Financial Stability and Optimal Interest-Rate Policy*

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Federal Reserve Board

February 2015

Abstract

We study optimal interest-rate policy in a New Keynesian model in which the economy is at risk of experiencing a financial crisis and the probability of a crisis depends on credit conditions. The optimal adjustment to interest rates in response to credit conditions is (very) small when the model is calibrated to match an estimated historical relationship between credit conditions, output, inflation and the likelihood of financial crises. Given the imprecise estimates of a number of key parameters, we also study optimal policy taking parameter uncertainty into account. We find that both Bayesian and robust central banks will respond more aggressively to financial stability risks when the probability and severity of financial crises are uncertain.

JEL Classification: E43, E52, E58, G01
Keywords: Financial Crises, Leverage, Optimal Policy

*The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of anyone else associated with the Federal Reserve System. Miguel Acosta, Timothy Hills and Jonathan Yu provided excellent research assistance. Andrea Ajello: andrea.ajello@frb.gov. Thomas Laubach: thomas.laubach@frb.gov. David López-Salido: david.lopez-salido@frb.gov. Taisuke Nakata: taisuke.nakata@frb.gov

1 Introduction

We study optimal monetary policy in a New Keynesian model augmented with a two-state crisis shock, which we interpret as the possibility of a financial crisis, and an endogenously time-varying crisis probability.[1] In this situation, when confronted with the possibility of a financial crisis, the policymaker faces a new intertemporal trade-off between stabilizing current real activity and inflation in normal times and mitigating the possibility of a future financial crisis. The adjustment to the policy rate that is optimal, compared to a setting without financial stability concerns, depends on three sets of parameters: the costs of suffering a financial crisis (and thus the benefits of avoiding this fate), the marginal effect of the policy rate on the crisis probability, and the current output and inflation losses arising from a policy response that averts future financial stability risks.

In our New Keynesian model the economy is at risk of experiencing a financial crisis and the probability of a crisis depends on credit conditions, as in Woodford (2012b). To make the exploration empirically relevant, we calibrate the relationship between the likelihood of financial crises and credit conditions to the U.S. experience, borrowing and adapting recent evidence on the cross-country historical data of Schularick and Taylor (2012). Our theoretical analysis shows that the optimal adjustment in the policy rate that arises from financial stability risks is (very) small, less than 10 basis points, when the model is calibrated to match the (estimated) historical relationships between credit conditions, output, inflation as well as the likelihood of a financial crisis.[2]

Nevertheless, reflecting the infrequent nature of crises episodes, the evidence linking credit conditions to financial crises and the effectiveness of interest-rate policy in preventing or reducing the impact of crises are subject to substantial uncertainty. More precisely, we find that a number of key parameters that control the transmission channel of monetary policy appear to be imprecisely estimated in the data. For this reason, we first consider the sensitivity of the optimal policy to alternative parameter values, and then analyse how the optimal policy is affected if the policymaker is confronted with uncertainty about some of parameters of the model.

Under alternative (plausible) assumptions regarding the value of key parameters, the optimal policy can call for larger adjustments to the policy rate than in a situation without financial stability concerns. For example, if we assume that the adjustment in the policy rate is two standard deviations more effective in reducing the crisis probability than in the baseline specification, the optimal adjustment in the policy rate can be as large as 50 basis points. Moreover, if we assume that the effects of a financial crisis on inflation and the output gap are comparable in magnitude to those observed during the Great Depression—as opposed to the Great Recession scenario used as

[1] Throughout the analysis we assume that the only policy tool available to the central bank is the short-term interest rate.

[2] Svensson (2014) uses the Riksbank DSGE model to perform a similar analysis and argues that the cost of "leaning against the wind" interest-rate policies in terms of current real activity far exceeded the benefits of financial stabilization in the recent Swedish experience. Clouse (2013) instead finds that policymakers may seek to reduce the variance of output by scaling back the level of accommodation in a stylized two-period model that is similar to ours in which loose interest-rate policy today can generate sizable future losses in output.

our baseline—the optimal policy will call for a riskless short-term interest rate that can be around 75 basis points higher than what would be optimal in the absence of financial stability concerns.

We then consider how the optimal policy is affected if the policymaker is uncertain about three sets of parameters. First, we look at uncertainty regarding the relationship between the crisis probability and aggregate credit conditions. As discussed in our empirical analysis, the parameters governing this relationship are estimated with wide confidence intervals, reflecting the infrequent nature of crises in history. Second, we consider uncertainty regarding the severity of the crisis. Recent studies have documented a large dispersion in the severity of crisis episodes across countries and times.[3] Finally, we look at uncertainty regarding the extent to which changes in the policy rate affects today's inflation and output. These parameters are subject to uncertainty since the structure of the economy and the monetary policy transmission channel can change over time.

We frame our optimal-policy problem under uncertainty following both of these approaches and consider two types of policymakers. The first type is a Bayesian central bank that aims to maximize the expected welfare of the economy for a given prior distribution of the parameters of the model. This approach originated from the seminal work of Brainard (1967).[4] We follow more recent work by Brock, Durlauf, and West (2003), Cogley, De Paoli, Matthes, Nikolov, and Yates (2011) and Svensson and Williams (2007) that incorporate Bayesian uncertainty into a linear quadratic framework and characterize optimal policy. This approach typically implies that the optimal policy exhibits some form of *attenuation*, as in Brainard (1967), compared with the case of no uncertainty, although this result has some exceptions.[5]

The second type of policymaker is a central bank that uses robust control methods aimed at protecting against worst-case scenarios. To do so, the central bank minimizes the maximum loss over a set of parameters, including those with only a low probability of being realized. Thus, an optimal policy is robust in the sense that it performs best in the worst-case configuration around the (single) reference model, providing a form of insurance against the least favorable scenarios. As in the case of Bayesian approach to model uncertainty, Brainard's principle can be overturned in this context: the robust policymaker will achieve higher welfare by responding more strongly in advance to forestall the development of future unfavourable outcomes (see Onatski and Stock (2002), and Tetlow and von zur Muehlen (2001), and Giannoni (2002)). That is, in this case, optimal policy might result in a more aggressive response than in the certainty-equivalent case.[6]

[3] See, for example, Reinhart and Rogoff (2009), Reinhart and Rogoff (2014), Jórda, Schularick, and Taylor (2013) and Romer and Romer (2014).

[4] A first step in the implementation of a Bayesian approach consists of building a crisp set of alternative elements of the transmission mechanism, or alternatively how different economic theories disagree over fundamental aspects of the economy. Then, modeling uncertainty requires the specification of a prior distribution over the space of models, and then propagates this uncertainty to the analysis of monetary policy problem by integrating monetary policy and models out from the posterior distribution. This is what is called Bayesian Model Averaging (e.g., Brock, Durlauf, and West (2003)).

[5] While Brainard's analysis is conducted in a static framework, in the dynamic models of Söderstrom (2002) and Giannoni (2002), for example, uncertainty about the persistence of inflation implies that it is optimal for the central bank to respond more aggressively to shocks than if the parameter were known with certainty. In our framework these intertemporal dimensions will arise endogenously from the effects of future likely crises on current outcomes.

[6] To our knowledge, none of the existing studies have considered the nonlinearity coming from the presence of

As discussed above, we examine three forms of uncertainty faced by the Bayesian and robust policymakers. First, our main finding is that uncertainty about the effectiveness of the interest-rate policy in reducing the probability of a crisis leads both the Bayesian and the robust policymakers to increase the policy rate by more than in the absence of uncertainty, so that the attenuation principle of Brainard (1967) fails. In the model with a Bayesian policymaker, the key to this result is related to the nonlinear properties of our crisis probability function: in our model the economy's likelihood of facing a financial crisis is increasing and convex in aggregate credit conditions and a higher sensitivity to aggregate credit conditions can make the probability of a crisis increase more rapidly for a given change in credit conditions. Uncertainty around this sensitivity parameter tends to make the *expected* probability of a crisis higher and more responsive to credit conditions and hence to the central bank's interest rate policy. In this context, given the higher marginal benefit associated with a tighter policy in lowering the expected future crisis probability (by reducing the availability of credit), the policymaker optimally decides to set the nominal rate higher than in the absence thereof. The same policy prescription follows from a robust control perspective since the hypothetical evil agent inside the head of the (robust) policymaker can maximize the welfare loss by increasing the sensitivity of the crisis probability to credit conditions.

Second, in the face of uncertainty about the severity of the crisis, measured in terms of output gap and inflation variability, the same result holds: This type of uncertainty leads both the Bayesian and the robust policymakers to set the policy rate higher than otherwise. In the model with a Bayesian policymaker, this result is driven by the nonlinearity of his/her quadratic utility function. In the model with the robust policymaker, this result is more general and does not hinge on the specification of a quadratic loss function.

Third, in the face of uncertainty about the response of today's inflation and output to the policy rate—the same uncertainty considered in Brainard (1967)—the attenuation principle holds for both types of policymakers: the presence of uncertainty leads policymakers to adjust the policy rate by less than otherwise.

The rest of the paper is organized as follows. Section 2 describes the model and discusses the parameterization used in our simulation exercise. Section 3 presents the results based on the baseline and some alternative calibrations. Section 4 formulates the problem of both Bayesian and robust-control policymakers and presents the results on how uncertainty about the parameters affects our previous prescriptions regarding optimal interest rate policy in the presence of financial stability concerns. A final section concludes. Extra material—including modeling, econometric analyses and an extension of the analysis that accounts for the presence of the zero lower bound constraint—is presented in different appendices at the end of the paper.

financial crises on the (optimal) nominal risk-free interest rate. In the appendix we sketch some of the potential implications for robust optimal policy of the effective lower bound on the short-term interest rate.

2 Financial Crises in a Simple New-Keynesian Model

Our stylized framework is a standard new-Keynesian sticky-price model augmented with an endogenous financial crisis event. The crisis follows a Markov process, with its transition probability governed by the evolution of aggregate financial conditions. Based on recent empirical work discussed below, we assume that periods of rapid credit growth raise the probability of transitioning from the non-crisis to the crisis state. In this sense, this basic setup closely resembles Woodford (2012a), but reduces the infinite horizon of that model to a two-period framework, which is convenient for computational and expositional reasons.[7] Thus, our framework allows us to evaluate the connection between optimal stabilization policy and the role of financial conditions that could potentially trigger a financial crisis.

2.1 Economic Structure and Policy Objectives

The following three equations describe the dynamics of the output gap y, inflation π, and credit conditions L.

$$y_1 = E_1^{ps} y_2 - \sigma \left[i_1 - E_1^{ps} \pi_2 \right] \tag{1}$$

$$\pi_1 = \kappa y_1 + E_1^{ps} \pi_2 \tag{2}$$

$$L_1 = \rho_L L_0 + \phi_i(i_1) + \phi_y y_1 + \phi_\pi \pi_1 + \phi_0. \tag{3}$$

From equation (1), the output gap in period one (y_1) depends on the expected output gap in period $t = 2$ ($E_1^{ps} y_2$), and on deviations of the period-one real rate, defined as $[i_1 - E_1^{ps} \pi_2]$, from its long-run equilibrium level (the relation between the private sector's expectations operator E^{ps} and rational expectations will be discussed below). From equation (2), inflation in period $t = 1$ depends on the current output gap and expected future inflation; while from equation (3), financial conditions in period $t = 1$ depend on their value in period $t = 0$, on the output gap, and on deviations of the nominal interest rate and inflation from their long-run targets. In particular, π denotes the deviation of inflation from the policymaker's inflation target; and i is the deviation of the riskless short-term nominal interest rate (the policy rate) from its long-run equilibrium rate. L is a proxy for aggregate credit conditions in the model. We choose L to be 5-year cumulative growth rate of real bank loans, expressed in decimal percentages (e.g., 0.2 corresponds to a 20% cumulative credit growth over the past 5 years). Credit conditions in period $t = 1$ depend on a constant ϕ_0 and on time $t = 0$ initial conditions L_0. Credit conditions can also respond to the current risk-free rate, i_1, the output gap, y_1, and inflation gap, π_1. We describe the choice of L in detail in the following section and relate it to the empirical literature on early predictors of financial crises.

To keep the analysis focused, we abstract from any direct effect of credit conditions on the

[7] It is computationally intensive to solve the infinite horizon version of the model due to the inherent nonlinearities introduced by the financial crisis and the zero lower bound. As discussed below, by suitably parameterizing the welfare loss in the "future" period, the model results can be interpreted as if they had been generated in the standard infinite-horizon setting. We plan to analyze the infinite-horizon version of this model in our future research.

output gap and inflation.[8] Instead, credit conditions only affect the probability γ_1 that controls the likelihood of the transition to a crisis state in period $t = 2$. Credit conditions, L_1, affect γ_1 according to the logistic function:

$$\gamma_1 = \frac{\exp(h_0 + h_1 L_1)}{1 + \exp(h_0 + h_1 L_1)} \tag{4}$$

where h_0 pins down the intercept probability when $L_1 = 0$, and h_1 is the sensitivity of the crisis probability to credit conditions.

Let $\pi_{2,c}$ and $y_{2,c}$ denote inflation and the output gap in the crisis state, while $\pi_{2,nc}$ and $y_{2,nc}$ denote their non-crisis-state values. Then inflation and the output gap outcomes in period $t = 2$ will take values:

$$(y_2, \pi_2) = \begin{cases} (y_{2,nc}, \pi_{2,nc}), & \text{with probability } = 1 - \gamma_1 \\ (y_{2,c}, \pi_{2,c}), & \text{with probability } = \gamma_1 \end{cases}$$

with $\pi_{2,c} < \pi_{2,nc} = 0$ and $y_{2,c} < y_{2,nc} = 0$.

Throughout our analysis, we assume that the private sector treats γ_1 as fixed and negligible in size and not as a function of L_1, implying that in this regard expectations are not rational. If expectations of γ_1 were modeled as rational, times of plentiful credit conditions would be associated with *reductions* of output and inflation, because the increased crisis probability reduces expected future inflation and output gaps, leading to lower inflation and a lower output gap today in the absence of any adjustment in the policy rate. This result seems inconsistent with much empirical evidence suggesting that times of buoyant financial conditions tend to be associated with private agents' expectations that these conditions will continue going forward (Shiller (2005), Shiller (2006)).[9]

We find evidence in support of this assumption in data from the Survey of Professional Forecasters (SPF) on expectations of future GDP growth and inflation. Appendix A shows that the median forecaster in the SPF assigned a probability close to 0% to the event that average real GDP and CPI inflation could fall during the course of 2008, in each quarter he was asked to forecast them over the course of 2007 and 2008. Similarly, the median SPF forecaster reported probabilities below 2% when asked to forecast the likelihood of negative growth for average real GDP in 2009, at least until the collapse of Lehman Brothers in 2008:Q3. Only at that point –between 2008:Q3 and in 2008:Q4– as more information on the severity of the financial crisis became available, did the median forecasted probability of negative growth and the median forecasted probability of CPI deflation in 2009 increase from 2% to 55% and from 0% to 10% respectively (see figures 13 to 16 in the appendix). We interpret these findings as evidence that expectations of financial market participants on the likelihood of a financial crisis and a prolonged downturn adjusted with a lag to the unfolding of the events over the course of the Great Recession, rather than responding preemptively, for example to the accumulation of financial imbalances over the course of the economic

[8] Appendix C discusses an extension of our model in which credit conditions have a positive effect on the output gap.

[9] Rational expectations will induce an extreme forward-looking precautionary saving component into the private sector intertemporal decision, inducing the optimal policy to be preemptively accommodative (Woodford, 2012a).

expansion of the 2000s.

In summary, we assume that private agents perceive the probability of the crisis to be different from γ_1 and to be constant and potentially negligible, i.e. a *tail-event*. Formally, we assume the following rule regarding private sector expectations:

$$E_1^{ps} y_2 = (1-\epsilon) y_{2,nc} + \epsilon y_{2,c} \tag{5}$$

$$E_1^{ps} \pi_2 = (1-\epsilon) \pi_{2,nc} + \epsilon \pi_{2,c} \tag{6}$$

where ϵ is arbitrarily small and does not depend on aggregate credit conditions.

Let WL denote the policymaker's loss function. The policy problem consists on choosing in period $t = 1$ the policy rate given initial credit conditions, L_0, the only endogenous state variable of the model. Formally, the problem of the central bank at time $t = 1$ is given by:

$$WL_1 = \min_{i_1} \; u(y_1, \pi_1) + \beta E_1[WL_2] \tag{7}$$

subject to the previous private sector equilibrium conditions (1) to (3), where

$$u(y_1, \pi_1) = \frac{1}{2}(\lambda y_1^2 + \pi_1^2) \tag{8}$$

and $WL_{2,c}$ and $WL_{2,nc}$ denote the welfare losses in the crisis and non-crisis states, respectively. $WL_{2,c}$ is related to inflation and the output gap in the crisis state by

$$WL_{2,c} = \frac{u(y_{2,c}, \pi_{2,c})}{1 - \beta\mu} \tag{9}$$

where μ is a parameter calibrated to capture the effects of the duration of financial crises on output and inflation, expressed in utility terms. This scaling-up is aimed at ensuring that the costs of financial crises are appropriately captured in our two-period framework.[10] The expected welfare loss at time $t = 2$ is then given by:

$$E_1[WL_2] = (1 - \gamma_1) WL_{2,nc} + \gamma_1 WL_{2,c} \tag{10}$$

2.2 Parameter Values

Table 1 shows the baseline parameter values. The values for the parameters pertaining to the standard New Keynesian model are chosen to be consistent with many studies in the literature, such as Woodford (2003). The annual inflation target is assumed to be 2 percent, and hence our

[10] One potential shortcoming of our two-period framework is that it may not take full account of the effects of the policy rate setting in period 1 on the crisis probability many periods into the future. In particular, our empirical estimates suggest that L is a highly persistent process, so that tolerating an increase in L in the current period raises the crisis probability persistently. The scaling parameter μ is meant to account for the the mean duration of financial crises, but will not capture the implications of a persistent increase in the crisis probability for optimal policy today. The results from the Great Depression calibration in section 3.2 may provide some approximation of this effect.

Table 1: Baseline Parameter Values

Parameter	Description	Parameter Value	Note
"Standard" Parameters			
β	Discount Factor	0.995	Standard
σ	Interest-rate sensitivity of output	1.0	Standard
κ	Slope of the Phillips Curve	0.024	Standard
λ	Weight on output stabilization	1/16	Equal weights on y and the annualized π
i^*	Long-Run Natural Rate of Interest	0.01	4% (Annualized)
Parameters for the equation governing the crisis probability			
h_0	Constant term	-3.396	
h_1	Coefficient on L	1.88	
Parameters for the equation governing the financial conditions			
ρ_L	Coefficient on the lagged L	19/20	
ϕ_0	Intercept	$(1-\rho_L)*0.2$	
ϕ_y	Coefficient on output gap	0.18	
ϕ_π	Coefficient on inflation gap	-1	
Parameters related to the second period			
$y_{2,nc}$	Output gap in the non-crisis state	0	
$\pi_{2,nc}$	Inflation gap in the non-crisis state	0	
$WL_{2,nc}$	Loss in the non-crisis state	0	
$y_{2,c}$	Output gap in the crisis state	-0.1	"Great Recession"
$\pi_{2,c}$	Inflation gap in the crisis state	$-0.02/4$	"Great Recession"
μ	Persistence of the crisis state	7/8	
$WL_{2,c}$	Loss in the crisis state	$\frac{u(y_{2,c},\pi_{2,c})}{1-\beta\mu}$	
Auxiliary parameters			
ϵ	Perceived crisis probability	0.05/100	Arbitrarily small

choice of the long-run equilibrium policy rate, i^*, of 4 percent implies an equilibrium real short-term rate of 2 percent. The weight $\lambda = \frac{1}{16}$ in the central bank's period loss function implies equal concern for annualized inflation gaps and output gaps.[11] We do not attempt to derive this objective from a representative household's utility, but are instead interested in the question of how a policymaker who pursues this objective would want to alter the macroeconomic stabilization in response to financial stability risks.

In the remainder of this section we will discuss the calibration of the probability of a financial crisis, γ_1, and the evolution of the credit conditions index, L. These are the parameters that influence our results most strongly and that may be considered more controversial in the debate about the appropriate response of interest rate policy to financial stability concerns. Finally, we will also discuss the choice of parameters that affect the severity of the crisis, a key determinant of

[11] In Appendix F we consider an alternative value for λ that is consistent with the one obtained under a second-order approximation of welfare, as in Woodford (2003).

the welfare losses associated with a crisis state.

2.2.1 A Simple Model of Crisis Probability and Credit Conditions: the U.S. Experience.

The ability to predict events such as currency, fiscal and financial crises by means of econometric models is hindered by the rarity of such episodes in the history of both advanced and emerging economies. Schularick and Taylor (2012) make a thorough attempt to understand the role of bank lending in the build-up to financial crises, using discrete choice models on a panel of 14 countries over 138 years (1870 - 2008). The paper characterizes empirical regularities that are common across crisis episodes for different countries and over time, trying to identify early predictors of financial crises. We use their data and analysis to inform the parameterization of our model.[12]

Schularick and Taylor (2012) assume and test that the probability of entering a financial crisis can be a logistic function of macro and financial predictors. Their baseline logit specification finds that the five annual growth rates of bank loans from $t-4$ to t are jointly statistically significant predictors of episodes of financial instability that start in period $t+1$. Other variables, such as measures of real activity, inflation, or stock price gains, have little explanatory power when added to their baseline regression that includes lagged real bank loan growth, suggesting that financial crises are in fact "credit booms gone bust."[13]

Let B_t denote the level of bank loans to domestic households and nonfinancial corporations (henceforth the "nonfinancial sector") in year t, and P_t the price level. Using the dataset of Schularick and Taylor (2012), we estimate a slightly simplified version of their model, in which the probability of a financial crisis occurring in country i and year t is $\gamma_{i,t} = \exp(X_{i,t})/(1+\exp(X_{i,t}))$, and $X_{i,t}$ is assumed to be related linearly to the financial condition variable, L_t:

$$X_{i,t} = h_0 + h_i + h_1 L_t \tag{11}$$

where h_0 is an intercept, h_i denote country-fixed effects and h_1 is the sensitivity of the crisis probability to the regressor L_t, as in model equation 4[14]. We define our predictor of financial crises episodes occurring at time $t+1$ as the 5-year cumulative growth rate of real banking loans from time $t-4$ to t:

$$L_t^a = \Sigma_{s=0}^{4} \Delta \log \frac{B_{i,t-s}}{P_{i,t-s}} \tag{12}$$

Appendix B provides details on how we adapt Schularick and Taylor's logit estimates, which are

[12] Schularick and Taylor (2012) also study how the role of monetary policy in sustaining aggregate demand, credit and money growth has changed after the Great Depression.

[13] Among related studies, Laeven and Valencia (2013) collect a comprehensive database on systemic banking crises and propose a methodology to date banking crises based on policy indices. Gourinchas and Obstfeld (2012) provide a similar study including developing countries and currency crisis episodes over the years 1973 - 2010. They find the share of aggregate credit over GDP to be a statistically significant predictor of financial and currency crises. Krishnamurthy and Vissing-Jorgensen (2012) note that short-term lending constitutes the most volatile component of credit over GDP and find that it plays a significant role in event-study logit regressions.

[14] For identification purposes the coefficient h_i for the United States is set to 0.

based on annual data, to U.S. quarterly data to calibrate the probability that a financial crisis materializes in period $t = 2$, labeled γ_1 in our model. We confirm that the variable L_t^a is a statistically significant predictor of financial crises for Schularick and Taylor's panel of countries. Equation (12) can be rewritten in quarterly form as the 20-quarter sum:

$$L_t^q := \sum_{s=0}^{19} \Delta\log\frac{B_{t-s}}{P_{t-s}}. \tag{13}$$

To limit the number of state variable of our model and help reduce the computational burden to find its solution, we approximate equation (13) by the recursive sum:[15]

$$L_t^q \approx \Delta\log\frac{B_t}{P_t} + \frac{19}{20}L_{t-1}^q \tag{14}$$

The components of the recursive sum in (14) are the quarterly growth rate of real bank loans that can be expressed as the difference between the nominal growth rate of bank loans and quarterly inflation:

$$\Delta\log\frac{B_t}{P_t} = \Delta\log B_t - \pi_t \tag{15}$$

To close the model, we estimate a reduced form equation governing the evolution of quarterly nominal credit growth, ΔB_t, on U.S. data for the post-war period. We begin by assuming that the quarterly growth rate of nominal bank loans depends on a constant, c, and can vary with the monetary policy instrument. i_t, and with the output gap, y_t:

$$\Delta\log B_t = c + \phi_i i_t + \phi_y y_t + \varepsilon_t^B \tag{16}$$

Estimating this reduced-form equation for growth of bank lending does not allow us to separately identify how shifts in the demand and supply of credit translate into loan growth. Moreover, the direction of causality between the left- and right-hand-side variables can be questioned. To ameliorate a potential simultaneity bias, we use lagged values of i_t and y_t as instruments for their current values. As discussed in Appendix B, we find that the coefficient on the policy rate is statistically insignificant, and therefore drop this term and reestimate the equation in the restricted form

$$\Delta\log B_t = c + \phi_y y_t + \varepsilon_t^B \tag{17}$$

Combining equations (17), (14), and (15), we obtain a simple dynamic equation describing the evolution of our credit conditions variable, L_t:

$$L_t \approx \rho_L L_{t-1} + \phi_0 + \phi_y y_t + \phi_\pi \pi_t + \varepsilon_t \tag{18}$$

[15] Figures 17 and 18 in the appendix display the differences between the financial condition indicators in equations (12), (13) (14) over the period 1960Q1-2008Q4.

which we adapt to our 2-period model notation as:

$$L_1 \approx \rho_L L_0 + \phi_0 + \phi_y y_1 + \phi_\pi \pi_1 + \varepsilon_1 \tag{19}$$

The value taken by L_1 enters the probability γ_1 that a financial crisis occurs between period 1 and 2, defined in equation 4. The estimated parameters of equations (19) and (4) are shown in the middle panel of Table 1. In particular, our estimates for h_0 and h_1 suggest that an increase of 10 percentage points of the 5-year real banking loan growth from 20% to 30% raises the annual probability of a financial crisis by one percentage point, from 4.9% to 5.6%. For robustness, in section 3.2 we consider alternative parameterizations in which the crisis probability is more responsive to the changes in credit conditions and economic outlook and (indirectly) to changes in the policy rate. For example, we describe optimal monetary policy decisions under higher sensitivity of the crisis probability to credit conditions, h_1, and higher sensitivity of credit conditions to the output gap, ϕ_y.

2.2.2 The Severity of the Crisis in the Baseline Calibration

Inflation and the output gap in the crisis state are chosen to roughly capture the severity of the Great Recession. In particular, we follow Denes, Eggertsson, and Gilbukh (2013) and assume that a financial crisis leads to a 10 percent decline in the output gap ($y_{2,c}$) and a 2 percent decline in inflation ($\pi_{2,c}$). We assume the expected duration of the crisis to be 8 quarters. The continuation loss in the crisis state, $WL_{2,c}$, is determined by the crisis-state inflation and output gap, as well as the by expected crisis duration. In section 3.2, we offer a sensitivity analysis that is intended to capture the severity of a Great-Depression-like scenario.

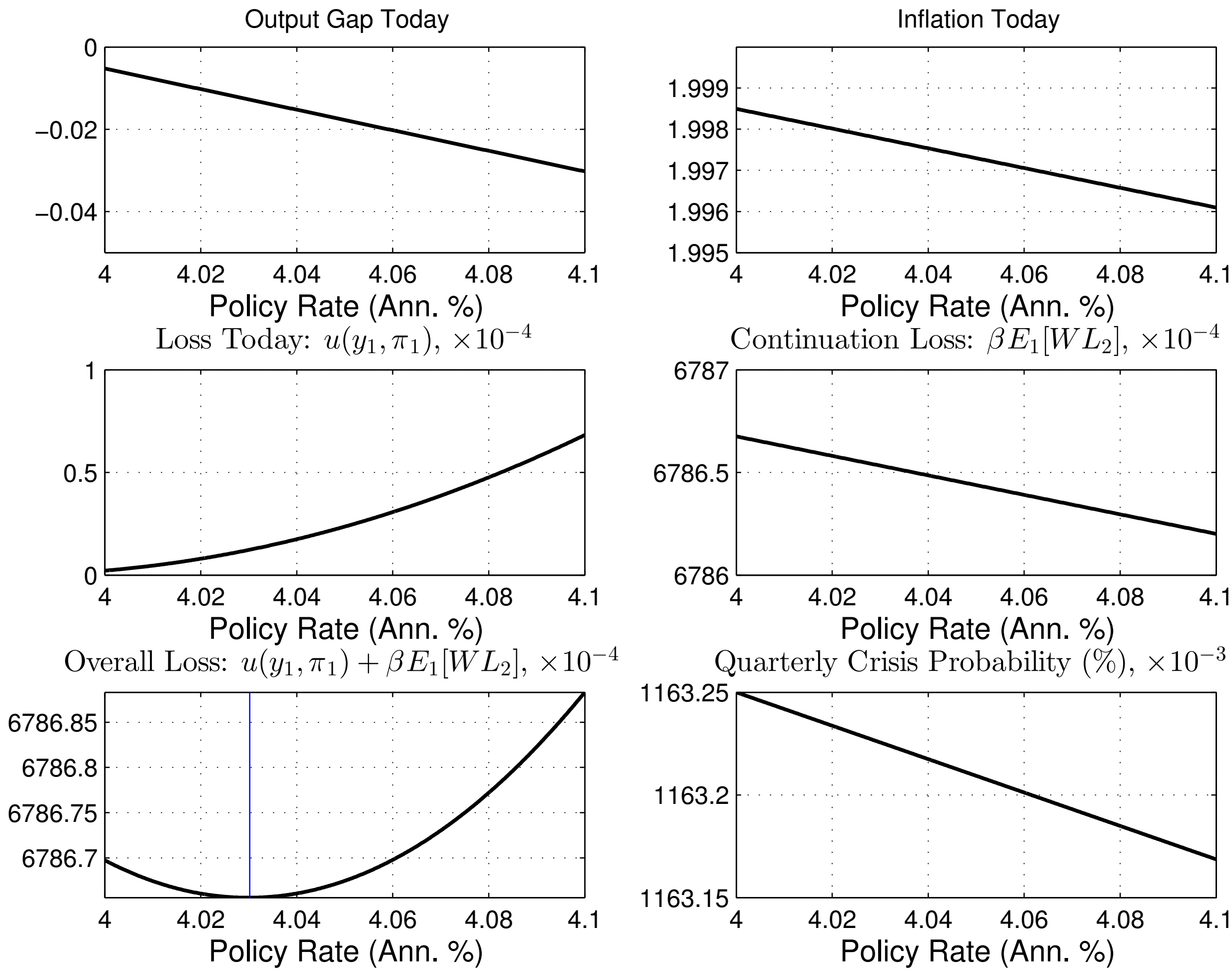
3 Optimal Policy and Financial Instability

In this section we describe the trade-off faced by the policymaker and describe the optimal policy results under our baseline calibration. We also perform some sensitivity analyses by varying key parameters that affect the monetary policy transmission in the model.

3.1 A Key Intertemporal Trade-off

We begin by illustrating the nature of the trade-off the central bank faces in choosing the policy rate in the model. For that purpose, the top two panels of Figure 1 show how the policy rate affects the output gap and inflation today. The middle panels shows how the policy rate affects today's loss (as a function of output gap and inflation today) and the continuation loss. The bottom-left panel shows how the policy rate affects the overall loss function, which is the sum of today's loss and the continuation loss. Finally, the bottom-right panel shows how the policy rate affects the probability that a financial crisis can occur tomorrow. In this figure, L_0 is set to 0.2, roughly corresponding to the average value of this variable in the U.S. over the past five decades.

Figure 1: A Key Trade-off Faced by the Central Bank



NOTE: In this figure, L_0 is set to 0.2, which is roughly the average value of this variable in the U.S. over the past five decades. In the bottom-left panel, the blue vertical line shows the optimal policy rate—the policy rate that minimizes the overall loss. The welfare losses are expressed as the one-time consumption transfer at time one that would make the household as well-off as the household in a hypothetical economy with efficient levels of consumption and labor supply, expressed as a percentage of the steady-state consumption, as described in Nakata and Schmidt (2014).

The top panels of Figure 1 show that as the central bank increases the policy rate, inflation and the output gap decline. In the absence of any changes in the policy rate from its natural rate, inflation and the output gap are slightly below 2% and zero, respectively, because households and firms attach a small probability to large declines in inflation and output next pereiod. Since the policy rate today decreases inflation and output gap linearly and the loss today is a quadratic function of these two variables, an increase in the policy rate increases today's loss quadratically (middle-left panel). On the other hand, the continuation loss decreases with the policy rate, as shown in the middle-right panel. This is because an increase in the policy rate, together with the associated declines in inflation and the output gap, worsens credit conditions at time $t = 1$, L_1, which in turn lowers the crisis probability, γ_1. The optimal policy rate balances the losses from lower economic activity today against the expected benefits from a reduced crisis probability next period. According to the bottom-left panel, under our baseline parameters, the overall loss is minimized when the policy rate is about 3 basis points above the long-run natural rate of 4%. This is the point at which the marginal cost of increasing the policy rate on today's loss equals the marginal benefit of increasing the policy rate.[16] In non-crisis times, the policymaker is willing to optimally keep the policy rate higher than its long-run natural rate, inducing a negative output gap and inflation lower than 2%, to reduce the probability of a financial crisis driven by exuberant credit conditions.

Our logit specification of the crisis probability equation implies that the effect of marginal changes in the policy rate on the crisis probability, and hence the continuation loss, depends on the lagged credit condition, L_0. To assess the effect of increasing concerns about financial stability on the optimal policy rate in the current period, we therefore vary in Figure 2 the level of L_0 along the horizontal axis. Because an increase in the policy rate reduces the crisis probability by more when credit growth is already high, the optimal policy rate increases with lagged credit conditions. When $L_0 = 0$, roughly the minimum of this variable observed in the U.S. over the past five decades—the optimal increase in the policy rate is about 2 basis points. When $L_0 = 0.5$, the peak observed in the U.S. in post-war data (see figures 17 and 18), the optimal increase in the policy rate is about 6 basis points.[17] Thus, even under conditions similar to those prevailing immediately prior to the onset of the financial crisis, the optimal adjustment to the short-term interest rate in response to potential financial stability risks would have been very small. The primary reason for this result is that the marginal effect of interest rate changes on the crisis probability, shown in the lower right panel of Figure 1, is minuscule under our baseline model calibration.

3.2 Alternative Scenarios

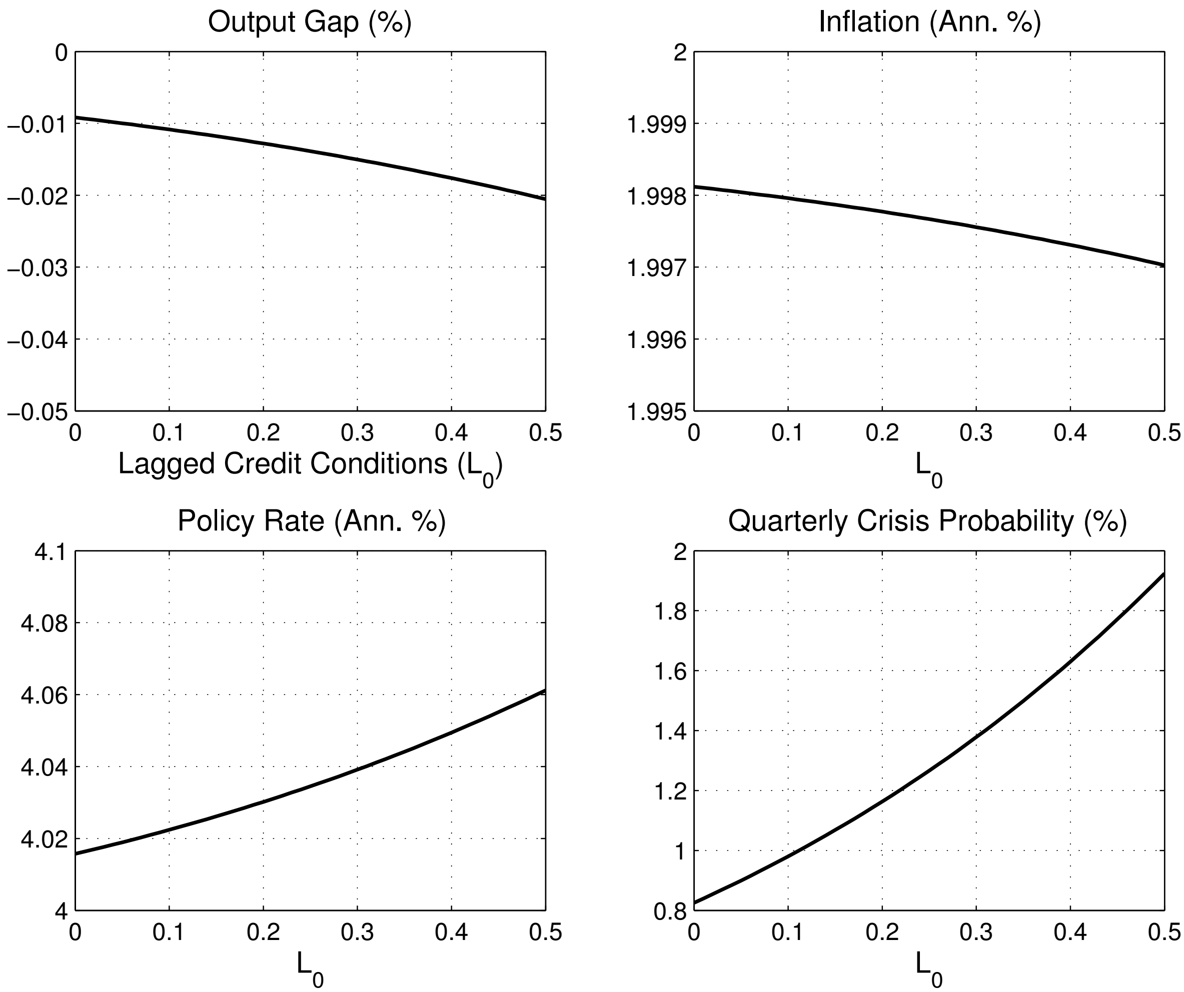
While the key parameters governing the crisis probability (4) and the law of motion for credit conditions (19) are based on empirical evidence, they are estimated with substantial uncertainty. In this section, we therefore examine the sensitivity of the result that financial stability considerations have little effect on optimal policy with respect to a range of alternative assumptions. In particular, we now analyze how the optimal policy rate and economic outcomes are affected by alternative assumptions regarding (i) the effectiveness of the policy rate in reducing the crisis probability, (ii) the severity of the crisis, and (iii) the alternative costs of increasing the policy rate on today's loss.[18] Table 2 reports the changes in the baseline parameters of the model that we adopt in our

[16] Under a standard Taylor rule, the policy rate is 4 basis points below the natural rate. At that rate, inflation and output gap are closer to their steady-state level, but the crisis probability is higher. See Figure 29 in the Appendix.

[17] This feature of optimal policy—the policy rate depending on the initial credit condition—would also arise even when the marginal crisis probability is constant if the severity of the crisis increases with the credit condition.

[18] In the Appendix we present an additional sensitivity analysis with respect to the parameter λ that controls the weight the central banker assigns to output stabilization. See Figures 22 and 23.

Figure 2: Leverage and Optimal Policy



NOTE: This figure shows the optimal policy as a function of the initial level of the credit condition variable, L_0. See also the note in Figure 1.

three sensitivity analyses. In section 4 we will consider how optimal policy is affected when the policymaker explicitly accounts for parameter uncertainty.

The columns of Figure 3 show the optimal policy rate and the implied outcomes in terms of the output gap and inflation as functions of initial credit conditions, L_0, under three model parameterizations that differ from the baseline. The left column of Figure 3 corresponds to a model in which monetary policy tightening is more effective in reducing the crisis probability. As shown in top panel of Table 2, we modify the sensitivity of the likelihood of a crisis to credit conditions, h_1, and the sensitivity of credit conditions to the output gap, ϕ_y, to be two standard deviations higher than the point estimates used in our baseline calibration. These higher sensitivities imply that an increase in the policy rate leads to a larger reduction in the crisis probability, and thus the

Table 2: Parameter Values for Sensitivity Analyses

Tightening More Effective in Reducing the Crisis Probability

h_1	Sensitivity of γ to L	3.0	+2 std. dev. from the baseline
ϕ_y	Sensitivity of L to y	0.258	+2 std. dev. from the baseline

A More Severe Crisis ("Great Depression")

$y_{2,c}$	Output gap in the crisis state	-0.3	30% drop in output gap
$\pi_{2,c}$	Inflation gap in the crisis state	$-0.1/4$	10% drop in annual inflation
$WL_{2,c}$	Loss in the crisis state	$\frac{u(y_{2,c},\pi_{2,c})}{1-\beta\mu}$	

Tightening Less Costly for Today's Inflation and Output Gap

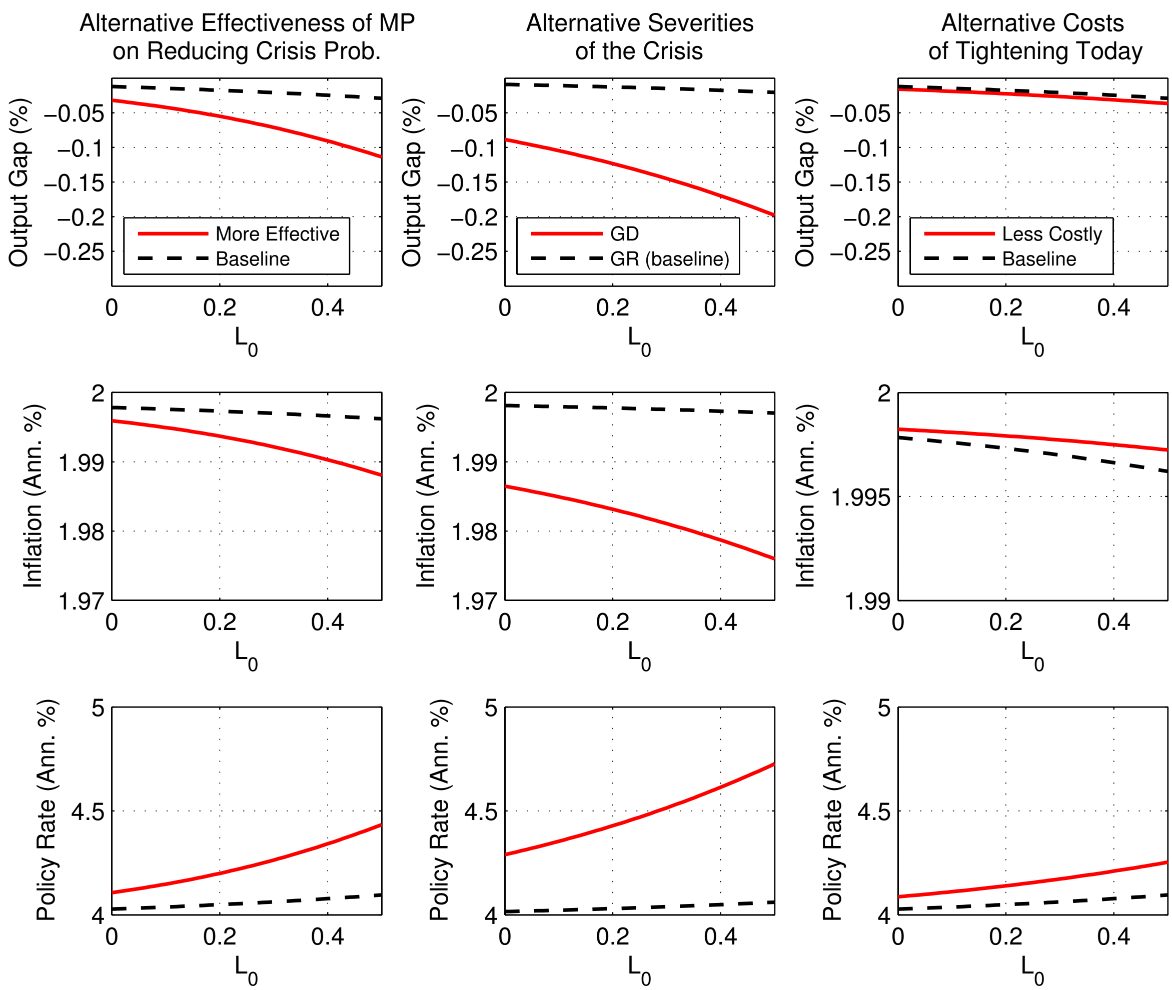
σ	Sensitivity of y to i	1/2	half of the baseline value
κ	Sensitivity of π to y	0.012	half of the baseline value

optimal policy rate is higher for any value of L_0. With $L_0 = 0.2$, the optimal policy rate is about 25 basis points higher than the long-run natural rate of 4%. With $L_0 = 0.5$, the optimal policy rate is about 45 basis points higher than 4%, as seen in the bottom panel of the left column of Figure 3. This additional incentive to tighten policy leads to lower inflation and output gap in the non-crisis state compared to our baseline, as shown in the top left panels of the figure, as well as to a model without financial stability considerations.

The middle column of Figure 3 shows the output gap, inflation, and the policy rate under optimal policy when the severity of the crisis is of a magnitude roughly similar to that of the Great Depression. As shown in the middle panel of Table 2, we assume that the output gap drops by 30% and inflation by 10% on an annual basis. A more severe crisis means that the benefit of raising the policy rate in reducing the continuation loss is larger, and thus the optimal policy rate is also higher for any values of L_0. With $L_0 = 0.2$, the optimal policy rate adjustment is about 30 basis points above the long-run natural rate of 4%. With $L_0 = 0.5$, the optimal policy rate adjustment is about 75 basis points over 4%, as seen in the bottom panel of the middle column of Figure 3.

Finally, the right column of Figure 3 shows the output gap, inflation, and policy rate under optimal policy when today's inflation and output gap are less affected by the change in the policy rate than under the baseline. As listed in the lower panel of Table 2, we assume that the sensitivity of the output gap to the policy rate and the sensitivity of inflation to the output gap are halved with respect to the baseline calibration. Less responsive inflation and output gap mean that the effect of raising the policy rate on today's loss is small, and thus the optimal policy rate is higher for any values of L_0. With $L_0 = 0.2$, the optimal policy rate adjustment is about 10 basis points over the long-run natural rate of 4%. With $L_0 = 0.5$, the optimal policy rate adjustment is more than 20 basis points above 4%.

Figure 3: Leverage and Optimal Policy under Alternative Scenarios



NOTE: This figure shows the optimal policy as a function of the initial level of the credit condition variable, L_0, under alternative calibrations of the model.

4 Optimal Policy under Parameter Uncertainty

We now consider how optimal policy is affected when the policymaker explicitly accounts for parameter uncertainty. We assume that the policymaker is uncertain about the value of specific parameters that affect the monetary policy transmission channel. Mirroring the sensitivity analysis in section 3.2, we assume uncertainty around: (i) the effectiveness of the policy rate in reducing the crisis probability, (ii) the severity of the crisis, and (iii) the alternative costs of increasing the policy rate on today's loss. We solve the model under two different assumptions on the policymaker's attitude towards uncertainty. We compute optimal interest-rate policy both under the assumption of a Bayesian policymaker, as in Brainard (1967), and of a robust policy maker, as in Hansen and Sargent (2008).

4.1 Sources of Uncertainty and Alternative Policymakers

Table 3 displays the prior distributions that we use to characterize uncertainty about of the parameters. The first type of uncertainty is about two parameters related to the effectiveness of the policy rate in reducing the crisis probability: h_1 and ϕ_y. In our analysis below, we consider uncertainty about these two parameters separately. In the "no-uncertainty" case, h_1 takes the value of $h_{1,base}$ with probability one. When there is uncertainty and the policymaker is Bayesian, h_1 follows a discrete uniform distribution that takes the values of $h_{1,min}$, $h_{1,base}$, and $h_{1,max}$, each with probability 1/3.[19] Notice that the expected values of h_1 is $h_{1,base}$.[20] When the policymaker is a robust decision maker, he considers the value of h_1 in the closed interval $[h_{1,min}, h_{1,max}]$. Uncertainty about ϕ_y follows a similar structure. Specific parameter values are listed in the top panel of Table 3.

The second type of parameter uncertainty is related to the severity of the crisis in terms of inflation and output outcomes in period $t = 2$: $\pi_{2,c}$ and $y_{2,c}$. Uncertainty regarding them is jointly analyzed and structured in the same way (see the middle panel of Table 3). Finally, we consider the effects of uncertainty about two parameters that directly control the effects of changes in the policy rate on today's inflation and output: σ and κ, respectively. Uncertainty regarding them is analyzed jointly and structured in the same manner as above (see the bottom panel of Table 3).

A Bayesian policymaker

The Bayesian policymaker problem at time one is given by

$$WL_1 = \min_{i_1} \; E_{1,\theta}\big[u(y_1, \pi_1) + \beta WL_2\big] \tag{20}$$

subject to the private sector equilibrium conditions described in the previous section and assuming that the private sector agents perceive the probability of the crisis as constant and negligible; but now the policymaker takes expectations of future welfare losses with respect to the joint distribution of future states and the uncertain subset of parameters θ.[21] This formulation of the problem follows that in the classic work of Brainard (1967) as recently restated by Brock, Durlauf, and West (2003) and Cogley, De Paoli, Matthes, Nikolov, and Yates (2011).

A Robust-control policymaker

The problem faced by a policymaker following a robust-control strategy is given by

$$WL_1 = \min_{i_1}\Big[\max_{\theta \in [\theta_{min}, \theta_{max}]} \; u(y_1, \pi_1) + \beta E_1[WL_2]\Big] \tag{21}$$

subject to the same set of private sector equilibrium constraints and private agents expectations.

[19] This is done for computational tractability; in future research we will consider parameter uncertainty in the form of a normal distribution, as opposed to a three-state discrete distribution.

[20] That is, these distributions imply mean-preserving spreads on these parameters.

[21] We adopt the simplified notation $E_{1,\theta}$ to denote the expectation of the policymaker with respect to the distribution of states and uncertain parameters.

Table 3: Calibration of Uncertainty

Parameter	Value	Probability
Uncertain Elasticity of Crisis Prob. to Credit Conditions		
$h_{1,min}$	0.74	1/3
$h_{1,base}$	1.88	1/3
$h_{1,max}$	3.02	1/3
Uncertain Elasticity of Credit Conditions to Output		
$\phi_{y,min}$	0.102	1/3
$\phi_{y,base}$	0.18	1/3
$\phi_{y,max}$	0.258	1/3
Uncertain severity of the crisis		
$\pi_{2,c,min}$	$-0.03/4$	1/3
$\pi_{2,c,base}$	$-0.02/4$	1/3
$\pi_{2,c,max}$	$-0.01/4$	1/3
$y_{2,c,min}$	-0.15	1/3
$y_{2,c,base}$	-0.1	1/3
$y_{2,c,max}$	-0.05	1/3
Uncertain effects of the interest-rate on today's π and y		
σ_{min}	0.5	1/3
σ_{base}	1	1/3
σ_{max}	1.5	1/3
κ_{min}	0.012	1/3
κ_{base}	0.024	1/3
κ_{max}	0.036	1/3

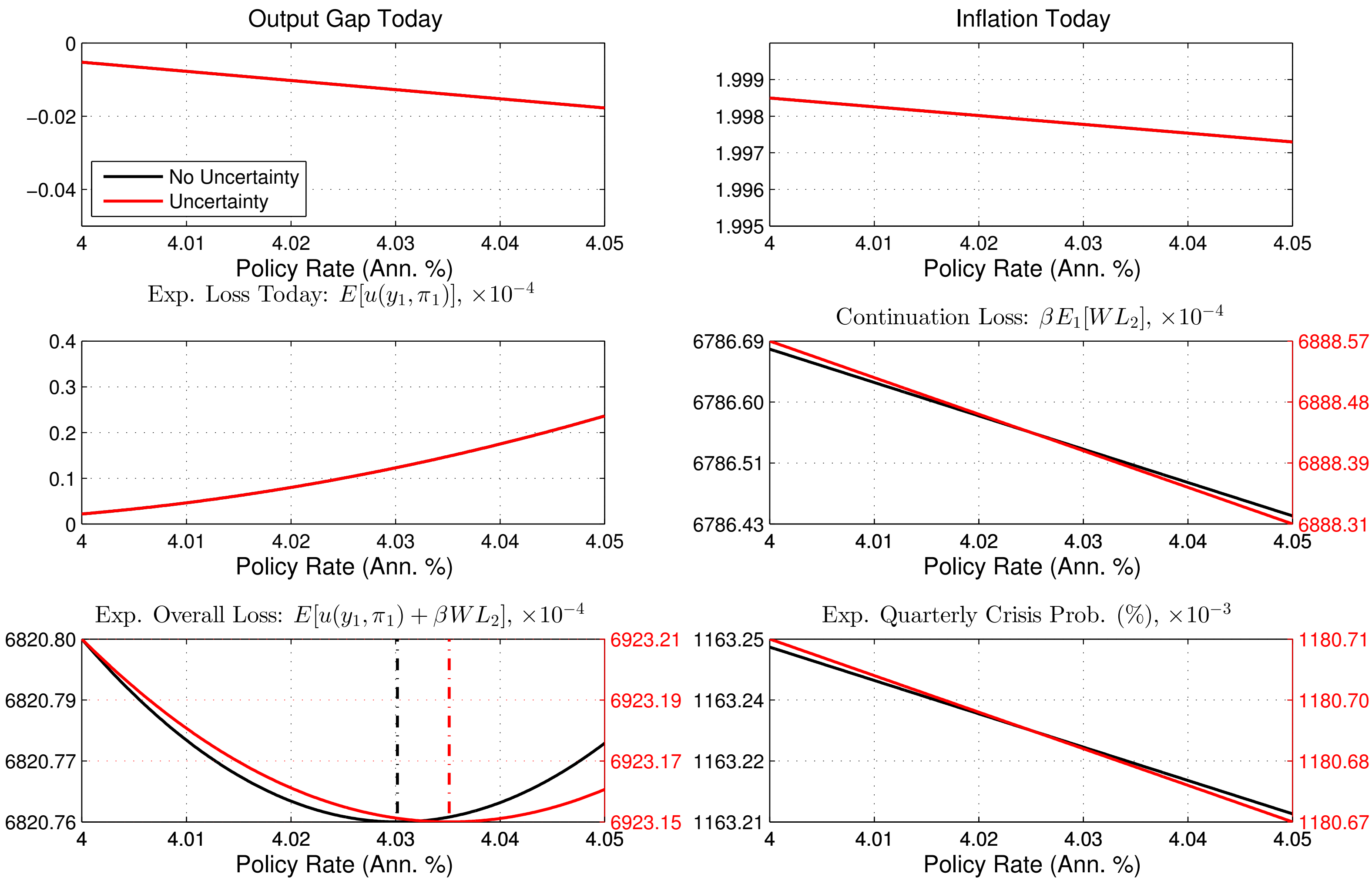
Following the literature, we will refer to the hypothetical agent who maximizes the welfare loss as the hypothetical evil agent resides inside the head of the robust policymaker. The vector of parameters θ is a subset of the model parameters that are subject to uncertainty, and θ_{min} and θ_{max} are the lower and upper bounds considered by the hypothetical evil agent when s/he maximizes the welfare loss, respectively. This *min-max* formulation is standard in the literature on robustness (Hansen and Sargent (2008)). While the robustness literature typically focuses on uncertainty arising from the distribution of exogenous shocks, uncertainty in our model comes from parameter values. Thus, our analysis closely follows those of Giannoni (2002) and Barlevy (2009) who also consider the problem of the robust decision maker under parameter uncertainty.[22]

[22] Hansen and Sargent (2014) also consider the problem of the robust policymaker under parameter uncertainty. In their work, a parameter is a random variable and the hypothetical evil-agent is allowed to twist the probability distribution of uncertain parameters. In our paper as well as in Giannoni (2002) and Barlevy (2009), a parameter is a scalar and the hypothetical evil-agent is only allowed to choose an alternative value for the parameter.

4.2 Uncertainty about the Crisis Probability

Figure 4 illustrates how the presence of uncertainty around the estimate of the sensitivity of the crisis probability to credit conditions, h_1, affects the intertemporal trade-off faced by the Bayesian policymaker. For each panel, red solid and black dashed lines refer to the cases with and without uncertainty, respectively.

Figure 4: The Trade-Off Facing the Bayesian Policymaker

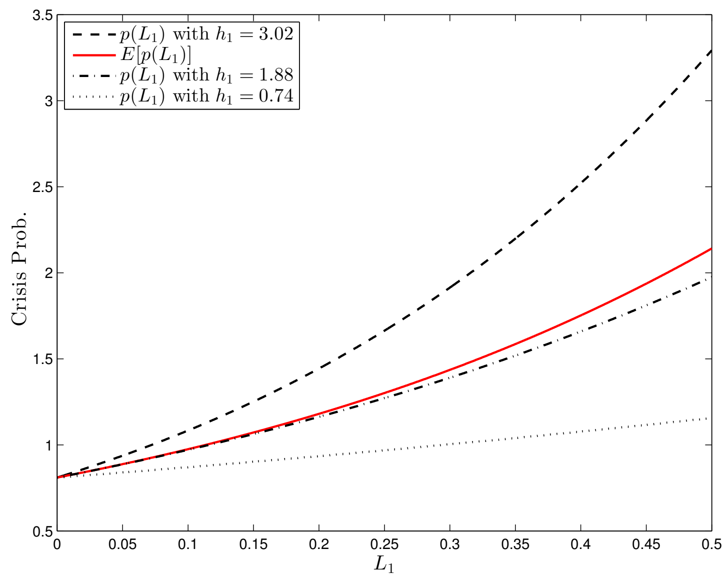


*In the bottom-left panel, vertical black dashed and red solid lines are for the optimal policy rates without and with uncertainty.

An increase in uncertainty regarding the effectiveness of interest-rate policy in reducing the crisis probability leads the Bayesian policymaker to adjust the policy rate by a larger amount, which can be seen in the bottom-left panel of Figure 4 for the case with initial credit conditions $L_0 = 0.2$.

The presence of uncertainty about the parameter h_1 does not alter the period $t = 1$ loss function since the crisis probability does not affect how the policy rate influences today's inflation and output outcomes. This can be seen in the middle left panel and top two panels of Figure 4. However, the presence of uncertainty does affect the expected continuation loss for period $t = 2$. As shown in the middle-right panel, the slope of the expected welfare loss function is steeper with uncertainty than without it. This means that the marginal gain of policy tightening is larger with uncertainty than without it. With the marginal costs of policy tightening unchanged in $t = 1$, this higher marginal gain of policy tightening translates into an optimal policy rate that is higher than in the absence of uncertainty even if just by a few decimals of a basis point, as seen in the middle-left panel.

Figure 5: The Effects of a Mean-Preserving Spread on h_1 for the Crisis Probability Function: $\gamma_1 = \frac{\exp(h_0+h_1L_1)}{1+\exp(h_0+h_1L_1)}$



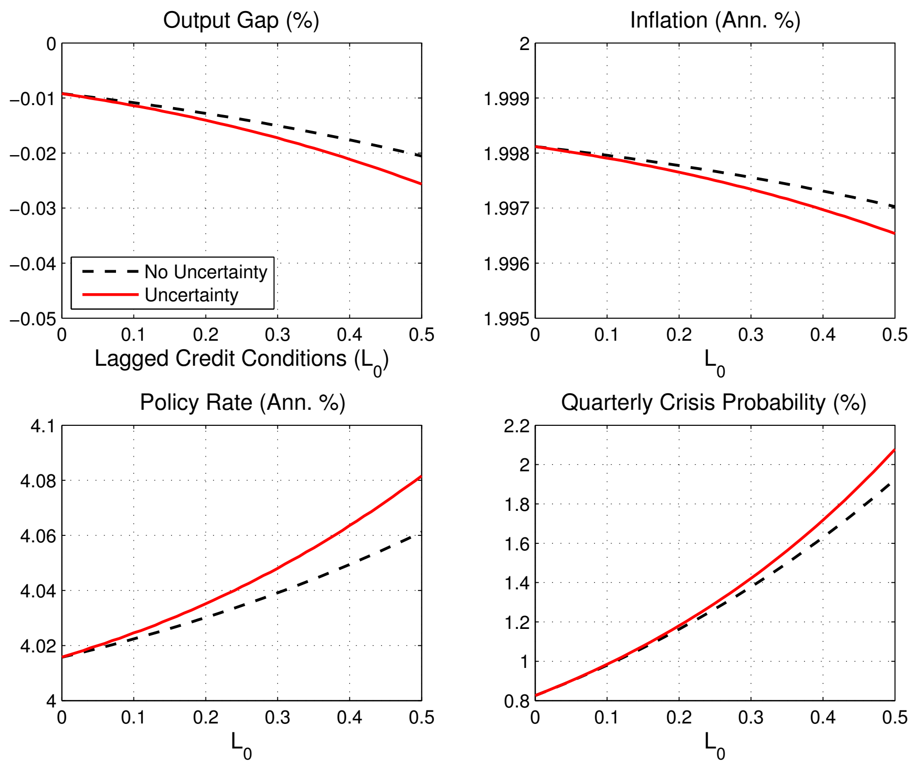
As shown in the middle-right panel of Figure 4, the slope of the expected continuation loss is steeper under uncertainty because the *expected* crisis probability under uncertainty is steeper than that of the (expected) crisis probability without uncertainty.[23] Why is the slope of the (expected) crisis probability function steeper under uncertainty? The reason is as follows. When h_1 increases, the slope as well as the level of the crisis probability function increases, which is captured in the steeper slope of the top black dashed line than that of the black dash-dotted line in Figure 5. When h_1 decreases, the slope, as well as the level, of the crisis probability function decreases, which is captured in the flatter slope of the bottom black dotted line than that of the black dash-dotted line in Figure 5. The convexity of the logit function implies that the increase in the slope of the crisis probability due to an increase in h_1 is larger than the decrease in the slope of the crisis probability due to a decrease in h_1 of the same magnitude. As a result, the slope of the *expected* crisis probability is steeper than that of the crisis probability function, which is captured by the fact that the slope of the red solid line is steeper than that of the black dash-dotted line. That is,

[23]Note that, since the non-crisis value is zero (i.e., $WL_{2,nc} = 0$), the expected continuation loss is a constant times crisis probability (i.e., $\beta E_1[WL_2] = \beta\big[(1-\gamma_1) * WL_{2,nc} + \gamma_1 * WL_{2,c}\big] = \beta WL_{2,c}\gamma_1$).

a mean-preserving spread in h_1 increases the slope of the (expected) crisis probability function.

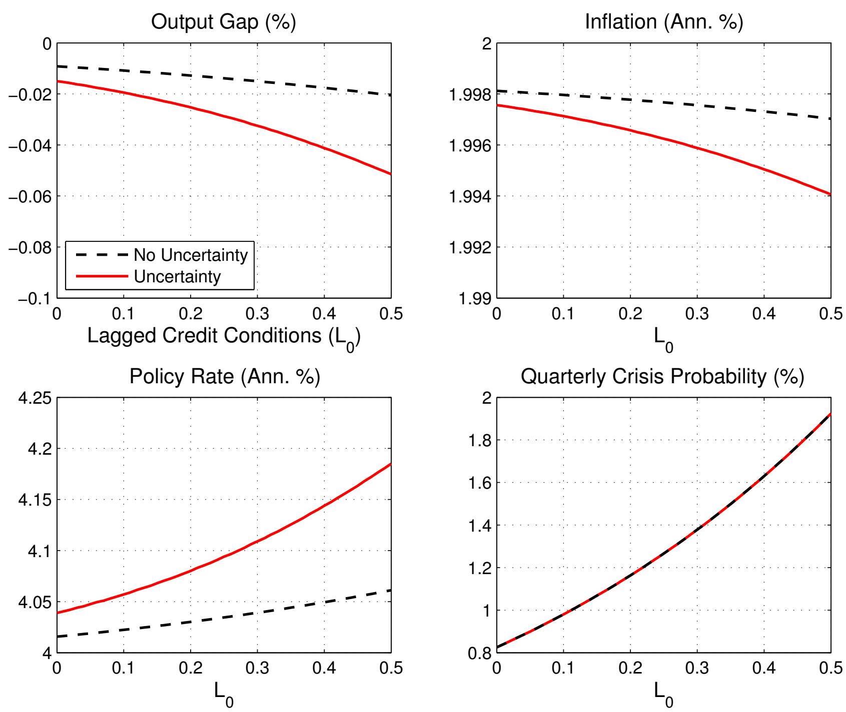
As demonstrated in Figure 6, this result does not depend on the level of credit conditions of the economy. The optimal adjustment of the policy rate is about 10-20 percent larger in the presence of uncertainty than in its absence and it is increasing in initial credit conditions, L_0.

Figure 6: Optimal Policy Under Uncertainty: Bayesian Policymaker
(uncertain effects of policy on the crisis probability)



Uncertainty about the effects of policy on the probability of a crisis also leads the robust policymaker to choose a higher policy rate, which is shown in Figure 7. The policymaker following robust control policies chooses the policy rate to minimize the welfare loss under the worst-case scenario. In the present context, the parameter value that leads to the maximum welfare loss is the highest h_1, as this implies higher crisis probability for *any given choice of* i_1. This is illustrated in Figure 8 that shows the payoff function of the hypothetical evil agent when the robust policymaker chooses the optimal policy rate under no uncertainty of 4.03 percent. By choosing the maximum possible h_1, the hypothetical evil agent can cause the largest damage to the robust policymaker. Thus, the robust policymaker chooses the policy rate in order to minimize the welfare loss, *anticipating* that the hypothetical evil agent would choose the highest possible h_1. A higher h_1 means that an increase in the policy rate leads to a larger decline in the continuation value. Thus, the robust policymaker adjusts the policy rate by more under uncertainty. In our calibration, the presence of uncertainty leads the robust policymaker to adjust the policy rate by 100-200 percent more. If, for example, initial credit conditions are particularly buoyant, with $L_0 = 0.5$, the robust policymaker would want to set the policy rate in the non-crisis state just below 4.2%, compared to 4.06% in the absence of uncertainty.

Figure 7: Optimal Policy Under Uncertainty: Robust Policymaker
(uncertain effects of policy on the crisis probability)



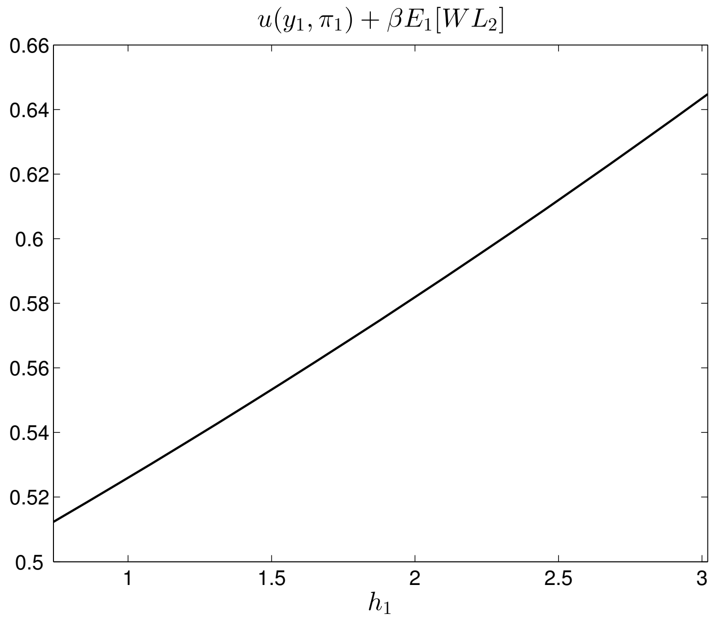
4.3 Uncertainty about Credit Conditions

Figure 9 shows how the uncertainty regarding the elasticity of credit conditions to output affects optimal policy. The left and right columns are for the Bayesian policymaker and the robust policymaker, respectively.

We verify numerically that the presence of uncertainty leads the Bayesian policymaker to choose a higher policy rate; however the difference between optimal policy with and without uncertainty is negligible, as shown by the overlapping black dashed and red solid lines in the left column. We find that uncertainty regarding the elasticity of credit conditions to output induces uncertainty about credit conditions today. This also makes the crisis probability uncertain. The convexity of the logit function implies that a mean-preserving spread in L_1 increases the level and slope of the (expected) crisis probability, which in turn increases the marginal benefit of policy tightening. However, in our calibration, this effect is very small.

The right column shows that this type of uncertainty leads the robust policymaker to choose a lower policy rate instead of a higher one. In this context, the parameter value that leads to the maximum welfare loss is the lowest possible value value for the parameter ϕ_y, as it implies a higher crisis probability *for any choice of* i_1. Thus, the robust policymaker sets the policy rate in order

Figure 8: The Objective Function of the Hypothetical Evil Agent inside the Head of the Robust Policymaker



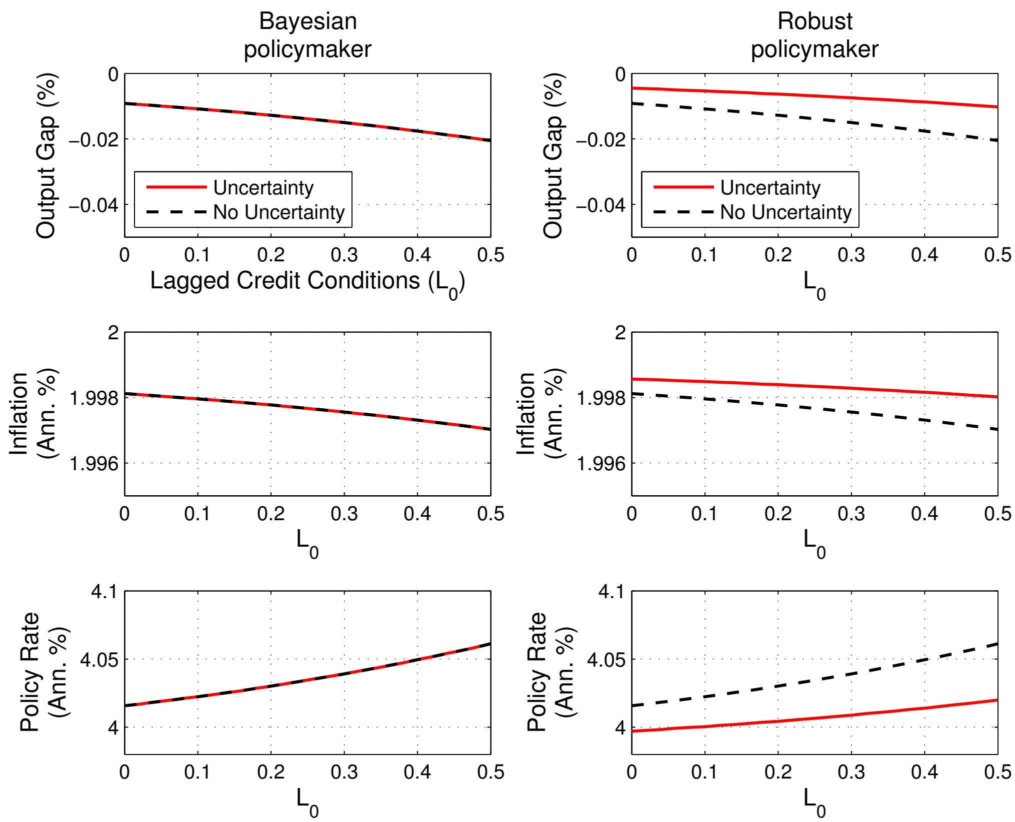
to minimize the welfare loss, *expecting* the hypothetical evil agent to choose the lowest possible ϕ_y. Notice that a low value of the parameter ϕ_y means that a policy tightening has, via aggregate demand, a weaker effect on credit conditions and the crisis probability. Facing a lower marginal benefit of policy tightening and a lower unchanged marginal cost, the policymaker chooses a lower policy rate than in the absence of uncertainty.

4.4 Uncertain Severity of the Crisis

Figure 10 shows how uncertainty regarding both inflation and output levels induced by a crisis, $(\pi_{2,c}, y_{2,c})$ affect the optimal policy under a Bayesian and robust policymaker, respectively. The figure shows that, regardless of the type, the policymaker chooses a higher policy rate in the presence of uncertainty than in the absence of it.

Why does uncertainty about the severity of the crisis lead the Bayesian policymaker to choose a higher policy rate? Uncertainty regarding the severity of the crisis does not affect today's output gap, inflation and loss. However, it does affect the (expected) continuation loss. In particular, the slope of the (expected) continuation value is steeper with uncertainty than without it. This is because the loss associated with the crisis state tomorrow is quadratic. As a result, an increase in the loss due to a decline in inflation is larger than a decline in the loss due to an increase in inflation of the same magnitude. Similarly, an increase in the loss due to a decline in output gap is larger than a decline in the loss due to an increase in output gap of the same magnitude. Thus, the presence of uncertainty regarding $\pi_{2,c}$ and $y_{2,c}$ increases the expected loss associated with the crisis state. The marginal benefits of policy tightening increases when the expected crisis loss

Figure 9: Optimal Policy Under Uncertainty:
Uncertain Elasticity of Credit Conditions to Output



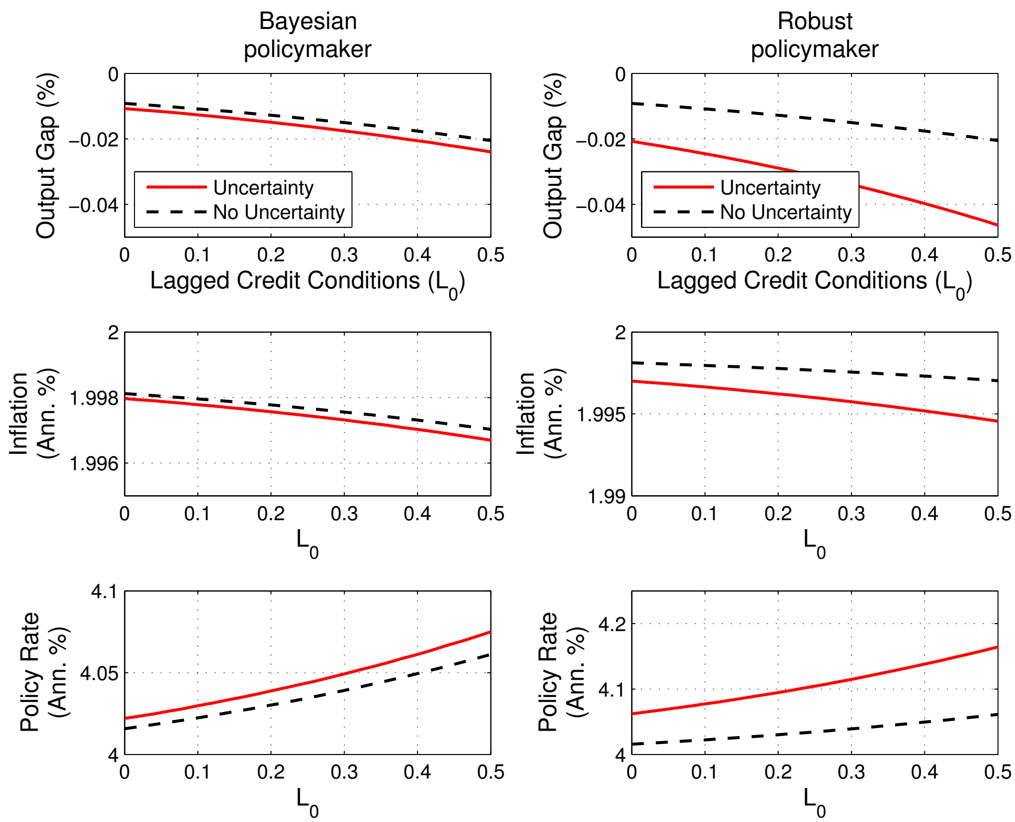
increases (i.e., $\beta E_1[WL_2] = \beta\gamma_1)E_1WL_{2,c})$. Accordingly, the marginal benefits of policy tightening is higher with uncertainty than without it. With the marginal cost of policy tightening unchanged, the higher marginal benefit of a reduced expected loss means that the optimal policy rate will be higher.

Similarly, the robust policymaker chooses a higher policy rate in the presence of this uncertainty. The hypothetical evil agent can reduce the welfare by choosing the largest possible declines in inflation and output gaps in the crisis state. This means that, for the robust policymaker, the marginal change in the continuation value associated with an adjustment of the policy rate is larger under uncertainty. Accordingly, the presence of uncertainty leads the robust policymaker to adjust the policy rate by more under uncertainty to avoid the unpleasant crisis scenario, as shown in the right-hand side panels of Figure 10.

4.5 Uncertain Effects of Policy on Today's Inflation and Output

Figure 11 shows the effect on the optimal policy of uncertainty over the parameters (σ, κ), that is the effects of interest rates on today's inflation and output. The two columns correspond to the Bayesian policymaker and the robust policymaker, respectively. They show that both types of

Figure 10: Optimal Policy Under Uncertainty:
Uncertain Severity of the Crisis

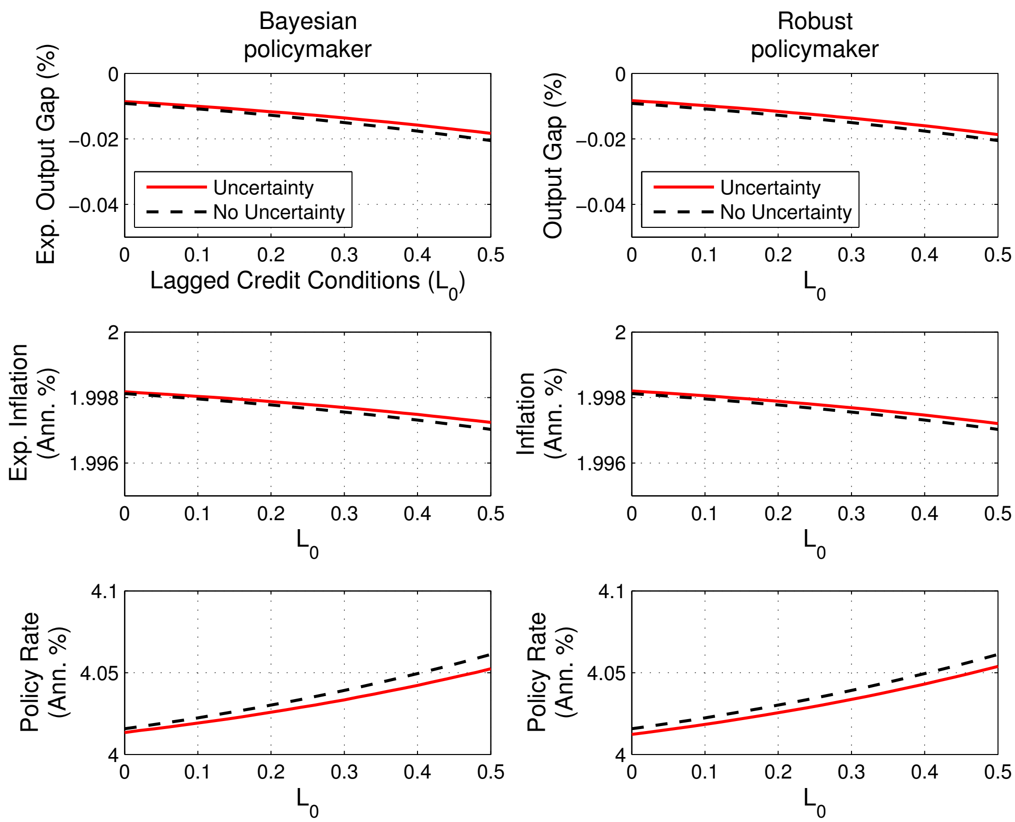


agents choose a lower policy rate in the presence of uncertainty than in the absence of it. This is the same type of uncertainty considered in Brainard (1967) and our result is consistent with his conclusion.

Why does uncertainty lead the Bayesian policymaker to choose a lower policy rate? As shown in the top two panels of Figure 12, uncertainty about the parameters σ and κ does not change the expected inflation and output gap today. This is because today's inflation and output depend linearly on the policy rate. However, this uncertainty does affect the expected loss today. This is because the central bank's welfare loss today is quadratic in inflation and output. As shown in the middle-left panel, the expected loss is larger with uncertainty than without it, and so is the marginal cost of policy tightening. While the presence of uncertainty has some effects on the (expected) continuation loss, they are negligible and the marginal benefits of policy tightening are essentially unchanged under uncertainty. Accordingly, the central bank will optimally set the policy rate lower in the presence of uncertainty than in the absence of it.

The robust policymaker also chooses a lower policy rate under uncertainty. In our calibration, the hypothetical evil agent inside the head of the central banker chooses the smallest possible σ and the largest possible κ. While a smaller σ increases welfare through higher (or less negative) output gap and inflation, it decreases welfare through higher L_1 and crisis probability. The hypothetical

Figure 11: Optimal Policy Under Uncertainty:
Uncertain Effects of Monetary Policy on Today's Inflation and Output

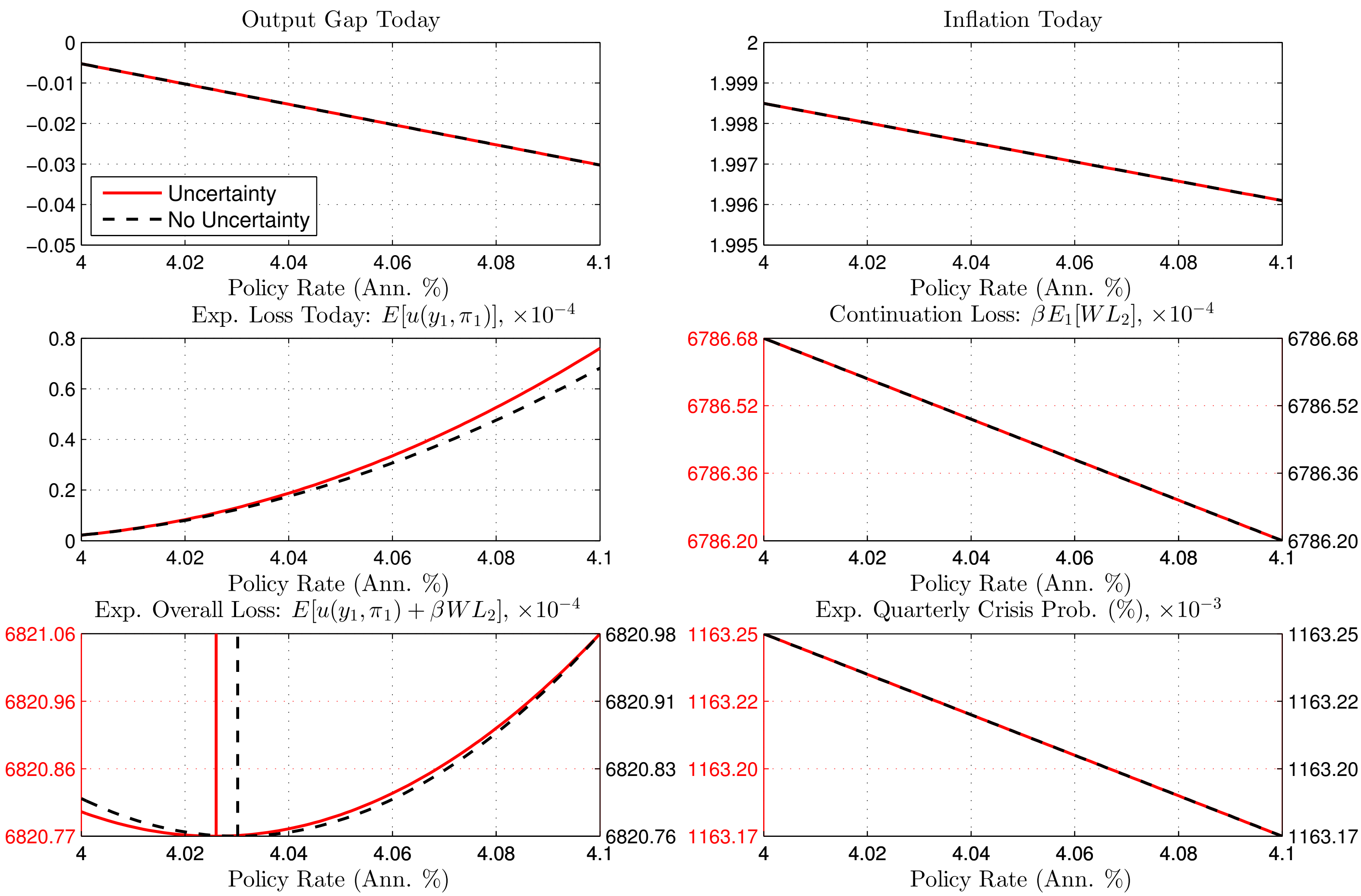


evil agent chooses the smallest possible σ because the second force dominates the first. The evil agent chooses the highest possible κ because a higher κ is associated with a lower (more negative) inflation and a higher L_1, both of which reduce welfare. Anticipating that the hypothetical evil agent would choose a smaller σ, the robust policymaker has an incentive to adjust the policy rate by more because an increase in the policy rate has a smaller consequence on today's output. Anticipating that the evil agent would choose a higher κ, the robust policymaker has an incentive to adjust the policy rate by less because an increase in the policy rate has a larger consequence on today's output. In our calibration, the second effect dominates the first, leading the robust policymaker to choose a lower policy rate under uncertainty, as shown in the right column of Figure 11.

4.6 Discussion

The effect of uncertainty over the parameters σ and κ that control the effects of policy on today's inflation and output is consistent with Brainard's *attenuation principle*: the policymaker optimally sets the policy rate lower than in the absence of uncertainty. However, our analysis shows that this principle does not generalize to other types of uncertainty. There are several reasons why this

Figure 12: The Trade-Off Facing the Bayesian Policymaker
Uncertainty Effects of Policy on Today's Allocations



*In the bottom-left panel, vertical black dashed and red solid lines are for the optimal policy rates without and with uncertainty.

difference arises. On the one hand, in Brainard's work uncertainty increases the marginal cost of monetary policy tightening today with negligible changes in the marginal expected loss tomorrow, so that the policymaker chooses a lower optimal policy rate in equilibrium. On the other had, we find that uncertainty that increases the future marginal benefit of monetary policy interventions (either because the policymaker is unsure about how credit conditions affect the probability of a crisis or is uncertain about the size of the output and inflation drops in the crisis state) tends to amplify the preemptive response of the policymaker. In these cases, uncertainty calls for a higher optimal rate due to the nonlinearity of the expected future loss derived either from the logit function or the quadratic nature of the per-period loss.

5 Conclusions

We have analyzed how the central bank should respond in normal times to financial imbalances in a stylized model of financial crises. For the version of the model that is calibrated to match the historical correlation of credit booms and financial crises in advanced economies, we find that the optimal increase in the policy rate due to financial imbalances is negligible. We also take an additional step to identify circumstances that would lead the central bank to adjust the policy rate more aggressively. We show that if (i) the severity of the crisis is comparable to that of the Great Depression, or (ii) the crisis probability is twice more responsive to financial conditions in the economy then the optimal adjustment to the policy rate can be as large as, or can even exceed, 50 basis points. Finally, we demonstrated that parameter uncertainty can induce a Bayesian and a robust policymaker to respond more aggressively to financial crises by setting the policy rate higher than in absence of uncertainty. This happens if the source of uncertainty can increase the expected marginal benefit of policy interventions aimed at reducing the likelihood of a crisis and its expected welfare loss.

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Appendices

A Output Growth and Inflation Expectations in the Great Recession: Evidence from the SPF

In this appendix we report evidence of how professional forecasters' expectations over future output growth and inflation evolved before and during the Great Recession.

Every quarter, participants in the Survey of Professional Forecasters (SPF) report the probability distribution of the growth rate of real average GDP expected over the current and next calendar years. Survey participants are asked to assign probabilities to the events that the growth rate of average real GDP between years 0 and 1 will fall within pre-determined ranges.

Since 1992:Q1, participants could explicitly indicate the likelihood that the growth rate of average real GDP (RGDP) be lower than 0%.[24]:

$$PRGDP_{y1} = Pr\left[100 \times \ln\left(\frac{\frac{RGDP^{y1}_{Q1}+RGDP^{y1}_{Q2}+RGDP^{y1}_{Q3}+RGDP^{y1}_{Q4}}{4}}{\frac{RGDP^{y0}_{Q1}+RGDP^{y1}_{Q2}+RGDP^{y0}_{Q3}+RGDP^{y0}_{Q4}}{4}}\right) < 0\%\right]$$

We concentrate on the Great Recession episode and study how expectations of professional forecasters behaved before and during the period of financial turmoil that built up to the downturn and to two years of negative growth for average real GDP: 2008 and 2009.

Realized average real GDP fell by -0.29% in 2008, and then fell again by -2.81% in 2009. Figure 13 shows that the median forecaster in the SPF (purple line) attached probabilities close to 0% to the event that average real GDP could fall during the course of 2008, in each quarter he was asked to forecast it, over the course of 2007 and 2008. Similarly, figure 14 shows that the median forecaster (purple solid line) reported probabilities below 2% when asked to forecast the likelihood of negative growth for average real GDP in 2009, at least until the collapse of Lehman Brothers in 2008:Q3. After this point, the median probability of negative growth in 2009 increased from 2% in 2008:Q3 up to 55% in 2008:Q4, and later converged to 100% by the second half of 2009 as more information on the severity of the financial crisis became available. The graphs also report the interquartile range for the same probabilities, as well as mean probabilities and the NBER-dated recession period is highlighted in grey.

We conduct a similar exercise using the SPF data for the probability distribution of the growth rate of average CPI over the same time frame. We are particularly interested in the forecasters' view on the likelihood of a prolonged deflationary scenario during the Great Recession. Figure 15 shows that the median forecaster (purple solid line) reported a probability of negative growth of average CPI to be 0% for 2008, over the course of the forecasting period (2007 and 2008). Similarly, figure 16 shows that the median forecaster kept the expected probability of deflation for 2009 equal to 0% until realized CPI inflation recorded a negative entry in 2008:Q4 (-2.3%, not shown in the figures). At that point the median forecaster increased the expected likelihood of a deflationary scenario to 3%, only to converge back to 0% once the temporary effect of lower energy prices faded out and realized CPI inflation went back into positive territory.

It is interesting to notice that the mean, together with the third quartile (green dash-dotted line) of the distribution of SPF participants included in the graphs, point out that a number of professionals did forecast a higher likelihood of a prolonged drop in real GDP and prices for 2008 and

[24] Prior to 1992:Q1, the upper bound of the lowest range in the survey was 2%. Moreover, prior to 1981:Q3, participants were surveyed about the probability distribution of nominal (and not real) GDP growth.

2009. Nonetheless, the third quartile forecast of how likely the drop in average real GDP would last through 2009 hovers around 10% and only increases rapidly after the collapse of Lehman Brothers. Deflation expectations show a similar pattern. We interpret this as interesting evidence of how agents did not anticipate the occurrence and the effects of the financial crisis of 2007-2009. Agents' expectations of the likelihood of a prolonged recession adjusted with a lag to the unfolding of the events on financial markets, rather than, for example, responding to the accumulation of financial imbalances over the course of the economic expansion of the 2000s.

B Credit Conditions and Crisis Probability

In this appendix we provide further details on our use of Schularick and Taylor's data and our adaptation of some of their results.

The Logit Model

Schularick and Taylor (2012) assume that the probability that a a given country, i, will fall into a financial crisis in period from period t and $t+1$ can be expressed as a logistic function $\gamma_{i,t}$ of a collection of predictors $X_{i,t}$:

$$\gamma_{i,t} = \frac{e^{X_{i,t}}}{1+e^{X_{i,t}}}$$

Their baseline specification for $X_{i,t}$ includes a constant, c, country fixed effects, α_i, and five lags of the annual growth rate of loans of domestic banks to domestic households, $B_{i,t}$, deflated by the CPI, $P_{i,t}$:

$$X_{i,t} = h_0 + h_i + h_{1,L}\Delta \log \frac{B_{i,t}}{P_{i,t-1}} + h_{2,L}\Delta \log \frac{B_{i,t-1}}{P_{i,t-1}} + h_{3,L}\Delta \log \frac{B_{i,t-2}}{P_{i,t-2}} + h_{4,L}\Delta \log \frac{B_{i,t-3}}{P_{i,t-3}} + h_{5,L}\Delta \log \frac{B_{i,t-4}}{P_{i,t-4}}$$

The model is estimated on annual data.

In order to reduce the number of lags and state variables in our model, we re-estimate a simplified version of Schularick and Taylor's model using the cumulative 5-year growth rate of bank loans from time $t-4$ to t, denoted as L_t, as predictor of a financial crisis in period $t+1$, instead of the five lags separately.

$$X_{i,t} = h_0 + h_i + h_1 L_t^a \tag{22}$$

where:

$$L_t^a = \Sigma_{s=0}^{4} \Delta \log \frac{B_{i,t-s}}{P_{i,t-s}}.$$

The estimated coefficients for this equation are significant and shown in Table 1 (the country fixed effect for the United States is set to zero, for identification purposes).

Table 1 - Estimates of the Schularick and Taylor Model for the U.S.
Regressor L_t: 5-year Cumulative Growth Rate

EQUATION	VARIABLES
h_1	1.880***
	(0.569)
h_0	-3.396***
	(0.544)
Observations	1,253

To adapt the results to our model calibrated to quarterly data, we assume that the annual probability of a crisis $\gamma_{i,t}$ is uniformly distributed over the 4 quarters within the year, so that the quarterly probability $\gamma_{i,t}^q$ is equal to:

$$\gamma_{i,t}^q = \frac{\gamma_{i,t}}{4}$$

We define a recursive approximation of L_t as the recursive sum of the quarterly growth rates recorded from time $t-19$ up to quarter t:

$$\Sigma_{s=0}^{19} \Delta \log \frac{B_{t-s}^q}{P_{t-s}} \approx L_t^q = \Delta \log \frac{B_t^q}{P_t} + \frac{19}{20} L_{t-1}^q \tag{23}$$

Figure 17 shows the cumulative annual regressor and its recursive counterpart defined in equations (B). Figure 18 shows the quarterly actual and recursive sums defined in equation (23). Figures 19 and 20 show the corresponding fitted probabilities using the logit coefficients in Table 1.[25] The series are remarkably similar. As expected, the recursive sums are less volatile than the actual 5-year growth rate both for the quarterly and annual series (the standard deviation of the quarterly actual and recursive sums in Figure 18 are 11% and 13.5% respectively).

Figure 20 shows the quarterly fitted probability that a crisis arises in period t (hence computed using the quarterly growth rates of bank loans over the past 5 years of data, up to quarter $t-1$), from equation (23). The cyclical properties of the quarterly series are the same as the ones of the annual series.[26]

Quarterly Bank Loan Growth

We assume that the quarterly growth rate of nominal bank loans is a function of the nominal federal funds rate i_t and of the output gap y_t:

$$\Delta \log B_t = c + \phi_i i_t + \phi_y y_t + \varepsilon_t^B \tag{24}$$

Estimating this reduced-form equation for growth of bank lending does not allow to separately identify how shifts in the demand and supply of credit translate into loan growth. Moreover, the direction of causality between the left- and right-hand-side variables can be questioned. To ameliorate a potential simultaneity bias, we use lagged values of the monetary policy instrument and of the output gap, i_{t-1} and y_{t-1}, as instruments for their current values, i_t and y_t. The output of the first-stage regressions (not reported, but available upon request) shows that the lagged monetary policy instrument, i_{t-1}, enters significantly and with a negative sign in the determination of the fitted contemporaneous output gap, $\hat{y}_t$, and with a positive sign (close to unity) in the equation for the fitted value of the contemporaneous monetary policy, $\hat{i}_t$ instrument.

Column 1 of table 2 shows the results of the second-stage regression. That coefficient on the fitted policy rate $\hat{i}_t$ appears to be statistically insignificant. We drop this term and reestimate the equation in the restricted form:

$$\Delta \log B_t = c + \phi_y \hat{y}_t + \varepsilon_t^B \tag{25}$$

Column 2 of table 2 shows that the output gap enters with a positive coefficient in the equation for quarterly credit growth: as expected economic expansions are characterized by a higher growth rate for banking loans. In particular, a positive output gap of 1 percentage point prompts a 0.18 percentage point higher growth rate for bank loans at time t. Our model features negative responses of the output gap to positive changes in the federal funds rate in equation 1. As a result monetary

[25]The series are built at both annual and quarterly frequencies for the U.S. using the total loans and leases and security investments of commercial banks from the Board of Governors of the Federal Reserve H.8 release

[26]The quarterly observations include infra-annual information. The last observation of 2008 shows a decline in the the growth rate due to the inclusion of the negative surprises in the the third quarter of 2008. The 2008 value of the annual series in figure 17 instead only contains information up to the end of 2007.

Table 2 - Nominal Credit Growth Process

VARIABLES	$\Delta \log L_t$	
ϕ_i	0.025	–
	(0.035)	(–)
ϕ_y	0.183***	0.180***
	(0.039)	(0.039)
c	2.190***	2.328***
	(0.205)	(0.095)
Observations	164	164
R-squared	0.18	0.15

policy will affect the growth rate of nominal loans as well.[27]

As long as our model features negative responses of the output gap to positive changes in the federal funds rate, monetary policy will affect the growth rate of nominal loans as well. In that case, at least for now, we can remain agnostic on the sign and magnitude of the effect of the monetary policy instrument on bank lending growth and set ϕ_i equal to zero.

C A Model with Credit-Output Linkages

In our baseline model, financial conditions affect the economy only *via* its effect on crisis probability. However, credit booms are often associated with output booms. Accordingly, we consider a model in which increases in financial conditions lead to an increase in output. Specifically, we modify the aggregate demand equation to include financial conditions as follows.

$$y_1 = E_1^{ps} y_2 + \sigma(i_1 - E_1^{ps}\pi_2) + \alpha_L(L_1) \tag{26}$$

Figure 21 shows the optimal policy in this model with credit-output linkage. The optimal policy rate increases with financial conditions. This is because financial conditions act as demand shocks in this version of the model. The central bank can offset the effects of increases in financial conditions on the output gap by increasing the policy rate. Thus, the credit-output linkage gives the central bank another incentive to raise the policy rate during a credit boom, over and above the crisis probability motive described earlier. The central bank would raise the policy rate in response to credit booms by less if the crisis probability was hypothetically constant. This is shown in the bottom-right panel of Figure 21 that shows the additional increase in the policy rate due to financial stability concerns. Consistent with our results in the baseline model, financial stability concerns imply a very small additional increase in the optimal policy rate.

[27]In a recent paper, Jimenez, Ongena, Peydro, and Saurina (2012) carefully identify the exogenous effects of monetary policy and aggregate economic conditions on the demand and supply of banking loans in Spain, using loan-level data. They find that positive changes in the nominal interest rate and negative output growth reduce the likelihood that banks approve loans request. The effect is larger for banks with poor fundamentals. They use these findings as evidence in support of the bank-lending transmission channel of monetary policy.

D The Details of the Optimal Policy

The central bank faces the following optimization problem:

$$WL_1 = \min_{i_1, y_1, L_1} u(y_1, \pi_1) + \beta[1 - p(L_1)]WL_{2,nc} + \beta p(L_1)WL_{2,c} \tag{27}$$

subject to the following constraints defining the private sector equilibrium conditions:

$$\begin{aligned} y_1 =& -\sigma i_1 + \sigma[(1-\epsilon)\pi_{2,nc} + \epsilon\pi_{2,c}] \\ &+ [(1-\epsilon)c_{2,nc} + \epsilon y_{2,c}] \end{aligned} \tag{28}$$

$$\pi_1 = \kappa y_1 + \beta[(1-\epsilon)\pi_{2,nc} + \epsilon\pi_{2,c}] \tag{29}$$

$$L_1 = \rho_L L_0 + \phi_i i_1 + \phi_y y_1 + \phi_\pi \pi_1 + \phi_0 \tag{30}$$

and where

$$\begin{aligned} &u(y_1, \pi_1) = -\frac{1}{2}(\lambda c_1^2 + \pi_1^2), \quad WL_{2,nc} = 0, \quad WL_{2,c} = \frac{u(y_{2,c})}{1-\beta\mu} \\ &p(L_1) = \frac{q * exp(h_0 + h_1 L_1)}{1 + exp(h_0 + h_1 L_1)} \Rightarrow p'(L_1) = \frac{q\, h_1 * exp(h_0 + h_1 L_1)}{(1 + exp(h_0 + h_1 L_1))^2} \end{aligned} \tag{31}$$

First-order necessary conditions: Let ω_1, ω_2 and ω_3 be the Lagrange multipliers on the constraints (28), (29) and (30).

$$\frac{\partial}{\partial i_1} = \omega_1 \sigma - \omega_3 \phi_i = 0 \tag{32}$$

$$\Leftrightarrow \omega_1 = \frac{\omega_3 \phi_i}{\sigma} \tag{33}$$

$$\frac{\partial}{\partial y_1} = \frac{\partial u(y_1, \pi_1)}{\partial y_1} + \omega_1 - \omega_2 \kappa - \omega_3 \phi_y = 0 \tag{34}$$

$$\Leftrightarrow u_{y_1} + \frac{\omega_3 \phi_i}{\sigma} - \omega_2 \kappa - \omega_3 \phi_y = 0$$

$$\Leftrightarrow u_{y_1} + \omega_3 \frac{\phi_i - \sigma\phi_y}{\sigma} - \omega_2 \kappa = 0$$

$$\Leftrightarrow \omega_2 = \frac{\sigma u_{y_1} + \omega_3(\phi_i - \sigma\phi_y)}{\kappa\sigma} \tag{35}$$

$$\frac{\partial}{\partial \pi_1} = \frac{\partial u(y_1, \pi_1)}{\partial \pi_1} + \omega_2 - \omega_3 \phi_\pi = 0 \tag{36}$$

$$\Leftrightarrow u_{\pi_1} + \frac{\sigma u_{y_1} + \omega_3(\phi_i - \sigma\phi_y)}{\kappa\sigma} - \omega_3 \phi_\pi = 0 \tag{37}$$

$$\frac{\partial}{\partial L_1} = -\beta p'(L_1)WL_{2,nc} + \beta p'(L_1)WL_{2,c} + \omega_3 = 0 \tag{38}$$

Solving for $y_1, i_1, L_1, \pi_1, \omega_3$. We obtain that

$$\begin{aligned} y_1 =& -\sigma i_1 + \sigma[(1-\epsilon)\pi_{2,nc} + \epsilon\pi_{2,c}] \\ & + \alpha_c y_0 + [(1-\epsilon)y_{2,nc} + \epsilon y_{2,c}] \end{aligned} \tag{39}$$

$$\pi_1 = \kappa y_1 + \beta[(1-\epsilon)\pi_{2,nc} + \epsilon\pi_{2,c}] \tag{40}$$

$$L_1 = \rho_L L_0 + \phi_i i_1 + \phi_y y_1 + \phi_\pi \pi_1 + \phi_0 \tag{41}$$

$$\frac{\partial}{\partial L_1} = -\beta p'(L_1) W L_{2,nc} + \beta p'(L_1) W L_{2,c} + \omega_3 = 0 \tag{42}$$

$$\frac{\partial}{\partial \pi_1} = u_{\pi_1} + \frac{\sigma u_{y_1} + \omega_3(\phi_i - \sigma\phi_y)}{\kappa\sigma} - \omega_3\phi_\pi = 0 \tag{43}$$

$$\Leftrightarrow \pi_1 = \frac{\sigma u_{y_1} + \omega_3(\phi_i - \sigma\phi_y)}{\kappa\sigma} - \omega_3\phi_\pi \tag{44}$$

and hence:

$$y_1 = -\sigma i_1 + \sigma[(1-\epsilon)\pi_{2,nc} + \epsilon\pi_{2,c}] + [(1-\epsilon)y_{2,nc} + \epsilon y_{2,c}] \tag{45}$$

$$\frac{\sigma u_{y_1} + \omega_3(\phi_i - \sigma\phi_y)}{\kappa\sigma} - \omega_3\phi_\pi = \kappa y_1 + \beta[(1-\epsilon)\pi_{2,nc} + \epsilon\pi_{2,c}] \tag{46}$$

$$L_1 = \rho_L L_0 + \phi_i i_1 + \phi_y y_1 + \phi_\pi\left(\frac{\sigma u_{y_1} + \omega_3(\phi_i - \sigma\phi_y)}{\kappa\sigma} - \omega_3\phi_\pi\right) + \phi_0 \tag{47}$$

$$-\beta p'(L_1) W L_{2,nc} + \beta p'(L_1) W L_{2,c} + \omega_3 = 0 \tag{48}$$

E The Details of the Optimal Policy under Uncertainty

The Bayesian policymaker:

We solve the optimization problem of the Bayesian policymaker numerically. For each L_0, we evaluate the welfare loss on 1001 grid points of the interest-rate on the interval $[x - 0.1/400, x + 0.1/400]$ where x is the optimal policy rate in the absence of uncertainty, and choose the policy rate that minimize the welfare loss.

The robust policymaker:

We solve the optimization problem of the robust policymaker numerically. For each L_0, we evaluate the welfare loss on 1001 grid points of the interest-rate on the interval $[x - 0.1/400, x + 0.1/400]$ where x is the optimal policy rate in the absence of uncertainty, and choose the policy rate that minimize the welfare loss. For a given interest-rate, we need to solve the optimization problem of the hypothetical evil agent inside the head of the policymaker. We do so again numerically by evaluating the objective function of the evil agent. In particular, when only one parameter is uncertain, we compute the objective function on 21 grid points on the interval $[\theta_{min}, \theta_{max}]$ and choose the parameter value that maximizes the welfare loss. When two parameters are uncertain, we compute the objective function on 21-by-21 grid points on the interval $[(\theta_{1,min}, \theta_{2,min}), (\theta_{1,max}, \theta_{2,max})]$ where θ_1 and θ_2 are two parameters under consideration, and choose the combination of parameter values that maximizes the welfare loss.

F More Sensitivity Analysis

Optimal policy rate is higher with a smaller weight on the output stabilization term in the central bank's objective function. See Figures 22 and 23.

G The Zero Lower Bound Constraint

G.1 The Policy Trade-off without Parameter Uncertainty

In our baseline model, we asked the question of "how should the central bank respond to a credit boom" when the economy is in the non-crisis state today (at time $t = 1$). In this section, we modify the model in order to ask the same question, but when the economy is in a recession and the policy rate is at the zero lower bound (ZLB).

The aggregate demand equation is modified so that there is a negative demand shock, Ω_1, at time $t = 1$.

$$y_1 = E_1^{ps} y_2 + \sigma(i_1 - E_1^{ps}\pi_2 - i^*) - \Omega_1 \tag{49}$$

where the variable Ω_1 is set so that the optimal shadow policy rate is minus 50 basis points at $L_0 = 0.2$ ($\Omega_1 = 0.0113$), that is, the policy maker is constrained by the zero lower bound.

As shown in Figure 24, the trade-off facing the central bank is the same as described in the previous section. In addition, since the optimal shadow policy rate is negative, the constrained-optimal policy for the nominal short-term interest rate is zero: the constrained-optimal policy rate is zero for $0 \leq L_0 \leq 1$ (Figure 25). As shown in Figure 26, the optimal actual policy rate can be positive for a sufficiently large L_0. In our model, this happens when the severity of the crisis is comparable to that of the Great Depression.

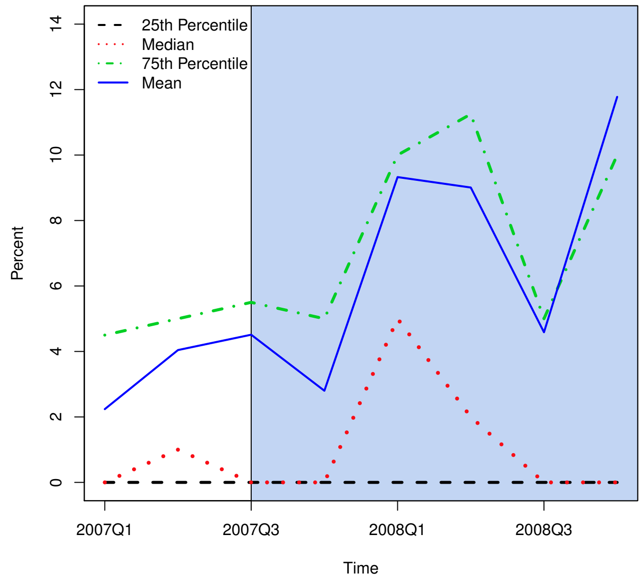
G.2 The Zero Lower Bound and Parameter Uncertainty

Uncertainty regarding the effectiveness of interest-rate policy ininfluencing the crisis probability affects both types of policymakers—the Bayesian and the robust policymakers—already facing a large contractionary shock in the same way as it affects the two types of policymakers in normal times. As shown in the left-column of Figure 27, the unconstrained optimal policy rate is higher in the presence of uncertainty than in the absence of it under the Bayesian policymaker. As shown in the left-column of Figure 28, the unconstrained optimal policy rate is higher in the presence of uncertainty than in the absence of it under the robust policymaker. For both types of policymakers, the unconstrained optimal policy rates remain below zero, and as a result, the actual optimal policy rate remains at zero.

Uncertainty regarding the severity of the crisis also affects the two types of policymakers facing a large contractionary shock in the same way as it affects them in normal times. As shown in the middle column of Figure 27 and 28, the unconstrained optimal policy rate is higher in the presence of uncertainty than in the absence of it. Since the unconstrained optimal policy rate remains below zero, the actual optimal policy rate remains zero.

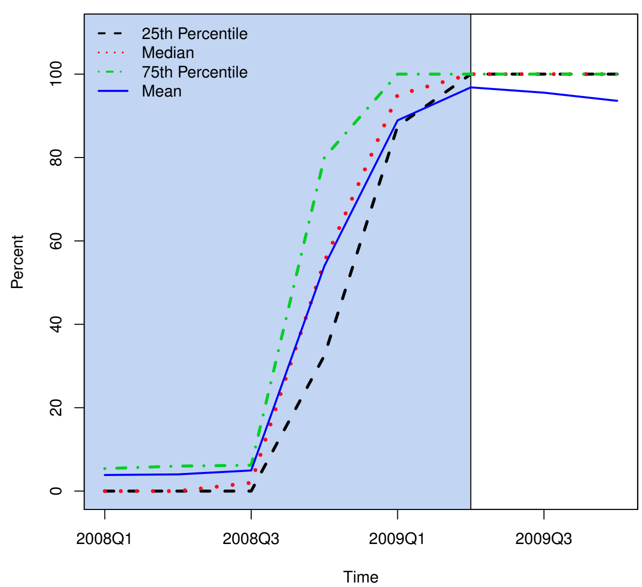
The Bayesian policymaker facing a large negative demand shock reduces the policy rate by less under uncertainty regarding the effect of policy on today's inflation and output, as shown in the right columns of Figure 27. This is a manifestation of the Brainard's attenuation principle: the Bayesian policymaker responds to the negative demand shock by reducing the policy rate by less under uncertainty. In our calibration, the optimal policy rate becomes positive. Uncertainty regarding the severity of the crisis affects the robust policymaker facing a large negative demand shock in the same way as in normal times. As shown in the right column of Figure 27, the unconstrained optimal policy rate is slightly lower in the presence of uncertainty than in the absence of it. The unconstrained optimal policy rate is below zero, and the actual optimal policy rate is zero.

Figure 13: Probability of Negative Growth of Average Real GDP in 2008



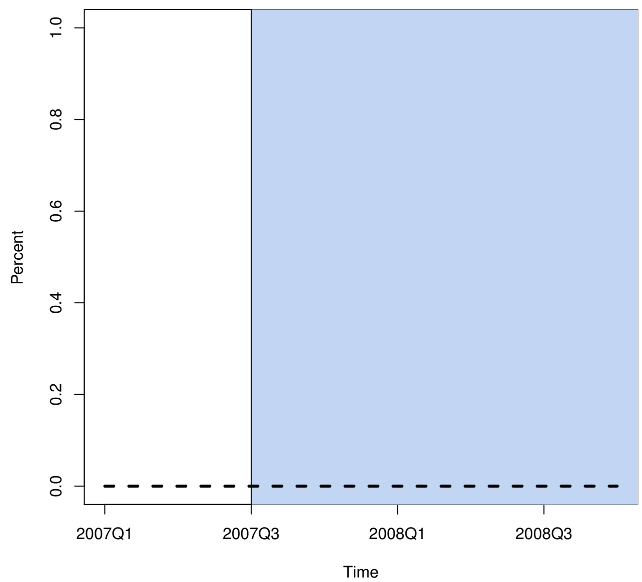
Note: See the text for details. The grey area identifies the Great Recession according to NBER dates. Data source: Survey of Professional Forecasters, Federal Reserve Bank of Philadelphia.

Figure 14: Probability of Negative Growth of Average Real GDP in 2009



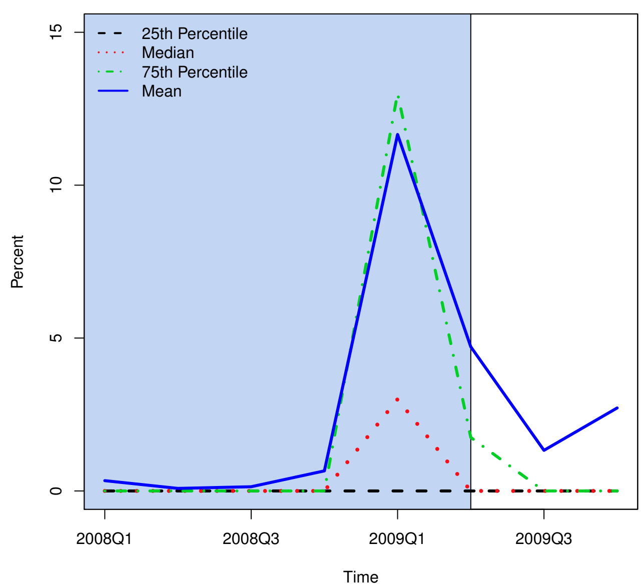
Note: See the text for details. The grey area identifies the Great Recession according to NBER dates. Data source: Survey of Professional Forecasters, Federal Reserve Bank of Philadelphia.

Figure 15: Probability of Negative Growth of Average CPI in 2008



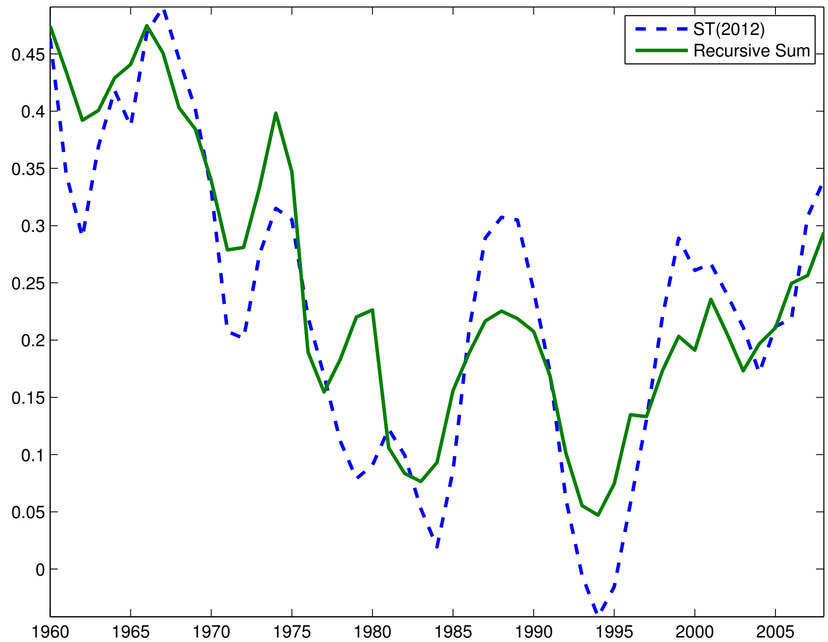
NOTE: See the text for details. The grey area identifies the Great Recession according to NBER dates. Data source: Survey of Professional Forecasters, Federal Reserve Bank of Philadelphia.

Figure 16: Probability of Negative Growth of Average CPI in 2009



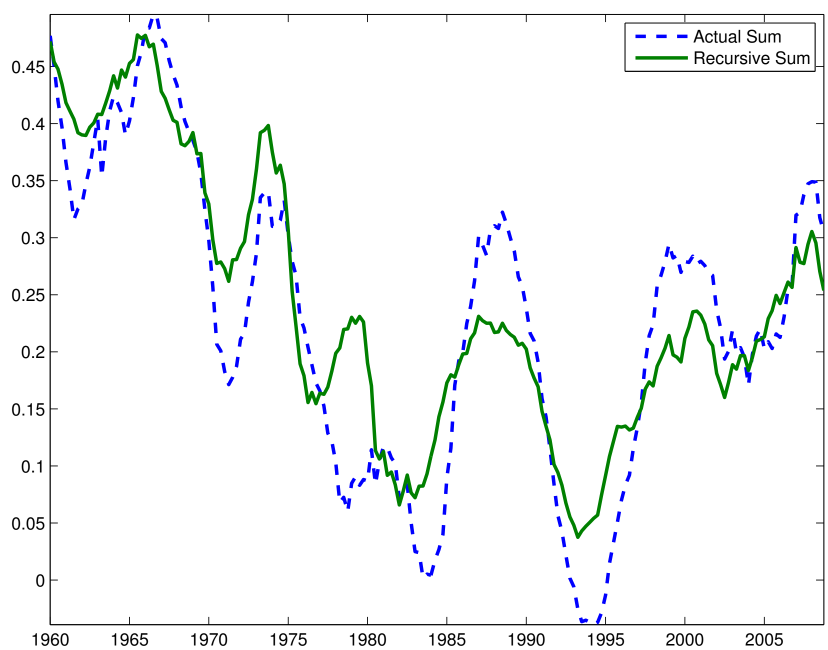
NOTE: See the text for details. The grey area identifies the Great Recession according to NBER dates. Data source: Survey of Professional Forecasters, Federal Reserve Bank of Philadelphia.

Figure 17: Annual 5-year Growth Rate of Real Banking Loans: Actual vs. Recursive Sum, 1960:2008



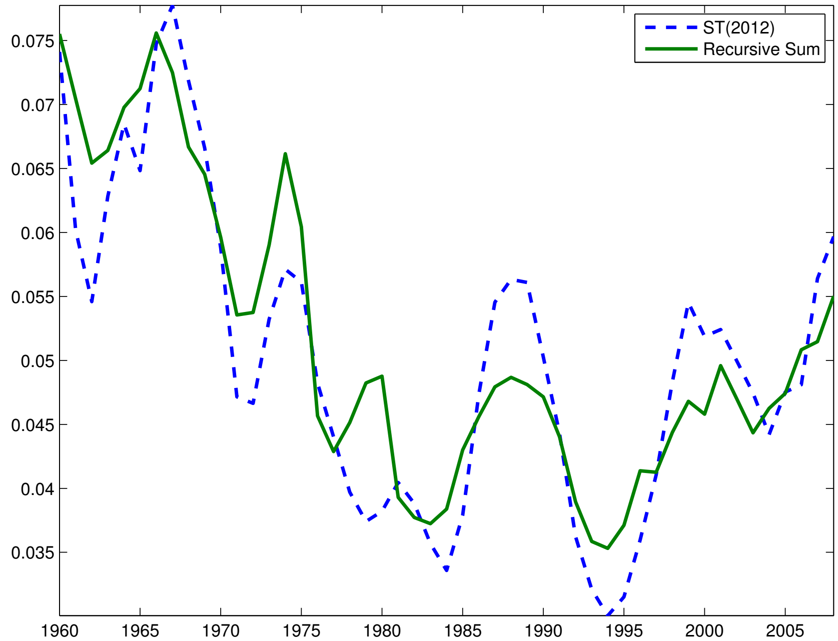
NOTE: Data source: Total loans and leases and security investments of commercial banks from the Board of Governors of the Federal Reserve H.8 release.

Figure 18: Quarterly 5-year Trailing Growth Rate of Real Banking Loans: Actual vs. Recursive Sum, 1960Q1:2008Q4



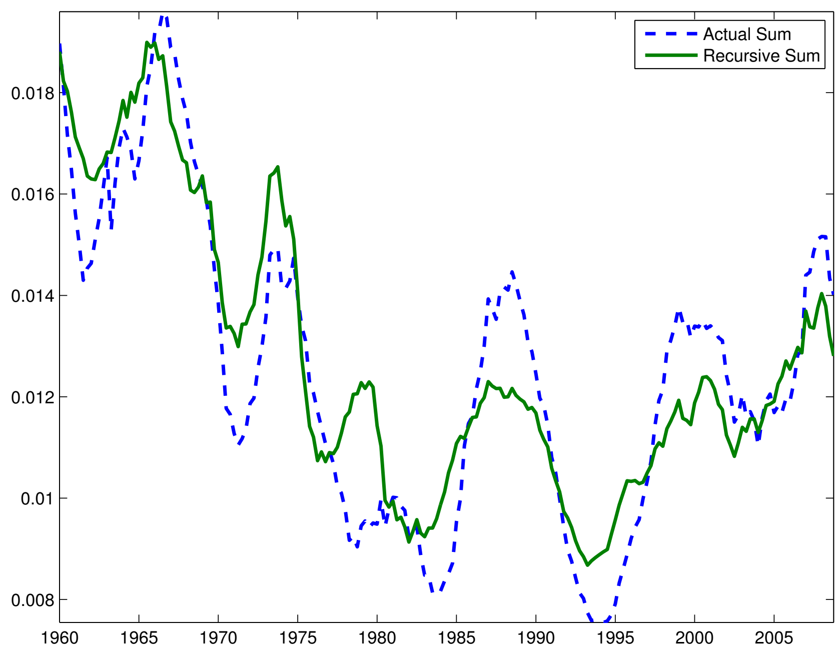
NOTE: Data source: Total loans and leases and security investments of commercial banks from the Board of Governors of the Federal Reserve H.8 release.

Figure 19: Annual Fitted Crisis Probabilities, 1960:2008



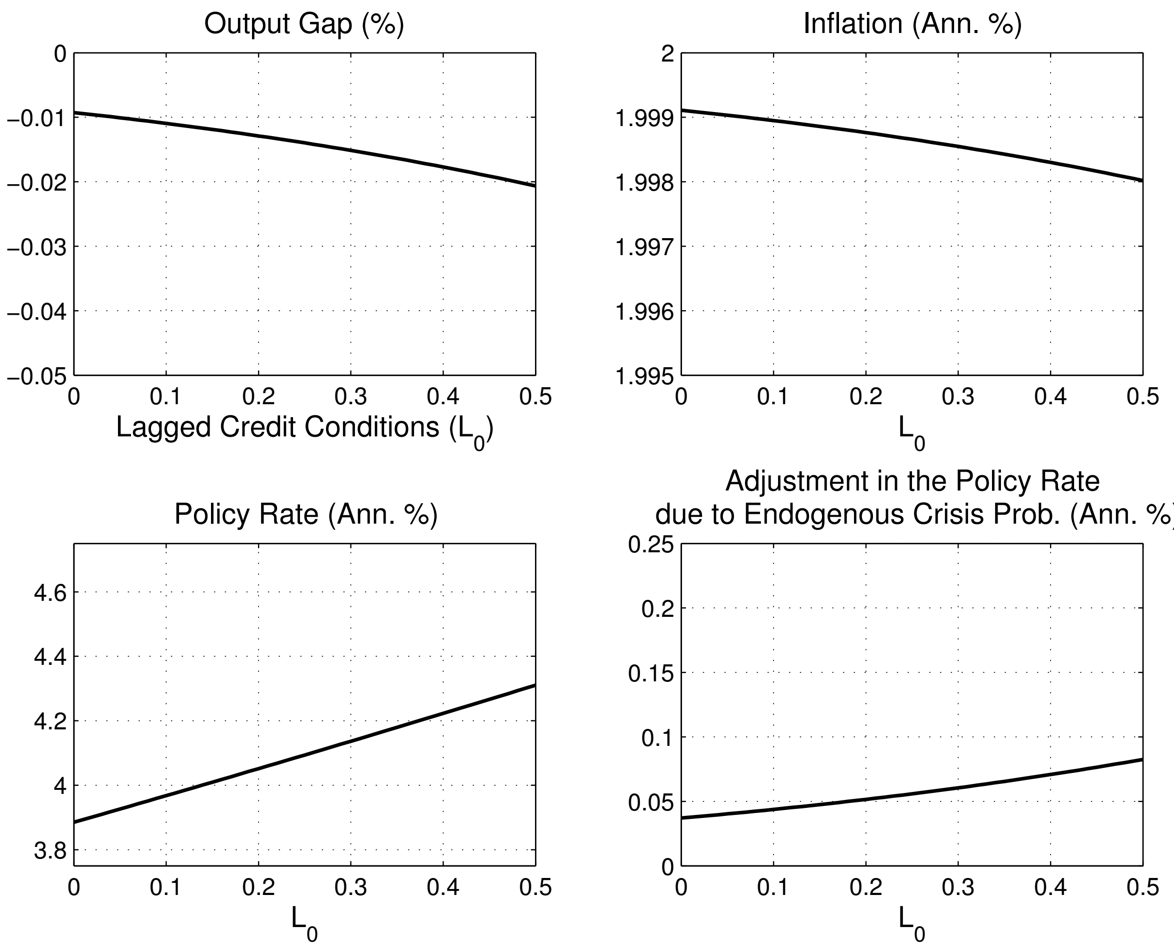
NOTE: Data source: Total loans and leases and security investments of commercial banks from the Board of Governors of the Federal Reserve H.8 release.

Figure 20: Quarterly Fitted Crisis Probabilities, 1960Q1:2008Q4



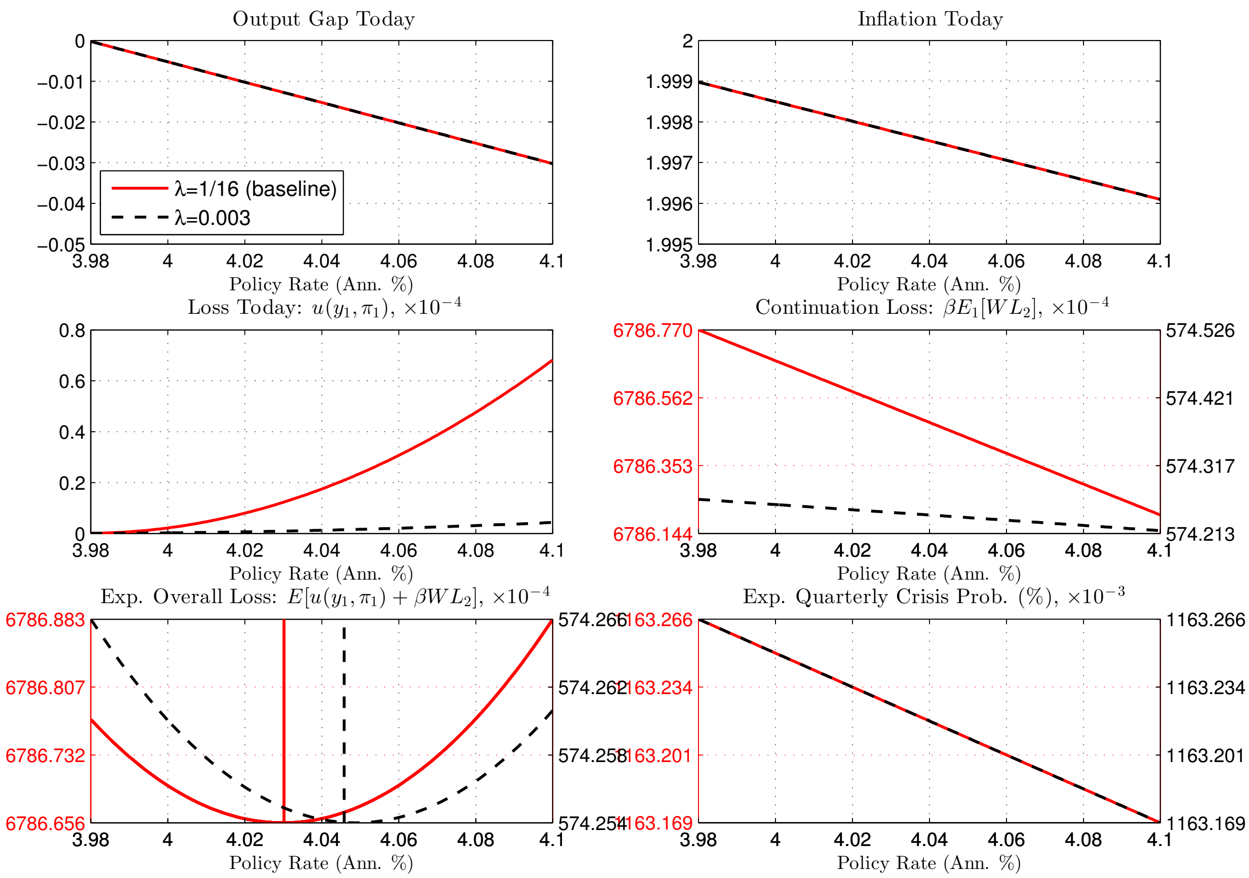
NOTE: Data source: Total loans and leases and security investments of commercial banks from the Board of Governors of the Federal Reserve H.8 release.

Figure 21: A Model with Credit-Output Linkage



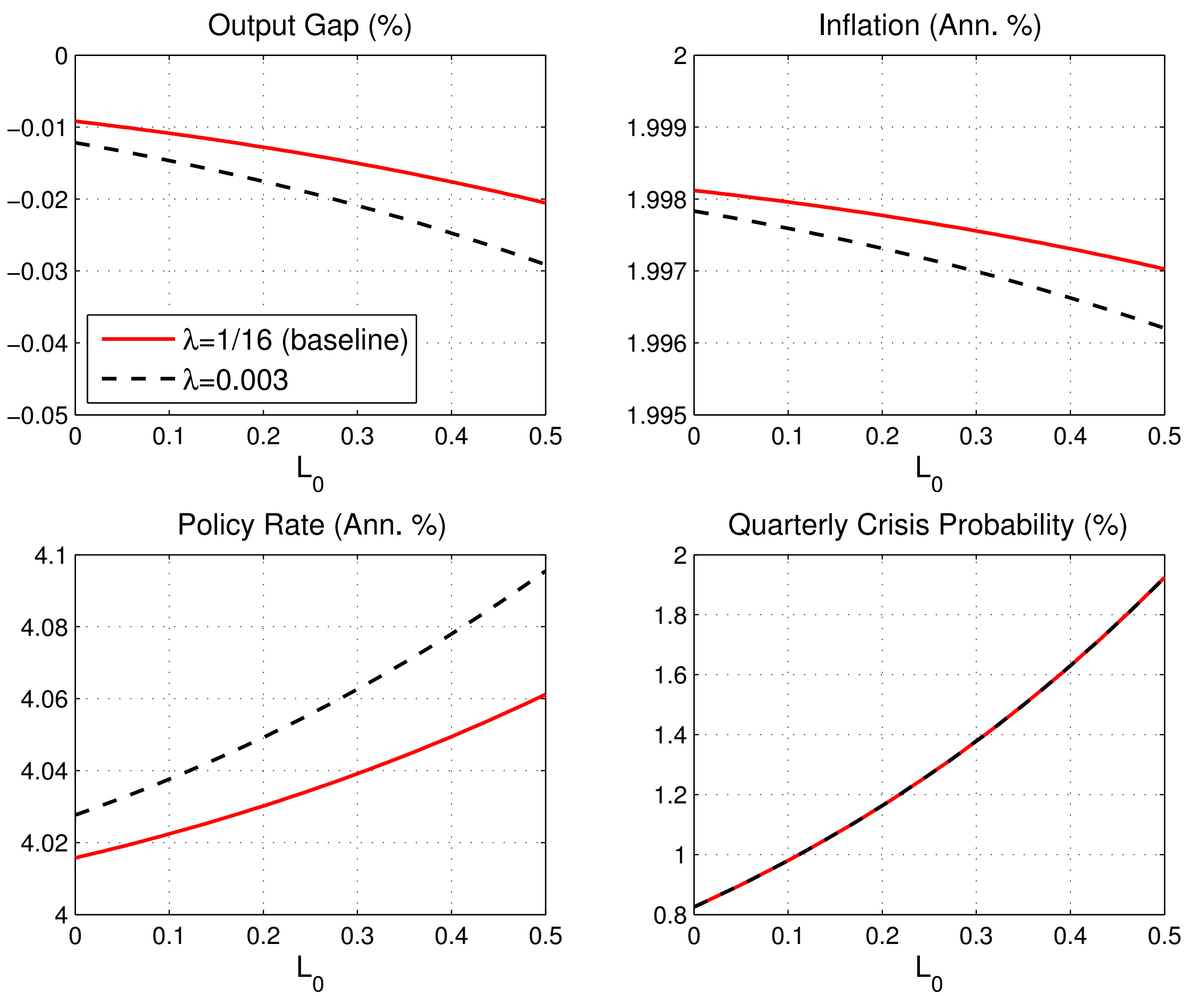
NOTE: This figure shows the optimal policy as a function of the initial level of the credit condition variable, L_0.

Figure 22: A Key Trade-off Faced by the Central Bank:
Alternative Weights on Output Stabilization



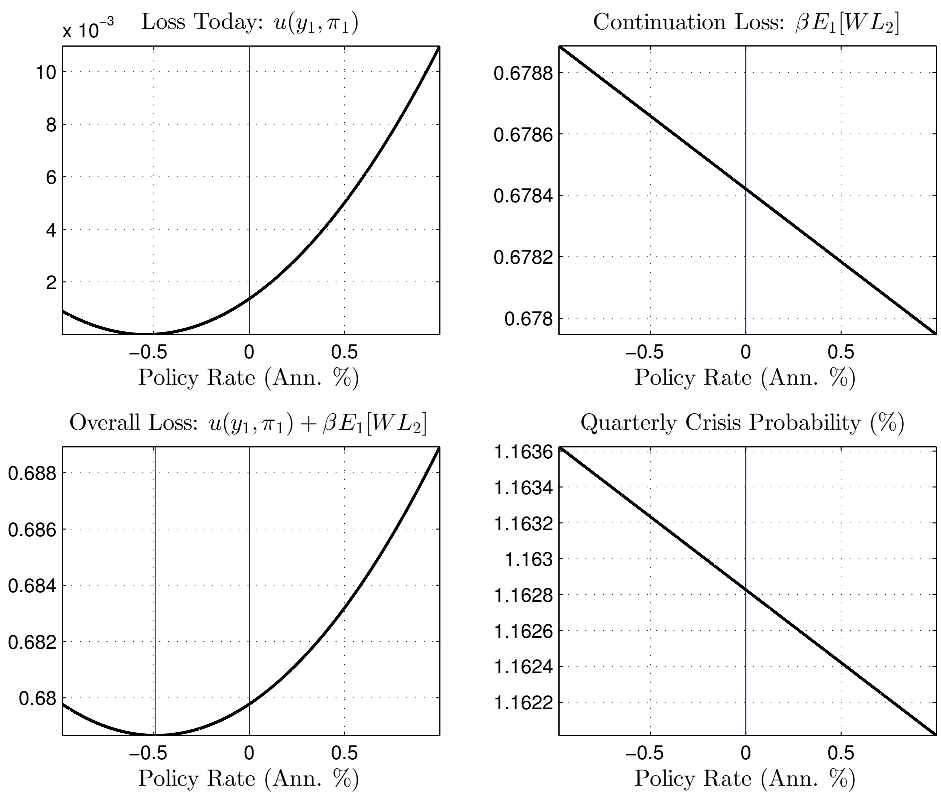
NOTE: In this figure, L_0 is set to 0.2, which is roughly the average value of this variable in the U.S. over the past five decades. In the bottom-left panel, the vertical lines show the optimal policy rate—the policy rate that minimizes the overall loss. The welfare losses are expressed as the one-time consumption transfer at time one that would make the household as well-off as the household in a hypothetical economy with efficient levels of consumption and labor supply, expressed as a percentage of the steady-state consumption, as described in Nakata and Schmidt (2014).

Figure 23: Leverage and Optimal Policy:
Alternative Weights on Output Stabilization



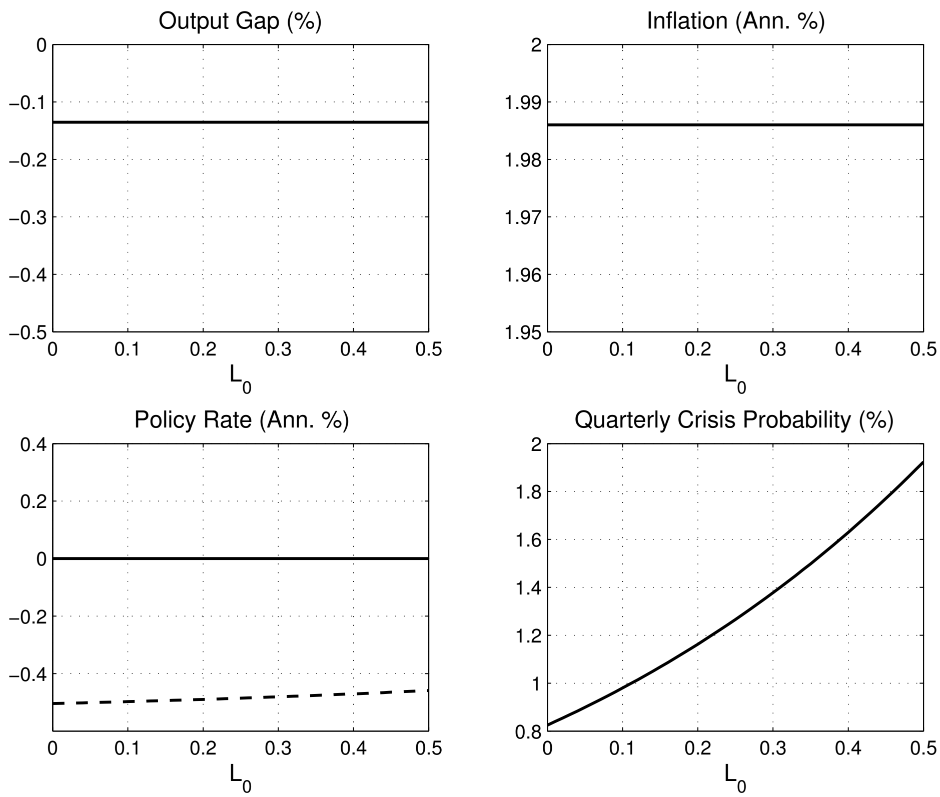
NOTE: This figure shows the optimal policy as a function of the initial level of the credit condition variable, L_0.1.

Figure 24: Optimal Policy Trade-off and the Zero Lower Bound Constraint



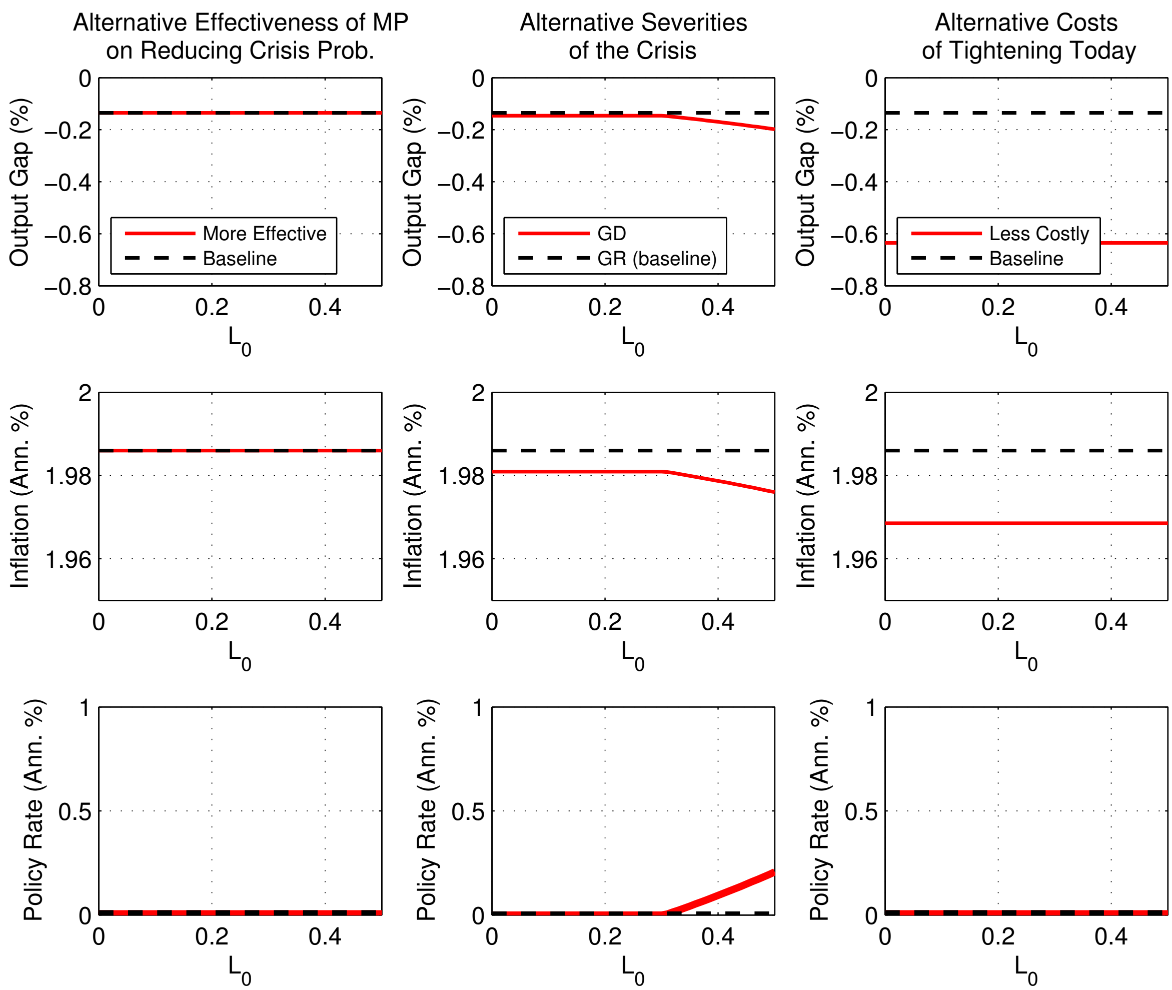
NOTE: In this figure, L_0 is set to 0.2, which is roughly the average value of this variable in the U.S. over the past five decades. In the bottom-left panel, the red vertical line shows the optimal shadow policy rate—the negative policy rate that would minimize the overall loss. The blue vertical line shows the constrained-optimal policy rate at the zero lower bound.

Figure 25: Optimal Policy at the ZLB



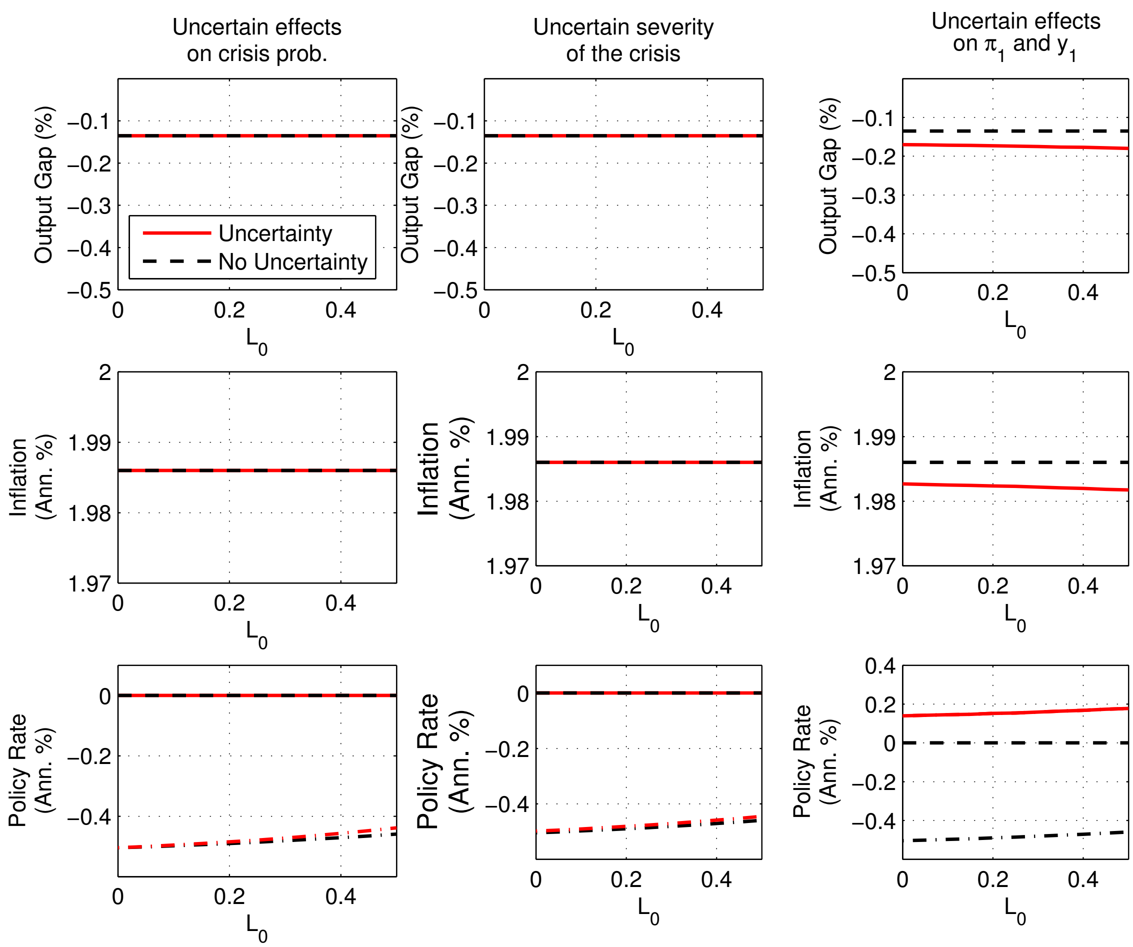
NOTE: In the bottom panels, the solid and dashed black lines are respectively for the actual and shadow optimal policy rates.

Figure 26: Optimal Policy and the ZLB: Alternative Scenarios



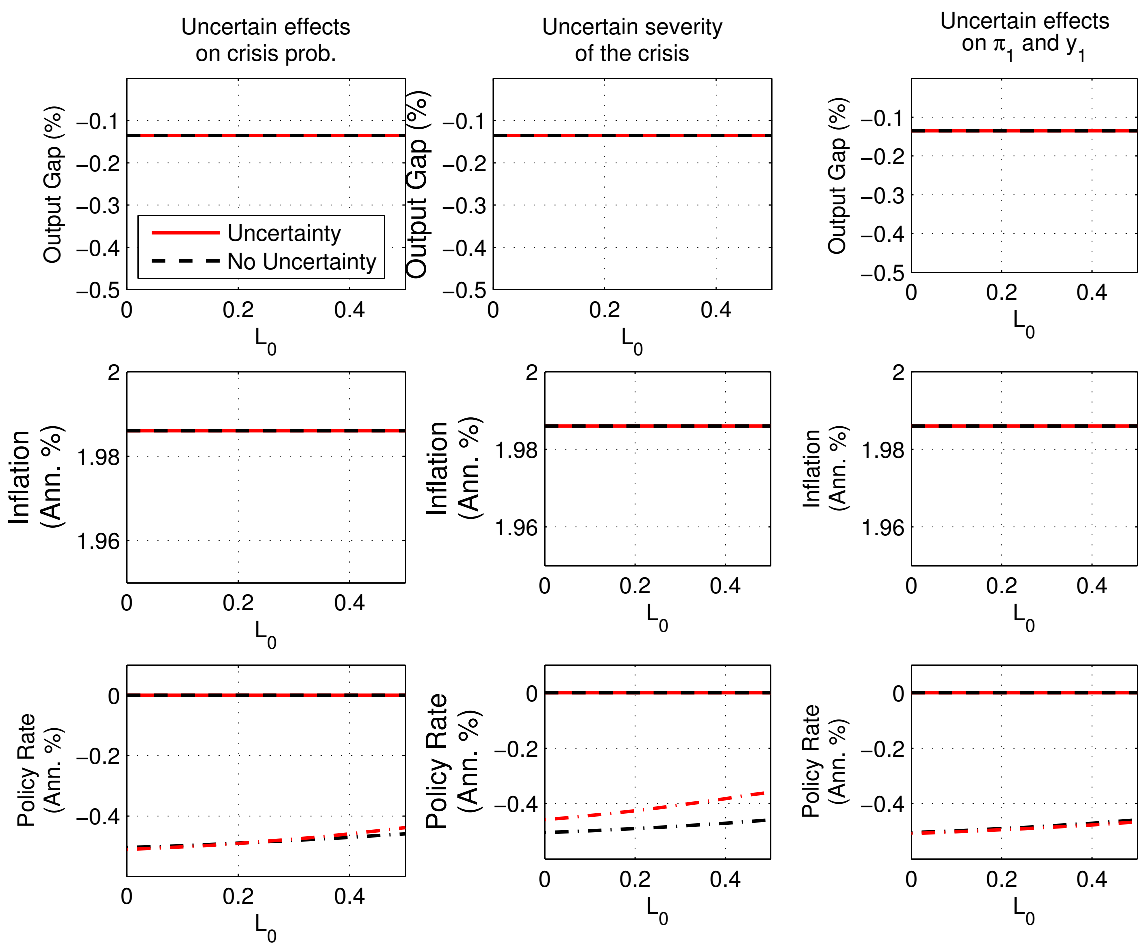
NOTE: This figure shows the optimal policy as a function of the initial level of the credit condition variable, L_0, under alternative calibrations of the model.

Figure 27: Optimal Policy Under Uncertainty at the ZLB: Bayesian Policymaker



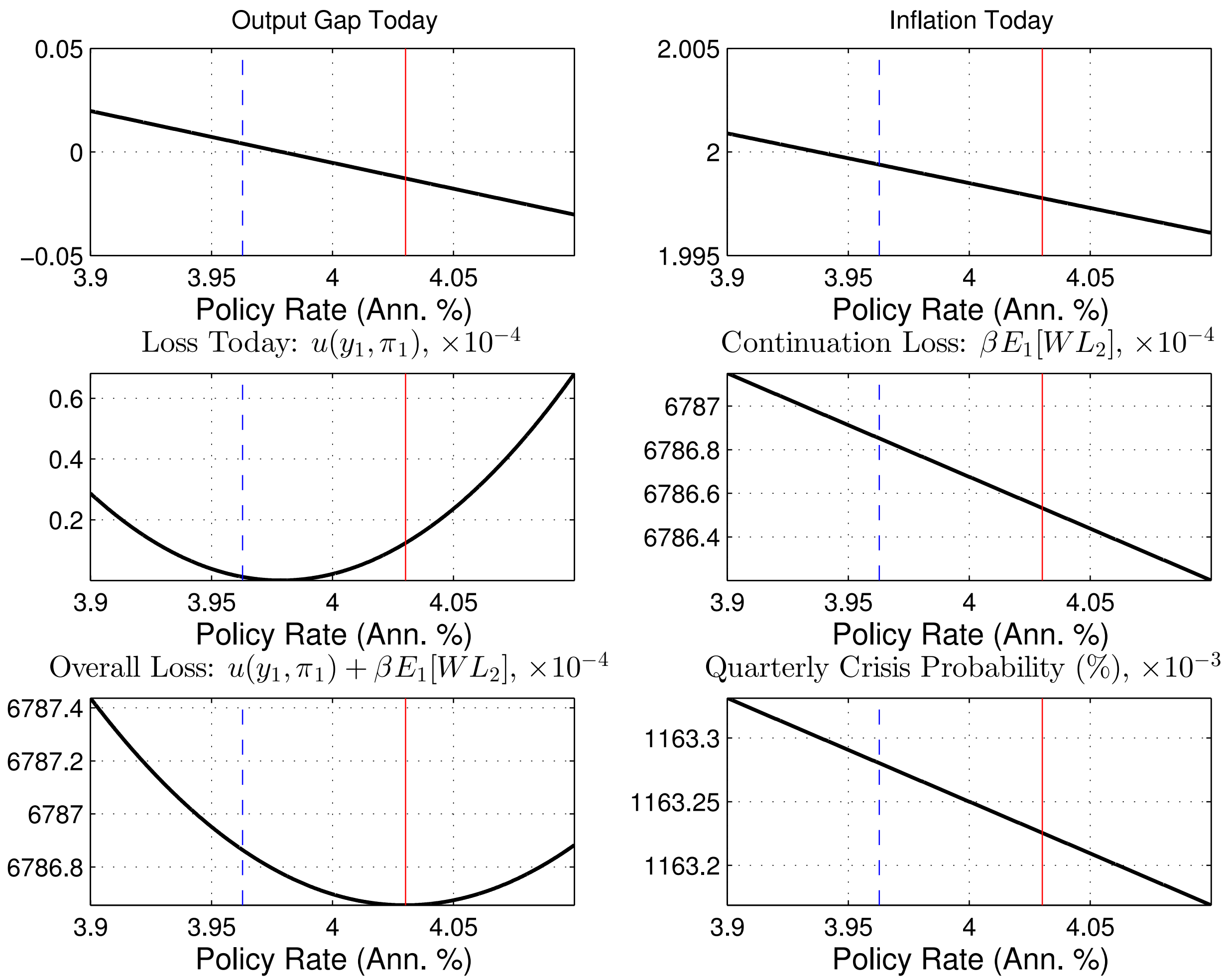
NOTE: In the bottom panels, the dash-dotted lines correspond to the shadow optimal policy rates.

Figure 28: Optimal Policy Under Uncertainty at the ZLB: Robust Policymaker



NOTE: In the bottom panels, the dash-dotted lines correspond to the shadow optimal policy rates.

Figure 29: A Key Trade-off Faced by the Central Bank (with a Taylor Rule)



NOTE: In this figure, L_0 is set to 0.2, which is roughly the average value of this variable in the U.S. over the past five decades. In the bottom-left panel, the red solid vertical line shows the optimal policy rate—the policy rate that minimizes the overall loss while the blue dashed line shows the rate implied by a simple Taylor rule. The welfare losses are expressed as the one-time consumption transfer at time one that would make the household as well-off as the household in a hypothetical economy with efficient levels of consumption and labor supply, expressed as a percentage of the steady-state consumption, as described in Nakata and Schmidt (2014).