{\infty} \mathrm{E}_0 \left[\beta^t U(X_t)\right] \simeq \text{constant } -\sum_{t=0}^{\infty} \mathrm{E}_0 \left[\beta^t X_t' W^H X_t\right], \tag{1}$$

where X_t is a $N \times 1$ vector with the model variables measured as their deviation from the steady state; therefore, $X_t' W^H X_t$ is referred to as the quadratic approximation of the household utility function $U(X_t)$.

We define Ramsey policy as a policy which maximizes (1) subject to the $N-1$ constraints of the economy. While N is the number of variables, there are only $N-1$ constraints provided by the SW model because the monetary policy rule is omitted. Unlike the efficient steady-state case of Rotemberg and Woodford (1998), second-order terms of the constraints do influence the construction of the W^H matrix in (1), and as detailed in Appendix A, we made assumptions on the functional forms for the various adjustment functions (for example, the capital utilization rate, the investment adjustment cost function, and the Kimball aggregators) that are consistent with the linearized behavioral equations in SW.

Since the constant term in (1) depends only on the deterministic steady state of the model, which is invariant across different policies considered in this paper, the optimal policy implemented by a Ramsey planner can be solved as

$$\tilde{X}_t^* \left(W^H; \tilde{X}_{t-1}\right) \equiv \arg\min_{X_t} \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t X_t' W^H X_t\right], \tag{2}$$

[12] Even when theoretical research papers imposed these assumptions, most prominent empirically oriented papers including Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003, 2007) did not assume the existence of such positive subsidies.

where the minimization is subject to the $N-1$ constraints in the economy, which are omitted for brevity. Following Marcet and Marimon (2012), the Lagrange multipliers associated with the constraints become state variables. Accordingly $\tilde{X}_t' \equiv [X_t', \ \varpi_t']$ now includes the Lagrange multipliers ϖ_t as well. For expositional ease, we denote these laws of motion more compactly as $\tilde{X}_t^*\left(W^H\right)$.

Using (1) to evaluate welfare would require taking a stance on the initial conditions. Doing so is particularly challenging when Lagrange multipliers are part of the vector of state variables because these are not readily interpretable. We therefore adopt the unconditional expectations operator as a basis for welfare evaluation.[13] The loss under Ramsey optimal policy is then defined by

$$Loss^R = \mathrm{E}\left[\left(X_t^*\left(W^H\right)\right)' W^H \left(X_t^*\left(W^H\right)\right)\right]. \tag{3}$$

Our choice of an unconditional expectation as the welfare measure is standard in the literature (see for instance Woodford, 2003). Furthermore, when the discount factor is close to unity—as is the case in our calibration—unconditional and conditional welfare are also quite similar.[14]

The Ramsey policy is a useful benchmark. Obviously, in theory a society could design a mandate equal to the Ramsey objective (1). But in practice most societies do not; instead, most central banks are subject to a mandate involving only a few variables. To capture this observation, we assume that a society provides the central bank with a loss function

$$\mathrm{E}_0\left[\sum_{t=0}^{\infty}\beta^t X_t' W^{CB} X_t\right], \tag{4}$$

where W^{CB} is a sparse matrix with only a few non-zero entries. The matrix W^{CB} summarizes the simple mandates and will be specified in detail in our analysis. Given a simple mandate, the optimal behavior of the central bank is

$$\tilde{X}_t^*\left(W^{CB};\tilde{X}_{t-1}\right) = \underset{X_t}{\arg\min}\mathrm{E}_0\left[\sum_{t=0}^{\infty}\beta^t X_t' W^{CB} X_t\right]. \tag{5}$$

When the simple mandate does not coincide with the Ramsey policy, we have that $W^{CB} \neq W^H$ and therefore that $\tilde{X}_t^*\left(W^{CB}\right) \neq \tilde{X}_t^*\left(W^H\right)$. To compute the extent to which the simple mandate

[13] See Jensen and McCallum (2010) for a detailed discussion about this criterion—with a comparision to the timeless perspective. They motivate the optimal unconditional continuation policy based on the presence of time inconsistency, since the policy would reap the credibility gains successfully. We note, however, that our approach does not exactly follow theirs in that their optimal steady state could be different from the steady state under the Ramsey policy in a model with steady-state distortions.

[14] The unconditional criterion is equivalent to maximization of the conditional welfare when the society's discount factor, $\tilde{\beta}$ in the expression $\left(1-\tilde{\beta}\right)^{-1}\mathrm{E}_0\left[\sum\tilde{\beta}^t\left[\tilde{X}_t^{CB}\left(W^{CB};\tilde{X}_{t-1}\right)\right]' W^{society}\left[\tilde{X}_t^{CB}\left(W^{CB};\tilde{X}_{t-1}\right)\right]\right]$, is approaching unity. In our case, we have that $\beta\gamma^{-\sigma_c} = 0.993$ based on the parameter values in Table A.1.

of the central bank approximates optimal policy, one can calculate its associated loss according to the formula:

$$Loss^{CB}\left(W^{CB}\right) = \mathrm{E}\left[\left(X_t^*\left(W^{CB}\right)\right)' W^H \left(X_t^*\left(W^{CB}\right)\right)\right]. \tag{6}$$

The welfare performance of the simple mandate is then found by taking the difference between $Loss^{CB}$ in eq. (6) and $Loss^R$ in eq. (3). In our presentation of the results, we express this welfare difference in consumption equivalent variation (CEV) units as follows:

$$CEV = 100\left(\frac{Loss^{CB} - Loss^R}{\bar{C}\left(\frac{\partial U}{\partial C}|_{s.s.}\right)}\right), \tag{7}$$

where $\bar{C}\left(\frac{\partial U}{\partial C}|_{s.s.}\right)$ can be interpreted as how much welfare increases when consumption in the steady state is increased by one percent. That is, CEV represents the percentage point increase in households' consumption, in every period and state of the world, that makes them in expectation equally well-off under the simple mandate as they would be under Ramsey policy.[15] Moreover, (7) makes it clear that our choice to neglect the policy-invariant constant in (1) when deriving the Ramsey policy in (2) is immaterial for the results in our paper since all alternative policies are evaluated as difference from the loss under Ramsey.

So far we have proceeded under the assumption that the law governing the behavior of the central bank specifies both the variables and the weights in the quadratic objective, i.e. W^{CB} in (4). But in practice, the mandates of central banks are only indicative and not entirely specific on the weights that should be attached to each of the target variables. A straightforward way to model this is to assume that society designs a law Ω that constrains the weights on some variables to be equal to zero, without imposing any restriction on the exact weight to be assigned to the remaining variables. When determining the simple mandate consistent with the law Ω, we assume the central bank is benevolent and selects a weighting matrix, W^{CB^*}, which minimizes the expected loss of the society. Formally,

$$W^{CB^*} = \underset{W\in\Omega}{\arg\min}\mathrm{E}\left[\left(X_t^*\left(W\right)\right)' W^H \left(X_t^*\left(W\right)\right)\right], \tag{8}$$

where the weighting matrix W^H is defined by (1).

[15] Given presence of habits, there are two ways to compute CEV. One can choose whether the additional consumption units do or do not affect the habit component (lagged consumption in each period). Consistent with the convention (see e.g. Lucas, 1988, and Otrok, 2001) of increasing the steady-state consumption in all periods, our chosen measure is calibrated to the case where both current and lagged consumption are increased. It is imperative to understand that the ranking of the mandates is invariant with respect to which measure is used. The only difference between the two measures is that the other measure is 3.4125 times smaller, refecting that accounting for the habit component requires a larger steady-state compensation. In the limit when the habit coefficient $\varkappa$ is set to unity, households would need to be compensated in terms of consumption growth.

To sum up, our methodology can examine the performance of simple mandates that central banks are typically assigned with. This statement is true whether the simple mandate specifies both the target variables and the exact weights, or whether the target variables are specified but the weights are loosely defined. In this latter case, our exercise can inform central banks of the optimal weights, and ultimately society about whether bounds on certain weights should be relaxed or not.

3 Benchmark Results

In Table 1, we report our benchmark results. The benchmark simple mandate we consider reflects the standard practice of monetary policy, and is what Svensson (2010) refers to as "flexible inflation targeting." Specifically, we use the framework in Woodford (2003) and assume that the simple mandate can be captured by the following period loss function

$$L_t^a = (\pi_t^a - \pi^a)^2 + \lambda^a x_t^2 \tag{9}$$

where π_t^a denotes the annualized rate of quarterly inflation, and x_t is a measure of economic activity with λ^a denoting its corresponding weight.

Based on the deep parameters in his benchmark model, Woodford (2003) derives a value of 0.048 for λ^a when x_t is a welfare-relevant output gap.[16] As for the first row of Table 1, we apply Woodford's weight on three different measures of economic activity. Our first measure is the output gap ($y_t^{gap} = y_t - y_t^{pot}$), i.e. the difference between actual and potential output, where the latter is defined as the level of output that would prevail when prices and wages are fully flexible and inefficient markup shocks are excluded.[17] The second measure we consider is simply the level of output (as deviation from the deterministic labor-augmented trend, i.e. $y_t - \bar{y}_t$). Finally, we also consider annualized output growth in the spirit of the work on "speed-limit" policies by Walsh (2003).

Turning to the numbers in the first row, we see—as expected—that adopting a target for output gap volatility yields the lowest loss, even when the weight on the resource utilization measure is

[16] Woodford's (2003) quarterly weight of $\lambda^q = 0.003$ translates into an annualized weight of $\lambda^a = 16\lambda^q = 0.048$. Throughout this paper, we will report annualized values.

[17] We follow the terminology of Justiniano et al. (2013). This measure of potential output is below the efficient level (roughly by a constant amount) because we do not assume that steady-state subsidies remove the output distortion induced by the price and wage markups at the steady state. Another—perhaps more traditional—definition of potential output is based on the noninflationary maximum level of output; a popular defintion by the Congressional Budget Office is based on this concept, and Plosser (2014) deals with both this concept and our welfare-relevant concept from a policy perspective.

quite low. Another observation from the first row of the table is that the magnitudes of the CEV numbers are moderate, which given the previous literature on the welfare costs of business cycles (e.g. the seminal work by Lucas, 1987, and subsequent work of Otrok, 2001) was to be expected. Even so, the CEV values—both relative to Ramsey and between different mandates—are large when taking into account similar studies on optimal monetary policy; for instance, the welfare losses are larger than the 0.05 percent threshold used in Schmitt-Grohe and Uribe (2007).

Table 1: Benchmark Results for "Flexible Inflation Targeting" Mandate in eq. (9).

	x_t: Output gap		x_t: Output (dev from trend)		x_t: Output growth (Ann.)	
Simple Mandate	λ^a	CEV (%)	λ^a	CEV (%)	λ^a	CEV (%)
Woodford (2003)	0.048	0.471	0.048	0.554	0.048	0.611
Dual Mandate	0.250	0.140	0.250	0.276	0.250	0.404
Optimized Weight	1.042	0.044	0.542	0.244	2.943	0.302

Note: CEV denotes the consumption equivalent variation (in percentage points) needed to make households indifferent between the Ramsey policy and the simple mandate under consideration according to eq. (7). The "Dual Mandate" refers to a weight of unity for the unemployment gap in the loss function (9), which translates into $\lambda^a = 0.25$ when applying a variant of Okun's law. Finally, "Optimized Weight" refers to minimization of eq. (6) w.r.t. λ^a in eq. (9).

The second row of Table 1 examines the dual mandate. Prominent academics like Svensson (2011) have interpreted this mandate as a simple loss function in inflation and the unemployment gap (i.e. actual unemployment minus the NAIRU) where the weight placed on economic activity is substantially higher than Woodford's (2003) value. And in a recent work, Yellen (2012) and senior Federal Reserve Board staff—including Reifschneider, Wascher and Wilcox (2013) and English, López-Salido and Tetlow (2013)—assigned equal weights for annualized inflation and the unemployment gap in the Federal Reserve's loss function.

Yellen (2012) also stipulates that the Federal Reserve converts the unemployment gap into an output gap according to a value of roughly 0.5. This value is based on the widely spread empirical specification of the Okun's law:

$$u_t - u_t^{pot} = \frac{y_t - y_t^{pot}}{2}. \tag{10}$$

Accordingly, the unit weight on the unemployment gap converts into a weight of $\lambda^a = 0.25$ on the output gap.[18] This value is roughly five times bigger than the value derived by Woodford, and indicates a lack of consensus regarding the weight that real activity should receive.

Interestingly, we can see from the second row in Table 1 that increasing the weight on real activity from Woodford's to the value consistent with the dual mandate reduces welfare losses by roughly a factor of two for output level and output growth. For our benchmark measure of economic

[18] Moreover, Gali, Smets and Wouters (2011) argue within a variant of the SW model with unemployment that fluctuations in their estimated output gap closely mirror those experienced by the unemployment rate. Therefore, the Okun's law we apply can also find support in a structural modeling framework.

activity (the output gap) the loss under the dual mandate is more than three times smaller. Based on the 0.05 percent CEV cut-off value adopted by Schmitt-Grohe and Uribe (2007), the reduction in all three cases should be deemed significant.

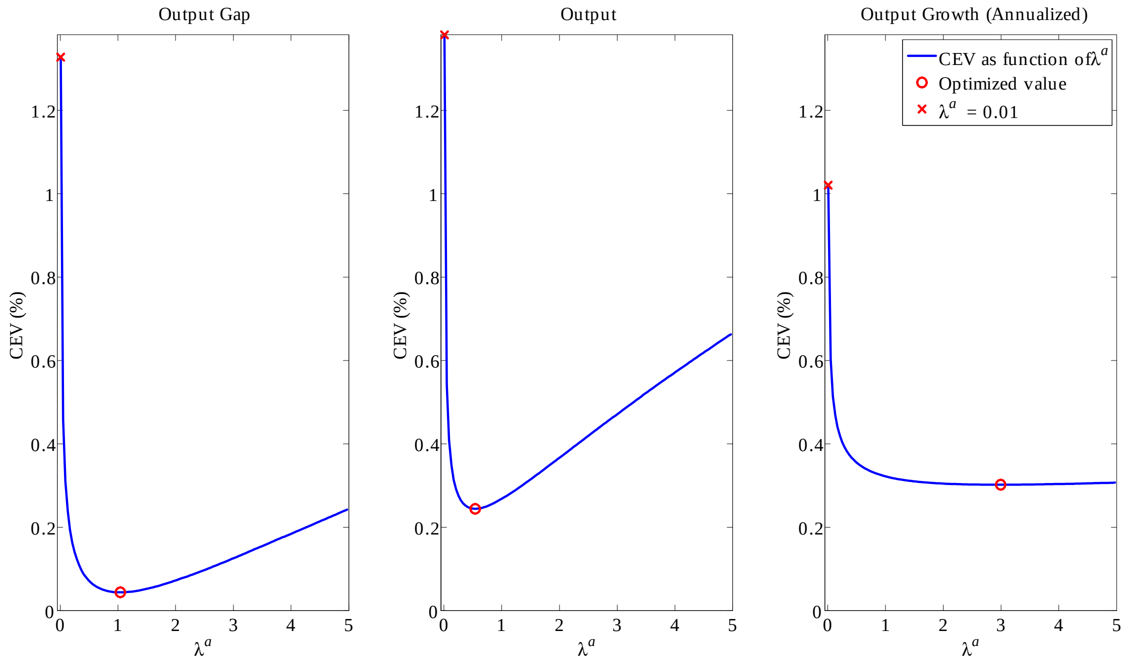
The last row in Table 1 displays the results when the weight λ^a is optimized. The optimized coefficient for the output gap is 1.042—much higher than in the two preceding loss functions. Coincidentally, it is also very similar to the unit weight on the unemployment gap as used in Yellen (2012). When the level of output replaces the output gap, the optimized coefficient is about 0.5. In the case of output growth, the optimized coefficient is even higher (around 2.9), which essentially is a so-called speed-limit regime (see Walsh, 2003). Responding to the model-consistent output gap is the preferred measure from a welfare perspective, and our analysis suggests that a large weight should be assigned to stabilize economic activity in addition to inflation regardless of the chosen resource utilization measure.[19]

To gauge the sensitivity of the CEV with respect to the weight assigned to resource utilization, Figure 1 plots the CEV as a function of λ^a for the three resource measures. Consistent with the results in Table 1, we see that there is quite some curvature of the CEV function for small values of λ^a for all three measures. Moreover, for the output gap we see that values in the neighborhood of the optimum (the range of λ^a between 0.5 and 1.5) perform similarly well, whereas for the mandate with the level of output the curvature near the optimum is higher. For output growth, the figure shows that any value above unity yields virtually the same CEV.

As noted in Section 2, these results are based on a non-efficient steady state. The results in Table 1 and Figure 1, however, are robust to allowing for subsidies to undo the steady-state distortions stemming from the presence of external habits, as well as firms' and households' monopoly power in price and wage setting. For the output gap and output as deviation from its trend, the optimized λ^a is roughly unchanged or sometimes higher. In particular, for the case with an efficient steady state, the optimized weight on output gap is 2.34, with an associated CEV of 0.0119. For output growth, the optimized λ^a is substantially lower (0.43). Given the flatness of the CEV function in Figure 1, it is not surprising that the results for output growth can be somewhat sensitive to the specific assumptions. Even so, the optimized weight on resource utilization is still relatively large, reflecting the larger curvature for smaller values of λ^a.

[19] We have also analyzed loss functions with a yearly inflation rate, i.e. $\ln(p_t/p_{t-4})$, instead of the annualized quarterly inflation rate in eq. (9). Our findings are little changed by this alternative inflation measure. For example, in the output gap case, we obtain an optimized λ^a equal to 0.95 and an associated CEV of 0.044. These results are very close to our benchmark findings of $\lambda^a = 1.04$ and CEV$= 0.044$.

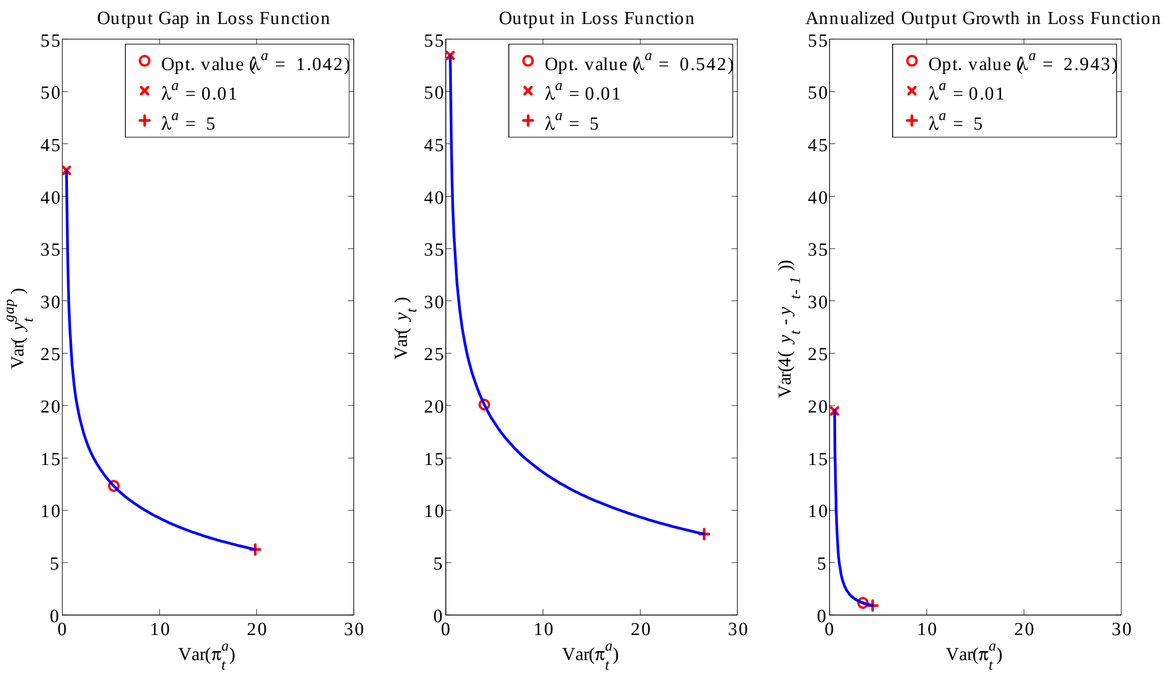
Figure 1: Consumption Equivalent Variation (percentage points) as Function of the Weight (λ^a) on Economic Activity.



Note: The figure plots the CEV (in %) for the simple mandate with inflation and: output gap (left panel), output level (middle panel), output growth (right panel) The coordinate with an '×' mark shows the CEV for $\lambda^a = 0.01$, the 'o' mark shows the CEV for the optimized weight.

To understand the curvature of the CEV for the various resource utilization measures in Figure 1, it is useful to depict variance frontiers. Notably, variance frontiers have been used by Taylor (1979), Erceg, Henderson and Levin (1998), and Clarida et al. (1999) as a way to represent a possible trade-off between inflation and output stabilization. Following Taylor (1979) and Clarida et al. (1999), we plot the efficient frontier with the variance of inflation on the horizontal axis and the variance of the resource utilization measure on the vertical axis. The slope of the curve is referred to as the trade-off between the two variances, and in a simple bivariate loss function (9) the slope equals $-1/\lambda^a$. In Figure 2, the line shows the combination of inflation and resource utilization volatilities when λ^a varies from 0.01 to 5. The coordinate with an '×' mark shows the volatility for $\lambda^a = 0.01$, the 'o' mark shows the volatility for the optimized weight, and the '+' mark shows the volatility for $\lambda^a = 5$. The figure shows that the trade-off between stabilizing inflation and economic activity is most favorable when the resource utilization measure is output growth (right panel); the variance of annualized output growth can be reduced to nearly 1 percent without $\text{Var}(\pi_t^a)$ increasing by much. Moreover, the flatness of the CEV witnessed in the right panel of Figure 1 for values of λ^a higher than optimal can be readily explained by the fact that Figure 2 shows that such values induce only small changes in the volatilities of inflation and output growth. Turning back to the results for output and the output gap, the figure shows that the trade-off is more pronounced, especially for output (middle panel). Accordingly, values of λ^a higher than optimal translate into a higher curvature of the CEV function in Figure 1.

Figure 2: Variance Frontier for Alternative Resource Utilization Measures.



Note: The figure plots the variance frontier for the simple mandate with inflation and: output gap (left panel), output level (middle panel), output growth (right panel). The coordinate with an '×' mark shows the volatility for $\lambda^a = 0.01$, the 'o' mark shows the volatility for the optimized weight, and the '+' mark shows the volatility for $\lambda^a = 5$.

Apart from helping to explain the optimized values in Table 1, another key feature of Figure 2 is the important trade-off between stabilizing inflation and the output gap in the SW model. This finding is seemingly at odds with Justiniano et al. (2013) who argued that there is little evidence that stabilizing the output gap comes at the cost of higher inflation volatility. In the next section, we address this issue together with the reasons for the importance of real activity.

3.1 The Importance of Real Activity

The key message from Table 1 is that the rationale for targeting some measure of real activity is much more important than previously thought either in policy circles or in previous influential academic work (e.g. Woodford (2003) and Walsh (2005)). By perturbing the parameter values (i.e.

turning off some bells and whistles) in the model, we seek to nail down why the model suggests that a high weight on real economic volatility improves household welfare.

We begin the analysis by using the SW parameters in Table A.1 to recompute λ^a according to the analytic formula provided in Woodford (2003):

$$\lambda^a \equiv \frac{16\kappa_x}{\left(\frac{\phi_p}{\phi_p-1}\right)}, \tag{11}$$

where κ_x is the coefficient for the output gap in the linearized pricing schedule (i.e. in the New Keynesian Phillips curve), and $\frac{\phi_p}{\phi_p-1}$ is the elasticity of demand of intermediate goods. In the SW model, the NKPC is given by

$$\pi_t - \iota_p \pi_{t-1} = \beta\gamma^{1-\sigma_c}\left(\mathrm{E}_t \pi_{t+1} - \iota_p \pi_t\right) + \frac{\left(1-\beta\gamma^{1-\sigma_c}\xi_p\right)\left(1-\xi_p\right)}{\xi_p\left(\left(\phi_p-1\right)\epsilon_p+1\right)} mc_t + \varepsilon_{p,t}. \tag{12}$$

However, because the SW model features endogenous capital and sticky wages, there is no simple mapping between the output gap and real marginal costs within the fully fledged model. But by dropping capital and the assumption of nominal wage stickiness, we can derive a value of $\kappa_x = 0.143$ in the simplified SW model.[20] From the estimated average mark-up ϕ_p, we then compute $\lambda^a = 0.87$. This value is considerably higher than Woodford's (2003) value of 0.048 for two reasons. First, Woodford's κ_x is substantially lower due to the assumption of firm-specific labor (the Yeoman-farmer model of Rotemberg and Woodford, 1997). Second, the estimated mark-up in SW implies a substantially lower substitution elasticity ($\frac{\phi_p}{\phi_p-1} = 2.64$) compared to Woodford's value (7.88).

The analytical weight on the output gap is robust to some key alterations of the model. Importantly, Galí (2008) shows that it remains unchanged even when allowing for sticky wages following Erceg, Henderson and Levin (2000). Still, this analysis is only suggestive, as by necessity it only considers a simplified model without some of the key features in the fully fledged model. As a consequence, the obtained λ^a will only partially reflect the true structure of the fully fledged SW model. Yet, the analysis suggests that a large part of the gap between Woodford's (2003) value and our benchmark finding of $\lambda^a = 1.042$ in the output-gap case stems from differences in household preferences and the estimated substitution elasticity between intermediate goods.

With these results in mind, we turn to exploring the mechanisms within the context of the fully fledged model. Our approach is to turn off or reduce some of the frictions and shocks featured in the model one at a time to isolate the drivers of the results. The findings are provided in Table 2.

[20] More specifically, we derive $\pi_t - \iota_p \pi_{t-1} = \beta\gamma^{1-\sigma_c}\left(\mathrm{E}_t \pi_{t+1} - \iota_p \pi_t\right) + \kappa_x\left[x_t - \frac{\varkappa}{1+\sigma_l(1-\varkappa)} x_{t-1}\right] + \varepsilon_{p,t}$ where x_t is the output gap and the slope coefficient κ_x equals $\frac{\left(1-\beta\gamma^{1-\sigma_c}\xi_p\right)\left(1-\xi_p\right)}{\xi_p\left(\left(\phi_p-1\right)\epsilon_p+1\right)}\left(\frac{1+\sigma_l(1-\varkappa)}{1-\varkappa}\right)$.

The first row restates the baseline results with the optimized weight. The second row presents the optimized weight on the real-activity term when dynamic indexation in price- and wage-setting is shut down, i.e. ι_p and ι_w are calibrated to zero. All the other parameters of the model are kept unchanged. As can be seen from the table, the calibration without indexation lowers the optimized weight for the output gap to roughly 0.3—about a third of the benchmark value. In the other columns where real activities are captured by the level and the growth rate of detrended output, the optimized weights are also found to be about a third of the benchmark values.

Table 2: Perturbations of the Benchmark Model.

	x_t: Output gap		x_t: Output (dev from trend)		x_t: Output growth (Ann.)	
Simple Mandate	λ^a	CEV (%)	λ^a	CEV (%)	λ^a	CEV (%)
Benchmark	1.042	0.044	0.542	0.244	2.943	0.302
No Indexation	0.318	0.042	0.179	0.220	0.817	0.285
No ε_t^p Shocks	0.914	0.039	0.343	0.220	1.235	0.278
No ε_t^w Shocks	2.094	0.020	0.355	0.213	1.267	0.226
Small ε_t^p and ε_t^w Shocks	1.268	0.024	0.112	0.167	0.157	0.180
No ε_t^p and ε_t^w Shocks	Large	0.016	0.161	0.150	0.025	0.134

Note: "No Indexation" refers to setting $\iota_p = \iota_w = 0$; "No ε_t^p (ε_t^w) Shocks" refers to setting the variance of the price markup shock (wage markup shock) to zero; "Small ε_t^w and ε_t^p Shocks" means that the std of these shocks are set to a 1/3 of their baseline values; and "No ε_t^w and ε_t^p Shocks" refers to setting the variance of both shocks to zero. "Large"means that the optimized value is equal or greater than 5.

To understand why indexation makes the real-activity term much more important than in a model without indexation, it is instructive to consider a simple New Keynesian model with indexation and sticky prices only. If we compute a micro-founded welfare-based approximation to the household utility function following Woodford (2003), such a model would feature the following terms in the approximated loss function

$$(\pi_t - \iota_p \pi_{t-1})^2 + \lambda \left(y_t^{gap}\right)^2, \tag{13}$$

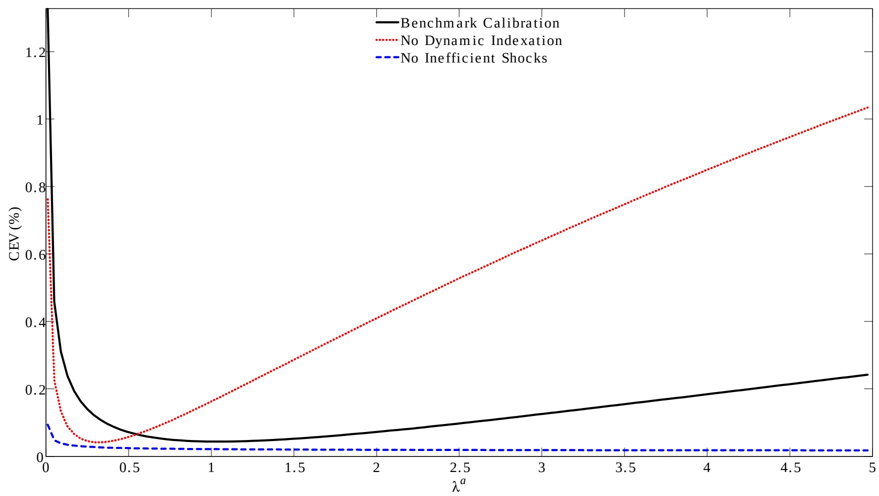
where ι_p is the indexation parameter in the pricing equation. Suppose further, for simplicity, that inflation dynamics in equilibrium can be represented by an AR(1) process $\pi_t = \rho\pi_{t-1} + \varepsilon_t$. In this simple setup, the welfare metric could be expressed as

$$\mathrm{E}_0 \left[(\rho - \iota_p)^2 (\pi_{t-1})^2 + \lambda \left(y_t^{gap}\right)^2 \right]. \tag{14}$$

Intuitively, in economies where prices have a component indexed to their lags, the distortions arising from inflation are not as severe. Consequently, there is less need to stabilize inflation.

In more empirically relevant models like SW, inflation persistence (ρ) is in large part explained by the indexation parameters (ι_p and, in our sticky-wage framework, ι_w matter as well). Therefore, these two parameter values tend to be similar and the coefficient on the inflation term is accordingly smaller. Hence, in a loss function like ours (eq. 9) where the inflation coefficient is normalized to unity, the coefficient on real activity tends to become relatively larger—as evidenced in Table 1.

Figure 3: CEV (in percentage points) as Function of λ^a for Alternative Calibrations.



Note: The figure plots the CEV (in %) as a function of λ^a for three different calibrations. The solid line refers to the benchmark calibration. The dotted line refers to the calibration in which $\iota_p = \iota_w = 0$. The dashed line refers to the calibration in which $var\,(\varepsilon_t^w) = var\,(\varepsilon_t^p) = 0$.

Notably, even when we remove indexation to lagged inflation in price and wage settings, the optimal value of λ^a still suggests a very large role for targeting economic activity; in fact, the optimal value is still slightly higher than the value implied by the dual mandate.[21] Moreover, one can observe from Figure 3 that dropping dynamic indexation is associated with a rather sharp deterioration in the CEV when λ^a is below 0.2. This finding suggests that a vigorous response to economic activity is indeed important even without indexation. Additionally, it is also important to point out that we kept all other parameters unchanged in this analysis; had we reestimated the model it is conceivable that the other parameters would change as to better account for the high degree of inflation persistence prevailing in the data, and accordingly inducing a higher λ^a again.[22]

Rows 3–6 in Table 2 examine the role of the inefficient markup shocks in the model. By comparing the CEV results in the third and fourth rows, we see that the wage markup shock contributes the most to the welfare costs of the simple mandate. But the key point is that even

[21] Indexation to lagged inflation in wage-setting (ι_w) matters more than dynamic indexation in price-setting in the model. Setting $\iota_p = 0$ but keeping ι_w unchanged at 0.65 results in an optimized $\lambda^a = 0.82$, close to our benchmark optimized value.

[22] SW showed that excluding indexation to lagged inflation in price and wage setting is associated with a deterioration in the empirical fit (i.e. reduction in marginal likelihood) of the model.

when one of these shocks is taken out of the model, the central bank should still respond vigorously to economic activity in order to maximize household welfare. Only when the standard deviation of both shocks are reduced or taken out completely (rows 5 and 6), λ^a falls for output and output growth. For the loss function with the model-consistent output gap, the weight λ^a is large when shocks are reduced (row 5), and is still large but hard to pin down when the standard deviation of both inefficient shocks are set to nil (row 6).

When both shocks are set to nil, any $\lambda^a > 0.1$ produces roughly the same CEV of about 0.016 although a $\lambda^a \geq 5$ generates the lowest welfare loss relative to Ramsey as can be seen from Figure 3. So, in the absense of price- and wage-markup shocks, this finding suggests that there is only a weak trade-off between inflation stabilization and stabilization of the output gap. Even so, the divine coincidence feature noted by Blanchard and Galí (2007) only holds approximately as the SW model features capital formation and sticky wages; see Woodford (2003) and Galí (2008).

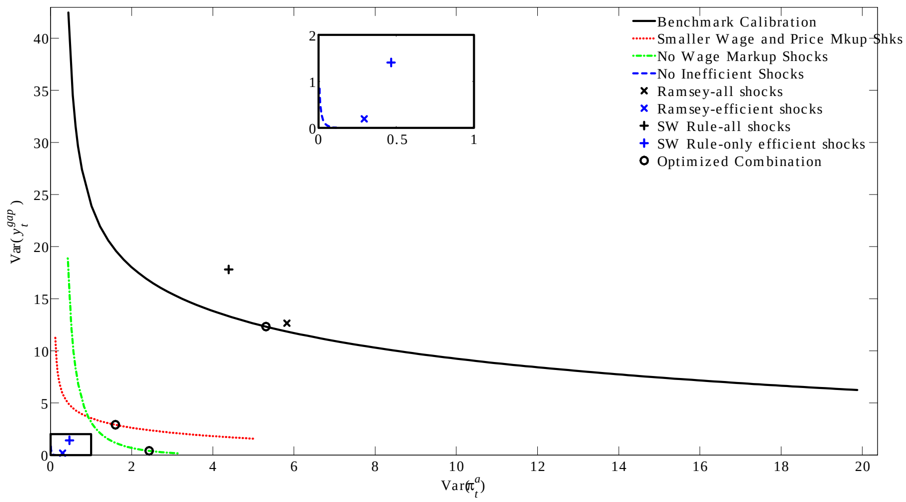
In Figure 4, we depict variance frontiers when varying λ^a from 0.01 to 5 for alternative calibrations of the model. We also include the implied $\{\mathrm{Var}\left(\pi_t^a\right), \mathrm{Var}\left(y_t^{gap}\right)\}$ combinations under Ramsey policy and the estimated SW policy rule with all shocks (marked by black 'x' marks) and without the inefficient shocks (the blue '+' marks). As expected, we find that both the estimated rule and the Ramsey policy are outside the variance frontier associated with the simple mandate (solid black line), but the locus of $\{\mathrm{Var}\left(\pi_t^a\right), \mathrm{Var}\left(y_t^{gap}\right)\}$ for the optimized λ^a is very close to the Ramsey policy. We interpret this finding as providing a strong indication that the simple mandate approximates the Ramsey policy well in terms of equilibrium output-gap and inflation, and not just CEV as seen from the results for the output gap in Table 1.[23]

Further, there is a noticeable trade-off between inflation and output gap volatility even when we set the standard deviation of the wage markup shocks to nil (dash-dotted green line) following the baseline model of Justiniano et al. (2013). The reason why the central bank has to accept a higher degree of inflation volatility in order to reduce output gap volatility in this case is that we still have the price markup shock active in the model. When the inefficient price markup shocks are excluded as well (dashed blue line in Figure 4), there is only a negligible inflation-output volatility trade-off (as shown in more detail in the small inset box). In this special case, we reproduce the key finding of Justiniano et al. (2013) that a shift from the estimated historical rule to Ramsey policy is a free lunch as it reduces output gap volatility without the expense of higher inflation

[23] It is imperative to understand that, although the Ramsey policy is associated with higher inflation and output gap volatility, the simple inflation-output gap mandate we consider is nevertheless inferior in terms of the households' welfare.

volatility.[24] Notably, this result does not arise in the case when any or both types of inefficient markup shocks are included; in this case, a shift from the estimated rule to Ramsey policy will be associated with a decline in output gap volatility but rising inflation volatility—thus the trade-off due to this shift.

Figure 4: Variance Frontiers for Alternative Calibrations.



Note: The figure plots the variance frontier for several calibrations: benchmark (solid line), $var\left(\varepsilon_t^w\right)$ and $var\left(\varepsilon_t^p\right)$ set to 1/3 of baseline values (dotted line), $var\left(\varepsilon_t^w\right) = 0$ (dashed-dotted line), and $var\left(\varepsilon_t^w\right) = var\left(\varepsilon_t^p\right) = 0$ (dashed line). The 'o' mark shows the volatility for the optimized weight and benchmark calibration. The coordinates with an '×' and the '+' mark denote the Ramsey and SW policy rule, respectively. The box in the graph zooms in the case with $var\left(\varepsilon_t^w\right) = var\left(\varepsilon_t^p\right) = 0$.

It is important to note that even the variant of our model without the inefficient shocks—which features only a very limited trade-off (in terms of the variance frontier) between stabilizing inflation and the output gap—warrants a relatively high λ^a (see Table 2 and Figure 3), although the choice of λ^a will obviously be less important from a welfare perspective, consistent with Justiniano et al. (2013) finding of no relevant trade-off.

Since substantial uncertainty remains about the importance of markup shocks over the business cycle, it is important to consider the more likely case where at least a small proportion of the observed variation in inflation and wages are in fact driven by inefficient price- and wage-markup shocks. The fifth row in Table 2 reports results in which the standard deviations of both the inefficient markup shocks have been set to a third of their baseline values. For the wage-markup shock, this alternative calibration can be motivated by the empirical work by Galí, Smets and

[24] To account for inflation persistence without correlated price markup shocks, Justiniano et al. (2013) allow for serially correlated shocks to the Fed's inflation target which are subsequently excluded in their optimal policy exercises.

Wouters (2011), who can distinguish between labor supply and wage markup shocks by including the unemployment rate as an observable when estimating a model similar the SW model. For the price markup shock, our choice is more arbitrary, which follows Justiniano et al. (2013) and assumes that almost 90 percent of the markup shock variances are in fact variations in the inflation target.

Even in this case, the table shows that the resulting λ^a is still high for the output gap. The reason is that if all shocks are efficient, then the choice of λ^a is not as relevant—even though a high λ^a is still optimal—and if all shocks are inefficient, then a high λ^a is required. Therefore, a high λ^a is optimal as long as a small proportion of shocks is indeed inefficient, and is also a robust choice if there is uncertainty about the inefficiency of the shocks.

4 Robustness Analysis

In this section, we explore the robustness of our results along some key dimensions. We first examine to what extent adding labor market variables, such as hours worked and wage inflation, to the loss function improves welfare. Second, we consider the merits of speed limit policies analyzed by Walsh (2003) and price- and wage-level targeting. Third and finally, we consider the extent to which the implied interest rate movements for the simple mandates under consideration are reasonable.

4.1 A Quest for a Different Objective: Should Labor Market Variables be considered?

One of the reasons for the popularity of inflation targeting comes from the results in the New Keynesian literature—importantly Clarida et al. (1999) and Woodford (2003)—that inflation in the general price level is costly to the economy. The old Keynesian literature, however, emphasized the importance of wage inflation.[25] Recent influential theoretical papers support that literature by suggesting to add wage inflation as an additional target variable, see e.g. Erceg, Henderson and Levin (2000) and Galí (2011). In the SW model employed in our analysis, both nominal wages and prices are sticky. It is therefore conceivable that wage inflation may be equally or even more important to stabilize than price inflation. In addition to studying nominal wage inflation, it is of interest to examine to what extent other labor market variables like hours worked can substitute for overall economic activity within the model. Hence, we propose to study the following augmented

[25] See Kim and Henderson (2005) for a more detailed discussion and references.

loss function:

$$L_t^a = \lambda_\pi^a \left(\pi_t^a - \pi^a\right)^2 + \lambda^a x_t^2 + \lambda_{\Delta w}^a \left(\Delta w_t^a - \Delta w^a\right)^2 + \lambda_e^a e_t^2, \tag{15}$$

where Δw_t^a denotes annualized nominal wage inflation (and Δw^a its steady state rate of growth), and e_t involves a measure of activity in the labor market.

In Table 3, we report results for this augmented loss function (15) when x_t is given by the output gap and e_t is given by the hours worked per capita gap l_t^{gap}, respectively. The first row re-states the benchmark results, i.e. with the optimized weight on y_t^{gap} in Table 1. The second row adds wage inflation to the loss function. Relative to the unit weight on inflation, the optimized objective function would ask for a weight of roughly 3.2 for the output gap term, and a weight of about 1.5 for nominal wage inflation volatility, which is higher than the normalized weight on price inflation volatility. The level of welfare when adding Δw_t is substantially higher (by 32.8 percent, when measured by the decrease in loss) than under the benchmark case.

Table 3: Variations of the Loss Function: Gap Variables in (15).

Loss Function	λ_π^a: π_t^a	λ^a: y_t^{gap}	$\lambda_{\Delta w}^a$: Δw_t^a	λ_e^a: l_t^{gap}	CEV (%)	Gain
Benchmark	1.000	1.042	–	–	0.044	–
Adding Δw_t^a	1.000	3.216	1.485	–	0.029	32.8%
Adding Δw_t^a, impose $\lambda^a = 0.01$	1.000	0.01*	0.013	–	1.260	−2673.6%
Replacing π_t with Δw_t^a	–	1.546	1.000	–	0.032	27.3%
Adding l_t^{gap}	1.000	0.880	–	0.518	0.043	1.6%
Replacing y_t^{gap} with l_t^{gap}	1.000	–	–	3.250	0.050	−14.3%
Replacing $[\pi_t, y_t^{gap}]$ with $[\Delta w_t^a, l_t^{gap}]$	–	–	1.000	4.044	0.016	63.3%

Note: The table reports variations of the simple objective (15). y_t^{gap} is used as measure for x_t, and l_t^{gap} is used as measure of e_t. The numbers in the "Gain" column are computed as $100\left(1 - \frac{\text{CEV}_{LFalt}}{0.044}\right)$ where CEV_{LFalt} is the CEV for the alternative loss function and 0.044 is the "Benchmark" objective CEV (row 1). A "*" after a coefficient implies that the value of this coefficient has been imposed.

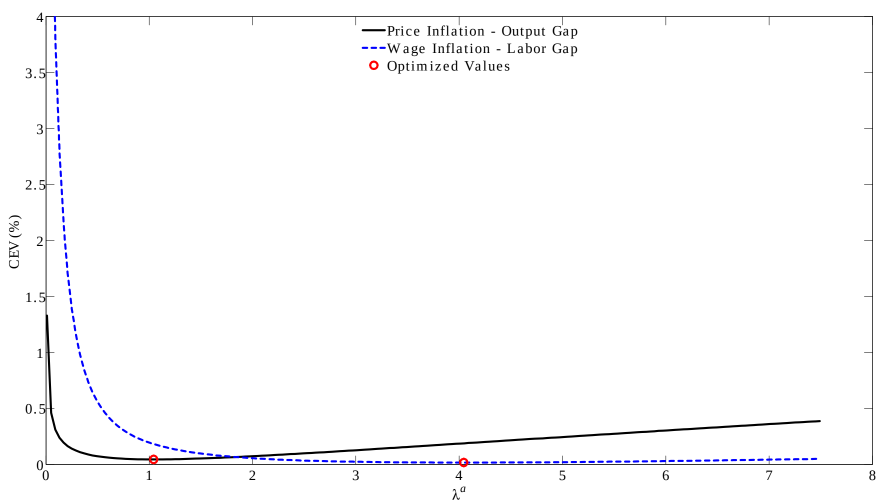
In our framework with inefficient cost-push shocks and capital accumulation, the introduction of Δw_t^a in the loss function does not make the presence of y_t^{gap} irrelevant in contrast to Erceg, Henderson and Levin (2000): the third row makes this clear by showing that the welfare loss is extremely high for a specification which includes both price and wage inflation but imposes a low weight on the output gap.[26] Moreover, we learn from the fourth row in the table that, although Δw_t^a receives a larger coefficient than π_t^a, responding to price inflation is still sizeably welfare

[26] Because the obtained weight for nominal wage inflation is close to nil when λ^a is fixed to 0.01, the CEV reported in Table 3 is about the same as the CEV reported for the benchmark calibration in Figure 3, recalling that this figure shows CEV's when varying λ^a between 0.01 and 5. Accordingly, it follows from the discussion of the results in Figure 3 that a loss function with only price and wage inflation is not observationally equivalent to a loss function in which the output gap is included even when the inefficient markup shocks are excluded; the optimized weight on the output gap is large even in this case. However, the absolute difference in CEV is much smaller in these cases: CEV is 0.03 for the pure price-wage inflation mandate, and very close to zero when the output gap is included with a large weight.

enhancing; when dropping π_t^a the welfare gain is somewhat lower compared to the trivariate loss function. Also, the optimal weight on economic activity remains high.

Next, we introduce the labor market gap, defined as $l_t^{gap} = l_t - l_t^{pot}$, as an additional target variable in the fifth column of Table 3. Such a labor market gap differs from the output gap because of the presence of capital in the production function. Unlike wage inflation, the inclusion of the labor market gap by itself does not increase welfare much. Moreover, given that price inflation is the nominal anchor, replacing the output gap with the labor gap results in a welfare deterioration of about 14 percent relative to our benchmark specification as can be seen from the sixth row. However, when price inflation is also replaced by wage inflation as a target variable, the labor gap performs much better, and this labor market oriented simple mandate generates a substantial welfare gain of 63 percent relative to our benchmark specification.

Figure 5: CEV (in percentage points) as Function of λ^a for Alternative Simple Mandates.



Note: The figure plots the CEV (in %) for the simple mandate with price inflation and output gap (solid line) and wage inflation and labor gap (dashed line). The coordinate with an 'o' mark shows the CEV for the optimized weight.

In Figure 5, we plot CEV as a function of λ^a for a simple mandate targeting price inflation and the output gap as well as a mandate targeting wage inflation and the labor market gap. Interestingly, we see from the figure that λ^a has to exceed 2 in order for the wage-labor simple mandate to dominate. So although the wage-labor gap mandate dominates the inflation-output gap mandate, the figure makes clear that a rather large λ^a is required for this to happen; strict nominal wage inflation targeting is thus very costly for society in terms of welfare. On the other hand, a beneficial aspect of the wage inflation-labor gap is that if λ^a indeed exceeds this threshold, then the CEV stays essentially flat instead of slightly increasing as is the case for the inflation-output gap mandate.

We also examine the role of labor market variables when only observable variables are included; hence, we consider levels instead of gap variables. As shown in Table 4, the role played by nominal wage inflation is not as prominent when x_t in (15) is represented by the level of output (as deviation from trend) instead of the output gap. The welfare gain relative to the benchmark case is only 5.3 percent higher when wage inflation is included. Accordingly, welfare is reduced by one percent—the third row—when price inflation is omitted. On the other hand, adding hours worked per capita enhances the welfare of households by nearly 30 percent. Finally, we see from the last row that a mandate with only wage inflation and hours worked performs the best, reducing the welfare cost associated with the simple mandate by nearly 34 percent relative to the benchmark objective.

Table 4: Variations of the Loss Function: Level Variables in (15).

Loss Function	λ_π^a: π_t^a	λ^a: $y_t - \bar{y}_t$	$\lambda_{\Delta w}^a$: Δw_t^a	λ_e^a: $l_t - \bar{l}$	CEV (%)	Gain
Benchmark	1.000	0.544	–	–	0.244	–
Adding Δw_t^a	1.000	0.954	0.463	–	0.230	5.3%
Replacing π_t with Δw_t^a	–	1.054	1.000	–	0.246	−1.0%
Adding $l_t - \bar{l}$	1.000	0.392	–	1.344	0.171	29.8%
Replacing $y_t - \bar{y}_t$ with $l_t - \bar{l}$	1.000	–	–	2.947	0.210	13.8%
Replacing $[\pi_t, y_t - \bar{y}_t]$ with $\left[\Delta w_t^a, l_t - \bar{l}\right]$	–	–	1.000	3.475	0.161	33.8%

Note: The table reports variations of the simple objective (15). $y_t - \bar{y}_t$ is used as measure for x_t, and $l_t - \bar{l}$ is used as measure of e_t. The numbers in the "Gain" column are computed as $100\left(1 - \frac{\text{CEV}_{LFalt}}{0.2440}\right)$ where CEV_{LFalt} is the CEV for the alternative loss function and 0.2440 is the "Benchmark" objective CEV (row 1).

Our conclusion is that while a standard objective with price inflation and the output gap generates small welfare losses relative to the Ramsey policy (just above 0.04% of the steady-state consumption), it makes sense within the SW model—which features substantial frictions in the labor market—to target wage inflation and a labor market gap instead. Doing so will reduce the welfare costs of the simple mandate even further. Moreover, we have shown that this conclusion is robust even if one considers the level of output and hours worked instead of their deviations around potential. Furthermore, regardless of whether the objective focuses on price or wage inflation, we always find a robust role for responding vigorously to economic activity (may it be output or hours worked), in line with our benchmark results in Table 1.

4.2 Another Quest for a Different Objective: Speed Limit Policies & Price- and Wage-Level Targeting

In this subsection, we examine the performance of speed limit policies (SLP henceforth) advocated by Walsh (2003) and price- and wage-level targeting.

We start with an analysis of SLP. Walsh's formulation of SLP considered actual growth relative to potential (i.e. output gap growth), but we also report results for actual growth relative to its steady state to understand how contingent the results are on measuring the change in potential accurately. Moreover, since the results in the previous subsection suggested that simple mandates based on the labor market performed very well, we also study the performance of SLP for a labor market based simple mandate.

We report results for two parameterizations of the SLP objective in Table 5.a. In the first row, we use the benchmark weight derived in Woodford (2003). In the second row, we adopt a weight that is optimized to maximize household welfare. Interestingly, we see that when replacing the level of output growth with the growth rate of the output gap (Δy_t^{gap}), welfare is increased substantially conditional on placing a sufficiently large coefficient on this variable. However, by comparing these results with those for y_t^{gap} in Table 1, we find it is still better to target the level of the output gap.

Turning to the SLP objectives based on nominal wage inflation and hours, we see that they perform worse than the standard inflation-output objectives unless the weight on the labor gap is sufficiently large. As is the case for output, the growth rate of the labor gap is preferable to the growth rate of labor itself. But by comparing with our findings in Table 3 we see that targeting the level of the labor gap is still highly preferable in terms of maximizing welfare of the households.

Table 5.a: Sensitivity Analysis: Merits of Speed Limit Policies.

	Price Inflation Objective				Wage Inflation Objective			
	x_t: Δy_t		x_t: Δy_t^{gap}		x_t: Δl_t		x_t: Δl_t^{gap}	
Parameterization	λ^a	CEV (%)	λ^a	CEV (%)	λ^a	CEV (%)	λ^a	CEV (%)
Woodford	0.048	0.611	0.048	0.525	0.048	0.885	0.048	0.817
Optimized	2.943	0.302	11.21	0.079	18.60	0.212	21.68	0.058

Note: The loss function under price inflation is specified as in (9), while the loss function with the annualized nominal wage inflation rate wage is specified as $(\Delta w_t^a - \Delta w^a)^2 + \lambda^a x_t^2$, where Δw^a denotes the annualized steady state wage inflation rate; see eq. (15). Δy_t denotes annualized output growth as deviation from the steady state annualized growth rate $(4(\gamma - 1))$. Δy_t^{gap} is the annualized rate of growth of output as deviation from potential, i.e. $4\left(\Delta y_t - \Delta y_t^{pot}\right)$. The same definitions apply to hours worked. See notes to Table 1 for further explanations.

Several important papers in the previous literature have stressed the merits of price level targeting as opposed to the standard inflation targeting loss function, see e.g. Vestin (2006). Price level targeting is a commitment to eventually bring back the price level to a baseline path in the face of shocks that creates a trade-off between stabilizing inflation and economic activity. Our benchmark flexible inflation targeting objective in eq. (9) can be replaced with a price level targeting objective as follows:

$$L_t^a = (p_t - \bar{p}_t)^2 + \lambda^a x_t^2, \tag{16}$$

where p_t is the actual log-price level in the economy and $\bar{p}_t$ is the target log-price level path which grows with the steady-state net inflation rate π according to $\bar{p}_t = \pi + \bar{p}_{t-1}$. When we consider wage level targeting we adopt a specification isomorphic to that in (16), but replace the first term with $w_t - \bar{w}_t$ where w_t is the nominal actual log-wage and $\bar{w}_t$ is the nominal target log-wage which grows according to $\bar{w}_t = \ln(\gamma) + \pi + \bar{w}_{t-1}$, where γ is the gross technology growth rate of the economy (see Table A.1).

In Table 5.b, we report results for both price- and wage-level targeting objectives. As can be seen from the table, there are no welfare gains from pursuing price-level targeting relative to our benchmark objective in Table 2, regardless of whether one targets the output or the hours gap. For wage-level targeting, we obtain the same finding (in this case, the relevant comparison is the wage-inflation hours-gap specification in Table 3 which yields a CEV of 0.016). These findings are perhaps unsurprising, given that the welfare costs in our model are more associated with changes in prices and wages (because of indexation) than with accumulated price- and wage-inflation rates.

Table 5.b: Sensitivity Analysis: Merits of Price and Wage Level Targeting.

	Price-Level Targeting				Wage-Level Targeting			
	x_t: y_t^{gap}		x_t: l_t^{gap}		x_t: y_t^{gap}		x_t: l_t^{gap}	
Parameterization	λ^a	CEV (%)	λ^a	CEV (%)	λ^a	CEV (%)	λ^a	CEV (%)
Woodford	0.048	0.542	0.048	0.776	0.048	0.502	0.048	0.688
Optimized	9.187	0.092	28.41	0.095	11.37	0.064	36.14	0.067

Note: The loss function under price-level targeting is given by (16), while the loss function with the nominal wage level is specified as $L_t^a = (w_t - \bar{w}_t)^2 + \lambda^a x_t^2$. See notes to Table 1 for further explanations.

4.3 Volatility of Interest Rates

In addition to inflation and some measure of resource utilization, simple objectives often include a term involving the volatility of interest rates; see e.g. Rudebusch and Svensson (1999). In practice, this term is often motivated by reference to "aversion to interest-rate variability" and financial stability concerns. From a theoretical perspective, Woodford (2003) derives an extended version of (9) augmented with an interest rate gap term $\lambda_r (r_t^a - r^a)^2$ when allowing for monetary transactions frictions ($r_t^a - r^a$ is the deviation of the annualized nominal policy rate r_t^a around the steady-state annualized policy rate r^a).

As an alternative, some researchers (e.g. Rudebusch and Svensson, 1999) and policymakers (e.g. Yellen, 2012) instead consider augmenting the objective function with the variance of the *change* in the short-run interest rate, i.e. $\lambda_r \left(\Delta r_t^a\right)^2$. By allowing for a lag of the interest rate in the loss function, the specification introduces interest rate smoothing, as the reduced-form solution will feature the lagged interest rate in the central bank's reaction function. Both specifications, however, will reduce volatility of policy rates because the central bank will, ceteris paribus, tend to be less aggressive in the conduct of monetary policy when $\lambda_r > 0$.

The first row in Table 6 considers the standard Woodford (2003) specification with only x_t as an additional variable to inflation as in (9). The second row in the table includes the $\left(r_t^a - r^a\right)^2$ term in the loss function and uses Woodford's (2003) weights for economic activity and the interest rate (0.048 and 0.077, respectively). The third row reports results for Yellen's (2012) specification of the loss function which includes the $\left(\Delta r_t^a\right)^2$ term in the loss function instead of $\left(r_t^a - r^a\right)^2$ and uses the weights (0.25 and 1.00, respectively). Finally, the last two rows present results when the coefficient on x_t and the interest rate gap—row 4—and the change in the interest rate gap—row 5—are optimized to maximize welfare of the households.

Turning to the results, we see by comparing the first and second rows in the table that the CEV is not much affected by the introduction of the interest term for the output gap and output. For output growth, however, including the interest rate term reduces the welfare costs by more than a factor of 2. Comparing the third row—the Yellen parameterization—with the Woodford specification in the second row, we see that while welfare improves considerably for all three different x_t variables, it is only for output growth that this improvement stems from the interest rate term. For the output gap and output, the improvement is mostly due to the higher λ^a, which can be confirmed by comparing the dual mandate row in Table 1 with the third row in Table 6.

When allowing for optimal weights (the last two rows in Table 6), we find that the optimized weight on the interest rate term in both cases are driven towards zero for the output gap, implying that the welfare consequences are marginal. Only for output and output growth do we find modest welfare improvements from including any of the two interest rate terms (compared to our benchmark results in Table 1 where CEV equaled 0.244 and 0.302 for output and output growth, respectively). However, in all cases our key finding holds up—some measure of real activity should carry a large weight.

Table 6: Sensitivity Analysis: Minimization of (9) with an interest rate term.

	x_t: Output Gap			x_t: Output (dev from trend)			x_t: Output Growth (Ann.)		
Loss Function	λ^a	λ_r	CEV (%)	λ^a	λ_r	CEV (%)	λ^a	λ_r	CEV (%)
Woodford: only x_t	0.048	–	0.471	0.048	–	0.554	0.048	–	0.611
Woodford: $r_t^a - r^a$	0.048	0.0770	0.462	0.048	0.0770	0.452	0.048	0.0770	0.523
Yellen: Δr_t^a	0.250	1.0000	0.186	0.250	1.0000	0.242	0.250	1.0000	0.547
Optimized: $r_t^a - r^a$	1.042	0.0001	0.044	0.461	0.1067	0.215	2.364	0.0926	0.280
Optimized: Δr_t^a	1.042	0.0001	0.044	0.530	0.0261	0.216	2.700	0.0827	0.285

Note: The loss function with the level of the interest rate is specified as $(\pi_t^a - \pi^a)^2 + \lambda^a x_t^2 + \lambda_r (r_t^a - r^a)^2$, while the loss function with the change in the interest rate is specified as $(\pi_t^a - \pi^a)^2 + \lambda^a x_t^2 + \lambda_r (\Delta r_t^a)^2$. See notes to Table 1 for further explanations.

One of the concerns for financial stability is that the nominal interest rate is conventionally the key instrument of monetary policy. In this vein, high volatility of interest rates could be problematic for financial markets if such policies were implemented. An additional concern is whether the probability distribution of nominal rates for the mandates under consideration covers the negative range in a nontrivial way. One of the advantages of specifying a simple mandate, rather than a simple interest rate rule, is that the central bank can choose to use a variety of instruments to implement the desired objective. Besides nominal interest rates, such instruments can include forward guidance, reserve requirements, asset purchases, money instruments and others. So even though the zero lower bound on nominal interest rates *per se* is less of a concern in our analysis, we still want examine to what extent our results are robust to limiting the short-term variability of monetary policy.

In what follows, we use a standard approach to limit the standard deviation of the nominal interest rate: Rotemberg and Woodford (1998) adopted the rule of thumb that the steady-state nominal rate minus two standard deviations (std) for the rate should be non-negative. Others, like Adolfson et al. (2011) adopted a three std non-negativity constraint. Since our parameterization of the SW model implies an annualized nominal interest rate of 6.25 percent, the allowable std is 3.125 under the Rotemberg and Woodford's rule of thumb and slightly below 2.1 under the stricter three-std criterion adopted by Adolfson et al. (2011). So although inclusion of $r_t^a - r^a$ or Δr_t^a does not improve welfare much, we are interested in examining to what extent our optimized simple mandates without interest rate terms are associated with excessive interest rate volatility and to what extent including the interest rate terms mitigates such excessive volatility.

Table 7: Interest Rate Volatility for Output Gap in Loss Function.

Loss Function	λ^a	λ_r	CEV (%)	std(r_t^a)
Woodford	0.048	–	0.471	8.92
Dual Mandate	0.250	–	0.140	8.76
Optimized	1.042	–	0.044	9.00
Woodford: $r_t^a - r^a$	0.048	0.0770	0.462	0.98
Yellen: Δr_t^a	0.250	1.0000	0.186	1.24
Optimized*: $r_t^a - r^a$	1.161	0.0770*	0.076	2.24
Optimized*: Δr_t^a	1.110	1.0000*	0.084	2.04

Note: std(r_t^a) denotes the standard deviation for the annualized nominal interest rate. y_t^{gap} is used as measure of x_t in the loss function. The * in the last two rows denote that these values have been fixed, and are hence not optimized.

Table 7 reports the result of our exercise. For brevity of exposition we focus on the output gap only, but the results are very similar for output level and output growth. As seen from the first three rows in the table, the objective functions in Table 1 that involve only inflation and the output gap are indeed associated with high interest rate volatility. The std's are all around 9 percentage points—a few times bigger than our thresholds. Hence, these loss functions are contingent on unrealistically large movements in the short-term policy rate. Turning to the fourth and fifth rows, which report results for the Woodford and Yellen loss functions augmented with interest rate terms, we see that the std's for the policy rate shrink by almost a factor of ten; these specifications are hence clearly consistent with reasonable movements in the stance of monetary policy.

The last two rows in the table report results when we re-optimize the weight on the output gap (λ^a) given a weight of 0.077 for $(r_t^a - r^a)^2$ (next-to-last row) and 1 for $(\Delta r_t^a)^2$ (last row) in the loss function. As seen from the last column, these policies generate considerably lower interest volatility relative to the optimized loss function which excludes any interest rate terms, and the obtained std's are in line with even the three-std threshold applied by Adolfson et al. (2011). To compensate for the interest rate terms, the optimization generates a slightly higher λ^a compared to the simple loss function with the output gap only. Overall, the lower flexibility to adjust policy rates is associated with a lower welfare; the CEV roughly doubles in both cases.[27] But it is notable that the CEV does not increase to the same extent as std(r_t^a) is reduced, reflecting that the central bank—which is assumed to operate under commitment—can still influence the long-term interest rate effectively by smaller but persistent movements of the short-term policy rate. Even so, we conclude that our benchmark result of a large weight on the real activity term holds for a plausible degree of interest rate volatility.

[27] It should be noted that the good performance of the nominal wage growth-labor gap simple mandate in Table 3 is also contingent on a relatively high interest rate volatility. However, when we augment the wage-labor loss function with an interest rate term, we find that the CEV is about twice as low as the inflation-output gap based objective which imposes the same interest rate volatility. Thus, the labor based mandate still outperforms the inflation-output gap mandate conditional on much less volatile policy rates.

5 Simple Interest Rate Rules

Up to this point, we have considered how simple mandates fare in terms of welfare and tried to find the best simple mandate among a certain class of such mandates. In this section, we turn our attention to simple interest rate rules. We do so for two reasons. First, we are interested in knowing to what extent simple and widely used interest rate rules like the Taylor (1993) rule can approximate the Ramsey policy compared with simple loss functions. Second, we are interested in knowing how a simple policy rule should be designed (in terms of variables and their associated weights) to *mimic* the Ramsey policy as closely as possible.

To be concrete, the central bank is posited to implement monetary policy by following a certain simple interest rate rule. Once a rule is adopted the central bank is assumed to be able to choose the response coefficients in the simple rule to maximize household welfare. If we denote the economy as $X_t(R; X_{t-1})$, where R represent a specific policy rule, the optimized simple rule that maximizes household welfare is defined by

$$R^*(\Lambda) = \underset{R \in \Lambda}{\arg\min} \mathrm{E}\left[(X_t(R; X_{t-1}))' W^{society} (X_t(R; X_{t-1}))\right], \tag{17}$$

where Λ is the set of possible parameterizations of the rule (17). The resulting loss for the society is

$$Loss^{rule}(\Lambda) = \mathrm{E}\left[(X_t(R^*(\Lambda)))' W^{society} (X_t(R^*(\Lambda)))\right]. \tag{18}$$

Given our previous findings that an objective with inflation and the output gap provides a good approximation of households' welfare, we consider variants of the following simple rule:

$$r_t^a - r^a = (1-\rho_r)\left[\varrho_\pi (\pi_t^a - \pi^a) + \varrho_y (y_t^{gap}) + \varrho_{\Delta y} 4 \Delta y_t^{gap}\right] + \rho_r \left(r_{t-1}^a - r^a\right). \tag{19}$$

However, since the results in Table 3 showed that an objective with wage inflation and the hours gap provided an even better approximation of Ramsey policy, we also entertain variants of the following simple interest rate rule:

$$r_t^a - r^a = (1-\rho_r)\left[\varrho_{\Delta w} (\Delta w_t^a - \Delta w^a) + \varrho_l (l_t^{gap}) + \varrho_{\Delta l} 4 \Delta l_t^{gap}\right] + \rho_r \left(r_{t-1}^a - r^a\right). \tag{20}$$

The results of this exercise are reported in Table 8. In the table, Panel A reports results when the objective is to minimize (18) and the rule is given by (19). Panel B, on the other hand, reports results when the rule is given by (20).

Table 8: Performance of Simple Rules (19).

Panel A: Inflation and Output Gap Rule (19)

Parameterization	ϱ_π	ϱ_y	$\varrho_{\Delta y}$	ρ_r	CEV (%)	std(r_t^a)
Taylor (1993)	1.50	0.50	–	–	0.399	5.43
Taylor (1999)	1.50	1.00	–	–	0.768	7.53
Optimized, Taylor	2.23	0.49	–	–	0.254	4.30
Optimized, $\varrho_{\Delta y}$= 0	11.78	5.76	–	0.99	0.216	2.08
Optimized, uncon.	20.20	0.40	56.52	0.48	0.033	7.81
Optimized, constr.	29.28	0.79	54.81	0.99	0.082	2.08

Panel B: Labor Market Rule (20)

Parameterization	$\varrho_{\Delta w}$	ϱ_l	$\varrho_{\Delta l}$	ρ_r	CEV (%)	std(r_t^a)
Optimized, uncon.	49.18	1.74	204.59	0.71	0.014	7.83
Optimized, constr.	11.40	1.72	3.53	0.98	0.104	2.08

Note: The "Optimized" coefficients in the panels are found by maximizing household welfare, i.e. minimizing CEV. In the optimizations, we always constrain the smoothing coefficient ρ_r to be between $0 - 0.99$. For the "Optimized, constr." optimizations, we have restricted the coefficients of the rule to imply a standard deviation for the policy rate less or equal to 2.0833.

Turning to the results in the first panel in Table 8, we see that neither the Taylor (1993) rule—which sets $\varrho_\pi = 1.5, \varrho_y = 0.5$ and $\varrho_{\Delta y} = \rho_r = 0$ in (19)—nor the Taylor (1999) rule—which doubles ϱ_y to 1—can approximate the Ramsey policy well using the 0.05 percent CEV cut-off value advocated by Schmitt-Grohe and Uribe (2007). Both policy rules increase CEV substantially relative to the simple mandate with inflation and the output gap. Perhaps counterintuitively to the simple mandate results that emphasized the importance of responding vigorously to economic activity, the Taylor (1999) rule is associated with higher loss than the Taylor (1993) rule.[28] As can be seen from the third row in the table—which reports results for an optimized Taylor type rule—it seems that the Taylor (1993) rule strikes a good compromise between responding to inflation and the output gap. Evidently, an optimized λ^a of unity in the loss function (9) does not translate into equal coefficients for $\pi_t^a - \pi^a$ and y_t^{gap} in (19). When we re-optimize the coefficients while allowing for interest rate smoothing—the fourth row in the table—we see that welfare is improved somewhat, but not sufficiently to close the welfare gap to the optimized simple inflation-output gap mandate in Section 3.

The key to improve the welfare performance of the simple rule is to include the annualized growth rate of the output gap in the simple rule (fifth row). When doing so, we find that the $\varrho_{\Delta y}$ coefficient becomes extremely large (56.52) and that welfare is improved markedly. This finding is consistent with Orphanides and Williams (2008), who argued in favor of responding to the growth rate of the output gap in optimized simple rules within the context of learning and model uncertainty.

[28] Debortoli and Nunes (2014) show that a larger weight on economic activity in the objective function does not necessarily map into a higher weight of economic activity in the interest rate rule.

However, we see from the last column that the standard deviation of the policy rate is substantially higher than the threshold value suggested by Adolfson et al. (2011) discussed earlier (see Section 4.3). And when we impose that the optimized rule should satisfy the constraint that std(r_t^a) is lower or equal to 2.08 (i.e. the Adolfson et al. (2011) threshold which we used in Section 4.3), we find that the welfare gains are more modest ("Optimized, constr.") relative to the unconstrained rule ("Optimized, uncon."). Although this rule also features a very large long-term coefficient on the labor market gap (54.81), it should be noted that the short-term coefficient ($(1 - \rho_r)\, \varrho_{\Delta y}$) is reduced sharply (from 29.4 to a meager 0.5).[29] Nevertheless, note that optimized simple rules perform about as well as their simple-mandate peers when it comes to maximizing households' welfare, although extreme parameterizations are needed for this to be the case.

Turning to Panel B in the table, we see that the performance of (20) is similar to the standard inflation-output gap based rule in Panel A. Unconstrained, the optimized rule calls for an extremely large response to the change in the labor gap. Interestingly, this unconstrained rule, which is associated with a CEV of a mere 0.0143 percent, performs better than any of the simple mandates or rules studied so far in the paper. However, a drawback with the optimized rule is the associated high interest rate volatility. And when imposing the restriction that std(r_t^a) should be less than the Adolfson et al. (2011) threshold, the short- and long-term coefficients on the change in the labor gap are much smaller and the CEV of the rule deteriorates slightly below that found for the inflation-output gap based simple rule in Panel A. This finding demonstrates the importance of imposing reasonable constraints on the coefficients in the rule and suggests that some rules, which may perform very well in theory, may not be as desirable to implement in practice.

These results show that even if a certain set of variables performs well with simple objectives, the same set of variables may not perform well with simple interest rate rules. For instance, Table 1 reports that the optimized simple dual inflation-output gap mandate yields a loss of 0.044. However, the third row in Table 8 shows that the loss for an optimized simple interest rate with the same variables increases roughly five times to 0.216, even when we allow for interest rate smoothing. Only when designing the simple interest rate rule carefully by including the change in the output gap (fourth row), does the optimized rule yield a welfare loss of 0.033, which is lower than the loss of the optimized simple mandate.

Our results demonstrate that inferring outcomes from simple mandates to simple interest rate

[29] The size of $\varrho_{\Delta y}$ and the performance of the rule in terms of welfare are diminished if the policy maker responds to the actual growth rate instead of the change in the gap; according to the model it is crucial to take the growth rate in potential output into account.

rules can be difficult. Both interest rate rules and simple mandates can perform very well when designed carefully, although somewhat extreme parameterizations appear to be required for interest rate rules to work well. Even so, a common theme for both optimized simple mandates and rules is that a vigorous response to economic activity is warranted.

6 Conclusions

There appears to be broad consensus among academics and policymakers that central banks should primarily focus on price stability and devote only modest attention to measures of real economic activity. Many influential studies in the monetary policy literature show that such a behavior would deliver the best possible policy from a social welfare perspective.

This paper revisits this issue within the context of an estimated medium-scale model for the US economy, and shows that the validity of earlier prescriptions may not be warranted. Looking at measures of economic activity seems to be more important than previously recognized in academia and in policy circles. This result is particularly relevant to economies affected by non-trivial real rigidities and inefficient shocks, thus displaying a relevant trade-off between stabilizing inflation and economic activity. In practice, it is difficult to assess the importance of real rigidities and the role inefficient shocks may play to magnify policy trade-offs. But that argument does not invalidate our main conclusion. Responding vigorously to measures of economic activity is a robust policy, in the sense that it would deliver good economic outcomes even in the absence of relevant policy trade-offs. Even so, we recognize that the importance of targeting economic activity hinges importantly on the role that inefficient cost-push shocks play for economic fluctuations, and research on their importance are warranted but beyond the scope of our paper.[30]

During the recent financial crisis many central banks, including the Federal Reserve and the Bank of England, cut policy rates aggressively to prevent further falls in resource utilization although the fall in inflation and inflation expectations were modest. By traditional metrics, such as the Taylor (1993) rule, these aggressive and persistent cuts may be interpreted as a shift of focus from price stability to resource utilization by central banks' during and in the aftermath of the recession. Our results make the case for a stronger response to measures of economic activity even during normal times. In our model, the policy trade-offs mainly arise from imperfections in

[30] Following Nekarda and Ramey (2013), we define the markup as the inverse of the labor share and find that shocks to the markup exert a significant influence on output using a medium-sized VAR similar to Christiano, Eichenbaum and Evans (2005). These results, available upon request, suggest that price markup shocks may indeed be relevant for business cycle fluctuations.

goods and labor markets. Considering an economy where inefficiencies are primarily associated with frictions in the financial markets constitutes an interesting extension to address some of the recent debates.

Finally, our analysis postulated that central banks operate in an ideal situation. In this respect our approach could be extended to study the design of simple policy objectives in more realistic situations, where central banks face uncertainty about the structure of the underlying economy or cannot implement their desired policies because of implementation lags or credibility problems.

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Appendix A The Smets and Wouters (2007) Model

Below, we describe the firms' and households' problem in the model, and state the market clearing conditions.[A.1]

A.1 Firms and Price Setting

Final Goods Production The single final output good Y_t is produced using a continuum of differentiated intermediate goods $Y_t(f)$. Following Kimball (1995), the technology for transforming these intermediate goods into the final output good is

$$\int_0^1 G_Y\left(\frac{Y_t(f)}{Y_t}\right) df = 1. \tag{A.1}$$

Following Dotsey and King (2005) and Levin, López-Salido and Yun (2007) we assume that $G_Y(.)$ is given by a strictly concave and increasing function; its particular parameterization follows SW:

$$G_Y\left(\frac{Y_t(f)}{Y_t}\right) = \left(\frac{\phi_p}{1-(\phi_p-1)\epsilon_p}\left[\left(\frac{\phi_p+(1-\phi_p)\epsilon_p}{\phi_p}\right)\frac{Y_t(f)}{Y_t}+\frac{(\phi_p-1)\epsilon_p}{\phi_p}\right]^{\frac{1-(\phi_p-1)\epsilon_p}{\phi_p-(\phi_p-1)\epsilon_p}}+\left[1-\frac{\phi_p}{1-(\phi_p-1)\epsilon_p}\right]\right), \tag{A.2}$$

where $\phi_p \geq 1$ denotes the gross markup of the intermediate firms. The parameter ϵ_p governs the degree of curvature of the intermediate firm's demand curve. When $\epsilon_p = 0$, the demand curve exhibits constant elasticity as with the standard Dixit-Stiglitz aggregator. When ϵ_p is positive—as in SW—the firms instead face a quasi-kinked demand curve, implying that a drop in its relative price only stimulates a small increase in demand. On the other hand, a rise in its relative price generates a large fall in demand. Relative to the standard Dixit-Stiglitz aggregator, this introduces more strategic complementarity in price setting which causes intermediate firms to adjust prices less to a given change in marginal cost. Finally, notice that $G_Y(1) = 1$, implying constant returns to scale when all intermediate firms produce the same amount.

Firms that produce the final output good are perfectly competitive in both the product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index Y_t, taking as given the price $P_t(f)$ of each intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price P_t, and hence solve the following problem:

$$\max_{\{Y_t, Y_t(f)\}} P_t Y_t - \int_0^1 P_t(f)\, Y_t(f)\, df, \tag{A.3}$$

[A.1] For a more detailed description of the model, we refer the reader to the on-line appendix of the Smets and Wouters paper, which is available online at http://www.aeaweb.org/aer/data/june07/20041254_app.pdf.

subject to the constraint (A.1). The first order conditions for this problem can be written

$$\begin{aligned}
\frac{Y_t(f)}{Y_t} &= \frac{\phi_p}{\phi_p-(\phi_p-1)\epsilon_p}\left(\left[\frac{P_t(f)}{P_t}\frac{1}{\Lambda_t^p}\right]^{-\frac{\phi_p-(\phi_p-1)\epsilon_p}{\phi_p-1}}+\frac{(1-\phi_p)\epsilon_p}{\phi_p}\right), \qquad (A.4)\\
P_t\Lambda_t^p &= \left[\int P_t\left(f\right)^{-\frac{1-(\phi_p-1)\epsilon_p}{\phi_p-1}}df\right]^{-\frac{\phi_p-1}{1-(\phi_p-1)\epsilon_p}},\\
\Lambda_t^p &= 1+\frac{(1-\phi_p)\epsilon_p}{\phi_p}-\frac{(1-\phi_p)\epsilon_p}{\phi_p}\int\frac{P_t(f)}{P_t}df,
\end{aligned}$$

where Λ_t^p denotes the Lagrange multiplier on the aggregator constraint (A.1). Note that for $\epsilon_p=0$ and $\Lambda_t^p=1$ in each period t, the demand and pricing equations collapse to the usual Dixit-Stiglitz expressions

$$\frac{Y_t\left(f\right)}{Y_t}=\left[\frac{P_t\left(f\right)}{P_t}\right]^{-\frac{\phi_p}{\phi_p-1}}, P_t=\left[\int P_t\left(f\right)^{\frac{1}{1-\phi_p}}df\right]^{1-\phi_p}. \qquad (A.5)$$

Intermediate Goods Production A continuum of intermediate goods $Y_t(f)$ for $f\in[0,1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces the demand schedule in eq. (A.4) from the final goods firms through the solution to the problem in (A.3), which varies inversely with its output price $P_t\left(f\right)$ and directly with aggregate demand Y_t.

Each intermediate goods producer utilizes capital services $K_t\left(f\right)$ and a labor index $L_t\left(f\right)$ (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t\left(f\right)=\varepsilon_t^aK_t(f)^\alpha\left[\gamma^tL_t(f)\right]^{1-\alpha}-\gamma^t\Phi, \qquad (A.6)$$

where γ^t represents the labour-augmenting deterministic growth rate in the economy, Φ denotes the fixed cost (which is related to the gross markup ϕ_p so that profits are zero in the steady state), and ε_t^a is total factor productivity which follows the process

$$\ln\varepsilon_t^a=(1-\rho_a)\ln\varepsilon^a+\rho_a\ln\varepsilon_{t-1}^a+\eta_t^a, \eta_t^a\sim N\left(0,\sigma_a\right). \qquad (A.7)$$

Firms face perfectly competitive factor markets for renting capital and hiring labor. Thus, each firm chooses $K_t\left(f\right)$ and $L_t\left(f\right)$, taking as given both the rental price of capital R_{Kt} and the aggregate wage index W_t (defined below). Firms can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output.

The prices of the intermediate goods are determined by Calvo-Yun (1996) style staggered nominal contracts. In each period, each firm f faces a constant probability, $1-\xi_p$, of being able to reoptimize its price $P_t(f)$. The probability that any firm receives a signal to re-optimize its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, it adjusts its price by a weighted combination of the lagged and steady-state rate of inflation, i.e., $P_t(f) = (1+\pi_{t-1})^{\iota_p}(1+\pi)^{1-\iota_p} P_{t-1}(f)$ where $0 \leq \iota_p \leq 1$ and π_{t-1} denotes net inflation in period $t-1$, and π the steady-state net inflation rate. A positive value of ι_p introduces structural inertia into the inflation process. All told, this leads to the following optimization problem for the intermediate firms

$$\max_{\tilde{P}_t(f)} \mathrm{E}_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \frac{\Xi_{t+j} P_t}{\Xi_t P_{t+j}} \left[\tilde{P}_t\left(f\right)\left(\Pi_{s=1}^{j}\left(1+\pi_{t+s-1}\right)^{\iota_p}\left(1+\pi\right)^{1-\iota_p}\right) - MC_{t+j}\right] Y_{t+j}\left(f\right), \quad \text{(A.8)}$$

where $\tilde{P}_t\left(f\right)$ is the newly set price. Notice that with our assumptions all firms that re-optimize their prices actually set the same price.

It would be ideal if the markup in $(A.2)$ can be made stochastic and the model can be written in a recursive form. However, such an expression is not available, and we instead directly introduce a shock ε_t^p in the first-order condition to the problem in (A.8). And following SW, we assume the shock is given by an exogenous ARMA(1,1) process:

$$\ln \varepsilon_t^p = \left(1-\rho_p\right)\ln \varepsilon^p + \rho_p \ln \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p, \eta_t^p \sim N\left(0, \sigma_p\right). \quad \text{(A.9)}$$

When this shock is introduced in the non-linear model, we put a scaling factor on it so that it enters exactly the same way in a log-linearized representation of the model as the price markup shock does in the SW model.[A.2]

A.2 Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector; that is, goods-producing firms regard each household's labor services $L_t(h)$, $h \in [0,1]$, as imperfect substitutes

[A.2] Alternatively, we could have followed the specification in Adjemian et al. (2008) and introduced the shock as a tax on the intermediate firm's revenues in the problem (A.8) directly. The drawback with this alternative approach is that the log-linearized representation of the model would have a different lead-lag structure from the representation in SW. In a later section, we perform robustness analysis with respect to the price- and wage-markup shocks and show that our main result holds.

for the labor services of other households. It is convenient to assume that a representative labor aggregator combines households' labor hours in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labor is equal to the sum of firms' demands. The aggregated labor index L_t has the Kimball (1995) form:

$$L_t = \int_0^1 G_L\left(\frac{L_t(h)}{L_t}\right) dh = 1, \tag{A.10}$$

where the function $G_L(.)$ has the same functional form as (A.2), but is characterized by the corresponding parameters ϵ_w (governing convexity of labor demand by the aggregator) and ϕ_w (gross wage markup). The aggregator minimizes the cost of producing a given amount of the aggregate labor index L_t, taking each household's wage rate $W_t(h)$ as given, and then sells units of the labor index to the intermediate goods sector at unit cost W_t, which can naturally be interpreted as the aggregate wage rate. From the FOCs, the aggregator's demand for the labor hours of household h—or equivalently, the total demand for this household's labor by all goods-producing firms—is given by

$$\frac{L_t(h)}{L_t} = G_L'^{-1}\left[\frac{W_t(h)}{W_t}\int_0^1 G_L'\left(\frac{L_t(h)}{L_t}\right)\frac{L_t(h)}{L_t}dh\right], \tag{A.11}$$

where $G_L'(.)$ denotes the derivative of the $G_L(.)$ function in eq. (A.10).

The utility function of a typical member of household h is

$$\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left[\frac{1}{1-\sigma_c}\left(C_{t+j}(h) - \varkappa C_{t+j-1}\right)\right]^{1-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} L_{t+j}(h)^{1+\sigma_l}\right), \tag{A.12}$$

where the discount factor β satisfies $0 < \beta < 1$.[A.3] The period utility function depends on household h's current consumption $C_t(h)$, as well as lagged aggregate per capita consumption to allow for external habit persistence. The period utility function also depends inversely on hours worked $L_t(h)$.

Household h's budget constraint in period t states that its expenditure on goods and net purchases of financial assets must equal its disposable income:

$$\begin{aligned} & P_t C_t(h) + P_t I_t(h) + \frac{B_{t+1}(h)}{\varepsilon_t^b R_t} + \int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h) \\ & = B_t(h) + W_t(h) L_t(h) + R_t^k Z_t(h) K_t^p(h) - a(Z_t(h)) K_t^p(h) + \Gamma_t(h) - T_t(h). \end{aligned} \tag{A.13}$$

Thus, the household purchases part of the final output good (at a price of P_t), which it chooses either to consume $C_t(h)$ or invest $I_t(h)$ in physical capital. Following Christiano, Eichenbaum, and

[A.3] Note that we deviate slightly from the notation in SW by using h to index households and using $\varkappa$ to denote the degree of habit formation.

Evans (2005), investment augments the household's (end-of-period) physical capital stock $K_{t+1}^{p}(h)$ according to

$$K_{t+1}^{p}(h) = (1-\delta)K_{t}^{p}(h) + \varepsilon_{t}^{i}\left[1 - S\left(\frac{I_{t}(h)}{I_{t-1}(h)}\right)\right]I_{t}(h). \tag{A.14}$$

The extent to which investment by each household h turns into physical capital is assumed to depend on an exogenous shock ε_{t}^{i} and how rapidly the household changes its rate of investment according to the function $S\left(\frac{I_{t}(h)}{I_{t-1}(h)}\right)$, which we specify as

$$S(x_t) = \tfrac{\varphi}{2}\left(x_t - \gamma\right)^2. \tag{A.15}$$

Notice that this function satisfies $S(\gamma) = 0$, $S'(\gamma) = 0$ and $S''(\gamma) = \varphi$. The stationary investment-specific shock ε_{t}^{i} follows

$$\ln \varepsilon_{t}^{i} = \rho_{i} \ln \varepsilon_{t-1}^{i} + \eta_{t}^{i}, \eta_{t}^{i} \sim N(0, \sigma_{i}). \tag{A.16}$$

In addition to accumulating physical capital, households may augment their financial assets through increasing their government nominal bond holdings (B_{t+1}), from which they earn an interest rate of R_t. The return on these bonds is also subject to a risk-shock, ε_{t}^{b}, which follows

$$\ln \varepsilon_{t}^{b} = \rho_{b} \ln \varepsilon_{t-1}^{b} + \eta_{t}^{b}, \eta_{t}^{b} \sim N(0, \sigma_{b}). \tag{A.17}$$

We assume that agents can engage in frictionless trading of a complete set of contingent claims to diversify away idiosyncratic risk. The term $\int_{s} \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h)$ represents net purchases of these state-contingent domestic bonds, with $\xi_{t,t+1}$ denoting the state-dependent price, and $B_{D,t+1}(h)$ the quantity of such claims purchased at time t.

On the income side, each member of household h earns after-tax labor income $W_t(h)L_t(h)$, after-tax capital rental income of $R_{t}^{k} Z_t(h) K_{t}^{p}(h)$, and pays a utilization cost of the physical capital equal to $a(Z_t(h))K_{t}^{p}(h)$ where $Z_t(h)$ is the capital utilization rate, so that capital services provided by household h, $K_t(h)$, equals $Z_t(h)K_{t}^{p}(h)$. The capital utilization adjustment function $a(Z_t(h))$ is assumed to be given by

$$a(Z_t(h)) = \frac{r^k}{\tilde{z}_1}\left[\exp\left(\tilde{z}_1\left(Z_t(h) - 1\right)\right) - 1\right], \tag{A.18}$$

where r^k is the steady state net real interest rate $(\bar{R}_{t}^{K}/\bar{P}_{t})$. Notice that the adjustment function satisfies $a(1) = 0$, $a'(1) = r^k$, and $a''(1) \equiv r^k \tilde{z}_1$. Following SW, we want to write $a''(1) = z_1 = \psi/(1-\psi) > 0$, where $\psi \in [0, 1)$ and a higher value of ψ implies a higher cost of changing the utilization rate. Our parameterization of the adjustment cost function then implies that we need to set $\tilde{z}_1 \equiv z_1/r^k$. Finally, each member also receives an aliquot share $\Gamma_t(h)$ of the profits of all firms, and pays a lump-sum tax of $T_t(h)$ (regarded as taxes net of any transfers).

In every period t, each member of household h maximizes the utility function (A.12) with respect to its consumption, investment, (end-of-period) physical capital stock, capital utilization rate, bond holdings, and holdings of contingent claims, subject to its labor demand function (A.11), budget constraint (A.13), and transition equation for capital (A.14).

Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described previously. Thus, the probability that a household receives a signal to re-optimize its wage contract in a given period is denoted by $1-\xi_w$. In addition, SW specify the following dynamic indexation scheme for the adjustment of the wages of those households that do not get a signal to re-optimize: $W_t(h) = \gamma (1+\pi_{t-1})^{\iota_w} (1+\pi)^{1-\iota_w} W_{t-1}(h)$. All told, this leads to the following optimization problem for the households

$$\max_{\tilde{W}_t(h)} \mathrm{E}_t \sum_{j=0}^{\infty} (\beta\xi_w)^j \frac{\Xi_{t+j} P_t}{\Xi_t P_{t+j}} \left[\tilde{W}_t(h) \left(\Pi_{s=1}^{j} \gamma (1+\pi_{t+s-1})^{\iota_w} (1+\pi)^{1-\iota_w}\right) - W_{t+j}\right] L_{t+j}(h), \quad \text{(A.19)}$$

where $\tilde{W}_t(h)$ is the newly set wage; notice that with our assumptions all households that reoptimize their wages will actually set the same wage.

Following the same approach as with the intermediate-goods firms, we introduce a shock ε_t^w in the resulting first-order condition. This shock, following SW, is assumed to be given by an exogenous ARMA(1,1) process

$$\ln \varepsilon_t^w = (1-\rho_w) \ln \varepsilon^w + \rho_w \ln \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w, \eta_t^w \sim N(0, \sigma_w). \quad \text{(A.20)}$$

As discussed previously, we use a scaling factor for this shock so that it enters in exactly the same way as the wage markup shock in SW in the log-linearized representation of the model.

A.3 Market Clearing Conditions

Government purchases G_t are exogenous, and the process for government spending relative to trend output, i.e. $g_t = G_t / \left(\gamma^t Y\right)$, is given by the following exogenous AR(1) process:

$$\ln g_t = \left(1-\rho_g\right) \ln g + \rho_g \left(\ln g_{t-1} - \rho_{ga} \ln \varepsilon_{t-1}^a\right) + \varepsilon_t^g, \varepsilon_t^g \sim N(0, \sigma_g). \quad \text{(A.21)}$$

Government purchases have no effect on the marginal utility of private consumption, nor do they serve as an input into goods production. The consolidated government sector budget constraint is

$$\frac{B_{t+1}}{R_t} = G_t - T_t + B_t. \quad \text{(A.22)}$$

By comparing the debt terms in the household budget constraint in eq. (A.13) with the equation above, one can see that receipts from the risk shock are subject to iceberg costs, and hence do not add any income to the government.[A.4]

Total output of the final goods sector is used as follows:

$$Y_t = C_t + I_t + G_t + a\left(Z_t\right)\bar{K}_t, \tag{A.23}$$

where $a\left(Z_t\right)\bar{K}_t$ is the capital utilization adjustment cost.

Finally, we need to specify the aggregate production constraint. To do that, we note that the unweighted sum of the intermediate firms' output equals

$$Y_t^{sum} = \int_0^1 Y_t\left(f\right)df, \tag{A.24}$$

which from eq. (A.6) can be rewritten as

$$\begin{aligned} Y_t^{sum} &= \int_0^1 \left[\varepsilon_t^a K_t(f)^\alpha \left[\gamma^t L_t(f)\right]^{1-\alpha} - \gamma^t\Phi\right] df \\ &= \varepsilon_t^a \left(\frac{K_t}{\gamma^t L_t}\right)^\alpha \int_0^1 \gamma^t L_t(f)df - \gamma^t\Phi, \end{aligned} \tag{A.25}$$

where the second equality follows from the fact that every firms capital-labor ratio will be the same in equilibrium.

From the first-order conditions to the final goods aggregator problem (A.4), it follows that

$$Y_t^{sum} = Y_t \int_0^1 \frac{\phi_p}{\phi_p-(\phi_p-1)\epsilon_p}\left(\left[\frac{P_t\left(f\right)}{P_t}\frac{1}{\Lambda_t^p}\right]^{-\frac{\phi_p-(\phi_p-1)\epsilon_p}{\phi_p-1}} + \frac{(1-\phi_p)\epsilon_p}{\phi_p}\right)df, \tag{A.26}$$

so that

$$\varepsilon_t^a\left(\frac{K_t}{\gamma^t L_t}\right)^\alpha \gamma^t \int_0^1 L_t(h)dh - \gamma^t\Phi = Y_t \int_0^1 \frac{\phi_p}{\phi_p-(\phi_p-1)\epsilon_p}\left(\left[\frac{P_t\left(f\right)}{P_t}\frac{1}{\Lambda_t^p}\right]^{-\frac{\phi_p-(\phi_p-1)\epsilon_p}{\phi_p-1}} + \frac{(1-\phi_p)\epsilon_p}{\phi_p}\right)df.$$

By inserting the expression for the unweighted sum of labor, $\int_0^1 \gamma^t L_t(h)dh$, into this last expression, we can finally derive the aggregate production constraint which depends on aggregate technology, capital, labor, fixed costs, as well as the price and wage dispersion terms.[A.5]

[A.4] But even if they did, it would not matter as we follow SW and assume that the government balances its expenditures each period through lump-sum taxes, $T_t = G_t + B_t - B_{t+1}/R_t$, so that government debt $B_t = 0$ in equilibrium. Furthermore, as Ricardian equivalence holds in the model, it does not matter for equilibrium allocations whether the government balances its debt or not in each period.

[A.5] We refer the interested reader to Adjemian, Paries and Moyen (2008) for further details.

A.4 Model Parameterization

When solving the model, we adopt the parameter estimates (posterior mode) in Tables 1.A and 1.B of SW. We also use the same values for the calibrated parameters. Table A1 provides the relevant values.

Table A.1: Parameter Values in Smets and Wouters (2007).

Panel A: Calibrated

Parameter	Description	Value	Parameter	Description	Value
δ	Depreciation rate	0.025	ϵ_p	Kimball Elast. GM	10
ϕ_w	Gross wage markup	1.50	ϵ_w	Kimball Elast. LM	10
g_y	Gov't G/Y ss-ratio	0.18			

Panel B: Estimated

Parameter	Description	Value	Parameter	Description	Value
φ	Investment adj. cost	5.48	α	Capital production share	0.19
σ_c	Inv subs. elast. of cons.	1.39	ψ	Capital utilization cost	0.54
$\varkappa$	Degree of ext. habit	0.71	ϕ_p	Gross price markup	1.61
ξ_w	Calvo prob. wages	0.73	π	Steady state net infl. rate	0.0081
σ_l	Labor supply elas.	1.92	β	Discount factor	0.9984
ξ_p	Calvo prob. prices	0.65	$\bar{l}$	Steady state hours worked	0.25
ι_w	Ind. for non-opt. wages	0.59	γ	Steady state gross growth	1.0043
ι_p	Ind. for non-opt. prices	0.22			

Panel C: Shock Processes

Shock		Persistence	MA(1)		Std. of Innovation (%)	
Neutral Technology	ρ_a	0.95	-		σ_a	0.45
Risk premium	ρ_b	0.18	-		σ_b	0.24
Gov't spending	ρ_g	0.97	ρ_{ga}	0.52	σ_g	0.52
Inv. Specific Tech.	ρ_i	0.71			σ_i	0.45
Price markup	ρ_p	0.90	μ_p	0.74	σ_p	0.14
Wage markup	ρ_w	0.97	μ_w	0.88	σ_w	0.24
Monetary policy	ρ_r	-	-		σ_r	-

Note: SW estimates $\rho_r = 0.12$ and $\sigma_r = 0.24$, but in our optimal policy exercises these parameters are not present.

There are two issues to notice with regards to the parameters in Table A1. First, we adapt and re-scale the processes of the price and wage markup shocks so that when our model is log-linearized it matches exactly the original SW model. Second, we set the monetary policy shock parameters to nil, as we restrict our analysis to optimal policy.