Optimal Disability Insurance and Unemployment Insurance With Cyclical Fluctuations

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Abstract

This paper studies the optimal joint design of disability insurance and unemployment insurance in an environment with moral hazard, when health status is private information, and cyclical fluctuations. I show how disability benefits and unemployment benefits vary with aggregate economic conditions in an optimal contract. In a special case of the model, I first show the optimal contract can be solved explicitly up to a system of non-linear equations. I then demonstrate that the optimal joint insurance system can be implemented by allowing workers to save or borrow using a bond and by providing flow payments and lump-sum transfers (or payments), where the interest rates and the amounts paid (transferred) depend on the employment or health status of the agent and the state of the economy. Finally, I consider a calibrated version of the full model and study the quantitative implications of both the current system and the optimal system. In the optimal system, disability benefits are designed such that the system punishes workers who stay unemployed for a long time. I consider the welfare impact of changing from the current system to the optimal one when both systems provide the same ex-ante utility to the worker. The cost savings incurred from incentive problems are substantial, and the unemployment rate will be reduced by roughly 40 percent.

1   Introduction

Disability insurance (DI) and unemployment insurance (UI) are two important social insurance programs that provide relief when people suffer from income fluctuations. As of December 2013,

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8.9 million disabled individuals received DI benefits, which corresponds to 5.7% of the population of workers in the labor force.[1] The unemployment rate was at 6.2%, which is higher than the pre-recession period, leaving 827 thousand workers who had exhausted their UI by the first quarter of 2014. While DI and UI both play significant roles in helping individuals to smooth consumption, both programs are subject to incentive problems. In fact, job search efforts on the part of applicants are hardly monitored, and, in 2001, approximately half of awards went to applicants with difficult- to-verify disabilities, such as mental disorders and diseases of the musculoskeletal system (Golosov and Tsyvisnski [2006]). In addition, both of these incentive problems respond differently to business cycles. In fact, both DI and UI applications and awards are reported to be countercyclical (Mueller et al, 2013). As in Figure 1, the cyclicality of SSDI applications and the unemployment rates are shown, and SSDI applications surge a year or two after unemployment peaks after the reform of SSDI in 1984. Although I do not show this information in the figure, it has been reported that SSDI awards have a similar pattern (Black, Daniel, and Sanders [2002]; Autor and Duggan [2003]; Duggan and Imberman [2009]; Coe et al. [2012]). If we think the health shocks are a-cyclical, why are the observed patterns of DI applications and awards countercyclical?
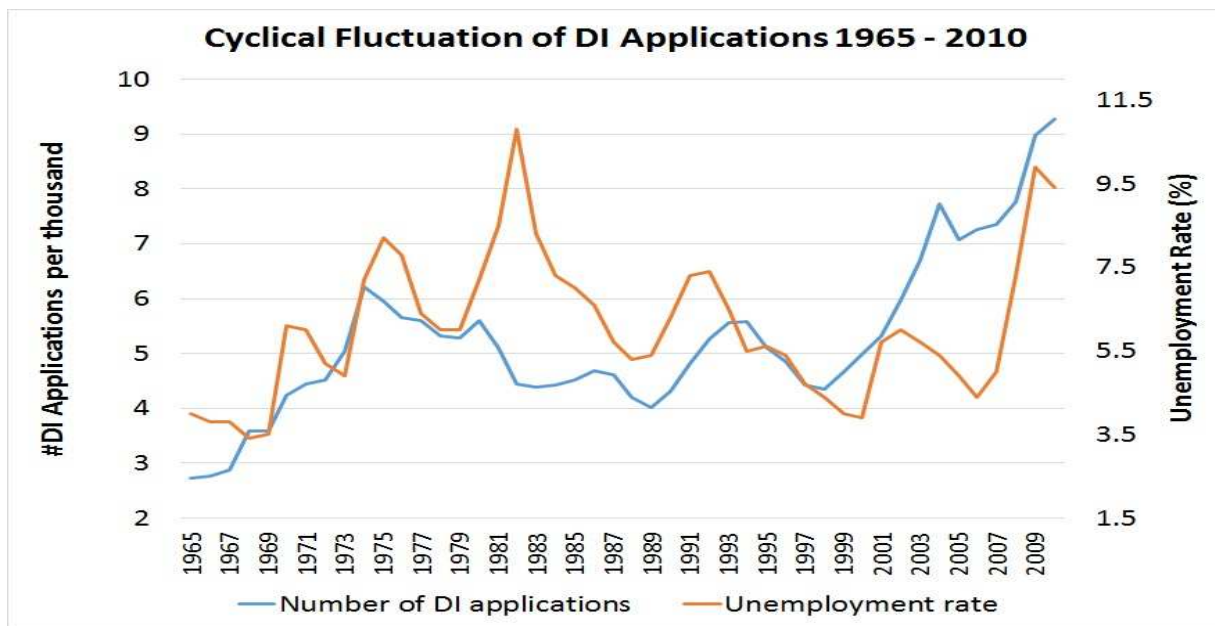

Figure 1: Disability Insurance Applications and the U.S. Unemployment Rate, 1965-2010

What is more, the design of unemployment insurance and disability insurance does not consider the incentive problems and cyclicality issues seriously, not to mention the interactions between these two insurance programs. In the United States, a typical unemployment insurance program

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[1]Source: Annual report Supplemental Security Income program, the Social Security Administration. US Bureau of Labor Statistics.

provides 26 weeks of benefits, but extended unemployment insurance programs would be adopted subsequently in recessions, providing some groups of people with different extended periods when their insurance benefits are exhausted. In the most recent recession, benefits were provided for a maximum of 99 weeks. In terms of disability insurance, the benefits are calculated with Primary Insurance Amount (PIA) formula, using the inflation-adjusted averaged monthly income and are progressive in the sense that low-income people receive higher benefits, but the amounts of benefits do not vary with aggregate economic state. From discussions above, we can observe that the patterns of disability insurance and unemployment insurance seem arbitrary and independent from each other. This paper will analyze the joint design problem with cyclical fluctuations.

To address the incentive problem associated with UI and DI and business cycles, this paper studies the optimal policy design of disability insurance and unemployment insurance in an environment where job finding and separation rates fluctuate with aggregate economic states. The objective of this paper is three-fold. First, I build a model of UI and DI to characterize optimal contract design. Second, I consider a special case of the model that can be solved explicitly and this allows me to demonstrate the implementation of the optimal contract via simple instruments. Last, I calibrate the model to the U.S. data and calculate the inefficiency in the current social insurance system. I then consider the potential cost savings from switching to the optimal system and conduct counterfactual analysis by presenting the impact of extended UI benefits. Thus, this paper suggests that the potential benefits for policy makers can be substantial if they adjust social insurance policies according to aggregate economic state changes and the joint design of the DI and UI.

In this paper, I build a tractable model that incorporates disability insurance into previous work by Hopenhayn and Nicolini [1997] and Li and Williams [2014]. Hopenhayn and Nicolini [1997] study the optimal unemployment insurance with moral hazard, which is extended by Li and Williams [2014], whose model takes business cycles into consideration. In our model, similarly to Li and Williams [2014], a risk-averse worker exerts costly job search efforts that increase job arrival rates, and job finding and separation rates depend on the aggregate economic climate. This paper diverts from the preceding literature by adding a health shock that makes a worker incapable of working, similar to Golosov and Tsyvisnski [2006]. The agency provides two kinds of insurance: unemployment insurance and disability insurance, and aims at minimizing the cost of providing insurance that satisfies a disabled workers participation constraint. The work incentive problem arises due to the fact that workers health conditions are unobservable to the agency in addition to their unobservable job search efforts, job finding and separation rates depend on aggregate economic conditions. By modeling DI and UI together with business cycles, our model enables us to develop the optimal design for when moral hazard and misreporting problems co-

exist and to compare the optimal social insurance schemes with the current programs through different aggregate economic states. While there are many studies of UI and, to a lesser extent, DI with respect to their independent effects on labor supply, few papers have studied how DI and UI jointly affect labor supply decisions with business cycles. To the best of my knowledge, no previous papers have developed an optimal policy design that considers the interaction between UI and DI with business cycles.

After laying out the model, I consider a special case with CARA utility functions and zero job separation rates. This special case allows me to derive the explicit solutions. I can then demonstrate simply and clearly how information frictions affect the optimal contract design. I find that the results are similar to the ones in Hopenhayn and Nicolini [1997] and Li and Williams [2014], where consumption decreases over the unemployment spell in the asymmetric information case in order to provide incentives. I then turn to consider the implementation of the contracts through a workers consumption-saving-effort problem. I find that the optimal contracts can be implemented by constant payments in each state through taxes while employed and subsidies while unemployed, and lump-sum transfers when state (employed/unemployed/disabled, boom/recession) switches. The implementation of the model allows me to understand the optimal pattern of the contract and analyze the comparative statistics as to how the optimal contract responds to changes in parameters, as well as how the agent would respond equivalently through borrowing and saving activities.

Third, I consider the quantitative implications of the model and study the impacts of DI on the unemployment insurance system. I first calibrate the model to obtain some key parameters. Then I study the following questions. First, I ask what is the amount of the cost reductions when the worker switches from the current system to the optimal one. Second, I consider the potential impact when the extended unemployment insurance benefits are adopted. I found that the cost reductions could be substantial and switching to the optimal system reduces unemployment rates by around 40%.

One thing that worths mentioning is that my cost-saving results differ in some fundamental ways from those in the literature. The substantial cost reduction implies that the government can actually generate positive revenue, if the optimal system is implemented. However, this also implies that healthy workers are willing to give up a lot of consumption in order to get compensated when the healthy shock arrives. The implied tax rate in the optimal system is huge in my quantitative section. Moreover, the huge implied tax rate is closely related to the "immiseration" result of Thomas and Worrall [1990] and Atkeson and Lucas [1992], which tells that the agent's promised utility tends to minus infinity under the optimal contract. Hence, computing the average consumption over a long period generates this large reduction.

Finally, I consider the impact of optimal systems on low-income workers and the impact of different policy reforms. First, as discussed in Autor and Duggan [2003, 2006], the rising replacement ratio of the DI is one of the reasons that causes the cyclicality. I first consider how effective the optimal system would be on the low-income workers because they are the group of workers with the highest replacement ratio of DI and have more incentive to apply for DI after a long unemployment spell. I found that the unemployment rate will be reduced around 60%. Second, I consider the impact of two policy reforms: (1) extending the maximum UI duration to 99 weeks in recessions, and (2) designing of the optimal DI but taking the current UI system as given. The first policy reform is motivated by the actions taken in the great recession where unemployed workers may receive UI up to 99 weeks. I found that there is not much difference in cost savings and unemployment rate between the system with standard extended UI duration and the extended one. The second experiment is intended to show the interdependence between the UI and DI. I found that even though the government can only change the DI system but not the UI system, there will be big differences in cost savings and unemployment rate.

The papers that are most closely related to this paper are Hopenhayn and Nicolini [1997], Li and Williams [2014], and Golosov and Tsyvisnski [2006]. Hopenhayn and Nicolini [1997] and Li and Williams [2014] focus on the optimal design of the unemployment insurance, while Li and Williams [2014] add business cycles on top of Hopenhayn and Nicolinis [1997] model. In Golosov and Tsyvisnski [2006], they study the optimal design of the disability insurance and focus on the asset-testing implementation mechanism. The solvable case and implementation of the optimal contract follow closely as the ones in Li and Williams [2014], but adding the disability state allows me to show the impact of UI on DI. My paper differs from those papers by considering the optimal joint design of disability and unemployment insurance with business cycles. The implications of this papers model are in line with the empirical literature. Among the papers that study the cyclicality property of DI are Autor and Duggan [2003], Rutledge [2012], and Mueller et al. [2013]. Autor and Duggan [2003] attribute the labor force exit propensity of displaced high school dropouts after 1984 to three major factors: reduced screening stringency of DI, declining demand for less skilled workers, and an unforeseen increase in the earnings replacement rate. They find that the sum of these forces can account for a decrease by one-half a percentage point in measured U.S. unemployment. Mueller et al. [2013] incorporate unemployment insurance to Autor and Duggans [2003] model by drawing on Rothsteins [2011] model of UI and job search. Given that the cyclicality of DI applications and awards is consistently found as a stylized fact, the findings of Mueller et al [2013] imply that there should be other channels to explain how a surge of DI applications and awards closely follows business cycles. In this paper, I am able to structurally analyze the cyclicality patterns of DI and UI in an optimal design framework as well

as explore the effects of earnings replacement rates on workers from different income groups.

The rest of the paper is organized as follows. In section 2, I lay out the model. Then I study the optimal contract design in section 3. Section 4 studies the solvable case and section 5 studies the implementation of the contract. The quantitative implications of the optimal design are demonstrated in section 6.

2   The Model

In this section, I lay out the model. The model is the continuous time version of the model that combines the UI model by Hopenhayn and Nicolini [1997] and the DI model by Golosov and Tsyvinski [2006], adding cyclical fluctuations seen in Li and Williams [2014]. In short, this model is based on the UI and business cycle model in Li and Williams [2014] and adds disability to this UI similar to Golosov and Tsyvinski [2006].

2.1   The Setup

I consider an infinitely lived agent (worker) who transitions between being employed and unemployed when healthy and could also become disabled. If employed, the agent receives a constant wage ω. If unemployed, the agent earns no income, but he may exert effort to find a job, with effort being costly to him but increasing the arrival rate of a job. When the agent becomes disabled, I assume that the job arrival rate and the wage is zero and disability is assumed to be an absorbing state: the disabled worker will not become healthy again. In addition, I assume that the economy switches between booms and recessions. In a boom, the job finding rate is higher, while the separation rate is lower. I will use $s_t \in \{B, R\}$ to denote the good and bad states.

When unemployed and in a state s_t, let $a_t \in [0, \bar{a}]$ be the search effort for the unemployed agent at time t. Then the job arrival rate is $q_s(a_t)$ with $q'_s(a) > 0$, $q''_s(a) \leq 0$. To simplify the computations, I assume the $q_s(a_t)$ is linear in effort:

$$q_s(a_t) = q_{s0} + q_{s1}a_t,$$

with $q_{s0} \geq 0$, $q_{s1} > 0$. In addition, $q_s(a_t)$ is assumed to have the following property:

$$q_R(a_t) \geq q_R(a_t), \ \forall a_t \in [0, \bar{a}],$$

which is intended to capture the assumption that the job finding rate is higher in booms than in recessions. Last, I assume that an employed worker loses his job with an exogenous separation rate p_s with $p_B < p_R$, that a healthy agent would become disabled with the rate λ_d, and that the rate that state s would transit to state s' is $\lambda_s, s \in \{B, R\}$.

2.2 Preferences and Incentive Compatible Contracts

I assume that an insurance agency ("the principal") provides unemployment and disability insurance to help the worker smooth his consumption. The workers employment status is publicly observable. However, the search effort taken by the unemployed worker and the health status of the disabled worker are not observable to the insurance agency. In other words, when a worker reports as disabled, the agency cannot tell if the agent is disabled or able to work or search for a job but shirking. By assuming health status is the private information for the agent, the agency cannot distinguish healthy workers from disabled workers. Hence, moral hazard and private information problem arise as the agency needs to offer insurance that induces the unemployed workers to exert effort and healthy workers not to misreport as disabled. In addition, I assume that the insurance agency cannot distinguish quitting from being laid off. Hence, the contract needs to induce the workers to take a job once it arrives and not to voluntarily quit.

I will define $j \in \{E, U\}$ being the employed and unemployed status with $E = 0$ and $U = 1$. Also, let $d \in \{H, D\}$ stand for healthy and disabled with $H = 0$ and $D = 1$. Let the worker's instantaneous utility be $u(c, a; j, d)$ if the consumption is c and effort taken is a in the state j and d, with u is strictly increasing and concave in c and decreasing and convex in a. I also assume the worker dies stochastically with rate κ. Let the subjective discount rate be $\hat{\rho}$ and thus the effective discount rate becomes $\rho = \hat{\rho} + \kappa$.

Next I will describe the contracts. A contract consists of a quadruple of processes $(c, a, j, d) = (\{c_t\}_{t=0}^{\infty}, \{a_t\}_{t=0}^{\infty}, \{j_t\}_{t=0}^{\infty}, \{d_t\}_{t=0}^{\infty})$, where c is the consumption process with c_t being the amount of consumption of the worker promised by the agency at time t, a is the process of effort level, j is the process of employment status, and d is the process of the reported health status, with a_t, j_t and d_t defined in a similar way. I assume $c_t \in [0, \bar{c}]$, $a_t \in [0, \bar{a}]$. The contract is history dependent in the sense that c_t and a_t depend on the entire history of the worker's employment status ($\{j_t\}_{t=0}^{\infty}$) and worker's health status ($\{d_t\}_{t=0}^{\infty}$). Invoking the revelation principle, I will focus on the truthfully reporting contracts, where the agent reports the true health status, would not voluntarily quit the job, and takes the recommended effort. Now I will describe the worker's maximization problem. Given (c, a, j, d), the worker chooses effort to maximize his lifetime expected utility:

$$\max_{\hat{a} \in A} E[\rho \int_0^{\infty} e^{-\rho t} u(c, \hat{a}) dt], \tag{1}$$

where E is the expectation operation, and $A = [0, \bar{a}]$. A contract is incentive compatible if and only if the worker (i) exerts the recommended search effort, (ii) truthfully reports health status, and (iii) would not quit the job that solves the problem (1).

Let $v(.)$ be the utility for the insurance agency. The objective of the agency is to design the

contract as follows.

$$\max_{(c,a,j,d)} E[-\rho \int_0^\infty r^{-\rho t} v(c_t - \mathbf{1}(\{\text{if the worker is employed}\})\omega)dt]$$

such that

$$(c, a, j, d) \text{ is incentive compatible}$$

and

$$E[\rho \int_0^\infty e^{-\rho t} u(c,a)dt] \geq W_0,$$

where W_0 is the reservation utility of the worker, $\mathbf{1}(.)$ is the indicator function.

3 Optimal Contract

In this section, I will show that optimal contracts can be derived by using the promised utility of the agent as states and controls. Then I will lay out the corresponding Hamiltonian-Jacobi-Bellman equations describing the optimal contracts.

3.1 Incentives and Promised Utility

In order to solve the optimal contracts, it is useful to first define the compensated martingales. I already defined the aggregate state $s_t \in \{B, R\}$, and now I will assign the numerical values as $R = 1$ being the recessions and $B = 0$ being the booms. Also, recalling that $j \in \{E, U\}$ are the employed and unemployed statuses, with $E = 0$ and $U = 1$, and $d \in \{H, D\}$ standing for healthy and disabled with $H = 0$ and $D = 1$. Let the associated compensated jump martingales be m_t^j, m_t^s, and m_t^d governing the jumps between 0 and 1, with m_t^j and m_t^s being observable to the agency while m_t^d is the reported process. The evolution for the processes of the compensated martingales can be written as

$$dm_t^j = (1 - d_t)(-(1 - j_t)[((1 - s_t)p_G + s_t p_B)] + j_t[(1 - s_t)q_R(a_t) + s_t q_R(a_t)])dt + \Delta j_t$$
$$dm_t^s = [-(1 - s_t)\lambda_R + s_t \lambda_R]dt + \Delta s_t$$
$$dm_t^d = (-(1 - d_t)\lambda_d)dt + \Delta d_t,$$

where Δ governs when the worker switches states. For example, for a healthy worker $(d = 0)$ who is in his unemployment spell $(j = 1)$ while the economy is in a boom $(s = 0)$, the compensated jump processes are:

$$dm_t^j = q_R(a_t)dt + \Delta j_t$$
$$dm_t^s = -\lambda_R dt + \Delta s_t$$
$$dm_t^d = -\lambda_d dt + \Delta d_t,$$

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where the negative term compensates the positive jumps, and it makes the process mean zero martingales.

Now I am ready to consider the incentive compatible contracts. Given a contract (c, a, j, d) and the arbitrary effort process $\hat{a}$, $\hat{j}$, and $\hat{d}$, I define the promised utility of the worker as

$$W_t \equiv E[\rho \int_t^\infty e^{-\rho t} u(c, \hat{a}) dt], \forall t \in [0, \infty],$$

which stands for the expected utility of a worker at time t given the contract (c, a, j, d) but exerting effort $\hat{a}$, reporting the employment status $\hat{j}$, and health status $\hat{d}$. I will first show the result using the martingale representation theorem.

Proposition 1. *Under a contract (c, a, j, d) and the chosen effort level $\hat{a}$, the chosen employment status $\hat{j}$, and the reported health status $\hat{d}$. Then there exists three $\mathbb{F}$-predictable[2] processes $g_t^{\hat{j}}$, g_t^s, and $g_t^{\hat{d}}$ such that*

$$E[\int_0^\infty e^{-\rho t} g_t^{\hat{j}} dt] < \infty, \ E[\int_0^\infty e^{-\rho t} g_t^s dt] < \infty, \ and \ E[\int_0^\infty e^{-\rho t} g_t^{\hat{d}} dt] < \infty,$$

and

$$dW_t = \rho(W_t - u(c_t, \hat{a}_t))) dt + \rho g_t^{\hat{j}} dm_t^j + \rho g_t^s dm_t^s + \rho g_t^{\hat{d}} dm_t^d.$$

Proof. See Appendix A.1. □

Next, I consider the conditions that guarantee the incentive compatible contracts.

Proposition 2. *Given the results in proposition 1, the contract is incentive compatible if and only if the following holds for all t:*

$$a_t \in \arg \max_{\tilde{a}_t} g_t^{\hat{j}} q_{s_t}(\tilde{a}_t) + u(c_t, \tilde{a}_t)$$

$$g_t^{\hat{j}} \leq 0$$

$$g_t^{\hat{d}} \leq 0.$$

Proof. See Appendix A.2. □

Proposition 1 is the standard method used in continuous time dynamic contracting literature, which is demonstrated in Sannikov [2008], Williams [2011], and Li [2012]. To solve the dynamic programming problem, I first use the appropriate martingales so that the objective function can be rewritten recursively. In proposition 2, I then show how to use the martingales derived from proposition 1 to express incentive compatible constraints. This way, we are able to write the problem recursively and impose the constraints on the incentive problems.

For the rest of this section, I will show how to derive the Hamilton-Jacobi-Bellman equations governing the optimal contracts using the propositions above.

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[2]$\mathbb{F}$-predictable stands for the sigma-algebra that is generated by the process of dm_t^j, dm_t^s, and dm_t^d.

3.2 Value functions and Optimal Contracts

In this section, I will derive the conditions of the value functions for the insurance agency. Defining $V(W, j, s)$ as the value functions for the agency with state j and s with promised utility W delivered to the worker when healthy and $V(W, d)$ as the value function with W delivered to the disabled worker. I first consider the boundary values of the value functions and promised utility and then the Hamilton-Jacobi-Bellman equations.

Before deriving the Hamilton-Jacobi-Bellman equations, let me consider the boundary points of value functions. Those boundary points will serve as the choice set for the HJB equations. Since I will use the promised utility as choices, I will first consider the possible sets given the boundaries of the parameter values such as consumption and effort. Since the ideas and arguments are similar to the ones in Li and Williams (2014), I explain the details in Appendix A.3.

3.2.1 The Hamilton-Jacobi-Bellman Equations

After deriving the boundary points, I am ready to specify the HJB equations that determine the optimal contracts. First, it is convenient to change the control variables using the promised utilities as variables. Considering the unemployed worker in the state s, if W_t^j is used as the worker's promised utility immediately after the change of the job status, and W_t^d as the worker's promised utility immediately after the change of the health status, the incentive compatible constraints become:

$$g_t^j \Delta j_t = \frac{W_t^j - W_t}{\rho}, \ g_t^d \Delta d_t = \frac{W_t^d - W_t}{\rho}$$

Then I can rewrite the constraints as

$$a^* \in \arg\max_{\hat{a}} u(c, \hat{a}) + \frac{W_t^j - W_t}{\rho} q_s(\hat{a}), \ W_t \geq W_t^d.$$

Similarly, considering the workers in state s, the constraints become

$$W_t \geq W_t^d, \ W_t^j \leq W_t \text{ when employed}, \ W_t^j \geq W_t \text{ when unemployed}.$$

Hence, the HJB equations can be specified as follows:

Proposition 3. *Suppose the value functions $(V(W, j, s), V(W, d))$ exist and the left and right*

boundaries are derived in proposition 4, then the value functions satisfy a system of HJB equations:

$$\rho V(W, u, s) = \max_{\substack{\hat{c} \in [0, \bar{c}], \\ W^j \in [W_l^{es}, W_r^{es}], \\ W^s \in [W_l^{us}, W_r^{us}], \\ W^d \in [W_l^d, W_r^d], \\ W^j \geq W, W^d \leq W}} -\rho v(\hat{c})$$

$$+ \rho V_W(W, u, s)[W - u(\hat{c}, a^*(W^j, W)) - q_s(a^*(W^j, W))\frac{W^j - W}{\rho} - \lambda_s \frac{W^s - W}{\rho} - \lambda_d \frac{W^d - W}{\rho}]$$

$$+ q_s(a^*(W^j, W))[V(W^j, e, s) - V(W, u, s)]$$

$$+ \lambda_s[V(W^s, u, s') - V(W, u, s)]$$

$$+ \lambda_d[V(W^d, d) - V(W, u, s)],$$

for $W \in [W_l^{us}, W_r^{us}]$ and $s = B, R$;

$$\rho V(W, e, s) = \max_{\substack{\hat{c} \in [0, \bar{c}], \\ W^j \in [W_l^{us}, W_r^{us}], \\ W^s \in [W_l^{es}, W_r^{es}], \\ W^d \in [W_l^d, W_r^d], \\ W^j \leq W, W^d \leq W}} -\rho v(\hat{c} - \omega) + \rho V_W(W, e, s)[W - u(\hat{c}) - p_s \frac{W^j - W}{\rho} - \lambda_s \frac{W^s - W}{\rho} - \lambda_d \frac{W^d - W}{\rho}]$$

$$+ p_s[V(W^j, u, s) - V(W, e, s)] + \lambda_s[V(W^s, e, s') - V(W, e, s)] + \lambda_d[V(W^d, d) - V(W, e, s)],$$

for $W \in [W_l^{es}, W_r^{es}]$ and $s = B, R$;

$$\rho V(W, d) = \max_{c \in [0, \bar{c}]} -\rho v(\hat{c}) + \rho V_W(W, d)[W - u(\hat{c})]$$

for $W \in [W_l^d, W_r^d]$.

Proof. See Appendix A.4. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ □

In Proposition 3, the contracting problems are reduced to solving a system of HJB equations. For the rest of the paper, I will discuss the implications from this system of HJB equations.

4   A Solvable Special Case

In this section, I will consider a special case that allows me to demonstrate the theoretical implications. By assuming the utility functions to be exponential, the solutions of the optimal contracts can be reduced to a system of non-linear algebraic equations. The purposes of making this simplification are two fold. First, solving a system of non-linear equations is easier, compared to the general HJB equations where I need to solve a system of partial differential equations. This simplification allows me to demonstrate the theoretical properties of the optimal contracts, and I am able to show the comparative statistics. Second, I can show that the optimal contracts

under exponential utilities can be implemented by a workers consumption-saving-effort model, which helps us to understand the properties of the optimal contracts by observing the workers self-insured behavior.

The arguments in this section follow closely the ones demonstrated in Li and Williams [2014]. In Li and Williams [2014], they show that using the exponential utilities and shutting down the channel of job separation yield solvable solutions and the optimal contracts can be implemented by allowing workers to save or borrow via different interest rates plus flow payments and lump-sum transfers. In this paper, I extend their work by showing that the methodology can be extended without shutting down the possibility of getting separated from jobs, and optimal contracts are implementable by a similar workers consumption-saving-effort model. The only difficulty comes from the indeterminacy of the flow payments; I will illustrate how to tackle this issue in the Appendix B.

In particular, I assume the workers' preferences are given by:

$$u(c, a) = \exp(-\theta_A(c - h(a))),$$

where h is increasing and convex with $h(0) = 0$, and $\theta_A > 0$ is the risk aversion coefficient. The agent's cost function is given by:

$$v(c) = \exp(\theta_P c),$$

where $\theta_P > 0$ is the risk aversion coefficient.

Although we do not specify the HJB equations for full information case, it is quite straightforward to extend from the HJBs for private information case. Denote $V_f(W, j, s)$, and let $V_f(W, d)$ be the value function for full information case. In the Appendix B, I show that solutions to the full information case take the following forms:

$$V_f(W, e, s) = -V_e(s)(-W)^{-\frac{\theta_P}{\theta_A}}, c_f(W, e, s) = \frac{\log V_e(s)}{\theta_P + \theta_A} - \frac{1}{\theta_A}\log(-W) + \frac{\theta_P}{\theta_P + \theta_A}\omega$$

$$V_f(W, u, s) = -V_u(s)(-W)^{-\frac{\theta_P}{\theta_A}}, c_f(W, u, s) = \frac{\log V_u(s)}{\theta_P + \theta_A} - \frac{1}{\theta_A}\log(-W) + \frac{\theta_A}{\theta_P + \theta_A}h(a(s))$$

$$V(W, d) = -(-W)^{-\frac{\theta_P}{\theta_A}}, c_f(W, d) = -\frac{1}{\theta_A}\log(-W),$$

where effort $a(s)$ depends only on the aggregate economic state, and $c_f(.)$ denotes the consumption in full formation case. Similarly, solutions to the private information case are:

$$V(W, e, s) = -V_e^*(s)(-W)^{-\frac{\theta_P}{\theta_A}}, c(W, e, s) = \frac{\log V_e^*(s)}{\theta_P + \theta_A} - \frac{1}{\theta_A}\log(-W) + \frac{\theta_P}{\theta_P + \theta_A}\omega$$

$$V(W, u, s) = -V_u^*(s)(-W)^{-\frac{\theta_P}{\theta_A}}, c(W, u, s) = \frac{\log V_u^*(s)}{\theta_P + \theta_A} - \frac{1}{\theta_A}\log(-W) + \frac{\theta_A}{\theta_P + \theta_A}h(a^*(s))$$

$$V(W, d) = -(-W)^{-\frac{\theta_P}{\theta_A}}, c(W, d) = -\frac{1}{\theta_A}\log(-W),$$

where $a^*(s)$ is the effort level in private information case, and $c(.)$ is the consumption.

For the next section, I will demonstrate how the results would change with respect to the changes in parameter values.

4.1 Analysis

In this section, I will illustrate the dynamics of the optimal contracts under different information structures, aggregate economic conditions, and responses to changes in parameter values. In Figures 2 and 3, I will use the parameter values from the calibration results in the later section: $q_B = 0.0037$, $q_R = 0.0032$, $\lambda_B = 0.0058$, $\lambda_R = 0.0078$, $\rho = 0.001$, and $\omega = 495$. In addition, I choose $\theta_A = 0.0015$, $\theta_P = 0.0005$, and $h(a) = \nu \frac{a^{1+\phi}}{1+\phi}$ with $\nu = 0.01$ and $\phi = 1.7$ for illustration purposes. In Figure 4, I will show how the effort and the consumption constant changes in response to the changes in parameter values.
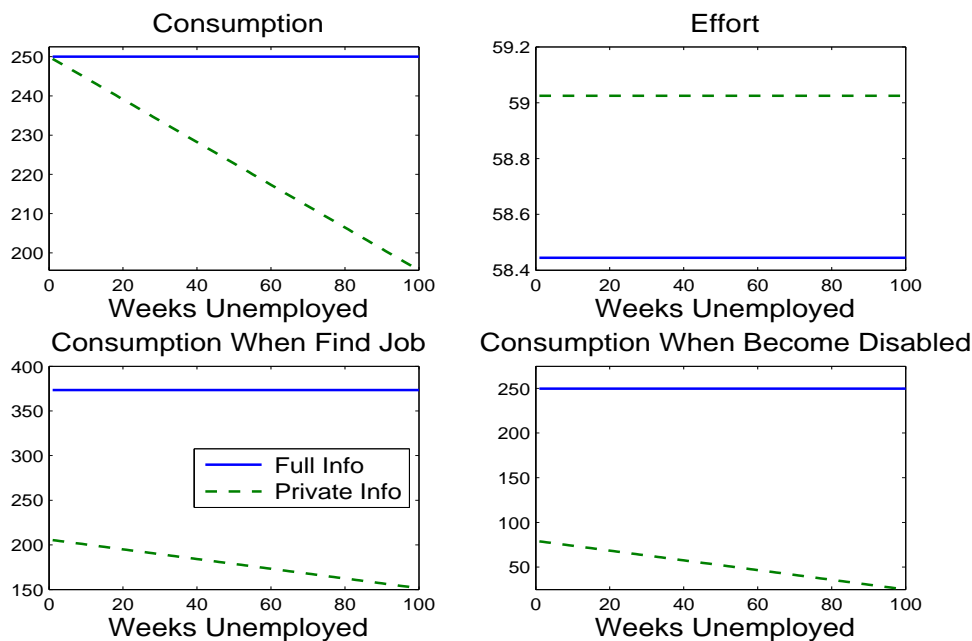

Figure 2: Responses of Selected Variables to Weeks Unemployed with Full information, Private Information, and Private Information without Disability

In Figure 2, I illustrate the differences between the contracts under full information and private information (at least for this set of parameter values). All variables are constant under full information case over the employed/unemployed spell, as the optimal contract effectively insures the workers since there is no information asymmetry. Under private information, consumption when unemployed decreases over the unemployment spell, and consumption upon finding a job or being disabled decreases over the spell. This intuition is transparent, as emphasized in Hopen-
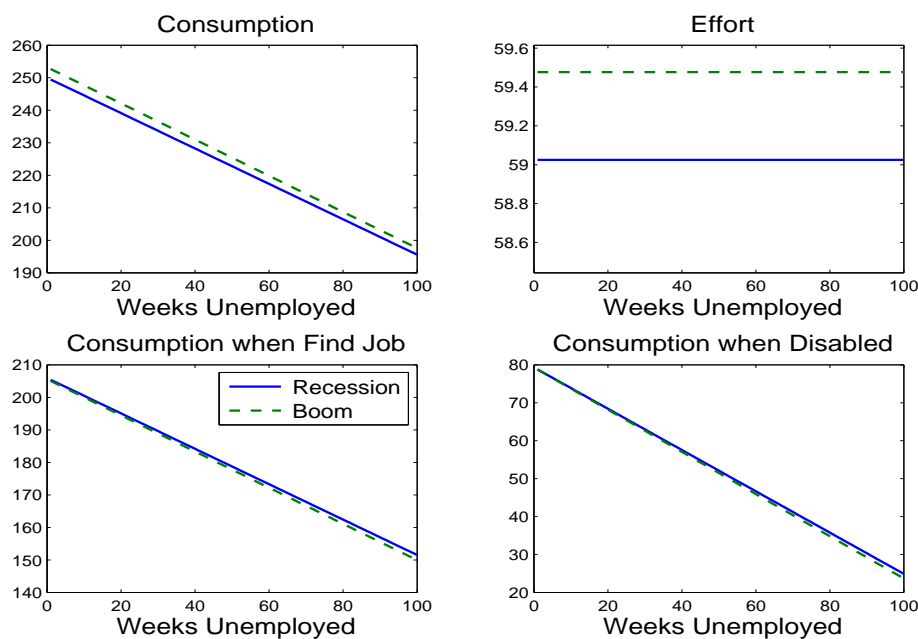
Figure 3: Responses of Selected Variables to Weeks Unemployed with Private Information in Booms and Recessions.

hayn and Nicolini [1997]. In order to induce the workers to search for jobs, consumption when unemployed decreases over the unemployment spell because the agency does not want to punish the unlucky workers at the beginning period of the unemployment spell. The same intuition works for the other property: in order to give incentives to the workers to leave unemployment status sooner, the agency would decrease the consumption when finding a job or becoming disabled when the worker stays in unemployment status for longer periods of time. Interestingly, effort level is higher under private information case than under full information case. By inducing workers to search for jobs harder under private information case, the cost from asymmetric information can be reduced since workers leave the unemployment pool faster.

In Figure 3, I plot the same variables, but I focus on the cyclical properties. As job search is more productive in booms, the agency would design the contract so that it induces the worker to exert higher effort, as higher consumption is provided during the unemployment spell. At the same time, the consumption upon finding a job or becoming disabled decreases at a faster rate over the unemployment spell in booms than in recessions for the same reason as search is more productive in booms. In addition, effort level is higher in booms than recessions, reflecting the situation that job finding rate is higher in booms and the optimal design would induce the workers to put forth higher search effort.

In Figure 4, I analyze how the effort $a^*(s)$ and consumption constant $c^*(s) + h(a^*(s))$ changes
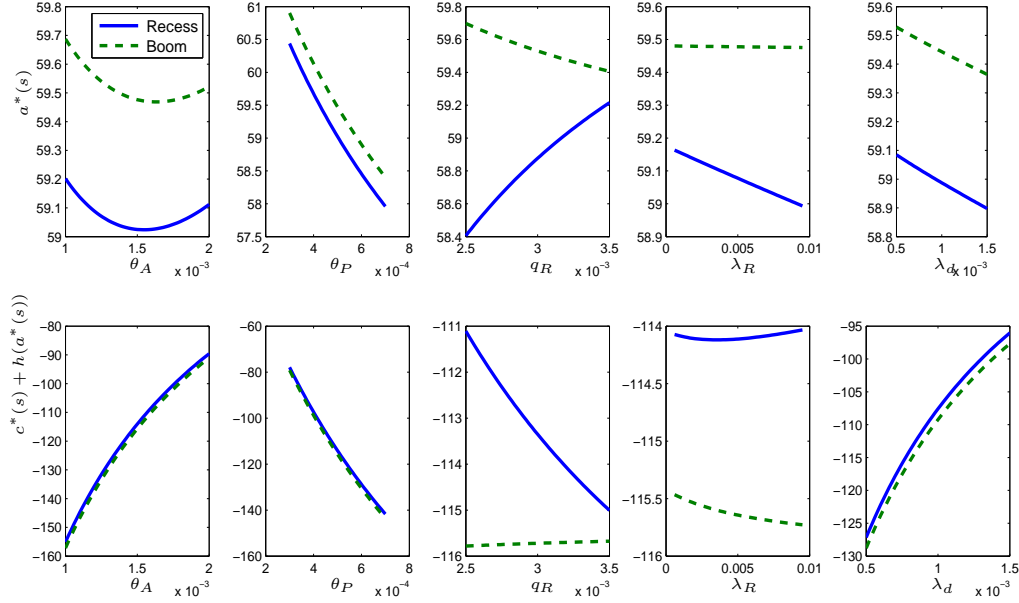
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Figure 4: Comparative Statistics of Effort $a^*(s)$ and Consumption Constant $c^*(s)$.

with respect to parameter changes. Recalling that the consumption takes the form of

$$c(W, u, s) = c^*(s) + h(a^*(s)) - \frac{1}{\theta_A} \log(-W).$$

The consumption constant captures the change in consumption independent of wealth. From the figures, effort and consumption constant are decreasing when the agency becomes more risk averse, as it is costly for the agency to provide incentives. However, the effect of risk coefficient parameter for the worker on effort is non-monotonic. In addition, the consumption constant increases when the worker is more risk averse, as consumption should be higher when the worker is more sensitive to risk. In the case when the worker is more likely to become employed in recessions (higher q_R), the information friction is less severe as both effort and the consumption constant become closer between the recessions and booms. Next, when it is easier to get out of the recession state (higher λ_R), the effects on effort and consumption are small. Last, when it is more likely for the worker to be hit by health shock, the effects are similar in booms and recessions, as health shock could affect the workers both in booms and in recessions.

5   Implementation of the Optimal Contract

In this section, I will show how the optimal contracts under exponential utilities can be implemented via some simple instruments – workers consumption-savings-effort model. In particular, the instruments are (1) allowing workers to save or borrow using a bond, (2) providing flow

payments and lump-sum transfers (or payments), where the interest rates and the amounts paid (transferred) depend on the employment or health status of the agent and the state of the economy. This allows me to gain insights on the properties of the optimal contracts, where the behavior of promised utility can be explained by a workers self-insured actions. As explained in the previous section, the ideas come from Li-Williams [2014], where they consider the implementation of optimal UI with business cycles. The main difference and difficulty come from the indeterminacy of the flow payments. Since the arguments are similar as the ones in Li-Williams [2014], I explain the details in the Appendix C. For the rest of this section, I first layout the model and then demonstrate the comparative analysis.

5.1 A Worker's Consumption-Savings-Effort Problem

I consider an environment where a worker has wealth x_t and has access to a bond with an instantaneous rate of return r^d when disabled, $r^e(s_t)$ when employed, and $r^u(s_t)$ when unemployed. In addition, this worker will receive a constant b^d when disabled, $b^e(s_t)$ when employed, and $b^u(s_t)$ when unemployed. Third, an employed worker receives a lump-sum transfer $B_d^e(s_t)$ if he becomes disabled, $B_u^e(s_t)$ if he becomes unemployed, and $A^e(s_t, x_t)$ when the aggregate economy state switches. An unemployed worker receives $B_d^u(s_t)$ when he becomes disabled, $B_e^u(s_t)$ when he finds a job, and $A^u(s_t, x_t)$ when the aggregate economy state switches.

The idea of implementation is that we allow the workers to self-insure, and the flow payments plus lump-sum transfers induce the correct incentives for the worker to search for the job and not apply for disability if healthy. Hence, the worker needs to decide how much to save (borrow) and consume and the effort level in each period.

In Appendix C, I will show the conditions when consumption-savings-effort model implements the optimal contracts. For the rest of this section, I will focus on demonstrating the implications from this implementation.

5.2 Illustrations

In this section, I will demonstrate the changes of interest rate and lump-sum transfers in response to changes in model parameters. The same parameter values will be used as in the previous section.

In Figure 5, I plot the effective interest rate when unemployed and employed. The effective interest rate is greater than the subjective discount rate $(r^u(s) > \rho)$ when unemployed, but greater than the subjective discount rate $r^e(s) < \rho$ when employed, so the contract provides an interest rate subsidy to the unemployed workers and taxes the employed workers. The subsidy or tax increase when the workers are more risk averse, but the subsidy increases while tax decreases
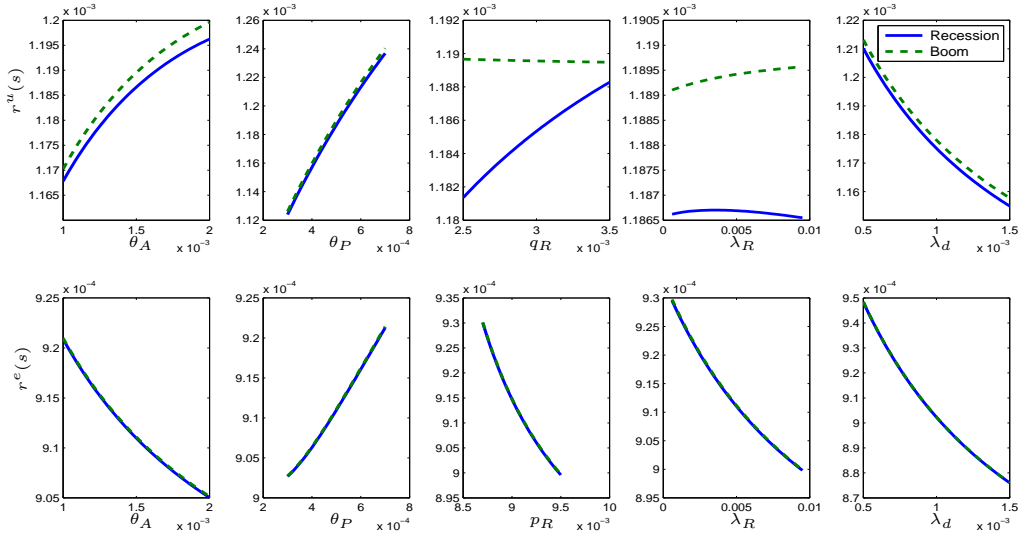
Figure 5: Comparative Statistics of Effective Interest Rate when Unemployed $r(s)$ and the Unemployed Benefits $B^u(s)$.

when the agency is more risk averse. The subsidy increases when it is easier to find the job, and the tax increases when it is easier to get separated from the job. Not surprisingly, subsidy and tax increase when it is easier for the workers to get hit by the health shock.
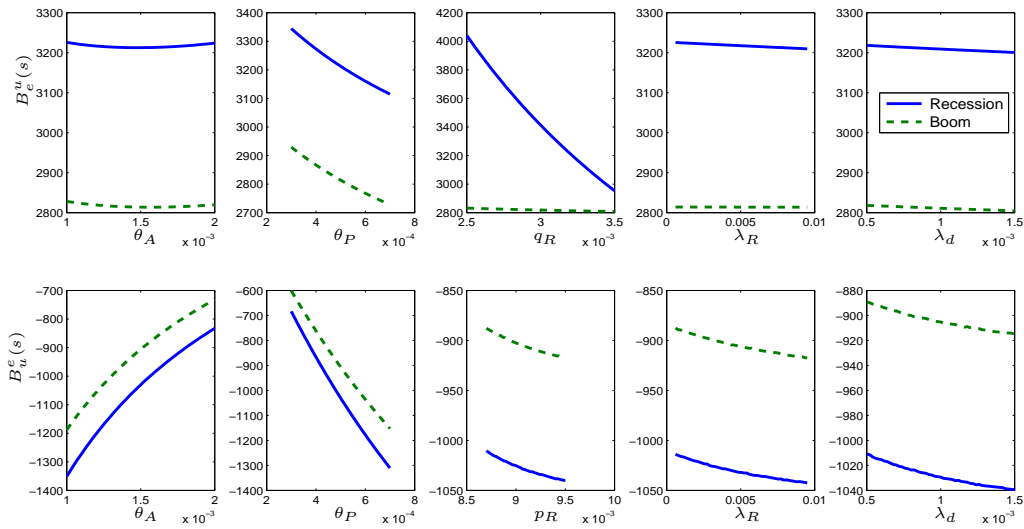

Figure 6: Comparative Statistics of Lump-Sum Transfers of $B^u_e(s)$ and $B^e_u(s)$ when a Worker's Employment Status Changes.

In Figure 6, I plot the lump sum payments $B^u_e(s)$ when the worker finds a job, and the lump

sum payment $B_u^e(s)$ when the worker gets separated from the job. The lump sum payments of $B_e^u(s)$ are positive and larger in recessions than in booms so as to provide incentives. The number also does not drop a lot as the probability of people who become disabled increases. In addition, the lump sum transfer $B_u^e(s)$ when the worker loses his job is negative, reflecting that worker is punished when separated from the job. Also, the transfers when the worker finds the job is higher than when the worker loses his job. This fact induces the worker to search for a job harder as he can then accumulate more of the wealth. This observation also explains the fact that workers with smaller average unemployment duration receive higher disability insurance benefits, as disability insurance provides extra incentives for the workers to search for jobs harder.
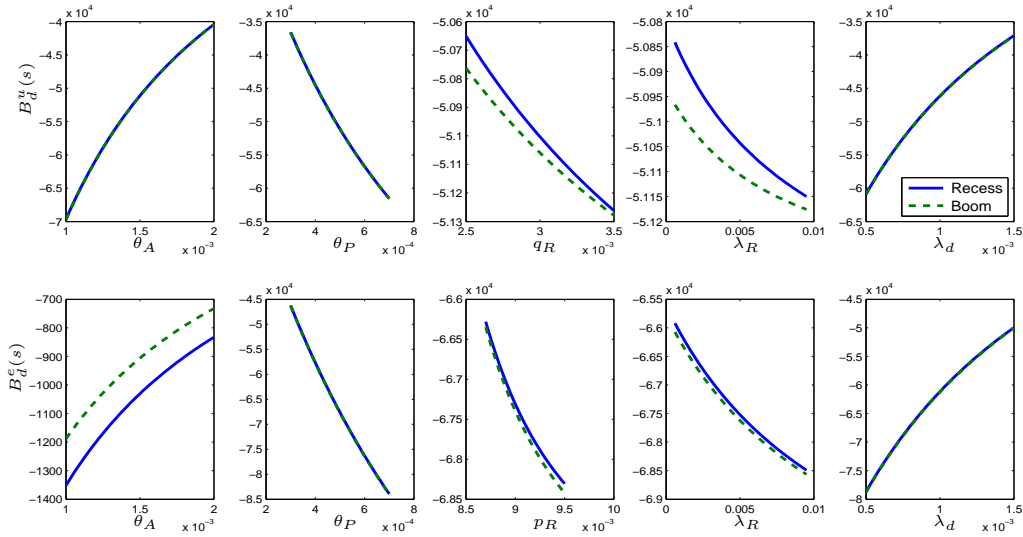

Figure 7: Comparative Statics of Lump-Sum Transfers $B_d^u(s)$, $B_d^e(s)$ when Became Disabled.

In Figure 7, I plot the lump-sum transfers when the worker becomes disabled from unemployed $B_d^u(s)$ and employed $B_d^e(s)$. The lump sum transfers when a worker becomes disabled are negative, meaning that the optimal contracts induce people to truthfully report is to lower the promised utility by a large amount. This negative amount is substantially larger when the probability of becoming disabled is lower. Also, a worker has to pay more when he transitions from employed to disabled, reflecting the fact that this transition is unfavorable to the agency in the optimal system.

6   A Quantitative Example

Although previous sections provide useful insights on the dynamics of the model and how it can be implemented, the assumptions in the previous example are significantly different from

what is generally used in the empirical literature. In this section, I will study the quantitative implications under a more complete and standard model, which allows for job separation.

Throughout this section, I will assume that the utility function of the worker takes the following form:

$$u(c, a) = \frac{c^{1-\gamma}}{1-\gamma} - \frac{a^{1+\phi}}{1+\phi}.$$

In addition, I will assume risk neutral agency where $v(c) = c$.

6.1 The Benchmark Contract and Cost of the Current System

I first consider a stylized version of the current system, which can be used to calibrate the model and measure the effects of switching to the optimal insurance system. I will call this stylized version, "the benchmark contract", where a worker receives constant unemployment benefits c^B for a fixed length of time and constant disability benefits c^d. Furthermore, I assume that the duration of the unemployment benefits is state dependent, wherein the worker can receive T_R periods in recessions and T_B periods in booms, where $T_B < T_R$.

The actual DI application process consists of several steps. First, the worker cannot earn more than a so-called "substantial gainful amount" and has a medical disability preventing him from working. Second , the worker has to apply for disability insurance and it takes at least three to five months before the awards are granted. In order to capture the complexities of the disability insurance screening process, I make the following assumptions. First, the worker has made the choice to apply for DI. Second, a worker with disability will be awarded the benefits, but a healthy worker will be accepted with probability π_d. Last, once being rejected, the worker cannot receive unemployment insurance or apply for DI with the same reason again unless being hit by the health shock or being employed. The last assumption is intended to capture the opportunity cost in reality when workers decide whether to apply for DI or not.

The detail derivations on solutions to the benchmark contract will be discussed in Appendix D. In the appendix, I will show that the utility level of a worker under the benchmark contract by solving the differential equations, and then replace the differential terms on those utilities that fall below the utility when workers become disabled. In addition, based on the status and choices of the workers, I will show how to calculate the corresponding cost to the agency of the benchmark contract.

6.2 Data and Calibration

The model period is one week. First, I will fix a few parameters following the literature. Following Hyponhayn and Nicolini [1997], the risk aversion is set to be $\gamma = 0.5$, and the weekly
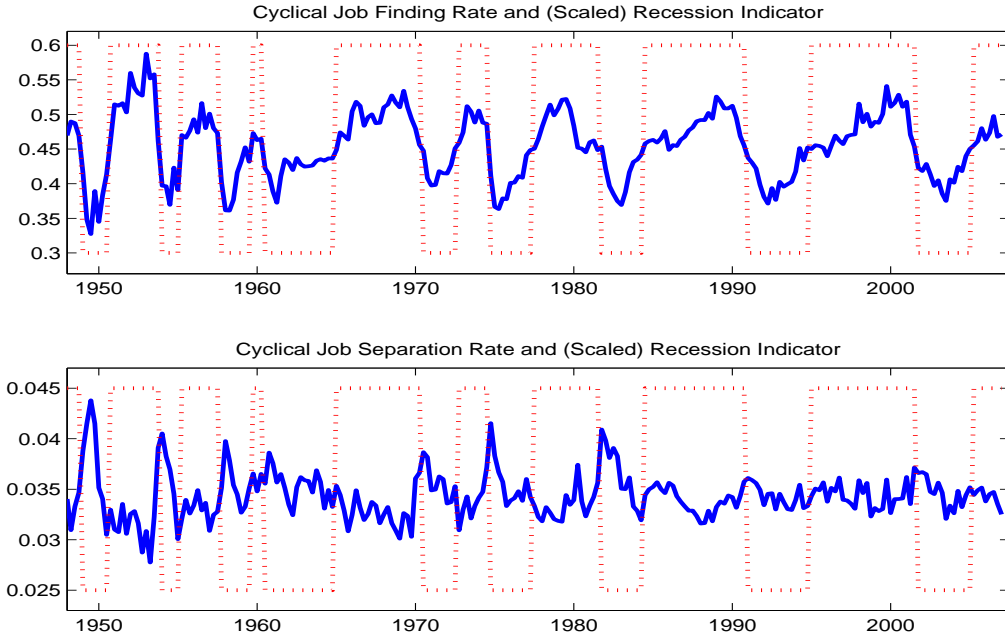
Figure 8: Cyclical job finding and unemployment rates, with estimated recession indicator.

discount rate is set to be $\rho = 0.001$, which corresponds an annual discount rate of 5%. Next, I will set the weekly wage to be $\omega = 495$, which corresponds to the median annual wage \$25,737 in 2007. In addition, I set the constant in the job finding rate to be $q_{s0} = 10^{-5}$, which prevents some singularity problems but has no impact on the main results. Also, I will set the maximum consumption that can be allocated to a worker as equal to wage: $\bar{c} = \omega$. This means that the choice set of the consumption is $[0, 2\omega]$. As for the health shock, I will set it as $\lambda_d = 0.0008$, which is taken from Low and Pistaferri (2014), and the acceptance rate for the healthy worker is 0.5, which is taken from Kitao (2014).

For the benchmark contract, I will set $T_B = 26$ weeks in booms, which is the average duration across U.S. states, and $T_R = 39$ in recessions, which corresponds to the regular federal extended unemployment benefits program. The replacement ratios for unemployed workers will be set to be $c^b = 0.47\omega$, which is consistent with the 47% average replacement ratio in U.S. in 2009. And the replacement ratio for workers with disability is set to be 33%, which is the average replacement ratio for a 40-year old worker in 2007 with median wage.

As for the Markov process for aggregate states and the corresponding job finding and separation rates, I will estimate a two-state Markov-switching process using the data from Shimer [2012]. This data set contains the quarterly averages of monthly job findings and separation rates from 1948Q1 to 2007Q1. I will focus on the job finding rates, as Shimer emphasized that

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the cyclicality in the data comes primarily from the job finding rates. In order to focus on the cyclicality components, I will first use the Hodrick-Prescott filter to remove the low-frequency trend from the job finding rate data. Then, I will estimate the two-state Markov-switching model, following the approach of Hamilton (1989). That is, the H-P filtered job finding rates (f_t) are estimated by

$$f_t = m_{s_t} + \epsilon_t.$$

From the estimation, I find that the mean job finding rates in booms and recessions are $m_B = 0.4875$ and $m_R = 0.4107$, with the transition rates 0.9332 and 0.9107. This gives the aggregate economic switching rates as $\lambda_B = 0.0058$ and $\lambda_R = 0.0078$. In addition, the estimated recession indicator, which is when the smoothed probability of a recession is greater than 0.5, is shown in Figure 8. I read the mean job separation rates in booms and recessions from the H-P filtered data, and this gives $p_B = 0.0086$ and $p_R = 0.0089$.

Last, I calibrate the rest of the parameters by simulating a population of 50,000 workers and computing the average job finding rates as well as the elasticities of unemployment duration with respect to an increase in the unemployment benefits. In this simulation, I assume workers start at age 40 and will work 25 years until they retire at age 65. For the elasticities, the typical range of estimates is between 0.5 and 1 (Landais et al. [2012], Chetty [2008]), and I will target the value at the middle of the rate at 0.7. This gives $q_{B1} = 0.0037$, $q_{R1} = 0.0032$, and effort cost function parameter $\phi = 0.145$.

6.3 Quantitative Implications

In this section, I demonstrate the quantitative implications of the optimal contracts. First, I will show the properties of the optimal contract. Then, I compare the differences between the benchmark system and the optimal contract, focusing especially on the cost reductions from adopting the optimal contracts. Last, I consider impacts of optimal contract on low-income workers and impacts of different policy reforms.

6.3.1 Characterizations of the Optimal Contracts

I will first demonstrate the properties of the optimal contract. Most of the results in the solvable special case are the same when I consider the full model with CRRA utility functions. This implies that the intuition from the solvable case can be applied in this quantitative example. The selected figures confirm this statement.

In Figure 9, I plot consumption when unemployed and upon finding a job or becoming disabled over an unemployment spell, in the full information case and the private information case.
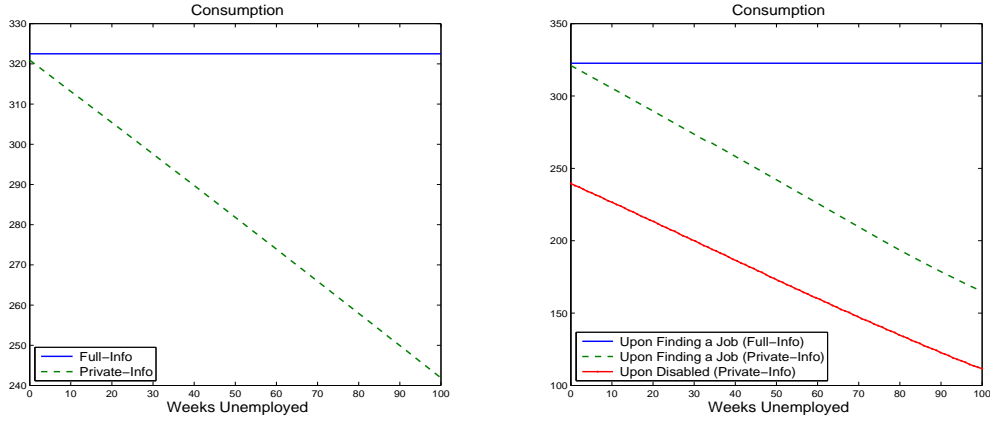
Figure 9: Consumption over an Unemployment Spell in a Recession under Full Information and Private Information.

Consumption is constant in all states under full information, but consumption when unemployed decreases over the unemployment spell under private information. Consumption upon finding a job or becoming disabled decreases over the unemployment spell. The intuition is transparent: in order to provide enough incentives, the consumption decreases over the unemployment spell as the unemployment state is the unfavorable state to the agency. At the same time, the agency does not want to punish the workers who are unlucky even though they put forth a lot of effort to search for a job. In addition, the agency will decrease consumption upon finding a job or becoming disabled over the unemployment spell because it does not want to give workers an incentive to stay in the unemployment status.
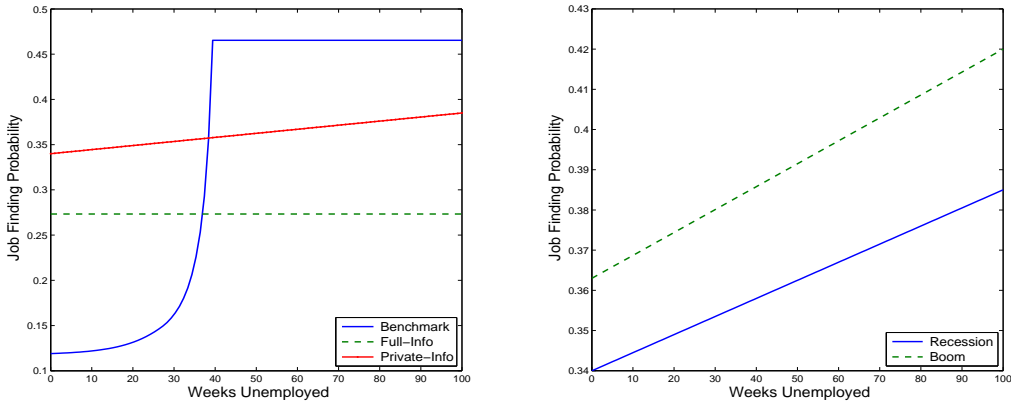

Figure 10: The Job Finding Rate over an Unemployment Spell.

Next, I show how the efforts differ between the benchmark model and the optimal contract. In Figure 10, I plot the job finding rates over an unemployment spell for the benchmark model

and optimal contracts under different information structures, as well as the job finding rates in booms and in recessions. Under full information, the job finding rate is constant over time, but the finding rate increases over the unemployment spell under the private information case. This reflects the impact of the incentives in the optimal contract. In addition, I can see that job finding rates are higher in booms than in recessions, but the patterns are parallel. The intuition behind these results is that it is more efficient to search for a job in booms and a higher periodicity of search gives a higher rate for the same search effort. Hence, it is easier for the agency to give incentives to workers in booms. As a result, the workers search harder when the economy is better. On the other hand, the graph shows that the incentive impact in booms and recessions functions in a similar way, even given these differences.

Last, I would like to discuss how DI affects workers search incentives. As shown in the implementation section, DI benefits increase upon finding a job and decrease upon losing a job. Also, DI benefits decrease over the unemployment spells. This indicates that workers with smaller average unemployment duration receive higher disability insurance benefits. In other words, the existence of DI gives workers more incentive to search for a job harder in order to obtain potential higher disability insurance benefits in the future.

6.3.2 Cost Reductions from the Optimal System

In this section, I will show the cost reductions and the potential gains of switching from the current system to the optimal system. Since I assume that only the agency has the techniques to transfer resources across time and that the worker cannot save or borrow, the potential cost savings may be considered as an upper bound on savings. In addition, switching and the comparisons are calculated when the benchmark contract and the optimal contract give the workers the same ex-ante utility. It means that the workers are indifferent between choosing the current system and the optimal system, and we can compare the difference in the government expenditure by implementing a different system.

First, I will show the summary statistics from simulations. In Table 1, I list the unemployment rates, unemployment durations, finding rates, and separation rates. Although I do not match the unemployment rate, the simulations from the benchmark contract match the data quite well, but the unemployment durations are different between the simulations and the data. The main reason is that I match the moments of job finding rates in booms and recessions, and it pins down the unemployment durations because there is not heterogeneity between workers. This problem can be fixed if we simulate different cohorts of workers. As in the optimal system, the unemployment rate drops around 40% and unemployed durations drop more than 50% as workers search harder in the optimal system.

	Benchmark		Optimal		Data	
	Boom	Recess	Boom	Recess	Boom	Recess
Unemp. Rates (%)	4.8	6.5	3.0	4.1	4.8	6.4
Unemp. Duration (weeks)	5.8	7.8	2.3	3.3	12.3	14.9
Finding Rate (months)	0.60	0.51	0.82	0.71	0.60	0.51
Sep. Rate (months)	0.033	0.035	0.033	0.035	0.033	0.035

Table 1: Summary Statistics from Simulations of the Benchmark Contract, the Optimal Contract and Data.

Government Expenditure (per week per worker)		
	Benchmark	Optimal
Avg Workers	73.3	-29.9
Savings (Cost Reductions)	–	140 %
By State		
Unemp (B)	231.8	154.4
Unemp (R)	232.4	234.3
Emp (B)	0	-282.7
Emp (R)	0	-228.7
Disabled	173.4	288.2

Table 2: Cost Comparisons between the Benchmark Contract and the Optimal Contract, Wage = 495/ week.

Next, I will show the potential gains of switching from the current system to the optimal system. As explained above, I consider that both the benchmark contract and the optimal contract will give the workers the same ex-ante utility, which means that the worker is indifferent between choosing the benchmark model and the optimal contract. Then, I compare the costs to the insurance agency. As in Table 2, the potential gains for the agency are substantial, around 140%. The agency actually collects benefits from the high-income workers.

In order to better understand where the cost savings come from, I break down the cost reductions by states. As in the optimal case, the workers pay big taxes when employed and receive higher disability benefits while receiving lower unemployment benefits in the optimal contracts than they do under the benchmark contract. This implies that the incentive problems in the benchmark contract can be handled in a different way without making the workers worse off, but cost reductions to the agency are still substantial. In addition, cost savings come mainly

from taxing the employed workers and there are more people working in the optimal system.

As mentioned earlier, the substantial cost reduction implies that the workers are willing to endure a big tax rate under the optimal system. As we can observe from the table 2, the healthy worker gives up around 250 dollars when employed and 100 dollars when unemployed so that he gets a compensation of 100 dollars when he is disabled. Although it seems less likely, one of the property of the optimal contract is the "immiseration" result shown in Thomas and Worrall [1990] and Atkeson and Lucas [1992], which implies that the agent's promised utility tends to minus infinity under the optimal contract. When the government considers the average consumption over a long period of time, a big reduction occurs because the promised utility tends towards the region where the incentive is less costly to provide.

6.4 Impact of the Optimal Contracts on Low-Income Workers

In this section, the impact of the optimal contracts on low-income workers will be shown. The motivation of this exercise comes from the discussions in Autor and Duggan [2003, 2006]. One reason for the increase in the DI applications is the rising replacement ratio of DI benefits, as workers have financial incentives to apply for DI if the replacement rate is high. I will show the results if the government implements the optimal system.

First, I will calculate the replacement ratios for disabled workers with different incomes. The benefits that a disabled worker can receive are calculated based on the Primary Insurance Amount formula (PIA), which uses the Average Indexed Monthly Earnings (AIME) and is progressive. The PIA formula is as follows:

$$
\text{PIA} = \begin{cases} 0.9 \times \text{AIME} & \text{if} \quad \text{AIME} \in [0, b_1] \\ 0.9 \times b_1 + 0.32 \times (\text{AIME} - b_1) & \text{if} \quad \text{AIME} \in [b_1, b_2] \\ 0.9 \times b_1 + 0.32 \times (b_2 - b_1) + 0.15 \times (\text{AIME} - b_2) & \text{if} \quad \text{AIME} > b_2, \end{cases}
$$

where Bend points (b_1, b_2) are rescaled each year by average growth in the economy.

In order to show and derive the replacement ratios for different disabled workers, I use data from all 40-year old workers in 2007 and divide them into five groups according to their wages and calculate the ratios using the formula above. The ratios for workers in different quintiles of wages are:

Earnings Percentile	0-20	20-40	40-60	60-80	80-100
Replacement Ratio	0.62	0.43	0.33	0.27	0.21

Table 3: Potential DI Income as % of Current Earnings for a 40-year Old Worker in 2007.

In Table 3, the replacement ratios range from 62% to 21% for low-income workers to highp-income workers. In this exercise, I will simulate the workers when the replacement ratio for DI is 62% and weekly wage equals $233. In addition, since it is more likely for the low income workers to apply for disability insurance, I will use the intensity of health shock as $\lambda_d = 0.004$ as estimated in Kitao (2014). Then I will compare the difference between the benchmark contract and the optimal contract.
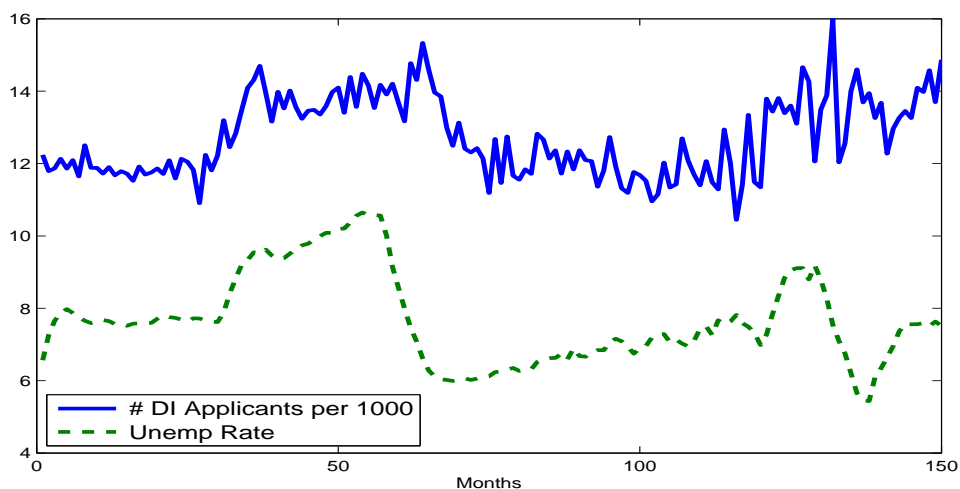

Figure 11: Cyclicality Property of the Benchmark Contract

Before showing the results, I will show the cyclicality property of the benchmark contract, as healthy workers have incentives to apply for DI in recessions after a long unemployment spell. In Figure 11, I demonstrate that cyclicality property can be observed. This cyclicality comes from the following two reasons: (1) workers on average are unemployed longer in recessions, and (2) utility when disabled is relatively higher when workers almost exhaust their UI benefits. Although I do not plot the results from the optimal contracts, it can be easily inferred that DI applications would look like a flat line since an optimal system induces healthy workers not to apply for DI.

In Table 4, I show the results of low-income workers in the benchmark system and the optimal system. First, unemployment rate is higher for low-income workers in the benchmark system, and the unemployment rate drops larger when the government implements the optimal system. In recessions, the unemployment rate drops from 9.6% in the benchmark system to 3.8% in the optimal system, reflecting that not only do workers search harder in the optimal system but healthy workers would not apply for disability.

As for the government expenditure, we can observe that the cost savings are smaller compared to the representative agents case and are around 3.6%. As we can observe in Table 4, low-income workers receive higher unemployment benefits and higher disability benefits. At the same time,

	Benchmark		Optimal	
	Boom	Recess	Boom	Recess
Unemp. Rates (%)	7.2	9.6	3.2	3.8
Unemp. Duration (week)	9.2	11.7	4.7	5.5

Government Expenditure (per week per worker)

	Benchmark	Optimal
Avg Workers	122.9	118.46
Savings	–	3.6%
Unemp (B)	108.8	158.5
Unemp (R)	108.0	135.9
Emp (B)	0	-80.0
Emp (R)	0	-103.9
Disabled	140.4	155.9

Table 4: Results for Lowest Quintile, Wage = 233/week.

there are more low-income workers as the shocks arrive at a higher intensity.

6.5   Impact of Different Policy Reforms

Last, let me consider the impacts of different policy reforms. The first exercise is motivated by the extended UI benefits during the Great Recession, when workers could receive up to 99 weeks of unemployment insurance. The second exercise is intended to show understand the interdependence of the unemployment insurance and disability insurance. As explained above, the existence of disability insurance provides the workers with extra search incentives, and I would like to show how much the search intensity can be provided by using the disability insurance alone.

For the first exercise, I will simulate the benchmark model with $T_R = 99$. This means that workers in recessions can receive the unemployment insurance up to 99 weeks, and I will compare the results to the case where $T_R = 39$, as workers can receive up to 39 weeks of unemployment insurance benefits in recessions. The results are shown in Table 5, where we can observe that differences are small in terms of unemployment rate and government expenditure between the system with 39 weeks of unemployment insurance benefits in recessions to the system and between the system with 99 weeks of UI in recessions. Although workers search slightly harder in recessions under the system of 39 weeks, most workers find a job before they exhaust their unemployment insurance. This result is similar to Rothstein [2011], and I further show that the government

	Benchmark ($T_R = 39$)		$T_R = 99$	
	Boom	Recess	Boom	Recess
Unemp. Rates (%)	4.8	6.5	4.7	6.7
Unemp. Duration (week)	5.8	7.8	5.7	8.1

Government Expenditure (per week per worker)

	Benchmark ($T_R = 39$)	$T_R = 99$
Avg Workers	73.3	73.5
Savings	–	0.3%
Unemp (B)	231.8	231.4
Unemp (R)	231.5	232.7
Emp (B)	0	0
Emp (R)	0	0
Disabled	163.4	163.4

Table 5: Results for Extending UI benefits to 99 Weeks during Recessions.

expenditure is similar between the two systems.

	Benchmark		Fix UI, Optimal DI	
	Boom	Recess	Boom	Recess
Unemp. Rates (%)	4.8	6.5	2.3	3.4
Unemp. Duration (week)	5.8	7.8	2.6	3.9

Government Expenditure (per week per worker)

	Benchmark	Fix UI, Optimal DI
Avg Workers	73.3	43.4
Savings	–	40.8%
Unemp (B)	231.8	219.7
Unemp (R)	232.4	224.6
Emp (B)	0	-165.9
Emp (R)	0	-169.0
Disabled	163.4	324.7

Table 6: Results for Taking UI as Given, Optimal DI.

Last, I will consider the second exercise: government takes the current unemployment in-

surance system as a given but considers the design of the optimal disability insurance. In this exercise, I assume that the government cannot change the unemployment insurance benefits c^b, and the maximum unemployment insurance durations $T_B = 26, T_R = 39$. However, the government can consider the design of optimal disability insurance. By doing this exercise, we can show the interdependence of the two insurance programs as well as how much search efforts can be motivated by the potential future disability insurance benefits.

In Table 6, the unemployment rate drops around 40% and the cost reductions are around 40%. This means that a big proportion of the search effort problem due to the incentives in the current system can be solved even if only the optimal disability insurance is implemented.

7 Conclusion

In this paper, I consider the optimal joint insurance system against unemployment and disability shocks, and I include business cycles. In order to understand the properties of the optimal contract, I consider a solvable special case and demonstrate how the optimal system can be implemented through saving and borrowing behavior with state-switching payments. Last, I consider a stylized version of the current system and evaluate the cost reductions when switching from the current system to the optimal system. I find that the cost reductions from the incentive problems are substantial, while extended UI benefits do not make the incentive problem serious. My results indicate that adjusting the system would have a bigger impact on the unemployment rate, and I would suggest the following policy reform: benefits should be contingent on the history of unemployment, average unemployment duration.

Appendix

Appendix A   Proofs

Appendix A.1   Proof for Proposition 1

Proof. Define

$$\phi_t(c,a) = E_t[\rho \int_0^\infty e^{-\rho s} u(c_s, a_s) ds | \mathbb{F}_t] = \rho \int_0^t e^{-\rho s} u(c_s, a_s) ds + e^{-\rho t} W_t.$$

Since $u(c,a)$ is bounded for the bounded sets of c and a. Also, $\phi_t(c,a)$ is uniform integrable. By the martingale representation theorem, there exist three $\mathbb{F}_t$-predictable square integrable processes g_t^j, g_t^s, g_t^d such that

$$d\phi_t(c,a) = \rho e^{-\rho t} g_t^j dm_t^j + \rho e^{-\rho t} g_t^s dm_t^s + \rho e^{-\rho t} g_t^d dm_t^d.$$

Combining the equations gives the result. □

Appendix A.2   Proof for Proposition 2

Proof. The incentive problems come from three parts: the workers effort is unobservable, employed workers can voluntarily quit their jobs, and healthy people can misreport as disabled. Since the incentive problems are state-contingent, i.e. unemployed workers cannot quit their jobs since they are not employed, I will prove this proposition state by state. I will start by showing the necessity first and then the sufficiency.

Let us first consider the case when the worker is unemployed. Given a contract $(c, a, j = 1, d = 0)$, I define $\phi_t(c, a')$ for some feasible a' and health status d' as

$$\phi_t(c, a') = \rho \int_0^t e^{-\rho s} u(c_s, a_s') ds + e^{-\rho t} W_t, \ \forall t \in [0, \infty]$$

It is clear that for any alternative (a', d'), $\phi_0(c, a') = W_0$. Differentiating the previous equation with respect to t gives:

$$d\phi_t(c, a') = \rho e^{-\rho t} u(c_t, a_t') dt - \rho e^{-\rho t} W_t dt + e^{-\rho t} dW_t$$
$$= \rho e^{-\rho t} [(u(c_t, a_t') - c(c_t, a_t)) dt + g_t^{j'} dm_t^{j'} + g_t^s dm_t^s + g_t^{d'} dm_t^d].$$

Let $dm_t^{j'}$ be the compensated jump martingale with a' and $dm_t^{d'}$ be the martingale associated with d'. Then

$$dm_t^j = ([(1 - s_t) q_B(a_t') + s_t q_R(a_t')] - [(1 - s_t) q_B(a_t) + s_t q_R(a_t)]) dt + dm_t^{j'}$$
$$dm_t^d = \lambda_d dt + dm_t^{d'}.$$

Hence,

$$d\phi_t(c, a') = \rho e^{-\rho t}[(u(c_t, a'_t) - c(c_t, a_t) + g^j_t([(1 - s_t)q_B(a'_t) + s_t q_R(a'_t)]$$

$$- [(1 - s_t)q_B(a_t) + s_t q_R(a_t)])dt + g^d_t \lambda_t)dt + g^j_t dm^{j'}_t + g^s_t dm^s_t + g^d_t dm^{d'}_t].$$

Under a', d', $dm^{j'}_t$, $dm^{d'}_t$ are martingales. Then, the drift term of $\phi_t(c, a')$ has the same sign as

$$[(u(c_t, a'_t) - c(c_t, a_t) + g^j_t([(1 - s_t)q_B(a'_t) + s_t q_R(a'_t)] - [(1 - s_t)q_B(a_t) + s_t q_R(a_t)]).$$

Thus, if

$$a_t \in \arg\max_{\tilde{a}_t} g^j_t q_{s_t}(\tilde{a}_t) + u(c_t, \tilde{a}_t)$$

$$g^d_t \leq 0,$$

$\phi_t(c, a')$ is a sub-martingale, so I have

$$E^{a',d'}[\phi_t(c, a')] > \phi_0(c, a') = W_0,$$

which implies that (a', d') dominates (a, d), hence it is not optimal. The case when the worker is employed can be proved in the similar way. Hence I prove the necessity.

For sufficiency, suppose (c, a, j, d) satisfies the conditions. Then, $\phi_t(c, a')$ is a super-martingale for any other alternative feasible choices. Since c_t, a_t, d_t, j_t are all bounded, I have

$$W_0 = \phi_t(c, a') \geq E[\phi_\infty(c, a')].$$

Hence (c, a, j, d) dominate all other feasible choices. $\qquad\square$

Appendix A.3   The Boundary Points of the Value Functions

Appendix A.3.1

I will show how the left and right boundaries are derived in this appendix. The arguments follow closely those in Li and Williams [2014].

First, the left boundaries are computed by considering the harshest contract that the principal can offer to the agent: the contract will give the agent the minimum level of consumption while forcing the agent to choose the highest effort level. The right boundaries are computed by solving the most generous contract that the agency can offer: giving the highest level of consumption to the agent. I will use $(W^{js}_r, V(W^{js}_r, j, s))$, $(W^d_r, V(W^d_r, d))$ for the right boundaries for the worker and the agency with status j or d and state s. The left boundaries are denoted as $(W^{js}_l, V(W^{js}_l, j, s))$, $(W^d_l, V(W^d_l, d))$. Then I will have the following proposition:

Proposition 4. *The bounds of the promised utility for a worker are $W_l^{js} = W_l^d = \underline{u}$, for $j = E, U$, and $s = B, R$. The corresponding value functions for the insurance agency $V(W_l^{js}, j, s)$, and $V(W_l^d, d)$ are*

$$\rho V(W_l^{us}, u, s) = \lambda_s(V(W_l^{us'}, u, s') - V(W_l^{us}, u, s)) + q_{s0}(V(W_l^{es}, e, s) - V(W_l^{us}, u, s))$$
$$+ \lambda_d(V(W_l^d, d) - V(W_l^{us}, u, s))$$
$$\rho V(W_l^{es}, e, s) = \rho v(\omega) + \lambda_s(V(W_l^{es'}, e, s') - V(W_l^{es}, e, s)) + p_s(V(W_l^{us}, u, s) - V(W_l^{es}, e, s))$$
$$+ \lambda_d(V(W_l^d, d) - V(W_l^{es}, e, s))$$
$$\rho V(W_l^d, d) = 0.$$

The upper bounds of the promised utility for the worker W_r^{js}, W_r^d satisfy the following:

$$\rho W_r^{us} = \max_{a \in [0, \bar{a}]} \rho u(\bar{c}, a) + \lambda_s(W_r^{us'} - W_r^{us}) + q_s(a)(W_r^{es} - W_r^{us}) + \lambda_d(W_r^d - W_r^{us})$$
$$\rho W_r^{es} = \rho u(\bar{c}_\omega) + \lambda_s(W_r^{es'} - W_r^{es}) + p_s(W_r^{us} - W_r^{es}) + \lambda_d(W_r^d - W_r^{es})$$
$$\rho W_r^d = \rho u(\bar{c}),$$

with $V(W_r^{js}, j, s) = V(W_r^d, d) = -\rho v(\bar{c})$.

Proof. Let us start with left boundaries. Since all the proofs are similar, let us focus on the case when the worker is unemployed and in state B. The other left boundaries can be proven in a similar way.

Let $\tau = \tau^j \wedge \tau^s \wedge \tau^d$, where τ^j is the time when the next job arrives, τ^s is the time when the next aggregate state changes, and τ^d is the next health shock hits. For any $\Delta > 0$, since consumption is *zero*, I have

$$V(W_l^{us}, u, s) = e^{-\rho(\Delta \wedge \tau)} \Pr(\tau > \Delta) V(W_l^{us}, u, s) + e^{-\rho(\Delta \wedge \tau)} [\Pr(\tau < \Delta, \tau = \tau^j) V(W_l^{es}, e, s)$$
$$+ \Pr(\tau < \Delta, \tau = \tau^s) V(W_l^{us'}, e, s') + \Pr(\tau < \Delta, \tau = \tau^d) V(W_l^d, d)].$$

Subtracting $V(W_l^{us}, u, s)$ and dividing Δ on both sides, I have

$$0 = e^{-\rho(\Delta \wedge \tau)} e^{-(\lambda_s + q_{s0} + \lambda_d)\Delta} [V(W_l^{us'}, u, s') - V(W^{us}, u, s)]$$
$$+ e^{-\rho(\Delta \wedge \tau)}(1 - e^{-(\lambda_s + q_{s0} + \lambda_d)})[\frac{q_{s0}}{\lambda_s + q_{s0} + \lambda_d} V(W_l^{es}, e, s)$$
$$+ \frac{\lambda_s}{\lambda_s + q_{s0} + \lambda_d} V(W_l^{us'}, e, s') + \frac{\lambda_d}{\lambda_s + q_{s0} + \lambda_d} V(W_l^d, d)].$$

Taking $\Delta \to 0$, I have the result.

Next, I consider the right boundaries. Let us consider the case of the unemployed worker in state s. All the other cases can be proven in a similar way.

The unemployed worker in state s faces the following problem:

$$\max_{a \in A} E_0[\rho \int_0^{\tau \wedge \Delta} e^{-\rho t} u(\bar{c}, a) dt] + e^{-\rho(\Delta \wedge \tau)}[e^{-(\lambda_s + q_s(a_t) + \lambda_d)\Delta} W_r^{us}$$

$$+ (1 - e^{-(\lambda_s + q_s(a_t) + \lambda_d)\Delta})(\frac{\lambda_s}{\lambda_s + q_s(a_t) + \lambda_d} W_r^{us'} + \frac{q_s(a_T)}{\lambda_s + q_s(a_t) + \lambda_d} W_r^{es} + \frac{\lambda_d}{\lambda_s + q_s(a_t) + \lambda_d} W_r^d].$$

The solution to the problem is W_r^{us}. Hence for any $a \in A$, I have:

$$W_r^{us} \geq \max_{a \in A} E_0[\rho \int_0^{\tau \wedge \Delta} e^{-\rho t} u(\bar{c}, a) dt] + e^{-\rho(\Delta \wedge \tau)}[e^{-(\lambda_s + q_s(a_t) + \lambda_d)\Delta} W_r^{us}$$

$$+ (1 - e^{-(\lambda_s + q_s(a_t) + \lambda_d)\Delta})(\frac{\lambda_s}{\lambda_s + q_s(a_t) + \lambda_d} W_r^{us'} + \frac{q_s(a_T)}{\lambda_s + q_s(a_t) + \lambda_d} W_r^{es} + \frac{\lambda_d}{\lambda_s + q_s(a_t) + \lambda_d} W_r^d].$$

Subtracting $V(W_l^{us}, u, s)$, dividing Δ on both sides, and taking $\Delta \to 0$, I have the result. $\square$

Appendix A.4 Proof for Proposition 3

Proof. I will show the HJB equation when the economy is in a boom period and the worker is unemployed. Other HJB equations can be proven in a similar manner.

Assuming at time t, the promised utility of the worker is $W_t = W$. Let c be the consumption process, W^j be the adjusted utility when the employment status changes, W^s be the adjusted utility when the aggregate state changes, and W^d be the adjusted utility when the health shock hits. Let τ^j, τ^s, and τ^d be the stopping time when the next job, aggregate state changes, and health shock arrive respectively. Defining $\tau = \tau^j \wedge \tau^s \wedge \tau^d$. At time t and for small interval of time Δ, I have

$$V(W_t, u, B) \geq E_t[\rho \int_0^{(t+\Delta)\wedge\tau} e^{-rhos} v(c_s) ds | \mathbb{F}_{t-}] + e^{-\rho((t+\Delta)\wedge\tau)} \Pr(\tau > t + \Delta | a) V(W_{t+\Delta}, u, B)$$

$$+ e^{-\rho((t+\Delta)\wedge\tau)} \Pr(\tau < t + \Delta, \tau = \tau^j | a) V(W_{t+\Delta}^j, e, B)$$

$$+ e^{-\rho((t+\Delta)\wedge\tau)} \Pr(\tau < t + \Delta, \tau = \tau^s | a) V(W_{t+\Delta}^s, u, R)$$

$$+ e^{-\rho((t+\Delta)\wedge\tau)} \Pr(\tau < t + \Delta, \tau = \tau^d | a) V(W_{t+\Delta}^d, d).$$

Let Δ be small enough such that everything is well-defined, and effort is constant in the interval. Subtracting $V(W_t, u, G)$ on both sides and dividing both sides by Δ, I have

$$0 \geq -\frac{1}{\Delta} E_t[\rho \int_0^{(t+\Delta)\wedge\tau} e^{-rhos} v(c_s) ds | \mathbb{F}_{t-}] + \frac{1}{\Delta} e^{-\rho((t+\Delta)\wedge\tau)} e^{-(\lambda_B + q_B(a^*(W_t^J, W_t)) + \lambda_d)} [V(W_{t+\Delta}, u, B) - V(W_t, u, B)]$$

$$+ e^{-\rho((t+\Delta)\wedge\tau)} (1 - e^{-(\lambda_B + q_B(a^*(W_t^J, W_t)) + \lambda_d)}) \frac{q_B(a^*(W_t^j, W_t))}{\lambda_B + q_B(a^*(W_t^J, W_t)) + \lambda_d} V(W_{t+\Delta}^j, e, B)$$

$$+ e^{-\rho((t+\Delta)\wedge\tau)} (1 - e^{-(\lambda_B + q_B(a^*(W_t^J, W_t)) + \lambda_d)}) \frac{\lambda_B}{\lambda_B + q_B(a^*(W_t^J, W_t)) + \lambda_d} V(W_{t+\Delta}^s, u, R)$$

$$+ e^{-\rho((t+\Delta)\wedge\tau)} (1 - e^{-(\lambda_B + q_B(a^*(W_t^J, W_t)) + \lambda_d)}) \frac{\lambda_d}{\lambda_B + q_B(a^*(W_t^J, W_t)) + \lambda_d} V(W_{t+\Delta}^d, d).$$

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Let $\Delta \to 0$, and I have

$$0 \geq -\rho(v(c_t)$$

$$+ V(W_t, u, B)) + V_W(W_t, u, B)\rho[W_t - u(c_t, a_t) - q_B(a(W_t^j, W_t))\frac{W_t^j - W_t}{\rho} - \lambda_B\frac{W_t^s - W_t}{\rho} - \lambda_d\frac{W_t^d - W_t}{\rho}]$$

$$+ q_B(a(W_t^j, W_t))[V(W, e, B) - V(W, u, B)] + \lambda_s[V(W^s, u, R) - V(W, u, B)] + \lambda_d[V(W^d, d) - V(W, u, s)].$$

Since c_t, a_t, W_t^j, W_t^s, W_t^d are chosen optimally, the inequality holds with equality. $\qquad\square$

Appendix B   Calculations for the Solvable Case

Appendix B.1   Disabled Worker

Let us guess that the form of the value function for the agency is:

$$V(W, d) = -(-W)^{-\frac{\theta_P}{\theta_A}}.$$

The FOC for the consumption gives

$$-\theta_P \exp(\theta_P * c) = -\frac{\theta_P}{\theta_A}(-W)^{-\frac{\theta_P + \theta_A}{\theta_A}}\theta_A \exp(-\theta_A c)$$

$$\Rightarrow \theta_P * c = -\theta_A * c - \frac{\theta_P + \theta_A}{\theta_A}\log(-W)$$

$$\Rightarrow c = -\frac{1}{\theta_A}\log(-W)$$

Plugging everything back into the HJB equation, I can see that all terms cancel out, which verifies the guess is correct.

Appendix B.2   Full Information

Let me start with the employed workers. I conjecture that the form for the value function is:

$$V(W, e, s) = -V_e(s)(-W)^{-\frac{\theta_P}{\theta_A}}.$$

Using the conjectured form, the slope matching conditions gives

$$W^j = \left(\frac{V_e(s)}{V_u(s)}\right)^{-\frac{\theta_A}{\theta_P + \theta_A}} W$$

$$W^s = \left(\frac{V_e(s)}{V_e(s')}\right)^{-\frac{\theta_A}{\theta_P + \theta_A}} W$$

$$W^d = V_e(s)^{-\frac{\theta_A}{\theta_P + \theta_A}} W.$$

Hence I have the following:

$$V(W^j, u, s) - V(W, e, s) = [-V_e(s)^{\frac{-\theta_P}{\theta_P + \theta_A}} V_u(s)^{\frac{-\theta_A}{\theta_P + \theta_A}} + V_e(s)](-W)^{-\frac{\theta_P}{\theta_A}}$$

$$V(W^s, e, s') - V(W, e, s) = [-V_e(s)^{\frac{-\theta_P}{\theta_P + \theta_A}} V_e(s')^{\frac{-\theta_A}{\theta_P + \theta_A}} + V_e(s)](-W)^{-\frac{\theta_P}{\theta_A}}$$

$$V(W^d, d) - V(W, e, s) = [-V_e(s)^{\frac{-\theta_P}{\theta_P + \theta_A}} + V_e(s)](-W)^{-\frac{\theta_P}{\theta_A}}.$$

Next, the optimality conditions for c gives:

$$c = \frac{1}{\theta_P + \theta_A} \log(V_e(s)) - \frac{1}{\theta_A} \log(-W) + \frac{\theta_A}{\theta_P + \theta_A}\omega$$

Using the optimality condition for c and slope matching conditions, plugging them into the HJB equation, and canceling the W term gives

$$V_e(s)^{-\frac{\theta_A}{\theta_P + \theta_A}} \left[\rho \exp(\frac{\theta_P \theta_A}{\theta_P + \theta_A}\omega) + p_s V_u(s)^{-\frac{\theta_A}{\theta_P + \theta_A}} + \lambda_s V_e(s')^{-\frac{\theta_A}{\theta_P + \theta_A}} + \lambda_d\right] = (\rho + p_s + \lambda_s + \lambda_d),$$

Then I turn to the problems for unemployed workers. I conjecture that the form for the value function is:

$$V(W, u, s) = -V_u(s)(-W)^{-\frac{\theta_P}{\theta_A}}.$$

Using the conjectured form, the slope matching conditions gives

$$W^j = \left(\frac{V_u(s)}{V_e(s)}\right)^{-\frac{\theta_A}{\theta_P + \theta_A}} W$$

$$W^s = \left(\frac{V_u(s)}{V_u(s')}\right)^{-\frac{\theta_A}{\theta_P + \theta_A}} W$$

$$W^d = V_u(s)^{-\frac{\theta_A}{\theta_P + \theta_A}} W.$$

Hence I have the following:

$$V(W^j, e, s) - V(W, u, s) = [-V_u(s)^{\frac{-\theta_P}{\theta_P + \theta_A}} V_e(s)^{\frac{-\theta_A}{\theta_P + \theta_A}} + V_u(s)](-W)^{-\frac{\theta_P}{\theta_A}}$$

$$V(W^s, u, s') - V(W, u, s) = [-V_u(s)^{\frac{-\theta_P}{\theta_P + \theta_A}} V_u(s')^{\frac{-\theta_A}{\theta_P + \theta_A}} + V_u(s)](-W)^{-\frac{\theta_P}{\theta_A}}$$

$$V(W^d, d) - V(W, u, s) = [-V_u(s)^{\frac{-\theta_P}{\theta_P + \theta_A}} + V_u(s)](-W)^{-\frac{\theta_P}{\theta_A}}.$$

Next, the optimality conditions for c and a gives:

$$c = \frac{1}{\theta_P + \theta_A} \log(V_u(s)) - \frac{1}{\theta_A} \log(-W) + \frac{\theta_A}{\theta_P + \theta_A}h(a)$$

$$\rho(-\frac{\theta_P}{\theta_A}V_u(s)(-W)^{-\frac{\theta_P + \theta_A}{\theta_A}})(-\theta_A e^{-\theta_A(c-h(a))}h'(a) + \frac{q_s}{\rho}(W^j - W)) = q_s(V(W^j, e, s) - V(W, u, s))$$

Using the optimality condition for c and slope matching conditions, plugging them into the HJB equation, and canceling the W term gives

$$V_u(s)^{-\frac{\theta_A}{\theta_P + \theta_A}} \left[\rho \exp(\frac{\theta_P \theta_A}{\theta_P + \theta_A}h(\bar{a}(s))) + q_s \bar{a}(s) V_e(s)^{-\frac{\theta_A}{\theta_P + \theta_A}} + \lambda_s V_u(s')^{-\frac{\theta_A}{\theta_P + \theta_A}} + \lambda_d\right] = (\rho + q_s \bar{a}(s) + \lambda_s + \lambda_d),$$

In sum, $\{\bar{a}(s), V_u(s), V_e(s)\}$ can be solved by the following three equations:

$$V_u(s)^{-\frac{\theta_A}{\theta_P+\theta_A}}\left[\rho\theta_P\exp(\frac{\theta_P\theta_A}{\theta_P+\theta_A}h(\bar{a}(s)))h'(\bar{a}(s)) + (1+\frac{\theta_P}{\theta_A})q_sVe^{-\frac{\theta_A}{\theta_P+\theta_A}}\right] = q_s(1+\frac{\theta_P}{\theta_A}),$$

$$V_u(s)^{-\frac{\theta_A}{\theta_P+\theta_A}}\left[\rho\exp(\frac{\theta_P\theta_A}{\theta_P+\theta_A}h(\bar{a}(s))) + q_s\bar{a}(s)Ve^{-\frac{\theta_A}{\theta_P+\theta_A}} + \lambda_sV_u(s')^{-\frac{\theta_A}{\theta_P+\theta_A}} + \lambda_d\right] = (\rho + q_s\bar{a}(s) + \lambda_s + \lambda_d),$$

$$V_e(s)^{-\frac{\theta_A}{\theta_P+\theta_A}}\left[\rho\exp(\frac{\theta_P\theta_A}{\theta_P+\theta_A}\omega) + p_sV_u(s)^{-\frac{\theta_A}{\theta_P+\theta_A}} + \lambda_sV_e(s')^{-\frac{\theta_A}{\theta_P+\theta_A}} + \lambda_d\right] = (\rho + p_s + \lambda_s + \lambda_d).$$

Appendix B.3   Private Information

The solution to the employed workers problem is the same for the full information case and the private information case. Now I will turn to the problem for the unemployed worker. The goal is to show a system of equations that jointly determine the parameters.

Let us guess that the value function has the form:

$$V(W, u, s) = -V_u^*(s)(-W)^{-\frac{\theta_P}{\theta_A}},$$

and the consumption takes the form of:

$$c(W, u, s) = c^*(s) + h(a^*(s)) - \frac{1}{\theta_A}\log(-W).$$

Using the conjectured forms, I have the following results:

$$v(c) = \exp(\theta_P(c^*(s) + h(a^*(s))))(-W)^{-\frac{\theta_P}{\theta_A}}$$

$$u(c, a^*(s)) = -\exp(-\theta_Ac^*(s))(-W)$$

$$u_c(c, a^*(s)) = \theta_A\exp(-\theta_Ac^*(s))(-W)$$

$$u_a(c, a^*(s)) = -\theta_Ah'(a^*(s))\exp(-\theta_Ac^*(s))(-W)$$

$$u_{aa}(c, a^*(s)) = -[\theta_A^2h'(a^*(s))^2 + \theta_Ah''(a^*(s))]\exp(-\theta_Ac^*(s))(-W)$$

$$u_{ac}(c, a^*(s)) = \theta_A^2h'(a^*(s))\exp(-\theta_Ac^*(s))(-W)$$

Next, the incentive compatible constraint for the effort gives:

$$-u_a(c, a) = q_s\frac{W^j - W}{\rho}.$$

Using the above equation, it is easier to use the controls of W^j. Straightforward but tedious calculations show that using the optimality conditions for c and a as Ill as the HJB equation, I can solve the eight unknowns $\{a^*(s), V_u^*(s), c^*(s), V_e^*(s), s \in \{B, R\}\}$ by the following system of

eight equations:

$$\exp((\theta_P + \theta_A)c^*(s) + \theta_P h(a^*(s))) = V_u^*(s)(1 - a^*(s)\theta_A h'(a^*(s)))$$

$$+ a^*(s)\theta_A h'(a^*(s))V_e^*(s)\left(1 - \frac{\rho}{q_s}\theta_A h'(a^*(s))\exp(-\theta_A c^*(s))\right)^{-\frac{\theta_P+\theta_A}{\theta_A}} \tag{2}$$

$$a^*(s)\theta_P(\theta_A h'(a^*(s))^2 + h''(a^*(s)))(-V_u^*(s) + V_e^*(s)\left(1 - \frac{\rho}{q_s}\theta_A h'(a^*(s))\exp(-\theta_A c^*(s))\right)^{-\frac{\theta_P+\theta_A}{\theta_A}})$$

$$= \exp(\theta_A c^*(s))\frac{q_s}{\rho}(V_u^*(s) - V_e^*(s)\left(1 - \frac{\rho}{q_s}\theta_A h'(a^*(s))\exp(-\theta_A c^*(s))\right)^{-\frac{\theta_P}{\theta_A}}) \tag{3}$$

$$0 = (\rho + q_s a^*(s) + \lambda_s + \lambda_d)V_u^*(s) - \rho\exp((\theta_P + \theta_A)c^*(s)) + \rho\frac{\theta_P}{\theta_A}V_u^*(s)\left(1 - \exp(-\theta_A c^*(s))\right)$$

$$+ a\theta_A \exp(-\theta_A c^*(s))h'(a^*(s)) - \frac{\lambda_s}{\rho}\left(\left(\frac{V_u^*(s)}{V_u^*(s')}\right)^{-\frac{\theta_A}{\theta_P+\theta_A}} - 1\right) - \frac{\lambda_d}{\rho}\left((V_u^*(s))^{-\frac{\theta_A}{\theta_P+\theta_A}} - 1\right))$$

$$- q_s a^*(s)V_e^*(s)\left(1 - \frac{\rho}{q_s}\theta_A h'(a^*(s))\exp(-\theta_A c^*(s))\right)^{-\frac{\theta_P}{\theta_A}}$$

$$- \lambda_s V_u^*(s')^{\frac{\theta_A}{\theta_P+\theta_A}}V_u^*(s)^{\frac{\theta_P}{\theta_P+\theta_A}} - \lambda_d V_u^*(s)^{\frac{\theta_P}{\theta_P+\theta_A}}, \tag{4}$$

$$V_e^*(s)^{-\frac{\theta_A}{\theta_P+\theta_A}}\left[\rho\exp(\frac{\theta_P\theta_A}{\theta_P+\theta_A}\omega) + p_s V_u^*(s)^{-\frac{\theta_A}{\theta_P+\theta_A}} + \lambda_s V_e^*(s')^{-\frac{\theta_A}{\theta_P+\theta_A}} + \lambda_d\right] = (\rho + p_s + \lambda_s + \lambda_d). \tag{5}$$

Appendix C Calculations for the Implementation

From the model setup, the problem for the worker's consumption-savings-effort model is characterized by the following three HJB equations:

$$\rho J(x, d) = \max_c \rho u(c) + J_x(x, d)(r^d x - c + b^d)$$

$$\rho J(x, e, s) = \max_c \rho u(c) + J_x(x, e, s)[r^e(s_t)x_t - c_t + b^e(s_t)] + p_s[J(x_t + B_u^e(s_t), u, s) - J(x, e, s)]$$

$$+ \lambda_s[J(x + A^e(s, x), e, s') - J(x, e, s)] + \lambda_d(J(x + B_d^e(s), d) - J(x, e, s)),$$

$$\rho J(x, u, s) = \max_{c,a} \rho u(c, a) + J_x(x, u, s)[r^u(s_t)x_t - c_t + b^u(s_t)] + q_s a[J(x_t + B_e^u(s_t), e, s) - J(x, u, s)]$$

$$+ \lambda_s[J(x + A^u(s, x), u, s') - J(x, u, s)] + \lambda_d(J(x + B_d^u(s), d) - J(x, u, s)),$$

where $J(x, d)$, $J(x, e, s)$, $J(x, u, s)$ are the value functions for disabled workers, employed workers in state s and unemployed workers in state s respectively. In this section, I will show that the solutions to the value functions take the following form:

$$J(x, d) = -e^{-\theta_A r^d x}$$

$$J(x, e, s) = -J_e(s)e^{-\theta_A r^e(s)x}$$

$$J(x, u, s) = -J_u(s)e^{-\theta_A r^u(s)x}.$$

In addition, I will show the conditions when this consumption-savings-effort model implements the optimal contracts. To be more specific, I show how the (1) deposit interest rate: r^d, $r^e(s)$,

and $r^u(s)$, (2) flow payments: b^d, $b^e(s)$, and $b^u(s)$, (3) lump-sum transfers: $B^e_u(s)$, $B^e_d(s)$, $B^u_e(s)$, $B^u_d(s)$, $A^e(s,x)$, and $A^u(s,x)$ are determined.

As explained in the main text, the ideas come from Li and Williams [2014]. The main difference is that I will show how to pin down the indeterminacy of the flow payments, which depends on the state where the workers transition from.

Appendix C.1   A Disabled Worker

Denoting the value for the disabled worker as $J(x,d)$. The HJB equation then becomes

$$\rho J(x,d) = \max_c \rho u(c) + J_x(x,d)[r^d x_t - c_t + b^d].$$

The optimality condition for c is then

$$\rho u'(c) = J_x(x,d).$$

Let me guess the form for the disabled worker as:

$$J(x,d) = -J_d \exp(-\theta_A r^d x).$$

From the optimality condition for c, I have

$$c(x,d) = -\frac{1}{\theta_A} \log(\frac{r^d J_d}{\rho}) + r^d x.$$

Substituting everything back to the HJB equation gives:

$$J_d = \frac{\rho}{r^d} \exp(\frac{r^d - \rho - \theta_A r^d b^d}{r^d}).$$

Substituting everything back to consumption and value function gives the results as follows:

$$J(x,d) = -J_d \exp(-r^d \theta_A x),$$

where

$$J_d = \frac{\rho}{r^d} \exp(\frac{r^d - \rho - r^d \theta_A x b^d}{r^d}).$$

Also, the consumption is

$$c(x,d) = \frac{\rho - r^d}{\theta_A r^d} + b^d + r^d x.$$

For the implementation, I know that the promised utility must equal the value function. Hence,

$$W = J(x,d) = \frac{\rho}{r^d} \exp(\frac{r^d - \rho - r^d \theta_A x b^d}{r^d}).$$

Thus, the consumption in the solvable case then becomes

$$c(W(x),d) = \frac{1}{\theta_A} \log(J_d) + r^d x.$$

When this environment implements the optimal contracts, the consumption must equal: $c(W(x), d) = c(x, d)$. Thus, I obtain that

$$r^d = \rho, c(x, d) = b^d + \rho x.$$

The value function for the disabled worker is then

$$J(x, d) = -\exp(-\theta_A(b^d + \rho x)).$$

I can observe that there is indeterminacy in the implementation of the contracts, where b^d and x cannot be determined at this stage. I will resolve this problem when I solve the problem for the employed and unemployed worker in the later subsections.

Appendix C.2   An Employed Worker

As explained above, the value function for employed workers in state s is assumed to have the following form:

$$J(x, e, s) = -J_e(s) \exp(-\theta_A r^e(s) x)$$

For the problem of an employed worker, I again first assume the constant amount b^d that the disabled worker can get if he transitions from the unemployed status depends on the wealth, the current aggregate state, and the difference in the interest rate. Let $b^d = (r^e(s_t) - \rho)x_T - \frac{1}{\theta_A} \log J_e(s)$, where x_T is the wealth of this unemployed worker at time T when disability shock hits this worker. In addition, $b^u(s)$ will make the value function for the unemployed workers have the following form:

$$J(x, u, s) = -J_e(s) \exp(-\theta_A r^e(s) x_T).$$

Also, flow payment when transitioning from s to s' makes the value function for unemployed workers have the following form:

$$J(x, e, s') = -J_e(s) \exp(-theta_A r^e(s) x_T).$$

Exact forms of $b^u(s)$ and $b^e(s')$ will be explained later in this section. Thus I obtain:

$$J(x_T, d) = -J_e(s) \exp(-\theta_A r^e(s_t) x_T)$$
$$J(x_T, u, s) = -J_e(s) \exp(-\theta_A r^e(s_t) x_T).$$

The HJB equation for the unemployed workers is as follows:

$$\rho J(x, e, s) = \max_c \rho u(c) + J_x(x, e, s)[r^e(s_t)x_t - c_t + b^e(s_t)] + p_s[J(x_t + B_u^e(s_t), u, s) - J(x, e, s)]$$
$$+ \lambda_s[J(x + A^e(s, x), e, s') - J(x, e, s)] + \lambda_d(J(x + B_d^e(s), d) - J(x, e, s)).$$

The optimality conditions for c is

$$\rho u_c(c) = J_x(x, e, s).$$

I assume the $A^e(s, x)$ takes the form of

$$A^e(s, x) = (\frac{r^e(s)}{r^e(s')} - 1)x + \frac{r^e(s)}{r^e(s')} \hat{A}^e(s)$$

Following the guess, the consumption then becomes

$$c(x, e, s) = -\frac{1}{\theta_A} \log(\frac{r^e(s)}{\rho}) - \frac{1}{\theta_A} \log(J_e(s)) + r^e(s)x.$$

Using the guesses for the value function, I have

$$J(x + B_u^e(s), u, s) - J(x, e, s) = (1 - \exp(-\theta_A r^e(s)B_u^e(s)))J_e(s) \exp(-\theta_A r^e(s)x)$$

$$J(x + B_d^e(s), d) - J(x, e, s) = (1 - \exp(-\theta_A r^e(s)B_d^e(s)))J_e(s) \exp(-\theta_A r^e(s)x)$$

$$J(x + A^e(s, x), u, s') - J(x, e, s) = [1 - \exp(-\theta_A r^e(s)\hat{A}^e(s))]J_e(s) \exp(-\theta_A r^e(s)x).$$

To implement the contracts, I first set $W = J(x, e, s)$. Thus, the consumption in the optimal contracts becomes

$$c(W(x), e, s) = \frac{logV_e(s)}{\theta_A + \theta_P} + \frac{\theta_P}{\theta_A + \theta_P} w - \frac{1}{\theta_A} \log(-W)$$

To make the consumption equal: $c(x, e, s) = c(W(x), e, s)$, I must have

$$\frac{r^e(s)}{\rho} = \exp(-\frac{\theta_A \theta_P}{\theta_A + \theta_P} w) V_e(s)^{-\frac{\theta_A}{\theta_A + \theta_P}}$$

which pins down $r^e(s)$.

For the other variables, recalling that

$$x_t = -\frac{\log(W_t) + \log J_e(s)}{r^e(s_t)\theta_A} = \frac{X_t + \log J_e(s)}{r^e(s)\theta_A}.$$

By generalized Ito's lemma, I can derive the evolution of the wealth:

$$dx_t = -\mu_W^e(s_t)dt - \log(w_j^e(s_t))\Delta s_t^j - (\frac{\log(w_s^e(s_t))}{r^e(s')\theta_A} + (1 - \frac{r^e(s)}{r^e(s')}x_t))\Delta s_t^s - \log(w_d^e)\Delta s_t^d$$

However, the evolution for this consumption-savings-effort problem is

$$dx_t = (\frac{1}{\theta_A} \log(\frac{r^e(s_t)}{\rho}) + \frac{1}{\theta_A} \log(J_e(s)) + b^e(s_t))dt + B_u^e(s_t)\Delta s_t^j + A^e(s, x)\Delta s_t^s + B_d^e(s)\Delta s_t^d.$$

Hence I can pin down $b^e(s), B_u^e(s_t), B_d^e(s)$, and $\hat{A}^e(s_t)$ as

$$b^e(s_t) = -\frac{1}{\theta_A} \log(\frac{r^e(s_t)}{\rho}) - \frac{\mu_W^e(s_t)}{r^e(s_t)\theta_A} - \frac{1}{\theta_A} \log J_e(s)$$

$$B_u^e(s_t) = -\frac{\log(w_j^e(s_t))}{r^e(s_t)\theta_A}$$

$$\hat{A}(s_t) = -\frac{\log(w_s^e(s_t))}{r^e(s_t)\theta_A}$$

$$B_d^e(s_t) = -\frac{\log(w_d^e(s_t))}{r^e(s_t)\theta_A}.$$

I can show that the guess of the value function solves the implementation problem.

Using the previous results, and canceling the terms in x, the HJB equation becomes:

$$0 = \rho - r^e(s) + \theta_A r^e(s)\left(\frac{1}{\theta_A}\log(\frac{r^e(s)}{\rho}) + \frac{1}{\theta_A}\log J_e(s) + b^e(s)\right)$$

$$+ p_s[1 - \exp(-\theta_A r^e(s)B_u^e(s))] + \lambda_s[1 - \exp(-\theta_A r^e(s)\hat{A}^e(s))] + \lambda_d(1 - \exp(-\theta_A r^e(s)B_d^e(s))).$$

Substituting back the policies for $b^e(s)$ (up to a scale of $J_e(s)$), B_d^e, $B_u^e(s)$, $\hat{A}^e(s)$ into the HJB equation above allows me to verify that all terms cancel out, which verifies my guess and shows that the policy implements the optimal contract.

For the indeterminacy of the flow payments, $b^e(s')$ and $b^u(s)$ can be determined by the following:

$$-\theta_A b^e(s') - \log(\frac{r^e(s')}{\rho}) - \frac{\mu_W^e(s')}{r^u(s')} = \log J_e(s)$$

$$\theta_A((r^e(s) - r^u(s))x_T - b^u(s)) - \frac{\mu_W^u(s)}{r^u(s)} - \log(\frac{r^u(s)}{\rho}) = \log J_e(s).$$

Appendix C.3 An Unemployed Worker

As explained above, the value function for unemployed workers in state s is assumed to have the following form:

$$J(x, u, s) = -J_u(s)\exp(-\theta_A r^u(s)x)$$

For the problem of an unemployed worker, I again first assume the constant amount b^d that the disabled worker can get if he transits from the unemployed status depends on the wealth, the current aggregate state, and the difference in the interest rate. Let $b^d = (r^u(s_t) - \rho)x_T - \frac{1}{\theta_A}\log J_u(s)$, where x_T is the wealth of this unemployed worker at time T when disability shock hits this worker. In addition, $b^e(s)$ will make the value function for the employed workers have the following form:

$$J(x, e, s) = -J_u(s)\exp(-\theta_A r^u(s)x_T).$$

Also, flow payment when transitioning from s to s' makes the value function for unemployed workers have the following form:

$$J(x, u, s') = -J_u(s)\exp(-\theta_A r^u(s)x_T).$$

Exact forms of $b^e(s)$ and $b^u(s')$ will be explained later in this section. Thus I obtain:

$$J(x_T, d) = -J_u(s)\exp(-\theta_A r^u(s_t)x_T)$$

$$J(x_T, e, s) = -J_u(s)\exp(-\theta_A r^u(s_t)x_T).$$

The HJB equation for the unemployed workers is as follows:

$$\rho J(x, u, s) = \max_{c,a} \rho u(c, a) + J_x(x, u, s)[r^u(s_t)x_t - c_t + b^u(s_t)] + q_s a[J(x_t + B_e^u(s_t), e, s) - J(x, u, s)]$$

$$+ \lambda_s[J(x + A^u(s, x), u, s') - J(x, u, s)] + \lambda_d(J(x + B_d^u(s), d) - J(x, u, s)).$$

The optimality conditions for c and a are

$$\rho u_c(c, a) = J_x(x, u, s),$$

$$-\rho u_a(c, a) = q_s[J(x_t + B_e^u(s_t), e) - J(x, u, s)].$$

I assume the $A^u(s, x)$ takes the form of

$$A^u(s, x) = (\frac{r^u(s)}{r^u(s')} - 1)x + \frac{r^u(s)}{r^u(s')}\hat{A}^u(s)$$

Following the guess, the consumption then becomes

$$c(x, u, s) = -\frac{1}{\theta_A}\log(\frac{r^u(s)}{\rho}) - \frac{1}{\theta_A}\log(J_u(s)) + r^u(s)x + h(a).$$

Using the guesses for the value function, I have

$$J(x + B_e^u(s), e, s) - J(x, u, s) = (1 - \exp(-\theta_A r^u(s)B_e^u(s)))J_u(s)\exp(-\theta_A r^u(s)x)$$

$$J(x + B_d^u(s), d) - J(x, u, s) = (1 - \exp(-\theta_A r^u(s)B_d^u(s)))J_u(s)\exp(-\theta_A r^u(s)x)$$

$$J(x + A^u(s, x), u, s) - J(x, u, s) = [1 - \exp(-\theta_A r^u(s)\hat{A}^u(s))]J_u(s)\exp(-\theta_A r(s)x).$$

The optimality condition for a becomes

$$\rho \theta_A h'(a)\exp(-\theta_A(c - h(a))) = q_s(1 - \exp(-\theta_A r^u(s)B_e^u(s)))J_u(s)\exp(-\theta_A r^u(s)x),$$

which the terms in x can be canceled. This verifies the optimal $a = a^*(s)$ is independent of x.

To implement the contracts, I first set $W = J(x, u, s)$. Thus, the consumption in the optimal contracts becomes

$$c(W(x), u, s) = c^*(s) + h(a^*(s)) + r^u(s)x.$$

To make the consumption equal: $c(x, u, s) = c(W(x), u, s)$, I must have

$$c^*(s) = -\frac{1}{\theta_A}\log(\frac{r^u(s)}{\rho}),$$

which pins down $r^u(s)$.

For the other variables, recalling that

$$x_t = -\frac{\log(W_t) + \log J_u(s)}{r^u(s_t)\theta_A} = \frac{X_t + \log J_u(s)}{r^u(s)\theta_A}.$$

By generalized Ito's lemma, I can derive the evolution of the wealth:

$$dx_t = -\mu_W^u(s_t)dt - \log(w_j^u(s_t))\Delta s_t^j - (\frac{\log(w_s^u(s_t))}{r^u(s')\theta_A} + (1 - \frac{r^u(s)}{r^u(s')}x_t))\Delta s_t^s - \log(w_d^u)\Delta s_t^d$$

However, the evolution for this consumption-savings-effort problem is

$$dx_t = (\frac{1}{\theta_A}\log(\frac{r^u(s_t)}{\rho})+\frac{1}{\theta_A}\log(J_u(s))-h(a^*(s))+b^u(s_t))dt+B_e^u(s_t)\Delta s_t^j+A^u(s,x)\Delta s_t^s+B_d^u(s)\Delta s_t^d.$$

Hence I can pin down $b^u(s)$, $B_e^u(s_t)$,$B_d^u(s)$, and $\hat{A}(s_t)$ as

$$b^u(s_t) = -\frac{1}{\theta_A}\log(\frac{r^u(s_t)}{\rho}) - \frac{\mu_W^u(s_t)}{r^u(s_t)\theta_A} - \frac{1}{\theta_A}\log J_u(s) + h(a^*(s))$$

$$B_e^u(s_t) = -\frac{\log(w_j^u(s_t))}{r^u(s_t)\theta_A}$$

$$\hat{A}(s_t) = -\frac{\log(w_s^u(s_t))}{r^u(s_t)\theta_A}$$

$$B_d^u(s_t) = -\frac{\log(w_d^u(s_t))}{r^u(s_t)\theta_A}.$$

I can show that the guess of the value function solves the implementation problem.

Using the previous results, and canceling the terms in x, the HJB equation becomes:

$$0 = \rho - r^u(s) + \theta_A r^u(s)(\frac{1}{\theta_A}\log(\frac{r^u(s)}{\rho}) + \frac{1}{\theta_A}\log J_u(s) - h(a(s)) + b^u(s))$$

$$+ q_s a(s)[1 - \exp(-\theta_A r^u(s)B_e^u(s))] + \lambda_s[1 - \exp(-\theta_A r^u(s)\hat{A}^u(s))] + \lambda_d(1 - \exp(-\theta_A r^u(s)B_d^u(s))).$$

Substituting back the policies for $b^u(s)$ (up to a scale of $J_u(s)$), B_d^u, $B(s)$, $\hat{A}(s)$ into the HJB equation above allows me to verify that all terms cancel out, which verifies our guess and shows that the policy implements the optimal contract.

For the indeterminacy of the flow payments, $b^u(s')$ and $b^e(s)$ can be determined by the following:

$$-\theta_A b^u(s') - \log(\frac{r^u(s')}{\rho}) - \frac{\mu_W^u(s')}{r^u(s')} + \theta_A h(a^*(s')) = \log J_u(s)$$

$$\theta_A((r^u(s) - r^e(s))x_T - b^e(s)) - \frac{\mu_W^e(s')}{r^e(s)} - \log(\frac{r^e(s)}{\rho}) = \log J_u(s).$$

Appendix D   Benchmark Contract

I will now show how to calculate the promised utility for the worker under the benchmark contract. Because of the assumption of the absorbing state, it is easier to consider the case without misreporting and then add the choice of misreporting back into the calculations subsequently.

Let me begin by considering the case when misreporting is not allowed. First, I assume that the expected utility for the worker in state s under the benchmark contract is W_s^e, and that the

expected utility for a disabled worker is W^d. I will assume the utility is given, and I will show that solving the benchmark model is equivalent to solving a system of ordinary differential equations. Next, I consider the period after T_R, in which the unemployed worker receives zero unemployment benefits. Let $\tau = \tau^j \wedge \tau^s \wedge \tau^d$, where τ^s is the first date of a switch in aggregate state, τ^j is the first time that this worker finds a job, and τ^d is the first time that the disability shock arrives. I also denote the utility in this region by W_s^{u3}. Following similar steps as I did in deriving the HJB equation above, I can see that, for $t > T_R$ the values of W_s^{u3} are

$$\rho W_s^{u3} = \max_{a \in [0, \bar{a}]} \rho(u(0) - h(a)) + q_s(a)(W_s^e - W_s^{u3}) + \lambda_s(W_{s'}^{u3} - W_s^{u3}) + \lambda_d(W^d - W_s^{u3}).$$

Then, let us consider time in $[T_B, T_R]$, where the worker only receives benefits in a recession. I will denote the utility of the worker in state s during this period as $W_s^{u2}(t)$. Similar to the previous case, the pair of HJB equations is as follows:

$$\rho W_R^{u2}(t) - \frac{d}{dt} W_R^{u2}(t) = \max_{a \in [0, \bar{a}]} \rho(u(c^B) - h(a)) + q_R(a)(W_s^e - W_R^{u2}(t)) + \lambda_s(W_B^{u2}(t) - W_R^{u2}(t))$$

$$+ \lambda_d(W^d - W_R^{u2}(t))$$

$$\rho W_B^{u2}(t) - \frac{d}{dt} W_B^{u2}(t) = \max_{a \in [0, \bar{a}]} \rho(u(0) - h(a)) + q_B(a)(W_s^e - W_B^{u2}(t)) + \lambda_s(W_R^{u2}(t) - W_B^{u2}(t))$$

$$+ \lambda_d(W^d - W_B^{u2}(t))$$

with boundary conditions $W_B^{u2}(T_R) = W_B^{u3}$ and $W_R^{u2}(T_R) = W_R^{u3}$. Last, in the region of $[0, T_B]$, the promised utility which I denote by $W_s^{u1}(t)$ evolves as:

$$\rho W_s^{u1}(t) - \frac{d}{dt} W_s^{u1}(t) = \max_{a \in [0, \bar{a}]} \rho(u(c^B) - h(a)) + q_s(a)(W_s^e - W_s^{u1}(t)) + \lambda_s(W_{s'}^{u1}(t) - W_s^{u1}(t))$$

$$+ \lambda_d(W^d - W_s^{u1}(t))$$

with boundary conditions $W_s^{u1}(T_B) = W_s^{u2}(T_B)$.

Adding back in the choice of misreporting is straightforward, since I assume that disability is an absorbing state. In the promised utility solved in the equations above, I know that the agent would misreport if $W_s^i(t) < W^d$. As a consequence, $\frac{d}{dt} W_s^i(t) = 0$. In sum, the set of HJB and ordinary differential equations are

$$\rho W_s^e = \begin{cases} \rho u(\omega) + p_s(W_s^u(0) - W_s^e) + \lambda_s(W_{s'}^e - W_s^e) + \lambda_d(W^d - W_s^e), & \text{if } W_s^e \geq W^d \\ \text{report disabled,} & \text{otherwise} \end{cases}$$

$$\rho W_s^{u3} = \begin{cases} \max_{a \in [0, \bar{a}]} \rho(u(0) - h(a)) + q_s(a)(W_s^e - W_s^{u3}) + \lambda_s(W_{s'}^{u3} - W_s^{u3}) \\ \qquad\qquad + \lambda_d(W^d - W_s^{u3}), & \text{if } W_s^{u3} \geq W^d \\ \text{report disabled,} & \text{otherwise} \end{cases}$$

$$W^d = u(c^d).$$

$$\rho W_R^{u2}(t) - \frac{d}{dt}W_R^{u2}(t) = \begin{cases} \max_{a\in[0,\bar{a}]} \rho(u(c^B) - h(a)) + q_R(a)(W_R^e - W_R^{u2}(t)) \\ \quad + \lambda_s(W_B^{u2}(t) - W_R^{u2}(t)) + \lambda_d(W^d - W_R^{u2}(t)), \quad \text{if } W_s^{u3} \geq \text{report disabled} \\ \qquad\qquad \text{report disabled}, \qquad\qquad\qquad\qquad \text{otherwise} \end{cases}$$

$$\rho W_B^{u2}(t) - \frac{d}{dt}W_B^{u2}(t) = \begin{cases} \max_{a\in[0,\bar{a}]} \rho(u(0) - h(a)) + q_B(a)(W_B^e - W_B^{u2}(t)) \\ \quad + \lambda_s(W_R^{u2}(t) - W_B^{u2}(t)) + \lambda_d(W^d - W_B^{u2}(t)), \quad \text{if } W_s^{u3} \geq \text{report disabled} \\ \qquad\qquad \text{report disabled}, \qquad\qquad\qquad\qquad \text{otherwise} \end{cases}$$

$$\rho W_s^{u1}(t) - \frac{d}{dt}W_s^{u1}(t) = \begin{cases} \max_{a\in[0,\bar{a}]} \rho(u(c^B) - h(a)) + q_s(a)(W_s^e - W_s^{u1}(t)) \\ \quad + \lambda_s(W_{s'}^{u1}(t) - W_s^{u1}(t)) + \lambda_d(W^d - W_s^{u1}(t)), \quad \text{if } W_s^{u3} \geq \text{report disabled} \\ \qquad\qquad \text{report disabled}, \qquad\qquad\qquad\qquad \text{otherwise} \end{cases}$$

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