Cooperation Cycles: A theory of endogenous capital-embodied shocks[*]

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Abstract

We provide a theory of endogenous shocks to the marginal efficiency of investment that is based on a limited commitment friction in the creation of new capital. Inventors generate ideas but are inefficient at implementing them. When inventors collaborate with firms, their ideas can be implemented more efficiently. However, firms cannot commit to appropriately compensate inventors. The best ideas are those most at risk of theft, since reputational concerns are insufficient to always discipline firms. The fear of expropriation leads inventors to implement their best ideas inefficiently without firms. Good news about future technological progress increases the value of future business and thus disciplines firms away from expropriating better ideas, leading to increases in measured productivity and the returns to new investment. In contrast to standard models, this mechanism leads to an investment boom and increased economic growth in response to good news about future technologies.

Fluctuations in the marginal efficiency of investment have long been proposed as an explanation for macroeconomic fluctuations.[1] Recent research has uncovered a strong empirical relation between estimated 'investment shocks' and output and investment fluctuations (Greenwood, Hercowitz, and Krusell, 1997; Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010). However, the estimated investment shocks display substantial fluctuations, indicating that the efficiency of investment improves or declines at fairly high frequencies. Clearly, these investment shocks proxy for something much more than technological improvements embodied in new capital. This paper provides a theory of endogenous investment shocks, based on frictions in implementing new investment projects.

Our starting point is that the process of capital creation may involve the participation of multiple parties. Each of these parties bring different benefits to the table. For instance, inventors may bring ideas, whereas firms may bring capital and expertise in implementing these ideas. When these parties cooperate, the resulting economic value created is greater than if cooperation did not take place. For instance, if inventors develop new projects in partnership with established firms, the overall surplus created is likely greater than if the inventor developed the project on her own. Importantly, even though partnerships are efficient, they expose parties to the risk of expropriation. In our example, firms may refuse to compensate inventors for the value of their ideas.[2] If inventors anticipate being expropriated, they will choose to inefficiently implement their ideas on their own. The loss of firms' reputation – future partnership opportunities – will in some cases serve as a deterrent, allowing firms to commit not to expropriate inventors.

We build a theory of endogenous fluctuations in the marginal efficiency of investment based on this tradeoff. In particular, the projects that are at risk of being expropriated – and thus will be inefficiently implemented – are those for which their immediate economic gain exceeds the value of future partnerships to a firm. The best ideas are those most at risk of theft, since the mechanism that disciplines firms – the loss of future partnership opportunities – depends on the average quality

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[1]This idea goes back to Keynes (1936). Keynes defined the marginal efficiency of new capital (investment) as "the rate of discount which would make the present value of the series of annuities given by the returns expected from the capital asset during its life just equal its supply price". More generally, we define the marginal efficiency as the productivity of new investment in generating installed capital, akin to a productivity shock embodied in capital goods.

[2]This idea goes at least back to Arrow (1962), who writes: "There is a fundamental paradox in the determination of demand for information; its value for the purchaser is not known until he has the information, but then he has acquired it without cost."

of ideas. This fear of expropriation leads inventors to implement their best ideas inefficiently without firms. Importantly, the ratio of the present value of future partnerships to the economic value created by new projects determines the measure of projects that are efficiently implemented in a partnership. As this ratio increases, more projects are implemented in partnerships; since partnerships are more efficient, the return to new investment also increases.

We consider two examples of shocks that lead to fluctuations in the ratio of the value of future partnerships to current gains from implementation. First, we consider shocks to the surplus allocation rule between inventors and firms. This shock has the advantage that, absent the limited commitment friction, would not affect equilibrium outcomes. Second, we consider shocks to beliefs about the likelihood of *future* technological improvements. Regardless of whether these beliefs are rational or not, they affect the *perceived* cost and benefit of expropriating inventors, and thus, the measure of projects that are efficiently implemented in equilibrium.

Our model generates positive comovement in investment, consumption, capital utilization, hours and measured productivity. Capital utilization allows consumption to rise in response to an increase in the marginal efficiency of investment, leading to positive correlation between consumption and investment. This equilibrium behavior contrasts sharply with most existing business cycle models with news shocks, in which good news typically cause recessions through an income effect – households feel richer and therefore decide to reduce investment and hours worked, leading to a recession today. In the presence of the limited commitment friction, these shocks endogenously enhance the marginal efficiency of investment, possibly offsetting the strong wealth effect.

In sum, we provide a model of endogenous fluctuations in marginal efficiency; these fluctuations are driven by variation in the incentives to cooperate. This abstract tradeoff between cooperation and the possibility of expropriation can take many forms. In the paper, we label the strong party as 'firms' and the weaker party as the 'inventor'. Indeed, most independent inventors cannot successfully create an organization to take commercial advantage of their inventions and, therefore, must rely on another party. A salient friction in this context is that ideas, once they are communicated, can easily be stolen (Arrow, 1962).[3] Importantly, the stronger party in this arrangement can also be a

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[3]Idea theft is possible because intellectual property rights are not always well protected. Patents provide some measure of protection, however expropriation may still be possible. For instance, E. H. Armstrong pioneered FM

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financier that has expertise in evaluating the project but also the ability to appropriate the idea.[4] In addition, sometimes inventors can expropriate firms.[5] Last, this tradeoff exists in broader contexts. For instance, international investors in politically unstable or developing countries face the risk of expropriation of their project investments. Specifically, states can often exercise their sovereignty and appropriate capital, either on an individual basis or as part of a wider scale nationalisation program.[6]

Our work has implications about the measurement and identification of capital-embodied shocks. Our model mechanism implies that news about future technologies improve real investment opportunities, and therefore affect the marginal rate of transformation between consumption and investment today. Hence, these news have a similar effect on quantities as investment-specific technology shocks (Solow, 1960; Greenwood, Hercowitz, and Huffman, 1988; Greenwood, Hercowitz,

radio in the 1910s and 1920's. However, any of Armstrong's inventions were claimed by others. The regenerative circuit, which Armstrong patented in 1914 as a "wireless receiving system," was subsequently patented by Lee De Forest in 1916; De Forest then sold the rights to his patent to AT&T. Furthermore, once disclosed, the ideas can be implemented without the innovator, who is often not crucial to the success of the venture. For example, Robert Kearns patented the intermittent windshield wiper in 1967. He tried to interest the "Big Three" auto makers in licensing the technology. They all rejected his proposal, yet began to install intermittent wipers in their cars, beginning in 1969. Kearns ultimately won the patent lawsuit against Ford in 1978 and Chrysler in 1982. The possibility of expropriation affects the choice of whether the new invention is implemented inside an established firm (or venture capitalist) or by the innovator himself.

[4]Outright idea theft is not the only way that financiers can expropriate innovators; financiers can also appropriate significant rents by diluting the innovator's stake in the venture. Often, this happens after the founder has left the company or been terminated. This is often possible due to contractual features of the VC arrangement (see, for instance Kaplan and Stromberg, 2004). For instance, as described in Atanasov, Ivanov, and Litvak (2012), the founder of Pogo.com, an e-gaming company, sued the VCs on the board for issuing complicated derivative securities, effectively reducing his stake from 13% to 0.1%, and then refusing to redeem his stock in violation of prior agreement. Similarly, VCs are at times better informed than the innovators and this permits other opportunities to expropriate. For example, the founders of Epinions, a consumer product review website, sued three VC funds for fraudulently withholding information that caused them multimillion dollar losses. The founders alleged that the financiers persuaded them to give up their ownership interests after being led to believe that the value of their stake was zero. At the time, the VCs had indicated that the value the company was around $30 million, well below its $45 million liquidation preference. The founders alleged that, a year later, the implied value of the company was $300 million, partly due to a deal with Google and other financial results and projections that were not disclosed by the VCs.

[5]According to Bhide (1999), 71% of the founders of firms in the Inc 500 list of fast growing technology firms report that they replicated or modified ideas encountered through previous employment. For example, in the late 80s, software maker Peoplesoft and its founder David Duffield were sued by Integral Systems which claimed that its software was based on computer code that was stolen from the company while Mr. Duffield worked there.

[6]A recent example is Venezuela's expropriation of oil projects in the Orinoco Belt in 2007. Historically, the lack of appropriate mechanisms open to foreign investors to protect projects and the associated risks caused a restriction in the flow of international investment into certain countries. In order to overcome this difficulty, there have been an increasing number of contractual protections that offer some measure of protection. However, the efficiency of these measures is limited for several reasons. First, expropriation can take many indirect forms, such as changes to taxation, environmental protection or labor laws. Second, new governments can choose to default on contracts signed by their predecessors.

and Krusell, 1997; Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010, 2011; Papanikolaou, 2011). However, the effect of news on the equilibrium relative price of investment goods in our model is either positive or zero, since our effect operates through the demand for new capital, similar to Christiano, Motto, and Rostagno (2013). Hence, traditional methods of identifying IST shocks based on changes in the relative price of equipment will miss this channel. Our work thus provides a micro-foundation for the marginal efficiency of investment shock in Justiniano, Primiceri, and Tambalotti (2011). In sum, our work suggests that capital embodied shocks in real business cycle models could be interpreted more broadly to include news about the likelihood of future innovation in addition to contemporaneous technological advances.

Our paper is connected to the recent body of work that aims to disentangle whether news about future productivity or capital embodied shocks are the dominant source of business cycle fluctuations (Beaudry and Portier, 2006; Beaudry and Lucke, 2009; Fisher, 2009; Barsky and Sims, 2011; Schmitt-Grohe and Uribe, 2012). Many studies identify these two types of shocks separately by imposing orthogonality and long-run restrictions. Our framework casts doubt on such identification strategies that separate news from capital-embodied or standard TFP shocks. In our model, news about the future increase measured total factor productivity and the marginal efficiency of investment today. Hence, disentangling these two types of disturbances in the data may be quite challenging.

Our mechanism is related to models with financial constraints. Close to our paper is the work of Jermann and Quadrini (2007) and Chen and Song (2012), who show that in the presence of a standard financial friction – a collateral constraint – news about future productivity can generate an economic expansion today. Our work features important differences. First, a 'news' shock in our model represents information about the arrival of new technologies; in contrast, Chen and Song (2012) consider news about future TFP. In a model with financial frictions due to collateral constraints, good news about future technological vintages lower the value of existing capital and therefore are likely to lead to *tighter* constraints today. Second, in our setting, good news about the future lead to an increase in measured TFP today. Specifically, good news about the future improve firms' ability to commit not to expropriate ideas of better quality, hence the marginal idea implemented efficiently is of *higher* quality than before. By contrast, in models of financial

constraints, firms will implement their better projects first; hence, good news about the future will lead to a reduction of the quality of the marginal project implemented – and thus to a reduction in the marginal product of capital. Third, we consider a specification of preferences that allows for income effects on the supply of labor. Fourth, our model leads to different predictions.

Our model contributes to the literature incorporating news about the future in real business cycle models. Jaimovich and Rebelo (2009) highlight the difficulty of standard real business cycle models in generating positive responses in all of consumption, investment and labor supply; labor supply typically falls in response to good news about the future, while consumption and investment typically show opposite responses. The culprit for the former is the income effect on labor choice; Jaimovich and Rebelo (2009) propose preferences that do not have a strong income effect in the short run to generate an increase in labor supply. Our model is consistent with the behavior of economic quantities both at business cycle as well as at medium run frequencies. Hence, our work is consistent with the findings of Comin and Gertler (2006), who document the existence of substantial medium-run fluctuations in economic quantities.

The last three decades saw the rise in prominence of an alternative form of funding for innovative companies: venture capital. By connecting the share of projects in a partnership with a firm to expectations about the arrival of future technologies, our model provides an economic foundation for the existence of venture capital cycles (Gompers and Lerner, 2006). Gompers, Kovner, Lerner, and Scharfstein (2008) document that the venture capital industry undergoes investment cycles; VCs with the most experience – a likely proxy for reputation in our model – increase their investment more during these cycles relative to firms with little experience. Moreover, even though they increase investment, their performance is not worse. Further, Nanda and Rhodes-Kropf (2011) document that, conditional on these firms going public, startups receiving their initial funding in periods with more VC funding filed more patents – and those patens receive more citations – relative to startups funded in less active years. In addition, they document that, conditional on going public, startups funded in more active years were valued higher on the IPO date. These empirical facts are consistent with our model.

Several papers study the role of frictions in entrepreneurship. A large segment of the literature

focuses on credit frictions that prevent poorer but potentially highly productive entrepreneurs from entering the market (see, for instance Banerjee and Newman, 1993). Our work is closest to the work that combines RBC-style models with frictions in the sale of ideas (see, for example Silveira and Wright, 2010; Chiu, Meh, and Wright, 2011). In the models of Silveira and Wright (2010) and Chiu et al. (2011), firms need to pay inventors the value of the idea upfront – that is, they cannot commit to pay them once the idea is implemented. Since ideas cannot be collateralized, this friction creates a demand for liquidity on the part of firms and a role for intermediation. By contrast, in our setting, firms cannot commit to pay inventors anything, either upfront or later, and paying the inventors before the idea is disclosed is not feasible due to adverse selection. Last, our main mechanism that leads to inefficient implementation of the best ideas is closely related to Kondo and Papanikolaou (2015), who apply a static version of the same mechanism to models of limited arbitrage.

1 Motivation

This paper provides a micro foundation for fluctuations in the marginal efficiency of investment (investment wedge). In general, we define the investment wedge as the series χ that measures the distance between inputs (investment expenses) and installed capital K

$$K_t = \chi_t \, f(I_t, I_{t-1}, K_{t-1}) + (1 - \delta)K_{t-1}. \tag{1}$$

Here, the function f can take several form depending on the assumptions regarding the shape of adjustment costs in the economy.

However, these estimated invested wedges are difficult to interpret as advances in technology. Consider the estimated investment wedge (MEI shock) from Justiniano, Primiceri, and Tambalotti (2010). We plot the filtered series in figure 1 below. We see that the resulting series exhibits substantial high-frequency variation. Roughly half the sample years correspond to technological regress.

Most researchers interpret these shocks as capturing something more than fluctuations in the level of the technology frontier. Indeed, Justiniano, Primiceri, and Tambalotti (2011) postulate that

the filtered shocks might proxy for disturbances to the intermediation ability of the financial system. However, the underlying source of these disturbances is not obvious. In what follows, we provide a theory that links the efficiency at which projects are implemented to beliefs about future economic conditions.

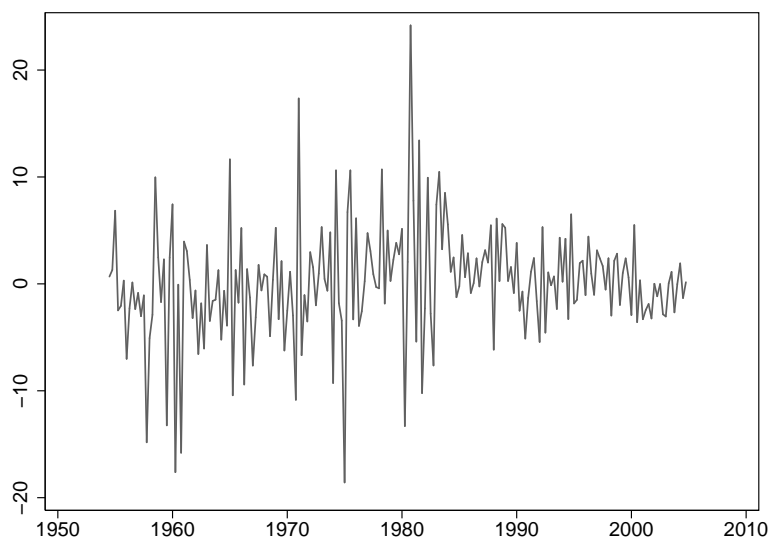**Figure 1**: Estimated Investment wedge $\log \chi_t$

Figure plots the filtered MEI shock from Justiniano, Primiceri, and Tambalotti (2010).

2   A Simple Model

To illustrate the main intuition behind our mechanism, we first present a greatly simplified version of the model. Time is continuous. There exist a set of firms of measure one and a group of entrepreneurs of measure one. Entrepreneurs have finite lives; they die each period with probability $\lambda \, dt$, and are replaced with a new entrepreneur. Upon entry into the economy, each entrepreneur is endowed with a new idea (blueprint) for a project. Both groups are risk-neutral and strategic. The interest rate is constant and equal to ρ.

Each idea can be implemented into a project. Implementation of an idea can either be done by the entrepreneur herself, or in a partnership with a firm. Once implemented, a project produces a

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constant output flow equal to

$$y = \rho \, p(\theta) \, \theta. \tag{2}$$

Here, θ indexes the quality of the investment opportunity: θ is i.i.d. over time, has c.d.f. $F(\theta)$ with support $[0, \infty)$. Further, $p(\theta)$ is a function taking the value $p = 1$ if the project is implemented in a partnership, and $p(\theta) = \phi < 1$ if the project is implemented by the entrepreneur herself. The partnership decision will depend on the quality of the idea, since θ is known to the entrepreneur when deciding whether to enter into a partnership or not.

Partnership is more efficient, but exposes the entrepreneur to the risk of expropriation. Most importantly, once the innovator decides to enter into a partnership with the firm, the latter can implement the project on its own and expropriate the innovator. If the firm expropriates the innovator, the latter obtains a payoff of zero. However, expropriating the innovator implies that future generations of entrepreneurs will refuse to do business with the firm. We next analyze this tradeoff.

For a given level of project quality θ a partnership is feasible between the entrepreneur and the firm if both parties obtain a higher payoff under the partnership than their outside option. Specifically, the entrepreneur needs to obtain a higher payoff than she would get by implementing the project alone,

$$\Pi^E(\theta) \geq \Pi^c(\theta) = \phi \, \theta. \tag{3}$$

Similarly, the financier should obtain a higher payoff to being in a partnership relative to expropriating the entrepreneur and incurring the loss of future business

$$\Pi^F(\theta) \geq \Pi^*(\theta) - V = \theta - V. \tag{4}$$

Here, V is the value of the relationship to the financier, and equals the present value of rents from interacting with future entrepreneurs

$$V_t \equiv \lambda \int_t^\infty \int_0^\infty e^{-\rho(s-t)} \, \Pi^F(\theta) \, dF(\theta) \, ds. \tag{5}$$

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Each instant dt a measure of $\lambda\,dt$ entrepreneurs is born and randomly matched to firms. Hence, each firm faces a probability $\lambda\,dt$ of meeting an entrepreneur each period.

Last, in a feasible partnership, the sum of the payoff to the entrepreneur and the firm has to be weakly less than the first-best level of profits,

$$\Pi^E(\theta) + \Pi^F(\theta) \leq \theta. \tag{6}$$

Equation (6) implies that a feasible partnership yields no cross-subsidization across current or future inventors.

Next, we examine the set of projects that are likely to be efficiently implemented in a partnership.

Definition 1 (A feasible partnership) *A feasible partnership rule $P(\theta), \Pi^F(\theta), \Pi^E(\theta)\}$ satisfies (3)-(6) $\forall \theta$ such that $P(\theta) = 1$.*

Last, we impose the stronger requirement that partnerships are efficient, which amounts to an interim Pareto optimality condition

Definition 2 (An efficient partnership) *A feasible partnership rule is efficient if (6) holds with equality and that there does not exist an equilibrium with an alternative partnership rule $\{\hat{P}(\theta), \hat{\Pi}^F(\theta), \hat{\Pi}^E(\theta)\}$ that satisfies (3)-(6), and $\exists(\theta, t)$ such that $\hat{P}_t(\theta) = 1$ and $P_t(\theta) = 0$.*

This restriction ensures that there does not exist an alternative partnership decision that would make everyone better off.

The resulting equilibrium depends on how the surplus is shared between the firms and the entrepreneurs. We assume Nash bargaining between the firm and the entrepreneur over the surplus and denote by η the fraction that goes to the entrepreneur.

Proposition 1 *The equilibrium is characterized by a threshold χ, such that partnership occurs, $P(\theta) = 1$ if $\theta \leq \chi$ and the entrepreneur implements the project alone, $P(\theta) = \phi$, otherwise.*

Equilibrium payoffs are given by

$$\Pi^F(\theta) = \begin{cases} \theta\,(1-\phi) - \eta\,(V - \phi\,\theta), & if \quad \theta \leq \chi \\ 0, & if \quad \theta > \chi \end{cases}, \tag{7}$$

$$\Pi^E(\theta) = \begin{cases} \phi\,\theta + \eta\,(V - \phi\,\theta), & if \quad \theta \leq \chi \\ \phi\,\theta, & if \quad \theta > \chi \end{cases}. \tag{8}$$

The threshold χ is the largest solution to

$$\phi\,\chi = V^\star(\chi), \tag{9}$$

where $V^\star(\chi)$ is the equilibrium value of the relationship value to the bank

$$V^\star(\chi) = \frac{\lambda\,(1 - (1-\eta)\,\phi)}{\rho + \eta\,\lambda\,F(\chi)} \int_0^\chi \theta\,dF(\theta) \tag{10}$$

Proposition 1 summarizes the intuition behind the financial friction in this paper. Relationships are limited in their ability to mitigate the hold-up problem between entrepreneurs and financiers. Intuitively, the benefits of expropriation to the firm are increasing in the quality of the project θ. By contrast, the costs of expropriation—the loss of future rents—depend on the *average* quality of projects that are supplied to firms, given by the integral in the right-hand side of (10). As a result, firms cannot commit to not expropriate an entrepreneur with a sufficiently high quality project. Entrepreneurs anticipate being expropriated, and thus refuse to enter a partnership agreement with the firm when their ideas are of sufficiently high quality $\theta \geq \chi$.

So far, implementing a project was costless; the full model includes investment. To foreshadow the main result in the full model, it is helpful to compute the average value of a new project,

$$\int_0^\infty P(\theta)\,\theta\,dF(\theta) = \bar{\theta} - (1-\phi) \int_\chi^\infty \theta\,dF(\theta). \tag{11}$$

The average value of implemented ideas is lower than $\bar{\theta}$ due to the hold-up friction. This wedge depends on the efficiency gains of partnerships relative to stand-alone projects $1 - \phi$ and is decreasing

in χ. If investment were endogenous, an increase in the equilibrium level of χ will increase the average value of new projects, and spur new investment.

In deriving these results we have made several assumptions, some of which are necessary while others are for convenience. The key assumptions driving our results are that: 1) the distribution of θ is unbounded; 2) the inventor's outside option is strictly increasing in θ; and 3) the relationship value to the firm (5) is finite. These assumptions imply that expropriation will be profitable for realizations of θ that are sufficiently higher than average – since the relationship value V depends on the average quality of a project.

Most of the other assumptions are in the interest of tractability. We restrict one party to be short-lived (or to interact only once) to simplify the dynamic game; more generally, we would have to keep track of multiple sets of continuation values. However, the central insight would survive: an increase in the total continuation value – now to be split among both parties – relaxes the incentive-compatibility constraints in the current period. Further, we have excluded explicit contracts in mitigating the agency friction – we have assumed that θ is not verifiable. This assumption allows us to focus on the endogenous time-variation in the set of implicit contracts that satisfy the participation (3) and incentive-compatibility (4) constraints along with the resulting fluctuations in the marginal efficiency of investment.

3   The Model

The model in the previous section has the feature that the productivity of a new unit of capital – an implemented idea, or project – is enhanced when entrepreneurs cooperate with firms. The key tradeoff in the model is that cooperation between parties is efficient, yet it exposes the weaker party in this arrangement to the possibility of expropriation. The weaker party will refuse to enter into arrangements that would result in ex-post expropriation by its partner. In this section, we embed this limited commitment friction into a dynamic general equilibrium model. We introduce an aggregate shock that affects the equilibrium level of cooperation – the threshold χ – and examine its effects on equilibrium outcomes.

3.1 Setup

We begin by first describing the setup of the model. We consider a continuous-time, infinite horizon economy, in which aggregate uncertainty is driven by a one-dimensional standard Brownian motion B.

3.1.1 Households

There is a continuum of households of measure one. Households have finite lives; they die with flow probability $\delta^h \, dt$. Households have preferences over consumption C and leisure N

$$J_t = E_t \int_t^\infty e^{-\rho(s-t)} \frac{\left(C_s N_s^\psi\right)^{1-\gamma}}{1-\gamma} \, ds. \tag{12}$$

The household discounts the future at a rate ρ, which includes the fact that they have finite lives.

Each period, households are endowed with a fixed unit of time that can be freely allocated between leisure N_t or labor L_t,

$$L_t + N_t = 1. \tag{13}$$

Exiting entrepreneurs are replaced by new entrepreneurs; each new entrepreneur is born endowed with a measure λ/δ^h of blueprints all with quality θ identical blueprints. By assumption, the total measure of new blueprints available each period is $\lambda \, dt$. Each inventor's blueprints differ in their quality θ, which is distributed on $(0, \infty)$ with cdf $F(\theta)$. The inventor knows the quality θ of her blueprints.

We assume that households share aggregate risks perfectly. Specifically, we assume that all inventors belong in a large 'family' or representative household, that consumes aggregate consumption C_t and leisure N_t.[7] The representative household can trade a complete set of state-contingent

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[7]This assumption greatly simplifies our analysis. A potential concern however is that, if inventors share risks, why do they worry about being expropriated? We could modify the assumption of large families by assuming that new inventors start with wealth that is proportional to the value of their own idea, scaled by the average value of all ideas at time t. Homotheticity implies that consumption and leisure are proportional to wealth; since households are risk averse and markets are complete, they will all hedge changes in their wealth subsequent to their entry into the economy, implying that they will all have the same marginal rate of substitution. Since households can trade claims on the path of B_t, market-based solutions are also possible, including hedging mortality risk as in Blanchard (1985). For our purposes, these assumptions are conservative. In particular, we have solved a version of our model with

claims contingent on the paths of B. We denote the equilibrium state price density by π_t, so that the representative household chooses consumption plans and make labor supply decisions to maximize (12) subject to the dynamic budget constraint

$$E_t \int_t^\infty \pi_s \, C_s \, ds = \pi_t \, W_t \tag{14}$$

where W_t is the amount of total wealth of the representative household.

3.1.2 Production

Ideas can be implemented into production units (projects) that produce a flow of capital services (an intermediate good). A project j produces a flow of capital services at time t equal to

$$x_{j,t} = u_{j,t} \, \theta_j^{1-\alpha} \, k_j^\alpha, \qquad \alpha \in (0,1). \tag{15}$$

The output of the project is increasing in the quality of the blueprint θ used in its production, and its scale of operation k. The output of the project is also affected by the rate u at which it is utilized. A higher rate of capital utilization increases the probability that the project depreciates. In particular, the probability that the project expires during the period t to $t + dt$ is a function of the rate of capital utilization $\delta(u)$, where $\delta'(u) > 0$ and $\delta''(u) > 0$. Variable capital utilization is not an essential feature of our model, but it helps generate positive comovement between investment and consumption growth.

The economy's flow of capital services produced at time t, is an aggregate of output of all of the existing projects,

$$X_t = \int_{\mathcal{J}_t} x_{j,t} \, dj \tag{16}$$

where we denote the set of all active projects in the economy by $\mathcal{J}_t$.

There is a continuum of infinitely lived competitive firms that own and operate the projects

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incomplete markets, in which new inventors cannot hedge risks with existing households. The main difference with our current setup is that this market incompleteness tends to ameliorate the wealth effect: since existing households are not the prime beneficiaries of improvements in the marginal efficiency of investment, the interest rate responds much less, and therefore M declines by less.

above. The boundary of the firm is not explicitly defined in our model. For simplicity, we assume that projects in partnership are operated by the firms that participated in their implementation. Projects that were implemented by entrepreneurs alone are subsequently sold to firms at their fair market value (which accounts for inefficient implementation). Beyond keeping the measure of firms constant, this assumption has no material impact.

The representative firm in the final goods sector combines the output of the intermediate good X (purchased at price p) with labor services L (purchased at price w) to produce the final good Y,

$$Y_t = X_t^\beta \left(e^{\mu t} L_t\right)^{1-\beta}. \tag{17}$$

Here, we allow for log labor productivity to grow deterministically at rate μ.

The final good can be allocated either towards consumption C or new investment I,

$$C_t + I_t = Y_t. \tag{18}$$

3.1.3 Investment, Blueprints and the Market for Ideas

New projects are created through the combination of blueprints (ideas) and machines. Blueprints are owned by inventors; machines are directly produced from investment goods.

Inventors choose between implementing the blueprint on their own, or in a partnership with a firm. Partnership with a firm implies that the project is implemented efficiently. Specifically, creating a project of scale k requires k machines. By contrast, if inventors choose to implement the project on their own, its implementation is less efficient: creating a project of scale k requires $\phi^{-\frac{\alpha}{1-\alpha}} k > k$ machines, where $\phi < 1$. As before, ϕ measures the efficiency loss of stand-alone implementation; if $\phi = 1$ partnerships offer no efficiency advantage.

Implementing the project in a partnership with a firm is more efficient. However, a partnership exposes the inventor to the risk of expropriation. As before, inventors first decide whether to implement their idea in partnership with a firm or not. After the partnership decision has been undertaken by the inventor, the firm decides whether to expropriate the inventor or not. For simplicity, we assume that the firm can fully appropriate the proceeds from the investment decision,

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leaving the inventor with a payoff of zero.

If the firm expropriates the inventor, it incurs a reputational cost, in the form of lost future partnering opportunities . In particular, if the firm expropriates an inventor, future generations of inventors will refuse to do partner with the firm. Denote the equilibrium payoff to the firm as a function of quality and the state of the economy to be $\Pi_t^F(\theta)$. The present value of these rents, discounted using the equilibrium state price density π_t,

$$V_t = \lambda \, E_t \int_t^\infty \frac{\pi_s}{\pi_t} \left(\int_0^\infty \Pi_s^F(\theta) \, dF(\theta) \right) ds, \tag{19}$$

is the cost of expropriation. Equation (19) is the analogue of (5) in the simple model.

As before, for a given level of project quality θ, a partnership is feasible between the innovator and the firm if the following conditions hold. First, the innovator needs to obtain a higher payoff under the partnership than her outside option (implementing the project herself). Second, the firm must prefer its payoff under the partnership to expropriating the inventor and losing the value of future business (19). Third, the sum of the payoffs to the inventor and the firm cannot exceed the value of the project under efficient implementation.

Last, we describe how the rents are split between inventors and firms. Denote by $NPV_t(\theta)$ the value of the project under efficient implementation, and by $\widehat{NPV}_t(\theta)$ the value of the project when it is implemented by the entrepreneur herself. The total surplus that is created by a partnership is equal to the first-best level of profits, minus the outside options of the two parties:

$$S_t(\theta) = NPV_t(\theta) - (NPV_t(\theta) - V_t) - \widehat{NPV}_t(\theta)$$
$$= V - \widehat{NPV}_t(\theta). \tag{20}$$

The requirement that a partnership is feasible reduces to the requirement that the surplus (20) is positive.

We assume that the firm and the inventor bargain over the share of the surplus created. Surplus is allocated according to Nash bargaining, with the inventor obtaining a fraction η_t of the surplus at

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time t.

$$\Pi_t^F(\theta) = NPV_t(\theta) - V_t + (1 - \eta_t)\left(V_t - \widehat{NPV}_t(\theta)\right) \tag{21}$$

$$\Pi_t^E(\theta) = \widehat{NPV}_t(\theta) + \eta_t\left(V_t - \widehat{NPV}_t(\theta)\right). \tag{22}$$

The share of the surplus η that goes to the inventor is stochastic and evolves according to

$$d\eta_t = \kappa(\bar{\eta} - \eta_t)\,dt + \sigma\,\sqrt{\eta_t(1 - \eta_t)}dZ_t, \tag{23}$$

where $\bar{\eta} \in (0, 1)$ is its long-run average and $1 - \kappa$ is its persistence. Our specification for the diffusion term ensures that the process η_t is bounded in $[0, 1]$.

Fluctuations in the bargaining power η between inventors and firms is the only source of randomness in our model. Variations in η_t can arise due to variations in the impatience of inventors or firms in the bargaining game. Shocks to η serve to illustrate the point that a variable that only affects the cost of expropriation – and therefore the degree to which firms can commit to implement projects – can lead to behavior akin to time variation in the marginal efficiency of investment. Examining shocks to η is attractive in our framework because, absent the limited commitment friction, these shocks have no effect on equilibrium outcomes.

However, we emphasize that we do not wish to argue that fluctuations in bargaining power among different parties involved in the creation of new capital are the main culprits for economic fluctuations. First, because our model is quite stylized; in reasonable extensions – for instance if we were to relax the assumption that the supply of new ideas is inelastic and independent of η – the equilibrium relation between χ and η would be ambiguous. Second, our mechanism is much more general; what matters is the relative value of relationships V (the cost of expropriation) to the net present value of new projects ν (the benefit of expropriation). Alternative shocks that affect V/ν could be shocks to beliefs about: future discount rates, the future efficiency of partnerships (changes in future ϕ), or the profitability of future innovations. Irrespective of whether these beliefs are rational or not, they will affect the cost of expropriating inventors and will lead to a qualitatively similar effect. In Section 5 we examine such a model, where we replace shocks to η with shocks

to beliefs regarding the mean of the distribution of future project quality θ. The model delivers qualitatively similar results.

3.2   Equilibrium

We next define the competitive equilibrium in the economy.

Definition 3 (Competitive Equilibrium) *A competitive equilibrium is a sequence of quantities* $\{C_t, N_t, Y_t, X_t, L_t\}$; *prices* $\{p_t, w_t, \pi_t\}$; *decisions on new investment* $\{k_t(\theta)\}$, *and capacity utilization* $\{u_t\}$; *a partnership rule,* $\{\mathcal{P}_t(\theta), \Pi_t^F(\theta), \Pi_t^E(\theta)\}$; *such that given the sequence of stochastic shocks* $\{\eta_t\}$:

1. *Households choose consumption plans* C_t *and labor services* L_t *to maximize utility* (12) *subject to* (13) *and the dynamic budget constraint* (14);

2. *The partnership rule is feasible and efficient, and surplus is distributed according to Nash bargaining with sharing rule* η_t: $\mathcal{P}_t(\theta) = 1 \Leftrightarrow$ (20) *is greater than zero and payoffs to the firm and the entrepreneur are given by* (21) *and* (22) *respectively.*

3. *Firms continuation value* (19) *equals the discounted present value of partnership profits*

4. *Final good firms choose their demand for capital* X *and labor services* L *to maximize profits*

5. *Investment is chosen to maximize the value of project using the equilibrium state price density* π

$$k_t(\theta) = \arg\max_k \ M_t \, \theta^{1-\alpha} k^{\alpha} - c_t(\theta)\, k, \tag{24}$$

   where M *is computed using the optimal policy of capital utilization* u *optimally chosen by the owners of the project*

$$M_t = \max_u E_t \int_t^{\infty} \exp\left(-\int_t^s \delta(u_\tau)\, d\tau\right) \frac{\pi_s}{\pi_t} \, p_s \, u_s \, ds. \tag{25}$$

   where

$$c_t(\theta) = \begin{cases} 1 & \text{if} \quad \mathcal{P}_t(\theta) = 1 \\ \phi^{-\frac{\alpha}{1-\alpha}} & \text{if} \quad \mathcal{P}_t(\theta) = 0 \end{cases}$$

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6. *The market for investment goods clears*

$$\lambda \int_0^\infty c_t(\theta) \, k_t(\theta) \, dF(\theta) = I_t. \tag{26}$$

7. *The resource constraints* (16)-(18) *are satisfied.*

We compute the equilibrium numerically. Before we discuss the model's predictions, we first highlight a

3.2.1 The Partnership Decision

We begin by examining the key tradeoff in the model, namely the choice between partnership and stand-alone investment. The decision of whether to implement the project in a partnership or not largely follows the logic of the simple the model in section 2. We begin our analysis by computing the equilibrium value of a project of quality θ when it is implemented efficiently

$$NPV_t(\theta) \equiv \max_k \; M_t \, \theta^{1-\alpha} k^\alpha - k = \theta \, v_t, \tag{27}$$

where v_t represents the net present value of a new project holding quality fixed

$$v_t = (1 - \alpha) \, M_t^{\frac{1}{1-\alpha}} \, \alpha^{\frac{\alpha}{1-\alpha}} \tag{28}$$

and M_t is defined by (25). Conversely, if the project is inefficiently implemented, its value – the inventors outside option – equals

$$\widehat{NPV}_t(\theta) \equiv \max_k \; M_t \, \theta^{1-\alpha} k^\alpha - \phi^{-\frac{\alpha}{1-\alpha}} k = \phi \, \theta \, v_t. \tag{29}$$

The firm's continuation value V determines its ability to commit not to expropriate the inventor; the firm's continuation value has to be sufficiently high so that the firm is willing to compensate the inventor for her outside option. A feasible partnership rule is characterized by a sequences $\{\chi_t\}$ such that if $\theta \leq \chi_t \Leftrightarrow \mathcal{P}_t(\theta)$. The quality of the marginal project in a partnership χ_t is given by the

condition that the surplus generated equals zero, $S_t(\chi_t) = 0$, which boils down to the condition

$$\phi \chi_t v_t = V_t. \tag{30}$$

Examining (30), we see that the partnership threshold χ_t is proportional to he ratio of the relationship value V (the cost of expropriation) to the value of a new idea v_t (the benefit of expropriation). Any shock that increases the the value of firms' reputation V relative to the benefits of expropriation v – for instance, a decrease in the surplus share to inventors η_t – will lead to higher cooperation today.

3.2.2 Marginal Efficiency of Investment

We next illustrate how our mechanism leads to endogenous investment wedges. Even though projects are heterogenous in their quality and scale, the mean value of their output (16) summarizes the relevant information for aggregate dynamics. To see why this is the case, we first note that the optimal scale of investment in a project of quality θ satisfies

$$k_t(\theta) = \theta \ (M_t)^{\frac{1}{1-\alpha}} \ \left(\frac{\alpha}{c_t(\theta)}\right)^{\frac{1}{1-\alpha}}, \tag{31}$$

Equation (31) bears similarities to the q-theory of investment (Hayashi, 1982). The optimal level of investment in a project of quality θ is a function of the ratio of the quality-adjusted value of a new project M to its marginal cost of implementation, where the marginal cost of implementing the project is a function of the partnership decision. Examining (31), we see that the optimal scale of implementation is increasing in the project quality θ; holding quality constant, the optimal scale is higher if the project is implemented in a partnership rather than by the inventor alone.

Next, consider the dynamics of the aggregate capital stock, defined as

$$K_t = \int_{J_t} \theta_j^{1-\alpha} \, k_j^\alpha \, dj. \tag{32}$$

Combining (31) with the market clearing condition for investment goods (26), we can write the

evolution of the current stock of capital services as

$$dK_t = I_t^\alpha \left(\lambda\, g(\chi_t) \right)^{1-\alpha} - \delta(u_t)\, K_t\, dt, \tag{33}$$

where

$$g(x) \equiv \bar{\theta} - (1-\phi) \int_x^\infty \theta\, dF(\theta), \tag{34}$$

is the familiar expression denoting the average return to new projects familiar from equation (11). Examining (33) we can see that our model aggregates to a fairly standard neoclassical growth model with decreasing returns to investment (or equivalently, convex installation costs, as in Abel (1983)).

More importantly, an increase in the partnership threshold χ_t has qualitatively the same effect as an improvement in the marginal efficiency of investment (Justiniano et al., 2010). Recalling the discussion following equation (11) in the simple model above, we can see here how an increase in cooperation (the partnership threshold χ_t) leads to more efficient implementation of blueprints, which manifests to an increase in the amount of installed capital X_t for a given amount of investment I_t.

4   Numerical Solution

Here, we discuss the implications of our model for equilibrium quantities and prices..

4.1   Parameter choice

We solve for equilibrium prices and quantities numerically. A full-scale estimation is outside the scope of this paper. We thus provide an illustrative numerical calibration that serves to illustrate the main point of the paper. Whenever possible, we calibrate the model using standard parameter values or by targeting standard moments. We calibrate the share of capital to one-third, $\beta = 1/3$. The share of leisure in the utility function is $\psi = 4$. The parameter α governs the degree of adjustment costs to investment; we choose $\alpha = 0.35$, which is close to quadratic adjustment costs. We choose $\mu = 0.8\%$ to match the average growth rate of output. We parameterize the project depreciation

rate as a function of capital utilization rate as $\delta(u) = \delta_0 + \delta_1/2\, u^2$; we choose the parameters δ_0 and δ_1 to generate an average depreciation rate of 2% per quarter, and to match the volatility of log changes of capital utilization (1.65% per quarter). For household preferences, we set $\gamma = 3$, which is consistent with typical calibrations of RBC models.

The remaining parameters are unique to our model and thus are difficult to calibrate using standard moments. We choose what we believe to be reasonable values. We choose $\lambda = 1$. We parameterize the distribution of project quality θ to be exponential with unit scale parameter, $F(\theta) = 1 - \exp(-\theta)$, so that mean project quality is equal to one, $E(\theta) = 1$. We set the long-run average of the surplus share rule to $\bar{\eta} = 1/2$. We assume that changes to η are volatile and persistent, $\sigma_\mu = 0.25$ and $\kappa = 0.025$. We set the parameter governing the degree of efficiency loss when the project is implemented by the inventor to $\phi = 0.75$, or equivalently, that the net present value of projects implemented in partnerships is about one-third higher relative to a project of the same quality that is implemented by the inventor.

4.2 Simulated Time Series

We first examine a simulated time-series from the model in Figure 2. Examining the first two panels, we see how the exogenous fluctuations in the surplus that accrues to firms $1 - \eta_t$ leads to endogenous fluctuations in the marginal efficiency of investment $g(\chi_t)$. These fluctuations lead to short-run fluctuations in output from it's linear trend, as we see in panels c and d. Consumption, investment, hours and utilization exhibit similar fluctuations (panels e and f, with investment more volatile than consumption, and utilization more volatile than hours worked.

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Figure 2: Simulated time-series data from the model

a. Sharing rule η_t

b. Investment wedge $(1 - \alpha) \log g(\chi_t)$

c. Output

d. Output and TFP
(band-pass filtered)

— Output

- - - Measured TFP

e. Consumption and Investment
(band-pass filtered)

— Investment

- - - Consumption

f. Labor and Capital Utilization
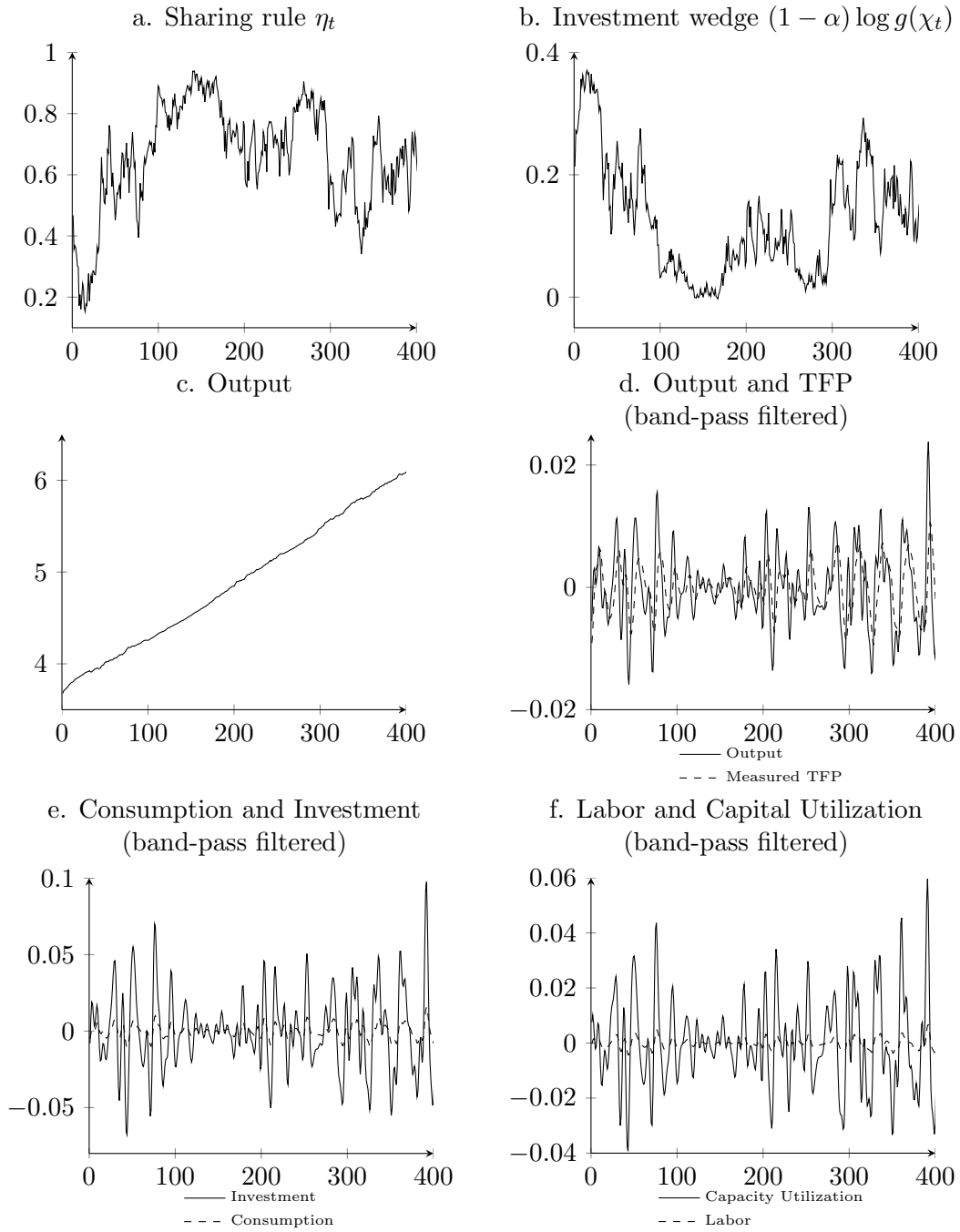(band-pass filtered)

— Capacity Utilization

- - - Labor

Figure plots the output of one simulation of our model. We use parameter values of $\alpha = 0.35$, $\beta = 1/3$, $\psi = 4$, $\delta_0 = -0.025$, $\delta_1 = 0.05$, $\delta_2 = 1.05$, $\gamma = 3$, $\rho = 0.03$, $\bar{\eta} = 1/2$, $\phi = 0.75$, $\sigma_\eta = 0.25$ and $\kappa_\eta = 0.025$.

4.3 Response to Shocks

To understand how the model generates comovement across output, productivity, investment, consumption and labor supply, we next examine the response of these quantities to a shock to the bargaining parameter η in the model. In figure 3, we examine the response to a one-time shock to the share of the surplus that goes to firms, $1 - \eta_t$. Since η_t is a persistence process, this shock decays slowly, as we see in panel (a).

Investment Efficiency. An increase in the share of the surplus that goes to firms, $1 - \eta_t$ implies that the value V_t of future partnerships increases, as we see in panel (b). This increase in V implies that the cost to firms of expropriating inventors rises, and given the indifference condition (30), leads to an increase in the partnership threshold χ_t, as we see in panel (c). As we see in panel (d), this increase in the partnership threshold leads to an endogenous increase in the marginal efficiency of investment $g(\chi_t)$.

Investment. The resulting improvement in the return to investment triggers a substitution effect, and leads to an increase in investment expenditures and thus to the capital stock X, as we see in panels (d)- (f). The resulting increase in investment spending is fairly short-lived due to an additional wealth effect: eventually households feel richer and desire to consume more. In particular, examining the first order condition for investment,

$$I_t = \lambda \, (\alpha \, M_t)^{\frac{1}{1-\alpha}} \; g(\chi_t), \tag{35}$$

we see that in addition to χ, it also depends on the marginal value of installed capital M. As we see in panel (g), the marginal value of installed capital drops. This drop is a standard feature of models with investment-specific shocks: as the average quality of new capital increases, the equilibrium value of old capital falls. This drop in M is in part due to an increase in the interest rate (as households anticipate an increase in consumption).

Utilization. As we see in panel (h), the equilibrium level of capital utilization u increases on impact. This increase in u is initially driven by the drop in M. In particular, the equilibrium level

of capital utilization u satisfies the first-order condition,

$$p_t = \delta'(u_t)\,M_t. \tag{36}$$

When determining u, firms trade off the benefits of increased capital utilization (which is proportional to the price of capital services p), versus the cost (the accelerated depreciation of the installed capital stock). As the marginal value of existing capital M falls, the costs of utilization drop, leading to an increase in u. Similar to investment, this increase is short-lived and is eventually reversed: as the capital stock starts declining back to trend, the rate of capital utilization drops to smooth consumption. Given that u_t is the same across all existing projects, we can write the total supply of capital services as

$$X_t = u_t\,K_t. \tag{37}$$

In panels (i)- (j), we see the response of factor prices p and w. The increase in the supply of capital services, due to both increased investment and higher capital utilization, implies a fall in the equilibrium price of capital services p_t. Since labor and capital are complements in production, the same increase in capital services implies an increase in the equilibrium wage on impact. Both prices subsequently revert to their stationary levels.

Hours. Labor supply is driven by the familiar intratemporal first order condition $U_N/U_C = w_t$, which can be re-written as

$$L_t = 1 - \psi\,\frac{C_t}{w_t}. \tag{38}$$

The increase in the equilibrium wage w leads to an initial increase in the labor supply. As households become richer – due to the accumulation of capital stock – the wealth effect starts dominating the substitution effect so labor supply declines before reverting back to steady state, as we see in panel k.

Output and Consumption. As we see in panel (l) The resulting increase in the capital stock, hours worked and labor supply implies that output increases on impact. Panel (m) shows that consumption increases with output; this increase is lower in the beginning, as more resources are allocated to investment. As households feel temporarily richer, the consumption-to-output ratio

rises due to the wealth effect.

Total Factor Productivity Improvements in the marginal efficiency of investment lead to increases in measured TFP, assuming the capital stock in imperfectly adjusted for quality. Consider the measured capital stock to evolve according to

$$d\hat{K}_t = I_t^\alpha \left(\lambda\right)^{1-\alpha} - \delta(u_t)\,\hat{K}_t\,dt, \tag{39}$$

The measured capital stock is constructed by accumulating investment expenses, adjusted for decreasing returns and variable depreciation (due to capital utilization). The difference between (39) and (33) is that it is not adjusted for variation in the average return to new investments. We then compute the measured total factor productivity as the log difference between observed output and the total output implied by (39) given the equilibrium level of capital utilization and labor supply. Measured total factor productivity equals

$$tfp_t = \beta \left(\log K_t - \log \hat{K}_t\right). \tag{40}$$

Examining (40), we can see that total factor productivity is a weighted average of past levels of the marginal efficiency of investment $g(\chi_t)$. As we see in panel (n), measured productivity rises as the partnership threshold rises in response to a decline in η_t.

Stock market. The market value of all firms is comprised by the market value of installed capital $M\,K$, plus the value of future partnerships V_t. On impact, the value of installed assets drops in (o) as M falls. However, as the economy accumulates more capital, the market value of installed capital $M\,K$ subsequently rises. Panel (p) shows that the value of the stock market $M\,K + V$ increases on impact.

Figure 3: Response to shock to η

(a) Surplus to firms, $1 - \eta$

(b) Relationship Value, V

(c) Fraction of ideas in partnership, $F(\chi)$

(d) Investment efficiency, $g(\chi)$

(e) Investment, I

(f) Capital stock, X

(g) Marginal value of installed capital, M

(h) Capital utilization, u

(i) price of capital services, p

(j) wage, w

(k) Labor supply, L

(l) Output, Y

(m) Consumption, C

(n) Measured Productivity

(o) Value of installed capital, $M K$

(p) Stock market Value, $M K + V$

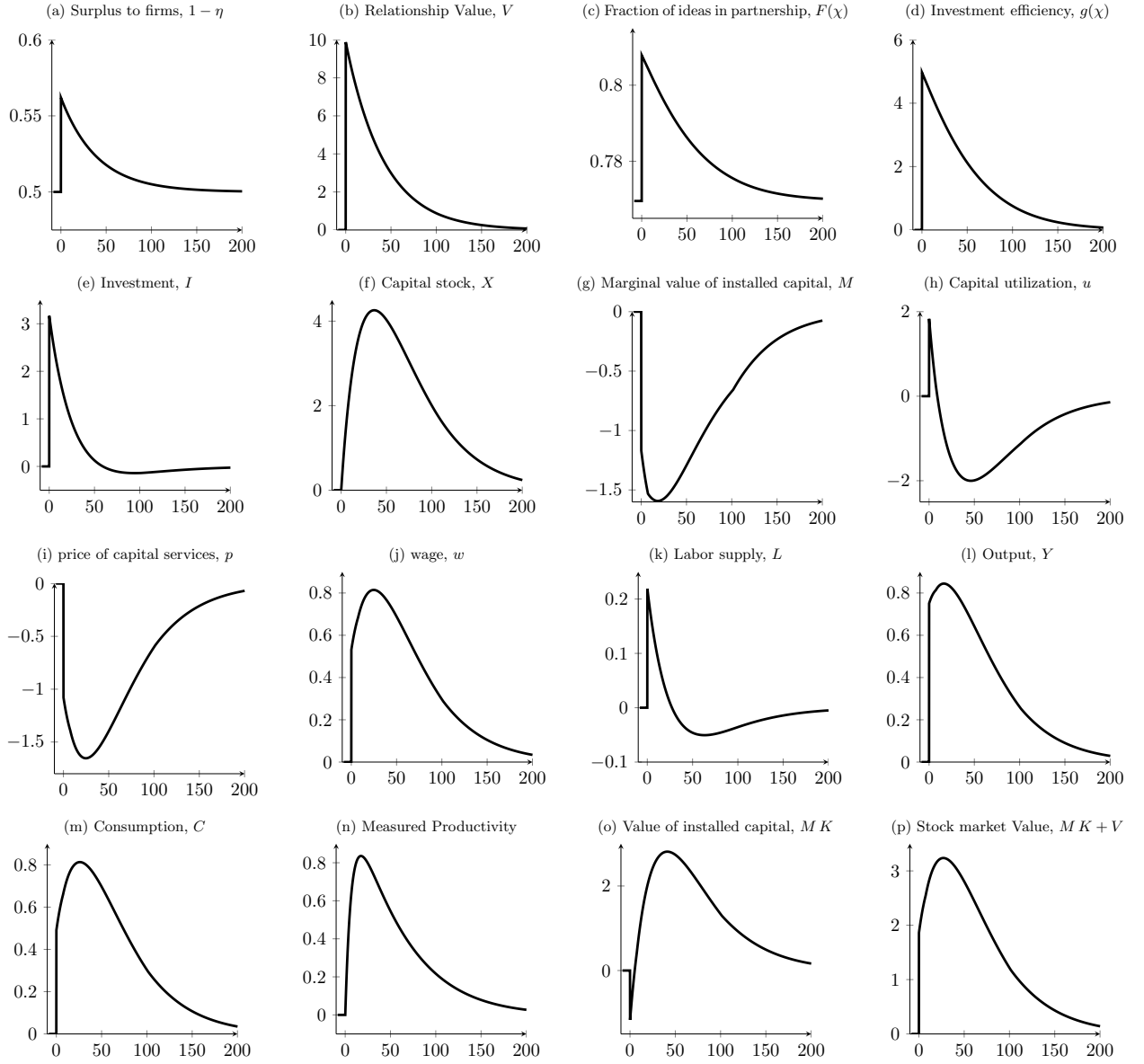Figure plots the response of equilibrium quantities and prices to a one-standard deviation change in η. We use parameter values of $\alpha = 0.35$, $\beta = 1/3$, $\psi = 4$, $\delta_0 = -0.025$, $\delta_1 = 0.05$, $\delta_2 = 1.05$, $\gamma = 3$, $\rho = 0.03$, $\bar{\eta} = 1/2$, $\phi = 0.75$, $\sigma_\eta = 0.25$ and $\kappa_\eta = 0.025$. We simulate the model at monthly frequencies, and then time aggregate to form quarterly observations. We compute impulse responses by computing the log deviation of quantities and prices from steady state values, averaged across 100,000 simulations.

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4.4 Comovement

Here, we explore the extent to which our model can quantitatively generate fluctuations and comovement in quantities consistent with the data. In Table 1, we compare the volatility and comovement of quantities between the model and the data, focusing on business cycle frequencies (6 to 32 quarters). Focusing on volatilities, we see that, with the exception of hours worked, the model quantities are approximately half as volatile as the data. The volatility of hours worked in the model is less than 10% of its empirical counterpart. Given that the estimates of Justiniano et al. (2010) imply that shocks to the marginal efficiency of investment can account for approximately half the volatility of aggregate fluctuations.

Further, as we see in the bottom panel of Table 1, investment, consumption, output and hours worked in the model comove at business cycle frequencies, similar to the data. One exception is that total factor productivity, adjusted for utilization, is mostly pro-cyclical in the model, whereas in the data is mostly counter-cyclical. This pattern, pointed out by Basu, Fernald, and Kimball (2006) is true for the utilization-adjusted TFP series. The unadjusted series is strongly pro-cyclical.

Table 1: Volatility and comovement, business cycle frequencies

	A. Data						B. Model					
	c	i	y	l	u	tfp	c	i	y	l	u	tfp
	0.88	5.89	1.52	1.78	3.24	0.84	0.41	2.45	0.60	0.16	1.52	0.33
			Correlation									
i	84.0						95.0					
y	89.8	94.8					99.3	98.0				
l	81.9	85.7	83.9				93.0	99.7	96.7			
u	72.8	89.4	83.6	83.0			73.1	88.5	80.0	90.9		
tfp	-16.4	-22.2	-10.7	-40.7	-34.3		55.5	29.4	46.1	23.8	-11.5	

Table compares moments of consumption, investment, output and hours in the data (Panel A) and in the model (B). All series are filtered using the bandpass filter at business cycle frequencies (6-32 quarters) (Christiano and Fitzgerald, 1999). Consumption, Investment, Output, Hours and Utilization data are from the BEA. Total factor productivity is utilization-adjusted TFP from Basu, Fernald, and Kimball (2006).

5 An alternative model with News shocks

The model in the previous section illustrates our main point through changes in the bargaining share between inventors and firms. This shock is convenient because absent the limited commitment friction has no effects on equilibrium outcomes. But of course, it is hard to interpret. In this section we should how more standard shocks – news about the future – have the same qualitative predictions.

To illustrate that our mechanism is much more general, consider the following modification to the model. We assume that the bargaining parameter η is a constant, but that agents' beliefs about the future vary over time. In particular, instead of equation (32), the output of a project is now given by

$$x_{j,t} = u_{j,t}\,\xi_s\,\theta_j^{1-\alpha}\,k_j^{\alpha}, \qquad \alpha \in (0,1). \tag{41}$$

where ξ_s is the level of the technological frontier at the time s the project is created—or equivalently the average quality of ideas supplied at time s. The frontier evolves according to

$$d\log\xi_t = s_\xi\,dN_t, \tag{42}$$

where N_t is a poisson jump process with a state dependent arrival rate $\mu_t \in \{\mu_L, \mu_H\}$, which follows a two-state continuous-time Markov chain with transition matrix

$$S = \begin{pmatrix} -p_L & p_L \\ p_H & -p_H \end{pmatrix}. \tag{43}$$

News in this economy corresponds to changes in the arrival rate μ. In particular, agents believe that with probability $\mu_t\,dt$ over the next instant, the level of frontier technology ξ increases by a proportional amount s_ξ, representing a technological revolution – or equivalently that the mean time until the next improvement is $1/\mu_t$. Whether these beliefs are rational or not is immaterial. What is important is that beliefs affect the ratio of V/ν – that is, the value of future partnerships relative to the benefits of expropriation – and therefore the partnership threshold χ_t.

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In contrast to the previous model, shocks to μ affect equilibrium outcomes even in the absence of the limited commitment friction. Specifically, an increase in μ induces a strong wealth effect that typically leads to an increase in consumption and leisure – and a corresponding drop in investment and hours worked. This is a common hurdle that real business cycle models with news about the future need to overcome in order to generate comovement. In our case, the problem is particularly astute, given that our preference specification implies a strong wealth effect on labor supply. To partially mitigate the strength of the wealth effect, we modify household preferences to allow the coefficient of risk aversion to be separate from the elasticity of intertemporal substitution. In particular, rather than (12), household preferences are now defined recursively according to

$$
J_t = E_t \int_t^\infty \frac{\rho}{1 - \theta^{-1}} \left(\frac{\left(C_s N_s^\psi \right)^{1 - \theta^{-1}}}{\left((1 - \gamma) J_s \right)^{\frac{\gamma - \theta^{-1}}{1 - \gamma}}} - (1 - \gamma) J_s \right) ds. \tag{44}
$$

As before, ψ is the preference weight on leisure and ρ is the subjective discount rate. Now, γ is the coefficient of relative risk aversion, and θ is the elasticity of intertemporal substitution (EIS). The preference specification (44) represents the continuous-time analog of Epstein and Zin (1989) utility (see e.g., Duffie and Epstein, 1992).

Examining Figure 4, we see that the model with news shocks generates qualitatively similar responses as the model with shocks to η. The magnitude of the response of consumption, investment and labor hours are fairly small, but this is due to the presence of a strong wealth effect. In the absence of the limited commitment friction, our model reduces to a standard business cycle model with news about future embodied shocks. To illustrate the strength of the wealth effect that our mechanism needs to overcome, in Figure 5 we plot the response of investment, consumption, hours and capacity utilization to the same news shock, μ.

Figure 4: Response to shock to μ

(a) Likelihood of technological innovation, μ

(b) Relationship Value, V

(c) Fraction of ideas in partnership, $F(\chi)$

(d) Investment efficiency, $g(\chi)$

(e) Investment, I

(f) Consumption, C
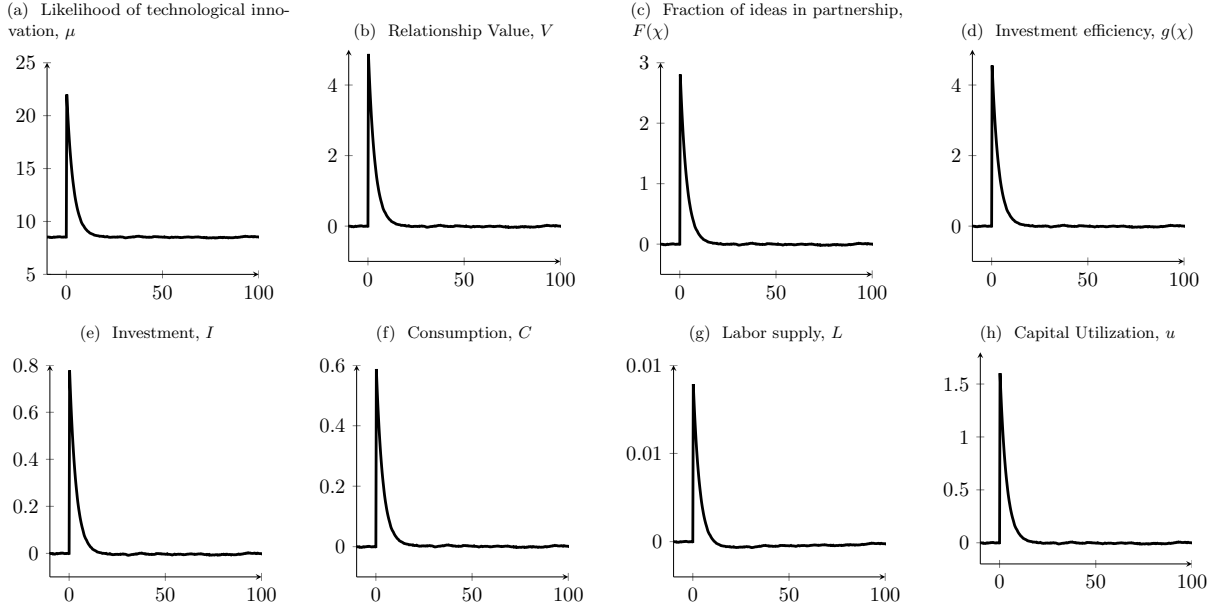
(g) Labor supply, L

(h) Capital Utilization, u

Figure plots the response of equilibrium quantities and prices to a one-standard deviation change in the likelihood of future technology improvements μ. We use parameter values of $\alpha = 0.35$, $\beta = 1/3$, $\psi = 4$, $\delta_0 = 0$, $\delta_1 = 0.15$, $\delta_2 = 2$, $\gamma = 1.5$, $\theta = 3.5$, $\rho = 0.01$, $\eta = 0.85$, $\phi = 0.74$, $p_H = 0.1$, $p_L = 0.2$, $\mu_H = 0.22$, $\mu_L = 0.01$, $\sigma_\xi = 0.2$. We simulate the model at monthly frequencies, and then time aggregate to form quarterly observations. We first simulate a path for μ_t given the transition probabilities (43). We then perturb this path by imposing a transition to $\mu_t = \mu_H$ at time 0. We compute impulse responses by computing the log deviation of quantities and prices from steady state values, and average these differences across 100,000 simulations.

Figure 5: Response to shock to μ – No limited commitment friction

(a) Investment, I (b) [Consumption, C (c) [Labor supply, L (d) [Capital Utilization, u
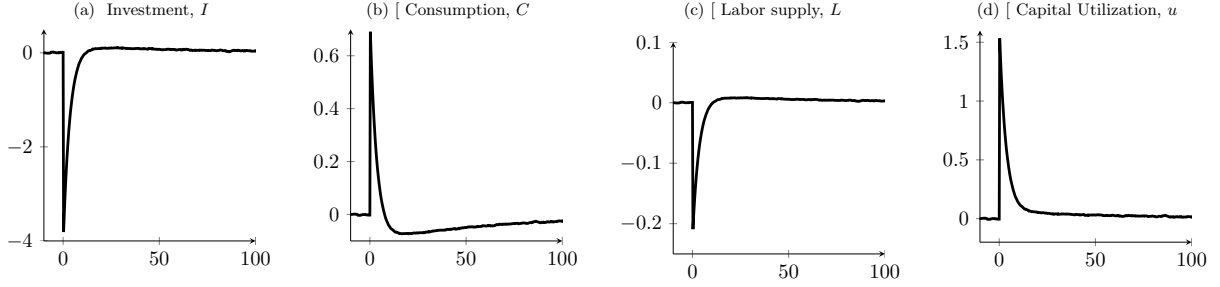
Figure plots the response of equilibrium quantities and prices to a one-standard deviation change in the likelihood of future technology improvements μ, in a model without the limited commitment friction (or, equivalently $\phi = 1$). We use parameter values of $\alpha = 0.35$, $\beta = 1/3$, $\psi = 4$, $\delta_0 = 0$, $\delta_1 = 0.15$, $\delta_2 = 2$, $\gamma = 1.5$, $\theta = 3.5$, $\rho = 0.01$, $\eta = 0.85$, $p_H = 0.1$, $p_L = 0.2$, $\mu_H = 0.22$, $\mu_L = 0.01$, $\sigma_\xi = 0.2$. We simulate the model at monthly frequencies, and then time aggregate to form quarterly observations. We first simulate a path for μ_t given the transition probabilities (43). We then perturb this path by imposing a transition to $\mu_t = \mu_H$ at time 0. We compute impulse responses by computing the log deviation of quantities and prices from steady state values, and average these differences across 100,000 simulations.

In the model without the limited commitment friction, good news about the future lead to a rise in consumption and capital utilization, but to a drop in investment and labor supply. Agents anticipate being wealthier in the future, hence, to smooth consumption they increase consumption and leisure by reducing investment and labor today. However, since in this example the technology frontier does not actually improve, output is lower in the medium run as the economy accumulates less capital – due to both lower investment and higher depreciation as a result of higher utilization.

In sum, this section demonstrates that our model mechanism is fairly general. Specifically, in the presence of the limited commitment friction, any shock that affects the relative value of relationships V to the marginal value of new projects ν will lead to endogenous movements in the level of cooperation χ and therefore to the marginal efficiency of investment.

6 Conclusion

We introduce a limited commitment friction in a relatively standard real business cycle. Firms implement projects in partnership with inventors. Firms cannot commit not to expropriate inventors whose ideas are of sufficiently high quality. This friction effectively restricts the supply of project

ideas and therefore affects the equilibrium demand for capital. Good news about future technological innovations endogenously increases the supply of new ideas, and therefore affects the demand for capital today. Our mechanism allows us to generate realistic, news-driven business cycle fluctuations and provides a theory of endogenous movements to the marginal efficiency.

We describe the implementation friction in the sale of new ideas. However, the insights are more general and could apply to any setting in which creating new productive units requires cooperation among multiple parties. Fluctuations in the degree of cooperation, either because the benefits or opportunity costs of cooperation vary over time will lead to time-variation in the return to new investment. States of the world in which cooperation is difficult will manifest as states in which the average productivity of new ideas appears low. This low productivity is the result of a larger mass of ideas being implemented inefficiently rather than shifts in the actual distribution of new ideas. Our mechanism can thus replicate apparent changes in the technology frontier even though no such changes have occurred and can help interpret periods of low measured productivity.

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Analytical Appendix

To conserve space, we provide the solution to a model that nests the models in Sections 3 and 5. That is, we consider the model in Section 5, in which η_t is time-varying according to (23).

We begin by examining the optimal investment policy in a project of quality θ given the partnership decision $P_t(\theta)$. Define $c_t(\theta) = 1$ if $P_t(\theta) = 1$ and $c_t(\theta) = \phi^{-\alpha/(1-\alpha)}$ otherwise. The market value at time t of a project of vintage s, scale k and quality θ is given by $M_t \xi_s \theta^{1-\alpha} k^\alpha$, where M_t is defined in (25). The owner of a project chooses scale k to maximize

$$NPV_t = \max_k \left\{ M_t \xi_t \theta^{1-\alpha} k^\alpha - c_t(\theta)k \right\} \tag{A.1}$$

yielding

$$k_t^*(\theta) = \theta \, (\alpha M_t \xi_t)^{\frac{1}{1-\alpha}} \left(\frac{1}{c_t(\theta)} \right)^{\frac{1}{1-\alpha}} \tag{A.2}$$

We guess and subsequently verify that the optimal partnership policy takes the form $P_t(\theta) = 1$ iff $\theta < \chi_t$. Next, we solve for the NPV of new projects. The total demand for new capital equals

$$\lambda \int_0^\infty c_t(\theta) \, k_t^*(\theta) dF(\theta) = \lambda \, (\alpha M_t \xi_t)^{\frac{1}{1-\alpha}} \, g(\chi_t) \tag{A.3}$$

where

$$g(\chi) \equiv \int_0^\chi \theta \, dF(\theta) + \phi \int_\chi^\infty \theta \, dF(\theta) = \bar{\theta} - (1 - \phi) \int_\chi^\infty \theta \, dF(\theta). \tag{A.4}$$

Demand for investment goods has to equal supply,

$$\lambda \, (\alpha M_t \xi_t)^{\frac{1}{1-\alpha}} \, g(\chi_t) = I_t. \tag{A.5}$$

Consider the aggregate capital stock

$$K_t = \int_{J_t} \xi_{s(j)} \theta_j^{1-\alpha} k_j^\alpha \, dj, \tag{A.6}$$

New capital created at time t is given by

$$\lambda \int_0^\infty \xi_t \theta^{1-\alpha} k_t^*(\theta)^\alpha dF(\theta) = \lambda \xi_t \, (\alpha M_t \xi_t)^{\frac{\alpha}{1-\alpha}} \, g(\chi_t)$$

$$= \xi_t I_t^\alpha \left(\lambda \, g(\chi_t) \right)^{1-\alpha}, \tag{A.7}$$

where the last equality comes from using the market clearing condition (A.5). We conjecture, and subsequently verify, that the rate of capital utilization is constant across projects and depends only on the aggregate state.

Hence, we can write the evolution of the capital stock as

$$dK_t = \xi_t \, I_t^\alpha \left(\lambda \, g(\chi_t) \right)^{1-\alpha} - \delta(u_t) \, K_t \, dt. \tag{A.8}$$

Notice how ξ_t and χ_t similarly affect the capital accumulation equation.

The NPV of a new project is equal to if implemented in a partnership,

$$NPV_t = \theta \, (1 - \alpha) \, (\xi_t \, M_t)^{\frac{1}{1-\alpha}} \, \alpha^{\frac{\alpha}{1-\alpha}}, \tag{A.9}$$

and

$$\widehat{NPV}_t = \phi \, \theta \, (1 - \alpha) \, (\xi_t \, M_t)^{\frac{1}{1-\alpha}} \, \alpha^{\frac{\alpha}{1-\alpha}}. \tag{A.10}$$

if implemented without the firm. Taking expectations over θ,

$$\int_0^{\chi_t} NPV_t dF(\theta) + \int_{\chi_t}^\infty \widehat{NPV}_t dF(\theta) = (1 - \alpha) \, g(\chi_t) \, (\xi_t \, M_t)^{\frac{1}{1-\alpha}} \, \alpha^{\frac{\alpha}{1-\alpha}}.$$

The next step is to solve for the state-dependent threshold χ_t. To do so, we need to derive the relationship value to the bank. If a project is implemented in a partnership, the firm obtains

$$\Pi_t^F(\theta) = NPV_t - \widehat{NPV}_t - \eta_t (V_t - \widehat{NPV}_t)$$
$$= \theta \, (1 - (1 - \eta_t) \, \phi) \, (1 - \alpha) \, (\xi_t \, M_t)^{\frac{1}{1-\alpha}} \, \alpha^{\frac{\alpha}{1-\alpha}} - \eta_t \, V_t \tag{A.11}$$

Averaging over projects,

$$\int_0^{\chi_t} \Pi_t^F(\theta) \, dF(\theta) = (1 - (1 - \eta_t) \, \phi) \, (1 - \alpha) \, (\xi_t \, M_t)^{\frac{1}{1-\alpha}} \, \alpha^{\frac{\alpha}{1-\alpha}} \int_0^{\chi_t} \theta \, dF(\theta) - \eta_t F(\chi_t) V_t. \tag{A.12}$$

The relationship value to the firm is

$$V_t = \lambda \, E_t \int_t^\infty \frac{\pi_s}{\pi_t} \left(\int_0^{\chi_t} \Pi_t^F(\theta) \, dF(\theta) \right) ds. \tag{A.13}$$

The firm can credibly commit not to expropriate as long as $\widehat{NPV}_t \leq V_t$. Hence, the quality of the marginal project χ_t is the solution to

$$\chi_t \, \phi \, (1 - \alpha) \, (\xi_t \, M_t)^{\frac{1}{1-\alpha}} \, \alpha^{\frac{\alpha}{1-\alpha}} = V_t. \tag{A.14}$$

Next, we solve for the evolution of the aggregate state variables. Since u_t is only depending on aggregate variables, we have that the supply of capital services equals

$$X_t = u_t \, K_t, \tag{A.15}$$

Aggregate investment and consumption are determined by (A.5) and the resource constraints (40)-(18). The

household's labor supply decision is intra-temporal, implying

$$L_t = 1 - \psi \frac{C_t}{w_t} = \left(1 + \frac{\psi}{1-\beta}(1 - i_t)\right)^{-1}. \tag{A.16}$$

where $i_t \equiv I_t/Y_t$ is the investment-to-output ratio, and we have used the standard condition that the equilibrium share of labor is $w_t L_t = (1 - \beta)Y_t$.

At this stage, it is helpful to write aggregate output as

$$Y_t = e^{z_t} \left(u_t^\beta L_t^{1-\beta}\right) \tag{A.17}$$

where

$$z_t = (1 - \beta)x_t + \beta \log K_t \tag{A.18}$$

$$x_t = \mu_x t. \tag{A.19}$$

We see that the trend is determined by z_t, whereas as we will see u_t and L_t will be stationary. Next, we revisit the equation determining the growth in the stock of capital

$$d \log K_t = \frac{\xi_t}{K_t} \left(e^{z_t} i_t \left(u_t^\beta L_t^{1-\beta}\right)\right)^\alpha \left(\lambda g(\chi_t)\right)^{1-\alpha} - \delta(u_t) dt. \tag{A.20}$$

We infer that the combination

$$\omega_t \equiv \log \xi_t - \log K_t + z_t = \log \xi_t + \alpha(1 - \beta)x_t - (1 - \alpha\beta) \log K_t \tag{A.21}$$

is likely to be stationary; that is, capital K_t is cointegrated with the two productivity shocks x and ξ. This leads us to write

$$\kappa(\omega_t, \mu_t, \eta_t) \equiv e^{\omega_t} \left(i_t \left(u_t^\beta L_t^{1-\beta}\right)\right)^\alpha \left(\lambda g(\chi_t)\right)^{1-\alpha} \tag{A.22}$$

given our conjecture – which we verify below – that hours L_t, utilization u_t, the investment-to-output ratio i_t and the threshold χ_t are functions of ω, μ and η. In what follows, we make this dependence explicit.

The next step involves computing M_t and V_t. Standard optimality results (Duffie and Skiadas, 1994) imply that the state price density π satisfies

$$\pi_t = \exp\left(\int_0^t h_J(C_s, N_t, J_s) ds\right) h_C(C_t, N_t, J_t) \tag{A.23}$$

We guess that the value function takes the form

$$J_t = \frac{1}{1-\gamma} e^{(1-\gamma) z_t} j(\omega_t, \mu_t, \eta_t) \tag{A.24}$$

The household's value function satisfies the HJB equation

$$0 = h(C, N, J) + \mathcal{D}J. \tag{A.25}$$

After substituting our guess (A.24) into (A.25) and simplifying, the terms containing z drop out and we obtain the following PDE for j

$$
0 = \Bigg\{ \rho \frac{1-\gamma}{1-\theta^{-1}} \left(u(\omega,\mu,\eta)^\beta \, L(\omega,\mu,\eta)^{1-\beta}(1-i(\omega,\mu,\eta)) \right)^{1-\theta^{-1}} j(\omega,\mu,\eta)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \left(1-L(\omega,\mu,\eta)\right)^{\psi(1-\theta^{-1})}
$$

$$
- \left[\rho \frac{1-\gamma}{1-\theta^{-1}} - (1-\beta)(1-\gamma)\mu_x - \beta(1-\gamma)\left(\kappa(\omega,\mu,\eta) - \delta(u(\omega,\mu,\eta)) \right) \right] j(\omega,\mu,\eta)
$$

$$
+ \left[\alpha(1-\beta)\mu_x - (1-\alpha\beta)\left[\kappa(\omega,\mu,\eta) - \delta(u(\omega,\mu,\eta)) \right] \right] \frac{\partial}{\partial\omega} j(\omega,\mu,\eta) + \mu\Big(j(\omega+s_\xi,\mu,\eta) - j(\omega,\mu,\eta) \Big)
$$

$$
+ \kappa_\eta(\bar\eta - \eta) \frac{\partial}{\partial\eta} j(\omega,\mu,\eta) + \frac{1}{2}\sigma_\eta\, \eta\,(1-\eta) \frac{\partial}{\partial\eta^2} j(\omega,\mu,\eta)
$$

$$
+ p_{LH}\Big(j(\omega,\mu_H,\eta) - j(\omega,\mu,\eta) \Big) + p_{HL}\Big(j(\omega,\mu_L,\eta) - j(\omega,\mu,\eta) \Big) \Bigg\} \tag{A.26}
$$

We can explicitly derive expressions for marginal utility along the equilibrium path, specifically,

$$
h_C(C_t^*, N_t^*, J_t^*) = \rho\, e^{-\rho\, z_t} A(\omega_t,\mu_t,\eta_t) \tag{A.27}
$$

where

$$
A(\omega,\mu,\eta) \equiv \left(\bar u(\omega,\mu,\eta)^\beta \, L(\omega,\mu,\eta)^{1-\beta}(1-i(\omega,\mu,\eta)) \right)^{-\theta^{-1}} \left(1-L(\omega,\mu,\eta)\right)^{\psi(1-\theta^{-1})} j(\omega,\mu,\eta)^{\frac{\gamma-\theta^{-1}}{\gamma-1}} \tag{A.28}
$$

and

$$
h_J(C_t^*, N_t^*, J_t^*) = -\rho \frac{\gamma-\theta^{-1}}{1-\theta^{-1}} \left(\left((1-i(\omega,\mu,\eta))\, u(\omega,\mu,\eta)^\beta L(\omega,\mu,\eta)^{1-\beta} \right)^{1-\theta^{-1}} j(\omega,\mu,\eta)^{\frac{1-\theta^{-1}}{\gamma-1}} (1-L(\omega,\mu,\eta))^{\psi(1-\theta^{-1})} + \frac{1-\gamma}{\gamma-\theta^{-1}} \right)
$$

Notice that h_J is a function of ω, μ and η.

The value of an existing project j is

$$
\max_u E_t \int_t^\infty e^{\int_t^s -\delta(u_\tau)d\tau} \frac{\pi_s}{\pi_t} p_s\, u_s\, \xi_s\, \theta_j^{1-\alpha} k_j^\alpha\, ds = M_t\, \xi_s\, \theta_j^{1-\alpha} k_j^\alpha \tag{A.29}
$$

where

$$
\pi_t M_t = \max_u E_t \int_t^\infty e^{\int_t^s -\delta(u_\tau)d\tau} \pi_s\, p_s\, u_s\, ds. \tag{A.30}
$$

Using the equation for the state price density (A.23), the fact that $p_t\,\bar u_t\,K_t = \beta Y_t$, along with the Feynman-Kac theorem with discounting, we get that

$$
M_t = e^{z_t} K_t^{-1} m(\omega_t,\mu_t,\eta_t) \left(A(\omega_t,\mu_t,\eta_t) \right)^{-1} \tag{A.31}
$$

where the function m solves the PDE

$$0 = \max_{u} \left\{ \beta A(\omega, \mu, \eta) \left(\frac{L}{\bar{u}} \right)^{1-\beta} u + \left[\alpha (1-\beta)\mu_x - (1-\alpha\beta)\Big(\kappa(\omega, \mu, \eta) - \delta(\bar{u}(\omega, \mu, \eta))\Big) \right] \frac{\partial}{\partial \omega} m(\omega, \mu, \eta) \right.$$

$$+ \kappa_\eta(\bar{\eta} - \eta) \frac{\partial}{\partial \eta} m(\omega, \mu, \eta) + \frac{1}{2}\sigma_\eta^2 \eta(1-\eta) \frac{\partial^2}{\partial \eta^2} m(\omega, \mu, \eta) + \mu \left(m(\omega + s_\xi, \mu, \eta) - m(\omega, \mu, \eta) \right)$$

$$- m(\omega, \mu, \eta) \left[-h_J(\omega, \mu, \eta) - (1-\beta)(1-\gamma)\mu_x - (\beta(1-\gamma)-1)\Big(\kappa(\omega, \mu, \eta) - \delta(\bar{u}(\omega, \mu, \eta))\Big) + \delta(u) \right]$$

$$\left. + p_{LH}\Big(m(\omega, \mu_H, \eta) - m(\omega, \mu, \eta) \Big) + p_{HL}\Big(m(\omega, \mu_L, \eta) - m(\omega, \mu, \eta) \Big) \right\} \tag{A.32}$$

When optimizing over capital utilization, the firm takes the aggregate utilization $\bar{u}$ s given. The first order condition for u^* is

$$\beta A(\omega, \mu, \eta) \left(\frac{L}{\bar{u}} \right)^{1-\beta} = m(\omega, \mu, \eta)\, \delta'(u^*) \tag{A.33}$$

Equation (A.33), along with the symmetry condition $\bar{u} = u$ pins down the equilibrium level of u.

Last, we solve for the relationship values V_t of the firm

$$\pi_t V_t = \lambda E_t \int_t^\infty \pi_s \left((1-(1-\eta_t)\phi)(1-\alpha) \left(e^{\xi_s} M_s \right)^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} \int_0^{\chi_s} \theta\, dF(\theta) - \eta_s F(\chi_s) V_s. \right) ds. \tag{A.34}$$

We conjecture that the relationship value takes the form

$$V_t = e^{z_t}\, v(\omega_t, \mu_t, \eta_t) \Big(A(\omega_t, \mu_t, \eta_t) \Big)^{-1}$$

Using the Feynman-Kac theorem with discounting, along with the equation for the state price density (A.23), we get a PDE for v

$$0 = \left\{ (1-(1-\eta)\phi)(1-\alpha) \left(e^\omega m(\omega, \mu, \eta) \right)^{\frac{1}{1-\alpha}} \left(\alpha A(\omega, \mu, \eta)^{-1} \right)^{\frac{\alpha}{1-\alpha}} \int_0^{\chi(\omega, \mu, \eta)} \theta\, dF(\theta) \right.$$

$$+ \left[\alpha(1-\beta)\mu_x - (1-\alpha\beta)\Big(\kappa(\omega, \mu, \eta) - \delta(u(\omega, \mu, \eta))\Big) \right] \frac{\partial}{\partial \omega} v(\omega, \mu, \eta)$$

$$+ \kappa_\eta(\bar{\eta} - \eta) \frac{\partial}{\partial \eta} v(\omega, \mu, \eta) + \mu \left(v(\omega + s_\xi, \mu, \eta) - v(\omega, \mu, \eta) \right) + \frac{1}{2}\sigma_\eta^2 \eta(1-\eta) \frac{\partial^2}{\partial \eta^2} v(\omega, \mu, \eta)$$

$$- v(\omega, \mu) \left(-h_J(\omega, \mu, \eta) - \beta(1-\gamma)\Big(\kappa(\omega, \mu, \eta) - \delta\big(u(\omega, \mu, \eta)\big)\Big) + \eta\lambda F(\chi(\omega, \mu, \eta)) \right)$$

$$\left. + p_{LH}\Big(v(\omega, \mu_H, \eta) - v(\omega, \mu, \eta) \Big) + p_{HL}\Big(v(\omega, \mu_L, \eta) - v(\omega, \mu, \eta) \Big) \right\} \tag{A.35}$$

where, as before, the z terms have cancelled out.

The last step is to verify our conjecture that the variables i_t, L_t, u_t, A_t and χ_t are only functions of ω, μ and η. The first order condition for investment yields

$$i(\omega,\mu,\eta)\left(u(\omega,\mu,\eta)^\beta L(\omega,\mu,\eta)^{1-\beta}\right) = \lambda\left(\alpha e^\omega m(\omega,\mu,\eta)\left(A(\omega,\mu,\eta)\right)^{-1}\right)^{\frac{1}{1-\alpha}} g(\chi(\omega,\mu,\eta)). \qquad (A.36)$$

while the equation that determines the threshold χ becomes

$$\chi(\omega,\mu,\eta)\,\phi\,(1-\alpha)\left(e^\omega m(\omega,\mu,\eta)\left(A(\omega,\mu,\eta)\right)^{-1}\right)^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}} = v(\omega,\mu,\eta)\left(A(\omega,\mu,\eta)\right)^{-1}. \qquad (A.37)$$

Examining the two equations above, along with (A.16), (A.28) and (A.33), our conjecture is verified.

The solution to the model in Section 3 can be obtained by setting $\theta = \gamma^{-1}$, $\xi = 1$ and $s_\xi = 0$. Conversely, the solution to the model in Section 5 can be obtained by setting $\eta_t = \bar{\eta}$, $\mu_x = 0$, and $\sigma_\eta = 0$.

We solve the model numerically by jointly solving the system of PDEs (A.26), (A.32), (A.35), the first order conditions (A.16), (A.33) and (A.36) and the condition determining the threshold (A.37). We use a finite difference scheme on an equally-spaced grid for ω and η. We iterate until convergence.