

Effects of New Goods and Product Turnover on Price Indexes*

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Abstract

This study analyzes the importance of new products for price measurements. Using large-scale retail scanner data in Japan, we construct a unit value price index and decompose its fluctuation into (1) standard price change effects, (2) substitution effects within continuing goods, and (3) turnover-new product effects. The aggregate unit price index exhibits different movements from the standard Laspeyres price index. After the 2014 change in Japan's tax rate, turnover-new product effects from the appearance of relatively expensive new goods increased by 1 percentage point, contributing to increases in unit value. However, the standard Laspeyres price index, which excludes information for new or disappearing goods, exhibited no large changes before and after the tax revision.

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1. Introduction

Establishing how the entry and exit of commodities affect cost of living indexes and economic welfare is a serious econometric problem. New products are introduced into markets almost daily, and a number of goods disappear. Most economic indicators of prices such as official consumer price indexes (CPI), however, are based on fixed bundles of commodities. That is, most of newly appeared goods are neglected in many official statistics unless the new products occupy significant amount of market share.

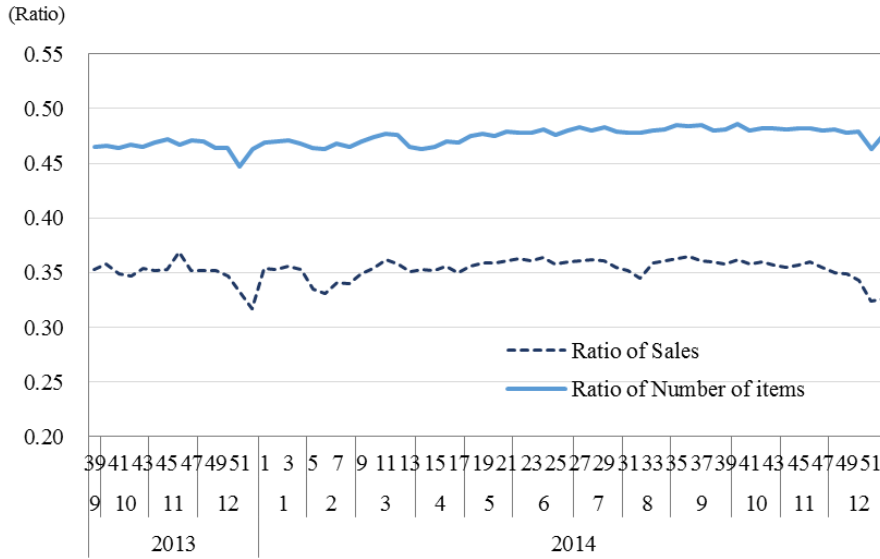
Although official CPIs are based on a limited number of products, economic data such as consumer expenditures and company sales cover all products that are traded. That is, the price index is based mostly on continuing goods, while expenditure data include new goods that just enter markets. This divergence in product space between expenditures and the price index could cause serious inconsistency when constructing “real” economic variables if the new goods are priced differently from incumbent goods.

Figure 1 shows the weekly appearance rate in Japan of new products in supermarkets, convenience stores, general merchandise stores, and drug stores.¹ More precisely, it shows the ratio of the number of products that did not exist during the same week in the previous year to total products.² Measured by sales, new products are about 35% of all products. Accordingly, by limiting the product bundle to products that remain in the market for more than one year, a significant quantity of sales information is neglected.

¹ Section 3 describes the data in detail.

² Figure 1 treats goods with identical commodity codes (Japanese Article Number: JAN) sold at different stores as different commodities. Therefore, the product appearance rate does not necessarily correspond to the appearance rate of new product brands.

Figure 1: Product Appearance Rate



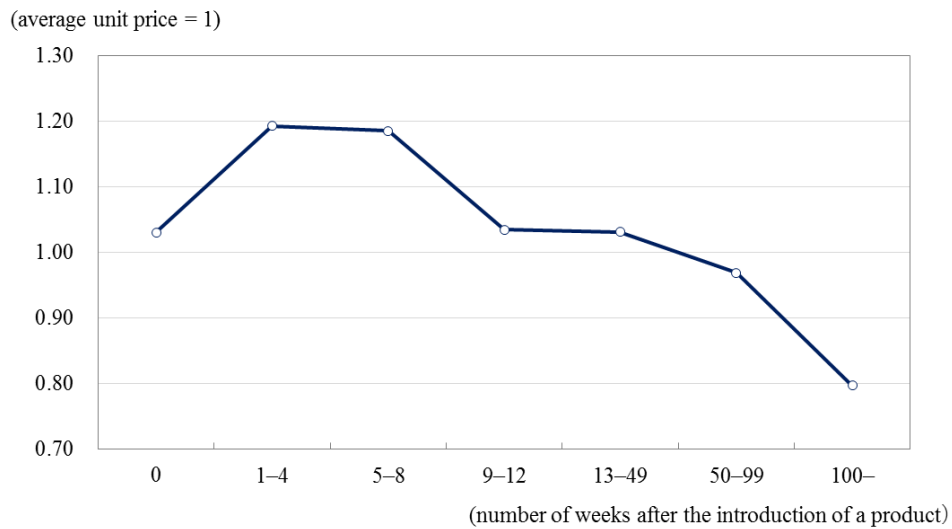
Notes: The ratios of sales and the number of items. Numerators are the sales or the number of items sold in the current period but not sold during that week one year earlier. Denominators are total sales or the total number of items sold in the current period. When constructing ratios, we calculated the ratios at store level, and then, aggregated over stores.

If prices of new products were not systematically different from those of incumbent goods, neglecting new products would not raise serious problems when estimating price indexes. Unfortunately, this is not the case. Figure 2 compares prices of new and incumbent products. The line shows the relative (unit) price of newly introduced cup noodles to incumbent cup noodles. Soon after the introduction of the new cup noodles, the price tends to be about 20% higher than that of the incumbent goods.³ That is, prices of new goods exceed prices of incumbent goods.⁴ How important are new products for price measurement? This study addresses that issue.

³ Pricing patterns over product life cycles and their relations to price indexes are an active research area. See Balk (2000), Klenow (2003), Bils (2009), and Melser and Syed (2013) for example. Recently, an increasing number of papers such as Bernard et al. (2010) and Bilbiie et al. (2012) investigate the relationship between product turnover and the business cycle. See Hamano (2013) for recent developments.

⁴ Of course, new products might have higher quality, which would reduce their “real” prices. Since the quality of processed foods and daily necessities is very hard to measure, the treatment of product-level quality changes of this paper is limited to changes in physical volume only. That is, this paper does not consider changes in tastes or packages.

Figure2: Difference in Unit Prices of New and Incumbent Cup Noodles



Notes: Relative price of items classified as cup noodles that are traded during September 2012 and December 2014. The horizontal axis shows the number of weeks passed after introduction of the items. The vertical axis is the relative price of the items. The average price normalizes at unity. Relatively low prices in Week 0 might reflect bargain sales to promote new goods.

The treatment of new products is a central issue in constructing price indexes. An entire chapter of the CPI Manual discusses quality adjustment methods for new products using hedonic and overlap approaches.⁵ Based on a constant-elasticity-of-substitution (CES) aggregator function, Feenstra (1994) and Feenstra and Shapiro (2003) derive a formula for a true cost-of-living index that the variety of products required for living changes over time. Also based on CES aggregator function, Broda and Weinstein (2010) find that new goods cause a significant “bias” in the price index. In their analysis, Broda and Weinstein (2010) assume that the market share of each commodity reflects its quality. Different from those previous studies, this paper does not assume any specific functional form for utility or expenditure functions. Rather, to evaluate the contribution of new and disappearing goods, we extend the decomposition technique developed by Silver (2009, 2010) and Diewert and Lippe (2010). Although Silver (2009, 2010) and Diewert and Lippe (2010) do not allow for the number of products to vary over time, we include changes in variety and derive a simple decomposition formula that is easy to calculate. In particular, we decompose changes in the unit value price index into changes in the standard product-level price change within continuing goods, the substitution effects within continuing goods, and the turnover-new product effects.

⁵ See Chapter 8 of ILO (2004).

The study yields several major findings. The aggregate unit value price index shows a higher rate of inflation than other price indexes such as the Törnqvist Index. Product turnover effects are generally positive, implying that new products are priced higher than disappearing or continuing goods. Substitution effects are generally negative, implying that volume shares and prices have negative correlation. When large macroeconomic shocks occur, the two effects move differently. Japan's consumption tax increased from 5% to 8% in April, 2014, its first major change since 1997. Immediately before the tax change, the substitution swelled, likely reflecting consumer stockpiling behavior that caused the unit value price inflation to drop substantially. On the other hand, changes in price effects measured by the Laspeyres formula hardly occurred before or after the tax revision, which implies that the product-level pricing behaviors remained unchanged during the sample period. We also observe that after the increase in the tax rate, turnover-new product effects from the appearance of relatively expensive new goods increased by 1 percentage point, which contributed to the increase in unit value prices.

This paper proceeds as follows. Section 2 explains how the unit price index can be decomposed into three price change effects. Section 3 describes scanner data used in this study. Section 4 discusses both aggregate and category-level results of our decomposition. Finally, Section 5 presents the conclusions to this study.

2. Price Index Decomposition

In this section, we explain the procedure to decompose changes in the unit value price index into changes in the standard product-level price change within continuing goods, the substitution effects within continuing goods, and the turnover-new product effects. That is, the goal of this section is to derive the formula that describes the unit value price inflation as the weighted sum of three effects such as,

Inflation Rate of Unite Value Price

$$= a_1 \textit{Price Change Effects} + a_2 \textit{Substituion Effects} + a_3 \textit{Turnover Effects} \\ + a_4 \textit{Cross Term}.$$

2.1. Classification of a product

First, we classify all commodities into (1) new goods, (2) old goods, and (3) continuing goods. Take two points of time, t and $t - y$. y is a fixed time interval such as one year. A commodity i is defined

as new good in period t if the commodity exists in period t and but not in period $t - y$. Similarly, a commodity i is old good in period t if the commodity does not exist in period t and but exists in period $t - y$. A commodity i is defined as continuing good in period t if the commodity exists both in period t and $t - y$. Define N_t, C_t and O_t as the set of new goods, the set of continuing goods, and the set of old goods. Finally, define the all items in category θ that exist in period t as $\theta_t (= N_t \cup C_t)$. It is worth noting that, by construction, θ_{t-y} is equal to the union of C_t and O_t , that is, $\theta_{t-y} = O_t \cup C_t$.

2.2. Calculation of a unit value price belonging to category θ

Let's denote q_t^i : quantity of item i sold in period t ,⁶ p_t^i : price of item i sold in period t . Then, the quantity-weighted average unit value price in period t (P_t^θ) can be expressed as the weighted sum of the unit value price of continuing goods (P_t^C) and new goods (P_t^N).

$$\begin{aligned}
P_t^\theta &\equiv \sum_{i \in \theta_t} \left[\left(\frac{q_t^i}{\sum_{j \in \theta_t} q_t^j} \right) p_t^i \right] \\
&= \sum_{i \in C_t} \left[\left(\frac{q_t^i}{\sum_{j \in \theta_t} q_t^j} \right) p_t^i \right] + \sum_{i \in N_t} \left[\left(\frac{q_t^i}{\sum_{j \in \theta_t} q_t^j} \right) p_t^i \right] \\
&= \left(\frac{\sum_{j \in C_t} q_t^j}{\sum_{j \in \theta_t} q_t^j} \right) \sum_{i \in C_t} \left[\frac{q_t^i}{\sum_{j \in C_t} q_t^j} \times p_t^i \right] + \left(\frac{\sum_{j \in N_t} q_t^j}{\sum_{j \in \theta_t} q_t^j} \right) \sum_{i \in N_t} \left[\left(\frac{q_t^i}{\sum_{j \in N_t} q_t^j} \right) p_t^i \right] \\
&= w_t^C P_t^C + w_t^N P_t^N,
\end{aligned} \tag{1}$$

where

$$w_t^C = \frac{\sum_{j \in C_t} q_t^j}{\sum_{j \in \theta_t} q_t^j}, w_t^N = \frac{\sum_{j \in N_t} q_t^j}{\sum_{j \in \theta_t} q_t^j}, P_t^C = \sum_{i \in C_t} \left[\frac{q_t^i}{\sum_{j \in C_t} q_t^j} \times p_t^i \right], \text{ and } P_t^N = \sum_{i \in N_t} \left[\left(\frac{q_t^i}{\sum_{j \in N_t} q_t^j} \right) p_t^i \right].$$

Note that, by construction, $w_t^N = 1 - w_t^C$.

Similarly, we can construct the unit value price index in period $t - y$ as the weighted sum of the unit value prices of continuing and disappearing goods.

⁶ This study measures prices and quantities for each product category in a common unit, such as price per milliliter. In actual scanner data, commodity-level data have unique volume information (v)—e.g., product A's price is ¥300 per milliliter. We transform the original price and quantity information into unit value price and quantity as follows: price = original_price/ v , quantity = original_quantity $\times v$.

$$\begin{aligned}
P_{t-y}^\theta &\equiv \sum_{i \in \theta_{t-y}} \left[\left(\frac{q_{t-y}^i}{\sum_{j \in \theta_{t-y}} q_{t-y}^j} \right) p_{t-y}^i \right] \\
&= \left(\frac{\sum_{j \in C_t} q_{t-y}^j}{\sum_{j \in \theta_{t-y}} q_{t-y}^j} \right) \sum_{i \in C_t} \left[\frac{q_{t-y}^i}{\sum_{j \in C_t} q_{t-y}^j} \times p_{t-y}^i \right] + \left(\frac{\sum_{j \in O_t} q_{t-y}^j}{\sum_{j \in \theta_{t-y}} q_{t-y}^j} \right) \sum_{i \in O_t} \left[\frac{q_{t-y}^i}{\sum_{j \in O_t} q_{t-y}^j} \times p_{t-y}^i \right] \\
&= w_{t-y}^C P_{t-y}^C + w_{t-y}^O P_{t-y}^O,
\end{aligned} \tag{2}$$

where $w_{t-y}^C = \frac{\sum_{j \in C_t} q_{t-y}^j}{\sum_{j \in \theta_{t-y}} q_{t-y}^j}$, $w_{t-y}^O = \frac{\sum_{j \in O_t} q_{t-y}^j}{\sum_{j \in \theta_{t-y}} q_{t-y}^j}$, $P_{t-y}^C = \sum_{i \in C_t} \left[\frac{q_{t-y}^i}{\sum_{j \in C_t} q_{t-y}^j} \times p_{t-y}^i \right]$, and

$$P_{t-y}^O = \sum_{i \in O_t} \left[\frac{q_{t-y}^i}{\sum_{j \in O_t} q_{t-y}^j} \times p_{t-y}^i \right].$$

2.3. Decomposition of the inflation rate of a unit value price

The inflation rate of the unit price index, $\pi_t^\theta (\equiv \frac{P_t^\theta - P_{t-y}^\theta}{P_{t-y}^\theta})$, can be written as follows:

$$\begin{aligned}
\pi_t^\theta &\equiv \frac{P_t^\theta - P_{t-y}^\theta}{P_{t-y}^\theta} \\
&= \left(\frac{P_{t-y}^C}{P_{t-y}^\theta} \right) \frac{w_t^C P_t^C - w_{t-y}^C P_{t-y}^C}{P_{t-y}^C} + \left(\frac{P_{t-y}^O}{P_{t-y}^\theta} \right) \frac{w_t^O P_t^O - w_{t-y}^O P_{t-y}^O}{P_{t-y}^O} \\
&= \left(\frac{P_{t-y}^C}{P_{t-y}^\theta} \right) w_{t-y}^C \frac{(w_t^C / w_{t-y}^C) P_t^C - P_{t-y}^C}{P_{t-y}^C} + \left(\frac{P_{t-y}^O}{P_{t-y}^\theta} \right) w_{t-y}^O \frac{(w_t^O / w_{t-y}^O) P_t^O - P_{t-y}^O}{P_{t-y}^O}.
\end{aligned} \tag{3}$$

Using the inflation rate of continuing goods captured by Laspeyres price index π_t^{CL} , the substitution effect among continuing goods, ϕ_t^C ,⁷ the effects of product turnover, and additional terms, we can express Equation (3) as the weighted sum of the three price effects and the cross term.⁸

⁷ We follow Diewert and Lippe (2010) for the definition of the substitution effect. This substitution effect can be written as the covariance between the change in volume share and relative prices. See A2 in Appendix 1 for details.

⁸ See Appendix 1 for the derivation.

$$\begin{aligned} \pi_t^\theta = & \left(\frac{P_{t-y}^C}{P_{t-y}^\theta} \right) w_{t-y}^C \pi_t^{CL} + \left(\frac{P_{t-y}^C}{P_{t-y}^\theta} \right) w_{t-y}^C \phi_t^C + \frac{w_{t-y}^O (P_t^C - P_{t-y}^O) + w_t^N (P_t^N - P_t^C)}{P_{t-y}^\theta} \\ & + \left(\frac{P_{t-y}^C}{P_{t-y}^\theta} \right) w_{t-y}^C \phi_t^C \pi_t^C, \end{aligned} \quad (4)$$

$$\text{where } \pi_t^C \equiv \frac{P_t^C - P_{t-y}^C}{P_{t-y}^C}.$$

The first of the R.H.S. is the standard price change effect measured by the Laspeyres formula. Note that if there were no product turnover, we would obtain $P_{t-y}^C = P_{t-y}^\theta$ and $w_{t-y}^C = 1$. Thus, the first term would be equal to the standard rate of price change, π_t^{CL} . The second term represents the substitution effects within continuing goods. If this term is negative, a product with relatively lower unit value price will increase its volume share. That is, the substitution effect captures the degree of shifts in demand caused by the difference in relative prices. The third term shows the contribution of product turnover to the unit value price index. The numerator of the third term is the weighted sum of the price differential between (1) new goods and continuing goods and (2) continuing goods and disappearing goods. Note that w_{t-y}^O is the ratio of disappearing goods within the total volume sold in period $t-y$, and $P_t^C - P_{t-y}^O$ shows the differences between the unit value prices of continuing goods and disappearing goods. Also note that w_t^N is the ratio of new goods in the total volume sold in period t , and that $P_t^N - P_t^C$ shows the differences between unit value prices of new and continuing goods. The third term can be interpreted as the substitution effects through product turnover. The final term represents the cross-effect of substitution effects and product price change effects that are supposed to be of second order.

2.4. Aggregation of Unit Value Price

The aggregate rate of change in the unit value price indexes is obtained via the Törnqvist formula in the following ways:

- The aggregate unit value price change rate:

$$\pi_t^{Total} = \exp \left\{ \sum_{\theta} \left[\frac{1}{2} \left(\frac{s_{t-y}^\theta}{\sum_{\theta} s_{t-y}^\theta} + \frac{s_t^\theta}{\sum_{\theta} s_t^\theta} \right) \ln(1 + \pi_t^\theta) \right] \right\} - 1, \quad (5)$$

- The aggregate price change effects of continuing goods:

$$\pi_t^{CL,Total} = \exp \left\{ \sum_{\theta} \left[\frac{1}{2} \left(\frac{s_{t-y}^\theta}{\sum_{\theta} s_{t-y}^\theta} + \frac{s_t^\theta}{\sum_{\theta} s_t^\theta} \right) \ln \left(1 + \left\{ \left(\frac{P_{t-y}^C}{P_{t-y}^\theta} \right) w_{t-y}^{C,\theta} \pi_t^{CL,\theta} \right\} \right) \right] \right\} - 1, \quad (6)$$

- The aggregate substitution effect for continuing goods:

$$\phi_t^{C,Total} = \exp \left\{ \sum_{\theta} \left[\frac{1}{2} \left(\frac{s_{t-y}^{\theta}}{\sum_{\theta} s_{t-y}^{\theta}} + \frac{s_t^{\theta}}{\sum_{\theta} s_t^{\theta}} \right) \ln \left(1 + \left\{ \left(\frac{P_{t-y}^C}{P_{t-y}^{\theta}} \right) w_{t-y}^{C,\theta} \phi_t^{C,\theta} \right\} \right) \right] \right\} - 1, \quad (7)$$

➤ The aggregate turnover–new product effect:

$$\pi_t^{T,Total} = \exp \left\{ \sum_{\theta} \left[\frac{1}{2} \left(\frac{s_{t-y}^{\theta}}{\sum_{\theta} s_{t-y}^{\theta}} + \frac{s_t^{\theta}}{\sum_{\theta} s_t^{\theta}} \right) \ln \left(1 + \left\{ \frac{w_{t-y}^{O,\theta} (P_t^{C,\theta} - P_{t-y}^{O,\theta}) + w_t^{N,\theta} (P_t^{N,\theta} - P_t^{C,\theta})}{P_{t-y}^{\theta}} \right\} \right) \right] \right\} - 1, \quad (8)$$

where s_t^{θ} , $P_{t-y}^{O,\theta}$, $P_{t-y}^{C,\theta}$, $\pi_t^{CL,\theta}$, and $\phi_t^{C,\theta}$ are the sales of category θ in period t , the unit value price of disappearing goods of category θ at time $t-y$, the unit value price of continuing goods of category θ at time $t-y$, the price change effects of continuing goods of category θ at time t , and the pure substitution effects of continuing goods of category θ at time t . And $[w_{t-y}^{O,\theta} (P_t^{C,\theta} - P_{t-y}^{O,\theta}) + w_t^{N,\theta} (P_t^{N,\theta} - P_t^{C,\theta})] / P_{t-y}^{\theta}$ is the product turnover contribution in the change rate of unit value price of category θ in period t .

3. Data

We use Japanese store–level weekly scanner data, known as the SRI,⁹ collected by INTAGE Inc. Sales records in the dataset cover sales of processed foods, daily necessities, and cosmetics and drugs that have JAN codes.¹⁰ The sampled period is between October 2012 and December 2014, which enables us to calculate the yearly rate of price changes between October 2013 and December 2014. The dataset cover various types of stores, such as general merchandise stores (218), supermarkets (1,051), convenience stores (818), drug stores (1,066), and specialized stores, such as liquor shops (639) located across Japan.¹¹ Table 1 gives a detailed description of the dataset used to calculate aggregated unit price indexes.

One noteworthy characteristic of the dataset is its detailed commodity classification. As is shown in the third column of Table 1, commodities are classified into more than 1,000 categories, about seven times larger than the number of classifications adopted by Japanese official statistics. As Diewert and Lippe (2010) stress, when constructing a unit value price index, we must wherever possible avoid aggregation over heterogeneous goods. We expect that the dataset’s highly detailed classification helps us to mitigate the aggregation bias indicated by Diewert and Lippe (2010).

To avoid the sample selection effect when calculating the rate of change of individual product prices, we limit the store and category space to a range such that stores and product categories exist in both the current week and the same week of the previous year. Furthermore, we use observations

⁹ SRI is the abbreviation for Japanese “Syakaichosa-kenkyujo Retail Index,” which translates as “Retail Index by The Institute of Social Research”.

¹⁰ Fresh foods are excluded in the dataset because they lack commodity codes.

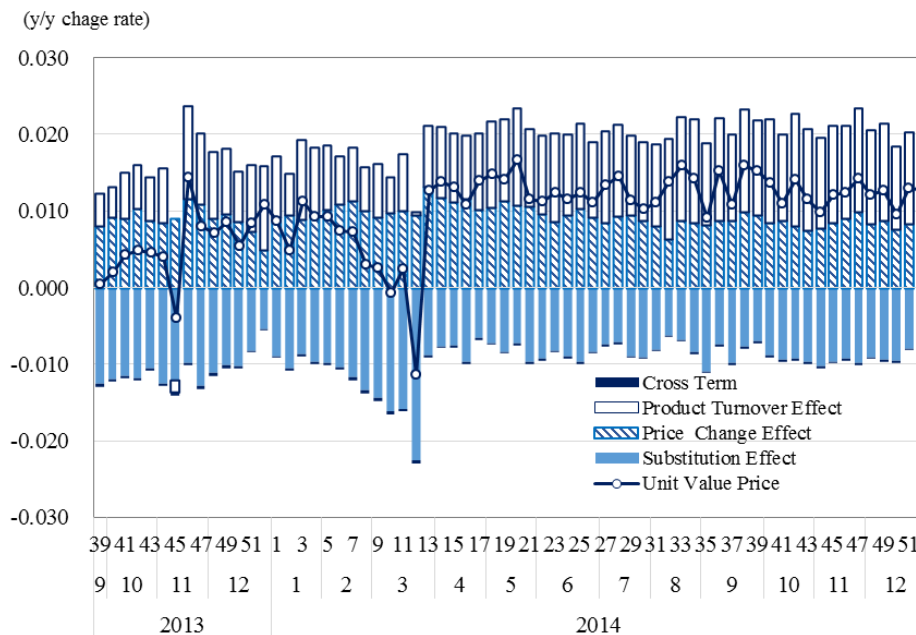
¹¹ Values in parentheses are the average number of stores during the sample period.

the three effects. Before interpreting Figure 3, we again note that on April 1, 2014, Japan's consumption tax rate increased from 5% to 8%, first major change since 1997.¹² Although the dataset used in this paper is pre-tax, we expect to see discontinuous changes before and after April 1, 2014.

From Figure 3, we observe that in most periods, turnover-new product effects are positive, a finding consistent with the case in Figure 2. This implies that the prices of new goods exceed the prices of continuing goods or that the prices of continuing goods exceed the prices of disappearing goods. The magnitude of the contribution of the turnover-new goods effects was about 1.5 percentage points. This figure is much larger than the estimates by Boskin et al. (1996), which indicated that the introduction of new and better products added 0.6 of a percentage point to annual consumption growth in the United States.

On the other hand, the substitution effects—i.e., the effects of the shift in demand from higher to lower priced products—are negative, implying that consumers increase expenditures on lower-priced goods. Compared with other price effects, the magnitude of the cross term is negligible.

Figure 3: Decomposition of Unit Value Price Growth Rate



Notes: Unit Value Price corresponds to Equation (9). The three types of bars correspond to the contributions of three price effects in Equations (10) – (12).

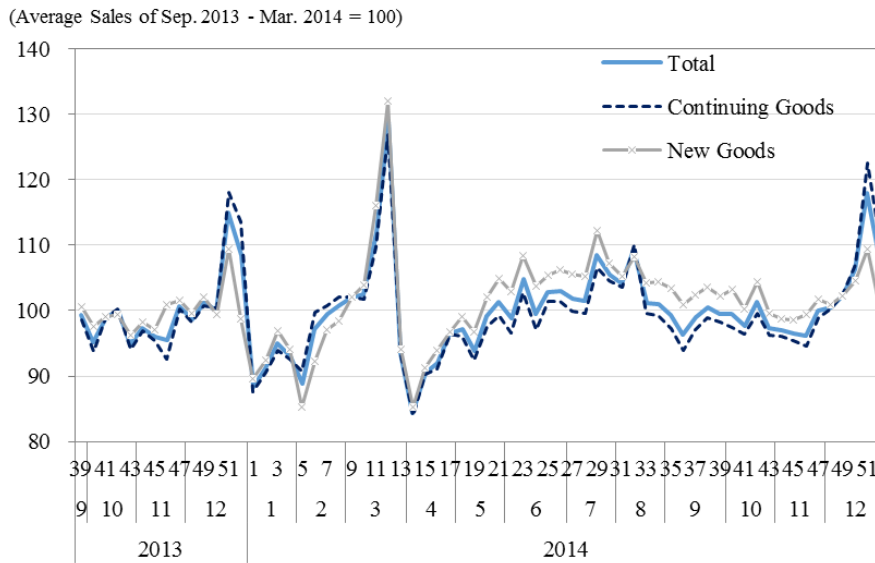
¹² Scanner data used in this study exclude consumption taxes. That is, price information in the dataset is pre-tax.

During our sampled period, the unit value price index exhibited large ups and downs in mid-November 2013. This turbulence occurred resulted from different dates for the Beaujolais Nouveau Day.¹³ From January until the end of March in 2014, the unit value price index declined, and then, it skyrocketed in the week that includes April 1, 2014. Compared with the large fluctuation in the unit value price index, the standard price change effects are quite stable. This implies that the frequency and/or magnitude of product-level price changes did not significantly vary during the sampled periods. The decline in the unit value price index during the first quarter in 2014 came with increasing substitution effects. Behind the dramatic decline in the substitution effect, there is a surge in total expenditures before the tax revision. Figure 4 shows that total sales surged by 20–30% just before the change in tax rate. We summarize that expecting consumers anticipated the tax increase and stockpiled lower-priced commodities, thereby increasing the substitution effects.

After the tax increase, the turnover-new goods effects surpass those evident before the tax revision, which contributed to the increase in the unit value price index. From Figure 4 we observe that the rate of sales growth for new goods exceeded that of continuing goods, suggesting an increasing economic role for new goods after the tax revision. After June, 2014, the contribution of the turnover-new goods effects became the largest of the three factors, implying relatively higher prices for new goods (Figure 2). Those facts suggest that introducing new goods is an instrument to increase product prices.

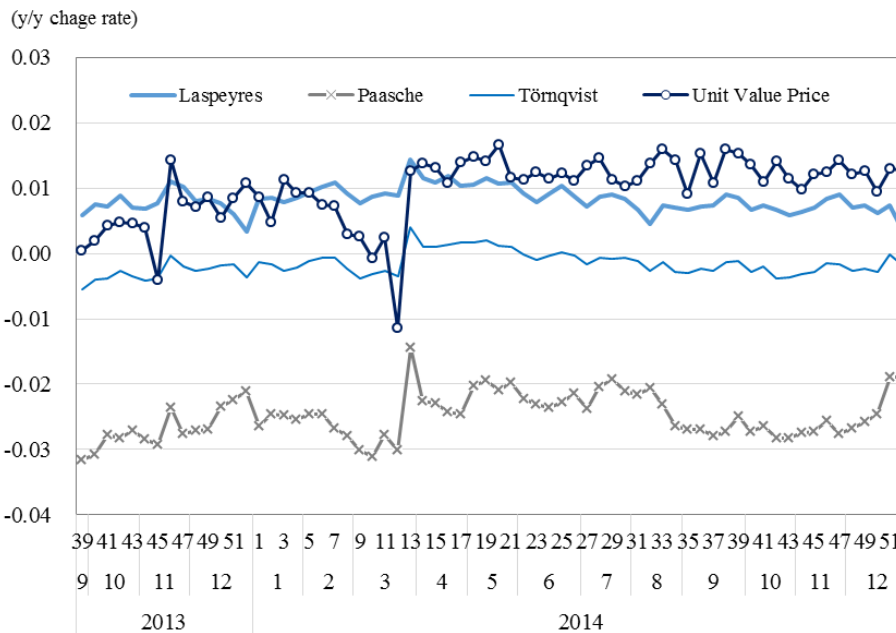
¹³ Beaujolais Nouveau Day is the third Thursday of November. In 2012, the third Thursday fell on November 15 during the second week of month, and in 2013, it fell on November 21 during the third week. When calculating the rates of change, we take differences from the same week in the previous year, for differences in dates of important events can strongly affect unit prices. Because Beaujolais Nouveau generally costs more than other average wines, unit prices during the second week of November 2013 were much lower than during the second week in November 2012. That circumstances caused huge ups and downs in the aggregate unit value price index in Figure 3.

Figure 4: Movements in Total Sales and Sales of New Goods



Notes: This figure is based on sales information in Table 1.

Figure 5: Comparisons of Several Price Indexes



Notes: Unit Value Price is identical to that in Figure 3. See Appendix 3 for details of the Laspeyres, Paasche, and Törnqvist indexes. When plotting the official CPI, we first chose 115 product categories that overlap those categories in the SRI with volume information. Then, we aggregated the category-level official CPIs to obtain the aggregate official CPI depicted in Figure 5.

Figure 5 compares the unit value price index to the standard price indexes: Laspeyres, Paasche, and Törnqvist indexes and the Japanese official CPI based on 115 product categories that overlap with SRI.¹⁴ First, it is significant that the discrepancy between Laspeyres and Paasche indexes is large. The Paasche index moves from -3% to -2% , while the Laspeyres index moves from 0% to 1% . One main cause of this departure is bargain sales. When commodities go on bargain sale, prices decline, sales quantities surge, and the weights of the commodities in the Paasche index are increased. Bargain sales in the current period do not change the weights in the Laspeyres index because they are by definition unchanged in the Laspeyres index.

The official CPI that is also constructed by Laspeyres formula tracks our Laspeyres index closely except for August–December 2014.¹⁵ This similarity indicates that factors excluded in the official CPI, such as bargain sales and substitution from higher to lower priced goods might not affect the general price level during the sampled period as long as we use the weight at base periods.

The unit value price index exhibits a generally higher rate of change than the Törnqvist index.¹⁶ The huge departure between the Törnqvist index based only on continuing goods and the unit value price index based on all traded goods implies significant roles for (1) the price differential between new and continuing goods and (2) substitution effects. For example, before the tax revision, the unit value price index declined, but the Törnqvist index was stable. Figure 3 reveals that, during the period, the substitution effects become larger than those in other periods.

4.2. Movements in Several Commodity Categories

This subsection analyzes the movements in the unit price index and its decomposition at the category level. Contributions of the three effects on the unit value price index vary across product categories to a large extent.

Figures 6 and 7 depict movements of several price indexes and the unit value price decomposition of heavy detergent and yogurt. The Törnqvist indexes exhibit very similar movements to those of the

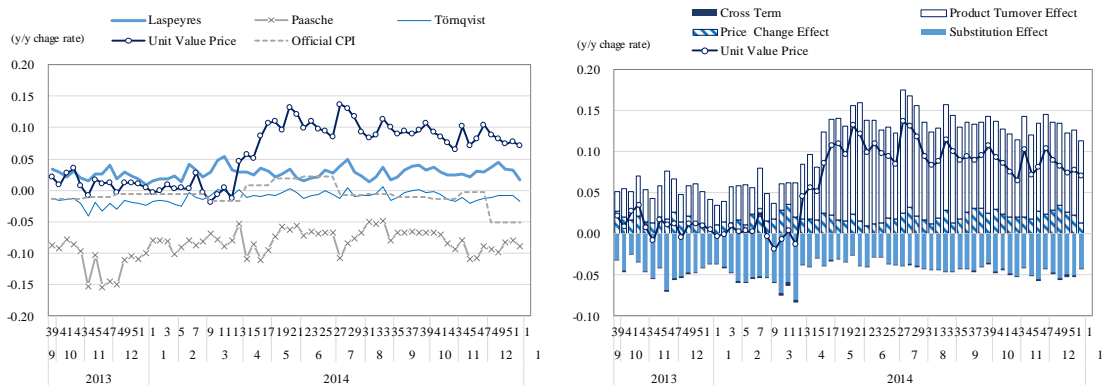
¹⁴ See Appendix 3 for the formula used to calculate these price indexes. The Fisher index exhibits movements almost identical to the Törnqvist index; hence, we exclude it from Figure 5.

¹⁵ The main differences between the official CPI and our Laspeyres index are as follows: (1) items adopted in the official CPI are one or a few popular products for each category, whereas SRI covers all products, (2) the official CPI does not incorporate information for bargain sales, whereas SRI includes sales and price information at bargain sales, and (3) the official CPI uses prices from the largest retailers for each region, whereas SRI contains different types of stores such as convenience stores, supermarkets, and general merchandise stores.

¹⁶ Note that we use the Törnqvist formula when aggregating categorical unit value price indexes.

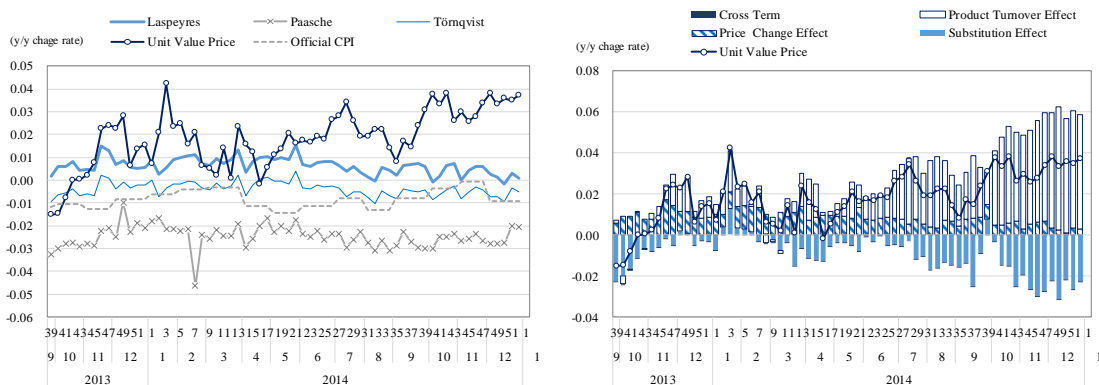
official CPI. After the tax revision, both goods exhibit very large positive turnover-new goods effects, which resulted in large discrepancies between the unit value price index and the official CPI.

Figure 6: Decomposition of Unit Price Index (Heavy Detergent)



Notes: See the notes for Figures 3 and 5 for details except for the official CPI.

Figure 7: Decomposition of Unit Price Index (Yogurt)



Figures 8 and 9 exhibit the cases of butter and facial tissues. Butter shows a rising trend, whereas facial tissues show a declining price trend. Contributions from the turnover-new goods effects and the substitution effects are minor in these cases. Most changes in the unit value price index come from changes in the product price change (π_t^{CL}). Thus, the unit value price indexes track the official CPIs very closely.

Figure 8: Decomposition of Unit Price Index (Butter)

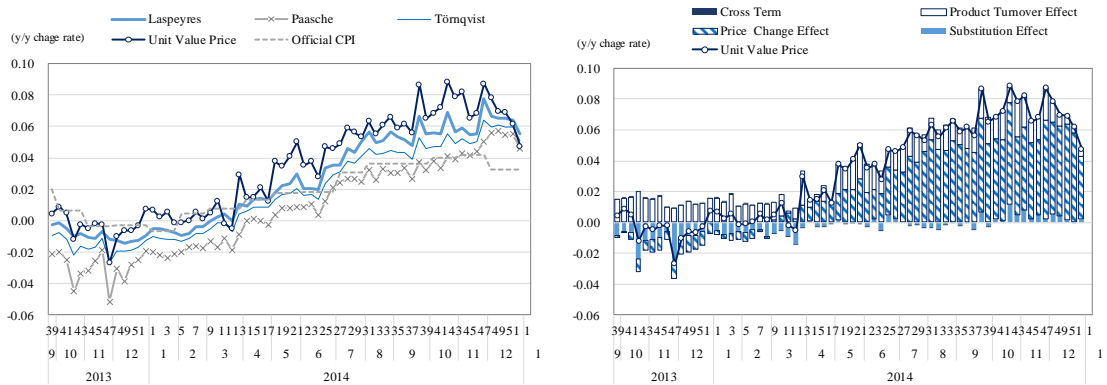
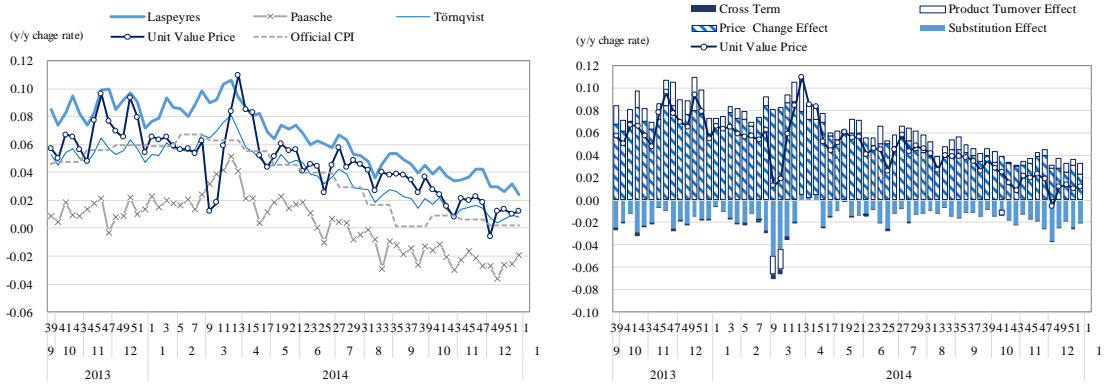


Figure 9: Decomposition of Unit Price Index (Facial Tissue)



Finally, Figures 10 and 11 present the cases of beer and salt. Both goods exhibit strong negative substitution effects just before the tax revision, whereas the turnover-new goods effects are moderate. Overall, the unit price of a case of beer (24 cans) is much lower than the price of the same beer sold separately. Similarly, the unit price of 1 kilogram of salt is much lower than that of 100 grams of the same salt sold as cooking salt. Both goods can be stored a long time. Thus, before the tax increase, consumers likely stockpiled those goods, reducing the unit value price index during the period.

Figure 10: Decomposition of Unit Price Index (Beer)

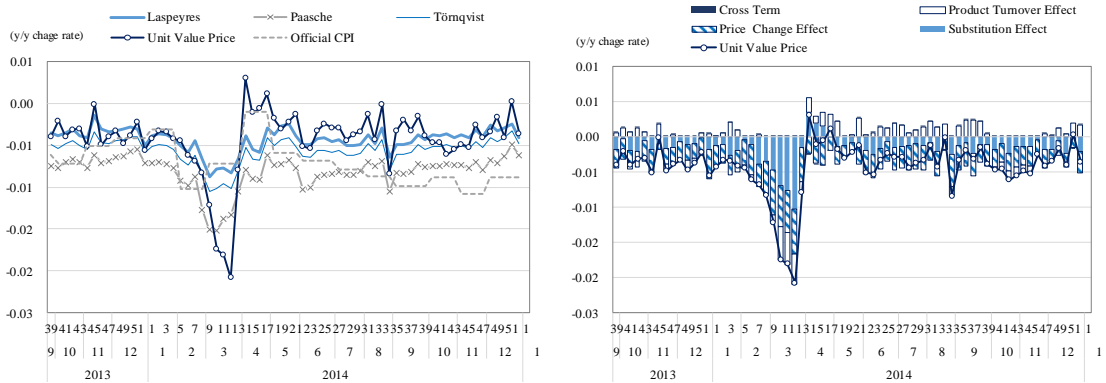
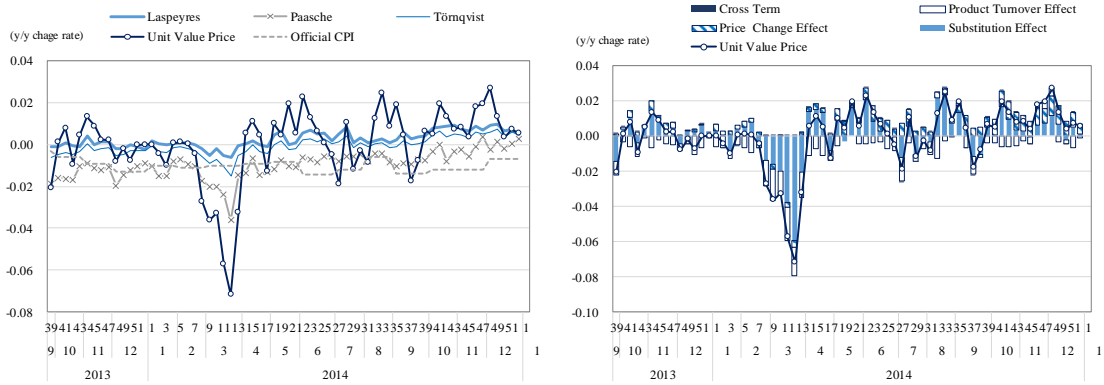


Figure 11: Decomposition of Unit Price Index (Salt)



5. Conclusion

This study has investigated unit price indexes based on large-scale retail scanner data. By extending the technique developed by Silver (2009, 2010) and Diewert and Lippe (2010), we decomposed changes in unit price into (1) turnover–new goods effects, (2) substitution effects, and (3) price change effects (Laspeyres price index). The aggregate unit value price index shows a higher rate of inflation than other price indexes, such as the Törnqvist index. Product turnover effects are generally positive, implying that new products are priced higher than disappearing or continuing goods. Substitution effects are generally negative, implying that negative correlation between volume shares and prices. Substitution effects strengthened just before the tax revision, likely reflecting consumer stockpiling behaviors, which reduced the unit value price inflation to a large extent. As measured by the Laspeyres formula, (pre-tax) commodity prices were scarcely changed before or after the tax revision, which implies that the patterns of product-level commodity price change were unchanged. After the tax rate being hiked, the turnover-new product effects are increased by 1 percentage point,

contributing to the increase in unit value prices.

Category-level analyses revealed that the influence of the three effects on the unit value price index varies greatly across product categories. Unit value price indexes exhibit movements similar to the official CPI for some goods with few turnover-new goods effects evident in such items as butter and facial tissue. However, some goods with large turnover-new goods effects such as heavy detergent and yogurt exhibit large discrepancies between unit value price indexes and official CPIs.

Many tasks related to our work remain. The increasing share of sales of new goods after the tax revision implies that the introduction of new goods to a certain extent is instrumental for price adjustment. The increase in the unit value price index after the tax revision was mainly caused by the introduction of high-priced new goods. If such factors as potential damage to product brands prevent producers from changing prices, introducing slightly different new goods can be more profitable than simply adjusting prices. Micro-analyses of price and product adjustments merit further investigation.

This study did not consider changes product quality such as taste or durability. In general, quality of processed foods and daily necessities is very difficult to measure. The Statistics Bureau of Japan does not adjust for quality of processed foods and daily necessities, except for volume (change in gram or milliliter), when constructing its consumer price index. Given that the information about characteristics, except for volume, is scarce for processed foods and daily necessities, it is difficult to employ a hedonic approach. If quality of new products surpasses that of incumbent goods, this study's estimates of unit price are vulnerable to upward bias. More detailed categorical-level investigations are needed to address the quality issue.

Other remaining tasks include the analysis of (1) the effects of the tax rate on the cycle of products introduced just before the tax reform, (2) possible measures for the cost of living index, such as the multilateral chained index proposed by De Haan and Van der Grient (2011), and (3) the impact on commodity prices of the large depreciation in the Japanese yen.

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Appendix 1: Derivation of the Unit Value Price Decomposition

A.1. Rate of Inflation of the Unit Value Price

The inflation rate of the unit value price index, $\pi_t^\theta (\equiv \frac{P_t^\theta - P_{t-y}^\theta}{P_{t-y}^\theta})$, can be written as follows:

$$\begin{aligned}\pi_t^\theta &\equiv \frac{P_t^\theta - P_{t-y}^\theta}{P_{t-y}^\theta} \\ &= \left(\frac{P_{t-y}^C}{P_{t-y}^\theta}\right) \frac{w_t^C P_t^C - w_{t-y}^C P_{t-y}^C}{P_{t-y}^C} + \left(\frac{P_{t-y}^O}{P_{t-y}^\theta}\right) \frac{w_t^N P_t^N - w_{t-y}^O P_{t-y}^O}{P_{t-y}^O} \\ &= \left(\frac{P_{t-y}^C}{P_{t-y}^\theta}\right) w_{t-y}^C \frac{(w_t^C/w_{t-y}^C)P_t^C - P_{t-y}^C}{P_{t-y}^C} + \left(\frac{P_{t-y}^O}{P_{t-y}^\theta}\right) w_{t-y}^O \frac{(w_t^N/w_{t-y}^O)P_t^N - P_{t-y}^O}{P_{t-y}^O}.\end{aligned}$$

We can rewrite this equation as:

$$\pi_t^\theta = \left(\frac{P_{t-y}^C}{P_{t-y}^\theta}\right) w_{t-y}^C \hat{\pi}_t^C + \left(\frac{P_{t-y}^O}{P_{t-y}^\theta}\right) w_{t-y}^O \hat{\pi}_t^T,$$

where $\hat{\pi}_t^C \equiv \frac{(w_t^C/w_{t-y}^C)P_t^C - P_{t-y}^C}{P_{t-y}^C}$ and $\hat{\pi}_t^T \equiv \frac{(w_t^N/w_{t-y}^O)P_t^N - P_{t-y}^O}{P_{t-y}^O}$.

A.2. Price Change Effects and Substitution Effects

Define the Laspeyres price index of the continuing goods (π_t^{CL}), as follows,

$$\pi_t^{CL} = \frac{\sum_{i \in C_t} \left[\frac{q_{t-y}^i}{\sum_{j \in C_t} q_{t-y}^j} \times p_t^i \right] - \sum_{i \in C_t} \left[\frac{q_{t-y}^i}{\sum_{j \in C_t} q_{t-y}^j} \times p_{t-y}^i \right]}{\sum_{i \in C_t} \left[\frac{q_{t-y}^i}{\sum_{j \in C_t} q_{t-y}^j} \times p_{t-y}^i \right]} = \frac{\sum_{i \in C_t} [q_{t-y}^i p_t^i]}{\sum_{i \in C_t} [q_{t-y}^i p_{t-y}^i]} - 1.$$

Since π_t^{CL} captures the effects of the price change of the continuing goods, we call this as the price change effects.

The substitution effect of continuing goods (ϕ_t^C), is defined as,

$$\phi_t^C \equiv \frac{P_t^C - \check{P}_t^C}{P_t^C},$$

$$\text{where } \check{P}_t^C = \sum_{i \in C_t} \left[\frac{q_{t-y}^i}{\sum_{j \in C_t} q_{t-y}^j} \times p_t^i \right].$$

Per Diewert and Lippe (2010), define covariance such as,

Define

$$\text{Cov}(x, y) = \frac{1}{T} \sum_i (x_i - x^*)(y_i - y^*) = \frac{1}{T} \sum_i (x_i)(y_i - y^*),$$

$$\text{where } s_t^i = \frac{q_t^i}{\sum_{j \in C_t} q_t^j},$$

$$p_t = [p_t^1, p_t^2, \dots, p_t^{T_t}],$$

$$s_t = [s_t^1, s_t^2, \dots, s_t^{T_t}],$$

T_t : #(θ_t), the number of products sold at time t .

The substitution effects (ϕ_t^C) can be transformed as follows,

$$\begin{aligned} \phi_t^C &\equiv \frac{P_t^C - \check{P}_t^C}{P_t^C} = \frac{\sum_{i \in C_t} p_t^i \left[\frac{q_t^i}{\sum_{j \in C_t} q_t^j} - \frac{q_{t-y}^i}{\sum_{j \in C_t} q_{t-y}^j} \right]}{\sum_{i \in C_t} \left[\frac{q_t^i}{\sum_{j \in C_t} q_t^j} \times p_t^i \right]} \\ &= \frac{\sum_{i \in C_t} p_t^i [s_t^i - s_{t-y}^i]}{\sum_{i \in C_t} \left[\frac{q_t^i}{\sum_{j \in C_t} q_t^j} \times p_t^i \right]} \end{aligned}$$

Because s_t^i and s_{t-y}^i are volume shares of good i , in times t and $t-y$, their averages are the same.

That is, $\sum_{i \in C_t} [s_t^i - s_{t-y}^i] = 0$.

Thus, we can obtain,

$$\sum_{i \in C_t} p_t^i [s_t^i - s_{t-y}^i] = T_t \text{Cov}(p_t, s_t - s_{t-y}).$$

Therefore, the substitution effects can be written by this covariance:

$$\phi_t^C = \frac{T_t \text{Cov}(p_t, s_t - s_{t-y})}{P_t^C}.$$

The R.H.S of the above equation is equivalent to the formula derived by Diewert and Lippe (2010). The interpretation of the covariance term is straightforward. If the price of good i exceeds the average price, its volume share is expected to decline. This substitution effect captures the degree of the negative correlation.

A.3. Decomposition of $\hat{\pi}_t^C$

To interpret the term with $\hat{\pi}_t^C$, we introduce variables $\tilde{\pi}_t^C$ as follows:

$$\begin{aligned}\tilde{\pi}_t^C &\equiv \hat{\pi}_t^C - \pi_t^{CL} \\ &= \frac{(w_t^C/w_{t-y}^C)P_t^C - P_{t-y}^C}{P_{t-y}^C} - \frac{\check{P}_t^C - P_{t-y}^C}{P_{t-y}^C} \\ &= (w_t^C/w_{t-y}^C - 1) \frac{P_t^C}{P_{t-y}^C} + \frac{P_t^C - \check{P}_t^C}{P_t^C} \frac{P_t^C}{P_{t-y}^C} \\ &= (w_t^C/w_{t-y}^C - 1)(1 + \pi_t^C) + \phi_t^C(1 + \pi_t^C),\end{aligned}$$

where $\pi_t^C \equiv \frac{P_t^C - P_{t-y}^C}{P_{t-y}^C}$.

Therefore, $\hat{\pi}_t^C$ can be expressed as,

$$\hat{\pi}_t^C = \pi_t^{CL} + \phi_t^C + (w_t^C/w_{t-y}^C - 1) + [(w_t^C/w_{t-y}^C - 1)\pi_t^C + \phi_t^C\pi_t^C].$$

$\hat{\pi}_t^C$ can be decomposed into four effects, (1) the price change effects of continuing goods, π_t^{CL} , (2) the substitution effects within continuing goods, ϕ_t^C , (3) the changes in weights of continuing goods between the period t and $t-y$, $(w_t^C/w_{t-y}^C - 1)$, and (4) the cross terms.

A.4. Decomposition of $\hat{\pi}_t^T$

$\hat{\pi}_t^T$ can be decomposed into three effects, (1) the changes in weights of new and disappearing goods, $(w_t^N/w_{t-y}^O - 1)$, (2) the price differential between new and disappearing goods, π_t^{NO} , and (3) the cross term as follows:

$$\begin{aligned}
\hat{\pi}_t^T &= \frac{(w_t^N/w_{t-y}^O)P_t^N - P_{t-y}^O}{P_{t-y}^O} \\
&= (w_t^N/w_{t-y}^O - 1) \frac{P_t^N}{P_{t-y}^O} + \frac{P_t^N - P_{t-y}^O}{P_{t-y}^O} \\
&= (w_t^N/w_{t-y}^O - 1)(1 + \pi_t^{NO}) + \pi_t^{NO} \\
&= (w_t^N/w_{t-y}^O - 1) + \pi_t^{NO} + (w_t^N/w_{t-y}^O - 1)\pi_t^{NO},
\end{aligned}$$

where $\pi_t^{NO} \equiv \frac{P_t^N - P_{t-y}^O}{P_{t-y}^O}$.

A.5. The Unit Value Price Decomposition

π_t^θ can be expressed as,

$$\begin{aligned}
\pi_t^\theta &= \left(\frac{P_{t-y}^C}{P_{t-y}^\theta}\right) w_{t-y}^C \frac{(w_t^C/w_{t-y}^C)P_t^C - P_{t-y}^C}{P_{t-y}^C} + \left(\frac{P_{t-y}^O}{P_{t-y}^\theta}\right) w_{t-y}^O \frac{(w_t^N/w_{t-y}^O)P_t^N - P_{t-y}^O}{P_{t-y}^O} \\
&= \left(\frac{P_{t-y}^C}{P_{t-y}^\theta}\right) w_{t-y}^C \{\pi_t^{CL} + \phi_t^C + (w_t^C/w_{t-y}^C - 1) + [(w_t^C/w_{t-y}^C - 1)\pi_t^C + \phi_t^C \pi_t^C]\} \\
&\quad + \left(\frac{P_{t-y}^O}{P_{t-y}^\theta}\right) w_{t-y}^O \{(w_t^N/w_{t-y}^O - 1) + \pi_t^{NO} + (w_t^N/w_{t-y}^O - 1)\pi_t^{NO}\} \\
&= \left(\frac{P_{t-y}^C}{P_{t-y}^\theta}\right) w_{t-y}^C \pi_t^{CL} + \left(\frac{P_{t-y}^C}{P_{t-y}^\theta}\right) w_{t-y}^C \phi_t^C + \left(\frac{P_{t-y}^O}{P_{t-y}^\theta}\right) w_{t-y}^O \pi_t^{NO} + \left(\frac{P_{t-y}^C}{P_{t-y}^\theta}\right) (w_t^C - w_{t-y}^C) \pi_t^C \\
&\quad + \left(\frac{P_{t-y}^O}{P_{t-y}^\theta}\right) (w_t^N - w_{t-y}^O) \pi_t^{NO} + \left(\frac{P_{t-y}^O - P_{t-y}^C}{P_{t-y}^\theta}\right) (w_t^N - w_{t-y}^O) + \left(\frac{P_{t-y}^C}{P_{t-y}^\theta}\right) w_{t-y}^C \phi_t^C \pi_t^C.
\end{aligned}$$

The third, fourth, fifth, and sixth terms of the R.H.S. of the above equation can be greatly simplified as,

$$\begin{aligned}
&\left(\frac{P_{t-y}^O}{P_{t-y}^\theta}\right) w_{t-y}^O \pi_t^{NO} + \left(\frac{P_{t-y}^C}{P_{t-y}^\theta}\right) (w_t^C - w_{t-y}^C) \pi_t^C + \left(\frac{P_{t-y}^O}{P_{t-y}^\theta}\right) (w_t^N - w_{t-y}^O) \pi_t^{NO} \\
&\quad + \left(\frac{P_{t-y}^O - P_{t-y}^C}{P_{t-y}^\theta}\right) (w_t^N - w_{t-y}^O) \\
&= \left(\frac{P_{t-y}^O}{P_{t-y}^\theta}\right) w_{t-y}^O \left(\frac{P_t^N - P_{t-y}^O}{P_{t-y}^O}\right) + \left(\frac{P_{t-y}^C}{P_{t-y}^\theta}\right) (w_t^C - w_{t-y}^C) \left(\frac{P_t^C - P_{t-y}^C}{P_{t-y}^C}\right) \\
&\quad + \left(\frac{P_{t-y}^O}{P_{t-y}^\theta}\right) (w_t^N - w_{t-y}^O) \left(\frac{P_t^N - P_{t-y}^O}{P_{t-y}^O}\right) + \left(\frac{P_{t-y}^O - P_{t-y}^C}{P_{t-y}^\theta}\right) (w_t^N - w_{t-y}^O)
\end{aligned}$$

$$\begin{aligned}
&= \left(\frac{P_t^N - P_{t-y}^O}{P_{t-y}^\theta} \right) w_{t-y}^O + \left(\frac{P_t^C - P_{t-y}^C}{P_{t-y}^\theta} \right) (w_t^C - w_{t-y}^C) + \left(\frac{P_t^N - P_{t-y}^O}{P_{t-y}^\theta} \right) (w_t^N - w_{t-y}^O) \\
&\quad + \left(\frac{P_{t-y}^O - P_{t-y}^C}{P_{t-y}^\theta} \right) (w_t^N - w_{t-y}^O) \\
&= \left(\frac{P_t^N - P_{t-y}^O}{P_{t-y}^\theta} \right) w_{t-y}^O + \left(\frac{-(P_t^C - P_{t-y}^C) + (P_t^N - P_{t-y}^O) + (P_{t-y}^O - P_{t-y}^C)}{P_{t-y}^\theta} \right) (w_t^N - w_{t-y}^O) \\
&\quad = \frac{w_{t-y}^O (P_t^N - P_{t-y}^O) + (w_t^N - w_{t-y}^O) (P_t^N - P_t^C)}{P_{t-y}^\theta} \\
&\quad = \frac{w_{t-y}^O (P_t^C - P_{t-y}^O) + w_t^N (P_t^N - P_t^C)}{P_{t-y}^\theta}.
\end{aligned}$$

Note that $w_t^C - w_{t-y}^C = 1 - w_t^N - (1 - w_{t-y}^O) = -(w_t^N - w_{t-y}^O)$.

Thus, we obtain

$$\pi_t^\theta = \left(\frac{P_{t-y}^C}{P_{t-y}^\theta} \right) w_{t-y}^C \pi_t^{CL} + \left(\frac{P_{t-y}^C}{P_{t-y}^\theta} \right) w_{t-y}^C \phi_t^C + \frac{w_{t-y}^O (P_t^C - P_{t-y}^O) + w_t^N (P_t^N - P_t^C)}{P_{t-y}^\theta} + \left(\frac{P_{t-y}^C}{P_{t-y}^\theta} \right) w_{t-y}^C \phi_t^C \pi_t^C.$$

Appendix 2: Formula for Price Indexes

Laspeyres, Paasche, and Törnqvist indexes depicted in Figure 5 are obtained using the following formula:

- Laspeyres index inflation rate:

$$\begin{aligned}\pi_t^{CL} &= \frac{\sum_{i \in C_t} [q_{t-y}^i p_t^i]}{\sum_{i \in C_t} [q_{t-y}^i p_{t-y}^i]} - \frac{\sum_{i \in C_t} [q_{t-y}^i p_{t-y}^i]}{\sum_{i \in C_t} [q_{t-y}^i p_{t-y}^i]} \\ &= \sum_{i \in C_t} \left[\frac{q_{t-y}^i p_{t-y}^i}{\sum_{j \in C_t} [q_{t-y}^j p_{t-y}^j]} \left(\frac{p_t^i}{p_{t-y}^i} \right) \right] - \sum_{i \in C_t} \left[\frac{q_{t-y}^i p_{t-y}^i}{\sum_{j \in C_t} [q_{t-y}^j p_{t-y}^j]} \left(\frac{p_{t-y}^i}{p_{t-y}^i} \right) \right] \\ &= \sum_{i \in C_t} \left[\frac{q_{t-y}^i p_{t-y}^i}{\sum_{j \in C_t} [q_{t-y}^j p_{t-y}^j]} \left(\frac{p_t^i - p_{t-y}^i}{p_{t-y}^i} \right) \right].\end{aligned}$$

- The Paasche index inflation rate:

$$\pi_t^{CP} = \frac{\sum_{i \in C_t} [q_t^i p_t^i]}{\sum_{i \in C_t} [q_t^i p_{t-y}^i]} - 1 = \frac{1}{\frac{\sum_{i \in C_t} [q_t^i p_{t-y}^i]}{\sum_{i \in C_t} [q_t^i p_t^i]}} - 1 = \frac{1}{\sum_{i \in C_t} \left[\frac{q_t^i p_t^i}{\sum_{j \in C_t} [q_t^j p_t^j]} \left(\frac{p_{t-y}^i}{p_t^i} \right) \right]} - 1.$$

- The Törnqvist index inflation rate:

$$\pi_t^{CT} = \exp \left\{ \sum_{i \in C_t} \left[\frac{1}{2} \left(\frac{q_{t-y}^i p_{t-y}^i}{\sum_{j \in C_t} [q_{t-y}^j p_{t-y}^j]} + \frac{q_t^i p_t^i}{\sum_{j \in C_t} [q_t^j p_t^j]} \right) \ln \left(\frac{p_t^i}{p_{t-y}^i} \right) \right] \right\} - 1.$$