Labor Market Dynamics: A Hidden Markov Approach

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Job Market Paper

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November 20, 2015

Abstract

This paper proposes a hidden state Markov model (HMM) that incorporates workers' unobserved labor market attachment into the analysis of labor market dynamics. Unlike previous literature, which typically assumes that a worker's observed labor force status follows a first-order Markov process, the proposed HMM allows workers with the same labor force status to have different history-dependent transition probabilities. I show that the estimated HMM generates labor market transition probabilities that match those observed in the data, while the first-order Markov model (FOM) and its many-state extensions cannot. Even compared with the extended FOM, the HMM improves the fit of the empirical transition probabilities by a factor of 30. I apply the HMM to (1) calculate the long-run consequences of separation from stable employment, (2) study evolutions of employment stability across different demographic groups over the past several decades, (3) compare the dynamics of labor market flows during the Great Recession to those during the 1981 recession, and (4) highlight the importance of looking beyond distributions of current labor force status.

*ishibata@uchicago.edu, University of Chicago: I would especially like to thank Robert Shimer, Steven Davis, and Loukas Karabarbounis for their great guidance and support. I would also like to thank Fernando Alvarez, Stephane Bonhomme, Thibaut Lamadon, Rafael Lopes de Melo, and the seminar participants at the University of Chicago's Applied Macro and Capital Theory Working Groups for their thoughts and comments.

1 Introduction

The Great Recession of 2007 featured an unprecedented increase in long-term unemployment and a sluggish labor market recovery. This has sparked a renewed interest in persistent labor market transitions and the long-term consequences of losing a job. Long-term unemployed workers have a significantly lower chance of holding a full-time job one year later than do their short-term counterparts.[1] An unemployment spell begets future unemployment, and returning to employment does not fully reset the clock for unemployed workers. This is inconsistent with most theoretical and empirical papers on workers' transitions, which assume that workers are homogeneous and their observed labor force status transition follows a first-order Markov process.[2]

In a first-order Markov model (FOM), a worker's transition dynamics depend only on her current labor force status. It does not admit the possibility of both heterogeneous and duration-dependent transition probabilities, because it imposes the same transition probabilities on all workers with the same labor force status, regardless of their differences in unobservable characteristics and labor force histories. As a result, the FOM with homogeneous workers fails to generate the persistent labor market dynamics observed in the data. It also underestimates the long-run reduction in probability of employment associated with separating from stable employment.

In this paper, I propose a Hidden Markov model (HMM) to capture observed persistent labor market dynamics by introducing *dynamic unobserved labor market attachment* to incorporate both heterogeneity and duration dependence for workers in all three labor force statuses: employment, unemployment, and nonparticipation. The introduction of dynamic unobserved labor market attachment is motivated by two empirical observations. First, workers even within the same labor force status are heterogeneous in their future labor market outcome.[3] This feature is already present in some models. For example, Blanchard and Diamond (1990) consider two types of workers. "Primary" workers are more likely than "secondary" workers to be employed, and less likely to leave the labor force upon losing a job. Loosely speaking, primary workers are more attached to the labor market than are secondary workers.

Second, workers' transitions are known to be history dependent. Most prominently, the probability of finding a job declines with the duration of unemployment because of human

[1]See, for instance, Hall (1995). Krueger, Cramer, and Cho (2014), for example, finds that the chance that the long-term unemployed will be employed one to two years later is 20 to 40 percent lower than that of the short-term unemployed.

[2]See, for instance, Abowd and Zellner (1985), Poterba and Summers (1986), Blanchard and Diamond (1990), Mortensen and Pissarides (1994), Shimer (2005), and Shimer (2008).

[3]See, for instance, Jones and Riddell (1999), and Barnichon and Figura (2013).

capital depreciation, employer discrimination in the hiring process, and lower search effort due to discouragement.[4] I introduce *labor market attachment* to capture the unobserved states that affect workers' future labor market transition dynamics. I also allow workers' attachment to change over time.

The model in this paper nests and improves upon the standard homogenous-worker FOM in several dimensions. First, the HMM significantly improves on the FOM in fitting the empirical transition probabilities among employment, unemployment and nonparticipation over a 15-month horizon. Even after extending the FOM to 29 different employment states, the HMM has a mean absolute deviation of transition probabilities which is 30 times smaller.[5]

Second, the HMM allows for heterogeneous transition probabilities among workers of the same current labor force status, an essential improvement on the FOM because the data suggest that workers' past labor force history significantly predicts her transition probability. In the FOM with homogeneous workers, a worker's future transition dynamics depend only on her current labor force status. As a result, the future labor market transitions for recently employed workers look the same regardless of their prior unemployment durations, which is inconsistent with the data. In contrast, a long-term unemployed worker in the HMM is more likely than her short-term counterpart to experience recurring unemployment before establishing stable employment. In the HMM, two workers of the same current labor force status could differ in their future transition probabilities due to differences in their current labor market attachment states (heterogeneity), and a given worker's transition probability changes over time due to changes in the labor market attachment state (duration dependence).

To estimate a Hidden Markov Model, I use the full panel of the Current Population Survey (CPS). This paper is one of the first to make full use of the CPS panel data, thus providing insight for future research.[6] Both theoretical and empirical papers with the first-order Markov assumption have used the CPS extensively, but previous authors with the CPS data on hand have generally either chosen to use only two consecutive months of data or the unemployment duration data alone.[7] The CPS, however, tracks workers' labor force history for eight months over a 16-month period. I use this full panel to estimate workers'

[4] See, for instance, Parsons (1972) for human capital, Kroft, Lange, Notowidigdo (2013) for employer discrimination in the hiring process of the long-term unemployed, and Krueger and Mueller (2011) for discouragement

[5] The extended FOM, as shown in Section 3.3, incorporates detailed labor force statuses including reason for unemployment.

[6] Other papers that use more than 2 consecutive months of employment data include Nekarda (2009), Krueger, Cramer, and Cho (2014), Elsby, Hobijn, and Sahin (2015), Kudlyak and Lange (2014), and Hall and Schulhofer-Wohl (2015).

[7] See for instance, Abowd and Zellner (1985), Poterba and Summers (1986), Fujita and Ramey (2009), Shimer (2012) for two consecutive months of the data, and Horsntein (2012), Ahn and Hamilton (2015), and Ahn (2014) for unemployment duration data.

unobserved labor market attachment, which captures heterogeneity and history dependence among workers across all three labor force statuses.

Additionally, my model treats history dependence more systematically than previous papers do. Most previous models only allow the probability of transitioning out of unemployment to depend on duration of unemployment and unobserved heterogeneity between workers. In my model, these considerations apply to labor market transitions between all three labor force statuses.[8] Accounting for history dependence for all three labor force statuses is important for a fuller understanding of labor market dynamics for two reasons. First, the data suggest that history-dependent transition probabilities are present not just for those classified as unemployed, but also employed and nonparticipating individuals.[9] Second, the nonparticipation margin also plays a key role in aggregate labor market dynamics. Approximately two-thirds of the workers who enter employment come directly from non-participation. In addition, approximately one-third of unemployment rate fluctuation is attributed to fluctuations on the participation margin (Elsby, Hobijn, and Sahin, 2015).

To better understand the unobserved labor market attachment states, I relate unobserved states to observable characteristics such as earnings, reason for unemployment, and detailed labor force types (*e.g.* full- and, part-time workers, and marginally attached nonparticipants). I find that the persistent employment attachment state is associated with higher wages and full-time jobs, and that the most persistent unemployment attachment state is associated with those who involuntarily lost their job (job losers). However, the detailed labor market statuses are not sufficient to fully capture either the labor market transitions over a 15-month period or the unobserved heterogeneity across workers. This is because heterogeneity exists even among workers who have the same detailed labor force status.

The HMM used in this paper can statistically assign individuals within the same labor force status to labor market attachment states based on their labor force status history. In doing so, this paper also unifies previous studies of categorizing workers within the labor force status into different types. For instance, Barnichon and Figura (2013) classify nonparticipants into marginal nonparticipants and other nonparticipants.[10] They find that marginal nonparticipants mostly enter the labor force through unemployment, while the other nonparticipants mostly move directly to employment. Kudlyak and Lange (2014) also find that current nonparticipants with recent employment have a much higher chance of finding job than those in nonparticipation for the three prior consecutive months. My model uses la-

[8]For instance, Horsntein (2012), Ahn (2014), and Ahn and Hamilton (2015) account for unobserved heterogeneity among the unemployed alone using the duration data in the CPS.

[9]See, for instance, Parsons (1972), Farber (1994), Jovanovic (1979) for history dependence (job tenure) of the employed workers, and Kudlyak and Lange, (2014) for history dependence among the nonparticipants.

[10]Marginal nonparticipants are those who desire to work, have looked for work within the last 12 months, but are not currently looking for work.

bor market attachment to precisely capture these behavioral patterns among workers with different labor force history.

To illustrate how the HMM can provide additional insight for future analysis of labor market dynamics, I apply the model to address four important issues in the area of labor market dynamics. First, I study whether the HMM can generate more realistic long-run consequences of unemployment than the FOM, which is known to underestimate it.[11] The HMM can indeed generate the long and persistent reduction in employment probabilities over the next 20 years when separating from stable employment, using only data spanning 16 months.

This extrapolative power of the HMM frees the researcher from reliance on a long panel. Existing estimates of the long-run costs of unemployment in the U.S. use Social Security data that include 20 or more years of earnings and employment information (*e.g.* Davis and von Wachter, 2011, Song and von Wachter, 2014). Rather than using up to 20 years of Social Security earnings records, as these previous studies do, I rely on short CPS panels that span only 16 months to estimate the long-run consequences of separating from stable employment. Because the CPS contains education information for individuals in addition to the age and sex information included in the Social Security data, the HMM can provide the first estimates of the long-term consequence of losing stable employment for more detailed demographic groups.

Second, I study how employment stability for different demographic groups changed over recent decades. My model can look beyond changes in the employment rate and study *employment stability*.[12] I find that while the average employment rate among men has declined since the 1970s, their employment stability has not.[13] Therefore, the observed decline in the employment rate masks sustained employment stability enjoyed by men in recent years. I also find that while women's employment stability improved until the mid-1990s, only more educated women experienced this improvement.

Third, I compare the distribution of unobserved labor market attachment states between the 1981 recession and the Great Recession of 2007-09. Both recessions featured high unemployment rates, but the recovery of the labor market was much more sluggish after the Great Recession. I find that this difference is attributable to larger inflows of workers in the most-persistent unemployment state, less-stable employment state, and the nonparticipan-

[11]See, for instance, Davis and von Wachter (2011)

[12]Hall (1982) Parsons (1972), Farber (1994) study the employment stability in terms of job tenure with a particular employer. Employment stability in this paper is more general than these previous studies. It includes job-to-job transition and measures the likelihood of continuous employment. The HMM can provide further insights for changes in employment stability for men and women, measured as the percent of employed workers in the most stable employment attachment state or the stable employment rate.

[13]Note that employment stability here is employment stability of employed men.

tion state associated with unemployment.

Lastly, I highlight the importance of looking beyond distributions of current observed labor force status, such as the unemployment rate and employment-population ratio. I consider a counterfactual exercise in which two economies share the same distribution of observed labor force status and differ only in unobserved states. A "good" economy is filled with workers with more association with employment and a "bad" economy is filled with workers with more association with unemployment. The future labor market dynamics look significantly different despite the fact that the two economies share the same initial distributions of observed labor force statuses. The bad economy experiences a high and persistent unemployment rate, while the good economy experiences a decline in the unemployment.

The rest of this paper is organized as follows. Section 2 reviews related literature. In Section 3, I first describe the CPS and empirical transitions data over a one-year horizon. I then show that both the FOM with homogenous worker and the extended FOM fail to fit the empirical transition probabilities. In Section 4, I introduce a hidden Markov model (HMM), the method of estimating it, and identification results. Section 5 presents the results. I first present the HMM's success in fitting the observed transition probabilities over a 15-month period. I also show how the HMM captures duration dependence and unobserved heterogeneity. In Section 5.3, to interpret the economic meanings of the unobserved states, I relate these unobserved states to observable characteristics. Section 6 presents four applications of the HMM described previously. Section 7 concludes.

2 Related Literature

There are several strands of literature that are related to the present research, including prior studies of history dependence in transition probabilities using more than two consecutive months of CPS matched data. Krueger, Cramer, and Cho (2014), for instance, use CPS panels spanning 16 months to study workers' labor market transitions over one year or more during the Great Recession. They find that long-term unemployed workers have around a 27 percent lower chance of being employed 15 months later than their short-term counterparts. Kudlyak and Lange (2014) study how labor force history affects the job-finding probability for non-participants, and find that workers who had a recent history of employment have a much higher chance of finding a job than someone who has been out of the labor force for the preceding three months. The model proposed in this paper captures the history dependence studied in those previous studies and summarizes workers' history dependence using labor market attachment states.

Several other papers also estimate unobserved heterogeneity using the Current Population Survey. Hornstein (2012), for instance, develops a dynamic accounting model of unemployment that allows for workers with high and low job-finding rates. His model allows ordered dynamic heterogeneity in which a high-exit type workers can become low-exit according to a Poisson process. Ahn and Hamilton (2015) and Ahn (2014) also develop a dynamic accounting model but incorporate duration dependence with a spline function. All three papers, however, only allow heterogeneity among the unemployed workers because they only use unemployment duration data.[14] On the other hand, I use the full CPS labor force history panel to estimate heterogeneity in transitions between all three labor force statuses.

This paper is also related to prior literature that uses a latent variable approach to correct for classification errors in which survey respondents misreport their true labor force status.[15] The HMM in this paper nests a variant of classification error models from the literature, where the number of unobserved states is equal to the number of observed labor force statuses (e.g Biemer and Bushery, 2000). The classification error papers treat unobserved states as true labor force status and estimate measurement errors for labor force history using three months of labor force history data. The present paper, however, allows for more than three unobserved states by using more than three months of data and interprets unobserved states as labor market attachments. The observation matrix in this model, therefore, still accounts for measurement errors in the CPS.

3 Data

The Current Population Survey (CPS), a national representative survey for the U.S., contains information regarding workers' labor force statuses, industries, and demographics (*e.g.* sex, age, race, geographical locations, and education) and is used to calculate national labor market statistics such as the unemployment rate. Since January 1976, the CPS has conducted monthly interviews of approximately 60,000 households and 100,000 to 150,000 individuals based on physical address. Households are interviewed for four consecutive months, and then rotated out for the next eight months before being interviewed again four more months. This structure allows researchers to track workers' labor force status for a maximum of eight months during a 16-month period.

Due to its survey design, each CPS monthly file contains eight rotation groups where a rotation group is defined by the number of months its members have been in the survey. In month t, only individuals belonging to rotation group 1 (those who just entered the survey)

[14] Some other recent papers study unobserved heterogeneity other labor force statuses besides unemployment. Browning and Carro (2014) study fixed unobserved heterogeneity with type-specific first-order Markov process for observed labor force statuses for employed and unemployed workers. Unlike the present model, their model, however, does not allow for duration dependence as the workers are not allowed to transition between unobserved types. Alvarez, Brovickova, and Shimer (2015) develop an optimal stopping time model for employed and non-employed workers.

[15] see, for instance, Feng and Hu (2013)

are eligible to be matched for the eight months over spanning the length of the next 15 months.[16] This limits the eligible group to approximately one-eighth of the total observations in each monthly file.

As in Shimer (2012), I follow the algorithm in Madrian and Lefgren (2000) to match CPS monthly files across months. Let τ be the number of months elapsed since the first entry into the survey. Thus, for an individual with a complete eight-month employment status history, I observe her labor force status in $\tau \in \mathbb{T} \equiv \{0, 1, 2, 3, 12, 13, 14, 15\}$. I match observations between month $\tau = 0$ and all the other months based on individuals' household identifier, household replacement number (which identifies whether the initial household has been replaced), personal identifier, sex, age, and race. Age is allowed to differ by increment of one between consecutive months. However, some individuals cannot be matched for various reasons including simple attrition and coding errors.

On average, each monthly file contains around 18,000 individuals belonging to rotation group 1 in month t. More than 80 percent of these individuals can be matched within the next four months, and around 65 percent of them can be matched for months 12 to 15. These numbers are slightly higher after a 1994 redesign and are comparable to the numbers reported in Drew, Flood, and Warren (2013). In Appendix, I show how different criteria affect the matching rates of observations between months. Lastly, I restrict the sample to the individuals between 16 and 64 years old in the initial survey month for whom I observe a complete eight-month labor force status history. This leaves me with 2,420,463 observations for the entire period and 6,320 average monthly observations.

Two main problems could arise due to failure to match individuals between months. First, the matched individuals do not necessarily represent the population. For instance, cross-sectional statistics implied by the matched individuals do not necessarily line up with the aggregate cross-sectional statistics because we can only match a subsample of the individuals in the survey. If the matching attrition is completely random and independent of any demographic characteristics, we can simply scale up the weights by the ratio of the total sample number in the original survey to the number in the matched sample. However, attrition is often not random. To ensure that demographic representations in the matched sample mirror those in the original survey, I reassign the sample weights of the matched individuals by multiplying them by the inverse of matching probability conditional on the individual's demographic characteristics and labor force status. Specifically, I run a logit model and calculate the predicted probability of matching conditional on gender, age, race, education, and labor force status. I then multiply the original weights by the inverse of the corresponding predicted probabilities.

[16] Counting the first month in the survey as month 0.

Another problem that arises from matching files across months is that there exists a rotation group bias–that is, differences in labor force statistics across rotation groups. Bailar (1975) first discusses rotation group bias in the CPS. A recent paper by Krueger, Mas, and Niu (2014) find that the unemployment rate among earlier rotation groups is higher than among later rotation groups. This rotation group bias is mitigated by limiting the first rotation group to those who responded in all eight months, as is done in this study. I also verify that the unemployment rates represented by my data match the official unemployment rate published by the Bureau of Labor Statistics.

The Current Population Survey (CPS) is also subject to classification errors. Classification errors occur when a worker misreports her true labor force status. Abowd and Zellner (1985) and Porterba and Summers (1986) are the two early seminal papers that addressed classification errors in the CPS. They propose that the CPS reinterview surveys be used to correct for classification errors. Their method, however, is also subject to certain criticisms (Feng and Hu, 2014). For instance, the reinterview survey itself is subject to classification errors. Furthermore, their classification error corrections are estimated based on the 1980s-era reinterview survey, and the same classification error correction may not be applicable to more recent data.

Some recent papers propose that classification errors should be corrected using employment status history data. For instance, Feng and Hu (2014) apply a latent-variable approach to estimate classification errors using three months of employment status history from the CPS. Elsby, Hobijn, and Sahin (2015) propose a mechanical way to correct for the classification errors. They propose that employment status history data that is recorded as out of the labor force (O), unemployment (U), and out-of-the-labor-force (O) should rerecorded as OOO, and that workers are recorded as unemployed (U), out of the labor force (O), unemployed (U) should be rerecorded as UUU. They show that the adjusted two-month transition flows closely resemble the adjusted transition probabilities based on the Abowd and Zellner correction. However, Kudlyak and Lange (2014) argue against this adjustment procedure. When they study labor force status histories for the four month period, the job-finding probabilities among workers out of the labor force significantly differ depending on their recent labor force histories. Their finding suggests that the reported labor force statuses indicate some underlying difference across workers. Because the aim of this paper is to estimate such underlying differences, which I call labor market attachment, I do not adjust the CPS using any of the procedures proposed in the previous literature. Furthermore, the observation matrix in my model captures classification errors studied by the error correction models.

Table 1: Labor Market Transitions (Historical Monthly Average)

Labor Force Status		Percent of Workers in t month		
$t = 0$ (6,320)	t month later	1 Month Later	3 Months	12 Months
Employment (4,434)	Employment	4,248 (95.8%)	4,159 (93.8%)	4,030 (90.9%)
	Unemployment	57 (1.3%)	93 (2.1%)	124 (2.8%)
	Out of the Labor Force	129 (2.9%)	182 (4.1%)	280 (6.3%)
Unemployment (312)	Employment	71 (22.9%)	112 (35.9%)	153 (49.3%)
	Unemployment	167 (53.5%)	119 (38.2%)	75 (24.0%)
	Out of the Labor Force	73 (23.5%)	80 (25.9%)	83 (26.7%)
Out of the Labor Force (1,574)	Employment	98 (6.2%)	162 (10.3%)	271 (17.2%)
	Unemployment	63 (4.0%)	69 (4.4%)	69 (4.4%)
	Out of the Labor Force	1,413 (89.8%)	1,343 (85.3%)	1,234 (78.4%)

3.1 Data on Worker Transitions

Table 1 shows the historical monthly average of workers' transitions over one, three and 12 months intervals between January 1976 and July 2014. For instance, out of 6,320 people in the monthly sample, on average, 4,434 people are employed during the first month they are interviewed.[17] Among initially employed workers, 4,248 (96 percent) are still employed one month later. After a year, 4,030 (91 percent) of them are employed. Two hundred sixty-seven people (54 percent) out of 312 initially unemployed workers are still unemployed one month later. After a year, among those 312 initially unemployed, 153 people (24 percent) are unemployed. Among 1,574 nonparticipants in the first month, 98 (6.2 percent) move to employment and 1,413 people (90 percent) remain out of the labor force. After a year, 1,234 (78 percent) of them are out of the labor force.

A canonical model in the search and matching literature assumes that the workers are ex-ante identical and their observed labor force statuses follow a first-order Markov process.[18] Given this assumption, the probability of transitioning between labor force status from one month to the next is calculated as the fraction of workers in the same current labor force status who will be in each of the three labor force statuses in the next month. For instance, between 1976 and 2014, if a worker is unemployed in one month, there is a 23 percent chance

[17] The sample is limited to the workers between 16 and 64 years old for whom I observe complete eight months of employment history.

[18] See, for instance, Mortensen and Pissarides (1994), Abowd and Zellner (1985), Poterba and Summers (1986), Blanchard and Diamond (1990).

that she will be employed in the following month, a 23 percent chance that she moves out of the labor force, and a 54 percent chance that she remains unemployed.

Let $\lambda_{y_t,y_{t+1}}$ be the probability of transitioning from labor force status y_t to y_{t+1} between months t and $t+1$ where $y_t \in \{$employed (E), unemployed (U), out-of-labor-force (O)$\}$. The first-order Markov model (FOM) implies that if a worker is in labor force status y_t this month, the chance that she is in labor force status y_{t+2} two months later is given by

$$\lambda_{y_t,y_{t+2}} = \sum_{y_{t+1}} \lambda_{y_t,y_{t+1}} \lambda_{y_{t+1},y_{t+2}} \tag{1}$$

Define C to be the 3 x 3 transition matrix with *(i,j)* element being λ_{ij} with $i, j \in \{E, U, O\}$. Then, the implied probability that a worker currently in labor force state $y_t \in \{E, U, O\}$ will be in labor force status $y_{t+\tau} \in \{E, U, O\}$ in month $t + \tau$, $\lambda_{y_t,y_{t+\tau}}$, can be expressed in matrix form as

$$\begin{bmatrix} \lambda_{E_t,E_{t+\tau}} & \lambda_{E_t,U_{t+\tau}} & \lambda_{E_t,O_{t+\tau}} \\ \lambda_{U_t,E_{t+\tau}} & \lambda_{U_t,U_{t+\tau}} & \lambda_{U_t,O_{t+\tau}} \\ \lambda_{O_t,E_{t+\tau}} & \lambda_{O_t,U_{t+\tau}} & \lambda_{O_t,O_{t+\tau}} \end{bmatrix} = C^{\tau} \tag{2}$$

For instance, the probability that an unemployed worker is unemployed two months later is given by $\lambda_{U_t,U_{t+2}} = \sum_{y_{t+1}} \lambda_{U_t,y_{t+1}} \lambda_{y_{t+1},U_{t+2}}$ or 30 percent. After a year, an unemployed worker would have a less than 5 percent chance of being unemployed. However, in the data, 24 percent of the workers who were unemployed in the first month were unemployed a year later. Figure 1 plots the probabilities that a worker whose labor force state is y_t in month t is in labor force status $y_{t+\tau}$ in month $t + \tau$. We observe that the probabilities derived from the FOM miss the empirical probabilities significantly over a longer horizon.

3.2 Why Does the FOM Fail?

Why do the FOM transition dynamics of homogeneous workers fail to match labor market transitions over 15 months in the data? One main reason is that the first-order Markov process assigns the same transition probabilities to workers of the same current labor force despite their differences in employment history as well as both observable (*e.g.* sex, education, age) and unobservable (*e.g.* human capital) characteristics.

First, workers' labor market transitions are history dependent for workers in all three labor force statuses. Most prominently, the job-finding probability declines with unemployment duration (see Figure 2). Analogously, the job-separation probability for employed workers declines with tenure. Moreover, Kudlyak and Lange, (2014) have recently documented history dependence among nonparticipants. They find that a nonparticipant who has been employed

Figure 1: Transition Probabilities: Data vs FOM

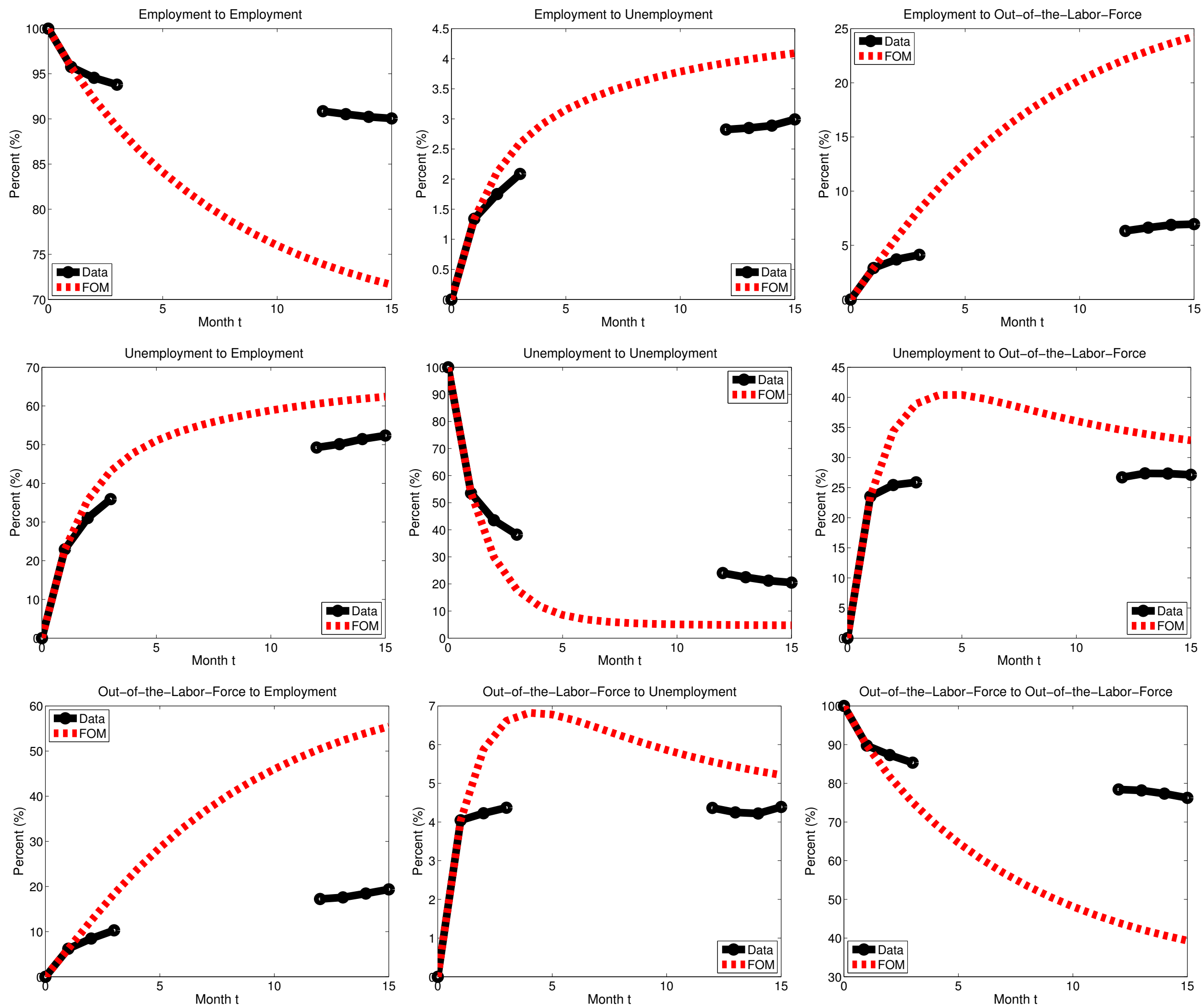
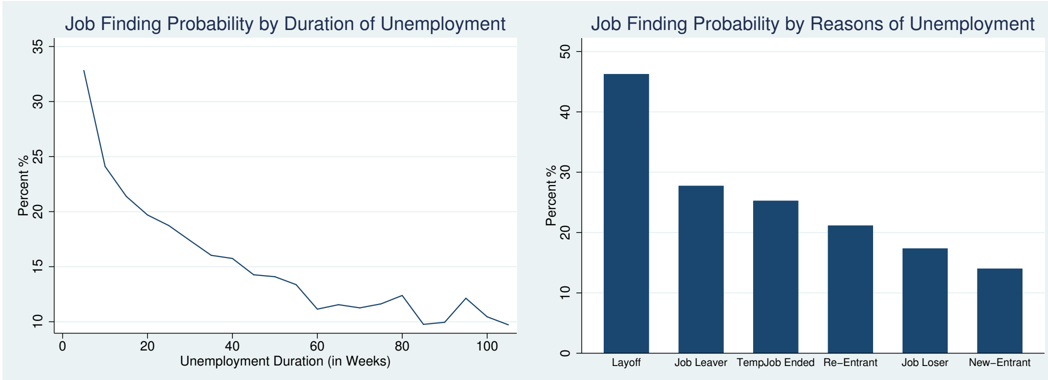


Figure 1 plots the probability of transitioning from labor force status y_0 to labor force status y_t between month *0* and month t over the next 15 months where $y \in \{\text{Employed,Unemployed,Out of the Labor Force}\}$. Black solid lines show the data from the CPS, and the red dashed lines show the transition probabilities implied by the first-order Markov model with homogenous workers.

Figure 2: Job Finding Probability by Duration and Reasons of Unemployment



within the last two months has a job-finding probability as high as 25-30 percent, while a worker who has been out of the labor force for the last three consecutive months has only a 2 percent chance of finding a job. This difference in job-finding probabilities cannot be accounted for by workers' heterogeneous reasons for being out of the labor force (*e.g.* discouraged, retired, disabled, or in school).

Second, the employment dynamics of workers of the same current labor force status could vary due to heterogeneity in both observable *(e.g.* sex, race, age, education) and unobservable (*e.g.* human capital) characteristics.[19] These characteristics can be either fixed or variable over time. For instance, job-finding probabilities among the unemployed differ depending on the individual's reason for separating from employment (see Figure 2). Unemployed workers who are "laid off"—temporarily out of work but able to be recalled by their former employer—have a more than 45 percent chance of finding a job, while those who are "job losers"—lost their jobs involuntarily—have a less than 20 percent chance of finding a job in the next month. Moreover, a declining empirical hazard rate of unemployment may be observed for two reasons. First, workers with relatively high exit rates may leave the unemployment state, so workers with relatively low exit rates would comprise an increasing share of the unemployment pool over time (heterogeneity). Second, the exit rate for a given worker may decline over the spell of unemployment due to factors such as human capital depreciation, employer statistical discrimination, and discouragement (genuine duration dependence).

3.3 FOM with Observable Heterogeneity

The previous section explained why the first-order Markov transition with homogeneous

[19]For instance, men are more likely to stay employed than women, and younger workers are more likely to lose their job but more likely to find a job than their older counterparts. In addition to workers' demographic characteristics, more detailed categories of labor force status can be observed in the CPS. Among the employed workers, for instance, full-time workers has a higher probability of staying employed next period than part-time workers.

workers cannot generate empirical labor market transition dynamics over the 15-month horizon. One natural question is whether incorporating observable heterogeneity and duration dependence of unemployment into the first-order transition matrix could improve the fit of the empirical transition probabilities.

The Current Population Survey (CPS) includes detailed data on workers' employment statuses, duration and reasons for unemployment. I extend the FOM to allow workers of the same current labor force status to differ based on these characteristics and study whether the extended FOM can fit empirical transition probabilities over 15 months. Employed workers can be further divided into full-time workers, part-time workers for economic reasons, and part-time workers for non-economic reasons. Unemployed workers are divided by both reasons for unemployment—(1) layoff, (2) job loser, (3) temporary job ended, (4) job leaver, (5) re-entrant, and (6) new-entrant—and duration—(1) less than 5 weeks, (2) 5-14 weeks, (3) 15-26 weeks, and (4) 27 or more weeks (Figure 2). This results in 6 x 4 = 24 types of unemployed workers. Finally, workers in out of labor force status are classified into the marginally attached—who looked for work within the last year but have not looked for work within the last four weeks—and other nonparticipants. This results in a total of 29 types of workers. Many of the entries in this extended first-order Markov transition matrix are zero by construction, and the effective number of parameters reduces to 212.[20]

Figure 3 plots the implied transition probabilities over the next 15 months for workers of different initial labor force status. While the extended FOM better captures the empirical labor market dynamics, it is still far from fitting the data. On average, the extended 29-state FOM transition probabilities miss their empirical counterparts by 9 percentage points. Therefore, even an extended 29-state first-order Markov transition matrix incorporating observable characteristics and unemployment duration still fails to generate the persistent labor market dynamics observed in the data.

Many previous papers have found different behavioral patterns among workers in the same current labor force status based on their response to detailed classification surveys (Jones and Riddell, 1999, Barnichon and Figura, 2013). Yet, these papers still fail to capture underlying differences in workers' degree of labor market attachment that would explain their empirically observed labor market transitions over the next 15 months. In the next section, I propose an

[20]With 29 types of labor force status, the total number of entries in the first-order Markov transition matrix turn out to be 29 x 29 =841. However, many of the entries in the first-order transition matrix are zero by construction. For instance, all the employed workers who become unemployed can only transition to less than 5 weeks of unemployment in the next month but not to unemployment with longer duration. Furthermore, workers who have been laid off cannot transition to job leavers or new entrant. Workers in the out of the labor force cannot also transition to unemployed workers with duration longer than 5 weeks, but there seem to be some workers who are in out of the labor force who transition back to unemployment as re-entrants in weeks longer than 5. Lastly, each row needs to sum to 1. After subtracting these entries, the number of parameters reduce to 212 parameters.

Figure 3: 29 State FOM Transition Dynamics

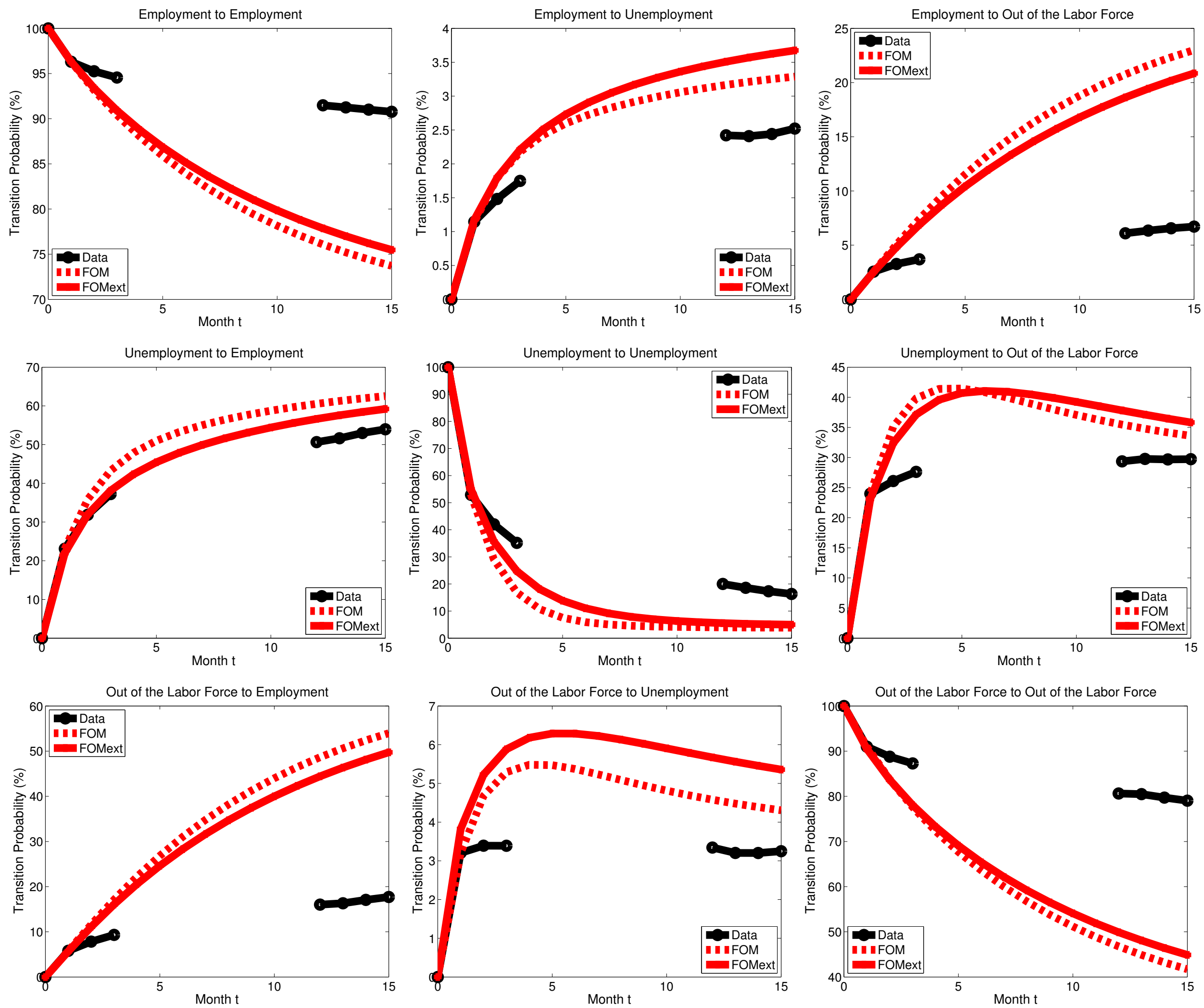
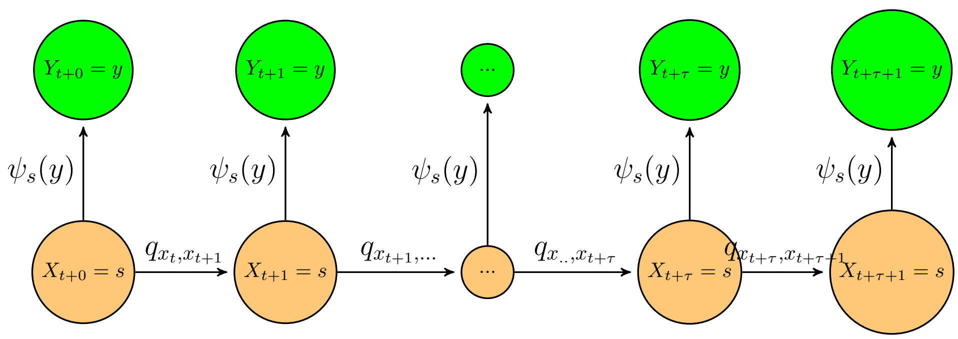


Figure 3 plots the probabilities of transitioning from labor force status y_0 to labor force status y_t between month *0* and month t over the next 15 months where y is either employment, unemployment, or out of the labor force. Black solid lines, "Data", show the data from the CPS, red solid lines, "FOM", show the first-order Markov model with homogenous workers and red dashed line, "FOMext", show the transition probabilities implied by the first-order Markov model.

Figure 4: Timing of the Model



alternative model that estimates the underlying unobserved heterogeneity across workers and admits the possibility that workers of the same current labor market status have different future labor market prospects. To this end, I introduce workers' *labor market attachment* into the model.

4 Hidden Markov Model

4.1 Model

In this section, I develop a statistical model of individual labor market transition, which incorporates workers' unobserved labor market attachment states. Time is discrete. The economy consists of L workers indexed by l, a set $\mathbb{Y}$ of *observable* labor market status, and a set $\mathbb{S}$ of S *unobserved* labor market attachment states indexed by $s \in \{1, \ldots, S\}$. The random variable X_t is defined to be a worker's unobserved labor market attachment state in time t.

Figure 4 illustrates the timing of the model. At $t = 0$, a worker is assigned to labor market attachment state s with probability $p_s = Pr(X_0 = s)$. In each period t, a worker transitions from labor market attachment state s to s' with probability $q_{ss'} = Pr(X_t = s'|X_{t-1} = s)$. If a worker is in attachment state s in period t, the probability that a worker will be in labor force status y in that period is $\psi_{sy} = Pr(Y_t = y|X_t = s)$.

Since there are S possible labor market attachment states and their transition follows a first-order Markov process, the transition across labor market attachment states is described by a $S \times S$ transition matrix Q where $q_{ss'} = Pr(X_{t+1} = s'|X_t = s)$ for $s \in \mathbb{S}$. Given M possible observable labor force statuses, the observation matrix is a $S \times M$ matrix Ψ: where $\psi_{sy} = Pr(Y_t = y|X_t = s)$ for $s \in \mathbb{S}$ and $y \in \mathbb{Y}$. Lastly, the initial distribution of unobserved labor market attachment states is given by a $1 \times S$ vector P:

For this paper, worker's observed labor force status y is either employed (E), unemployed (U), or out of the labor force (O), thus $M = 3$.[21] The number of unobserved labor market attachment states is $S = 9$. The number of unobserved states is determined by the combinations of statistical criteria, stability of the parameters, and economic interpretations. First, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) suggest that the number of unobserved labor market attachment states should be 10. However, I choose 9 unobserved states because Monte Carlo Simulations of the model show that parameters are not stable in the case of 10 unobserved states. Estimates of the model with 9 states, however, are stable and show economically meaningful unobserved labor market attachment states.

There are several points that are noteworthy. First, a HMM nests the FOM with homogeneous workers. If the number of unobserved labor force attachment states is equal to the number of labor force statuses (*i.e.* $S = M$) and the observation matrix is an identity matrix (*i.e.* $\Psi = I_M$), then the HMM is identical to the FOM with homogeneous workers. In this special case, the labor market attachment states are exactly same as the observed labor force statuses, and the transition matrix for unobserved states is equal to the transition matrix for the observed statuses (*i.e.* $Q = C$).

Second, to obtain the baseline estimation, I assume that the parameters of the model—the initial distribution, P; transition dynamics for the labor market attachment states, Q; and the observation matrix, Ψ—are the same for all workers in the data. This allows me to estimate the model parameters with a dataset that has a relatively short time period and a large cross section, such as the Current Population Survey.

4.2 Estimation

To estimate a hidden-state Markov model (HMM), I employ the Maximum Likelihood Estimation (MLE) with the Expectation-Maximization (EM) algorithm. Using the HMM model parameters, I can express the probability of observing a specific sequence labor market statuses for an individual worker. I can then construct the likelihood by expressing the probability of simultaneously observing labor market histories for all workers in the economy. I apply the EM algorithm to this likelihood function to estimate the model parameters. While the baseline model imposes common parameters across individuals, I could also estimate the model separately for different demographic groups by relaxing this assumption.

The model parameters consists of three objects $\Lambda = \{P, Q, \Psi\}$: P is a $1 \times S$ vector of initial distribution of unobserved labor market attachment states, Q is a $S \times S$ transition matrix, and Ψ is a $3 \times S$ observation matrix. The data consists of employment histories for

[21]Note that the model can be estimated with more detailed labor force statuses, such as full-time workers.

L individuals in the economy: $\{y^l_\tau\}^{l\in\{1,\dots,L\}}_{\tau\in\mathbb{T}}$ where y^l_τ is individual l's labor force status in month τ, the months elapsed since l's first entry into the CPS. Given the model parameters $\Lambda = \{P, Q, \Psi\}$, the likelihood function of observing individual l's employment history, y^l, is expressed in a matrix form as

$$\mathcal{L}^l(\Lambda|y^l) = P\hat{\Psi}_{\cdot y^l_0}\left[\prod_{\tau=1}^{3} Q\hat{\Psi}_{\cdot y^l_\tau}\right] Q^8 \left[\prod_{\tau=12}^{T} Q\hat{\Psi}_{\cdot y^l_\tau}\right] \mathbf{1}'_{\mathbf{S}} \tag{3}$$

where $\hat{\Psi}_{\cdot y}$ is a diagonal matrix of a column vector of Ψ corresponding to labor force status $y \in \{E, U, O\}$ and $\mathbf{1}'_{\mathbf{S}}$ is a $S \times 1$ vector of ones. The likelihood function of simultaneously observing labor force history for all individuals $\{y^l\}_{l=1}^L$ in the economy is then expressed as:

$$\mathcal{L}(\Lambda|\{y^l\}_{l=1}^L) = \prod_{l=1}^{L}\mathcal{L}^l(\Lambda|y^l) = \prod_{l=1}^{L} P\hat{\Psi}_{\cdot y^l_0}\left[\prod_{\tau=1}^{3} Q\hat{\Psi}_{\cdot y^l_\tau}\right] Q^8 \left[\prod_{\tau=12}^{T} Q\hat{\Psi}_{\cdot y^l_\tau}\right] \mathbf{1}'_{\mathbf{S}} \tag{4}$$

I maximize the likelihood function in equation (4) to estimate the parameters. The details of this estimation procedure is in the Appendix.

4.3 Identification

This section presents identification results for the HMM. Identification of the parameters is necessary because my goal is not only to better fit the empirical transition probabilities in Figure 1 but also to interpret the resulting labor market attachment states in an economically meaningful way. Allman, Matias, and Rhodes (2009) prove the identification results for a hidden Markov Model using Kruskal's algebraic results (1976, 1977). Theorem 6 in their paper shows the maximum number of unobserved states that can be identified given the number of time periods and the number of observable labor force statuses:

Theorem 6 of Allman, Matias, and Rhodes (2009)

The parameters of an HMM with S hidden states and M observable states are generically identifiable from the marginal distribution of $T = 2k + 1$ consecutive variables provided k satisfies

$$\begin{pmatrix} k + M - 1 \\ M - 1 \end{pmatrix} \geq S$$

Their identification results show that with three observable labor forces, ($M = 3$) and seven months of observation ($T = 7$), we could identify up to 10 unobserved states ($S = 10$). Based on the economically meaningful results and stability of the parameters, I choose the number of hidden states to be 9.

Table 2: Maximum Number of Labor Market Attachment States S

Three Observed Labor Force Statuses (M=3)							
# of Time Periods: T	3	5	7	9	11	13	15
# of Possible Employment History	27	243	2,187	19,683	177,147	1,594,323	14,348,907
Max # of Unobserved States: S	3	6	10	15	21	28	36
# of Parameters	14	47	119	254	482	839	1367

The identification is not trivial.[22] For example, simultaneous permutations of the model parameter matrices, Q, Ψ, and P do not change the underlying model structure (known as *label swapping*). For instance, swapping the labor market attachment states s and s' and applying corresponding permutation for Q, Ψ, and P would not change the transition probabilities implied by the model. As a result, a hidden Markov model can be identified up to *label swapping.* Refer to Allman, Mathias, and Rhodes (2009) for further details.

Table 2 shows the maximum number of identifiable unobserved labor market attachment states and their corresponding numbers of parameters and data points (*i.e.* possible employment histories) for different time periods T, using three observed labor force statuses. A longer time period, T, allows more unobserved states but increases the possible employment histories to be targeted. Therefore, it is not guaranteed that the HMM can fit the data.

5 Results

This section presents the HMM results. First, I evaluate how well the HMM fits the data by calculating mean absolute deviations of transition probabilities over 15 months. I also argue that this success in fitting the data is not simply achieved through the increase in model parameters. Second, I present the HMM estimates and describe the unobserved labor market attachment states. Third, I discuss how labor market attachment states admit the possibility that two workers with the identical observed labor force status have different transition probabilities (unobserved heterogeneity). Fourth, I describe how a worker's unobserved state evolves as she remains in the same labor force status. I show that the HMM admits the possibility that the same worker could transition from a less persistent labor market attachment state to a more persistent one, even as her labor market status remains the same (genuine duration dependence).

5.1 Persistent Labor Market Dynamics

This subsection presents HMM's fit of empirical transition probabilities over a 15-month period. Figure 5 plots empirical and HMM estimates of transition probabilities for seven

[22]It cannot be obtained by a simple accounting of number of unknowns and number of model parameters to be estimated.

months over the 15-month period.[23] The HMM fits the data, including transitions that the FOM failed to predict, fairly well. For instance, the FOM does not accurately predict one-year transitions from unemployment to unemployment, nor from out of the labor force to employment (Figure 5 and Table 11 in the Appendix). In the data, 24 percent of unemployed workers are unemployed a year later, while only 16 percent of nonparticipants are employed a year later. However, the FOM predicts the corresponding probabilities to be less than 5 percent and approximately 50 percent, respectively. The HMM predicts the corresponding probabilities to be 20 percent and 17 percent, which are much closer to the data. The HMM's estimates of the other one-year transition probabilities are also much closer to the data than those of the FOM.

To formally evaluate the HMM's performance, I calculate the mean absolute deviations of transition probabilities for seven months over the 15-month period. The mean absolute deviation shows the average percentage point difference between the transition probabilities predicted by the model and their empirical counterparts. With three labor force statuses, there are nine possible transition probabilities for seven months, $\tau \in \mathbb{T} \equiv \{0, 1, 2, 3, 12, 13, 14, 15\}$, totaling $9 \times 7 = 63$ data points (Figure 5). The mean absolute deviation of the model from the empirical counterparts is expressed as:

$$MAD_{HMM} = \sum_{y' \in \mathbb{Y}} \sum_{y \in \mathbb{Y}} \sum_{\tau \in \mathbb{T}} \mid Pr(Y_\tau = y' | Y_0 = y)_{HMM} - Pr(Y_\tau = y' | Y_0 = y)_{Data} \mid \tag{5}$$

where $Pr(Y_\tau = y' \mid Y_0 = y)$ is the probability that a worker whose initial observed labor force status was y transitions into labor force status y' in month $\tau \in \mathbb{T}$. The mean absolute deviation of the HMM estimates is 0.3 percentage points, whereas that of the extended 29-state FOM is 8.9 percentage points. Therefore, the HMM reduces deviation by a factor of 30.

Note that while the baseline FOM with homogeneous workers uses only the first two months of the data, the HMM uses more data. Thus, unlike those of the HMM, the mean absolute deviations of the baseline FOM mostly consist of out-of-sample mean deviations. I also consider different specifications of the FOM that utilizes all eight months of transitions, but these versions still fail to reduce mean absolute deviations. For example, I estimate the maximum likelihood of the FOM over all eight months of the transitions (FOMmle in Table 3). I also calculate the FOM using the 8–month average of transition probabilities or transitions that vary month by month. Yet, none of these modifications to the FOM generate the persistent labor market dynamics observed in the data. In the Appendix, I also consider

[23] $Pr(Y_\tau = y' | Y_o = y)$ where $y,' y \in \{E, U, O\}$ and $\tau = \{0, 1, 2, 3, 12, 13, 14, 15\}$

Figure 5: Transitions over 15 months

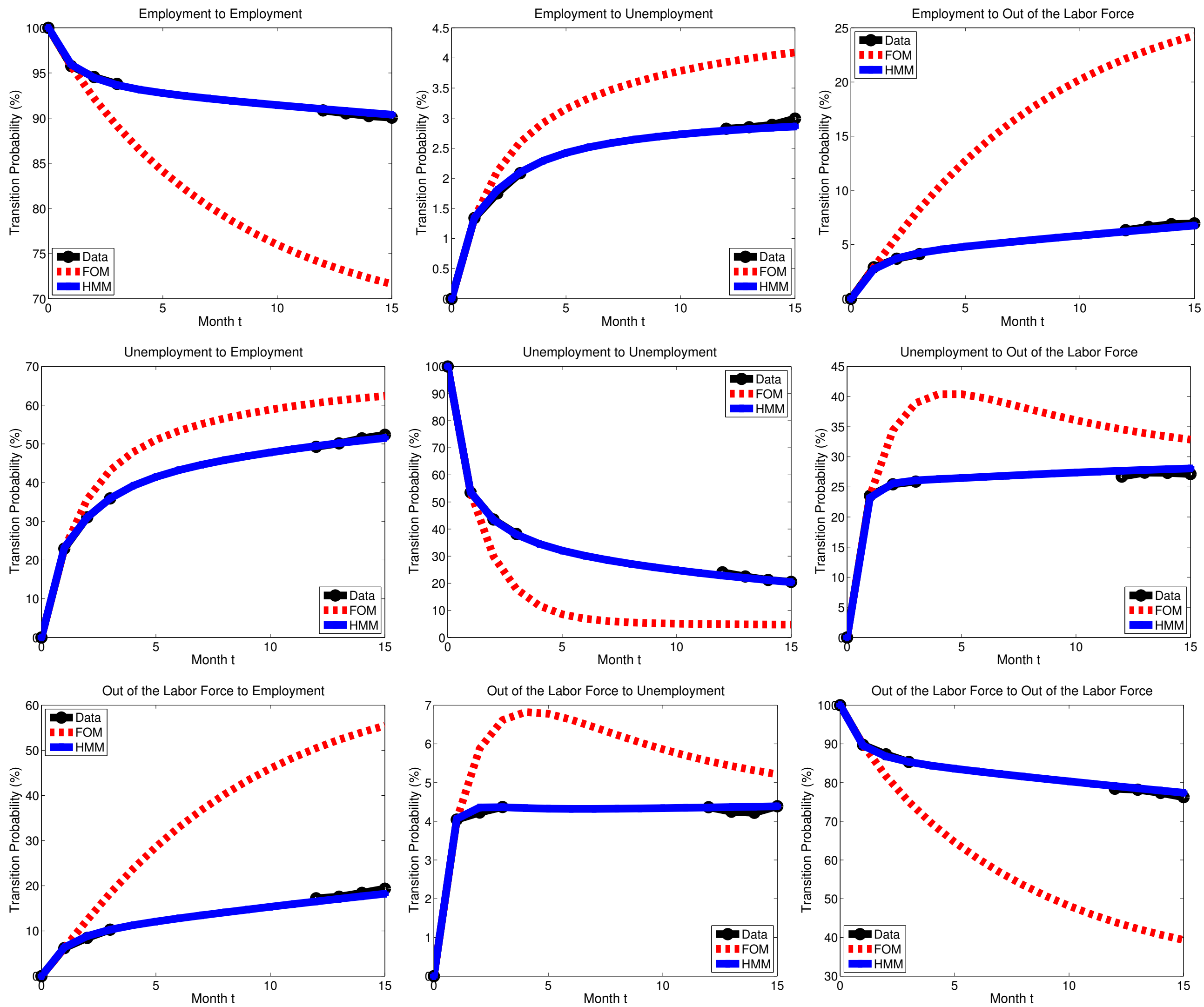


Figure 5 plots the transition probabilities implied by the data (black dotted line), the FOM (red dashed line), and the HMM (blue solid line).

Table 3: Mean Absolute Errors (in Percentage Points)

Hidden Markov Model			First-Order Markov Model		
	MAD	# of Parameters	Model	MAD	# of Parameters
HMM3	2.92	14	FOMbase	10.08	6
HMM4	2.10	23	FOMmle	9.53	6
HMM5	1.45	34	FOMext	8.86	212
HMM6	1.37	47			
HMM7	0.89	62			
HMM8	0.54	79			
HMM9	0.29	98			
HMM10	0.26	119			

Table 3 shows mean absolute deviations between the data and the transition probabilities predicted by models with different specifications. For example, HMM3 shows the HMM with the number of hidden states equal to 3. FOMbase is the baseline FOM with three labor force statuses using the first two months of the data. FOMmle is the maximum likelihood estimation of the FOM using all eight months of the data (7 transitions). FOMext is the FOM with 29 employment types studied in Section 3.

alternative models that allow for unobserved heterogeneity, specifically the finite mixture model and the fixed unobserved heterogeneity FOM as in Browning and Carro (2014).[24] The mean absolute deviations of these models with unobserved heterogeneity are still higher than the HMM with similar parameter sizes. Furthermore, these models do not admit the possibility that workers could be changing types (unobserved states) over time.

Moreover, the HMM's better performance is not solely attributable to its greater number of model parameters. Table 3 compares the mean absolute deviations of different FOM and HMM specifications and their corresponding number of parameters.[25] While the extended 29-state FOM with 212 parameters still misses the empirical transition probabilities by 8.9 percentage points on average, the HMM with only three unobserved states and 14 parameters reduces the mean absolute deviation of transition probabilities to 2.9 percentage points.[26] The HMM with 9 unobserved labor market attachment states reduces the mean absolute deviation of transition probabilities down to 0.3 percentage points. This result shows that the HMM's accurate results are attributable to its flexibility in allowing a worker's labor

[24]Furthermore, I also show that ignoring history dependence and unobserved heterogeneity for employment and out of the labor force status will result in mean absolute deviations of 7.0 percentage points.

[25]This mean absolute deviation is not weighted. The mean absolute deviation weighted by the fraction of population in each employment status (E,U, or O) is in Table 9 in the Appendix. The HMM's mean absolute deviation further decreases when we use weights, because the HMM is better at capturing employment status, which comprises a large share.

[26]Even if we use all eight month of average transitions, the mean absolute deviation actually gets worse. The mean absolute deviation of the extended FOM using the 8 month average becomes 10.36 percentage points.

market attachment states to govern the distribution of her current labor force status.[27]

The HMM assumes that a worker's observed labor force status depends only on her unobserved labor market attachment states but not on her labor force status in the previous month. This assumption is not as restrictive as it may seem. Table 10 in the Appendix shows one-month transition probabilities calculated using both the HMM and FOM, which are, by construction, equal to the empirical probabilities. The HMM estimates of one-month transitions are different from those based on the FOM by only less than 0.3 percentage points on average. In the FOM, the worker's current labor force status depends only on her labor force status in the previous month. In the HMM, however, the worker's labor force status in the previous month predicts the distribution of worker's current labor force status only through her unobserved labor market attachment state. Thus, the conditional independence assumption for the observed labor force status is not restrictive.

Moreover, while the HMM in this paper uses only information on the three labor force statuses, we could in principle incorporate more observed states into the model. In the Appendix, I show how incorporating additional detailed information on employment, (full-time employment (F), part-time employment (P), unemployment (U), and nonparticipation (O)) could improve the fit for the transition probabilities between three labor force statuses (E,U,O). Table 12 in the Appendix shows that using four observed states (F,P,U,O) would reduce the mean absolute deviations for transitions between three labor force statuses (E,U,O) by a factor of 1.5-2.8. The gain becomes smaller when I allow more unobserved states.

5.2 HMM Estimates

Table 4 summarizes HMM estimates of nine labor market attachment states. Column 1 shows observation matrix Ψ where $\psi_{sy} = Pr(Y = y|X = s)$. Column 2 shows the probability distribution of observed labor force status after 6 months conditional on the unobserved labor market attachment states in month 0. In the Appendix, I present the transition matrix for labor market attachment states, the standard errors of the estimates based on a bootstrap method, and the Monte Carlo Simulation results and show that the estimated parameters are reasonable (Table 13, 15, and 16).

The observation matrix Ψ in Column 1 in Table 4 shows the distribution of current labor force status conditional on unobserved labor market attachment states: $\psi_{iy} = Pr(Y = y|X = i)$. Workers with some attachment states are more likely to be employed. Likewise, in other attachment states, workers are most likely to be unemployed or out of the labor force in the

[27] When the number of unobserved states are three, the HMM is equivalent to the FOM with homogeneous workers if the observation matrix is an identity matrix $\Psi = I_3$. The only differences between the HMM with three unobserved states and the FOM with homogenous workers comes from the fact that workers in the same labor market attachment states could have different observed labor force status, that is $\Psi \neq I_3$.

Table 4: Labor Market Attachment States

Labor Market Attachment	Column 1: Dist of LFS Current Month Ψ			Column 2: Distribution of Observed Labor Force Status 6 Months Later $Pr(Y_{t+6} \mid X_t)$		
	E	U	O	$Y_{t+6} = E$	$Y_{t+6} = U$	$Y_{t+6} = O$
E1	99.7	0.0	0.3	98.8	0.1	1.1
E2	98.0	1.3	0.7	82.7	13.1	4.3
E3	98.4	1.6	0.0	53.8	4.4	41.9
U1	5.4	84.9	9.7	69.8	21.3	8.9
U2	3.9	87.9	8.2	16.4	62.2	21.4
U3	8.9	86.8	4.3	18.0	18.8	63.3
O1	0.0	2.5	97.5	44.7	5.1	50.3
O2	0.0	0.0	100.0	14.3	20.9	64.8
O3	0.4	0.1	99.5	2.6	0.9	96.5

current month. I call this first dimension of attachment "*prone*" and label unobserved states associated with, say, employment "employment-prone (*E-prone*) states". Likewise, I label unobserved states associated with unemployment and out of the labor force, unemployment-prone (*U-prone)* and out-of-the-labor-force-prone states (*O-prone)*, respectively.

Second, the labor market attachment states also differ in terms of their predicted future outcomes. This accords with unobserved heterogeneity among workers of the same current labor force status. Column 2 in Table 4 shows the distribution of observed labor force status six months later conditional on workers' attachment states in month 0: $Pr(Y_{t+6}|X_t)$. For example, workers in some *employment-prone* (*E-prone*) state are more likely to be employed, while others are less attached to employment and are more likely to become unemployed or leave the labor force.

Thus, I further divide workers *within* the same prone states. I assign 1 if a worker is more likely to become employed in the future, 2 if a worker in that state is more likely to become unemployed, and 3 if a worker is more likely to move out of the labor force. For example, the E1 state is the most persistent employment state because not only is a worker in E1 likely to be employed in this period (employment-prone), she has the highest chance of being employed in the future (99 percent) among the workers in all E-prone states (E1,E2, and E3). Likewise, a worker who was initially in E2, the second-most persistent employment state, still has an 83–percent chance of being employed six months later but also has a 13 percent chance of becoming unemployed. The E2 state is associated with the highest probability of future unemployment among the E-prone states. Lastly, a worker who was initially in state E3 will have the highest (42 percent) chance of moving out of the labor force among the workers in

Table 5: Monthly Job Finding/Separation Probability by Labor Market Attachment

Job Finding Probability (in %)		
HMM	U1	35.6
	U3	9.6
	U2	3.8
FOM	U	23.0

Job Separation Probability of E-prone States (in %)		
HMM	E2	6.4
	E3	2.0
	E1	.01
FOM	E	1.3

the E-prone states.

Similarly, six-month outcomes also differ among workers who are initially in unemployment prone-states. If a worker's current unobserved state is U1, she has the highest chance (70 percent) of being employed six months later among all workers initially in U-prone states. A worker initially in the U2 state has the highest chance of being unemployed six months later. Therefore, U2 is the most persistent unemployment attachment state. A worker who is initially in U3 has a higher (63 percent) chance of leaving the labor force.

A similar pattern is observed for the workers in the out of the labor force attachment states. Nonparticipants in the O1 state are more likely to become employed six months later (45 percent) than those in the other two O-prone states. While a worker in the O2 state is more attached to the labor force (21 percent chance of being unemployed six months later) than an O3 worker, the O3 state is most persistent out of the labor force status (96 percent).

The chance that a worker in the O3 state is employed six months later is a mere 2.6 percent. This is consistent with the findings by Kudlyak and Lange (2014) who show large differences in in job-finding probabilities among non-participants depending on their labor force histories. Nonparticipants who have been out of the labor force for the three consecutive months are more likely to be in the most persistent O-prone state and are very unlikely to move back to employment. On the other hand, nonparticipants who have been employed for at least one month out of the last three have unobserved O states that indicate higher attachment to employment or unemployment (O1 or O2). Therefore, the previously observed heterogeneity among workers in the same current labor force status is well captured by the labor market attachment states in this paper.

5.2.1 Unobserved Heterogeneity

How do workers in the same current labor force status behave differently in the future within an HMM framework? Unlike the FOM, which assigns the same transition probability to all workers with the same current labor force status, the HMM's unobserved labor market attachment states allow heterogeneity in transition probabilities across the workers in the same current labor force status. In the FOM, the monthly job-finding probability for any unemployed worker is 23 percent. On the other hand, in the HMM, different unemployed and employed workers have vastly different job-finding probabilities. An unemployed worker in the U1 state is much more likely to be employed in the next month than his counterparts in the U3 or U2 states. While an unemployed worker in the U3 state has a 9.6 percent chance of finding a job in the next month, an unemployed worker in the most unemployment persistent state, U2, has merely a 3.8 percent chance of becoming employed. Similarly, while the job separation probability in the FOM is 1.3 percent for all employed workers, job separation probabilities among employed workers in the HMM differ depending on their unobserved states. If an employed worker is in E2, E3, or E1, the probability that she will lose her jobs is 6.4, 2.0, or .01 percent, respectively.

5.2.2 Duration Dependence

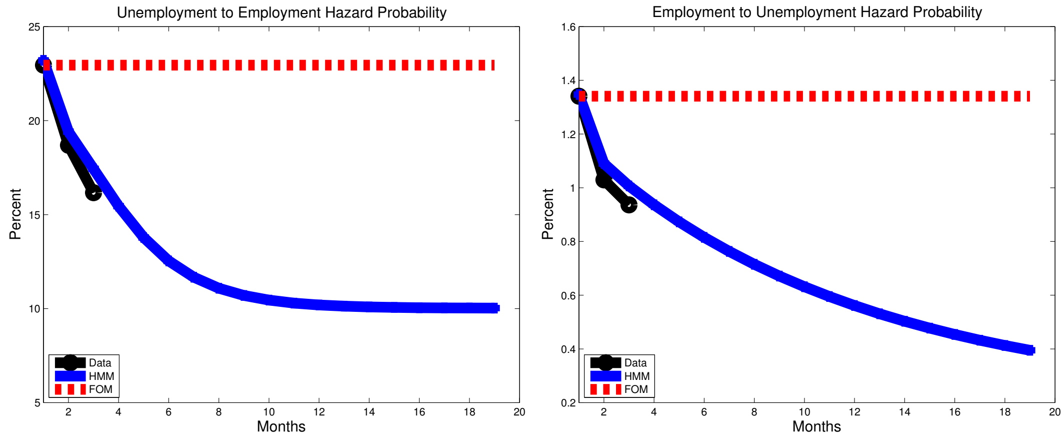
How does an HMM incorporate history dependence–the changes in transition probability of a given worker over the duration of the current labor force status? An HMM captures duration dependence through dynamic unobserved labor market attachment states. A worker who has been unemployed (employed) in the past is more likely to be in a more persistent unemployment-(employment-)prone state than a worker who recently became unemployed (employed). This allows a worker who has remained in the current state longer to have a lower exit probability. Figure 6 plots both the unemployment-to-employment exit probability and the employment-to-unemployment exit probability against the duration in the current state. Figure 6 shows the HMM estimates, observed data, and FOM estimates. The data show that the unemployment-to-employment exit probability, *i.e.* job-finding probability, declines in the duration of unemployment. However, the FOM with homogenous workers implies that the unemployment-to-employment transition probability is constant and independent of unemployment duration: $Pr(Y_t = E|Y_{t-1} = U) = Pr(Y_t = E|Y_{t-1} = U, \ldots, Y_{t-\tau} = U) = \lambda_{UU}$. The HMM fits fairly well the observed decline in unemployment (unemployment) exit probability over duration.

To calculate duration dependence, I use the Bayes rule to obtain the probability of each labor market attachment state given the sequence of employment history and the initial distribution P of attachment states. If a worker is unemployed, the probability that the worker is in the unobserved state s is expressed as:

$$Pr(X_t = s|Y_t = U) = \frac{p_s \psi_{jU}}{\sum_{i=1}^{S} p_s \psi_{iU}} \equiv \theta_j^{U_1} \tag{6}$$

More generally, if a worker is unemployed for τ months, then the probability of being in

Figure 6: Hazard Rate, Data, FOM, and HMM



state s is can be expressed as:

$$Pr(X_t = s | Y_t = U, \dots, Y_{t-\tau} = U) = \underbrace{\left(\frac{1}{P\hat{\Psi}_{\cdot U} \left(Q\hat{\Psi}_{\cdot U} \right)^{\tau-1} 1_S} \right)}_{1\times 1} \underbrace{\left(P\hat{\Psi}_{\cdot U} \left(Q\hat{\Psi}_{\cdot U} \right)^{\tau-1} \right)}_{1\times S} \theta_s^{U_\tau} \quad (7)$$

A similar expression can be obtained for any sequence of employment history $y = (y_t, y_{t-1}, \dots, y_{t-\tau})$. Note, however, that the assumption that the underlying labor market attachment state follows the first-order Markov process would imply that the initial conditional unobserved state probability $p_s^{Y_\tau}$ eventually converges to the steady state. As a result, the empirical hazard rate becomes flat after that point. However, the data also seem to suggest that there is a limiting exit probability as well. As long as we have enough time periods to estimate a HMM allowing for many states, this is not a strong restriction.

5.2.3 Distribution of E/U/O Durations by Labor Market Attachment States

What is the composition of labor market attachment states among workers who have experienced a continuous spell of employment, unemployment, or out of the labor force status? Figure 7 plots the distribution of E/U/O duration among workers with each labor market attachment state. Panel A shows the percent of employed workers who remain employed and the distribution of labor market attachment states. For example, 90 percent of workers employed in month 0 remain employed for five consecutive months. As the duration of these workers' employment increases, they shift from less–persistent employment state (E2, E3) to the most–persistent employment state (E1).[28]

[28]Note that unlike in models with fixed unobserved heterogeneity, here a worker's unobserved state changes

Similarly, Panel B shows the distribution of unemployment duration by labor market attachment state. Around 20 percent of unemployed workers in month 0 remain unemployed after five months, and initially unemployed workers' attachment states change over time. While the short-term unemployed workers have a mix of different persistent levels (U1, U2, and U3), as unemployment duration increases, workers who remain unemployed comprise increasingly those in the most persistent unemployment state. The unemployed workers who initially have a higher chance of moving to employment (U1) or out of the labor force (U2) either exit unemployment or transition to a more persistent unemployment state.

Panel C shows the distribution of out of the labor force duration by labor market attachment. Workers who are most persistently out of the labor force (O3) comprise approximately 70 percent of workers out of the labor force during the first month. In month 6, 80 percent of them remain out of the labor force. Among these workers, the fraction in the most persistent O-prone state workers (O3) increases to approximately 90 percent. The distribution of duration in each labor force status thus show that the workers whose status does not change comprise an increasing share of workers in the most persistent state associated with that labor force.

5.3 What are the Labor Market Attachment States?

What, then, do the estimated labor force attachment states capture? To understand the economic meanings of the unobserved labor market attachment states, this section relates the unobserved labor market attachment states to the observable data in the CPS. Specifically, I relate the unobserved labor force attachment states to an individual's earnings, detailed labor force type, reason for unemployment, and demographic characteristics. This analysis provides three major findings. First, the unobserved labor market attachment states are associated with observables. Second, the mapping between the unobservable labor market attachment states and observables is not one to one. There exists unobserved heterogeneity even among workers with the same observable characteristics. Third, the attachment states capture the persistent labor market dynamics and thus provide accurate predictions of a worker's future labor market outcome.

Table 6 presents average monthly earnings of workers in different employment-prone states after controlling for their education, experience, race, sex, and industry.[29] The average employed worker in the most persistent labor market attachment state (E1) earns $3,994

over time. Thus, the employed workers in the most persistent employment state in each month comprise not only those who have already been in the most persistent state in the previous month but also those who switched from less persistent employment state in the previous month to a more persistent employment state this month.

[29] I run a mincer regression controlling for worker's education, experience, race, sex and industries and rescale the residual wages to match up with the average earnings.

Figure 7: Distribution of E/U/O Duration by Workers of Each Labor Market Attachment

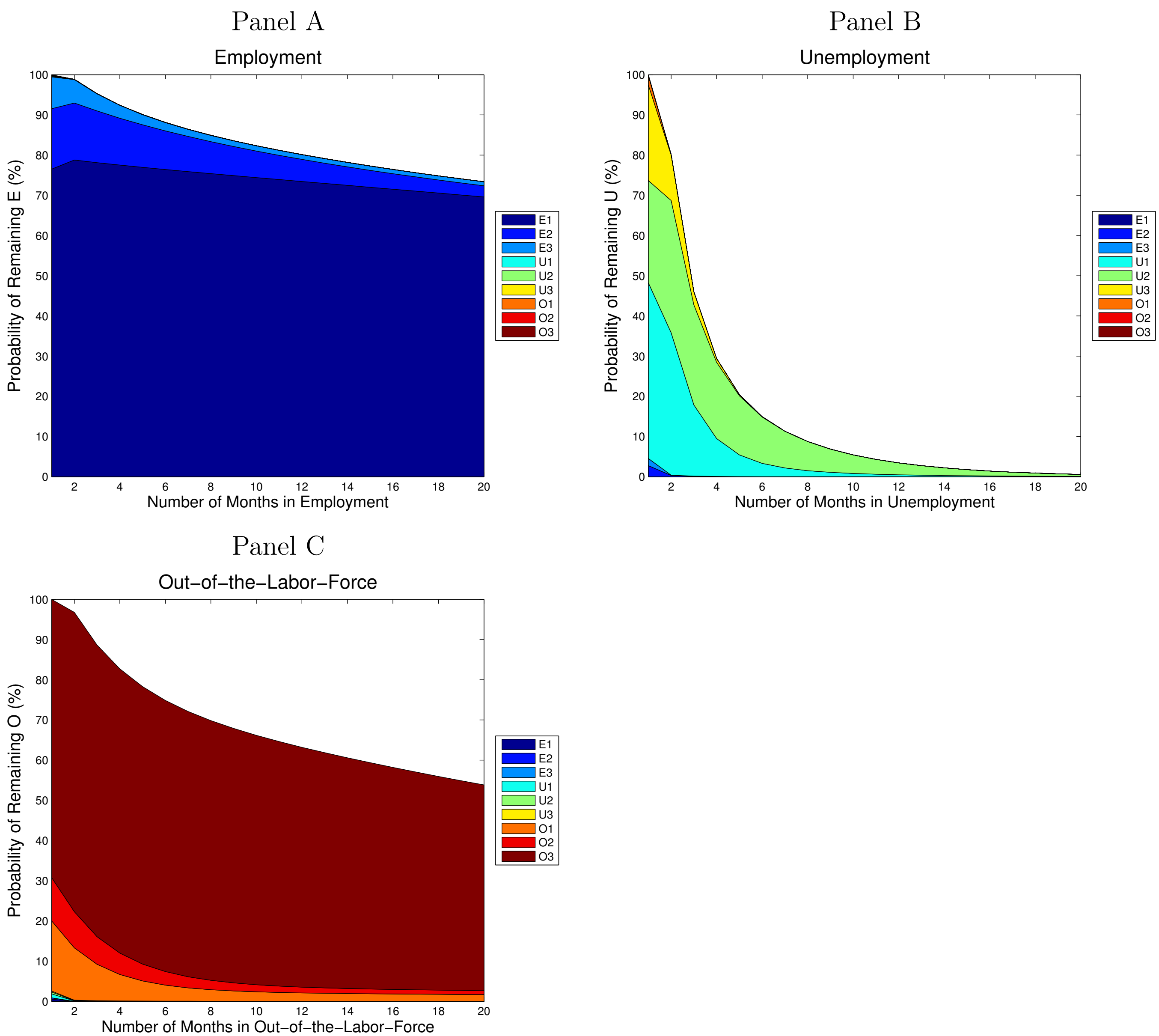


Figure 7 plots the distribution of duration in employment, unemployment, and out of the labor force by workers of each labor market attachment–the percent of remaining employed/unemployed/out of the labor force for t consecutive months and the compositions of the labor market attachment in month t. Panel A shows the distribution of duration by the employed of each labor market attachment. Panel B and Panel C show the same statistics for the unemployment and out of the labor force, respectively.

Table 6: Key Characteristics of Labor Market Attachment States *(Across)*

	Labor Market Attachment		
	E1	*E2*	*E3*
Average Monthly Earnings (2005 US$)	3,994	3,118	2,129
Employment Types	*E1*	*E2*	*E3*
Full Time	82.2	13.2	4.6
Part-Time for Economic Reasons	63.0	26.4	10.6
Part-Time for Non Economic Reasons	66.0	16.8	17.3
Share in E-prone (in Percent)	78.9	14.3	6.8
Unemployment by Reasons	*U1*	*U2*	*U3*
Temporary Layoff	73.3	18.4	8.3
Job Loser	49.5	39.6	10.9
Re-Entrant / New-Entrant	31.5	26.1	33.9
Share in U-prone (in Percent)	48.8	31.7	19.5
Out of the Labor Force	*O1*	*O2*	*O3*
Marginally Attached	29.1	39.7	31.2
Others	17.9	8.9	73.2
Share in O-Prone (in Percent)	18.4	9.4	72.2
Industry	*E1*	*E2*	*E3*
Construction	73.8	19.8	6.3
Government	84.9	10.9	6.9
Share in E-prone (in Percent)	78.9	14.5	6.9
Demographic Groups	*E1*	*U2*	*O3*
Men	61.0	1.8	10.1
College +	68.9	0.8	9.1
Married	59.6	1.0	16.3
Share in Population (in Percent)	54.1	1.5	16.6

per month, higher than workers in the E2 ($3,118) and E3 ($2,129) states. Thus, employed workers in the persistent employment-prone state enjoys not only higher job security but also higher earnings.

Table 6 shows the share of detailed observed types *across* labor market attachment states. In the Appendix, I also present the share of detailed observed types *within* labor market attachment states. Full-time workers are most closely associated with the most stable employment labor market attachment state (E1). While, on average, 79 percent of employed workers in E-prone states are in the E1 state, 82 percent of full-time workers are in E1 state. Part-time workers for economic reasons (involuntary part-time workers) tend to be in the E2 state and are thus more likely to move to unemployment, rather than leave the labor force, if they lose their job.[30] While 14 percent of all E-prone workers are in the E2 state, 26 percent of part-time workers for economic reasons belong to the E2 state. Lastly, part-time workers for non-economic reasons tend to be in the E3 state and are therefore more likely to move out of the labor force in the event of losing a job.[31] Seventeen percent of part-time workers for non-economic reasons are in the E3 state, versus 7 percent of all E-prone workers.

Unemployed workers with different reasons for separation also tend to have different labor market attachment states. Temporarily laid-off workers are more likely to be in the U1 state and thus more likely to move to employment in the future. This is consistent with the fact that unemployed workers on temporary layoff are much more likely to find a job (Figure 2). Job losers, on the other hand, are more likely to be associated with the most-persistent unemployment state (U2). Approximately 40 percent of job losers are in the U2 state, as opposed to 32 percent of all unemployed workers. Among unemployed workers in the U2 state, 44 percent are job losers (Table 17 in the Appendix).

The two distinct groups of non-participants in the Bureau of Labor Statistics (BLS) dataset, marginally attached and other non-participants, also show distinct patterns across labor market attachment states. Marginally attached workers are those who are available to work and, have searched for a job within the last 12 months, but not in the last 4 weeks. Barnichon and Figura (2013) show that marginally attached workers are more attached to the labor market than others. That is, they are more likely to enter the labor force. For marginally attached non-participants, the average monthly probability of transitioning into the labor force between 1994 and 2010 was 62 percent. The same probability was only 5 percent for other non-participants. However, conditional on entering the labor force, marginally attached workers are more likely to enter through unemployment, while the other non-participants are

[30]Involuntary part-time workers who wish to work full time but working full time due to slack economic conditions etc.

[31]Part-time workers for non-economic reasons desire to work part-time instead of full time, roughly translated as voluntary part-time workers.

more likely to move directly to employment.

The O-prone states capture this behavioral pattern. As shown above, workers in the O1 (O2) state are more likely to move to employment (unemployment) while workers in the O3 state are more likely to remain out of the labor force. Marginally attached workers are more likely to be in the O1 state (29 percent) or the O2 state (40 percent), while other non-participants are more likely to be in the most persistent O state (O3) (73 percent). If they are not in the most O-persistent state, marginally attached workers are more likely to be in O2 (39.7/(29.1+39.7)=58 percent), where the most common transition is to unemployment than other non-participants (8.9/(17.9+8.9)=33 percent). This pattern is consistent with the findings by Barnichon and Figura (2013).

Lastly, different industries and demographic groups also show some differences in labor market attachment states. For example, the construction industry has the largest share of workers in the least stable employment state (20 percent in E2), and government workers are more likely to be in the most stable employment state (85 percent in E1). Men are more likely than women to be in a more stable employment relationship (61 percent in E1), more likely to be in the most persistent unemployment state (1.8 percent in U2), and more likely to be attached to the labor force (10.1 percent in O3). College–educated workers are more likely to enjoy stable employment (69 percent in E1), less likely to suffer persistent unemployment (0.8 percent in U2), and less likely to stay out of the labor force for a long time (9.1 percent in O3). Married workers are also likely to be in stable employment (59.6 percent in E1) and less likely to suffer persistent unemployment (59.6 percent in U2).

This section has shown that my model's labor market attachment states are associated with observable characteristics and the patterns of association are in line with the findings of previous studies. However, there still exists unobserved heterogeneity among the workers with the same observed labor force status and other characteristics. This finding also suggests that this model's labor market attachment states provide more accurate measures than the FOM for understanding the conditions of individual workers, as well as the aggregate labor market.

6 Applications

In this section, to illustrate how the HMM can improve insight into labor market dynamics, I apply the HMM to study four important questions regarding the labor market. The first application considers HMM's improvement on the FOM in estimating the long-term consequences of separating from employment. The FOM is known to underestimate the loss in earnings and the reduction in future employment probabilities associated with separating from employment (*e.g.* Davis and von Wachter, 2011). I find that the HMM generates the reduction close to the level estimated using the U.S. Social Security data (Song and von Wachter, 2014).

In the second application, I study the evolution of employment stability for workers of different demographic groups over recent decades. The HMM can look beyond changes in the employment rate and study *employment stability*. I find that, while the average employment rate for men has declined, employment stability has not. While the average employment stability of women increased over time until the mid-1990s, only highly educated women experienced this improvement.

In the third application, I study the evolution of labor market attachment during the Great Recession of 2007-09 and the 1981 recession. While both recessions featured high unemployment rates, the labor market recovered much more slowly after the Great Recession. I find that a more sluggish recovery during the Great Recession is attributable to larger and more persistent inflows of unemployed workers in the more-persistent unemployment state (U2), employed workers in less stable employment states (E2), and nonparticipants more attached to unemployment (O2).

In the final application, I use a hypothetical situation to highlight the importance of unobserved heterogeneity. I consider two economies, "good" and "bad" economies with an identical distribution of current labor force status but with different distributions of labor market attachment. I show that the future labor market dynamics depend significantly on the distribution of underlying labor market attachments. While a "good" economy experiences a decline in unemployment, a "bad" economy experiences persistently high unemployment.

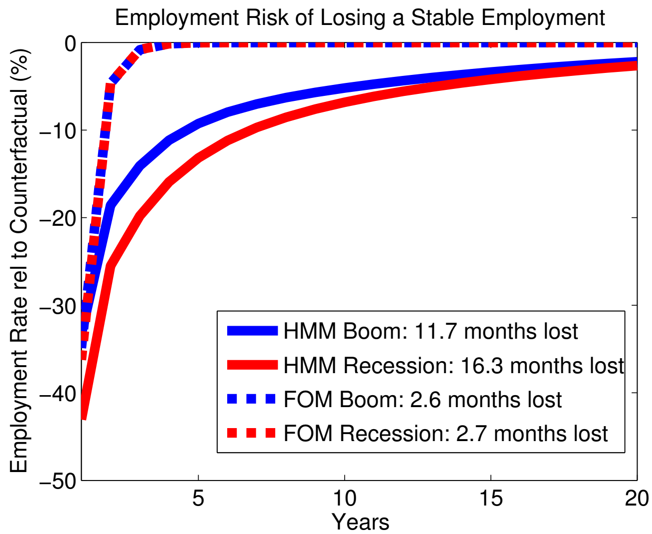
6.1 Separation from Stable Employment

Separation from stable employment results in a large and persistent reduction in employment probabilities relative to a control group who did not experience separation.[32] This reduction is more severe if separation happens during a recession than during a boom. Yet, the standard search model is known to underestimate the consequences from separating from employment (Davis and von Wachter, 2011). Song and Wachter (2014) use U.S. Social Security data to estimate the reductions in employment probabilities due a mass-layoff for men younger than 50 between 1980 and 2005. They find that the reduction in employment over the next 20 years is 1.33 years during an economic expansion and 1.48 years during a recession.

Figure 8 plots reductions in employment probabilities experienced by workers who sep-

[32] See for instance, Jacobson, Lalonde, and Sullivan (1993), von Wachters, Song, and Manchester (2011) and Davis and von Wachter (2011).

Figure 8: Employment Risk of Losing a Stable Employment



arated from stable employment relative to those who did not. I call this employment risk. First, while the FOM transition implies a short-lived reduction in employment after separation, the HMM implies a persistent reduction. Even after 20 years, workers who separated after three years of continuous employment still have a 5 percent lower employment probability than the control group who did not separate.

Second, it has been previously observed that the effect of separation from stable employment is larger if the separation occurs during a recession rather than a boom (Song and Wachter, 2014). This pattern is also observed in the HMM estimations. The employment risk relative to a control group is consistently greater for workers who become unemployed during a recession. Over the next 20 years, the cumulative loss in years of employment relative to a control group who did not experience a separation is 1.35 years in a recession and .97 years in a boom, which is close to the data. The FOM, on the other hand, implies that the cumulative loss in years in employment is only .23 years during recessions and 0.22 years during booms.

To make this exercise comparable to Song and von Wachter (2014) and Davis and von Wachter (2011), I first restrict the sample to male workers between 25 and 50 years old between 1980 to 2008 and re-estimate the HMM separately for the expansionary and recessionary periods. Davis and von Wachter (2011) restrict their sample to male workers 50 years and younger who experienced at least three consecutive years of employment prior to the job displacement. Thus, I limit my sample to male workers 25 years and older because these workers may have worked for at least three years before becoming unemployed. I define recession based on NBER recession periods and boom as a nonrecessionary period where the unemployment rate is below 6 percent. The three years of continuous employment provides information about the probability distribution of worker's labor force attachment states at the time of transition into unemployment. I use separate HMM estimates for periods of recession and boom in calculating the employment risk and compute the annual average of employment risk from the monthly series.

There are several differences between the employment risk estimated in Song and von Wachter (2014) and that estimated by the HMM. First, the frequency of the data is different. While employment is reported annually in the Social Security data, the HMM in this paper uses monthly CPS data. Second, the workers in the sample are different. In Song and von Wachter (2014), workers are restricted to men who had belonged to a large firm (50 or more employees) and experienced mass-layoff events. On the other hand, the sample here does not include these restrictions. In my sample, workers could have had job-to-job transitions as long as they were continuously employed for the last three years. Workers in my sample could also have been separated from employment for reasons other than a mass-layoff event. Despite the aforementioned differences, it is remarkable that the HMM, which uses just eight months of labor force transition history over 16 months, can accurately predict the persistent reduction in employment probabilities associated with separation from stable employment observed in the Social Security data.

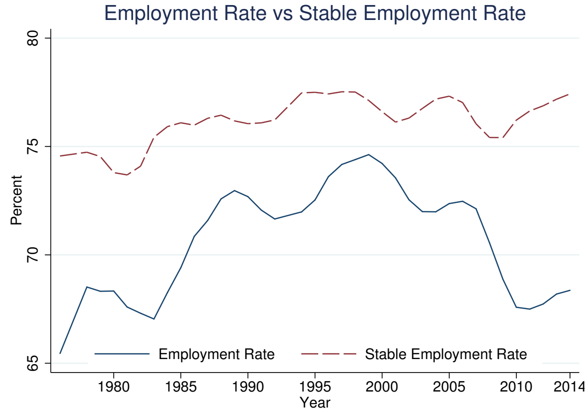
While Social Security has been the main data source for studies of long-term consequences of unemployment in the U.S., its demographic information is limited to age and gender. The CPS, however, contains more demographic information, such as education, but only contains 8 months of labor force history over a 16 month period. The extrapolative power of the HMM makes it possible to estimate employment risk over 20 years using only 16 months of labor force history. Thus, estimating the HMM using CPS data makes it possible to obtain the first estimates of long-run consequences of separating from stable employment for different education groups. I reestimate the HMM for the various sex $\times$ age $\times$ education groups and calculate the employment risk for those demographic groups over recent decades. Table 18 in the Appendix shows the results.

6.2 Employment Stability

In this section, I study how employment stability has evolved since the 1970s. While it is well known that the female labor force participation has increased over the past several decades, while that of men has declined, the employment stability trends are less well understood.[33] The HMM allows us to measure employment stability by looking at the percentage of

[33] Hall (1982), Farber (2004), Gottschalk, Moffitt (1999), and Jaeger and Stevens (1999) discuss the employment stability of workers in terms of job tenure at a particular employer. The job stability in my paper focuses on general employment stability allowing job-to-job transitions without going through unemployment. The stable-employment rate is calculated as follows. I use the stationary HMM estimates, Q, Ψ and

Figure 9: Employment Rate vs Stable Employment Rate (Three Year Moving Average)



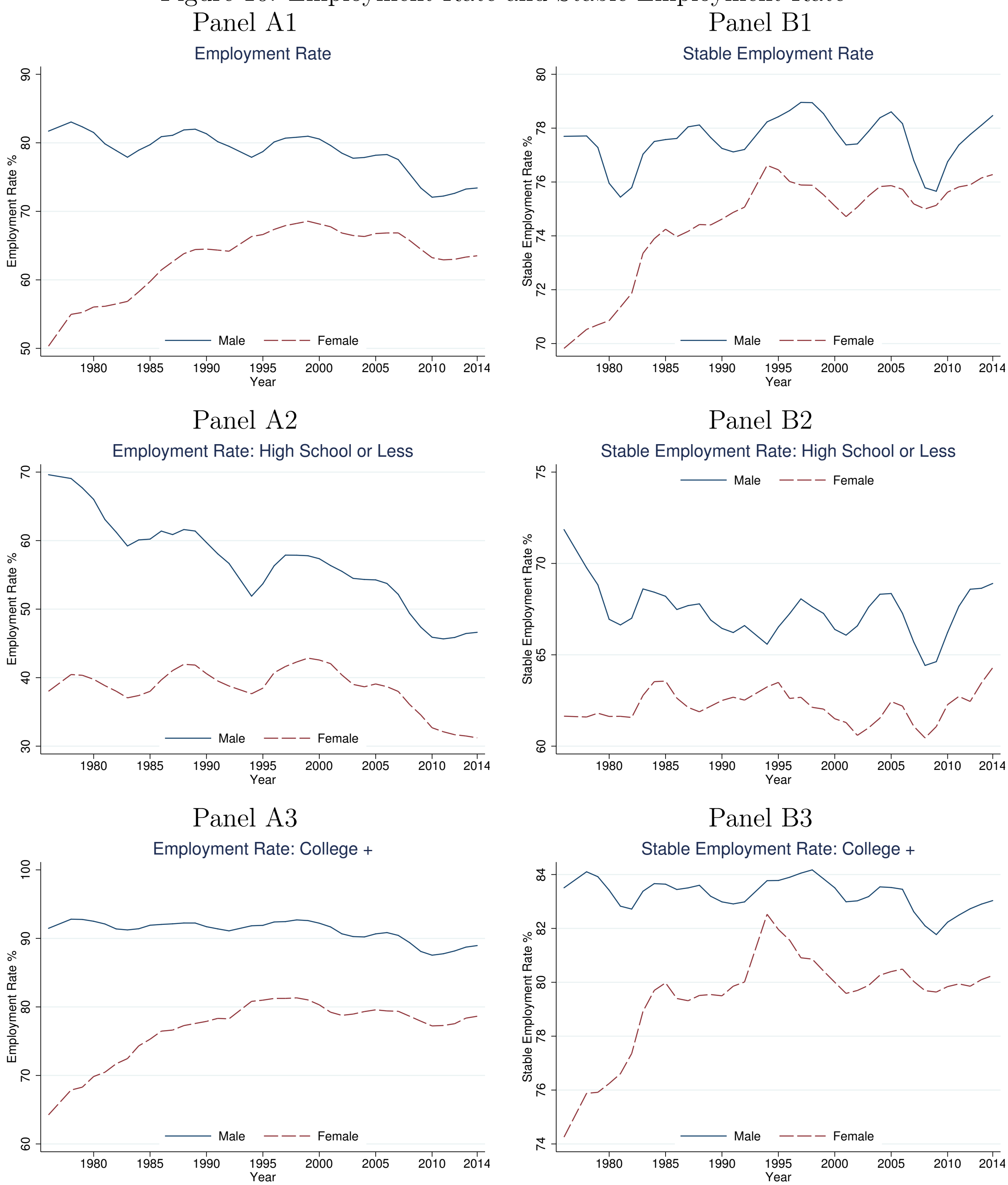
employed workers in the most persistent employment state (E1) over time. Figure 9 plots the employment rate and the stable employment rate between 1976 and 2014. While the aggregate employment rate increased from the 1970s until 2000 and then declined, the employment stability increased slightly until 1995 and has remained relatively flat since then. Even during the Great Recession, employment stability did not decline as sharply as the employment rate and recovered relatively quickly. In other words, while many workers lost employment during the Great Recession, those who remained employed did not suffer a decline in employment stability.

What has driven the changes in employment rate and employment stability since the 1970s? To answer this question, I study how the employment rate and employment stability have changed for different demographic groups. Figure 10 plots employment rates and employment stability for different demographic groups. Panel A1 in Figure 10 shows that the employment rate for men has steadily declined while the employment rate for women increased until the mid-1990s and has stayed relatively flat since then. On the other hand, the employment rate for women increased between the late 1970s and the mid-1990s and remained relatively constant since. In contrast, Panel B1 in Figure 10 plots employment stability by sex over the same period. While men's employment stability did not deteriorate very much, employment stability among women increased until the early 1990s and has not changed significantly since then.

However, when we look closely at employment stability by education, we see further variations in stability improvement. Panel A2 in Figure 10 plots the employment rates for men and women with a high school education or less. The employment rate for men with

P and apply Baye's rule in equation (7) to infer a worker's unobserved states in the first month given her eight-month employment history over 16 months. I then aggregate up to obtain the percent of employed workers in stable-employment rate.

Figure 10: Employment Rate and Stable-Employment Rate



this education level has steadily declined from 70 percent to 50 percent since 1976. The employment rate for women the same education level remained fairly constant at around 40 percent until 2005, and declined after the Great Recession.

Panel B2 in Figure 10 plots employment stability for men and women with a high school education or less. Unlike their employment rate, which has declined, employment stability for men with high school degree or less stayed fairly constant, at around 67 percent until the Great Recession. For women with this education level, on the other hand, employment stability has stayed fairly flat since the 1970s and has slightly increased after the Great Recession.

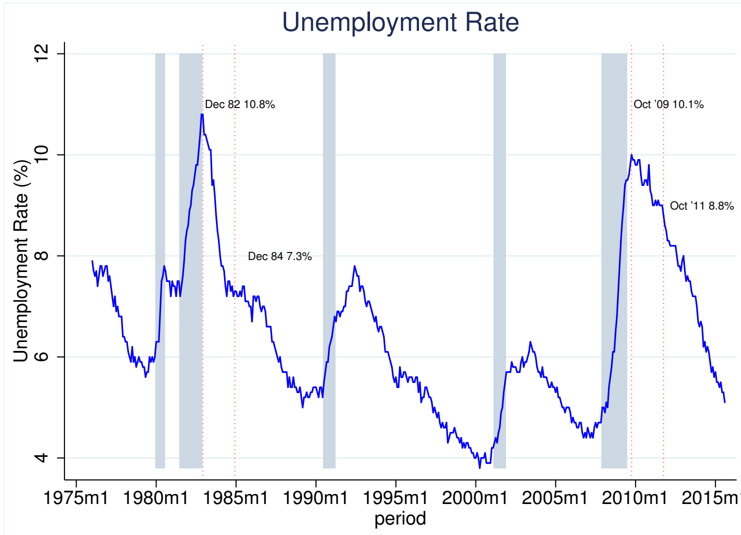
Different patterns emerge, however, for workers with a college degree. Panel A3 in Figure 10 plots employment rates for men and women with a college degree. Employment rates for college-educated men did not decrease as much as those of their high-school educated counterparts over the last 40 years. Their employment stability was also relatively constant throughout the period (Panel B3 in Figure 10). On the other hand, both the employment rate and employment stability for women with a college education steadily increased until the mid–1990s and has stayed relatively flat since then. Therefore, we find that the decline in employment rate for men was concentrated among men with less education while improvements in both the employment rate and employment stability among women were concentrated among those with more education. In the Appendix, I present a detailed analysis of how the employment rate and employment stability have changed over time across different age groups, following Davis and Haltiwanger (2014).

6.3 1981 Recession vs 2007 Recession

After both the 1981 and 2007 recessions, the U.S. unemployment rate climbed above 10 percent. However, the Great Recession in 2007 was followed by much more persistent unemployment. The unemployment rate, which peaked at 10.1 percent in October 2009, remained at 8.8 percent two years later. After the 1981 recession, on the other hand, the unemployment rate declined from 10.8 percent in December 1982 to 7.3 two years later (Figure 11). Were there more unemployed workers in a persistent attachment state during the Great Recession than during the 1981 recession? Were there are more employed workers at risk of losing a job? Were there more non-participants with a high attachment to unemployment?

To answer these questions, I study the evolution of labor market attachment state distribution during the 1981 and 2007 recessions. During a recession, not only do more workers tend to move to a more persistent unemployment state, but unemployed workers already in the most persistent unemployment state are also tend to remain there longer. To incorporate such changes in transition rates among labor market attachment states into my analysis, I

Figure 11: US Unemployment Rate



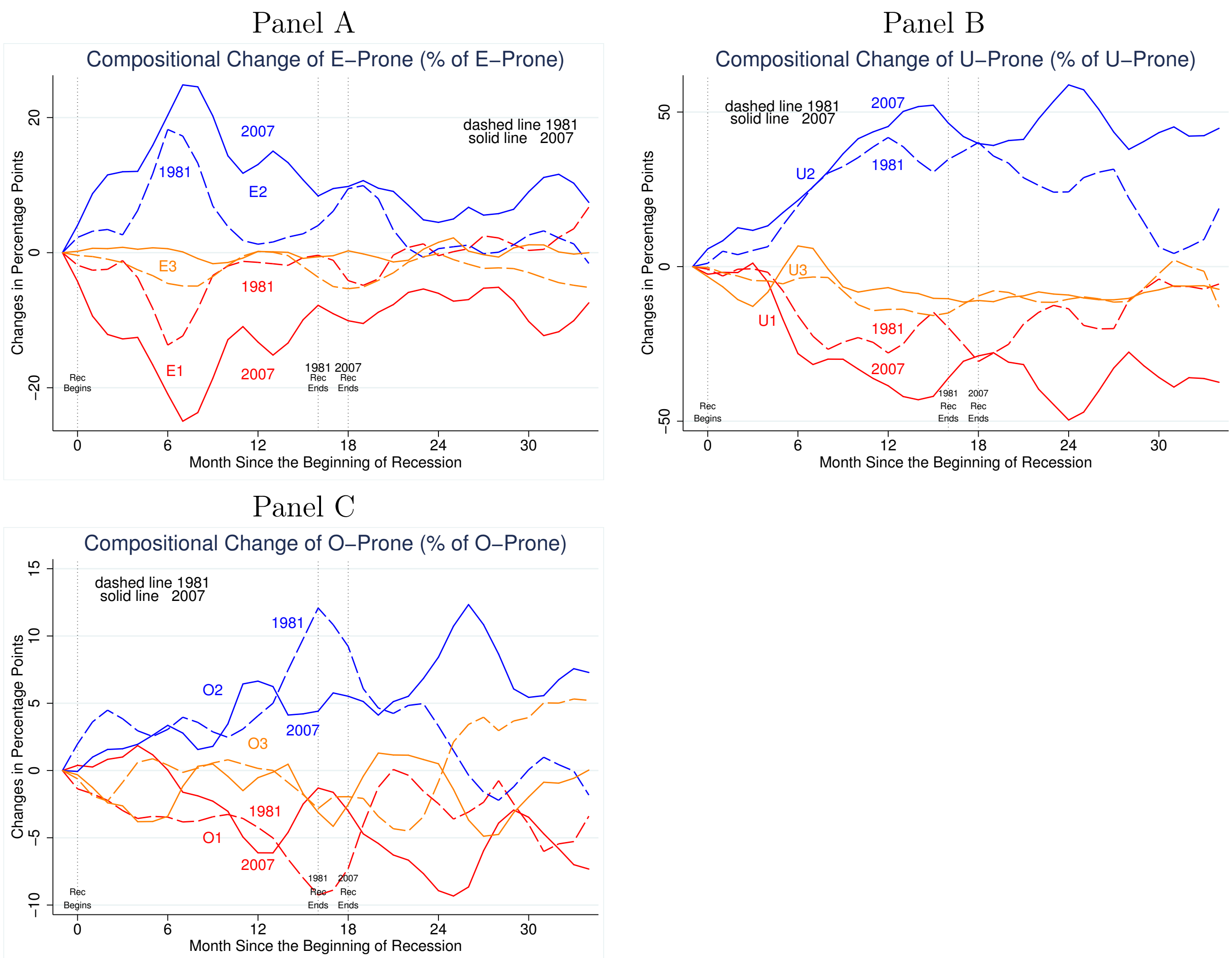
allow both the transition probabilities and initial distributions of labor market attachment states to change for each cohort-calendar month.[34] I re-estimate the HMM with the fixed observation matrix Ψ while allowing the initial distribution P_t and the transition matrix for the labor market attachment states Q_t to vary at each cohort-calendar month.

Figure 12 plots changes in the distribution of labor market attachment states relative to the beginning of each recession.[35] Panel A in Figure 12 plots the changes in composition for each employment-prone state. The two recessions showed similar patterns, but the changes were stronger and more persistent for the 2007 recession. In the first six months of each recession, the share of workers in the most-stable employment state (E1) decreased while the share in the less-stable employment state (E2) increased. While the percentage of workers in the most-stable employment state (E1) declined by 14 percentage points during the 1981 recession, the same number declined by 25 percentage points during the 2007 recession. Similarly, while the share of workers in the less-stable employment state (E2) increased by 18 percentage points during the 1981 recession, the same share increased by 25 percentage points during the 2007 recession. The 2007 recession's consequences for employment stability were thus more severe than those of the 1981 recession. The Great Recession entailed a slower recovery of employment stability as well. While the share of E1 returned almost to pre-recession levels within 12 months during the 1981 recession, it remained around 10

[34]Since each month corresponds to cohort entering that month, the initial distribution, P, and the transition probabilities, Q, are cohort-specific.

[35]Figure 17 in the Appendix plots the evolutions of E-prone,U-prone, and O-prone states around the recessions.

Figure 12: Comparison between the 1981 Recession and 2007 Recession



percentage points lower than the pre-recession level even after 30 months during the Great Recession.

Panel B in Figure 12 plots the changes in the composition of unemployment-prone states after the onset of each recession. Changes in the share of unemployment prone states during the two recessions were similar during their first six months, but diverged gradually thereafter. For example, the share of individuals in the most-persistent unemployment state (U2) increased by 40 percentage points during the first 12 months of both recessions. However, the share of U2 stayed high, at around 45 percentage points above pre-recession levels, during the 30 months after the beginning of the 2007 recession, while the same share gradually declined to around 5 percentage points above pre-recession levels during the same period after the 1981 recession. This indicates that the increase in the share of the most-persistent unemployment state (U2) was slightly greater and much more lasting during the Great Recession.

Panel C in Figure 12 plots the changes in the composition of out-of-the-labor-forcc-prone states after the onset of each recession. The compositional changes of O-prone states are not much different between the two recessions during the recession's first year. While the share of individuals in the out-of-the-labor-force state most attached to unemployment (O2) peaked in the 16th month of the 1981 recession, the same share peaked in the 26th month of the 2007 recessions. While this share decreased to its pre-recession level 2 years after the onset of the 1981 recession, it did not return to its pre-recession level even 30 months after the 2007 recession began. Thus, the compositional shift toward (O2) happened much later during the 2007 recession and was more persistent.

A close look at evolutions of labor market attachment states during the 1981 and 2007 recessions suggests that the compositional shifts toward states associated with future unemployment happened across all three labor force statuses during both periods, but the magnitudes and persistence of such shifts were greater during the Great Recession in 2007. This conclusion contrasts with that of Song and von Wachter (2014), which argues that the incidence and duration of long-term nonemployment during the Great Recession was comparable to those of previous recessions.

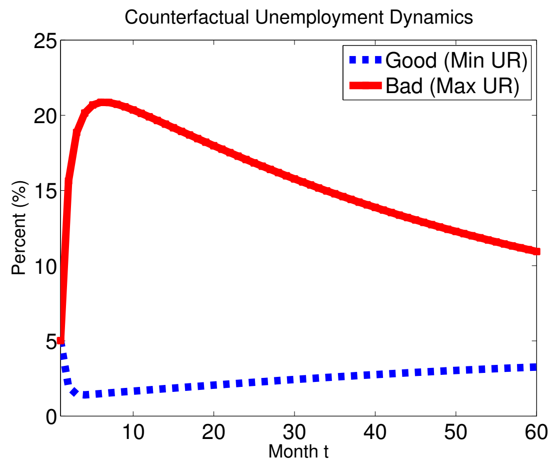
6.4 Beyond the Current Labor Force Statuses

One key implication of the HMM is that knowledge of worker's current labor force status is not sufficient to understand the labor market dynamics. To highlight this point through the lens of HMM, I conduct the following hypothetical exercise. Suppose there are two economies, "good" and "bad", identical in terms of their distributions of observable labor force status. In the first month, the unemployment rate is at 5 percent in both economies. The populations of both economies consist of 70 percent employed workers, 3.7 percent unemployed workers,

Table 7: Distribution of Labor Market Attachment

Distribution of Labor Market Attachment in Month *t*	*E1*	*E2*	*E3*	*U1*	*U2*	*U3*	*O1*	*O2*	*O3*
Good Economy	69.7	0	0	0	0	4.2	0	0	26.1
Bad Economy	0	71.3	0	0	3.2	0	0	25.5	0
Steady State	60.4	8.1	4.7	1.7	1.0	0.6	4.5	1.7	17.3

Figure 13: Counterfactual Unemployment Rate Dynamics



and 26.3 percent non-participants. However, the good economy is filled with workers who have high labor market attachment, whereas the bad economy is filled with workers who have low labor market attachment. Table 7 shows the distribution of workers by unobserved attachment state in the two economies. In the good economy, most employed workers are in the most-persistent employment-prone state (E1), most unemployed workers are in the unemployment-prone state that is most likely to move out of the labor force in the future (U3), and most non-participants are in the most-persistent O-prone state (O3). In contrast, the bad economy consists of employed workers who are in less-stable employment and are likely to become unemployed in the future (E2), unemployed workers who are in the most-persistent unemployment-prone state (U2), and non-participants who are more likely move to unemployment than employment (O2). How would the labor market dynamics of these two economies change over time?

Figure 13 plots unemployment rate dynamics in the two economies over 5 years, and the results are drastically different. In the bad economy, the unemployment rate spikes up from 5 percent to more than 20 percent and stays above 14 percent even after six years. On the other hand, the unemployment rate in the good economy goes down to 1.4 percent and then slowly increases.

This hypothetical exercise highlights the importance of looking beyond the distribution of current labor force statuses. The unobserved labor market attachment states used in this paper are critical for understanding labor market dynamics because the distribution of observed labor force status in an economy does not provide a full understanding of its labor market conditions.

7 Conclusion

This paper cast doubt on the long tradition of analyzing labor market dynamics assuming that observed labor force status follows the first-order Markov process. As a possible alternative model, I proposed a Hidden Markov Model that introduces *unobserved labor market attachment states* to capture unobserved heterogeneity for all three labor force statuses: employment, unemployment, and out of the labor force. The model in this paper captures empirically observed labor market dynamics over a 15-month horizon, which the FOM and its extensions cannot. By incorporating unobserved heterogeneity and duration dependence, the proposed model also allows for heterogeneous transition probabilities among workers in the same observed labor force status.

This alternative model suggests that the distributions of observed labor force status do not provide information sufficient to fully understand labor market dynamics. For instance, ignoring the distribution of workers among labor market attachment states can provide misleading predictions of future labor market outcomes. This proposed model facilitates more in-depth analysis of the labor market than the standard FOM. For instance, the HMM can replicate the empirical large and persistent reduction in employment probabilities associated with separating from stable employment. The HMM can also analyze employment stability in addition to simply employment rates. Thus, this paper suggests that we must reassess the standard empirical and theoretical models of labor market dynamics going forward.

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8 Appendix

8.1 CPS Matching Rates

This section documents how different criteria affect the success rates of matching observations in the Current Population Survey (CPS). Following Madrian and Lefgren (2000) and Shimer (2012), I use matching criteria based on (1) household and personal identifiers (ID), (2) sex, (3) race, and (4) age. Table 8 shows the percent of matched observations between month 0 and month t (Panel 8A) and for consecutive months from 0 through t (Panel 8B). From Column 1-4, one criterion is added to the previous list of criteria. For instance, Column 1 (ID) in Panel 8A reports the percent of observations matched based solely on the household and personal identifiers[36] between month 0 and t. On average, 92 percent of observations are matched between month 0 and month 1. Similarly, 78 percent of observations are matched between month 0 and a year later ($t = 12$). Column 2 (Sex) in Panel 8A imposes additional criterion that the sex of two observations in two different monthly files must also match. Column 3 in Panel 8A shows matching rates when identifier, sex, and race are used as matching criteria. Lastly, Column 4 and 5 in Panel 8A show corresponding matching rates when age allowing for +/- 2 year variations and age allowing for +1 age difference are added to the matching criteria, respectively. Panel 8B in Table 8 shows the percentage of observations that are matched from month 0 through t for different criteria. For instance, Column 4 (Age +/- 2) and Row 4 ($t \leq 12$) in Panel 8B shows that 65 percent of observations are matched for all the months $t = 1, 2, 3, 12$.

[36](the household identifier (HHID), personal line number (LINENO), and household number (HHNUM))

Table 8: Monthly Average % of Matched

Obs between/through month 0 and τ: 1976-2014					
Panel 8A: between month *0* and τ					
month\criterion	1. ID	2. Sex	3. Race	4. Age +/- 2	5. Age +1
$\tau = 1$	91.7	90.7	90.5	89.8	89.0
$\tau = 2$	89.1	87.0	86.8	86.7	86.0
$\tau = 3$	87.3	85.0	84.6	84.2	83.6
$\tau = 12$	77.9	75.3	74.3	69.7	66.3
$\tau = 13$	77.7	75.1	74.1	68.8	66.4
$\tau = 14$	77.1	74.1	73.0	67.8	65.7
$\tau = 15$	76.7	73.8	72.6	66.9	65.0
Panel 8B: all months through *0* to *t*					
month\criterion	1. ID	2. Sex	3. Race	4. Age +/- 2	5. Age +1
$\tau \leq 1$	91.7	90.7	90.5	89.8	89.0
$\tau \leq 2$	87.4	85.5	85.2	84.9	84.1
$\tau \leq 3$	83.8	81.4	81.2	80.8	80.2
$\tau \leq 12$	72.0	69.6	69.0	64.9	62.0
$\tau \leq 13$	69.3	67.1	66.5	62.3	59.4
$\tau \leq 14$	67.2	65.0	64.4	60.1	57.4
$\tau \leq 15$	65.4	63.3	62.7	58.2	55.6

8.2 Detailed Estimation Methods: EM Algorithm

This section shows the detailed algorithm to estimate the HMM.

1. E Step: Compute the conditional expectations of the missing data given the observations and the current estimates $\hat{\Lambda}_t$

2. M Step: Maximize the log likelihood replacing the missing data by their conditional expectations with respect to Λ.

Necessary Ingredients for an EM algorithm

For exposition purposes, if we only have one individual in the data, then the likelihood function would be $L(\Lambda|Y^l) = p_{x_1^l}\psi_{x_1^l}(y_1^l)q_{x_1^l x_2^l}\psi_{x_2^l}(y_2^l)\cdots q_{x_{T-1}^l x_T^l}\psi_{x_T^l}(y_T^l)$.

Forward Algorithm

I can instead calculate the above equation recursively as follows. Let $\alpha_t(s)$ be the probability of the observed sequence up to time t for $t = 1, 2, \ldots, T$, $s = 1, 2, \ldots, S$ with the unobserved state in t equal to s:

$$\alpha_t(s) = Pr(Y_1 = y_1, Y_2 = y_2, Y_3 = y_3, ., Y_t = y_t, X_t = s|\Lambda)$$

In the initial period, let $\alpha_1(i) = p_s\psi_s(y_1)$ for $s = 1, \ldots, S$ For $t = 2, \ldots T$ and $s = 1, \ldots, S$, we can compute $\alpha_t(s') = [\sum_{s=1}^{s} \alpha_{t-1}(s)q_{ss'}]\,\psi_{s'}(y_t)$ Then the probability of observed sequence given the parameters of the model Λ is $Pr(Y|\Lambda) = \sum_{s=1}^{S} \alpha_T(s)$. Furthermore, since we have the probability of observing the state for L individuals in the economy, the above equations become $\prod_{i=1}^{L} Pr(Y^l|\Lambda) = \prod_{l=1}^{L}\left(\sum_{s=1}^{S} \alpha_T(s)\right)$. I can then employ the backward algorithm to find the most likely hidden states given the parameters $\Lambda = (Q, \Psi, P)$ and observed sequence, Y.

Backward Algorithm

For $t = 1, \ldots, T$ and $s = 1, \ldots, S$, $\beta_t(s) = Pr(Y_{t+1}, Y_{t+2}, \ldots, Y_T|X_t = s, \Lambda) = \sum_{s'=1}^{N} q_{ss'}\psi_{s'}(y_{t+1})\beta_{t+1}(s')$ In period T, we let $\beta_T(s) = 1$ for $s = 1, \ldots, S$. For $t = 0, \ldots, T-1$ and $s = 1, \ldots, S$, we have $\beta_t(s) = \sum_{s'=1}^{N} a_{ss'}\psi_{s'}(y_{t+1})\beta_{t+1}(s')$. We now want to find the most likely hidden state. Define

$$\delta_t(s) \equiv Pr(x_t = s|Y, \Lambda) = \frac{\alpha_t(s)\beta_t(s)}{\sum_{i=1}^{S} \alpha_T(s)} = \frac{\alpha_t(s)\beta_t(s)}{P(Y|\Lambda)}$$

The most likely state is $s^* = arg\max_{i\in S} \delta_t(s)$ for $t = 1, \ldots, T$. (called local decoding). Finally, we want to estimate the parameter values $\Lambda = (Q, \Psi, P)$. Define $\delta_t(s, s')$ to be the probability of being in hidden state s in period t and s' in $t+1$, $Pr(X_t = s, X_{t+1} = s'|\Lambda)$ as follows: $\delta_t(s, s') == Pr(X_t = s, X_{t+1} = s'|Y, \Lambda) = \frac{\alpha_t(i)q_{ss'}\psi_{s'}(y_{t+1})\beta_{t+1}(s')}{\sum_{s=1}^{S} \alpha_T(s)}$. Summing up, we obtain

$$\begin{aligned}
\sum_{s'=1}^{S} \delta_t(s, s') &= \sum_{s'=1}^{S} \frac{\alpha_t(s)q_{ss'}b_{s'}(y_{t+1})\beta_{t+1}(s')}{\sum_{s=1}^{S} \alpha_T(s)} \\
&= \frac{\alpha_t(s)}{\sum_{s=1}^{S} \alpha_T(s)} \underbrace{\sum_{s'=1}^{S} q_{ss'}\psi_{s'}(y_{t+1})\beta_{t+1}(s')}_{\beta_t(s)} \\
&= \frac{\alpha_t(s)\beta_t(s)}{\sum_{s=1}^{S} \alpha_T(s)}
\end{aligned}$$

$$= \quad \delta_t(s)$$

EM algorithm

Therefore, we can estimate the $q_{ss'}$ and $\psi_{s'}(y_k)$ as follows for L individuals. For $s = 1, \ldots, S$,

$$p_s = \frac{\sum_{l=1}^{L} \delta_0^l(s)}{\sum_{i=1}^{S} \sum_{l=1}^{L} \delta_0^l(s)}$$

For $s = 1, \ldots, S$ and $s' = 1, \ldots, S$, we compute

$$q_{ss'} = \frac{\sum_{l=1}^{L} \sum_{t=1}^{T-1} \delta_t^l(s,s')}{\sum_{l=1}^{L} \sum_{t=1}^{T-1} \delta_t^l(s)} = \frac{\sum_{l=1}^{L} \sum_{t=1}^{T-1} \delta_t^l(s,s')}{\sum_{l=1}^{L} \sum_{t=1}^{T-1} \sum_{j=1}^{S} \delta_t^l(s,s')} \tag{8}$$

For $s = 1, \ldots, S$ and $k = 1, \ldots, M$

$$\psi_s(k) = \frac{\sum_{l=1}^{L} \sum_{t=1, Y_t = y_k}^{T-2} \delta_t^l(s)}{\sum_{l=1}^{L} \sum_{t=1}^{T-2} \delta_t^l(s)} \tag{9}$$

Iteration process can be as follows: (1) Initialize $\Lambda = (Q, \Psi, P)$. (2) Compute $\alpha_t^l(s)$, $\beta_t^l(s)$,$\delta_t^l(s,s')$, $\delta_t^l(s)$ for $l = 1, \ldots, L$. (2) Re-estimate the model $\Lambda = (Q, \Psi, P)$ using equations (8) and (9).

If $Pr(Y|\Lambda)$ increases, go to 2. The log-likelihood is given by:

$$\begin{aligned} \log L &= \log \left(\prod_{l=1}^{L} p_{x_1^l} \left(\prod_{t=2}^{T} q_{x_{t-1}^l, x_t^l} \prod_{t=1}^{T} \psi_{x_t^l}(y_t^l) \right) \right) \\ &= \sum_{l=1}^{L} \log(p_{x_1^l}) + \sum_{l=1}^{L} \sum_{t=2}^{T} \log q_{x_{t-1}^l, x_t^l} + \sum_{l=1}^{L} \sum_{t=2}^{T} \log \psi_{x_t^l}(y_t^l) \end{aligned}$$

$$\begin{array}{rccccc} u_s^l(t) = 1 & \text{iff} & x_t = s & \text{for } t = 1, 2, \ldots, T & \text{for } l = 1, 2, \ldots, L & \\ v_{ss'}^l(t) = 1 & \text{iff} & x_{t-1} = s, x_t = s' & \text{for } t = 2, 3, \ldots, T & \text{for } l = 1, 2, \ldots, L & \\ D_s^l(t) = 1 & \text{iff} & y^l(t) = s & \text{for i=1,...,M} & \text{for } l = 1, 2, \ldots L & \text{for } t = 1, 2, \ldots, T \end{array}$$

$$\log L \quad = \quad \sum_{l=1}^{L} \sum_{s=1}^{S} u_s^l(t) \log(p_s) + \sum_{l=1}^{L} \sum_{t=2}^{T} \sum_{s=1}^{S} \sum_{s'=1}^{S} v_{ss'}^l(t) \log q_{s,s'} + \sum_{l=1}^{L} \sum_{t=2}^{T} \sum_{s=1}^{S} \sum_{k=1}^{M} u_s^l(t) D_k^l(t) \log \psi_s(y_t^l)$$

E Step:

Since the unobserved states $X^{(T)}$ are not directly observable, we replace $v_{ss'}^l(t)$ and $u_s^l(t)$ with their conditional expectations given the observations $\mathbf{Y^{(T)l}} = (y_1^l, \ldots, y_T^l)$. $\hat{u}_s^l(t) = P(x_t = s|\mathbf{Y^{(T)l}}) = \frac{\alpha_t(s)\beta_t(s)}{L_T}$.

$\hat{v}^l_{ss'}(t) = P(x^l_{t-1} = s, x^l_t = s'|Y^{(T)l}) = \frac{\alpha_{t-1}(s)a_{ss'}b_k(y_t)\beta_t(s')}{L_T}$.

$$\log L = \underbrace{\sum_{l=1}^{L}\sum_{s=1}^{S}\hat{u}^l_s(1)\log(p_s)}_{F^1(P)} + \underbrace{\sum_{l=1}^{L}\sum_{t=2}^{T}\sum_{s=1}^{S}\sum_{s'=1}^{S}\hat{v}^l_{ss'}(t)\log q_{ss'}}_{F^2(Q)} + \underbrace{\sum_{l=1}^{L}\sum_{t=2}^{T}\sum_{s=1}^{S}\sum_{k=1}^{M}\hat{u}^l_s(t)D^l_k(t)\log\psi_s(y^l_t)}_{F^3(\Psi)}$$

M Step:

The log likelihood functions are additively separable in P, Q, Ψ $(F^1(P), F^2(Q), F^3(\Psi))$. Maximization with respect to P: $\max_\pi F^1(P)$ s.t. $\sum_{s=1}^{S} p_s = 1$ gives

$$p_s = \frac{\sum_{l=1}^{L}\hat{u}^l_s(t)}{\sum_{s=1}^{S}\sum_{l=1}^{L}\hat{u}^l_j(t)} = \frac{\sum_{l=1}^{L}\delta^l_0(s)}{\sum_{s=1}^{S}\sum_{l=1}^{L}\delta^l_0(s)}$$

Maximization with respect to Q: $\max_{Q=\{a_{ss'}:s,s'=1,\ldots,S\}} F^2(Q)$ s.t. $\sum_{s'=1}^{S} q_{ss'} = 1$ gives

$$q_{ss'} = \frac{\sum_{l=1}^{L}\sum_{t=2}^{T}\hat{v}^l_{ss'}(t)}{\sum_{s'=1}^{S}\sum_{l=1}^{L}\sum_{t=2}^{T}\hat{v}^l_{ss'}(t)} = \frac{\sum_{l=1}^{L}\sum_{t=1}^{T-1}\delta^l_t(s,s')}{\sum_{l=1}^{L}\sum_{t=1}^{T-1}\sum_{s'=1}^{S}\delta^l_t(s,s')}$$

Maximization with respect to Ψ: $\max_{\Psi=\{\psi_s(k):k=1,\ldots,M,s=1,\ldots,S\}}\sum_{l=1}^{L}\sum_{t=2}^{T}\sum_{s=1}^{S}\sum_{k=1}^{M}\hat{u}^l_s(t)D^l_k(t)\log\psi_s(y^l_t)$ s.t. $\sum_{k=1}^{M}\psi_s(k) = 1$ gives

$$\psi_s(k) = \frac{\sum_{l=1}^{L}\sum_{t=2}^{T}\hat{u}^l_s(t)D^l_k(t)}{\sum_{k=1}^{M}\sum_{l=1}^{L}\sum_{t=2}^{T}\hat{u}^l_s(t)D^l_k(t)} = \frac{\sum_{l=1}^{L}\sum_{t=1,}^{T-2}D(Y^l_t = k)\delta^l_t(s)}{\sum_{k=1}^{M}\sum_{l=1}^{L}\sum_{t=1}^{T-2}D(Y^l_t = k)\delta^l_t(s)}$$

8.3 Alternative Model with Unobserved Heterogeneity

In this Appendix, I consider several alternative models with unobserved heterogeneity. I consider (1) Finite Mixture Model and (2) First Order Markov Model with Fixed Unobserved Heterogeneity as in Browning and Carro (2014). I show that the HMM outperforms these alternative models in capturing the persistent labor market dynamics.

8.3.1 Fixed Mixture Model

A Fixed Mixture Model is nested by the HMM considered in this paper. I consider nine unobserved states, equal to the number considered by the HMM in this paper. This model is equal to a HMM with the restriction that $Q = I_S$. In this case, the model parameters would be of the initial distribution and the observation matrix $\underbrace{S-1}_{P} + \underbrace{S \times (M-1)}_{\Psi} = 26$.

The likelihood function for the Finite Mixture Model is given by:

$$\mathcal{L}(\Lambda|\{y^l\}_{l=1}^L) = \prod_{l=1}^{L} \mathcal{L}^l(\Lambda|y^l) = \prod_{l=1}^{L} P\hat{\Psi}_{\cdot y_0^l} \left[\prod_{\tau=1}^{3} \hat{\Psi}_{\cdot y_\tau^l}\right] I_S^8 \left[\prod_{\tau=12}^{T} \hat{\Psi}_{\cdot y_\tau^l}\right] \mathbf{1_S'}$$

The mean absolute deviation of the model is 4.08 percentage points.

8.3.2 First Order Markov model with Fixed Unobserved Heterogeneity

A First Order Markov model with three types of unobserved heterogeneity is equivalent to having $P = \begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix}$, $\Psi = \begin{bmatrix} I_M \\ I_M \\ I_M \end{bmatrix}$, $Q = \begin{bmatrix} Q_1 & 0_M & 0_M \\ 0_M & Q_2 & 0_M \\ 0_M & 0_M & Q_3 \end{bmatrix}$ where $P_s = (p_{s,E}, p_{s,U}, p_{s,O})$ is the probability that a worker is assigned to type s and employment status in the first period, I_M is a $M \times M$ identity matrix, Q_s is the transition matrix for type $s \in \{1,2,3\}$, 0_M is a $M \times M$ null matrix, and $p_{s,y} = Pr(Y = y, S = s)$. This corresponds to the FOM of observable labor force status with 3 unobserved types. The mean absolute deviation is 1.88 percentage points. In this case the number of parameters is given by $\underbrace{(S-1)}_{P} + \underbrace{(3 * (M-1))}_{Q_{s'}} * 3 = 26$ parameters. This model specification, however, does not admit the possibility that a given worker could be changing her transition probability over time (genuine duration dependence). Furthermore, Browning and Carro (2014) finds that without additional covariates of observables (*e.g.* demographic information) the model performs poorly for a long-term projection. HMM with $S = 6$ has lower mean absolute deviation 1.37 with 47 parameters. However, the effective number of parameters (non-zero parameters) is 36 because many entries are effectively zero (<0.0001).

Table 9: Weighted Mean Absolute Deviations

Hidden Markov Model			First Order Markov Model		
	MAD	# of Para	Model	MAD	# of Para
HMM3	2.04	14	FOMbase	9.48	6
HMM4	0.96	23	FOMmle	8.40	6
HMM5	0.72	34	FOMext	7.90	212
HMM6	0.37	47			
HMM7	0.25	62			
HMM8	0.24	79			
HMM9	0.21	98			
HMM10	0.19	119			

8.4 Weighted Mean Absolute Deviation

Table 9 shows the weighted mean absolute deviations. The HMM performs better here than when evaluated based on the simple mean absolute deviation. The formula to calculate this object is expressed as:

$$MAD_{HMM} = \sum_{y' \in \mathbb{Y}} \sum_{y \in \mathbb{Y}} \sum_{\tau \in \mathbb{T}} Pr(Y_0 = y) \times \mid Pr(Y_\tau = y' | Y_0 = y)_{HMM} - Pr(Y_\tau = y' | Y_0 = y)_{Data} \mid \tag{10}$$

Table 10: One Month Ahead Transition Probability FOM vs HMM
One Month Ahead Transition Probability

FOM		month 2		
		Employment	Unemployment	Out-of-the-Labor-Force
month 1	Employment	96.07	1.25	2.68
	Unemployment	23.35	53.93	22.72
	Out-of-the-Labor-Force	6.04	3.70	90.26
HMM		month 2		
		Employment	Unemployment	Out-of-the-Labor-Force
month 1	Employment	96.21	1.25	2.54
	Unemployment	24.05	53.62	22.33
	Out-of-the-Labor-Force	6.28	3.70	90.00

Table 11: Transition Probability One Year Later FOM vs HMM

$Pr(Y_{12} = y_{12} \mid Y_1 = y_1)$ for $y \in \{E, U, O\}$					
			y_{12} : Labor Force Status in month 12		
			Employed (E)	Unemployed (U)	Out of the Labor Force (O)
y_0: LFS in Month 0	Data	E	90.8	2.8	6.3
	FOM	E	73.9	3.9	22.2
	HMM	E	91.0	2.8	6.2
	Data	U	49.3	24.0	26.7
	FOM	U	60.6	4.9	34.6
	HMM	U	49.6	22.9	27.6
	Data	O	17.2	4.4	78.4
	FOM	O	50.5	5.6	45.0
	HMM	O	16.7	4.4	79.0

Table 12: Hybrid Model with More Observed States to Fit Transitions for Three Labor Force Status

Hidden Markov Model				
	Column A		Column B	
# of Observed	EUO	# of Para	FPUO	# of Para
HMM3	2.92	14	1.06	17
HMM4	2.10	23	0.70	27
HMM5	1.45	34	0.67	39
HMM6	1.37	47	0.44	53
HMM7	0.89	62	0.34	69
HMM8	0.54	79	0.32	87
HMM9	0.29	98	0.19	107
HMM10	0.26	119	0.17	129

Table 12 shows mean absolute deviations for transition probabilities between employment, unemployment, and out of the labor force. "FPUO" in Column B incorporates additional information on full-time and part-time workers in predicting transitions between employment, unemployment, and out of the labor force. It shows that incorporating additional observed states would also improve the fit.

Table 13: HMM Estimates P and Q

P: Initial Distribution of Labor Market Attachment	
E1	53.8
E2	10.8
E3	5.6
U1	2.5
U2	1.4
U3	1.3
O1	4.5
O2	2.7
O3	17.4

Q: Transition Matrix for Labor Market Attachment									
	E1	E2	E3	U1	U2	U3	O1	O2	O3
E1	99.5	0.1	0.4	0.0	0.0	0.0	0.0	0.0	0.0
E2	1.2	91.9	0.8	6.1	0.0	0.0	0.0	0.0	0.0
E3	4.5	1.4	66.5	0.2	0.0	0.0	27.3	0.0	0.0
U1	0.0	32.1	0.7	60.3	4.6	0.0	1.2	1.1	0.0
U2	0.0	0.0	0.0	5.3	90.5	0.0	0.0	4.2	0.0
U3	0.0	0.0	6.4	0.0	0.0	36.6	5.6	51.5	0.0
O1	0.0	0.0	26.6	1.8	0.0	1.1	65.2	0.2	5.1
O2	0.0	0.1	0.0	1.9	1.2	20.0	1.7	72.6	2.5
O3	0.0	0.0	0.0	0.0	0.0	0.0	1.1	0.5	98.4

Table 14: Boot Strap: Standard Deviation and 95 % Confidence Interval (Bootstrap)

P: Initial Distribution	
SD(P)	CR(95%)
0.4	[54.8,53.3]
0.4	[11.3,9.8]
0.1	[5.9,5.5]
0.1	[2.6,2.4]
0.1	[1.6,1.3]
0.2	[2.0,1.3]
0.1	[4.6,4.2]
0.4	[3.9,2.5]
0.5	[17.6,15.8]

Ψ: Observation Matrix					
SD(P)			CR(95%)		
0.0	0.0	0.0	[99.7,99.8]	[0.0,0.0]	[0.2,0.3]
0.2	0.2	0.1	[97.6,98.4]	[0.9,1.6]	[0.6,0.8]
0.1	0.1	0.0	[98.3,98.8]	[1.2,1.7]	[0.0,0.0]
0.5	0.5	0.3	[4.3,6.1]	[84.0,86.1]	[9.2,10.3]
0.2	0.8	0.8	[3.6,4.3]	[85.6,88.7]	[7.5,10.5]
0.9	9.5	10.4	[5.6,9.9]	[54.0,91.4]	[0.0,40.6]
0.0	0.3	0.3	[0.0,0.0]	[2.1,3.7]	[96.3,97.9]
0.0	0.5	0.5	[0.0,0.0]	[0.0,0.0]	[100.0,100.0]
0.0	0.0	0.0	[0.4,0.5]	[0.0,0.1]	[99.4,99.5]

Q: Transition Matrix: Standard Deviations								
0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4	0.7	0.1	0.4	0.0	0.0	0.0	0.0	0.0
0.1	0.2	0.5	0.1	0.0	0.1	0.4	0.0	0.0
0.0	1.6	0.3	2.0	0.4	0.0	0.2	0.2	0.0
0.0	0.0	0.0	0.4	0.4	0.0	0.0	0.2	0.0
0.0	0.0	0.5	0.0	0.0	6.6	1.4	5.2	0.0
0.0	0.0	0.5	0.1	0.0	0.2	0.7	0.6	0.3
0.0	0.1	0.0	0.2	0.2	2.4	0.5	2.9	0.7
0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.2

Q: Transition Matrix: CR(95%)								
[99.6,99.3]	[0.2,0.0]	[0.5,0.4]	[0.0,0.0]	[0.0,0.0]	[0.0,0.0]	[0.0,0.0]	[0.0,0.0]	[0.0,0.0]
[2.2,0.7]	[92.7,90.2]	[0.9,0.6]	[7.0,5.6]	[0.0,0.0]	[0.0,0.0]	[0.0,0.0]	[0.0,0.0]	[0.0,0.0]
[4.9,4.3]	[1.8,1.0]	[67.5,65.4]	[0.4,0.0]	[0.0,0.0]	[0.2,0.0]	[28.1,26.4]	[0.0,0.0]	[0.0,0.0]
[0.0,0.0]	[35.5,29.3]	[1.4,0.2]	[63.6,56.1]	[5.4,3.9]	[0.0,0.0]	[1.7,0.7]	[1.5,0.7]	[0.0,0.0]
[0.0,0.0]	[0.0,0.0]	[0.0,0.0]	[6.1,4.4]	[91.6,89.7]	[0.0,0.0]	[0.0,0.0]	[4.5,3.9]	[0.0,0.0]
[0.0,0.0]	[0.0,0.0]	[7.2,5.1]	[0.0,0.0]	[0.0,0.0]	[61.2,33.4]	[7.0,0.4]	[54.2,32.9]	[0.0,0.0]
[0.0,0.0]	[0.0,0.0]	[28.0,25.9]	[1.9,1.6]	[0.0,0.0]	[1.4,0.4]	[66.2,63.7]	[1.9,0.0]	[5.3,4.3]
[0.0,0.0]	[0.5,0.0]	[0.0,0.0]	[2.2,1.5]	[1.4,0.6]	[21.3,11.3]	[3.4,1.1]	[82.6,71.0]	[2.9,0.0]
[0.0,0.0]	[0.0,0.0]	[0.0,0.0]	[0.0,0.0]	[0.0,0.0]	[0.0,0.0]	[1.2,0.8]	[0.5,0.1]	[99.1,98.4]

Table 15: Monte Carlo Exercise: P and Ψ

Initial Distribution: P		
Bias $(E(P)-P)$	$SD(P)$	CR(95%)
-2.6	1.6	[59.7 , 53.6]
1.4	1.5	[11.9 , 6.5]
1.3	0.6	[5.7 , 3.7]
0.3	0.3	[2.6 , 1.7]
-0.2	0.2	[2.1 , 1.3]
-0.1	0.2	[1.9 , 1.2]
0.9	0.5	[4.5 , 2.8]
0.1	0.3	[3.4 , 2.0]
-1.0	0.5	[19.3 , 17.3]

Observation Matrix: Ψ								
Bias $(E(\Psi)-\Psi)$			$SD(\Psi)$			CR(95%)		
0	0	0	0	0	0	[99.7,99.7]	[0,0.1]	[0.3,0.3]
0.2	-0.1	0	0.3	0.3	0.1	[97.1,98.4]	[0.9,1.9]	[0.6,0.9]
0.4	-0.1	-0.2	0.5	0.1	0.5	[96.5,98.6]	[1.4,2]	[0,1.7]
0.2	0	-0.2	0.5	0.5	0.2	[4.3,6.2]	[83.7,85.9]	[9.5,10.3]
0	0.1	0	0.1	0.3	0.2	[3.7,4.2]	[87.3,88.4]	[7.8,8.6]
0.4	2.4	-2.9	0.9	8.3	9.1	[6.5,9.8]	[66.5,91.7]	[0,26.5]
-0.3	-0.3	0.6	0.6	0.3	0.7	[0,2.1]	[2.3,3.3]	[94.9,97.7]
-0.2	-1.6	1.8	0.2	2	2.2	[0,0.7]	[0,6.2]	[93,100]
0	0	0	0	0	0	[0.4,0.5]	[0.1,0.2]	[99.4,99.5]

To see how accurately the MLE with an EM algorithm can estimate the true parameter values, I present Monte Carlo results for the HMM with 9 unobserved states. I assume that the HMM estimates in the paper are the true data-generating process. I simulate 500 sample sets with size equal to the number of observations in the data: N=2,420,463. I estimate the HMM model using the MLE with an EM algorithm and calculate the summary statistics of the estimates from the simulated data. Table 15 presents the bias and the standard deviation (SD), and 95 percent confidence intervals (CR(95%)).

Table 16: Monte Carlo Exercise: Q

Q: Transition Matrix																		
	Bias ($E(Q)-Q$) (in %)									$SD(Q)$ (in %)								
States	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1	0.0	0.1	0.3	-0.1	0.0	0.0	-0.2	0.0	0.0	0.1	0.0	0.2	0.1	0.0	0.0	0.1	0.0	0.0
2	0.4	0.3	0.5	-0.4	-0.2	-0.1	-0.5	0.0	0.0	0.7	0.7	0.3	1.1	0.4	0.1	0.3	0.0	0.0
3	2.9	0.9	-0.3	0.2	0.0	0.0	-0.5	-0.9	-2.3	2.0	0.6	0.7	0.1	0.0	0.1	2.9	0.7	1.9
4	-3.2	0.4	0.6	0.9	0.7	-0.2	0.5	0.7	-0.4	3.4	4.4	0.3	1.4	1.8	0.3	0.4	0.5	0.3
5	-0.1	-1.4	0.0	1.8	-0.2	-0.2	0.0	0.3	-0.1	0.2	2.0	0.0	2.3	0.4	0.7	0.1	0.7	0.2
6	-0.8	-1.6	0.9	-0.6	-0.2	-1.4	3.1	4.2	-3.7	0.6	1.4	0.7	1.0	0.7	4.7	2.1	6.0	2.7
7	-2.9	-0.9	0.9	0.1	-0.1	0.5	-0.1	0.2	2.2	1.8	0.6	2.8	0.2	0.1	0.6	0.8	0.1	2.1
8	-0.1	0.0	-1.3	0.7	-0.1	-0.1	0.6	-1.0	1.4	0.2	0.1	1.1	0.9	0.4	2.7	0.6	2.0	1.2
9	0.0	0.0	-0.4	-0.1	0.0	-0.3	0.7	0.3	-0.2	0.1	0.0	0.3	0.0	0.0	0.2	0.5	0.2	0.1

Q: Transition Matrix CR(95%) (in %)									
	1	2	3	4	5	6	7	8	9
1	[99.6 , 99.4]	[0.1 , 0.0]	[0.4 , 0.0]	[0.2 , 0.0]	[0.0 , 0.0]	[0.0 , 0.0]	[0.4 , 0.0]	[0.0 , 0.0]	[0.1 , 0.0]
2	[2.2 , 0.0]	[92.7 , 90.1]	[0.9 , 0.0]	[8.7 , 4.7]	[1.2 , 0.0]	[0.3 , 0.0]	[0.9 , 0.0]	[0.1 , 0.0]	[0.1 , 0.0]
3	[5.1 , 0.0]	[1.7 , 0.0]	[68.7 , 65.8]	[0.3 , 0.0]	[0.1 , 0.0]	[0.2 , 0.0]	[33.3 , 22.3]	[2.2 , 0.0]	[5.3 , 0.0]
4	[9.0 , 0.0]	[40.9 , 24.3]	[1.0 , 0.0]	[61.9 , 56.4]	[5.7 , 0.0]	[1.0 , 0.0]	[1.5 , 0.1]	[1.5 , 0.0]	[0.9 , 0.0]
5	[0.8 , 0.0]	[5.1 , 0.0]	[0.1 , 0.0]	[5.7 , 0.0]	[91.5 , 90.0]	[2.4 , 0.0]	[0.3 , 0.0]	[4.6 , 1.8]	[0.6 , 0.0]
6	[2.1 , 0.0]	[4.5 , 0.0]	[7.0 , 4.3]	[2.8 , 0.0]	[3.2 , 0.0]	[48.0 , 33.3]	[6.4 , 0.0]	[58.5 , 36.4]	[7.5 , 0.0]
7	[5.1 , 0.0]	[2.0 , 0.0]	[32.3 , 21.5]	[1.9 , 1.3]	[0.4 , 0.0]	[1.6 , 0.0]	[67.4 , 64.3]	[0.3 , 0.0]	[5.4 , 0.0]
8	[0.6 , 0.0]	[0.4 , 0.0]	[3.1 , 0.0]	[2.6 , 0.0]	[1.9 , 0.0]	[25.2 , 15.1]	[2.2 , 0.0]	[78.2 , 70.5]	[3.2 , 0.0]
9	[0.1 , 0.0]	[0.1 , 0.0]	[0.8 , 0.0]	[0.1 , 0.0]	[0.0 , 0.0]	[0.5 , 0.0]	[1.1 , 0.0]	[0.6 , 0.0]	[98.7 , 98.4]

To see how accurately the MLE with an EM algorithm can estimate the true parameter values, I present Monte Carlo results for the HMM with 9 unobserved states. I assume that the HMM estimates in the paper are the true data generating process. I simulate 500 sample sets with size equal to the number of observations in the data: N=2,420,463. I estimate the HMM model using the MLE with an EM algorithm and calculate the summary statistics of the estimates from the simulated data. Table 16 presents the bias, the standard deviation (SD), and 95 percent confidence intervals (CR(95%)).

8.5 Expected Number of Years in Employment Lost from Separating from Stable Employment

Column A in Table 18 shows expected number of years in employment over the next 20 years if a worker remains employed, or becomes unemployed this month after three consecutive years of employment. Column B in Table 18 shows the expected number of years in employment lost by becoming unemployed relative to the control group who stayed employed (Column B). We first observe that across education groups, prime-aged workers (aged 25-39 and 40-54) have the longest expected number of years in employment in the next twenty years if they were employed for the past three consecutive years. Across different age groups, more-educated workers enjoy more-stable employment in the future than less educated workers. For example, if a male, 40-54–year-old college-educated worker has been employed for the last three years and remains employed this month, the worker is expected to be employed for 19 years out of the next 20. A male 40-54-year-old worker with a high school degree or less is expected to be employed for less than 17 years over the next 20. The same pattern holds for women as well. In the 1970s, while a 40-54–year–old college–educated female worker is expected to be employed for 19 years, a 40-54–year–old female worker with a high school degree or less is expected to be employed for less than 16 years over the next 20. Moreover, young workers with a high school education or less have suffered a decline in the expected number of years in employment. In the 1980s, if a 25-39-year-old male worker with high school education or less was expected to be employed for 18 years, but this number declined to 17 years in the 2000s. However, in general, both male and female workers in most age and education groups did not see large changes in the expected number of years in employment for the next 20 years over the past three decades.

We now look at the expected number of years in employment lost due to separating from stable employment (Column B). We find that the number of years in employment lost is smaller for younger workers than older workers across educational groups. For instance, in the 1980s, while a 25-39–year–old man with high school degree or less suffers one fewer year employment relative to the control group who did not experience unemployment, a 40-54 year old man with the same education level would suffer 2 years of loss in employment over the next 20 in the event of separation. In addition, the expected number of years in employment lost after separating from a stable employment is generally smaller for workers with more education. For example, in the 2000s, for a female worker between 25 and 39 years old, the expected loss in employment due to separation is 2.7 years if she has a high school degree or less. For a female worker with some college, this number is 1.8 years. If she has a college degree, it is 1.4 years. Thus, this section showed the new estimates of expected years in

Table 17: Within Labor Market Attachment Statistics

Key Characteristics of Labor Market Attachment States *(Within)*

	Labor Market Attachment			
Employment Types	*E1*	*E2*	*E3*	Share in E
Full Time	85.1	76.3	56.4	81.8
Part-Time for Economic Reasons	2.7	6.4	5.4	3.4
Part-Time for Non Economic Reasons	12.2	17.3	38.2	14.8

Unemployment by Reasons	*U1*	*U2*	*U3*	Share in U
Temporary Layoff	19.3	7.4	5.2	13.2
Job Loser	36.6	44.4	19.3	34.2
Re-Entrant / New-Entrant	24.6	30.8	63.0	10.5

Out of the Labor Force	*O1*	*O2*	*O3*	Share in O
Marginally Attached	15.0	24.4	11.4	2.1
Others	85.0	75.6	88.6	97.9

employment for the next twenty years by sex $\times$ age $\times$ education groups. We find that the more–educated prime-age workers enjoy stable employment, and the expected number of years lost in employment due to separation is also generally greater for less–educated older workers.

Table 18: Expected Years of Employment in Next 20 Years After 3 Years of Continuous Employment

			Column A Expected # of Years in Employment in next 20 Years after 3 Years of Continuous Employment						Column B Expected # of Years in Employment Lost in Next 20 Years		
			If Stayed Employed			If Became Unemployed					
Sex	Education	Age	1980s	1990s	2000s	1980s	1990s	2000s	1980s	1990s	2000s
Men	High School and Less	16-24	16.1	15.9	14.5	15.1	15.2	13.3	.98	.73	1.24
		25-39	18.2	17.9	16.9	17.2	16	15	1.02	1.86	1.84
		40-54	16.7	16.7	16.4	14.6	14.6	13.1	2.11	2.15	3.33
		55-64	8.7	10.9	10.4	6.3	7.3	7.4	2.45	3.59	3.05
	Some College	16-24	16.5	17.2	16.3	15.6	16.3	14.8	.91	.88	1.48
		25-39	18.9	18.6	18.4	17.8	17.5	16.8	1.11	1.11	1.61
		40-54	17.8	17.9	16.8	14.6	16.2	14.9	3.25	1.64	1.95
		55-64	10	11.6	12.8	6	9	8.2	4.05	2.57	4.64
	College and more	16-24	18.2	18.4	17	17.5	17.5	14.6	.79	.86	2.32
		25-39	19.4	19.3	18.9	18.5	18.6	18	.85	.7	.88
		40-54	18.9	18.9	18.5	17.8	17.7	16.7	1.07	1.17	1.81
		55-64	11.7	12.7	13.6	7.5	10.2	10.1	4.22	2.51	3.47
Women	High School and Less	16-24	14.9	14.4	12.9	13.8	13.6	11.9	1.11	.81	1.03
		25-39	15.7	17.2	15.5	13.6	14.1	12.8	2.06	3.1	2.72
		40-54	15.5	16	17.8	13	13	12.9	2.5	3.02	4.91
		55-64	11.7	9.8	10.3	6.4	6.8	7.3	5.33	3.04	2.98
	Some College	16-24	16.3	17.5	17.3	15.4	16.6	15.6	.93	.93	1.65
		25-39	16.7	16.8	16.1	14.8	15.3	14.2	1.97	1.57	1.83
		40-54	17.1	16.8	16.6	14.8	15.2	14.3	2.29	1.59	2.31
		55-64	10.3	11.5	13.5	7	8.5	9	3.33	3.01	4.58
	College and more	16-24	18.5	19.1	14.6	17.3	18.1	12.6	1.17	1.06	2.01
		25-39	17.4	17.2	16.7	15.9	16	15.3	1.48	1.15	1.4
		40-54	19.2	18.2	17.8	17.2	17	16.3	2.06	1.29	1.53
		55-64	11.7	14.3	12.9	8.2	5.9	9.4	3.49	8.33	3.48

8.6 Detailed Employment Stability Analysis

Davis and Haltiwanger (2014) look at how employment rates have changed over the last several decades for different demographic groups.[37] Their analysis answers how the incidence of employment has changed but does not answer whether or not employed workers are experiencing more-stable employment. In this section, I apply the HMM estimates to answer the second question. Specifically, I document how stable-employment rates of employed workers—percent of employed workers in the most stable employment state (E1)—have evolved during the last several decades across different demographic groups.[38] I find that while employment rates for men have declined and employment rates for women have increased over time, changes in employment stability are rather small.

I first review incidence of employment for different demographic groups. I follow the analysis of Davis and Haltiwanger (2014) and focus on the three year average for the following periods: 1977-1979, 1987-1989, 1998-2000, and 2012-2014.[39] Panel A in Figure 10 plots the employment rates of men by age and education groups over different time periods. Employment rates are higher for more educated workers across age groups. Employment rates have an inverse U shape over age with the employment rates being higher for prime age workers (between 25 to 50 years old) than for younger (less than 25 years old) and older (50 + years old) workers across all education group. Since the late 1970s, the employment rates for men fell across all education groups. The decline in employment rates in the past several decades is more pronounced for older men with less education. On the other hand, the employment rate did not fall much for men with college or more education. For example, between 1977-1979 and 1987-1989, the average employment rate of men with less than a high school education has declined from 82 percent to 74 percent while the employment rate for men with college degrees has declined from 92 percent to 90 percent.[40]

[37]See, for instance, Hall (1982), Farber (2004), Gottschalk, Moffitt (1999), and Jaeger and Stevens (1999) discuss the employment stability of workers in terms of job tenure at a particular employer. The job stability in my paper focuses on general employment stability allowing job-to-job transitions without going through unemployment.

[38]The stable-employment rates are calculated by applying the HMM estimates to the eight month labor force status history for workers during the different time periods studied here.

[39]Note that aggregate employment rates were similar for these periods (with slightly lower employment rate in 2009-2011). As a robustness check, instead of dividing the periods into 1977-1979, 1987-1989, 1998-2000, and 2012-2014, I have also divided periods into the 1970s, the 1980s, the 1990s, and the 2000s, and I still observe similar patterns in age profile of the employment rates by education groups of men and women. Since I limit the sample to the population that for whom I observe complete 8 months of labor force status, there are more noises for each age bin than in Davis and Haltiwanger (2014) who use all eight rotational groups. I smooth the noise in the data by calculate the average workers of of workers +/- 2 years of age. For example, employment rate at age 30 in the figure means the average employment rate for workers between 28 and 32 years old. Thus, the interpretation of employment rate at each age is the average of +/- 2 age group.

[40]For average employment rate, I calculate the average employment for workers older than 22 years old. The purpose of this is to make the number comparable across different education groups because most college

Figure 14: Employment Rate and Stable-Employment Rate

Panel A Panel B

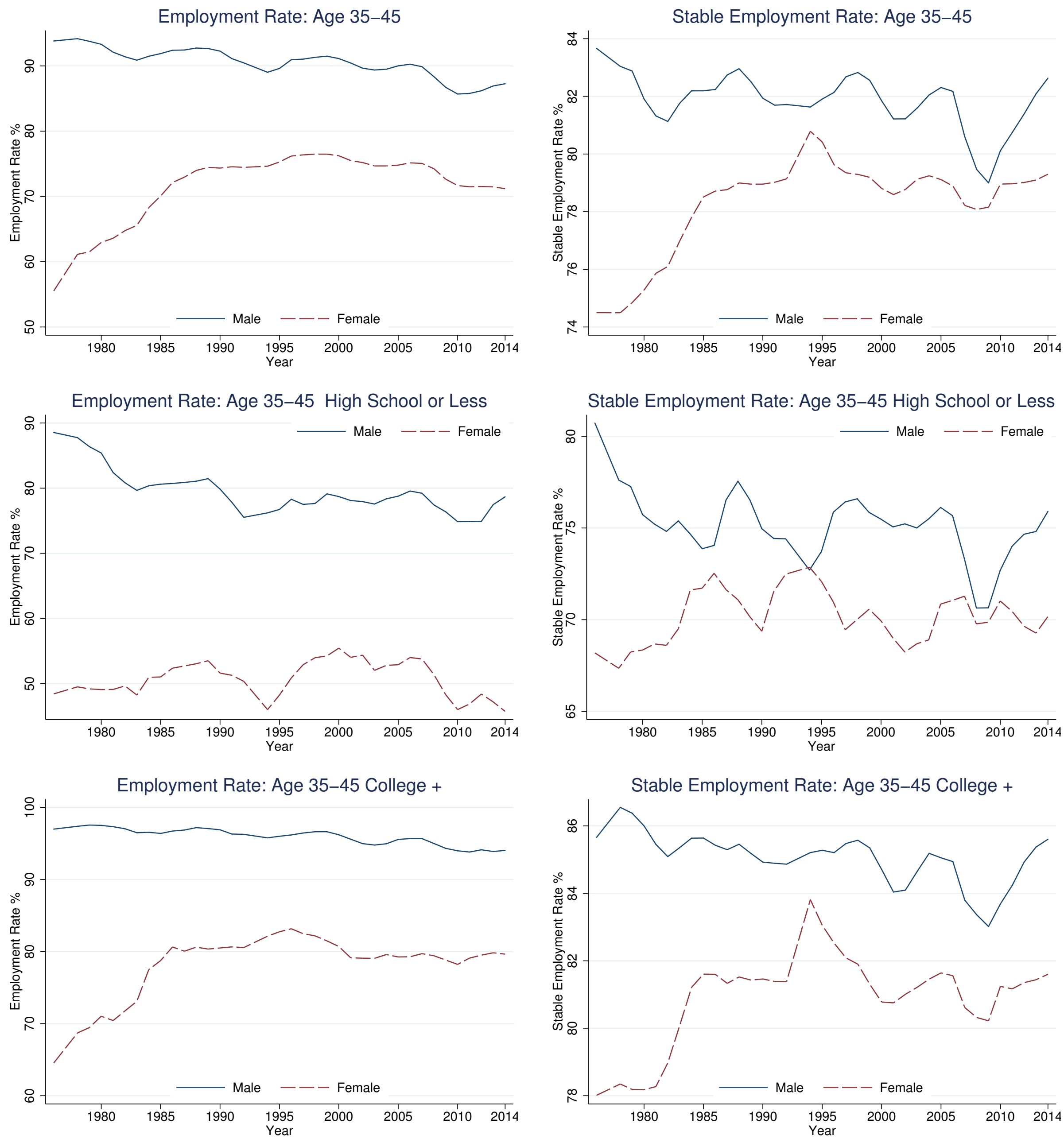

Figure 14 plots the employment rate and stable-employment rates for men and women between 35 and 45 years old, analogous to Figure 10.

This observation from employment rates alone could give an impression that all men within the same education category have suffered a deterioration in labor market conditions. However, employment stability rates, the percent of employed workers in the most stable employment state (E1) based on the HMM estimates, provide more in-depth insights into the changes in labor market conditions over time. Panel B in Figure 10 plots the employment stability rates of men by age and education group over different time periods. First, while employment rates for men across education groups reach the peak over around age 25, the employment stability rates do not reach their peak until ages 45 to 50 years for less educated men (high school or less). Men with some college or more education enjoy not only a high employment rate starting at age 25, they also enjoy stable employment when they are younger. Second, stable-employment rates do not decline as sharply as employment rates for older men across all education groups. Indeed, for men with a high school education or less, while employment rates are lower for men between 50 and 55 years old than men between 30 and 40 years old, employment stability is slightly higher for men between 50 and 55 years old than men between 30 and 40 years old. In other words, among the less–educated men, while a smaller fraction of men older than 50 years old are employed than their younger counterparts, conditional on being employed, older employed workers are more likely to have stable employment. For example, between 1987-1989, while employment rate of men with high school degree declines from 90 percent for 50 years old to close to 60 percent for 55 years old, the stable-employment rate for these age groups only declines from 80 percent to 75 percent.

We now look at changes in stable-employment rates for men over time. Except for the 2012-2014 decline in employment stability after the Great Recession, we do not observe a clear sign of deterioration in employment stability across all education groups. There are small decline in employment stability for older workers, but the changes are not as stark as the employment rates in Panel A. For example, between 1977-1979 and 1987-1989, the stable-employment rate—the percent of employed workers in the most stable employment state—for men with a high school education has declined only by 1.4 percentage points from 81 percent to 79 percent. For men with a college education, the stable-employment rate declined only by 0.8 percentage points from 84 percent to 83 percent during the same period. Thus, we can conclude that while men, especially less–educated men, have suffered a decline in the employment rates, conditional on being employed, men seem to enjoy similar employment stability in the recent decade than in the late 1970s.

We now turn to changes in employment conditions for women. Panel A in Figure 16 plots the employment rates of women by age and education group over different time periods.

educated workers do not enter the labor force until 22 years old.

Employment rates are higher for women with more education than women with less education. Since the late 1970s, the employment rates for women have increased across education groups. For example, between 1977-1979 and 1987-1989, the employment rate for 30–year–old women with some college education has increased from 65 percent to 76 percent. For the same period, the employment rate for 30 year old women with a high school diploma has also increased from 56 percent to 65 percent.

However, stable-employment rates of women show a different picture of the labor market conditions for women. Panel B in Figure 16 plots the stable-employment rates of women by age and education group over different time periods. Similar to men, while employment rates for older women are lower than for their younger counterparts, conditional on being employed, the older female workers enjoy more stable employment. While employment stability for younger workers improves slightly, employment stability for older female workers did not improve much. For example, between 1977-1979 and 1987-1989, while the stable-employment rate for 25 year old women with some college education increased by 5 percentage points (from 72 percent to 77 percent), the stable-employment rate for 50 year old women with some college education increased by less than 0.5 percentage points (from 79.3 to 79.8 percent). On average, the stable-employment rate between 1977-1979 and 1987-1989 for women with some college saw a mild improvement (a 2.8 percentage point increase). Since the late 1980s, however, the employment stability of women has not seen significant improvement. Therefore, while employment rates for women generally have improved since the late 1970s, improvement in their employment stability in more recent decades has been mild.

Therefore, while the large decline in employment rates for men suggests that employment situations for men have deteriorated, the mild decline in employment stability suggests that male employed workers still enjoy relatively stable employment. In other words, while less men are working, conditional on working, men still do seem to enjoy stable employment. On the other hand, while more women are working, and employed women are enjoying more stable employment than in the late 1970s, since the 1980s, employment conditions for working women do not seem to have changed much. In sum, this section has shown that while looking at employment rates alone suggests that labor market conditions for men have deteriorated while those for women have improved, employed men continue to enjoy stable employment while female workers' employment stability did not drastically improve since the 1990s.

Figure 15: Age Profile of Employment Rates by Education for Sub-Periods: Men

Panel A | Panel B

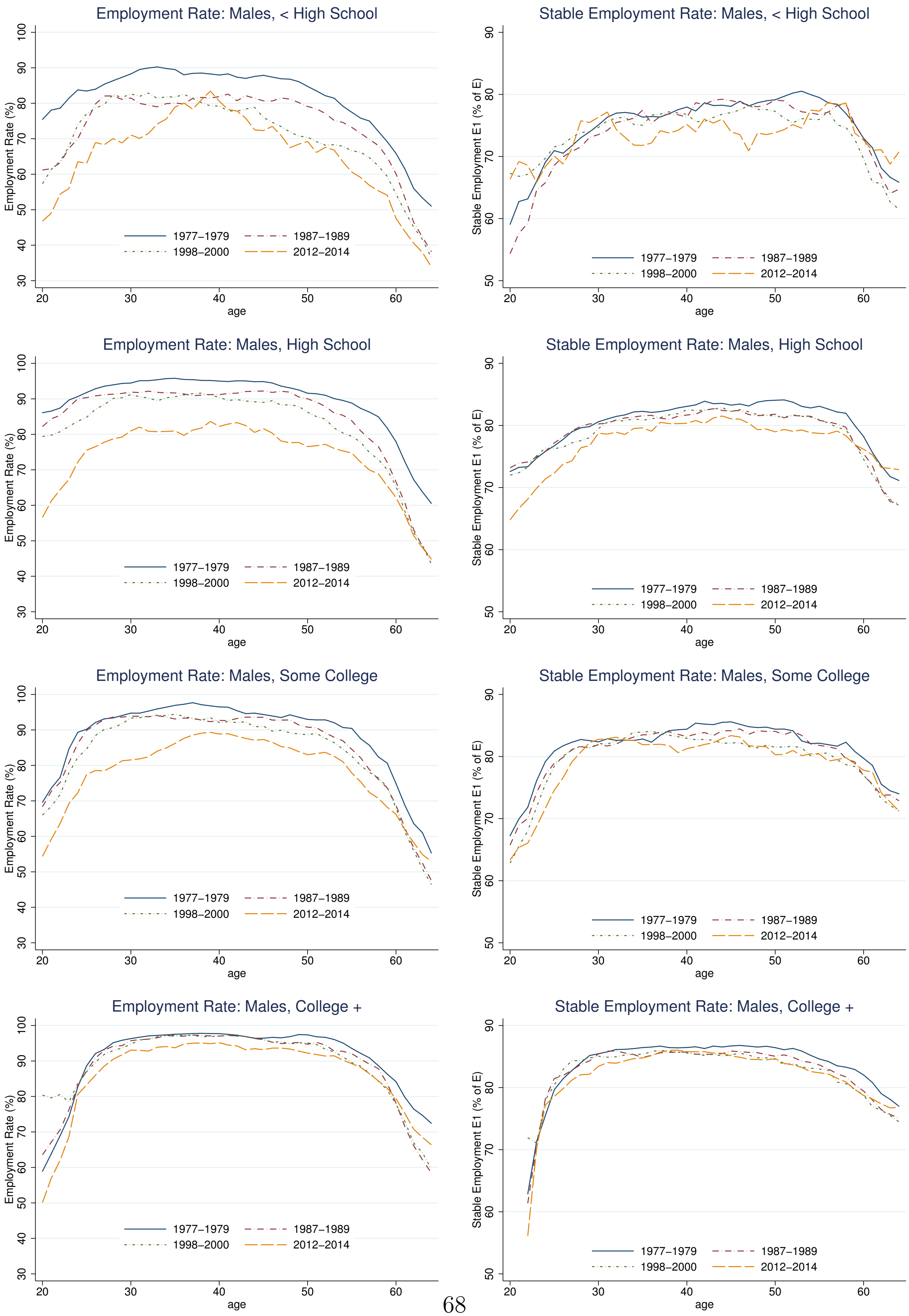

Figure 16: Age Profile of Employment Rates by Education for Sub-Periods: Women

Panel A | Panel B

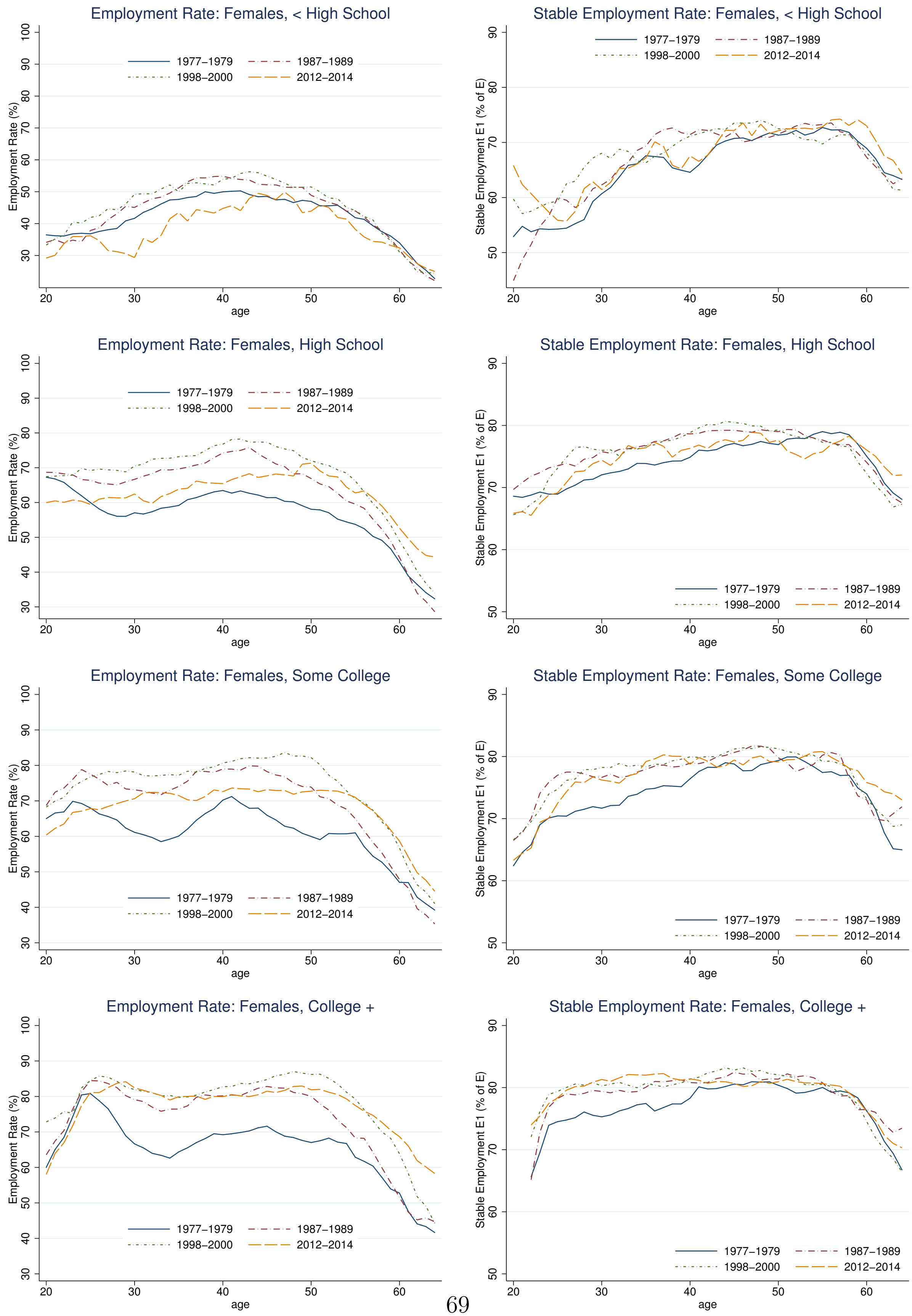

Figure 17: Evolution of Labor Market Attachments Around the 1981 and 2008 Recessions

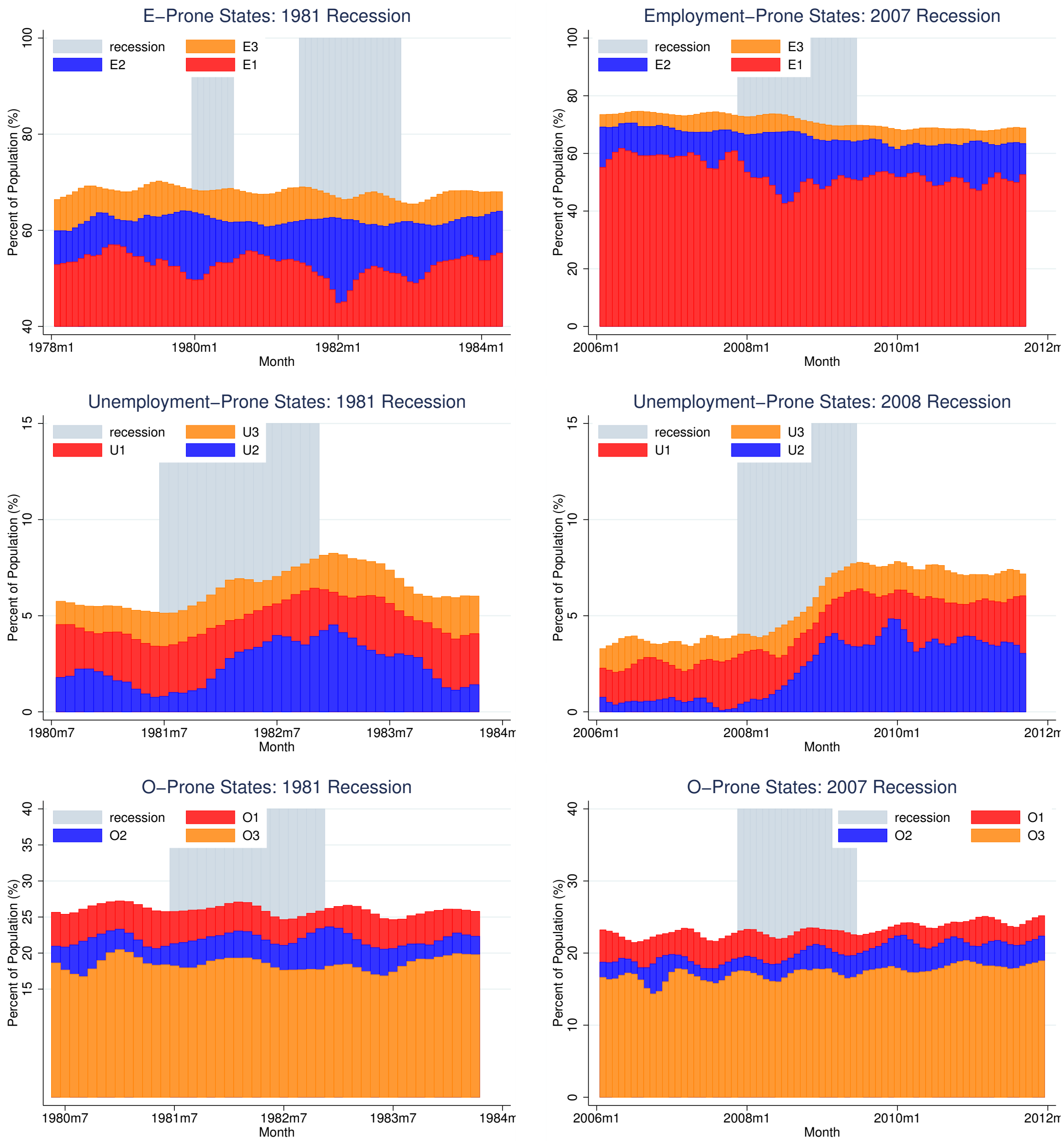

Figure 17 plots the evolution of inflows of workers in different attachment states as percent of population around the 1981 and 2007 recessions. At the onset of both recessions, the share of workers in the most stable employment (E1) declined, and employed workers with less stable employment (E2) comprised an increasing share of employed workers. Toward the end of the recession, the share of workers with most stable employment (E1) recovered and the share of employed workers with less stable employment (E2) subdued. Among the workers in the unemployment prone states, the share of the most unemployment prone states increased as the economy went deeper into recession and continued to increase even after the official date of recession ended. Among the workers in the out of the labor force prone states, the share of workers more attached to unemployment (O2) increased along with workers who are in the most persistent O state (O3).