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Uncertainty in a Borderline: Evidence from a Field Experiment*

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Abstract

Using a theoretical and experimental approach, this paper investigates the role of borderline uncertainty between success and failure in determining the input of effort by university students. By conducting a randomized experiment in an actual economics course in a Japanese university, we demonstrate that rank information feedback improves the performance of students with only average scores in the midterm examination, but harms the performance of high achieving students. We also construct a theoretical model of an uncertain borderline to interpret the experimental results. Our contribution is to demonstrate that the rank information can relate to the recognition of the borderline. In this case, the rank information feedback acting as an incentive scheme should vary for different types of students as a way to improve their motivation to learn.

Keywords: education, experiment, information feedback, relative rank, borderline

JEL Classification: C93, D03, D81, I21

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1 Introduction

Recently, many studies have shed light on the role of relative performance information in providing an incentive for individuals to make effort in firms and other organizations, including schools. For instance, Azmat and Iriberri (2010), Tran and Zeckhauser (2012), and Ashraf et al. (2014) explore the effects of relative performance information feedback on student academic achievement. Although these all demonstrate that relative performance information feedback has a statistically significant effect on the incentive of students to learn, the signs of this feedback process vary. For example, Azmat and Iriberri (2010) and Tran and Zeckhauser (2012) identify positive effects and argue that the relative performance information feedback motivates students because they inherently prefer competition. In contrast, Ashraf et al. (2014) find negative effects and relate the results to a model of optimal expectations in which low-ability students exert less effort as a means to avoid information about their poorer relative ability. In all these experiments, the rewards for students are via a piece-rate payment system, that is, students who study harder obtain more rewards. However, in some actual setups, student rewards are through a relative evaluation system, or a so-called tournament.

In this paper, we conduct a field experiment to explore the effect of relative performance information feedback on students' incentives to learn in an actual schooling environment. In our experiment, and in most academic courses, the performance of students in the midterm and final examinations together determines student grades. In our experiments, we evaluate students relatively and there is an uncertain borderline between success and failure. Following the empirical framework adopted by Tran and Zeckhauser (2012), we find that for students with intermediate (high) midterm examination scores, the provision of their rank information for the midterm examinations yields higher (lower) scores in the final examination. These results contrast with those of the abovementioned literature.

To interpret the experimental results, we develop a theoretical model of an uncertain borderline. If there is an uncertain borderline in a relative evaluation system, the performance ranking of students in the midterm examination serves as a source of information for students about their likelihood of passing the course. Therefore, we consider information on the performance ranking of students in the midterm examination as a proxy for grading uncertainty. Our theoretical model is consistent with the experimental result.

That is, for marginal students whose scores in the midterm examination lie on the borderline, ranking information serves as a signal indicating that they will need to work harder to pass the course. For students with sufficiently high scores in the midterm examination, ranking information is a signal that they can pass the course with only moderate effort. Therefore, knowledge of performance ranking information from the midterm examination affects student attitudes to the final examination.

The remainder of the paper is organized as follows. In Section 2, we describe the experimental design and the estimation results. Section 3 sets out the theoretical framework. Section 4 provides some concluding remarks.

2 An experiment and estimation

Experimental design We performed an experiment using first-year students enrolled in an introductory economics course during the second semester of 2012 (hereafter, we refer to this as Economics II) in a Japanese university. Prior to taking Economics II, these students had already taken an earlier introductory economics course during the first semester of 2012 (hereafter, we refer to this as Economics I). Credits for Economics I and II are requirements for graduation. Therefore, whether students can pass the Economics II course, as well as obtain a higher grade, is one of their prime concerns. Students were obliged to attend one of four classes for Economics I and II. Students with top-40 scores obtained from a mathematical test conducted immediately after entrance into the university (hereafter, we denote this Pretest of Mathematics) were placed in a more mathematically orientated economics class. We randomly allocated the other students to the remaining three classes, with all classes fixed across the first and second semesters.

Homework and an examination score determine student grades in this course, which is the weighted average of their scores in the midterm and the final examination.[1] All students sat the same examinations at the same time regardless of their class. According to the university's guidelines, the standard pass score for the course is 60. However, if students are strictly graded by this guideline, it frequently occurs that the pass rate is

[1] In the case of second semester 2012, "40% of the midterm examination score" + "60% of the final examination score" + "the number of homework submissions (1 point per submission × 10)". Full scores for both examinations were 110. We assigned a score of 100 out of 110 for the economics component, and to encourage the students to study mathematics as part of their remedial education, assigned a score of the remaining 10 out of 110 to basic mathematics.

inappropriately low. Consequently, in many cases, the decision on whether students pass or fail the course depends on their relative scores as a means to avoid this situation, with instructors adjusting the final scores to obtain a reasonable distribution of grades. In other words, instructors determine the borderline between success and failure. Students are aware of this particular evaluation system because the course instructors announce it at the beginning of the semester.[2]

The details of the experiment are as follows.[3] Using a random number generator, we randomly assigned all students to a treatment group or a control group. Following the midterm examination, we sent a letter to students, personally delivered during class time, which revealed their total score for the midterm examination and their score for the mathematics problems. In addition, letters to students in the treatment group also reported information on the rank of their score in the midterm examination (i.e. the rank of each student in relation to other students in the course and the total number of students in the course). We did not include this information in the letters to those students in the control group. Figures 1 and 2 reproduce the information provided to the students in the treatment and control groups, respectively.[4]

[Insert Figure 1 here.]

[Insert Figure 2 here.]

As discussed in Tran and Zeckhauser (2012), we cannot rule out the possibility that students attempted to communicate their own rank information to a classmate. However, it would be difficult for a student in the control group to identify a student in the treatment group with exactly the same score. Therefore, while students in the treatment group know their rank exactly, students in the control group will only have a vague knowledge about their rank. Thus, our experiment can reveal whether the difference in the precision of rank information accounts for the difference in the final examination scores.

[2] In addition, students have already taken Economics I in the first semester, and only 58.2% of the students (171 of 292 students) passed. Therefore, students know that relative performance is important for passing the examinations in Economics II.

[3] This experiment was approved by the university's research ethics committee (Application Number H26-002).

[4] Our letters are similar to those used in Ashraf et al. (2014).

Descriptive statistics Table 1 provides the descriptive statistics. With the full sample, the mean value of the midterm examination score is 51.76. Comparing the mean values for the control and treatment groups, we can see that the mean score for the control group (53.35) is three points higher than that for the treatment group (50.15). With the full sample, the mean value of the final examination score is 67.17. We can see that the mean score of the final examination for the control group (67.61) is one point higher than that for the treatment group (66.71). The mean value of the difference between the midterm and the final examination for the treatment group (16.57) is two points higher than that for the control group (14.26).

[Insert Table 1 here.]

To visually better understand the relationship between the midterm and the final examination scores, we plot a scatterplot for the control and the treatment groups in Figure 3. As shown, we can see that the improvements in the final examination scores depend on the range of the midterm examination scores. For example, students with a score of 60 or more in the midterm examination in the control group tended to obtain higher scores in the final examination than comparable students in the treatment group. Moreover, control group students with a midterm examination score less than 30 improved their final examination score more than did students in the treatment group. These observations suggest that the impact of information on a student's relative rank in the midterm examination on the extent of improvement in the final examination varied by the midterm examination score.

[Insert Figure 3 here.]

Estimation procedure We wish to estimate the impact of the rank information in the midterm examination on the score in the final examination. Following Tran and Zeckhauser (2012), we rely on the following empirical framework:

$$Y_i = \boldsymbol{X}_i \boldsymbol{\alpha} + \beta D_i + \gamma + \epsilon_i, \tag{1}$$

where i indexes the students and Y_i are the scores in the final examination for student i. The vector $\boldsymbol{X}_i$ contains the student characteristics, including the scores for the midterm examination, Economics I, and the Pretest of Mathematics. We also include dummy variables identifying males and the different classes. Table 1 provides descriptive statistics for these variables. $\boldsymbol{\alpha}$ is the impact of these characteristics on their scores. D_i is a dummy variable equal to one if student i is given information on their relative rank in the midterm examination (that is, the student is in the treatment group), and zero if student i is not given this information (that is, in the control group). β is the effect of the information on the relative position in the midterm examination for the final examination. γ is the constant term and ϵ_i is the disturbance. We assume ϵ_i is distributed $N(0, \sigma^2)$.

Results We ran a regression of the final examination scores on the treatment dummy variable and the other student characteristics following equation (1). The results are in columns (1a) and (1b) in Table 2. In addition, we also ran a regression for various intervals across the midterm examination score distribution. In particular, we divided the sample into three subsamples, corresponding to groups of students whose scores in the midterm examination were low, intermediate, and high. The results in columns (2a) and (2b) in Table 2 are for the subsample of students whose midterm examination scores were less than the 20th percentile. The results in columns (3a) and (3b) in Table 2 are for the subsample of students whose midterm examination scores were between the 20th and 65th percentiles. Lastly, the results in columns (4a) and (4b) are for the subsample of students whose midterm examination scores were more than the 65th percentile.

In regression specifications (1a), (1b), (2a), and (2b), the rank information has no significant effects on the final examination scores.[5] On the other hand, in regression specification (3a) and (3b), the rank information feedback has significant positive effects on the final examination scores. The results imply that for those students with an intermediate score in the midterm examination, information about their rank in the midterm examination improved their score in the final examination. In contrast, in regression specifications (4a) and (4b), the rank information feedback has significant negative effects on the final examination scores. That is, for students with high scores for the midterm examination, information on their rank in the course could reduce their incentive to study

[5]For students with low scores for the midterm examination, we need to pay careful attention to the interpretation of these estimation results. For example, 9% of students who sat the midterm examination were excluded from the sample because they did not actually sit the final examination. The average score for these students in the midterm examination was 31.3. This suggests that sample attrition correlates with lower scores in the midterm examination, and this leads to a downward bias (toward zero) in the estimated treatment effects.

for the final examination.[6]

[Insert Table 2 here.]

Discussion Azmat and Iriberri (2010) and Tran and Zeckhauser (2012) found that the second scores of students with rank information were higher than those of the students without rank information, regardless of their first scores. On this basis, they argued that rank information improves student scores because it stimulates competitiveness among students. On the other hand, Ashraf et al. (2014) found that the rank information decreased student performance. They interpreted this result using a model of optimal expectations in which low-ability students exert low effort to avoid obtaining information about their relative ability.

Unlike these existing findings, our results demonstrate that the rank information on final scores has opposite effects depending on whether the students' midterm scores are intermediate or high. While the results are surprising, they seem reasonable if students recognize the importance of passing a borderline between success and failure. In this manner, a report on student ranks in the midterm examination decreases grading uncertainty for students because it provides them with useful information on their relative position in the course regarding performance. In the next section, we construct a simple theoretical model to interpret our experimental results.

3 Theoretical interpretation

Setup Consider a course where the assessment comprises a midterm examination and a final examination. The course grade awarded for each student depends on a final score, which is the weighted average of the scores obtained in the two examinations: $\phi s_0 + (1-\phi)s$, where s_0 and s are the scores in the midterm and the final examination, respectively, and $\phi \in (0,1)$ is the weight given to the midterm examination. Before the course commences, the pass score is set to $S \in \mathbb{R}$ and is known to all students. However, as described in the previous section, students know that the actual borderline for passing can change according to the profile of all student final scores to realize a reasonable distribution of student grades. Therefore, S is not a rigorous criterion for course success,

[6] In Appendix A, we confirm that the results are robust with respect to the alternative way of dividing the sample.

rather an approximate standard expected for the passing score. A student with score s_0 in the midterm examination then believes that the average required score in the final examination is equal to[7]

$$\bar{s} \equiv \frac{S - \phi s_0}{1 - \phi}.$$

However, the actual borderline, denoted by $\hat{s}$, is a random variable of a form such as

$$\hat{s} = \theta \bar{s}, \tag{2}$$

where θ is a random variable that follows a uniform distribution among $[1 - \varepsilon, 1 + \varepsilon]$ and $\varepsilon \in (0, 1)$.

Following the midterm examination, every student in the class knows their own score $s_0 \in \mathbb{R}$ for the examination. Each student then makes effort $e \geq 0$ in preparing for the final examination, and obtains a score

$$s = e \tag{3}$$

in the final examination, but at a cost of

$$c = e^2. \tag{4}$$

Finally, we assume students are concerned only whether they pass the course.[8] For any final examination scores s, the utility of the student is given by

$$u(s) = \begin{cases} 1 & \text{if } s \geq \hat{s}, \\ -1 & \text{if } s < \hat{s}. \end{cases}$$

[7]When there is no uncertainty about the borderline, the condition for passing the examination would be $\phi s_0 + (1 - \phi)s \geq S$. This is equivalent to $s \geq \bar{s}$.

[8]The reward system in our experiment is a tournament because students are evaluated relatively. However, because there are many participants and many winners, we assume that participants do not care about the performance of the other participants directly but rather pay attention to the borderline between win and lose. Of course, in reality, some students will attempt to obtain the best grade possible. However, to keep the model simple and understand the role of a borderline as clearly as possible, we assume this preference relation. In the context of tournaments, Aoyagi (2010) and Edere (2010) theoretically explore the relationship between an information feedback and an agent's incentives.

Equilibrium The students select their level of effort e to maximize their expected utility. In a mathematical formulation, the optimization problem for students is

$$\begin{aligned} \text{Maximize} \quad & U(e) \equiv \mathbb{E}[u(s) - c], \\ \text{subject to} \quad & (3), (4), e \geq 0. \end{aligned}$$

Through simple calculations, we obtain the functional form of U[9]:

$$U(e) = \begin{cases} -1 - e^2 & \text{if} \quad 0 \leq e < (1-\varepsilon)\bar{s}, \\ -\frac{1}{\varepsilon} + \frac{1}{\varepsilon \bar{s}} e - e^2 & \text{if} \quad (1-\varepsilon)\bar{s} \leq e < (1+\varepsilon)\bar{s}, \\ 1 - e^2 & \text{if} \quad e \geq (1+\varepsilon)\bar{s}. \end{cases}$$

We adopt the following assumption to focus on the solution most relevant for the empirical analysis.

Assumption 1 *Uncertainty for students is sufficiently strong:*

$$\varepsilon > \frac{1}{3}.$$

If ε is small, then each student selects either a zero or minimal input for passing the examination regardless of s_0. This is because students can deterministically control their grades because of weak uncertainty and hence weigh the certain benefits of qualification and the required effort costs. In contrast, if ε is sufficiently large, then students cannot do this when their scores in the midterm examination are of an intermediate value. Because the latter case is relevant for the experimental results, we adopt Assumption 1 to rule out the former irrelevant case.

Proposition 1 *Let* $\underline{s_0} = \phi^{-1}[S - \frac{1}{2}(1-\phi)(\varepsilon(1-\varepsilon))^{-\frac{1}{2}}]$ *and* $\bar{s_0} = \phi^{-1}[S - (1-\phi)(2\varepsilon(1+\varepsilon))^{-\frac{1}{2}}]$*. In equilibrium, the optimum* e^* *is characterized by*

$$e^* = \begin{cases} 0 & \text{if } s_0 < \underline{s_0}, \\ \frac{1}{2\varepsilon \bar{s}} & \text{if } \underline{s_0} \leq s_0 < \bar{s_0}, \\ (1+\varepsilon)\bar{s} & \text{if } s_0 \geq \bar{s_0}. \end{cases}$$

[9]The derivation is shown in Appendix B.

Proof See Appendix C.

The intuition underlying Proposition 1 is as follows. If a student obtains a low score in the midterm examination (that is, $s_0 < \underline{s_0}$), they will make no effort because an unacceptably large effort is required in the final examination to pass the course. In contrast, if their grade in the midterm examination is sufficiently high (that is, $s_0 \geq \bar{s_0}$), the student will minimize their effort under the constraint of a passing score because this can be at a very small effort cost. Finally, in the intermediate case (that is, $\underline{s_0} \leq s_0 < \bar{s_0}$), the student selects the inner solution as a means of balancing the uncertain benefit and certain cost. In this case, the student cannot predict with certainty the result of the final examination.

Interpreting the empirical results We focus on the relationship between the final examination scores and the grading uncertainty. The equilibrium marginal effect of an increase in uncertainty is

$$\frac{\partial s}{\partial \varepsilon} = \frac{\partial e^*}{\partial \varepsilon}$$

given each student's production function is given by $s = e$. Immediately from Proposition 1, we obtain the relationship between the degree of uncertainty and the final examination scores.

Proposition 2 *A decrease in the degree of uncertainty improves the final examination scores of students with intermediate scores in the midterm examination ($\underline{s_0} \leq s_0 < \bar{s_0}$). However, this exerts a detrimental effect on the final examination scores for students with high scores in the midterm examination ($s_0 \geq \bar{s_0}$).*

Proof From Proposition 1, the sign of $\frac{\partial e^*}{\partial \varepsilon}$ is negative if $\underline{s_0} \leq s_0 < \bar{s_0}$ but positive if $s_0 \geq \bar{s_0}$. (Q.E.D.)

Proposition 2 demonstrates that the effects of information feedback on the final examination scores vary according to the range of midterm examination scores. In particular, the information feedback has positive (negative) effects for students with intermediate (high) midterm scores. Greater transparency thus encourages students with intermediate scores in the midterm examination. This is because it reduces the risk of a fail in the final examination and therefore induces them to increase their required effort to achieve a passing score. However, this also expands the opportunity for students with high scores to economize on the required level of effort to pass the final examination.

4 Conclusion

In this paper, we conduct a randomized experiment and find that for students with intermediate (high) midterm examination scores, the more precise the information provided, the higher (lower) their score in the final examination. In our theoretical model, students study to pass the course, but they have to pay a cost in the effort required to obtain a higher score. We also assume that students are concerned only whether they pass the course. In this setup, the effect of additional information on effort depends on the midterm examination scores. A decrease in uncertainty then raises (lowers) the final examination scores of students with intermediate (high) scores from the midterm examination. This theoretical explanation is consistent with the results of our randomized experiment.

Our experimental results are almost consistent with the result of Eriksson et al. (2009), who conducted randomized experiments in a laboratory situation. They find that in a tournament setup, the underdog subjects increase their efforts when informed of their relative performance continuously. However, for the frontrunner subjects, they do not find any significant effects of relative performance information on their efforts. The difference in award system appears to account for the difference in results. In our experiment, those who obtain a sufficiently high score in the midterm examination can pass the course with only a moderate effort. However, only one of a matched pair is rewarded in the laboratory experiment in Eriksson et al. (2009). Therefore, the frontrunner subjects who know their performance is higher than that of their competitors will not slacken off. Therefore, our findings confirm the results of Eriksson et al. (2009) in a real environment.

Our theoretical and experimental results have the following policy implications. For students with intermediate midterm examination scores, it is beneficial to inform them of their performance ranking as a means of decreasing uncertainty. However, performance ranking information may be harmful for students with high midterm examination scores.[10]

[10] Students may also need to make effort when studying other courses. In this case, ranking information is beneficial for students whose scores are sufficiently high, because it enables them to spend more time

Therefore, the nature of the incentive scheme should vary according to different types of students to improve their motivation to learn.

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Appendix

A Robustness checks

In the main results, we arbitrarily divide the sample into three subsamples, corresponding to groups of students with low, intermediate, and high scores in the midterm examination. To confirm whether the results are robust against an alternative way of dividing the sample, we ran regressions of the final examination scores on the treatment dummy variable

completing other important tasks.

and the other explanatory variables for other intervals across the midterm examination score distribution. Columns (1a), (1b) and (2a), (2b) in Table A.1 include students whose midterm examination scores were less than the 15th and 25th percentiles, respectively. The results show that the rank coefficients are statistically insignificant.[11]

The results for columns (3a) and (3b) in Table A.1 are for students whose midterm examination scores were more than the 15th percentile and less than the 70th percentile. The results in columns (4a) and (4b) in Table A.1 are for students whose midterm examination scores were more than the 25th percentile and less than the 60th percentile. The results in columns (3a)–(3b) and (4a)–(4b) are cases where the ranges of intermediate scores are wider and narrower than the main results, respectively. All of the results demonstrate that the rank coefficients display statistically significant positive values. The results for columns (5a) and (5b) in Table A.1 are for students whose midterm examination scores were more than the 60th percentile. The results for columns (6a) and (6b) in Table A.1 are for students whose midterm examination scores were more than the 70th percentile. All of the results indicate that the rank coefficients have statistically significant negative values, except for that in (5b). These results demonstrate that the estimation results are robust, even when we divide the sample differently.

[Insert Table A.1 here.]

B Derivation of the functional form of $U(e)$

For the case of $s = e < (1-\varepsilon)\bar{s}$, students fail the examination for certain. Therefore, $U(e) = -1 - e^2$. At the same time, $s = e \geq (1-\varepsilon)\bar{s}$ ensures that the students pass the examination, that is, $U(e) = 1 - e^2$. If $(1-\varepsilon)\bar{s} \leq s = e < (1+\varepsilon)\bar{s}$, whether students pass or fail the examination depends on the realized $\hat{s}$. Because $\hat{s} \leq s = e \Leftrightarrow \theta \leq \frac{e}{\bar{s}}$, $U(e)$ can be written as follows:

$$\begin{aligned} U(e) &= \int_{1-\varepsilon}^{\frac{e}{\bar{s}}} 1 \times \frac{1}{2\varepsilon} d\theta + \int_{\frac{e}{\bar{s}}}^{1+\varepsilon} (-1) \times \frac{1}{2\varepsilon} d\theta - e^2 \\ &= -\frac{1}{\varepsilon} + \frac{1}{\varepsilon\bar{s}} e - e^2. \end{aligned}$$

[11] As described in footnote 5, we need to consider this when interpreting the results for the subsample with low midterm scores owing to sample attrition.

C Proof of Proposition 1

Put $\tilde{U}(e) = -\frac{1}{\varepsilon} + \frac{1}{\varepsilon\bar{s}}e - e^2$. Let $\tilde{e}$ be the unique maximizer of $\tilde{U}$, which satisfies $\tilde{U}'(\tilde{e}) = 0$.[12] As U is decreasing in $[0, (1-\varepsilon)\bar{s})$ and $[(1+\varepsilon)\bar{s}, +\infty)$, U can be maximized only at 0, $\tilde{e}$ or $(1+\varepsilon)\bar{s}$.

Case 1: $\tilde{e} < (1-\varepsilon)\bar{s}$

In this case, $e^* = 0$ because U is decreasing throughout the domain. We obtain that $\tilde{e} < (1-\varepsilon)\bar{s}$ if and only if

$$s_0 < \phi^{-1}\Big[S - (1-\phi)\big(2\varepsilon(1-\varepsilon)\big)^{-\frac{1}{2}}\Big] \equiv s_0^1.$$

Case 2: $(1-\varepsilon)\bar{s} \leq \tilde{e} < (1+\varepsilon)\bar{s}$

The condition $(1-\varepsilon)\bar{s} \leq \tilde{e} < (1+\varepsilon)\bar{s}$ is equivalent to

$$s_0^1 \leq s_0 < \phi^{-1}\Big[S - (1-\phi)\big(2\varepsilon(1+\varepsilon)\big)^{-\frac{1}{2}}\Big] \equiv s_0^2.$$

In this case, $e^* = 0$ or $\tilde{e}$ because U is decreasing for $e \geq \tilde{e}$. The condition for $e^* = \tilde{e}$ is $U(0) \leq U(\tilde{e})$, which can be reduced to

$$s_0 \geq \phi^{-1}\Big[S - 2^{-1}(1-\phi)\big(\varepsilon(1-\varepsilon)\big)^{-\frac{1}{2}}\Big] \equiv s_0^3.$$

It is obvious that $s_0^1 < s_0^3$. Besides, $s_0^3 < s_0^2$ under Assumption 1: $\varepsilon > \frac{1}{3}$.

Therefore, $e^* = 0$ if $s_0 < s_0^3$ and $e^* = \tilde{e}$ if $s_0^3 \leq s_0 < s_0^2$.

Case 3: $(1+\varepsilon)\bar{s} \leq \tilde{e}$

By Case 2, $(1+\varepsilon)\bar{s} \leq \tilde{e}$ if and only if $s_0 \geq s_0^2$. In this case, $e^* = 0$ or $\bar{e}$ because U is increasing in $[(1-\varepsilon)\bar{s}, (1+\varepsilon)\bar{s})$. The condition for $e^* = \bar{e}$ is $U(\bar{e}) \geq U(0)$ and it is equivalent to

$$s_0 \geq \phi^{-1}\Big[S - 2^{\frac{1}{2}}(1-\phi)(1+\varepsilon)^{-1}\Big] \equiv s_0^4.$$

By long but straightforward calculations, we find that $s_0^4 < s_0^2$ under Assumption 1. Hence, $e^* = \bar{e}$ if $s_0 \geq s_0^2$.

In sum, we obtain the characterization of e^* in Proposition 1 by putting $s_0^3 = \underline{s}_0$ and $s_0^2 = \bar{s}_0$ (Q.E.D.)

[12] We can find that $\tilde{e} = \frac{1}{2\varepsilon\bar{s}}$.

Table 1: Descriptive statistics

	Variables	Definition	Ave.	S.D.	Min.	Max.
Total	Final score	Score in the final examination	67.17	14.84	16	100
$n = 239$	Mid score	Score in the midterm examination	51.76	16.64	12	102
	Rank	Report on the relative position of the score in the mid examination	0.50	0.50	0	1
	Male	=1 if male, =0 if female	0.87	0.34	0	1
	Class2	=1 if in the mathematically orientated economics class, =0 elsewhere	0.16	0.37	0	1
	Class3	=1 if in the normal economics class A, =0 elsewhere	0.26	0.44	0	1
	Class4	=1 if in the normal economics class B, =0 elsewhere	0.29	0.45	0	1
	Math	The scores in Pretest of Mathematics	37.39	13.56	11	87
	Econ I	The scores in Economics I	63.57	13.43	26	96
Treatment group	Final score		66.72	14.19	16	92
$n = 119$	Mid score		50.15	14.94	12	95
	Rank		1.00	0.00	1	1
	Male		0.87	0.34	0	1
	Class2		0.13	0.33	0	1
	Class3		0.27	0.45	0	1
	Class4		0.31	0.46	0	1
	Math		35.81	10.72	12	67
	Econ I		62.43	12.90	26	95
Control group	Final score		67.61	15.50	26	100
$n = 120$	Mid score		53.35	18.09	18	102
	Rank		0.00	0.00	0	0
	Male		0.87	0.34	0	1
	Class2		0.19	0.40	0	1
	Class3		0.26	0.44	0	1
	Class4		0.27	0.44	0	1
	Math		38.96	15.78	11	87
	Econ I		64.71	13.89	31	96

Table 2: Estimation results: the impacts of information of the student's rank in the midterm examination on the final examination score

	Full sample		under 20th %tile		20th–65th %tile		over 65th %tile	
	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)	(4a)	(4b)
Rank	-0.886	1.185	-1.967	-6.233	6.609***	6.107***	-7.978***	-5.109**
	[1.922]	[1.446]	[5.592]	[4.567]	[2.258]	[1.981]	[2.263]	[2.078]
Mid score		0.288***		1.222**		0.204		0.296***
		[0.056]		[0.519]		[0.164]		[0.083]
Male		0.476		2.637		-3.048		1.409
		[2.007]		[5.509]		[2.702]		[2.876]
Class2		1.506				5.750		2.217
		[2.952]				[3.921]		[3.643]
Class3		1.252		6.721		1.463		-2.082
		[1.924]		[5.616]		[2.898]		[2.690]
Class4		3.344*		5.556		2.946		4.438
		[1.998]		[5.126]		[2.595]		[3.852]
Math		0.059		-0.230		0.070		-0.049
		[0.085]		[0.372]		[0.124]		[0.099]
Econ1		0.450***		0.638***		0.469***		0.323***
		[0.072]		[0.217]		[0.099]		[0.105]
Constant	67.608***	18.931***	53.412***	-6.327	61.673***	21.582**	79.438***	34.372***
	[1.415]	[4.722]	[3.697]	[13.327]	[1.707]	[9.926]	[1.354]	[8.746]
Observations	239	239	35	35	119	119	85	85
Log likelihood	-983.2	-905.6	-147.0	-133.6	-465.7	-448.6	-316.4	-302.4
F-test	0.212	25.49***	0.124	7.903***	8.565***	7.242***	12.43***	8.238***
R-squared	0.001	0.478	0.004	0.536	0.069	0.302	0.135	0.377

Notes:

1) Standard errors in bracket are heteroskedasticity robust.

2) *, ** and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

3) F-test reports F statistics for testing whether all the coefficients except the constant are jointly zero.

Table A.1: Estimation results for robustness checks

	under 15th %tile		under 25th %tile		15th–70th %tile		25th–60th %tile		over 60th %tile		over 70th %tile	
	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)	(4a)	(4b)	(5a)	(5b)	(6a)	(6b)
Rank	-8.224	-8.925	1.729	1.061	5.248**	5.485***	5.791**	5.778**	-6.182***	-2.960	-6.230**	-4.029*
	[6.351]	[5.131]	[4.675]	[4.323]	[2.189]	[1.877]	[2.550]	[2.413]	[2.075]	[1.955]	[2.474]	[2.250]
Mid score		1.488**		0.747*		0.065		0.191		0.315***		0.291***
		[0.702]		[0.420]		[0.139]		[0.247]		[0.076]		[0.099]
Male		0.681		-1.322		-1.609		-2.557		0.588		0.324
		[6.796]		[4.127]		[2.447]		[3.234]		[2.663]		[3.685]
Class2						7.841*		11.281**		1.059		0.300
						[3.980]		[4.660]		[3.378]		[3.974]
Class3		7.387		5.306		1.410		3.678		-0.980		-2.390
		[6.520]		[4.826]		[2.677]		[3.593]		[2.594]		[2.809]
Class4		3.618		-0.082		3.723		4.648		5.138		3.516
		[6.552]		[5.109]		[2.480]		[2.835]		[3.148]		[4.910]
Math		0.161		0.182		0.011		-0.090		0.040		-0.004
		[0.394]		[0.275]		[0.116]		[0.137]		[0.095]		[0.111]
Econ1		0.478*		0.518***		0.514***		0.552***		0.291***		0.315***
		[0.271]		[0.185]		[0.091]		[0.118]		[0.096]		[0.118]
Constant	53.308***	-13.179	52.227***	0.553	62.349***	26.083***	62.209***	21.059	77.982***	31.246***	79.364***	34.785***
	[3.743]	[14.713]	[3.135]	[11.746]	[1.769]	[7.988]	[1.856]	[14.497]	[1.355]	[8.097]	[1.399]	[10.780]
Observations	25	25	45	45	140	140	94	94	100	100	74	74
Log likelihood	-103.3	-93.87	-186.8	-176.3	-552.5	-528.5	-368.6	-356.3	-373.8	-356.9	-275.1	-264.4
F-test	1.677	7.842***	0.137	5.159***	5.747**	9.453***	5.157**	5.194***	8.875***	8.623***	6.340**	5.152***
R-squared	0.069	0.561	0.003	0.376	0.042	0.320	0.053	0.272	0.084	0.347	0.086	0.316

Notes:

1) Standard errors in bracket are heteroskedasticity robust.

2) *, ** and *** indicate statistical significance at the 10%, 5% and 1% levels, respectively.

3) F-test reports F statistics for testing whether all the coefficients except the constant are jointly zero.

Figure 1: The letter to students in the treatment group (originally written in Japanese)

Introductory Economics II, Second Semester, 2012
A report on the result of your midterm examination

Student ID **7 digit ID** Name **Name of the student**
Class **Instructor's name**

Your score in the midterm examination is **49** /110.
Your score in problems of mathematics is **6** /10.

Within four classes, you were **146**th out of 285 students.

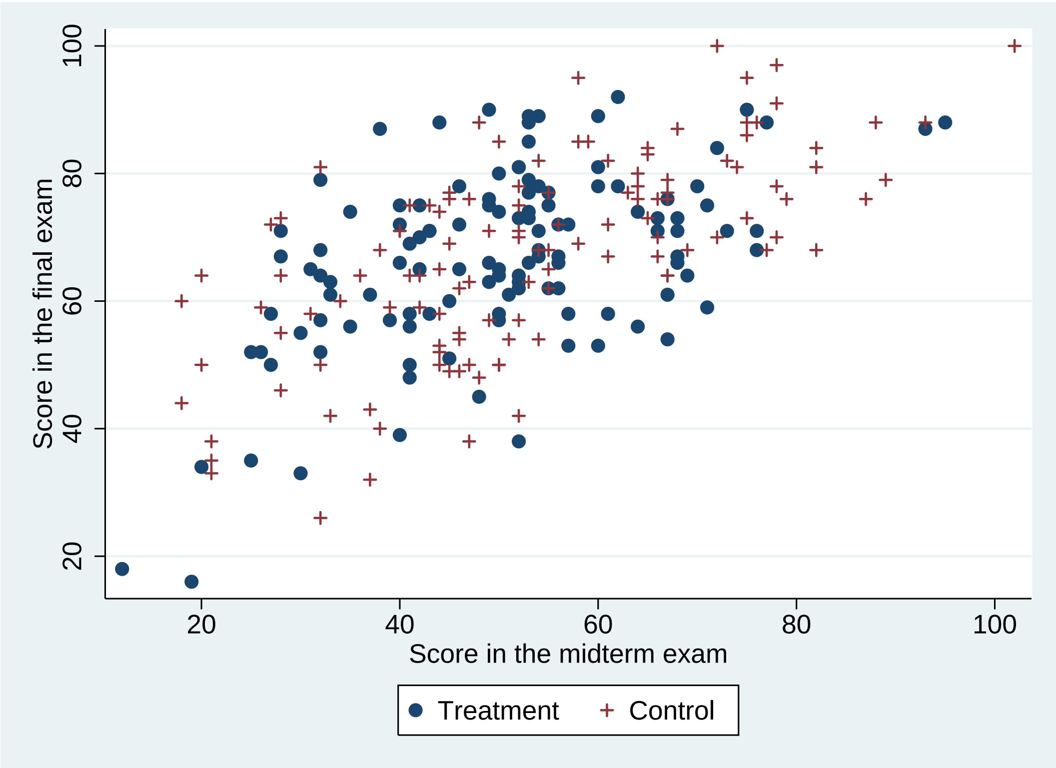
Figure 2: The letter to students in the control group (originally written in Japanese)

Introductory Economics II, Second Semester, 2012
A report on the result of your midterm examination

Student ID **7 digit ID** Name **Name of the student**
Class **Instructor's name**

Your score in the midterm examination is **49** /110.
Your score in problems of mathematics is **6** /10.

Figure 3: The relationship between the midterm and the final examination scores



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