# Payment Instruments and Collateral in the Interbank Payment System

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June 30, 2015

**Abstract**

This paper presents a three-period model to analyze why banks need bank reserves despite the presence of other liquid assets like Treasury securities. The model highlights the fact that the interbank market is an over-the-counter market. It shows that the large value payment system operated by the central bank can be regarded as a collateralized contract to save liquidity to settle bank transfers. In this interpretation, bank reserves are the balances of collateral. The optimal contract is the floor system. Whether a private clearing house can replace the central bank depends on the range of collateral it can accept.

*Email: tomura.hajime@gmail.com. I thank Hiroshi Fujiki, Hiroki Arato, Masaya Sakuragawa, Hidehiko Ishihara, Jamie McAndrews, and participants in various seminars for their comments. I am also grateful for financial support from JSPS Grant-in-Aid for Scientific Research (24223003).

# 1 Introduction

Base money consists of cash and bank reserves. Banks hold bank reserves not merely to satisfy a reserve requirement, but also to make interbank payments to settle bank transfers between depositors. In fact, the daily transfer of bank reserves in a country tends to be as large as a sizable fraction of annual GDP.[1] Also, several countries have abandoned a reserve requirement.[2] Banks in these countries still use bank reserves to settle bank transfers everyday.

Why do banks need bank reserves for interbank payments? Theoretically, banks should be able to pay other liquid assets, such as Treasury securities. If different issue dates and maturities make it inconvenient to use Treasury securities for payment instruments, then banks can set up a special purpose vehicle that issues security accounts backed by a pool of Treasury securities. Thus, a simple consideration of physical transaction costs does not imply that the central bank is essential for interbank payments.

To further investigate this question, this paper constructs a hypothetical model to analyze how banks would settle bank transfers without the central bank in the current environment. The model has four key features. First, banks' commitments are limited, so that banks need payment instruments to settle bank transfers. Second, there exist liquid bonds that are transferable at no physical transaction cost, so that the need for bank reserves does not simply stem from a security transaction cost. Third, without the central bank, banks need to negotiate the terms of settlement of bank transfers bilaterally. This assumption reflects the fact that the interbank market is an over-the-counter (OTC) market. Finally, a bank must pay a penalty if it fails to complete bank-transfer requests from its depositors on time. This assumption is due to the fact that banks make interbank payments on behalf of depositors.

The model shows that the OTC settlement of bank transfers causes a hold-up problem,

[1]For example, the average daily transfer of bank reserves in the U.S. was 20.2% of annual GDP in 2014, and the figure in Japan was 24.3% in 2013.

[2]These countries include Australia, Canada, Denmark, Mexico, New Zealand, Norway, Sweden, and the U.K.

because a penalty for failed bank transfers weakens the bargaining position of an originating bank.[3] As a result, a receiving bank can require an originating bank to pay a premium, which increases the amount of liquidity necessary for the settlement of bank transfers.

The central bank can eliminate this hold-up problem if it can prevent ex-post bargaining over the settlement of bank transfers. One way to do this is to supply legal tender. This policy works if depositors use bills of exchange drawn on banks to send bank transfers, because receiving banks must accept legal tender at face value when originating banks present it to settle its financial obligations. This result is consistent with the historical use of bills of exchange and cash in the interbank settlement of bank transfers.

Currently, banks pay bank reserves, which are usually not designated as legal tender. Yet, an originating bank can settle bank transfers unilaterally by remitting the equivalent nominal value of bank reserves to a receiving bank under a rule set by the central bank. This paper replicates this feature of the interbank payment system operated by the central bank, so-called a large value payment system, by characterizing it as an interbank settlement contract. It is shown that the use of such a contract saves banks' liquidity to settle bank transfers, because it eliminates ex-post bargaining over the settlement of bank transfers by specifying the terms of settlement in advance. Furthermore, banks need a custodian of collateral to write a pledgeable contract because of their ex-ante symmetry. These results explain why a third party like the central bank must operate the large value payment system, and also why banks swap liquid bonds like Treasury securities for bank reserves, which can be regarded as the balances of collateral in the contract.

The optimal contract corresponds to the floor system. This result is not only due to the Friedman's rule, but also because the floor system obviates the need for an OTC interbank money market. Also, this paper discusses whether a private clearing house can take over the role of the central bank. The model indicates two issues. First, to operate the large value payment system, a clearing house must be able to commit to returning collateral to

[3] An originating bank is a bank sending bank transfers to another bank on request from its depositors.

banks after the settlement of bank transfers. Historical evidence suggests that this is not a significant challenge for a private clearing house. Second, a private clearing house dominates the central bank if it's eligible collateral is wider than the central bank's. In this regard, a private clearing house may need to have enough ability to supervise member banks to replace the central bank.

Just to clarify, this paper does not aim to replicate the historical evolution of the interbank payment system from the time before the foundation of the central bank. Instead, this paper models a hypothetical interbank payment system without the central bank, taking as given the current environment, such as the existence of other liquid assets than bank reserves and an OTC interbank market. An ample supply of such liquid assets may be only a recent phenomenon, perhaps after the large-scale issuance of Treasury securities in the 1970s. This approach is necessary to draw policy implications for the current interbank payment system.

## 1.1 Related literature

The distinction between bank reserves and other liquid assets is related to the legal restriction theory of money. Wallace (1983) discusses why money is necessary despite the presence of interest-bearing Treasury securities. He points out that the non-negotiability and the large denomination of Treasury securities provide a role for money as the medium of exchange. This paper brings this question to interbank payments, highlighting the fact that the interbank market is an OTC market. In the recent literature, Piazzesi and Schneider (2015) analyze asset prices and monetary policy with a constraint that banks must settle bank transfers by paying bank reserves between them. This paper derives the use of bank reserves endogenously.

There exists an extensive literature that analyzes central-bank policies that replace illiquid assets with money. Freeman (1996) analyzes the effect of a discount window that replaces illiquid IOUs with money. Other papers on this issue include Green (1997), Fujiki (2003, 2006), Mills (2004), Gu, Guzman and Haslag (2011), and Chapman and Martin (2013).[4] Also, there is a search-theoretic literature on money and illiquid collateral, such as Shi (1996), Ferraris and Watanabe (2008), and Andolfatto, Berentsen and Waller (2013).[5] In the business-cycle literature, Kiyotaki and Moore (2012) analyze the effect of money supply that replaces illiquid private securities. In contrast to these papers, this paper analyzes why the central bank needs to replace liquid assets with bank reserves.

The analysis of OTC settlement of bank transfers is related to the paper by Martin and McAndrews (2010), which raises an open question on the need for the overnight interbank money market. This paper shows that it is optimal to eliminate the overnight interbank money market as the floor system does, because it prevents a hold-up problem in an OTC market. Another related paper is the work by Ennis and Weinberg (2013), which analyzes the effect of stigma in the OTC interbank money market. Also, Afonso and Lagos (2014) constructs a dynamic search model to analyze the U.S. federal funds market.

This paper also adds to the literature on private interbank payment systems. Kahn (2013) analyzes how the competition between a public and a private interbank payment system limits the central bank's ability to manipulate monetary policy. Kahn (2009) brings this issue to cross-border settlement. Kahn and Roberds (2009) analyze the vertical integration of a public and a private interbank payment system through tiering. From a broader perspective, this paper is related to the literature on electronic money, such as Skeie (2009).

The remainder of the paper is organized as follows. The practical features of the interbank payment system are briefly reviewed in section 2. The baseline model is presented in section

[4]Green (1997) discusses whether a private clearing house can take over the role of the central bank. Fujiki (2003, 2006) analyzes the effect of the central bank's liquidity provision policies on cross-border settlement. Mills (2004) proposes an alternative mechanism to the discount window based on collateralized lending. Gu, Guzman and Haslag (2011) analyze the optimal intraday interest rate. Chapman and Martin (2013) investigate the role of tiering to limit the central bank's exposure to credit risk.

[5]Shi (1996) shows useless assets except for the owner can serve as collateral to facilitate intertemporal exchange in a money-search model. Ferraris and Watanabe (2008) analyze the co-existence of money and credit by introducing money loans secured by illiquid capital. Andolfatto, Berentsen and Waller (2013) analyze optimal monetary policy with money backed by illiquid capital.

3. The central bank is introduced into the model in sections 4 and 5. Section 6 discusses whether a private clearing house can replace the central bank. Section 7 concludes.

# 2 Brief overview of the interbank payment system in practice

This section briefly summarizes the background of the baseline model in this paper. Typically there are two tiers in the interbank payment system in a country. In the first tier, small-valued bank transfers from depositors enter an automated clearing house (ACH).[6] At this stage, an ACH processes a large number of small-valued bank transfers to calculate the net balance of bank transfers for each bank. Then in the second tier, the banks with outgoing net bank transfers remit the corresponding nominal values of current-account balances to the ACH's account at the central bank. These balances are so-called bank reserves.[7] The ACH passes on the received bank reserves to the banks with incoming net bank transfers, so that it maintains a zero net position of bank reserves. The transfer of bank reserves in the second tier settles gross bank transfers bundled in the first tier. The system that processes the transfers of bank reserves in the second tier is called a large value payment system.[8]

Banks can obtain bank reserves from the central bank in exchange for liquid assets, such as Treasury securities and high-quality private securities, through open market operations. In each day, however, some banks may run short of bank reserves because of an imbalance between incoming and outgoing bank transfers for each bank. They fulfill the shortfalls of bank reserves by borrowing bank reserves overnight in an OTC interbank money market.[9]

[6]If a depositor sends a large-valued bank transfer, then it is directly settled at the large value payment system without going through an ACH.

[7]The formal name of bank reserves differs across countries. For example, they are called reserve balances in the U.S. and settlement balances in Canada.

[8]This system is called Fedwire in the U.S., TARGET2 in the Eurozone, CHAPS in the U.K., and BoJ-NET in Japan.

[9]The central bank normally allows banks to run negative balances of bank reserves during the daytime through daylight overdrafts. Banks can fulfill expected shortfalls in bank reserves in the interbank money market in each morning, and unexpected shortfalls at the end of each day.

In the following section, this paper presents a baseline model to analyze how banks would settle bank transfers without the central bank. To draw policy implications for the current interbank payment system, the model runs this thought experiment given the current environment; thus, the model maintains the existence of liquid assets other than bank reserves and also an OTC interbank market, in which banks can settle negative positions of bank transfers at the end of each day (see Figure 1). The central bank will be introduced later into the model to clarify its role in the current interbank payment system.

# 3 Baseline model of an interbank payment system without the central bank

Time is discrete and indexed by $t = 0, 1, 2$. There are two banks indexed by $i = A, B$. Each bank receives a unit amount of goods from its depositors in period 0. For simplicity, assume that goods are perishable and that the deposit interest rate is set to zero.[10]

Banks can transform deposited goods into bank loans and bonds. Bank loans generate an amount $R_L$ of goods in period 2 per invested good. Similarly, the gross rate of return on bonds in period 2 is $R_B$. Assume that

$$R_L > R_B > 1, \tag{1}$$

in which one equals the gross rate of return on deposits.

In period 1, bank $i$ for $i = A, B$ receives depositors' orders to remit a fraction $\lambda_i$ of its total deposits to the other bank.[11] The joint probability distribution of $\lambda_A$ and $\lambda_B$ is

$$(\lambda_A,\ \lambda_B) = \begin{cases} (\eta,\ 0) & \text{with probability } 0.5, \\ (0,\ \eta) & \text{with probability } 0.5, \end{cases} \tag{2}$$

[10] A zero deposit interest rate can be derived as an endogenous equilibrium outcome. See Appendix A for the formal assumption about depositors.

[11] Assume that depositors cannot withdraw goods from banks in period 1, as banks cannot produce any good by terminating bank loans or bonds in period 1.

where $\eta \in (0,1)$.[12] Thus, banks are symmetric ex-ante. Call the bank with $\lambda_i = \eta$ the "originating bank", and the bank with $\lambda_i = 0$ the "receiving bank".

In reality, the flows of bank transfers are confidential information for each bank.[13] Given this fact, assume that the realization of $(\lambda_A,\ \lambda_B)$ cannot be made public due to a state verification cost. Thus, the court cannot enforce a contingent contract such that the receiving bank increases its deposit liabilities by $\eta$ in period 1 without any compensation from the originating bank.[14] As a result, banks must determine the terms of settlement of bank transfers after the realization of $(\lambda_A,\ \lambda_B)$ in period 1.[15]

Assume that the interbank market is an OTC market; so banks bilaterally bargain over how much amounts of assets the originating bank must transfer to the receiving bank to settle bank transfers. The outcome of bargaining is determined by Nash bargaining with equal bargaining power for each bank.

Bonds are transferable at no physical transaction cost. In contrast, if a bank sells its bank loans to the other bank in period 1, then the gross rate of return on the transfered bank loans becomes $\delta$ ($\in (0, R_L]$). The difference between $R_L$ and $\delta$ can be interpreted as a loan monitoring cost incurred by the bank purchasing the bank loans.[16]

If banks do not reach any agreement, then no bank transfer is made. In this case, the originating bank must incur a cost $\gamma\eta$ ($\gamma > 0$) in period 2. This cost can be interpreted as a long-term cost due to loss of reputation with depositors, or a cost payable in period 2 due to a litigation filed by depositors for failed payments. For normalization, the receiving bank

[12] It is implicitly assumed that overlapping gross flows of bank transfers between banks are automatically canceled out at an ACH, so that banks only need to settle a net flow of bank transfers at the end of period 1.

[13] For example, usually only a limited number of central-bank staff can see the identities of banks involved with each payment flow in the large value payment system.

[14] This contract would be the first-best if there were no state verification cost, because banks would be able to invest all the deposits into bank loans in period 0 for the highest rate of return without worrying about the resale of assets.

[15] Later, it will be shown that cross-posting collateral between banks does not make a contract pledgeable.

[16] The loan interest rate, $R_L$, can be interpreted as the rate of return on bank loans net of the loan monitoring cost for the originator bank. Thus, this assumption does not imply that an originator bank does not have to monitor bank loans.

does not face any penalty. All the results shown below remain the same if the penalty on the receiving bank is smaller than that on the originating bank. This assumption reflects the fact that a deposit contract includes the right to send a bank transfer on demand, for which the originating bank is liable, but the receiving bank is not.[17]

In period 2, each bank receives returns on its bank loans and bonds, repays deposits given a zero deposit interest rate, and consumes the residual as its profit. Banks are risk-neutral; thus each bank chooses its portfolio of bank loans and bonds in period 0 to maximize the expected profit in period 2. An equilibrium is a Perfect Bayesian Nash equilibrium for the two banks. See Table 1 for the summary of events in the model.

## 3.1 Parametric assumptions

Throughout the paper, assume that

**Assumption 1.** $R_B > (1+\gamma)\,\eta$.

This assumption ensures that it is possible to settle bank transfers if a bank invests into a sufficiently large amount of bonds. Also, assume that

**Assumption 2.** $\gamma > 4\left(\dfrac{R_L}{R_B} - 1\right)$,

so that banks choose to settle bank-transfer requests in any equilibrium considered below.

## 3.2 Hold-up problem in the OTC settlement of bank transfers

Let us start from the case in which bank loans are transferable at no transaction cost:

**Assumption 3.** $\delta = R_L$.

Solve the model backward. Under Assumption 3, the bargaining problem between the originating and the receiving bank in period 1 takes the following form:

$$\max_{l\in[0,k],\; b\in[0,a]} [-(R_L l + R_B b - \eta) - (-\gamma\eta)]^{0.5}(R_L l + R_B b - \eta)^{0.5}, \tag{3}$$

[17] All the results of the model hold if the penalty for failed bank transfers is higher for the originating bank than for the receiving bank.

where: $k$ and $a$ are the amounts of bank loans and bonds, respectively, held by the originating bank at the beginning of period 1; $l$ and $b$ denote the amounts of bank loans and bonds, respectively, that the originating bank pays to the receiving bank; and $\eta$ is the face value of bank transfers in period 1.

The left square bracket is the trade surplus for the originating bank, and the right parenthesis is that for the receiving bank. The first term in the left square bracket, $-(R_L l + R_B b - \eta)$, is a change in profit in period 2 for the originating bank in case of the successful settlement of bank transfers.[18] The second term in the left square bracket, $-\gamma\eta$, is the penalty for failed settlement of bank transfers. This penalty determines the threat point for the originating bank. In contrast, the trade surplus for the receiving bank equals the profit from receiving bank transfers, $(R_L l + R_B b - \eta)$, given no penalty on the receiving bank for failed settlement of bank transfers.

The solution for the bargaining problem is

$$R_L l + R_B b = \eta + \frac{\gamma\eta}{2}, \tag{4}$$

which is feasible under Assumption 1.[19] Thus, the originating bank must pay an extra value of assets, $\gamma\eta/2$, besides the face value of bank transfers, $\eta$. This result is due to bilateral bargaining in an OTC interbank market. The originating bank must complete the bank transfers within period 1 to avoid a penalty for failed settlement of bank transfers. Taking advantage of this time constraint, the receiving bank can charge the originating bank a premium for the settlement of bank transfers.

This premium can be regarded as the interbank overnight interest rate, for which the principal is the originating bank's debit position of bank transfers, $\eta$. For example, the premium is a repo rate if the originating bank provides its assets as collateral to the receiving bank in period 1, and then repays the principal and the interest in period 2 from the return

[18] In this case, the originating bank pays assets worth $R_L l + R_B b$, while its deposit liabilities declines by $\eta$.

[19] Given Assumption 1 and the flow of funds constraint for each bank in period 0, $k + a = 1$, there exists a pair of $l$ and $b$ satisfying (4), $l \leq k$, and $b \leq a$ for every possible pair of $k$ and $a$.

on collateral. Here, a repo and a spot sale are equivalent because assets mature in period 2. Alternatively, the premium is an unsecured call rate if the originating bank commits to repaying the principal and the interest from the return on its assets in period 2. While it is assumed that a contract contingent on the flow of bank transfers is unverifiable due to a state verification cost, this assumption does not prohibit a non-contingent debt contract, such as an unsecured call loan. Thus, it is indifferent in the model if the originating bank transfers assets to the receiving bank in period 1, or commits to paying the return on those assets in period 2.

### 3.3 Efficiency of an interbank payment system without the central bank in case of liquid bank loans

Now move back to period 0. The profit maximization problem for each bank in the period is:

$$\begin{aligned} \max_{\{k \geq 0,\, a \geq 0\}} \; & R_L k + R_B a - 1 + \frac{1}{2}\frac{\gamma\eta}{2} + \frac{1}{2}\left(-\frac{\gamma\eta}{2}\right), \\ \text{s.t. } & k + a = 1, \end{aligned} \tag{5}$$

where the constraint is a flow of funds constraint that the sum of investments into bank loans, $k$, and bonds, $a$, must equal the total amount of deposits, 1, at each bank in period 0. The first two terms in the objective function are the returns on bank loans and bonds in period 2. The third term is the face value of deposit liabilities issued in period 0. The last two terms are the expected net gain and loss due to incoming and outgoing bank transfers, i.e., $\pm(R_L l + R_B b - \eta)$, as implied by (4).

Given $R_L > R_B > 1$ as assumed in (1), the solution for this problem is

$$(k,\ a) = (1,\ 0). \tag{6}$$

Thus, each bank invests only into the assets with the highest rate of return:

**Proposition 1.** Suppose Assumption 1 holds. Under Assumption 3, each bank chooses the efficient resource allocation, (6), in period 0.

### 3.4 Inefficiency of an interbank payment system without the central bank in case of illiquid bank loans

The efficiency result described above is overturned if bank loans are illiquid. Now suppose that the cost of liquidating bank loans, $R_L - \delta$, is sufficiently high:

**Assumption 4.** $\delta < \dfrac{R_L}{1+\gamma}$.

This assumption implies $\delta < R_B$ given Assumption 2; thus, the rate of return on bank loans becomes smaller than that on bonds if transfered.

Under Assumption 4, the bargaining problem for the settlement of bank transfers in period 1 takes the following form:

$$\max_{l\in[0,k],\ b\in[0,a]} [-(R_L l + R_B b - \eta) - (-\gamma\eta)]^{0.5} (\delta l + R_B b - \eta)^{0.5}. \tag{7}$$

The left square bracket and the right parenthesis contain the trade surpluses for the originating and the receiving bank, respectively. Note that the gross rate of return on transferred bank loans, $l$, in the right parenthesis is changed from $R_L$ to $\delta$.

Denote the changes in profit for the originating and the receiving bank as a result of the bargaining by $\theta(a)$ and $\phi(a)$, respectively. Both $\theta(a)$ and $\phi(a)$ are the functions of the amount of bonds held by the originating bank, $a$, given the flow of fund constraint on the bank in period 0, $k = 1 - a$. The solution for the bargaining problem implies that the following result holds for all $\delta \in (0, R_L)$ under Assumption 1:

$$\begin{aligned}&(\theta(a),\ \phi(a)) \\ &= \begin{cases} (-\gamma\eta,\ 0), & \text{if } R_B a - \eta < -\frac{\delta\gamma\eta}{R_L-\delta}, \\ (-[R_L l(a) + R_B b(a) - \eta],\ \delta l(a) + R_B b(a) - \eta), & \text{otherwise,} \end{cases}\end{aligned} \tag{8}$$

where $l(a)$ and $b(a)$ denote the solutions for $l$ and $b$, respectively, given $a$:

$$(l(a),\ b(a)) = \begin{cases} \left(\frac{\delta\gamma\eta-(R_L+\delta)(R_B a-\eta)}{2R_L\delta},\ a\right), & \text{if } R_B a - \eta \in \left[-\frac{\delta\gamma\eta}{R_L-\delta},\ \frac{\delta\gamma\eta}{R_L+\delta}\right], \\ (0,\ a), & \text{if } R_B a - \eta \in \left[\frac{\delta\gamma\eta}{R_L+\delta},\ \frac{\gamma\eta}{2}\right], \\ \left(0,\ \frac{1}{R_B}\left(\eta + \frac{\gamma\eta}{2}\right)\right), & \text{if } R_B a - \eta > \frac{\gamma\eta}{2}. \end{cases} \tag{9}$$

See Appendix B for the proof.

These equations imply that banks fail to agree on the settlement of bank transfers (i.e., $\theta(a) = -\gamma\eta$), if $a$ is too small. In this case, the amount of bank loans that must be liquidated to settle bank transfers is too large, given a high loan liquidation cost, $R_L - \delta$, under Assumption 4.

If $a$ is sufficiently large, then the originating bank settles bank transfers by paying assets worth more than the face value of bank transfers, i.e., $R_L l(a) + R_B b(a) > \eta$. This result holds because the receiving bank can charge the originating bank a premium, given the time constraint that the originating bank must complete bank-transfer requests within period 1. This hold-up problem is as same as the reason behind the second term on the right-hand side of (4).

In period 0, the profit maximization problem for each bank can be written as

$$\begin{aligned} &\max_{\{k\geq 0,\ a\geq 0\}} R_L k + R_B a - 1 + \frac{1}{2}\theta(a) + \frac{1}{2}\phi(a'), \\ &\text{s.t. } k + a = 1, \end{aligned} \tag{10}$$

where $a'$ denotes the amount of bonds held by the other bank at the end of the period, which each bank takes as given. Under Assumptions 2 and 4, each bank invests into the just enough amount of bonds in period 0 to avoid liquidation of bank loans in period 1:

**Proposition 2.** Suppose Assumptions 1, 2 and 4 hold. Each bank chooses

$$(k,\ a) = \left(1 - a,\ \frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L + \delta}\right)\right), \tag{11}$$

in period 0. Given this value of $a$, the originating bank pays only bonds for the settlement of bank transfers in period 1:

$$(l,\ b) = \left(0,\ \frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L + \delta}\right)\right). \tag{12}$$

*Proof.* See Appendix C. □

Thus, the originating bank must invest into an extra value of bonds besides the face value of bank transfers, $\eta$, because of a hold-up problem in the OTC settlement of bank transfers.

# 4 Liquidity-saving effect of legal tender

Now introduce the central bank into the baseline model with illiquid bank loans. For simplicity, assume the classical dichotomy holds, so that it is sufficient to describe the model in real terms. This section covers two cases. In the first case, the central bank issues interest-bearing liabilities backed by bonds, which corresponds to the Friedman's rule. In the second case, the central bank designates its liabilities as legal tender. It will be shown that the efficiency of the interbank payment system improves only in the second case, as a hold-up problem in the settlement of bank transfers is prevented only in that case.

## 4.1 Introduction of central-bank liabilities just as interest-bearing assets

Suppose that the central bank issues liabilities in exchange for bonds held by banks in period 0. The central bank repays its liabilities by the whole return on its bonds in period 2; thus the central bank issues interest-bearing liabilities. These liabilities are transferable between banks at no physical transaction cost, just like bonds. The central bank cannot accept bank loans, because it does not have enough ability to monitor bank loans.

In this case, bank reserves and bonds are identical assets. Thus, no change occurs to the baseline model. Hence, Propositions 1 and 2 remain to hold.

This result illustrates that the Friedman's rule is not enough to improve the efficiency of the interbank payment system. This result holds because, unlike retail payers, banks can use interest-bearing bonds for payment instruments. Thus, just supplying central-bank liabilities as interest-bearing assets does not make any change to the interbank payment system.

## 4.2 Introduction of central-bank liabilities as legal tender

Next, suppose that the central bank does not only issue interest-bearing liabilities backed by bonds, but also designates its liabilities as legal tender. Consider the following arrangement in period 1. After the realization of bank-transfer requests to each bank (i.e., $\lambda_{i,t}$ for $i = A, B$), the originating bank issues bills of exchange (BOE) to its depositors requesting bank transfers. These depositors, in turn, send BOE to their payees, so that the payees can present BOE to their bank, i.e., the receiving bank. The receiving bank, then, presents BOE to the originating bank (see Figure 2).

If the originating bank has exchanged its bonds for central-bank liabilities of value $\eta$ in period 0, then it can settle its outgoing net bank transfers by paying the central-bank liabilities to the receiving bank in period 1. The receiving bank cannot bargain the terms of settlement in this case, because a creditor is obliged by law to accept legal tender at face value as repayment of the payer's financial obligation.[20] Thus:

**Proposition 3.** If the central bank issues legal tender in exchange for bonds at par value in period 0, then each bank only invests into an amount $\eta/R_B$ of bonds to obtain legal tender of value $\eta$ in period 0.

Comparison between Propositions 2 and 3 implies that the supply of legal tender reduces the amount of liquidity necessary for the settlement of bank transfers. This liquidity-saving effect of legal tender obtains because the use of BOE and legal tender eliminates ex-post bargaining between banks over the settlement of bank transfers. This result coincides with the historical fact that banks issued BOE, such as checks and bank drafts, for depositors to send bank transfers, and then settled the net balances of interbank payment obligations by cash, including gold and legal tender notes.[21]

[20]The definition of legal tender is such that a creditor cannot sue a debtor for non-repayment if a debtor pays his debt in legal tender.

[21]See Cannon (1900, Ch. 6) for the U.S. example.

# 5 Large value payment system as a collateralized interbank settlement contract

Currently, banks settle bank transfers by remitting bank reserves in the large value payment system. While bank reserves are convertible with cash, they themselves are not legal tender.[22] Yet, an originating bank can settle bank transfers unilaterally by remitting the corresponding nominal balance of bank reserves to a receiving bank under a rule set by the central bank.[23] This section replicates this feature of the large value payment system by introducing an interbank settlement contract into the baseline model.

An interbank settlement contract is a contingent contract that specifies the terms of settlement of each possible realization of bank transfers in advance. Such a contract, however, is unverifiable as assumed above; thus, banks must post collateral to make an interbank settlement contract pledgeable without enforcement by the court.

Banks, however, cannot write a pledgeable interbank settlement contract by cross-posting collateral between themselves. Even if banks swap some amounts of bank loans and bonds as collateral between them in period 0, they take an equal amount of collateral from each other, given the ex-ante symmetry between them in period 0. As a result, a bank does not lose anything by reneging on a contract after the realization of bank transfers in period 1, because it can cancel out the collateral taken by, and from, the other bank.

Now consider a contract in which the central bank acts as the custodian of collateral. A contract,

$$f : (\hat{\lambda}_A,\ \hat{\lambda}_B) \in \{\emptyset,\ \lambda_A\} \times \{\emptyset,\ \lambda_B\} \mapsto (b_A(\hat{\lambda}_A,\ \hat{\lambda}_B),\ b_B(\hat{\lambda}_A,\ \hat{\lambda}_B)) \in \mathbb{R}^2_+, \tag{13}$$

maps the outflows of bank transfers reported by bank A, $\hat{\lambda}_A$, and bank B, $\hat{\lambda}_B$, to a contingent flow of bonds, $b_i(\hat{\lambda}_A,\ \hat{\lambda}_B)$, from bank $i$ to the central bank for $i = A, B$. A negative value

[22] Usually, legal tender status is limited to minted coins and central-bank notes. See 31 U.S.C. §5103 for example.

[23] For example, see Regulation J of the Federal Reserve System.

of $b_i(\hat{\lambda}_A,\ \hat{\lambda}_B)$ indicates a flow of bonds from the central bank to bank $i$. If $\hat{\lambda}_i = \emptyset$, then it implies that bank $i$ opts out of the contract in period 1. Otherwise, $\hat{\lambda}_i = \lambda_i$, that is, bank $i$ must report bank-transfer requests from its depositors truthfully. The central bank does not have any endowment in any period. Thus:

$$\sum_{i=A,B} b_i(\hat{\lambda}_A,\ \hat{\lambda}_B) = 0 \quad \text{for all } (\hat{\lambda}_A,\ \hat{\lambda}_B). \tag{14}$$

This constraint implies that the central bank is the central counterparty, which maintains a zero net flow of bonds.

The central bank offers only a symmetric contract between banks, given their ex-ante symmetry in period 0. Hence:

$$\hat{b} \equiv b_A(\eta,\ 0) = b_B(0,\ \eta), \tag{15}$$

$$b_A(0,\ \eta) = b_B(\eta,\ 0) = -\hat{b}, \tag{16}$$

where $\hat{b}$ denotes the value of bonds to be transfered from the originating bank to the receiving bank through the central bank. The central bank aims to maximize each bank's expected profit in period 2.

To implement the contract, the central bank requires each bank to pledge an amount $\hat{b}$ of bonds in period 0. The central bank, then, transfers bond balances between banks according to the contract in period 1. Assume that the central bank can commit to returning the resulting balance of bonds to each bank only in period 2, even if a bank opts out of the contract in period 1. Also assume that the central bank cannot accept bank loans as collateral, so that the role of the central bank does not simply arise from the replacement of illiquid assets with liquid central-bank liabilities through the contract.

Because a contract contingent on the flow of bank transfers is unverifiable, banks can always bail out of the contract without enforcement by the court:

**Assumption 5.** If either bank rejects the offer of the contract in period 0, or opts out of the contract in period 1, then banks settle bank transfers via bilateral bargaining in period 1.

Thus, the central bank cannot enforce the contract if either bank's ex-post payoff from bilateral bargaining is higher than that from the contract. See Table 2 for the summary of the model with an interbank settlement contract offered by the central bank.

## 5.1 Optimal interbank settlement contract

Under Assumption 5, the contract must ensure that the receiving bank does not incur a loss from the settlement of bank transfers, because the receiving bank would not incur a loss even if no bank transfer were settled. Thus,

$$R_B\hat{b} - \eta \geq 0, \tag{17}$$

where the left-hand side is the receiving bank's ex-post payoff from the contract in period 1.

Guess and verify that no bank has incentive to opt out of the contract in period 1. Given this conjecture, the optimal contract problem for the central bank is specified as follows:

$$\begin{aligned} \max_{\{k\geq 0,\, a\geq 0,\, \hat{b}\}} \; & R_L k + R_B a - 1 - \frac{1}{2}(R_B\hat{b} - \eta) + \frac{1}{2}(R_B\hat{b} - \eta), \\ \text{s.t. } & k + a = 1, \\ & R_B\hat{b} - \eta \geq 0, \\ & a \geq \hat{b}, \end{aligned} \tag{18}$$

where: $k$ and $a$ are the amounts of bank loans and bonds, respectively, that each bank invests into in period 0; and $\hat{b}$ is the amount of bonds to be transferred from the originating bank to the receiving bank in period 1. The first constraint is the flow of funds constraint for each bank in period 0. The second constraint is the incentive-compatibility constraint for the receiving bank to remain in the contract in period 1, i.e., (17). The third constraint indicates that each bank must invest into a sufficient amount of bonds to pledge to the central bank under the contract, $\hat{b}$. The solution for this problem is characterized by

$$(k,\ a,\ \hat{b}) = \left(1 - a,\ \hat{b},\ \frac{\eta}{R_B}\right), \tag{19}$$

which is feasible under Assumption 1.

Now let us verify that no bank has incentive to deviate from this contract in period 1. Suppose that banks enter into the contract characterized by (19) in period 0, but one of the banks opts out of the contract to initiate bilateral bargaining in period 1. In this case, banks can transfer only bank loans between them, because the central bank keeps their entire bond holdings until period 2, given $a = \hat{b}$. Thus, the bargaining problem in this case is

$$\max_{\tilde{l} \in [0,k]} [-(R_L \tilde{l} - \eta) - (-\gamma\eta)]^{0.5} (\delta \tilde{l} - \eta)^{0.5}, \tag{20}$$

where the left square bracket and the right parenthesis are the trade surpluses for the originating and the receiving bank, respectively, and $\tilde{l}$ is the amount of bank loans transfered from the originating bank to the receiving bank.

Under Assumption 4, the total trade surplus, $(1+\gamma)\eta/R_L - \eta/\delta$, is negative due to a high loan liquidation cost. Thus, banks do not settle bank transfers outside the contract. Also, each bank has a weakly higher payoff from the contract than no settlement of bank transfers, because the contract characterized by (19) leaves each bank break-even, while the originating and the receiving bank's payoff are $-\gamma\eta$ and 0, respectively, in case of no settlement of bank transfers. Hence, no bank has incentive to opt out of the contract in period 1.

Given Assumptions 2 and 4, the comparison between (11) and (19) implies that banks participate into the optimal contract, (19), in period 0 because they can save the amount of bonds necessary for the settlement of bank transfers. Thus:

**Proposition 4.** Suppose that Assumptions 1, 2, 4, and 5 hold. Banks participate into the interbank settlement contract characterized by (19).

The key reason for this result is that the contract eliminates ex-post bargaining between banks for the settlement of bank transfers, because it specifies the terms of settlement in advance. Also, note that the central bank does not need to commit to confiscating the bonds of a bank opting out of the contract. To implement the contract, it only needs to retain bonds until period 2. Thus, $b_i(\emptyset,\ \cdot) = b_i(\cdot,\ \emptyset) = 0$ for $i = A, B$ in the optimal contract.

### 5.2 Large value payment system as a collateralized interbank settlement contract

This contract replicates the fact that banks can settle bank transfers unilaterally by remitting bank reserves through the large value payment system. This result implies that the system can be characterized as a collateralized interbank settlement contract. It further indicates that the system improves the efficiency of the interbank payment system, because it prevents a hold-up problem in the settlement of bank transfers.

The model also shows that a third party like the central bank must operate the large value payment system, because the system needs a custodian of collateral. In light of this result, bank reserves are the balances of collateral under the custody of the central bank. This result in turn explains why banks swap liquid bonds, such as Treasury securities, for bank reserves through open market operations, despite that both are liquid assets. It is because the acquisition of bank reserves is not the purchase of independent liquid assets, but the submission of collateral to participate into an interbank settlement contract.

In addition, the central bank in the model can implement the contract only by retaining pledged collateral until the end of the settlement of bank transfers. This result is consistent with the fact that the central bank usually does not have a rule to confiscate bank reserves when a bank opts out of the large value payment system, while it also does not guarantee to exchange bank reserves for its assets on demand from banks.

### 5.3 Implementation of the optimal contract by the floor system

The optimal contract in the model shares the key features of the floor system. In the floor system, the central bank supplies a sufficiently large amount of bank reserves for interbank payments in advance, so that banks do not need to borrow bank reserves in the interbank money market. To give banks incentive to hold the supplied amount of bank reserves, the central bank pays interest on bank reserves. Consequently, this interest rate determines the short-term nominal interest rate in the financial market. This system has been adopted by New Zealand since July 2006.

In the optimal contract in the model, banks pledge to the central bank the enough amount of bonds to settle possible bank transfers in the future. Thus, banks do not settle any bank transfer in an OTC interbank market. Also, the central bank passes on to banks the whole return on bonds pledged as collateral. This policy is equivalent to interest payments on bank reserves. Moreover, the interest paid by the central bank equals that on bonds, i.e., the short-term interest rate in the financial market.

This result on the optimality of the floor system is not solely due to the Friedman's rule. While interest payments on bank reserves are necessary to minimize the cost for banks to hold bank reserves, merely replacing liquid bonds with interest-bearing bank reserves does not have any effect, as shown in section 4.1. The optimality of the floor system rests on the presence of the large value payment system, which allows to eliminate the OTC settlement of bank transfers, or an OTC interbank money market, by an ample supply of bank reserves.

It is possible to extend the model to compare the optimal contract with another contract featuring the channel system. In the channel system, the central bank pays interest on a overnight credit position of bank reserves, but imposes a higher interest rate on a overnight debit position of bank reserves. Given the threat point created by the two central-bank interest rates, banks borrow and lend bank reserves overnight in an OTC interbank money market at the end of each day. It can be shown that a contract featuring the channel system requires banks to prepare a larger amount of bonds in period 0 than the optimal contract, because of the active use of the OTC interbank money market. Thus, the floor system has a larger liquidity-saving effect than the channel system. See appendix D for more details.

### 5.4 Robustness of the model to the convertibility between cash and bank reserves

The model abstracts from the fact that banks can convert bank reserves into cash at par any time. Given the status of cash as legal tender, depositors in reality can withdraw cash from banks to pay it to their sellers even if bank transfers are unavailable. Cash payments, however, are imperfect substitutes to bank transfers because of a higher physical transaction cost. Thus, the convertibility between cash and bank reserves does not eliminate a penalty that depositors impose on banks for failed bank transfers. The transaction cost would be also high if banks withdrew cash from the central bank and settled a large value of bank transfers with cash outside the large value payment system. It is implicitly assumed in the model that banks do not have incentive to pay such a high transaction cost to settle bank transfers outside the large value payment system in period 1.

## 6 Can a private clearing house replace the central bank?

So far, the custodian of collateral in the interbank settlement contract has been assumed to be the central bank. Can a private clearing house take over the role of the central bank? The model makes it possible to ask this question, as it derives the central bank's large value payment system as an endogenous contract. This question is also motivated by recent developments of private electronic interbank payment systems, such as the Clearing House Interbank Payment System (CHIPS) and CLS.[24] While CHIPS and CLS are currently only netting mechanisms in which the net balance of bank transfers for each bank is settled through a transfer of bank reserves in each day, their existence implies that it is technologically possible to replace the large value payment system operated by the central

[24]These private interbank payment systems are also called large value payment systems. CHIPS clears large-valued bank transfers related to foreign exchange transactions. The net balances of bank transfers in CHIPS are settled by the transfers of bank reserves at the Fedwire at the end of each day. CLS allows a payment versus payment settlement between multiple currencies. See Kahn and Roberts (2001) for more details on CLS.

bank.

For this question, the model implies that the custodian of collateral must be able to commit to returning collateral to banks after the settlement of bank transfers. Because the resulting balance of collateral depends on the flow of bank transfers for each bank, the non-verifiability of the flow of bank transfers requires the custodian of collateral to make a commitment without a contingent contract enforceable by the court.

In practice, the central bank occasionally releases its assets by absorbing bank reserves through open market operations. This action can be interpreted as the return of collateral to banks. In history, the New York Clearinghouse, the first private clearing house in the U.S. preceding the Federal Reserve, issued specie certificates in exchange for gold, so that member banks could settle checks with specie certificates rather than a physical transfer of gold (Gorton 1984.) Thus, historical evidence suggests that a private clearing house can install a proper governance structure, such as mutual ownership by member banks, to commit to returning collateral to its member banks.

Another issue is the range of eligible collateral. The model implies that the efficiency of the interbank payment system increases as the custodian of collateral accepts a wider range of collateral.

Indeed, collateral policies differ among clearing houses. For example, while the Federal Reserve limits eligible collateral to government-guaranteed securities in normal time, the European Central Bank accepts private securities for a large portion of collateral on a regular basis.[25] As to private clearing houses, CHIPS and CLS accept only bank reserves for collateral.[26] In history, the New York Clearinghouse issued loan certificates against member banks' portfolios during banking panics between 1857 and 1914, i.e., until the Federal Reserve was formed. Banks could use the loan certificates to settle checks among them, so as to save gold for the repayment of deposits during the banking panics.

[25] See Cheun, von Köppen-Mertes, and Weller (2009) for more details.

[26] In these systems, participating banks must prepay bank reserves to run negative net positions of bank transfers in the course of each day.

It is an open question what determines the range of eligible collateral for a clearing house. For this question, an interesting fact is that when the New York Clearinghouse issued loan certificates, it had power to supervise member banks to expell unsound banks from its membership. As described by Gorton (1984), it needed to monitor the financial health of its member banks regularly to prevent troubled member banks from covertly submitting non-performing assets as collateral.[27] Thus, a clearing house's ability to supervise its member banks may determine the range of collateral it can accept.

Currently, CHIPS and CLS are mutually owned by large commercial banks. Given this ownership structure, they may have difficulty in monitoring member banks' portfolios, as large banks are unlikely to reveal detailed balance-sheet information to fellow banks through a clearing house. Also, it may be difficult for a private clearing house to supervise its influential share holders. If these obstacles limit the monitoring ability of a private clearing house, then the central bank established as an independent public institution has advantage over a private clearing house in terms of the range of collateral it can accept. A further investigation into this question is left for future research.

# 7 Conclusions

This paper shows that a hold-up problem in the OTC settlement of bank transfers results in an endogenous need for the large value payment system operated by the central bank. In light of this result, bank reserves are the balances of collateral in a contract characterizing the large value payment system. The central bank plays the role of the custodian of collateral in this contract. The optimal contract can be implemented by the floor system.

To clarify these results in a simple set-up, this paper abstracts from the fiscal cost of inter-

[27] A related issue is tiering. In this arrangement, the central bank limits the membership of its large value payment system to a small number of large banks. As a result, small banks settle bank transfers through the large banks. As analyzed by Kahn and Roberds (2009) and Chapman and Martin (2013), this arrangement can be seen as delegated monitoring by the central bank. Alternatively, this paper's model suggests that tiering occurs if some banks are too small to handle wholesale liquid assets used as collateral in the large value payment system.

est payments on bank reserves for the consolidated government. In the literature, Berentsen, Marchesiani and Waller (2014) compare the channel and the floor system with a financial constraint on the central bank. It is left for future research to analyze the optimal interbank payment system with such a constraint.

It also remains an open question if a private clearing house can replace the central bank. The model implies that it depends on whether a private clearing house can accept a wider range of collateral than the central bank. In this regard, historical experience suggests a possible linkage between a clearing house's ability to supervise member banks and the range of eligible collateral it can accept. In addition, the model abstracts from a financial crisis, focusing on the regular function of the interbank payment system. A further investigation into these issues is left for future research.

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# A Baseline model with a formal assumption about depositors

## A.1 Preference and technology

Time is discrete and indexed by $t = 0, 1, 2$. There are two banks indexed by $i = A, B$. Each bank has a fixed customer base consisting of a unit continuum of risk-neutral depositors. Each depositor is endowed with a unit of goods in period 0. A depositor can save its good in two ways. One is storage technology, in which a depositor can store its good without depreciation or appreciation between consecutive periods. The other is a bank deposit. If a depositor deposits its good in period 0, then the depositor's bank can transform the good into a bank loan or a bond in that period. A bank loan generates an amount $R_L$ of goods in period 2 per invested good. Similarly, the gross rate of return on a bond in period 2 is $R_B$ Assume that

$$R_L > R_B > 1, \tag{21}$$

the last term is the gross rate of return on storage.

Each depositor becomes a buyer or a seller due to an idiosyncratic shock in period 1. A buyer can consume goods produced by sellers at the other bank in period 1, but cannot consume goods in period 2. A seller can produce goods at a unit utility cost per good in period 1, and consume goods in period 2. Each depositor maximizes the following expected utility:

$$U = p_1 c_{b,1} + (1 - p_1)(-h_{s,1} + c_{s,2}), \tag{22}$$

where: $p_1$ is the probability to be a buyer in period 1 for each depositor in period 0; $c_{b,1}$ is the consumption in period 1 in case of becoming a buyer; and $h_{s,1}$ and $c_{s,2}$ are the production in period 1 and the consumption in period 2, respectively, in case of becoming a seller.

## A.2 Deposit contract

Depositors are anonymous to each other; thus, buyers cannot buy goods on credit in period 1. Banks can offer a deposit contract to depositors such that, to pay the price of goods in period 1, buyers can order their bank to remit the corresponding balance of deposits from their accounts, if any, to the sellers' bank accounts in period 1. The goods market in period 1 is competitive: every depositor takes the price of goods as given.

For simplicity, assume that banks cannot commit to any future behavior for depositors. At the same time, also assume that if a bank fails to complete the bank transfers requested by its depositors through deposit contracts, then it must incur a cost $\gamma$ per depositor ($\gamma > 0$). This cost can be interpreted as representing a long-term cost due to loss of reputation, or a cost payable in period 2 due to a litigation filed by depositors for failed payments. In contrast, the cost of failed settlement of bank transfers for the receiving bank, i.e., the bank with $\lambda_i = 0$, is normalized to zero. Thus, the originating bank must pay a higher penalty for failed settlement of bank transfers than the receiving bank. The underlying assumption is that a deposit contract includes the right to send a bank transfer on demand, for which the originating bank is liable, but the receiving bank is not.

Each bank sets the deposit interest rate for its depositors monopolistically in period 0. If a bank reneges on the redemption of a deposit in period 2, then depositors can seize the bank loans and bonds of the bank and convert them into goods in period 2. The gross rate of return on seized bank loans and bonds declines to one due to liquidation cost. Thus, the pledgeable deposit interest rate is zero. To satisfy the participation constraint for depositors, a bank cannot set a deposit rate lower than zero because depositors would be better off by storing goods by themselves in such a case. As a result, banks set the deposit interest rate to zero in period 0.

Neither depositor or bank can generate goods by terminating bank loans or bonds in period 1. Also, depositors cannot seize bonds and bank loans in period 1 due to a high asset management cost for them. Hence, the maturity of deposits comes in period 2.

## A.3 Settlement of bank transfers

The buyer fraction of depositors at each bank is stochastic. At each bank, a fraction $\lambda_i$ of depositors become buyers. The joint probability distribution of $\lambda_A$ and $\lambda_B$ is

$$(\lambda_A,\ \lambda_B) = \begin{cases} (\eta,\ 0) & \text{with probability } 0.5, \\ (0,\ \eta) & \text{with probability } 0.5, \end{cases} \tag{23}$$

where $\eta \in (0, 1)$. Given (2), the unconditional probability for each depositor to be a buyer, i.e., $p_1$, is

$$p_1 = 0.5\eta. \tag{24}$$

Assume that it is too costly to verify the flows of bank transfers in public, because it must breach the privacy of depositors. Thus, banks cannot implement a contingent contract such that the receiving bank increases its deposit liabilities by $\eta$ in period 1 without any compensation from the originating bank. Even if banks swap some amounts of bank loans and bonds as collateral between them in period 0, they take an equal amount of collateral from each other, given the ex-ante symmetry between them in period 0. As a result, a bank does not lose anything by reneging on an unverifiable contingent contract in period 1, because it can cancel out the collateral taken by, and from, the other bank.

Thus, banks need to pay bank loans or bonds to settle bank transfers between them after the realization of $\lambda_A$ and $\lambda_B$ in period 1. Assume that the interbank market is an OTC market; so banks determine the terms of settlement through bilateral bargaining. The outcome of bargaining is determined by Nash bargaining in which each bank has equal bargaining power. If banks do not reach an agreement, then no bank transfer is made. In this case, the originating bank receives a penalty, as assumed above.

Bonds are transferable at no cost between banks. In contrast, if a bank sells its bank loans to the other bank in period 1, then the bank buying the bank loans must monitor the bank loans by itself to generate returns. In this case, the net return per loan in period 2 becomes $\delta$ ($\in (0, R_L]$). The difference between $R_L$ and $\delta$ is due to a loan monitoring cost.Also, assume that a bank cannot commit to monitoring bank loans if its bank loans are submitted to the other bank as collateral for a repo. Thus, a repo and a spot sale are indifferent in the model.

### A.4 Each bank's objective and the definition of equilibrium

In period 2, each bank receives returns on its bank loans and bonds, repays deposits given a zero deposit interest rate, and consumes the residual as its profit. Each bank is risk-neutral, and chooses its portfolio of bank loans and bonds in period 0 to maximize the expected profit in period 2. An equilibrium is a perfect Bayesian Nash equilibrium for the two banks.

## B Proof for (8) and (9)

Given $\delta < R_L$, the first-order conditions for the bargaining problem, (7), with respect to $l$ and $b$ are:

$$-\frac{R_L}{-[R_L l + R_B b - \eta] + \gamma\eta} + \frac{\delta}{\delta l + R_B b - \eta} + \underline{\theta}_l - \overline{\theta}_l = 0, \tag{25}$$

$$-\frac{R_B}{-[R_L l + R_B b - \eta] + \gamma\eta} + \frac{R_B}{\delta l + R_B b - \eta} - \overline{\theta}_b = 0, \tag{26}$$

where $\underline{\theta}_l$, $\overline{\theta}_l$, and $\overline{\theta}_b$ are proportional to the non-negative Lagrange multipliers for $0 \leq l$, $l \leq k$, and $b \leq a$. The Lagrange multiplier for the other constraint, $0 \leq b$, is always zero, because it is positive only if $\underline{\theta}_l > \overline{\theta}_l$. Note that if $\underline{\theta}_l > \overline{\theta}_l$, then $b = l = 0$, under which $\delta l + R_B b - \eta$ is negative.

Given that the denominator in each side is the same across the two conditions and the assumption that $R_L \geq \delta$, $\underline{\theta}_l = \overline{\theta}_l = \overline{\theta}_b = 0$ cannot hold. Thus, there are four cases to consider: $\{l = 1 - a,\ b = a\}$; $\{l \in (0,\ 1 - a),\ b = a\}$; $\{l = 0,\ b = a\}$; and $\{l = 0,\ b \in (0, a)\}$.

In the first case, $\underline{\theta}_l = 0$ and $\overline{\theta}_l \geq 0$. For this case to happen, it must hold that

$$\frac{R_L}{-[R_L(1-a)+R_Ba-\eta]+\gamma\eta} \leq \frac{\delta}{\delta(1-a)+R_Ba-\eta}. \tag{27}$$

Given (26) and the assumption $R_L \geq \delta$, $\overline{\theta}_b > 0$.

In the second case, $\underline{\theta}_l = \overline{\theta}_l = 0$. In this case, (25) implies that

$$\exists l \in (0,\ 1-a),\ \text{s.t.}\ \frac{R_L}{-[R_Ll+R_Ba-\eta]+\gamma\eta} = \frac{\delta}{\delta l+R_Ba-\eta}. \tag{28}$$

Given (26) and the assumption $R_L \geq \delta$, $\overline{\theta}_b > 0$.

In the third case, $\underline{\theta}_l \geq 0$, $\overline{\theta}_l = 0$, and $\underline{\theta}_b \geq 0$. Thus, (26) implies

$$\frac{R_B}{-[R_Ba-\eta]+\gamma\eta} \leq \frac{R_B}{R_Ba-\eta}. \tag{29}$$

Also, (25) implies

$$\frac{R_L}{-[R_Ba-\eta]+\gamma\eta} \geq \frac{\delta}{R_Ba-\eta}. \tag{30}$$

In the fourth case, $\underline{\theta}_l \geq 0$, $\overline{\theta}_l = 0$, and $\underline{\theta}_b = 0$. Hence:

$$\exists b \in (0,\ a),\ \text{s.t.}\ \frac{R_B}{-[R_Bb-\eta]+\gamma\eta} = \frac{R_B}{R_Bb-\eta}. \tag{31}$$

This condition is sufficient for (25) under $l = 0$ and $\underline{\theta}_l \geq \overline{\theta}_l = 0$, given the assumption $R_L \geq \delta$.

Summarizing the four cases, the solutions for $l$ and $b$ under $\delta < R_L$ take the following form:

$$(l(a),\ b(a)) = \begin{cases} (1-a,\ a), & \text{if } R_Ba-\eta \leq \frac{\delta\gamma\eta-2R_L\delta(1-a)}{R_L+\delta}, \\ \left(\frac{\delta\gamma\eta-(R_L+\delta)[R_Ba-\eta]}{2R_L\delta},\ a\right), & \text{if } R_Ba-\eta \in \left(\frac{\delta\gamma\eta-2R_L\delta(1-a)}{R_L+\delta},\ \frac{\delta\gamma\eta}{R_L+\delta}\right), \\ (0,\ a), & \text{if } R_Ba-\eta \in \left[\frac{\delta\gamma\eta}{R_L+\delta},\ \frac{\gamma\eta}{2}\right], \\ \left(0,\ \frac{1}{R_B}\left[\eta+\frac{\gamma\eta}{2}\right]\right), & \text{if } R_Ba-\eta > \frac{\gamma\eta}{2}, \end{cases} \tag{32}$$

if both banks have non-negative trade surpluses in each case.

In the third and the fourth case, it is immediate that both banks have non-negative trade surpluses. In the second case, the necessary and sufficient condition for non-negative trade surpluses for both banks is

$$\delta\gamma\eta + (R_L - \delta)[R_B a - \eta] \geq 0. \tag{33}$$

In the first case, the necessary and sufficient conditions for non-negative trade surpluses are:

$$\gamma\eta \geq R_L(1-a) + R_B a - \eta, \tag{34}$$

$$\delta(1-a) + R_B a - \eta \geq 0. \tag{35}$$

If (33) and (34)-(35) are not satisfied in the second and the first case, respectively, then banks do not settle bank transfers in period 1.

Now show the following lemma:

**Lemma 1.** Under Assumption 1, $(l(a),\ b(a)) = (1-a, a)$ never occurs in equilibrium.

*Proof.* This lemma is equivalent to say that the first case of (32) does not exist for any $a \in [0,1]$, or violates (34) or (35). First, a necessary condition for the existence of the first case is that there exists $a \in [0,1]$ such that

$$R_B a - \eta \leq \frac{\delta\gamma\eta - 2R_L\delta(1-a)}{R_L + \delta}, \tag{36}$$

as implied by (32). Note that both sides of this condition are increasing functions of $a$ and also that the left-hand side is higher than the right-hand side at $a = 1$ under Assumption 1. Thus, there exists $a \in [0,1]$ satisfying (36) if and only if the intercept of the left-hand side is lower than that of the right-hand side:

$$-\eta < \frac{\delta\gamma\eta - 2R_L\delta}{R_L + \delta}. \tag{37}$$

If this condition is violated, then the first case does not exist for any $a \in [0,1]$.

Suppose that (37) holds. This condition is equivalent to

$$(\eta - \delta)R_L > \delta[R_L - (1+\gamma)\eta]. \tag{38}$$

Thus, $\eta > \delta$, and hence $R_B > \delta$, follows given Assumption 1. For the first case to exist in this case, both (34) and (35) must be satisfied. Given $R_B > \delta$, these two conditions can be written as

$$a \geq \max\left\{\frac{R_L - (1+\gamma)\eta}{R_L - R_B},\ \frac{\eta - \delta}{R_B - \delta}\right\}, \tag{39}$$

where the first and the second term in the max operator are derived from (34) and (35), respectively. Under (37) and Assumption 1, it can be shown that:

$$\begin{aligned}
&\frac{R_L - (1+\gamma)\eta}{R_L - R_B} - \frac{\eta - \delta}{R_B - \delta} \\
&\qquad \propto R_L(R_B - \delta) - (1+\gamma)\eta(R_B - \delta) - \eta(R_L - R_B) + \delta(R_L - R_B) \\
&\qquad = R_L(R_B - \eta) + (\eta - \delta)R_B - (1+\gamma)\eta(R_B - \delta) \\
&\qquad = R_L(R_B - \eta) + (\eta - \delta)R_B - (1+\gamma)\eta(R_B - \eta + \eta - \delta) \\
&\qquad = [R_L - (1+\gamma)\eta](R_B - \eta) + (\eta - \delta)[R_B - (1+\gamma)\eta] > 0.
\end{aligned} \tag{40}$$

The inequality holds due to $\eta > \delta$ under (37). Thus, (34) is sufficient for (35) under (37) and Assumption 1.

Finally, show that (34) is violated in the first case of (32), if (37) and Assumption 1 hold. In this case, the first case of (32) can exist only for $a \in [0, a^*]$ such that

$$R_B a^* - \eta = \frac{\delta\gamma\eta - 2R_L\delta(1 - a^*)}{R_L + \delta}. \tag{41}$$

The root for this equation can be explicitly solved as

$$a^* = \frac{R_L(\eta - \delta) - \delta[R_L - (1+\gamma)\eta]}{R_L(R_B - \delta) - \delta(R_L - R_B)}. \tag{42}$$

It can be shown that

$$
\begin{aligned}
& a^* - \frac{R_L - (1+\gamma)\eta}{R_L - R_B} \\
& \qquad \propto R_L(\eta - \delta)(R_L - R_B) - \delta[R_L - (1+\gamma)\eta](R_L - R_B) \\
& \qquad - R_L(R_B - \delta)[R_L - (1+\gamma)\eta] + \delta(R_L - R_B)[R_L - (1+\gamma)\eta] \\
& \qquad = R_L(\eta - \delta)(R_L - R_B) - R_L(R_B - \delta)[R_L - (1+\gamma)\eta] \\
& \qquad \propto \frac{\eta - \delta}{R_B - \delta} - \frac{R_L - (1+\gamma)\eta}{R_L - R_B} < 0, \quad (43)
\end{aligned}
$$

where the last inequality is implied by (40). Thus, $a^*$ is below the lower bound for $a$ that satisfies (34). Hence, banks cannot have non-negative trade surpluses in the first case of (32), if (37) holds. □

It remains to pin down the range of $a$ for the second case of (37). The necessary and sufficient condition for non-negative trade surpluses in the second case of (37), (33), implies

$$
a \geq \frac{1}{R_B}\left(\eta - \frac{\delta\gamma\eta}{R_L - \delta}\right). \quad (44)
$$

As shown in the proof for Lemma 1, (36) never holds for $a \in [0, 1]$ if (37) is violated. Thus, in this case, (44) becomes the lower bound for $a$ in the second case of (37).

If (37) is satisfied, it can be shown that the right-hand side of (44) is greater than $a^*$ in (42), that is, the root for (41):

$$
\begin{aligned}
& \frac{1}{R_B}\left(\eta - \frac{\delta\gamma\eta}{R_L - \delta}\right) - a^* \\
& \qquad \propto \eta[R_L - (1+\gamma)\delta][(R_L + \delta)R_B - 2\delta R_L] - \{\eta[R_L + (1+\gamma)\delta] - 2\delta R_L\}(R_L - \delta)R_B \\
& \qquad = \eta R_L(2\delta R_B - 2\delta R_L) - \eta(1+\gamma)\delta(2R_L R_B - 2\delta R_L) + 2\delta R_L(R_L - \delta)R_B \\
& \qquad = 2\delta R_L\left[-\eta(R_L - R_B) - \eta(1+\gamma)(R_B - \delta) + R_B(R_L - \delta)\right] \\
& \qquad \propto -\eta(R_L - R_B) - \eta(1+\gamma)(R_B - \delta) + R_B(R_L - R_B + R_B - \delta) \\
& \qquad = (R_L - R_B)(R_B - \eta) + (R_B - \delta)[R_B - (1+\gamma)\eta] > 0. \quad (45)
\end{aligned}
$$

The last inequality follows from $R_B > \delta$ under Assumption 1 and (37), as implied by (38). Thus, (44) is the lower bound for $a$ in the second case of (37) regardless of whether (37) is satisfied. Banks do not make any deal in period 1 if the value of $a$ is lower than the right-hand side of (44).

# C Proof for Proposition 2

If $l(a) = 0$ at the optimum of the bargaining problem (10), then each bank chooses the lower bound for $a$ such that $l(a) = 0$, because an increase in the bond holdings only results in a transfer of more bonds in case of an outflow of bank transfers as long as $l(a) = 0$, as implied by (8). Hereafter, denote the lower bound for $a$ such that $l(a) = 0$ by $\hat{a}$.

Next, compare $\hat{a}$ and the value of $a$ such that $l(a) > 0$. For the range of $a$ such that $l(a) > 0$, the objective function in the profit maximization problem for a bank in period 0, (10), can be written as

$$\begin{aligned}\Pi(a,\ a') \equiv R_L(1-a) + R_B a - 1 \\ + \frac{1}{2}\left\{-R_L \frac{\delta\gamma\eta - (R_L+\delta)(R_B a - \eta)}{2R_L\delta} - R_B a + \eta\right\} + \frac{1}{2}\phi(a'). \quad (46)\end{aligned}$$

The derivative of this function with respect to $a$ is

$$\begin{aligned}\frac{\partial \Pi(a,\ a')}{\partial a} &= -R_L + R_B + \frac{1}{2}\left[\frac{R_L(R_L+\delta)R_B}{2R_L\delta} - R_B\right] &(47)\\ &= -R_L + R_B + \frac{1}{2}\frac{R_L-\delta}{2\delta}R_B &(48)\\ &\propto R_L R_B - \delta(4R_L - 3R_B). &(49)\end{aligned}$$

Because the objective function in the profit maximization problem for a bank in period 0, (10), is continuous at $\hat{a}$ and the upper bound for $a$ such that $l(a) > 0$, choosing a value of $a$ such that $l(a) > 0$ is dominated by choosing $a = \hat{a}$ in period 0 if

$$\delta < \frac{R_L R_B}{4R_L - 3R_B}. \quad (50)$$

Finally, find the condition under which choosing $a = \hat{a}$ in period 0 dominates no settlement of outgoing bank transfers. If no settlement of outgoing bank transfers is optimal for a bank, then each bank sets $a = 0$ in period 0 because it does not need any liquidity for interbank settlement in period 1. Thus, choosing $a = \hat{a}$ in period 0 dominates no settlement of outgoing bank transfers if and only if

$$\begin{aligned} R_L \left[1 - \frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L+\delta}\right)\right] + R_B \left[\frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L+\delta}\right)\right] - 1 \\ + \frac{1}{2}\left[-R_B \frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L+\delta}\right) + \eta\right] + \frac{1}{2}\phi(a') \\ > R_L - 1 + \frac{1}{2}(-\gamma\eta) + \frac{1}{2}\phi(a'), \end{aligned} \tag{51}$$

where the left- and the right-hand side are the expected payoff for a bank with $a = \hat{a}$ and $a = 0$, respectively. This condition is equivalent to

$$\delta < \frac{R_L[(2+\gamma)R_B - 2R_L]}{2(R_L - R_B)(1+\gamma)}, \tag{52}$$

because

$$\begin{aligned} -\left(\frac{R_L}{R_B} - 1\right)\left(\eta + \frac{\delta\gamma\eta}{R_L+\delta}\right) + \frac{1}{2}\left(-\frac{\delta\gamma\eta}{R_L+\delta}\right) - \frac{1}{2}(-\gamma\eta) \\ \propto -2\left(\frac{R_L}{R_B} - 1\right)\eta[R_L + (1+\gamma)\delta] + \gamma\eta R_L \\ = -2\left(\frac{R_L}{R_B} - 1\right)\eta(1+\gamma)\delta + \eta R_L\left[\gamma - 2\left(\frac{R_L}{R_B} - 1\right)\right]. \end{aligned} \tag{53}$$

If both (50) and (52) hold, then it is optimal for a bank to choose $a = \hat{a}$ in period 0. Under Assumption 2, (50) is sufficient for (52) as

$$\begin{aligned} \frac{R_L[(2+\gamma)R_B - 2R_L]}{2(R_L - R_B)(1+\gamma)} - \frac{R_L R_B}{4R_L - 3R_B} \\ \propto [(2+\gamma)R_B - 2R_L](4R_L - 3R_B) - 2(R_L - R_B)(1+\gamma)R_B \\ = \gamma R_B(4R_L - 3R_B) - 2(R_L - R_B)(4R_L - 3R_B) \\ - 2(R_L - R_B)(1+\gamma)R_B \\ = (2R_L - R_B)[\gamma R_B - 4(R_L - R_B)]. \end{aligned} \tag{54}$$

Assumption 4, in turn, is sufficient for (50) under Assumption 2, because

$$\frac{R_L R_B}{4R_L - 3R_B} - \frac{R_L}{1+\gamma} \propto R_B(1+\gamma) - (4R_L - 3R_B) \tag{55}$$

$$= -4(R_L - R_B) + R_B\gamma. \tag{56}$$

Thus, Assumptions 2 and 4 are sufficient for $a = \hat{a}$ in period 0.

# D Comparison between the channel system and the floor system

This section confirms the optimality of the floor system in comparison to an alternative modern reserve-supply policy, the so-called channel system. This system has been adopted by Australia, Canada, the Euro area, and the U.K. In this system, the central bank supplies only a tiny, or even no, amount of bank reserves to banks overnight.[28] The central bank, however, allows a large volume of collateralized overdrafts of bank reserves during each day, so that banks can smoothly send bank reserves to each other to settle bank transfers between them. The settlement of bank transfers is final immediately after bank reserves are transferred to the receiving bank, because the central bank guarantees the transfer of bank reserves in any event. At the end of each day, an imbalance between outgoing and incoming bank transfers for each bank results in a distribution of debit and credit positions of bank reserves across banks. If these positions are left overnight, then the central bank charges a higher interest rate on a debit position than the interest rate that it pays on a credit position. Typically, banks arrange overnight loans of bank reserves in the interbank money market, so that no bank has a debit position of bank reserves overnight. The interbank interest rate falls between the two central-bank interest rates on bank reserves.

[28]For example, the target overnight balance of bank reserves in Canada was zero for March 2006 to May 2007.

## D.1 Extension of the model to nest the channel system

To nest the channel system in the model, consider the following contract. In period 0, the central bank requires each bank to pledge an amount $(1 + f)\eta/R_B$ of bonds as collateral, where $f$ is the central-bank interest rate on overnight credit positions of bank reserves, which will be defined below. This collateral corresponds to collateral for an intraday overdraft of bank reserves at the central bank.[29] In period 1, the central bank guarantees the settlement of bank-transfer requests to each bank, $(\lambda_A,\ \lambda_B)$, immediately after they are realized. This assumption reflects the fact that the central bank provides intraday overdrafts of bank reserves and the finalty of bank-reserve transfers in the channel system. If no action is taken afterwards, then the central bank charges the originating bank an interest rate, $f$, on the face value of bank transfers, $\eta$. This interest charge is subtracted from collateral pledged by the originating bank in period 0; thus no collateral is returned to the originating bank in this case. Here, it is assumed that the central bank cannot let a bank to have an overnight debit position of bank reserves unsecured. Also, the central bank adds an amount $(1 + d)\eta/R_B$ of bonds to the receiving bank's collateral, where $d$ is the overnight interest rate on a credit position of bank reserves. Assume $f \geq d \geq 0$, so that the central bank has enough bonds to pass to the receiving bank. The central bank returns the remaining balance of collateral to each bank in period 2.

If the receiving bank sends a reverse transfer of bank reserves, $\eta$, to the originating bank, then the net transfer of bank reserves becomes zero for each bank. In this case, the central bank neither charges or adds interest on any bank's collateral in period 1. Assume that after the realization of bank-transfer requests from depositors in period 1, banks can negotiate over the terms of a reverse transfer of bank reserves through Nash bargaining with equal bargaining power for both parties. This transaction corresponds to an overnight loan in the OTC interbank money market, as the spot sale of bonds, a repo, and an unsecured call loan

[29] The result is the same if each bank invests into this amount of bonds in period 0 and pledges the bonds to the central bank in period 1 only in case that it becomes an originating bank.

are indifferent in the model (see the end of section 3.2 for this property of the model.)

## D.2 Equilibrium with the channel system

Now solve the bargaining problem for a reverse transfer of bank reserves in period 1. This problem can be written as:

$$\max_{b'}\{-(R_B b' - \eta) - (-f\eta)\}^{0.5}[R_B b' - \eta - d\eta]^{0.5}, \tag{57}$$

where $b'$ is the amount of bonds that the originating bank pays to the receiving bank for a reverse transfer of bank reserves. The left curly and the right square bracket in (57) are the trade surpluses for the originating and the receiving bank, respectively. Note that the threat point is determined by interest rates on overnight debit and credit positions of bank reserves, $f$ and $d$, because bank transfers requested by depositors have been settled before the bargaining.[30]

The solution to this problem is

$$b' = \left(1 + \frac{f+d}{2}\right)\frac{\eta}{R_B}. \tag{58}$$

Thus, the interbank interest rate, $(f+d)/2$, falls between the two central-bank interest rates, $f$ and $d$.[31] Hence, paying $b'$ is feasible as it is less than the amount of collateral pledged to the central bank, $(1+f)\eta/R_B$.

[30] The payoffs for the originating and the receiving bank at the threat point are $-[(1+f)\eta - \eta]$ and $(1+d)\eta - \eta$, respectively, because deposit liabilities, $\eta$, are transfered from the originating bank to the receiving bank as a result of the settlement of bank transfers before the bargaining.

[31] To minimize the friction associated with the contract, assume that the originating bank can receive a reverse transfer of bank reserves to pay off an overdraft at the central bank at the same time as it takes back collateral from the central bank and pays it to the receiving bank. In reality, an originating bank in the channel system cannot make these transactions simultaneously. Thus, it needs to pay back bank reserves to the central bank before having a reverse transfer of bank reserves from a receiving bank. See Neville and McVanel (2006) for an example in Canada. The result described below shows that even without such a friction, the channel system is dominated by the floor system.

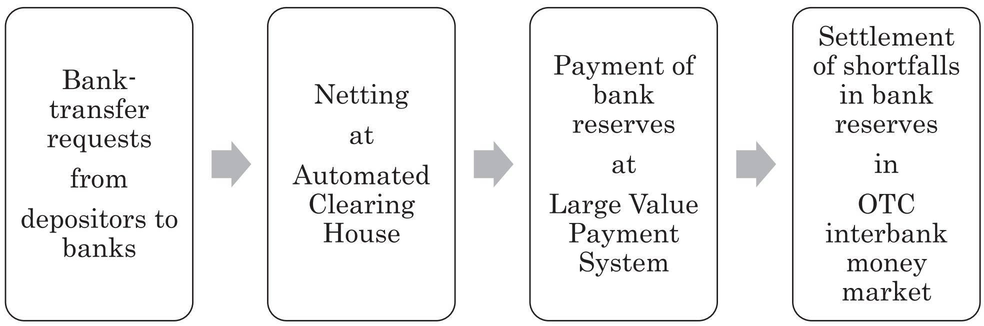
### D.3 Liquidity costs associated with the channel system

Overall, each bank in this contract must invest into an amount $(1+f)\eta/R_B$ of bonds in period 0. To minimize the amount of bonds necessary, the central bank must set $f = d = 0$. Note that this policy eliminates incentive for banks to trade in period 1, and makes the contract equivalent to the optimal contract in the previous section, which represents the floor system. Thus, the floor system dominates the channel system, given that the channel system involves an active OTC interbank money market.

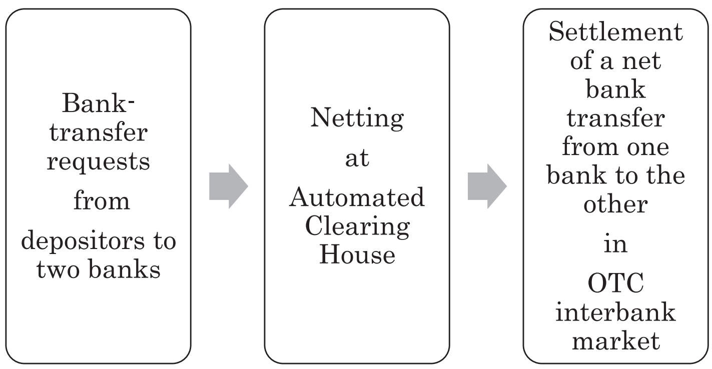
This result illustrates two liquidity costs associated with the channel system. First, allowing banks to have an intraday debit position of bank reserves raises a collateral requirement for banks, because the central bank must make sure that an overnight debit position of bank reserves is always secured even if the borrowing bank is not willing to post additional collateral.

Second, even if the central bank can enforce banks to pay off an intraday debit position of bank reserves without collateral, banks still need to hold the amount of bonds necessary to settle bank transfers in an OTC interbank money market in period 1, i.e., $b'$. If $f$ and $d$ are set positive as is usual in the channel system, then the value of $b'$ exceeds the amount of bonds necessary for the floor system, i.e., $\eta/R_B$. This result holds because raising $f$ and $d$ both strengthens the bargaining position of the receiving bank against the originating bank. Thus, the dual interest rate policy in the channel system does not completely eliminate a hold-up problem in an OTC interbank market as the floor system does. As a result, the liquidity-saving effect of the channel system becomes smaller than that of the floor system.

Figure 1: Comparison between the interbank payment system in reality and the baseline model

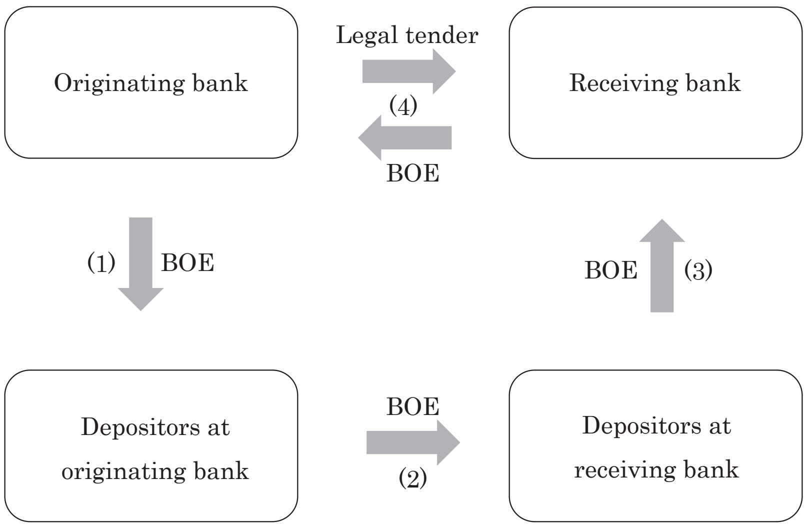



(a) Interbank payment system in reality




(b) Baseline model

Figure 2: Settlement of bank transfers by bills of exchange (BOE) and legal tender




Note: The numbers in parentheses indicate the order of transactions in period 1.

Table 1: Summary of events in the baseline model

| Period 0 | Period 1 | Period 2 |
|---|---|---|
| There are two banks; each bank receives a unit amount of goods from depositors, given a zero deposit interest rate. | One of the banks has an outflow of bank transfers, $\eta$, to the other bank. The probability to be the originating bank is 0.5 for each bank. | Banks receive returns on bank loans and bonds, repay deposits, and consume the residual. |
| Banks invest deposited goods into bank loans and bonds. | A bank must incur a penalty, $\gamma\eta$, if it fails to send bank transfers requested by its depositors within period 1. | The return of goods per loan equals $R_L$ if bank loans are not transferred in period 1, and $\delta$ ($\leq R_L$) if bank loans are transferred in the period. |
| | Banks bargain over how much amounts of bank loans and bonds the originating bank must pay to the receiving bank to settle bank transfers. | The return of goods per bond always equals $R_B$ ($< R_L$). |

Table 2: Summary of the model with an interbank settlement contract

| Period 0 | Period 1 | Period 2 |
|---|---|---|
| There are two banks; each bank receives a unit amount of goods from depositors, given a zero deposit interest rate. | One of the banks has an outflow of bank transfers, $\eta$, to the other bank. The probability to be the originating bank is 0.5 for each bank. | If banks enter into a contract in period 0, the central bank returns the remaining balance of bonds to each bank. |
| Banks invest deposited goods into bank loans and bonds. | A bank must incur a penalty, $\gamma\eta$, if it fails to send bank transfers from its depositors within period 1. | Banks receive returns on bank loans and bonds, repay deposits, and consume the residual. |
| The central bank offers an interbank settlement contract for banks, which requires each bank to pledge bonds to the central bank in period 0. | If neither bank rejected the offer of a contract in period 0 or opts out of a contract in period 1, then the central bank transfers bond balances between banks according to bank-transfer requests reported by each bank, as specified by the contract. | The return of goods per loan equals $R_L$ if bank loans are not transferred in period 1, and $\delta$ ($< R_L$) if bank loans are transferred in the period. |
| | Otherwise, banks bargain over how much amounts of bank loans and bonds the originating bank must pay to the receiving bank to settle bank transfers. | The return of goods per bond always equals $R_B$ ($< R_L$). |