Intergenerational Politics, Government Debt and Economic Growth*

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Abstract

This paper develops a two-period overlapping-generations model featuring endogenous growth and intergenerational conflict over fiscal policy. In particular, the paper characterizes a Markov-perfect political equilibrium of the voting game between generations, and shows the following results. First, in an unbalanced budget case, the government borrows or lends in the capital market depending on the share of capital in production. Second, when the government borrows in the capital market, an introduction of a balanced budget rule results in a higher growth rate. Third, to obtain a normative implication of the political equilibrium, the paper considers a benevolent planner with a commitment technology, and shows that the planner always chooses to lend in the capital market to save more for future generations and thus attains a higher growth rate than the government in a political equilibrium.

Keywords: Economic Growth; Government Debt; Overlapping Generations; Population Aging; Probabilistic Voting.

JEL Classification: D72, D91, H63.

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1 Introduction

Many OECD countries have experienced a decline in fertility rates and an increase in life expectancy over the past several decades (OECD, 2011). These demographic changes have created an increasing political power of the elderly in voting, which has been expected to increase government spending in favor of them. An aging population would also increase the tax burden on the young as a by-product of this political pressure. Although these predictions are controversial (Razin, Sadka and Swagel, 2002; Gradstein and Kaganovich, 2004), the changes in spending and tax burden definitely affect household savings, which in turn are expected to influence economic growth and welfare in the long run.

Several studies have attempted to investigate the political effects of demographic changes on government spending and economic growth. Examples are Gradstein and Kaganovich (2004), Holz-Eakin, Lovely and Tosun (2004), Tosun (2008), Iturbe-Ormaetxe and Valera (2012) and Kaganovich and Meier (2012). These studies assumed myopic agents who have no concern about the impact of their voting on future policies. However, recent studies suggest the importance of dynamic links between current and future political choices, and they argue there is a need to develop models that include forward-looking agents from the viewpoint of economic analysis and policy debate (see, for example, Hassler et al., 2003; Hassler et al., 2005; and Hassler, Storesletten and Zilibotti, 2007).

The forward-looking behavior of agents can be captured by employing politico-economic models based on the concept of Markov-perfect equilibrium (Krusell, Quadrini and Rios-Rull, 1997; Grossman and Helpman, 1998; Azariadis and Galasso, 2002). Examples include studies based on the neoclassical growth model (Beauchemin, 1998; Forni, 2005; Bassetto, 2008; Gonzalez-Eiras and Niepelt, 2008; Gonzalez-Eiras, 2011; and Song, 2011) and those based on the endogenous growth model (Gonzalez-Eiras and Niepelt, 2011; and Kuehnel, 2011). In particular, these studies assume overlapping generations and analyze the effect of intergenerational conflict over government spending and tax burden on growth and/or welfare. All of these studies assume a balanced government budget; that is, they ignore the possibility of government spending financed by government debt issue. However, every generation in their models might have an incentive to shift the burden to future generations by issuing government debt. Thus, from the practical viewpoint of fiscal policy, there is a need to undertake analysis in the presence of government debt.

With the above background in mind, this paper aims to consider the following three issues. First, how does an intergenerational conflict over fiscal policy affect economic growth and the state of financial balance via voting in an economy without a balanced budget rule? We consider this issue by characterizing a Markov-perfect political equilibrium to demonstrate an interaction between current and future policy variables. In particular, we assume probabilistic voting a la Lindbeck and Weibull (1987) where the objective of the government is to maximize the weighted sum of the utility of the young and old. Second, how does government debt issue affect government spending and economic growth? We answer this question by demonstrating a special case in which a balanced budget is required by statute, and comparing it to the unbalanced budget case. Third, what is the normative implication of the political equilibrium outcome? We characterize the Ramsey allocation by assuming a benevolent planner who takes care of the welfare of all generations and can commit to policies over long periods of time, and compare it to the political equilibrium outcome.

For the purpose of analysis, this paper uses a two-period-lived overlapping-generations model with AK technology a la Romer (1986) and Lucas (1988). The paper adopts the AK technology to simply demonstrate endogenous economic growth. Government spending is represented by public goods provision shared by the young and old. The spending is financed by tax on the young, and it can also be financed by government debt issue. In each period, tax, public goods provision and new debt issue are determined by probabilistic voting. These policy variables are conditioned by payoff-relevant state variables, that is, the beginning-of-period government debt and capital in the present framework. This implies that the expected level of public goods provision in the next period depends on the current government debt issue as well as next-period stock of capital, both of which are affected by current policy decision making. The forward-looking agents take account of this inter-temporal effect when they participate in voting.

Based on the abovementioned framework, we first characterize a Markov-perfect political equilibrium in an economy without a balanced budget rule: government spending is financed by tax on the labor income of the young, and it can also be financed by the issue of government debt. The government surely borrows in the capital market and thus households become lenders if the share of capital in production is below a critical value. However, the opposite result holds if the capital share is above a critical value: the government lends in the capital market, and thus households become borrowers.

Given the abovementioned feature of the government's state of financial balance, we find that the growth rate of capital is constant except for the initial period. In other words, the growth rate changes between the first two periods, depending on the government financial balance. In particular, the growth rate decreases if the government borrows in the capital market; the issue of government debt crowds out capital accumulation. The opposite result holds if the government offers loans to households and thus enhances their saving. We also find that population aging increases the growth rate for most set of parameters.

Second, a special case, called a balanced budget case, is considered: the government is prohibited from borrowing or lending in the capital market, and thus a balanced budget is required by statute. To consider the role of government debt, we compare the government spending-to-GDP ratio and the growth rate in the balanced budget case to those in the unbalanced budget case, and find the following result: the spending-to-GDP ratio and growth rate of capital are lower (higher) in the unbalanced budget case than in the balanced budget case if the government borrows (lends) in the capital market. This result suggests that the share of capital, which affects the financial status of government, is a key to determine the relative performance of the unbalanced budget case compared to the balanced budget case in terms of government spending and economic growth. The result also suggests that the introduction of a balanced budget rule enables us to attain a higher growth rate when the government borrows in the capital market in an unbalanced budget economy.

Third, to consider a normative implication of the political equilibrium, this paper characterizes a Ramsey allocation in which a benevolent planner with a commitment technology sets fiscal policy over time to maximize the welfare of all generations. The planner who attaches weights to all successive generations has an incentive to save more, spend less today and accumulate more capital for future generations than the government caring about only living two generations in the political equilibrium. The analysis shows that the planner provides loans to the present generation by lending in the capital market, and makes the present generation save more. Therefore, the planner's allocation attains a higher growth rate than the political equilibrium.

This paper is related to literature on debt politics in an overlapping-generations model: Cukierman and Meltzer (1989); Song, Storesletten and Zilibotti (2012) and Rohrs (2010). Cukierman and Meltzer (1989) consider the politics of government debt issue in an overlapping-generations model including two types of agents: bequest-unconstrained and bequest-constrained agents. Their focus is thus on intragenerational conflict over government debt issue, not on intergenerational conflict.

Song, Storesletten and Zilibotti (2012) successfully capture the intergenerational conflict over government debt issue in a multi-country framework and demonstrate cross-country differences in fiscal policy. However, the effects of population aging on fiscal policy are abstracted away in their analysis because they assume no fertility and longevity changes, and growth consequences are not predictable in their model because of the assumption of a small open economy. Rohrs (2010) extends their analysis by assuming a closed economy, but there is no capital accumulation and thus no growth prediction.

In contrast to these studies, this paper demonstrates the intergenerational conflict caused by population aging and its consequences for long-run economic growth. In particular, the present paper contributes to the literature in that it shows the difference between political equilibrium and Ramsey allocation in terms of economic growth, and identifies the mechanism behind the difference, which has not yet been fully demonstrated in previous studies.

This paper is organized as follows. Section 2 presents the model and characterizes economic equilibrium. Section 3 characterizes political equilibrium without a balanced budget rule, and compares it to the political equilibrium with a balanced budget rule. Section 4 demonstrates the Ramsey allocation and compares it to the political equilibrium outcomes. Section 5 provides concluding remarks. Proofs are given in Appendix A. Three extensions of the model are discussed in Appendix B.

2 The Model and Economic Equilibrium

Consider an infinite-horizon economy composed of identical agents, perfectly competitive firms, and perfect annuity markets. A new generation, called generation t, is born in each period $t = 0, 1, 2, ...$ Generation t is composed of a continuum of $N_t > 0$ units which are identical agents. We assume that $N_t = (1 + n)N_{t-1}$: the net rate of population growth is $n > -1$.

2.1 Preferences and Utility Maximization

Agents live a maximum of two periods, youth and old age. In youth, each agent is endowed with one unit of labor, which is supplied inelastically to firms, and obtains wages. An agent in generation t divides his or her wage w_t between his or her own current consumption c_t^y, saving, held as an annuity and invested into physical capital and/or government debt, for consumption in old age, s_t, and the payment of tax quoted as a proportion of his or her wage, $\tau_t w_t$; τ_t is the period-t tax rate on labor income. Thus, the budget constraint for a period-t young agent is $c_t^y + s_t \leq (1 - \tau_t)w_t$.

Agents are assumed to be faced with uncertain lifetimes. In particular, an agent dies at the end of youth with a probability of $1 - p \in (0, 1)$ and lives throughout old age with a probability of p. If an agent dies young, his or her annuitized wealth is transferred to the agents who live throughout old age via annuity markets. If an agent is alive in old age, he or she consumes the return from savings. The budget constraint for a period-$t+1$ old agent is given by $c_{t+1}^o \leq \tilde{R}_{t+1}s_t$ where c_{t+1}^o is consumption in old age and $\tilde{R}_{t+1}$ is the return from savings as an annuity.

Agents consume two goods: private and public goods. We assume additively separable logarithmic preferences over private and public goods. The utility of a young agent in period t is written as $\ln c_t^y + \theta \ln g_t + p\beta \cdot \left\{\ln c_{t+1}^o + \theta \ln g_{t+1}\right\}$ where g_t denotes the per capita period-t public goods provision, $\theta(> 0)$ captures the preference weight on public goods, and $\beta \in (0, 1)$ is a discount factor. Thus, the expected utility maximization problem of a period-t young agent can be written as:

$$\max_{\{c_t^y, s_t, c_{t+1}^o\}} \ln c_t^y + \theta \ln g_t + p\beta \cdot \left\{\ln c_{t+1}^o + \theta \ln g_{t+1}\right\}$$
$$\text{s.t. } c_t^y + s_t \leq (1-\tau_t) w_t,$$
$$c_{t+1}^o \leq \tilde{R}_{t+1} s_t,$$
$$\text{given } \tau_t, w_t, \text{ and } \tilde{R}_{t+1}.$$

Solving the problem leads to the following consumption and saving functions:

$$c_t^y = \frac{1}{1+p\beta}(1-\tau_t)w_t, c_{t+1}^o = \frac{p\beta \tilde{R}_{t+1}}{1+p\beta}(1-\tau_t)w_t, \text{ and } s_t = \frac{p\beta}{1+p\beta}(1-\tau_t)w_t.$$

In period 0, there are both young agents in generation 0 and initial old agents in generation -1. Each agent in generation -1 is endowed with s_{-1} units of goods, earns the return $\tilde{R}_0 s_{-1}$, and consumes it. The measure of the initial old agents is pN_{-1}. The utility of an agent in generation -1 is $\ln c_0^o + \theta \ln g_0$.

2.2 Technology and Profit Maximization

There is a continuum of identical firms. They are perfectly competitive profit maximizers that produce output with the use of a constant-returns-to-scale Cobb-Douglas production function, $Y_t = A_t(K_t)^\alpha (N_t)^{1-\alpha}$, where Y_t is aggregate output, A_t is the productivity parameter, K_t is aggregate capital, N_t is aggregate labor, and $\alpha \in (0,1)$ is a constant parameter representing capital share. The productivity parameter is assumed to be proportional to the aggregate capital per labor unit in the overall economy: $A_t = A(K_t/N_t)^{1-\alpha}$. Capital investment thus involves a technological externality of the kind often used in theories of endogenous growth (see, for example, Romer, 1986; Lucas, 1988). Capital is assumed to fully depreciate within a period.

In each period t, a firm chooses capital and labor in order to maximize its profits, $\Pi_t = A_t(K_t)^\alpha (N_t)^{1-\alpha} - R_t K_t - w_t N_t$, where R_t is the rental price of capital and w_t is the wage rate. The firm takes these prices as given. The first-order conditions for profit-maximization are given by:

$$K_t : R_t = \alpha A_t (K_t)^{\alpha-1} (N_t)^{1-\alpha},$$
$$N_t : w_t = (1-\alpha) A_t (K_t)^\alpha (N_t)^{-\alpha}.$$

2.3 Government Budget Constraint

Fiscal policy is determined through elections. Government debt is traded in a domestic capital market. Let B_t denote the aggregate inherited debt and G_t denote the aggregate spending on public goods. A dynamic budget constraint in period t is $B_{t+1} + N_t \tau_t w_t =$

$G_t + R_t B_t$ where B_{t+1} is the newly issued debt, $N_t \tau_t w_t$ is the aggregate tax revenue, and $R_t B_t$ is debt repayment.

Let $b_t \equiv B_t/N_t$ denote an inherited debt per capita and $g_t \equiv G_t/N_t$ denote a per capita period-t public spending.[1] Dividing both sides of the above constraint by N_t, we obtain a per capita form of the government budget constraint:

$$(1+n)b_{t+1} + \tau_t w_t = g_t + R_t b_t,$$

where $\tau_t > (<)0$ holds when the government imposes a tax on (provides a subsidy to) individuals; and $b_{t+1} > (<)0$ holds when the government borrows (lends) in the capital market. The present analysis allows that the government may offer a subsidy and/or loans to individuals.

Given b_t, the elected government in period t chooses the labor income tax τ_t, per capita public spending $g_t \equiv G_t/N_t$ and the newly issued debt b_{t+1} subject to the above constraint. We assume that the government in each period is committed to not repudiating the debt.[2]

2.4 Economic Equilibrium

A market clearing condition for capital is $K_{t+1}+B_{t+1} = N_t s_t$, which expresses the equality of the total savings by young agents in generation t, $N_t s_t$, to the sum of the stocks of aggregate physical capital and aggregate government debt. Dividing both sides by N_t leads to:

$$(1+n)\cdot(k_{t+1}+b_{t+1}) = s_t.$$

Since the market for capital is competitive, the following arbitrage condition holds under perfect annuity:

$$\tilde{R}_{t+1} = R_{t+1}/p \ \forall t.$$

Formally, an economic equilibrium is defined as follows.

Definition 1. An *economic equilibrium* is a sequence of prices, $\left\{w_t, R_t, \tilde{R}_t\right\}_{t=0}^{\infty}$, a sequence of allocations, $\{c_t^y, c_t^o, s_t\}_{t=0}^{\infty}$, a sequence of capital stock $\{k_t\}_{t=0}^{\infty}$ and government debt $\{b_t\}_{t=0}^{\infty}$ with the initial conditions $k_0 > 0$ and $b_0 = 0$, and a sequence of policies $\{\tau_t, g_t\}_{t=0}^{\infty}$, such that the following conditions are met: (i) the conditions of utility maximization with the budget constraints in youth and old age; (ii) the conditions of profit maximization; (iii) the government budget constraint; (iv) the capital-market-clearing condition; and (v) the no arbitrage condition.

[1] The public good in the present model does not satisfy the non-rivalry property. The definition of $g_t \equiv G_t/N_t$ implies that per capita public spending decreases as the population increases. The good is classified as an impure public good in a strict sense. However, in the following, we call it "a public good" for simplicity of description; and use "a public good" and "public spending" interchangeably.

[2] The multi-period debt structure more closely resembles reality. However, the present paper assumes a one-period debt structure to simplify the strategy space in voting and to derive analytical solutions of the model.

Under the assumption of productive externality, $A_t = A(K_t/N_t)^{1-\alpha}$, the first-order conditions for profit maximization are rewritten as:

$$R_t = R \equiv \alpha A \text{ and } w_t = (1-\alpha)Ak_t.$$

By using the saving function and the first-order conditions for profit maximization, we can rewrite the capital-market-clearing condition as follows:

$$(1+n)(k_{t+1}+b_{t+1}) = \frac{p\beta}{1+p\beta} \cdot (1-\tau_t)(1-\alpha)Ak_t. \tag{1}$$

In an economic equilibrium, the indirect utility of a young agent in period t, V_t^y, and that of an old agent alive in period t, V_t^o, respectively, can be expressed as functions of government policy, capital stock and government debt:

$$\begin{aligned} V_t^y &= (1+p\beta)\ln(1-\tau_t)(1-\alpha)Ak_t + \theta \ln g_t + p\beta\theta \ln g_{t+1} \\ V_t^o &= \ln(k_t + b_t) + \theta \ln g_t, \end{aligned}$$

where some irrelevant terms are omitted from the expressions. The first term of the young agent's indirect utility function corresponds to the utility of consumption in youth and old age; and the second and third terms show the utility of the first and second period public goods, respectively. The first term of the old agent's indirect utility corresponds to the utility of consumption and the second shows the utility of public goods.

3 Political Equilibrium

This paper assumes probabilistic voting in the demonstration of the political mechanism. In each period, the government in power maximizes a political objective function. Formally, the political objective function in each period t is given by:

$$\Omega_t = pV_t^o + (1+n)V_t^y,$$

where p and $(1+n)$ are the relative weights of old and young agents measured as a percentage of the population in the economy, respectively.[3] The government's problem in period t is to maximize Ω_t subject to the government budget constraint, given the two state variables, k_t and b_t.[4]

This paper restricts its attention to a stationary Markov-perfect equilibrium. Markov perfectness implies that outcomes are history-dependent only on the payoff-relevant state

[3] An alternative formulation of the objective function is to assume $\Omega_t = (p/(1+n+p))V_t^o + ((1+n)/(1+n+p))V_t^y$ where $p/(1+n+p)$ and $(1+n)/(1+n+p)$ are the shares of the old and the young in the population, respectively. The solutions are equivalent between the two formulations.

[4] An explicit microfoundation for this modeling is explained in Persson and Tabellini (2000, Chapter 3) and Acemoglu and Robinson (2005, Appendix).

variables, that is, capital, k, and government debt, b. The stationary part implies that our focus is on equilibrium policy rules, which are not indexed by time. Therefore, the expected level of public goods provision for the next period, g_{t+1}, is given by a function of the next period stocks of capital and debt, $g_{t+1} = G(k_{t+1}, b_{t+1})$. By the use of recursive notation with x' denoting next-period x, we can define a stationary Markov-perfect political equilibrium in the present model as follows:

Definition 2. A *stationary Markov-perfect political equilibrium* is a set of functions, $\langle T, G, B\rangle$, where $T : \Re_{++} \times \Re \to [0, 1]$ is a tax rule, $\tau = T(k, b)$, $G : \Re_{++} \times \Re \to \Re_{++}$ is a government expenditure rule, $g = G(k, b)$, and $B : \Re_{++} \times \Re \to \Re$ is a debt rule, $b' = B(k, b)$, such that:

(i) the capital market clears:

$$(1 + n)(k' + B(k, b)) = \frac{p\beta}{1 + p\beta}(1 - T(k, b)) \cdot (1 - \alpha)Ak, \tag{2}$$

(ii) given k and b, $\langle T(k, b), G(k, b), B(k, b)\rangle = \arg\max \Omega(k, b, g, b', g')$ subject to $g' = G(k', b')$, (2), and the government budget constraint,

$$(1 + n)B(k, b) + T(k, b)(1 - \alpha)Ak = G(k, b) + Rb, \tag{3}$$

where $\Omega(k, b, g, b', g')$ is defined by:

$$\begin{aligned}\Omega(k, b, g, b', g') \equiv\, & p\{\ln(k + b) + \theta \ln g\} + (1 + n)\{(1 + p\beta)\ln(1 - T(k, b))(1 - \alpha)Ak \\ & +\theta \ln g + p\beta\theta \ln g'\}\end{aligned}$$

A new state variable, x, is introduced to solve the problem in a tractable way:

$$x \equiv (1 - \alpha)Ak - Rb,$$

where x represents the labor income minus government debt repayment. With the use of this new variable, we can reformulate the problem in Definition 2(ii) as follows.

Lemma 1. *Assume $G(k', b') = G(x') \equiv G((1-\alpha)Ak' - Rb')$. The problem in Definition 2(ii) is reformulated as:*

$$\begin{aligned}\langle G(x), X(x)\rangle = \arg\max\, & \{(1 + n)(1 + p\beta)\ln(A \cdot (x - G(x)) - (1 + n) \cdot X(x)) \\ & +(p + 1 + n)\theta \ln G(x) + (1 + n)p\beta\theta \ln g'\} \\ \text{subject to } & g' = G(X(x)),\end{aligned}$$

where X is a mapping from $\Re$ to $\Re$.

Proof. See Appendix A.1.

The problem in Lemma 1 implies that we can solve the government's problem and thus find policy functions in the following ways. First, we find solutions to the reformulated problem, $g = G(x)$ and $x' = X(x)$. Second, we use the solutions, the capital market clearing condition and the government budget constraint to find the policy functions $b' = B(k, b)$ and $\tau = T(b, k)$, and the law of motion of capital, $k' = K(b, k)$.

The analysis proceeds as follows. In Section 3.1, a political equilibrium is characterized to investigate the effects of population aging on the growth rate of capital. Section 3.2 focuses on a special case in which the government runs no deficit; a balanced budget is required by statute. We compute the government spending-to-GDP ratio and the growth rate in the balanced budget case, and compare them to those in the unbalanced budget case.

3.1 Characterization of Political Equilibrium

In order to obtain a solution to the problem in Lemma 1, we conjecture the following linear function:

$$g' = G_0 \cdot x',$$

where $G_0 \in (0, \infty)$ is a constant parameter. Under this conjecture, we solve the problem and obtain the following policy functions:

$$G(x) = \frac{(p+1+n)\theta}{(1+n)\left\{1+p\beta(1+\theta)\right\}+(p+1+n)\theta}x, \tag{4}$$

$$X(x) = X_0 x, \tag{5}$$

where X_0 is a constant which is defined by:

$$X_0 \equiv \frac{\theta p \beta A}{(1+n)\left\{1+p\beta(1+\theta)\right\}+(p+1+n)\theta}.$$

These functions constitute a stationary Markov-perfect political equilibrium as long as $G_0 = (p+1+n)\theta \cdot [(1+n)\left\{1+p\beta(1+\theta)\right\}+(p+1+n)\theta]^{-1}$.

Policy function (5) states that the wage income minus debt repayment in the next period, $(1-\alpha)Ak' - Rb'$, depends on that in the current period, $(1-\alpha)Ak - Rb$. By the use of the capital market clearing condition in (2) and the government budget constraint in (3), we can find that each of k' and b' is determined as a function of $(1-\alpha)Ak - Rb$, and thus the ratio b'/k' becomes constant across periods. The tax rate is determined to satisfy the government budget constraint in each period. The following proposition formally states the finding demonstrated so far.

Proposition 1. *Consider an economy without a balanced budget rule. Given $k_0 > 0$ and $b_0 = 0$, a stationary Markov-perfect political equilibrium is characterized by the* *following policy functions,*

$$
\begin{aligned}
\tau = T(k,b) &\equiv \frac{(p+1+n)\theta - (1+n)\frac{p\beta}{1+\alpha p\beta}\left\{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)\right\}}{(1+n)\left(1+p\beta(1+\theta)\right) + (p+1+n)\theta} \\
&+ \frac{(1+n)\left(1+p\beta(1+\theta)\right) + (1+n)\frac{p\beta}{1+\alpha p\beta}\left\{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)\right\}}{(1+n)\left(1+p\beta(1+\theta)\right) + (p+1+n)\theta} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{b}{k}, \\
g = G(k,b) &\equiv \frac{(p+1+n)\theta}{(1+n)\left(1+p\beta(1+\theta)\right) + (p+1+n)\theta} \cdot \left\{(1-\alpha)Ak - Rb\right\}, \\
b' = B(k,b) &\equiv \frac{p\beta}{1+\alpha p\beta} \cdot \frac{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)}{(1+n)\left(1+p\beta(1+\theta)\right) + (p+1+n)\theta} \cdot \left\{(1-\alpha)Ak - Rb\right\},
\end{aligned}
$$

and the law of motion of capital,

$$
k' = \frac{p\beta}{1+\alpha p\beta} \cdot \frac{\theta + \alpha\left(1+p\beta(1+\theta)\right)}{(1+n)\left(1+p\beta(1+\theta)\right) + (p+1+n)\theta} \cdot \left\{(1-\alpha)Ak - Rb\right\},
$$

where

$$
\frac{b}{k} = \frac{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)}{\theta + \alpha\left(1+p\beta(1+\theta)\right)}
$$

holds $\forall t \geq 1$. *The government borrows (lends) in the capital market, that is,* $b' > (<)0$, *if and only if* $\alpha < (>)(1-\theta+p\beta)/\{1+p\beta(1+\theta)\}$.

Proof. See Appendix A.2.

Proposition 1 implies that the model economy has the following two features. First, the tax rate is constant across periods, which is a common feature in the literature (see, for example, Gonzalez-Eiras and Niepelt, 2008; Song, 2011). In the present framework, a constant tax rate comes from a linear policy function associated with the two specifications, the logarithmic preferences and the AK technology. Because of these specifications, the tax rate becomes a function of the debt-to-GDP ratio, b/k, and the ratio becomes constant across periods. Therefore, the tax rate is also constant across periods.

Second, the government borrows or lends in the capital market; the state of financial balance depends on the parameter α representing the share of capital in production. To understand the mechanism behind this result, recall the policy function $B(k,b)$ in Proposition 1, which can be rearranged as:

$$
\left\{\frac{(1-\alpha)A}{1+p\beta} + R\right\} b' = \frac{\overbrace{\frac{p\beta}{1+p\beta}\left(1+p\beta(1+\theta)\right)(1-\alpha)A}^{(a.1)} - \overbrace{p\beta\theta A}^{(a.2)}}{(1+n)\left(1+p\beta(1+\theta)\right) + (p+1+n)\theta} \cdot \left\{(1-\alpha)Ak - Rb\right\}. \quad (6)
$$

The expression says that individuals devote a part of their available resources, the wage income minus debt repayment, $(1-\alpha)Ak - Rb$, to saving, denoted by the term (a.1) in Eq. (6). With the use of fiscal policy, the government splits the saving into investing for the next period stock of capital, denoted by the term (a.2) on the right-hand side, and buying or selling government bonds, denoted by the term on the left-hand side.

Eq. (6) implies that the government borrows (lends) in the capital market if the saving, denoted by the term (a.1), is greater (less) than the investment in capital, denoted by the term (a.2). Their relative strength depends on the parameter α. If the capital share, α, is low and thus the labor share, $1-\alpha$, is high such that $\alpha < (1-\theta+p\beta)/\{1+\beta(1+\theta)\}$ holds, agents earn enough wage income for saving. They can afford to lend in the capital market, and the government becomes a borrower. However, if the capital share is high such that $\alpha > (1-\theta+p\beta)/\{1+\beta(1+\theta)\}$ holds, the opposite result holds: agents borrow in the capital market, and the government becomes a lender. Therefore, the parameter α, representing the share of capital, plays a key role in determining the government state of financial balance.

Based on the result in Proposition 1, we derive the growth rate of per capita capital, k'/k, and investigate how the growth rate is affected by population aging, that is, a lower population growth rate and greater longevity of agents. The following proposition summarizes the result.

Proposition 2. *Consider a political equilibrium in an unbalanced budget case.*

(i) *The growth rate of capital is:*

$$\frac{k_{t+1}}{k_t} = \begin{cases} \underbrace{\dfrac{p\beta}{1+\alpha p\beta}}_{(b.4)} \cdot \dfrac{\theta+\alpha\overbrace{(1+p\beta(1+\theta))}^{(b.1)+(b.3)}}{\underbrace{(1+n)(1+p\beta(1+\theta))}_{(b.1)+(b.3)}+\underbrace{(p+1+n)\theta}_{(b.2)}} \cdot (1-\alpha)A & \text{for } t=0 \\ \dfrac{\overbrace{p\beta\theta}^{(b.3)}}{\underbrace{(1+n)(1+p\beta(1+\theta))}_{(b.1)+(b.3)}+\underbrace{(p+1+n)\theta}_{(b.2)}} A & \text{for } t \geq 1. \end{cases}$$

For $t \geq 1$, $k_{t+1}/k_t \gtrless k_1/k_0$ holds if and only if $\alpha \gtrless (1-\theta+p\beta)/\{1+p\beta(1+\theta)\}$.

(ii) *The growth rate of capital is increased by a lower population growth rate.*

(iii) *For $t \geq 1$, the growth rate of capital is increased by greater longevity; for $t=0$, it is increased by greater longevity if $p < [(1+n)(1+\theta)/\alpha\beta\{(1+n)\beta(1+\theta)+\theta\}]^{1/2}$ holds.*

Proof. See Appendix A.3.

The growth rate of capital is constant across periods except the initial period. This is because the model exhibits a constant interest rate inherited from AK technology. However, the growth rate changes between the first two periods, i.e., period 0 and period

1, because the government starts to borrow or lend in the capital market in period 0. In particular, the growth rate decreases from period 0 to period 1 if the government borrows in the capital market (i.e., if $\alpha < (1-\theta+p\beta)/\{1+p\beta(1+\theta)\}$ holds). The issue of government bonds crowds out capital accumulation. The opposite result holds if the government lends in the capital market (i.e., if $\alpha > (1-\theta+p\beta)/\{1+p\beta(1+\theta)\}$ holds). A government offer of loans to households enhances their saving and thus promotes capital accumulation.

Next, let us consider how economic growth is affected by a lower population growth rate and greater longevity. To see the effect, recall the growth rate of capital demonstrated in Proposition 2; and also recall the political objective function demonstrated in Lemma 1:

$$\begin{aligned}\Omega = &\underbrace{(1+n)(1+p\beta)}_{(b.1)} \ln\left(A(x-G(x)) - (1+n)X(x)\right) \\ &+ \underbrace{(p+1+n)\theta}_{(b.2)} \ln G(x) + \underbrace{(1+n)p\beta\theta}_{(b.3)} \ln G(X(x))\end{aligned}$$

The terms (b.1), (b.2) and (b.3) in the political objective function correspond to those in the equation for the growth rate, and the term (b.4) in the equation of the growth rate shows the saving rate. The equation for the growth rate says that the government allocates the wage income, $(1-\alpha)Ak$, in period 0 and the output, Ak, in period $t \geq 1$ into consumption, current public goods provision, and investment in capital that contributes to the formation of future public goods. The allocation is affected by population growth and longevity via the above four terms.

The equation implies that a lower population growth rate definitely leads to a higher growth rate of capital because of the positive effect via the terms (b.1), (b.2) and (b.3) observed in the denominator of the equation. The equation also implies that greater longevity has competing effects on the growth rate. In period $t \geq 1$, greater longevity leads to a larger weight on future public goods provision, as shown by the term (b.3) in the numerator. This gives the politician an incentive to invest more in capital for future public goods provision, thereby producing a positive effect on the growth rate. On the other hand, greater longevity implies larger weights on consumption and public goods provision for the old, as shown by the terms (b.1) and (b.2) in the denominator. This provides politicians with an incentive to use the resources for current consumption and current public goods provision instead of investing in capital, thereby producing a negative effect on the growth rate. The analysis shows that the negative effect is outweighed by the positive effect, thereby resulting in a higher growth rate.

In period 0, the result is not straightforward. The positive effect described above is smaller than that in period $t \geq 1$. Therefore, the negative effect via the terms (b.1) and (b.2) outweighs the positive effect via the term (b.3) in period 0. However, greater longevity attaches a larger weight on future consumption and thus produces a higher saving rate: this positive effect, observed by the term (b.4), may outweigh the negative effects observed by the terms (b.1) and (b.2). The result in Proposition 2 demonstrates the condition for such a situation to arise.

3.2 Balanced Budget Case

So far we have assumed that government expenditures can be financed by issuing government debt. In a standard neoclassical growth model, the presence of government debt may crowd out capital and lower economic growth, and it may also affect the size of government spending via the government budget constraint.

In order to understand the role of government debt in the present political economy model, we focus here on a special case in which a balanced budget is required by statute: the government is unable to issue government bonds, and runs a balanced budget in each period. We compute the government spending-to-GDP ratio and the growth rate of a balanced budget case, and compare them to those in the unbalanced budget case. We then investigate how a balanced budget rule affects spending-to-GDP ratio and economic growth.

Given the presumption of the balanced budget rule, the government budget constraint becomes $g_t = \tau_t w_t$. Government expenditure g_t is financed by labor income tax revenue from the young, $\tau_t w_t$. The capital market clearing condition is $K_{t+1} = s_t L_t$, expressing the equality of total savings by young agents to the stock of aggregate capital. We divide both sides by N_t and substitute the saving function and the government budget constraint into the clearing condition to obtain the law of motion of capital for a given government expenditure:

$$(1+n)k_{t+1} = \frac{p\beta}{1+p\beta}\left\{(1-\alpha)Ak_t - g_t\right\}. \tag{7}$$

The indirect utility functions of the young and the old are now given by

$$\begin{aligned} V_t^y &= (1+p\beta)\ln\left((1-\alpha)Ak_t - g_t\right) + \theta\ln g_t + p\beta\theta\ln g_{t+1}, \\ V_t^o &= \theta\ln g_t, \end{aligned}$$

respectively, where the terms unrelated to political decisions are omitted from the expression. With the use of these functions, we can write the political objective function as:

$$\Omega_t = (1+n)(1+p\beta)\ln\left((1-\alpha)Ak_t - g_t\right) + (p+1+n)\theta\ln g_t + (1+n)p\beta\theta\ln g_{t+1}.$$

The objective function indicates that capital is a payoff-relevant state variable.

The government's problem in period t is to choose g_t subject to constraint (7) given k_t. Solving the problem leads to the following proposition.

Proposition 3. *Consider an economy with a balanced budget rule. Given $k_0 (> 0)$, a stationary Markov-perfect political equilibrium is characterized by the following policy functions,*

$$\tau_t = \frac{(p+1+n)\theta}{(1+n)\{1+p\beta(1+\theta)\}+(p+1+n)\theta} \in (0,1),$$
$$g_t = \frac{(p+1+n)\theta}{(1+n)\{1+p\beta(1+\theta)\}+(p+1+n)\theta}(1-\alpha)Ak_t,$$

and the law of motion of capital,

$$\frac{k_{t+1}}{k_t} = \frac{p\beta}{1+p\beta} \cdot \frac{1+p\beta(1+\theta)}{(1+n)\{1+p\beta(1+\theta)\}+(p+1+n)\theta} \cdot (1-\alpha)A.$$

Proof. See Appendix A.4.

As in the unbalanced budget case demonstrated in Section 3.1, the tax rate and the growth rate of capital are constant across periods. In addition, the solution in the balanced budget case matches with that in the unbalanced budget case if and only if $\alpha = (1-\theta+p\beta)/\{1+\beta(1+\theta)\}$, that is, if and only if there is no debt issue in an economy without a balanced budget rule. In order to consider the role of government debt, we compare spending-to-GDP ratio and the growth rate in the balanced-budget case to those in the unbalanced budget case, and obtain the following result.

Proposition 4. *Let $x|_{\text{Debt}}$ and $x|_{\text{Balanced}}$ denote the variable x in the unbalanced budget case and in the balanced budget case, respectively.*

(i) *For $t = 0$, $g_0/Ak_0|_{\text{Debt}} = g_0/Ak_0|_{\text{Balanced}}$ holds. For $t \geq 1$, $g_t/Ak_t|_{\text{Debt}} \lesseqgtr g_t/Ak_t|_{\text{Balanced}}$ holds if and only if $\alpha \lesseqgtr (1+p\beta-\theta)/(1+p\beta(1+\theta))$ for $t \geq 1$.*

(ii) *For $t \geq 0$, $k_{t+1}/k_t|_{\text{Debt}} \lesseqgtr k_{t+1}/k_t|_{\text{Balanced}}$ holds if and only if $\alpha \lesseqgtr (1+p\beta-\theta)/(1+p\beta(1+\theta))$.*

Proof. See Appendix A.5.

The mechanism behind the first result in Proposition 4 is as follows. In the initial period, the government spending-to-GDP ratios are equal between the unbalanced and balanced budget cases. The available resource for the initial period government is the labor income, $(1-\alpha)Ak$, for both cases. The government imposes a tax on labor income, and uses the revenue for government spending, g. However, for period $t \geq 1$, the ratios may differ between the two cases. In the unbalanced budget case, the available resources for the government are given by $(1-\alpha)Ak - Rb$, which are smaller (larger) than those in the balanced budget case when the government borrows (lends) in the capital market, that is, when $\alpha < (>) (1+p\beta-\theta)/(1+p\beta(1+\theta))$. Because of this difference in available resources, the ratio in the unbalanced budget case becomes higher or lower than that in the balanced budget case.

The second result implies that the growth rate in the unbalanced budget case becomes higher or lower than that in the balanced budget case, depending on the state of financial balance. When $\alpha < (1+p\beta-\theta)/(1+p\beta(1+\theta))$ such that the government borrows in the capital market, the government debt crowds out private investment and thus capital formation. However, when $\alpha > (1+p\beta-\theta)/(1+p\beta(1+\theta))$ such that the government lends in the capital market, the state lending enables the households to save more, thereby enhancing capital formation. Therefore, the share of capital, denoted by α, is one of the keys to determine the relative performance of the unbalanced budget compared to the balanced budget in terms of economic growth.

To check the empirical plausibility of the condition $\alpha < (>) (1+p\beta-\theta)/(1+p\beta(1+\theta))$, let us consider the following numerical example. We assume a generation to be 30 years in length, and a single-period discount rate of 0.99. Because individuals under the current assumption plan over generations that span 30 years, we discount the future by $(0.99)^{30}$. If we set $p = 0.8$ and $\theta = 0.1$, then the critical value is given by $\alpha = 0.271$. Therefore, the unbalanced budget case attains a higher (a lower) growth rate than the balanced budget case if the capital share is higher (lower) than 0.271.

4 Ramsey Allocation

This section characterizes a Ramsey allocation chosen by a benevolent planner. The planner has the ability to commit to all his or her future policy choices at the beginning of a time period subject to the competitive equilibrium constraints, which include the capital market clearing condition and the government budget constraint. We compare the Ramsey allocation with the political equilibrium demonstrated in Section 3. Then we evaluate the normative aspect of the political equilibrium in terms of economic growth and the government spending-to-GDP ratio.

The benevolent planner is assumed to value the welfare of all households. In particular, the planner has an objective function with dynastic discounting: the planner's weight on generations is assumed to reflect the discount factor of households as well as the cohort size. The planner's objective function, denoted by W, is therefore given by $W =$

$pV_0^o + (1+n)\sum_{t=1}^{\infty}\{\beta(1+n)\}^{t-1}V_{t-1}^y$, or:

$$W = \sum_{t=0}^{\infty}(\beta(1+n))^t\left[(1+n)(1+p\beta)\ln\{(1-\alpha)Ak_t - g_t - Rb_t + (1+n)b_{t+1}\}\right.$$
$$\left.+(p+1+n)\theta\ln g_t\right],$$

where the terms unrelated to political decisions are omitted from the expression. We assume $\beta(1+n) < 1$. Given $k_0(>0)$ and $b_0 = 0$, the planner's problem is to choose $\{g_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$ subject to the capital market clearing condition and the government budget constraint.

Using a method similar to that applied in the previous section, we can reformulate the above-mentioned objective function in terms of $x_t \equiv (1-\alpha)Ak_t - Rb_t$:

$$V(x_0) = \max_{\{g,x'\}}\sum_{t=0}^{\infty}(\beta(1+n))^t\left[(1+n)(1+p\beta)\ln\{A(x_t-g_t)-(1+n)x_{t+1}\} + (p+1+n)\theta\ln g_t\right]$$
given x_0,

where the unrelated terms on decision making are abstracted away from the value function $V(\cdot)$. The recursive formulation of this problem is:[5]

$$V(x) = \max_{\{g_t,x_{t+1}\}}\{(1+n)(1+p\beta)\ln\{A(x-g)-(1+n)x'\}$$
$$+(p+1+n)\theta\ln g + \beta(1+n)V(x')\}.$$

We solve the functional equation based on the guess-and-verify method and obtain the following policy functions:

$$x' = \beta Ax; \tag{8}$$
$$g = \frac{\{1-\beta(1+n)\}(p+1+n)\theta}{(1+n)(1+p\beta)+(p+1+n)\theta}x. \tag{9}$$

The tax rate is determined to satisfy the government budget constraint in each period. The Ramsey allocation is summarized in the following proposition.

Proposition 5. *Consider an economy without a balanced budget rule.*

[5]The state space is non-compact in the Ramsey problem because capital grows at a constant rate and approaches infinity. Therefore, there may exist a possibility that there are multiple value functions satisfying the functional equation. We assume here that the value function and the corresponding policy functions derived in the following analysis are equivalent to those of the original problem.

(i) *The Ramsey allocation is characterized by the following policy functions,*

$$\begin{aligned}
b' &= \frac{(-1)\beta}{1+\alpha p\beta} \cdot \frac{(1+p\beta)\left\{(1+n)(1+p\beta)-p\right\}+(p+1+n)\theta+\alpha p\left\{(1+p\beta)+\beta(p+1+n)\theta\right\}}{(1+n)(1+p\beta)+(p+1+n)\theta} \\
&\quad \times \left\{(1-\alpha)Ak-Rb\right\}, \\
\tau &= \left(1-\frac{(1+n)(1+p\beta)^2(1-\beta(1+n))}{(1+\alpha p\beta)\left\{(1+n)(1+p\beta)+(p+1+n)\theta\right\}}\right) \\
&\quad + \frac{\alpha}{1-\alpha}\cdot\frac{(1+n)(1+p\beta)^2(1-\beta(1+n))}{(1+\alpha p\beta)\left\{(1+n)(1+p\beta)+(p+1+n)\theta\right\}}\cdot\frac{b}{k}, \\
g &= \frac{\left\{1-\beta(1+n)\right\}(p+1+n)\theta}{(1+n)(1+p\beta)+(p+1+n)\theta}\left\{(1-\alpha)Ak-Rb\right\},
\end{aligned}$$

and the law of motion of capital,

$$k' = \frac{\beta\left[(1+p\beta)\{(1+n)+\alpha p\}+(1+\alpha p\beta)(p+1+n)\theta\right]}{(1+\alpha p\beta)\left\{(1+n)(1+p\beta)+(p+1+n)\theta\right\}}\left\{(1-\alpha)Ak-Rb\right\},$$

where:

$$\frac{b'}{k'} = -1+\frac{(1+p\beta)p(1-\beta(1+n))}{(1+p\beta)\{(1+n)+\alpha p\}+(1+\alpha p\beta)(p+1+n)\theta} < 0.$$

(ii) *The growth rate of capital in the Ramsey allocation is:*

$$\frac{k_{t+1}}{k_t} = \begin{cases} \frac{\beta[(1+p\beta)\{(1+n)+\alpha p\}+(1+\alpha p\beta)(p+1+n)\theta]}{(1+\alpha p\beta)\{(1+n)(1+p\beta)+(p+1+n)\theta\}}(1-\alpha)A & \text{for } t=0 \\ \beta A & \text{for } t\geq 1. \end{cases}$$

Proof. See Appendix A.6.

The most important feature is that the ratio of b'/k' takes a negative value. The Ramsey planner, who accounts for the welfare of all future generations, finds it optimal to reallocate resources from the present generation to future generations. For this aim, the planner gives loans to the present generation by lending in the capital market and makes the present generation save more, regardless of the rate of return on saving.

To evaluate the normative implication of the political equilibrium outcome, we compare the growth rate and the government spending-to-GDP ratio in the political equilibrium without a balanced budget rule to those in the Ramsey allocation, and obtain the following result.

Proposition 6. *Let* $x|_{\text{Ramsey}}$ *denote the variable* x *in the Ramsey allocation, and compare the Ramsey allocation and the political equilibrium allocation in an unbalanced budget case.*

(i) *For* $t = 0$, $g_0/Ak_0|_{\text{Ramsey}} < g_0/Ak_0|_{\text{Debt}}$ *holds. For* $t \geq 1$, $g/Ak|_{\text{Ramsey}} > g/Ak|_{\text{Debt}}$ *holds if and only if* $1 > \beta(1+n)(1+\theta)$ *and* $\alpha > \tilde{\alpha}$ *hold where:*

$$\tilde{\alpha} \equiv \frac{\theta\left\{(1-\beta(1+n))(1+p\beta(1+\theta)) - \beta(p+1+n)\theta\right\}}{\left\{(1-\beta(1+n))(1+p\beta(1+\theta)) - \beta(p+1+n)\theta\right\}(1+p\beta(1+\theta)) + \theta(1+p\beta)}.$$

(ii) *The growth rate is higher in the Ramsey allocation than in the political equilibrium:* $k'/k|_{\text{Ramsey}} > k'/k|_{\text{Debt}}$ *holds.*

Proof. See Appendix A.7.

To understand the difference in government spending-to-GDP ratios between the political equilibrium in the balanced budget case and the Ramsey allocation, recall the ratios for the two cases:

$$\left.\frac{g}{Ak}\right|_{\text{Ramsey}} = \frac{\{1-\beta(1+n)\}(p+1+n)\theta}{(1+n)(1+p\beta)+(p+1+n)\theta}\left\{(1-\alpha) - \alpha\left.\frac{b}{k}\right|_{\text{Ramsey}}\right\}A,$$
$$\left.\frac{g}{Ak}\right|_{\text{Debt}} = \frac{(p+1+n)\theta}{(1+n)(1+p\beta+p\beta\theta)+(p+1+n)\theta}\left\{(1-\alpha) - \alpha\left.\frac{b}{k}\right|_{\text{Debt}}\right\}A.$$

The following two factors contribute to the difference in the ratios: the term $1-\beta(1+n)$ in the numerator of the equation for $g/Ak|_{\text{Ramsey}}$; and the term $p\beta\theta$ in the denominator of the equation for $g/Ak|_{\text{Debt}}$. The first factor includes weights on all successive generations in the Ramsey allocation. The spending-to-GDP ratio in the Ramsey allocation becomes lower as the weight on future generations represented by the term $\beta(1+n)$ becomes larger. The second factor represents the weight of the young on future public goods provision in the political equilibrium. The spending-to-GDP ratio in the political equilibrium becomes lower as the weight on future public goods provision becomes higher. Therefore, both factors imply a negative impact on the spending-to-GDP ratio.

The effect of the former factor in the Ramsey allocation is outweighed by the effect of the latter factor in the political equilibrium. The planner in the Ramsey allocation, who takes care of all future generations, puts relatively less weight on the currently living generations. In other words, the planner puts less emphasis on the current public goods provision in the allocation of resources than the government in the political equilibrium. Therefore, the Ramsey allocation attains a lower spending-to-GDP ratio than the political equilibrium allocation in period 0.

However, this result may be reversed for $t \geq 1$. The planner always lends in the capital market as demonstrated in Proposition 5. The planner receives repayment from households, and this repayment generates a positive income effect on government spending. In particular, this positive effect becomes larger as the repayment rate (i.e., the share of capital, α) becomes larger. Therefore, there exists a critical value of α, denoted by $\tilde{\alpha}$, such that the spending-to-GDP ratio in the Ramsey allocation is larger than that in the political equilibrium if the capital share is above the critical value.

The argument above suggests that the Ramsey planner puts more emphasis on future capital (that is, x') than current public goods provision (that is, g). We can confirm this statement by focusing on the objective functions in the political equilibrium and in the Ramsey problem, both of which are given by:

$$\begin{aligned}
\Omega &= (1+n)(1+p\beta)\ln\left(A(x-g)-(1+n)x'\right)+(p+1+n)\theta\ln g+(1+n)p\beta\theta\ln x'; \\
V(x) &= (1+n)(1+p\beta)\ln\{A(x-g)-(1+n)x'\}+(p+1+n)\theta\ln g+\beta(1+n)V(x') \\
&= (1+n)(1+p\beta)\ln\{A(x-g)-(1+n)x'\}+(p+1+n)\theta\ln g \\
&+\beta(1+n)\underbrace{\frac{(1+n)(1+p\beta)+(p+1+n)\theta}{1-\beta(1+n)}\ln x'}_{(c.1)},
\end{aligned}$$

where the unrelated terms are eliminated from these expressions. The term (c.1) comes from the value function derived by solving the functional equation for the Ramsey planner.

The above expressions show that the coefficients of $\ln\{A(x-g)-(1+n)x'\}$ and $\ln g$ are equivalent between the two functions. However, the coefficient of $\ln x'$ in the function Ω is smaller than that in the function $V(\cdot)$. This property implies that the government in the political equilibrium attaches a smaller weight on the return from x' than the planner in the Ramsey allocation. Because of the lower weight on the return of x', the political equilibrium attains a lower growth rate compared to that in the Ramsey allocation.

The reason for a smaller weight on the return of x' in the political objective function is as follows. In the political equilibrium, the period-t government is concerned with only the currently living voters, that is, period-$t-1$ and period-t generations. A return of an increase in x' for the period-t government comes only from the next period public goods provision. However, in the Ramsey allocation, the planner is concerned with the currently living voters as well as unborn future generations. A return of an increase in x' for the planner comes from an increase in the next period public goods provision as well as utility gains of successive future generations. This difference in views between the government and the planner results in an increase in x' of one unit producing a higher marginal benefit for the central planner than for the period-t government. Therefore, the allocation of resources shifts toward x' in the Ramsey allocation.

5 Concluding Remarks

How does an intergenerational conflict over fiscal policy affect government spending and economic growth via voting? How does government debt issue affect government spending and economic growth? What are the normative implications of the political equilibrium outcome? This paper has attempted to answer these questions in an overlapping-generations model in which public goods provision is financed by tax and the issue of government debt, and fiscal policy is determined via probabilistic voting that captures an intergenerational conflict.

The paper shows the following three results. First, the government lends or borrow in the capital market depending on the capital share. In particular, if the capital share is below a critical value such that the government borrows in the capital market, an introduction of a balanced budget rule enables us to attain a higher government spending-to-GDP ratio as well as a higher growth rate. Second, an introduction of a balanced budget rule results in a higher growth rate if the government borrows in the capital market. Third, the political equilibrium outcome in an economy without a balanced budget rule is compared to the Ramsey allocation in which an infinitely lived planner commits to all his or her future policy choices. The analysis shows that the planner always lends in the capital market, and the growth rate is higher in the Ramsey allocation than in the political equilibrium.

The main contribution of this paper is to demonstrate the growth and normative implication of government debt issue in the presence of intergenerational conflict over fiscal policy. These implications have not been fully shown in previous studies (Cukierman and Meltzer, 1989; Song, Storesletten and Zilibotti, 2012; and Rohrs, 2011). To demonstrate the implications, this paper relies on a logarithmic utility function, AK technology, limited policy instruments and inelastic labor supply. Appendix B briefly discusses how relaxing these assumption would affect the predictions of the model, but further analysis is left as future work.

A Proofs

A.1 Proof of Lemma 1

First, we substitute the government budget constraint $(1+n)b' + \tau(1-\alpha)Ak = g + Rb$ into the capital market clearing condition $(1+n)(k'+b') = (p\beta/(1+p\beta))(1-\tau)(1-\alpha)Ak$ to replace τ by k, b and b':

$$(1+n)(k'+b') = \frac{p\beta}{1+p\beta}\left\{(1-\alpha)Ak - g - Rb + (1+n)b'\right\}.$$

This expression is reformulated as follows:

$$\begin{aligned}(1+n)b' &= \frac{p\beta}{1+p\beta}\left\{(1-\alpha)Ak - g - Rb\right\} - \frac{1+n}{(1-\alpha)A}\left\{(1-\alpha)Ak' - Rb'\right\} \\ &\quad - \frac{1+n}{(1-\alpha)A}\left\{Rb' - (1-\alpha)A\frac{p\beta}{1+p\beta}b'\right\}.\end{aligned}$$

We move the third term on the right-hand side to the left-hand side and rearrange the terms to obtain:

$$\begin{aligned}(1+n)b' &= \left[\frac{p\beta}{1+p\beta}\left\{((1-\alpha)Ak - Rb) - g\right\} - \frac{1+n}{(1-\alpha)A}\left\{(1-\alpha)Ak' - Rb'\right\}\right] \\ &\quad \times \left(\frac{R}{(1-\alpha)A} + \frac{1}{1+p\beta}\right)^{-1}. \end{aligned} \tag{10}$$

Next, we rewrite the indirect utility function of the young, $V^y = (1+p\beta)\ln(1-\tau)(1-\alpha)Ak + \theta \ln g + p\beta\theta \ln g'$, as follows:

$$\begin{aligned}V^y &= (1+p\beta)\ln\left\{((1-\alpha)Ak - Rb) - g + (1+n)b'\right\} + \theta\ln g + p\beta\theta\ln g' \\ &= (1+p\beta)\ln\left[((1-\alpha)Ak - Rb) - g + \left\{\frac{p\beta}{1+p\beta}\left\{((1-\alpha)Ak - Rb) - g\right\}\right.\right. \\ &\quad \left.\left. -\frac{1+n}{(1-\alpha)A}\left\{(1-\alpha)Ak' - Rb'\right\}\right\} \times \left(\frac{R}{(1-\alpha)A} + \frac{1}{1+p\beta}\right)^{-1}\right] + \theta\ln g + p\beta\theta\ln g',\end{aligned}$$

where the first equality comes from the substitution of the government budget constraint and the second equality comes from the substitution of (10). The above expression is rewritten as:

$$\begin{aligned}V^y &= (1+p\beta)\ln\left[A\left\{((1-\alpha)Ak - Rb) - g\right\} - (1+n)\left((1-\alpha)Ak' - Rb'\right)\right] \\ &\quad + \theta\ln g + p\beta\theta\ln g',\end{aligned} \tag{11}$$

where constant terms are omitted from the expression.

With the use of (11) and $x \equiv (1-\alpha)Ak - Rb$, the political objective function is now given by:

$$\Omega(x, g, x', g') = (1+n)(1+p\beta)\ln\left\{A(x-g) - (1+n)x'\right\} + (p+1+n)\theta\ln g + (1+n)p\beta\theta\ln g',$$

where the unrelated terms are omitted from the expression. Because the capital market clearing condition and the government budget constraint are included in $\Omega(x, g, x', g')$, the problem is now to maximize $\Omega(x, g, x', g')$ subject to $g' = G(k, b)$, given x, k and b. Therefore, the problem in Definition 2(ii) is reformulated as in the statement in Lemma 1 if we assume $G(k, b) = G(x) \equiv G((1-\alpha)Ak - Rb)$.

■

A.2 Proof of Proposition 1

Consider the reformulated problem demonstrated in Lemma 1. Given the guess of $g' = G_0 \cdot x'$, we obtain the following first-order conditions with respect to x' and g :

$$x' : (1+n)(1+p\beta)\frac{1+n}{A(x-g)-(1+n)x'} = \frac{(1+n)p\beta\theta}{x'}, \tag{12}$$

$$g : (1+n)(1+p\beta)\frac{A}{A(x-g)-(1+n)x'} = \frac{(p+1+n)\theta}{g}. \tag{13}$$

Conditions (12) and (13) lead to the following relation between g and x':

$$g = \frac{p+1+n}{Ap\beta}x'. \tag{14}$$

Substitution of (14) into (12) leads to the following optimality condition for x' :

$$x' = \frac{\theta p\beta A}{(1+n)\{1+p\beta(1+\theta)\}+(p+1+n)\theta}x. \tag{15}$$

With (14) and (15), the optimality condition for g becomes:

$$g = \frac{(p+1+n)\theta}{(1+n)(1+p\beta(1+\theta))+(p+1+n)\theta}x. \tag{16}$$

Therefore, the function $g' = G_0 x'$ constitutes a stationary Markov-perfect political equilibrium as long as $G_0 = (p+1+n)\theta \cdot [(1+n)(1+p\beta(1+\theta))+(p+1+n)\theta]^{-1}$ holds.

To find the policy functions $B(k, b)$ and $T(k, b)$, recall the capital market clearing condition and the government budget constraint, given by:

$$(1+n)(k'+b') = \frac{p\beta}{1+p\beta}(1-\tau)(1-\alpha)Ak, \tag{17}$$

$$(1+n)b' + \tau(1-\alpha)Ak = g + Rb, \tag{18}$$

respectively. Given k and b, the four variables, g, k', b' and τ, are determined by (15), (16), (17) and (18).

Substitution of (16) and (18) into (17) leads to:

$$(1-\alpha)Ak' = \frac{p\beta}{1+p\beta}\cdot\frac{\{1+p\beta(1+\theta)\}(1-\alpha)A}{(1+n)(1+p\beta(1+\theta))+(p+1+n)\theta}\cdot\{(1-\alpha)Ak - Rb\} - \frac{(1-\alpha)A}{1+p\beta}b'. \tag{19}$$

We substitute (19) into (15) and rearrange the terms to obtain the policy function $B(k,b)$:

$$b' = B(k,b) \equiv \frac{p\beta}{1+\alpha p\beta} \cdot \frac{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)}{(1+n)\left(1+p\beta(1+\theta)\right)+(p+1+n)\theta} \cdot \left\{(1-\alpha)Ak - Rb\right\}. \quad (20)$$

With the use of (19) and (20), we obtain the law of motion of capital:

$$k' = \frac{p\beta}{1+\alpha p\beta} \cdot \frac{\theta+\alpha\left(1+p\beta(1+\theta)\right)}{(1+n)\left(1+p\beta(1+\theta)\right)+(p+1+n)\theta} \cdot \left\{(1-\alpha)Ak - Rb\right\}. \quad (21)$$

(20) and (21) imply that b'/k' is constant across periods after period 1:

$$\frac{b'}{k'} = \frac{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)}{\theta+\alpha\left(1+p\beta(1+\theta)\right)} \ \forall t \geq 1.$$

Given $k' > 0$, this equation says that $b' \gtreqless 0$ holds if and only if $\alpha \lesseqgtr (1-\theta+p\beta)/\{1+p\beta(1+\theta)\}$.

To determine the policy function $T(k,b)$, recall the government budget constraint (18), which is rewritten as:

$$\begin{aligned}
\tau(1-\alpha)Ak &= g + Rb - (1+n)b' \\
&= \frac{(p+1+n)\theta}{(1+n)\left(1+p\beta(1+\theta)\right)+(p+1+n)\theta} \cdot \left\{(1-\alpha)Ak - Rb\right\} + Rb \\
&\quad - (1+n)\frac{p\beta}{1+\alpha p\beta} \cdot \frac{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)}{(1+n)\left(1+p\beta(1+\theta)\right)+(p+1+n)\theta} \cdot \left\{(1-\alpha)Ak - Rb\right\},
\end{aligned}$$

where the second equality is derived by using (16) and (20).

Dividing both sides by $(1-\alpha)Ak$ and rearranging the terms, we obtain:

$$\begin{aligned}
\tau = T(k,b) \equiv\ & \frac{(p+1+n)\theta - (1+n)\frac{p\beta}{1+\alpha p\beta}\left\{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)\right\}}{(1+n)\left(1+p\beta(1+\theta)\right)+(p+1+n)\theta} \\
+ & \frac{(1+n)\left(1+p\beta(1+\theta)\right) + (1+n)\frac{p\beta}{1+\alpha p\beta}\left\{(1-\theta+p\beta) - \alpha\left(1+p\beta(1+\theta)\right)\right\}}{(1+n)\left(1+p\beta(1+\theta)\right)+(p+1+n)\theta} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{b}{k},
\end{aligned} \quad (22)$$

where:

$$\frac{b}{k} = \begin{cases} 0 & \text{for } t = 0, \\ \frac{(1-\theta+p\beta)-\alpha(1+p\beta(1+\theta))}{\theta+\alpha(1+p\beta(1+\theta))} & \text{for } t \geq 1. \end{cases} \quad (23)$$

The remaining task is to show that $g > 0$ and $\tau < 1$ hold $\forall t \geq 0$. In period 0, given $b_0 = 0$, (16) and (22) imply that $g_0 > 0$ and $\tau_0 < 1$ hold for any set of parameters.

Next, consider g in period $t \geq 1$. Equation (16) implies that $g > 0$ holds if and only if $(1-\alpha) - \alpha b/k > 0$ holds. Given (23), the necessary and sufficient condition for $g > 0$ in period $t \geq 1$ becomes $\theta(1+\alpha p\beta) > 0$, which holds for any set of parameters.

Finally, consider τ in period $t \geq 1$. We substitute (23) into (22) and rearrange the terms to obtain:

$$\tau < 1 \Leftrightarrow (1+n) \cdot \left[\underbrace{(1+p\beta(1+\theta)) + \frac{p\beta}{1+\alpha p\beta} \{(1-\theta+p\beta) - \alpha(1+p\beta(1+\theta))\}}_{(*1)} \right] \times \left[\underbrace{\frac{\alpha}{1-\alpha} \cdot \frac{(1-\theta+p\beta) - \alpha(1+p\beta(1+\theta))}{\theta+\alpha(1+p\beta(1+\theta))} - 1}_{(*2)} \right] < 0, \tag{24}$$

where the sign of the term (*1) is positive whereas the sign of the term (*2) is negative. Therefore, the condition (24) holds for any set of parameters.

■

A.3 Proof of Proposition 2

(i) Recall the law of motion of capital demonstrated in Proposition 1. Given the initial condition $b_0 = 0$, the growth rate of capital in period 0, k_1/k_0, is immediately computed as demonstrated in Proposition 2(i).

Next, recall the law of motion of capital in period $t \geq 1$. Dividing both sides of the equation by k_t leads to:

$$\frac{k_{t+1}}{k_t} = \frac{p\beta}{1+\alpha p\beta} \cdot \frac{\theta+\alpha(1+p\beta(1+\theta))}{(1+n)(1+p\beta(1+\theta))+(p+1+n)\theta} \cdot \left\{(1-\alpha) - \alpha\frac{b_t}{k_t}\right\} A.$$

Substitution of the ratio b_t/k_t shown in Proposition 1 into the above expression leads to:

$$\frac{k_{t+1}}{k_t} = \frac{x_{t+1}}{x_t} = \frac{p\beta\theta}{(1+n)(1+p\beta(1+\theta))+(p+1+n)\theta} A \text{ for } t \geq 1.$$

Direct calculation leads to $k_{t+1}/k_t \gtreqless k_1/k_0 \Leftrightarrow \alpha \gtreqless (1-\theta+p\beta)/\{1+p\beta(1+\theta)\}$ for $t \geq 1$.

(ii) The result is immediate from the expression k_1/k_0 and $k_{t+1}/k_t (t \geq 1)$ in Proposition 2(i).

(iii) The growth rate of capital for $t \geq 1$ is reformulated as:

$$\frac{k_{t+1}}{k_t} = \frac{\beta\theta}{(1+n)\left(\frac{1}{p}+\beta(1+\theta)\right)+\left(1+\frac{1+n}{p}\right)\theta} A,$$

indicating that $\partial\left(k_{t+1}/k_t\right)/\partial p > 0$ holds for $t \geq 1$.

The differentiation of k_1/k_0 with respect to p yields:

$$\begin{aligned}
&[(1+\alpha p\beta)\{(1+n)(1+p\beta(1+\theta))+(p+1+n)\theta\}]^2\cdot((1-\alpha)A\beta)^{-1}\cdot\frac{\partial(k_1/k_0)}{\partial p}\\
&=\underbrace{\{\theta+\alpha(1+p\beta(1+\theta))\}\{(1+n)(1+p\beta(1+\theta))+(p+1+n)\theta\}}_{(*3)}\\
&+p\alpha\beta(1+\theta)(1+\alpha p\beta)\{(1+n)(1+p\beta(1+\theta))+(p+1+n)\theta\}\\
&-\underbrace{p(1+\alpha p\beta)\{\theta+\alpha(1+p\beta(1+\theta))\}\{(1+n)\beta(1+\theta)+\theta\}}_{(*4)}.
\end{aligned}$$

The expression indicates that $\partial(k_1/k_0)/\partial p>0$ holds if the term (*3) is greater than the term (*4). After some calculation, we find that:

$$(*3)>\ (*4)\Leftrightarrow p<\left[\frac{(1+n)(1+\theta)}{\alpha\beta\{(1+n)\beta(1+\theta)+\theta\}}\right]^{1/2}.$$

■

A.4 Proof of Proposition 3

In order to solve the problem, we conjecture the following linear policy function:

$$g_{t+1}=G_1\cdot k_{t+1},$$

where $G_1\in(0,\infty)$ is a constant parameter. Under this conjecture and the capital market clearing condition (7), we can reformulate the problem as:

$$\max_{\{g_t\}}(1+n)\{1+p\beta(1+\theta)\}\ln((1-\alpha)Ak_t-g_t)+(p+1+n)\theta\ln g_t.$$

Solving this problem leads to the following policy function:

$$g_t=\frac{(p+1+n)\theta}{(1+n)\{1+p\beta(1+\theta)\}+(p+1+n)\theta}(1-\alpha)Ak_t.$$

This function constitutes a stationary Markov-perfect political equilibrium as long as $G_1=(p+1+n)\cdot[(1+n)\{1+p\beta(1+\theta)\}+(p+1+n)\theta]^{-1}\cdot(1-\alpha)A$. The tax rate becomes:

$$\tau_t=\frac{g_t}{w_t}=\frac{(p+1+n)\theta}{(1+n)\{1+p\beta(1+\theta)\}+(p+1+n)\theta}\in(0,1).$$

We substitute the policy function into the constraint (7) to obtain the law of motion for capital:

$$\frac{k_{t+1}}{k_t}=\frac{p\beta}{1+p\beta}\cdot\frac{\{1+p\beta(1+\theta)\}}{(1+n)\{1+p\beta(1+\theta)\}+(p+1+n)\theta}\cdot(1-\alpha)A.$$

■

A.5 Proof of Proposition 4

(i) The first statement is immediate from the results in Propositions 1 and 3. To show the second statement, we make a direct comparison:

$$\left.\frac{g}{Ak}\right|_{\text{Debt}} \lesseqqgtr \left.\frac{g}{Ak}\right|_{\text{Balanced}} \Leftrightarrow \frac{Rb}{Ak} \gtreqqless 0 \Leftrightarrow b \gtreqqless 0 \Leftrightarrow \alpha \lesseqqgtr \frac{1+p\beta-\theta}{1+p\beta(1+\theta)},$$

where the last identity comes from the result in Proposition 1.

(ii) For $t \geq 0$, direct comparison leads to the result.

■

A.6 Proof of Proposition 5

Recall the capital market clearing condition and the government budget constraint, given by (17) and (18), respectively. Given k and b, an allocation (k', b', τ, g) is characterized by the first-order conditions with respect to x' and g, (8) and (9), the capital market clearing condition, (17), and the government budget constraint, (18).

Substitution of (9) and (18) into (17) leads to:

$$k' = \frac{\beta}{1+p\beta} \cdot \frac{(1+p\beta)+(p+1+n)\beta\theta}{(1+n)(1+p\beta)+(p+1+n)\theta} \cdot \{(1-\alpha)Ak - Rb\} - \frac{1}{1+p\beta} b'. \tag{25}$$

We reformulate the condition (8) as:

$$b' = \frac{1-\alpha}{\alpha} k' - \beta A \left\{ \frac{1-\alpha}{\alpha} k - b \right\},$$

and substitute (25) into this expression to obtain:

$$\begin{aligned}\left(1 + \frac{1-\alpha}{\alpha} \cdot \frac{1}{1+p\beta}\right) b' = \frac{\beta}{\alpha} &\left[\frac{p(1-\alpha)}{1+p\beta} \cdot \frac{(1+p\beta)+(p+1+n)\beta\theta}{(1+n)(1+p\beta)+(p+1+n)\theta} - 1 \right] \\ &\times \{(1-\alpha)Ak - Rb\}.\end{aligned}$$

After rearranging the terms, we obtain the policy function of b' demonstrated in Proposition 5(i).

Substitution of the policy function of b' into (17) leads to the law of motion of capital demonstrated in the proposition. The ratio of b/k is determined by using the equations for b' and k'. The tax rate is determined by substituting the policy functions of g and b' into the government budget constraint.

(ii) Recall the law of motion of capital demonstrated in Proposition 5(i). Given $b_0 = 0$, the growth rate of capital in period 0, k_1/k_0, is immediately obtained. Next, divide both sides of the law of motion of capital by k_t :

$$\frac{k_{t+1}}{k_t} = \frac{\beta\left[(1+p\beta)\{(1+n)+\alpha p\} + (1+\alpha p\beta)(p+1+n)\theta\right]}{(1+\alpha p\beta)\left\{(1+n)(1+p\beta)+(p+1+n)\theta\right\}} \left\{ (1-\alpha)A - \frac{Rb_t}{k_t} \right\}.$$

We substitute the ratio of b/k demonstrated in the first part of the proposition into the above expression and rearrange the terms. Then we obtain $k_{t+1}/k_t = \beta A$ for $t \geq 1$.

■

A.7 Proof of Proposition 6

(i) Consider the government spending-to-GDP ratio in period 0. By direct comparison, we see that:

$$\left.\frac{g_0}{Ak_0}\right|_{\text{Ramsey}} < \left.\frac{g_0}{Ak_0}\right|_{\text{Debt}} \Leftrightarrow 0 < (1+n)(1+p\beta(1+\theta)) + (1+n)\theta,$$

which holds for any set of parameters.

For $t \geq 1$, the ratios are given by:

$$\begin{aligned}\left.\frac{g}{Ak}\right|_{\text{Ramsey}} &= \frac{\{1-\beta(1+n)\}(p+1+n)\theta}{(1+n)(1+p\beta)+(p+1+n)\theta}\left\{(1-\alpha)-\alpha\left.\frac{b}{k}\right|_{\text{Ramsey}}\right\}A,\\ \left.\frac{g}{Ak}\right|_{\text{Debt}} &= \frac{(p+1+n)\theta}{(1+n)(1+p\beta(1+\theta))+(p+1+n)\theta}\left\{(1-\alpha)-\alpha\left.\frac{b}{k}\right|_{\text{Debt}}\right\}A.\end{aligned}$$

With the use of the results demonstrated so far, the term $(1-\alpha)-\alpha b/k$ is computed as:

$$\begin{aligned}(1-\alpha)-\alpha\left.\frac{b}{k}\right|_{\text{Ramsey}} &= \frac{(1+\alpha p\beta)\{(1+n)(1+p\beta)+(p+1+n)\theta\}}{(1+p\beta)\{(1+n)+\alpha p\}+(1+\alpha p\beta)(p+1+n)\theta},\\ (1-\alpha)-\alpha\left.\frac{b}{k}\right|_{\text{Debt}} &= \frac{\theta(1+\alpha p\beta)}{\theta+\alpha(1+p\beta(1+\theta))}.\end{aligned}$$

Therefore, $g/Ak|_{\text{Ramsey}}$ and $g/Ak|_{\text{Debt}}$ become:

$$\begin{aligned}\left.\frac{g}{Ak}\right|_{\text{Ramsey}} &= \frac{\{1-\beta(1+n)\}(p+1+n)\theta(1+\alpha p\beta)}{(1+p\beta)\{(1+n)+\alpha p\}+(1+\alpha p\beta)(p+1+n)\theta}A,\\ \left.\frac{g}{Ak}\right|_{\text{Debt}} &= \frac{(p+1+n)\theta(1+\alpha p\beta)\theta}{\{(1+n)(1+p\beta(1+\theta))+(p+1+n)\theta\}\{\theta(1+\alpha p\beta)+\alpha(1+p\beta)\}}A.\end{aligned}$$

By comparing these, we have:

$$\begin{aligned}&\left.\frac{g}{Ak}\right|_{\text{Ramsey}} \lesseqgtr \left.\frac{g}{Ak}\right|_{\text{Debt}}\\ &\Leftrightarrow \left[\underbrace{\{1-\beta(1+n)\}(1+p\beta(1+\theta))-\beta(p+1+n)\theta}_{(*5)}\right]\cdot[\theta(1+\alpha p\beta)+\alpha(1+p\beta)]\\ &\lesseqgtr \theta(1+p\beta)(1-\alpha), \qquad (26)\end{aligned}$$

where the sign of the term (*5) is:

$$(*5) \gtreqless 0 \Leftrightarrow 1 \gtreqless \beta(1+n)(1+\theta).$$

(a) Suppose that $1 \leq \beta(1+n)(1+\theta)$ holds: that is, (*5) ≤ 0 holds. The left-hand side of (26) is nonpositive, while the right-hand side is positive. Therefore, $g/Ak|_{\text{Ramsey}} < g/Ak|_{\text{Debt}}$ always holds.

(b) Alternatively, suppose that $1 > \beta(1+n)(1+\theta)$ holds: that is, (*5) > 0 holds. The condition (26) is rewritten as:

$$\left.\frac{g}{Ak}\right|_{\text{Ramsey}} \lesseqgtr \left.\frac{g}{Ak}\right|_{\text{Debt}} \Leftrightarrow \alpha \lesseqgtr \tilde{\alpha}$$

where $\tilde{\alpha}$ is defined as in the statement of the proposition.

(ii) Recall the growth rate in the Ramsey allocation and that in the unbalanced budget political equilibrium in period 0. By direct comparison, we see that:

$$\left.\frac{k_1}{k_0}\right|_{\text{Ramsey}} \gtrless \left.\frac{k_1}{k_0}\right|_{\text{Debt}} \Leftrightarrow \frac{(1+p\beta)\{(1+n)+\alpha p\} + (1+\alpha p\beta)(p+1+n)\theta}{(1+n)(1+p\beta)+(p+1+n)\theta}$$
$$\gtrless \frac{p\{\theta + \alpha(1+p\beta(1+\theta))\}}{(1+n)(1+p\beta(1+\theta))+(p+1+n)\theta},$$

where the numerator on the left-hand side is greater than the numerator on the right-hand side, and the denominator on the left-hand side is smaller than the denominator on the right-hand side. Therefore, $\left.\frac{k_1}{k_0}\right|_{\text{Ramsey}} > \left.\frac{k_1}{k_0}\right|_{\text{Debt}}$ holds for any set of parameters.

For $t \geq 1$, we see that:

$$\left.\frac{k'}{k}\right|_{\text{Ramsey}} \gtrless \left.\frac{k'}{k}\right|_{\text{Debt}} \Leftrightarrow 1 > \frac{p\theta}{(1+n)(1+p\beta(1+\theta))+(p+1+n)\theta},$$

which holds for any set of parameters.

■

B Discussion and Extension

In the main analysis of the paper, we have assumed that (i) a government does not tax capital income; (ii) labor supply is inelastic; and (iii) individual preferences are represented by a logarithmic utility function. In this appendix, we examine how the analysis and results would change if either of these assumptions is relaxed.

B.1 Capital Income Taxation

This section introduces capital income tax as an additional means to raise tax revenues. Although an analytical solution is unavailable for this case, we can identify the condition for a zero capital tax.

Constraints and optimality conditions are modified as follows. The budget constraint in old age is given by $c_{t+1}^o = \left(1-\tau_{t+1}^k\right)\tilde{R}_{t+1}s_t$ where $\tau_{t+1}^k \in [0,1]$ is the capital income tax in period $t+1$. Given this constraint, the solution to the utility maximization problem becomes:

$$c_t^y = \frac{1}{1+p\beta}\left(1-\tau_t\right)w_t, c_{t+1}^o = \frac{p\beta}{1+p\beta}\left(1-\tau_{t+1}^k\right)\tilde{R}_{t+1}\left(1-\tau_t\right)w_t, s_t = \frac{p\beta}{1+p\beta}\left(1-\tau_t\right)w_t.$$

Because of a logarithmic utility function, the capital income tax rate has no direct effect on saving. However, it has an effect on the old-age consumption via the return of saving. Firm's profit-maximization conditions are the same as before. The government budget constraint is modified as $(1+n)b_{t+1}+\tau_t w_t+\tau_t^k\tilde{R}_t s_{t-1} = g_t + R_t b_t$ where the third term on the left-hand side, $\tau_t^k\tilde{R}_t s_{t-1}$, denotes the revenue from the capital income tax.

With factor market clearing conditions and an arbitrage condition, the capital market clearing condition $(1+n)(k_{t+1}+b_{t+1}) = s_t$ is rewritten as:

$$k_{t+1} = \frac{1}{1+n}\frac{p\beta}{1+p\beta}\left((1-\alpha)Ak_t - g_t - Rb_t + \tau_t^k\frac{R}{p}(1+n)(k_t+b_t)\right) - \frac{1}{1+p\beta}b_{t+1}, \quad (27)$$

where the term $\tau_t^k(R/p)(1+n)(k_t+b_t)$ that appears in the parentheses shows the capital income tax revenue.

The indirect utility functions of the young and the old are given by, respectively:

$$\begin{aligned} V_t^y &= (1+p\beta)\ln\left((1-\alpha)Ak_t - g_t - Rb_t + (1+n)b_{t+1} + \tau_t^k\frac{R}{p}(1+n)(k_t+b_t)\right) \\ &\quad + p\beta\ln\left(1-\tau_{t+1}^k\right) + \theta\ln g_t + p\beta\theta\ln g_{t+1}, \\ V_t^o &= \ln\left(1-\tau_t^k\right) + \theta\ln g_t, \end{aligned}$$

where the terms unrelated to political decisions are omitted from the expressions above. There are three new terms in the expression of the indirect utility functions. The term $\tau_t^k\left(R/p\right)(1+n)(k_t+b_t)$ in V_t^y shows the capital income tax revenue returned to the current young; and the terms $\left(1-\tau_{t+1}^k\right)$ in V_t^y and $\left(1-\tau_t^k\right)$ in V_t^o show the capital income tax burden.

The political objective function in period t is $\Omega_t = pV_t^o + (1+n)V_t^y$. Recall the variable $x_t \equiv (1-\alpha)Ak_t - Rb_t$. By using this variable, the political objective function is written as:

$$\begin{aligned}\Omega_t &= p\ln\left(1-\tau_t^k\right) + (p+(1+n))\,\theta\ln g_t \\ &\quad + (1+n)(1+p\beta)\ln\left(x_t - g_t + (1+n)b_{t+1} + \tau_t^k\frac{R}{p}(1+n)(k_t+b_t)\right) \\ &\quad + (1+n)p\beta\ln\left(1-\tau_{t+1}^k\right) + (1+n)p\beta\theta\ln g_{t+1};\end{aligned}$$

and the condition (27) is rewritten as:

$$\begin{aligned}(1+n)b_{t+1} &= \left(\frac{R}{(1-\alpha)A} + \frac{1}{1+p\beta}\right)^{-1} \\ &\quad\times\left[\frac{p\beta}{1+p\beta}\left(x_t - g_t + \tau_t^k\frac{R}{p}(1+n)(k_t+b_t)\right) - \frac{1+n}{(1-\alpha)A}x_{t+1}\right].\end{aligned}$$

Substituting this condition into the objective function Ω_t leads to:

$$\begin{aligned}\Omega_t &= p\ln\underbrace{\left(1-\tau_t^k\right)}_{(d.1)} + (p+(1+n))\,\theta\ln g_t + (1+n)(1+p\beta)\ln\Bigg[(x_t-g_t) \\ &\quad + \left(\frac{R}{(1-\alpha)A} + \frac{1}{1+p\beta}\right)^{-1}\cdot\left\{\frac{p\beta}{1+p\beta}\left(x_t - g_t + \underbrace{\tau_t^k\frac{R}{p}(1+n)(k_t+b_t)}_{(d.2)}\right)\right. \\ &\quad \left.-\frac{1+n}{(1-\alpha)A}x_{t+1}\right\} + \underbrace{\tau_t^k\frac{R}{p}(1+n)(k_t+b_t)}_{(d.3)}\Bigg] + (1+n)p\beta\ln\underbrace{\left(1-\tau_{t+1}^k\right)}_{(d.4)} + (1+n)p\beta\theta\ln\underbrace{g_{t+1}}_{(d.5)}.\end{aligned}$$

The objective of the period-t government is to choose $\{\tau_t^k, \tau_{t+1}^k, g_t, g_{t+1}, x_{t+1}\}$ to maximize Ω_t given x_t, b_t, and k_t.

To solve the problem, we guess the following policy functions:

$$\tau_t^k = \tau^k \text{ and } g_{t+1} = G_2\cdot x_{t+1},$$

where $\tau^k \in [0,1]$ and $G_2 \in (0,\infty)$ are constant parameters. The reason for guessing that the capital tax rate is constant over time is that the tax rate goes to $+\infty$ or $-\infty$ if it depends on the state variable. Given the guess above, we derive the condition for $\tau_t^k = 0$ to be optimal:

$$\begin{aligned}\tau_t^k = 0 \text{ if } &(-1)p(1+(1+n)\beta) \\ &+(1+n)(1+p\beta)\frac{\left(1+\frac{1-\alpha}{1+p\beta}\right)A\frac{R}{p}(1+n)(k_t+b_t)}{A(x_t-g_t)-(1+n)x_{t+1}} \le 0. \qquad (28)\end{aligned}$$

When $\tau_t^k = 0$, the optimal levels of g_t and x_{t+1} are the same as those obtained in the case without capital income tax:

$$g_t = \frac{(p + (1+n))\,\theta}{(1+n)\left\{1 + p\beta(1+\theta)\right\} + (p + (1+n))\theta} x_t;$$
$$x_{t+1} = \frac{\theta p \beta A}{(1+n)\left\{1 + p\beta(1+\theta)\right\} + (p + (1+n))\theta} x_t.$$

Plugging these into the $\tau_t^k = 0$ condition in (28), we obtain:

$$\frac{b_0}{k_0} = 0 < \frac{p(1+(1+n)\beta)\frac{1-\alpha}{\alpha} - \left(1 + \frac{1-\alpha}{1+p\beta}\right)\frac{1+n}{p}\left[(1+n)\{1+p\beta(1+\theta)\} + (p+(1+n))\theta\right]}{p(1+(1+n)\beta) + \left(1 + \frac{1-\alpha}{1+p\beta}\right)\frac{1+n}{p}\left[(1+n)\{1+p\beta(1+\theta)\} + (p+(1+n))\theta\right]}. \tag{29}$$

Therefore, the zero capital income tax is chosen via politics if the first term on the numerator is greater than the second term. Such a situation holds true for a low value of α.

To understand the condition (29), recall the government objective function Ω_t. The function says that the introduction of the capital income tax incurs two kinds of costs: a tax burden on the old, represented by the term (d.1), and a tax burden on the young via the return of savings, represented by the term (d.4).

The function also says that the introduction of capital income tax decreases the labor income tax rate, increases after-tax labor income of the households, and thus creates two benefits: increase in consumption represented by the term (d.2) and (d.3), and an increase in the provision of future public goods via an increase in savings (i.e., capital), represented by the term (d.5).

These benefits depend on the rate of return on saving, R, that is, the capital share, α, as shown in the terms (d.2) and (d.3). In particular, the benefits become smaller as α becomes lower. We can find a critical value of α, such that the costs outweigh the benefits when α is below the critical value. That is, it is optimal to set $\tau_t^k = 0$ if the ratio α is below the critical value.

B.2 Endogenous Labor-Leisure Choice

The benchmark model assumes that every individual inelastically supplies one unit of labor in youth. We here modify the model by introducing an individual's labor-leisure choice decision. In particular, we assume that the utility of the young agent in period t is given by $\gamma \ln c_t^y + (1-\gamma)\ln l_t + p\beta \ln c_{t+1}^o$ where $l_t \in [0,1]$ is time for leisure, and $\gamma \in [0,1]$ and $1-\gamma$ are parameters representing the preferences for youthful consumption and leisure, respectively. The lifetime budget constraint is modified as $c_t^y + c_{t+1}^o/\tilde{R}_{t+1} \leq (1-\tau_t)w_t(1-l_t)$.

Under this modified setting, optimal consumption and leisure are:

$$c_t^y = \frac{\gamma}{1+p\beta}(1-\tau_t)w_t, c_{t+1}^o = \frac{p\beta}{1+p\beta}(1-\tau_t)w_t, l_t = \bar{l} \equiv \frac{1-\gamma}{1+p\beta},$$

where $\bar{l} = 1$ if $\gamma = 1$. The optimal level of leisure is independent of the disposable income $(1-\tau_t)w_t$ because an income effect is offset by a substitution effect under a logarithmic utility function.

The government budget constraint becomes $(1+n)b_{t+1} + \tau_t w_t(1-\bar{l}) = g_t + Rb_t$, or:

$$\tau_t w_t = \frac{1}{1-\bar{l}} \{g_t + Rb_t - (1+n)b_{t+1}\}.$$

By the use of this constraint and the factor market clearing conditions in Section 3, we obtain:

$$\begin{aligned}(1-\tau_t)w_t &= w_t - \frac{1}{1-\bar{l}}(g_t + Rb_t - (1+n)b_{t+1}) \\ &= \left((1-\alpha)Ak_t - \frac{1}{1-\bar{l}}Rb\right) - \frac{1}{1-\bar{l}}((g_t - (1+n)b_{t+1})).\end{aligned}$$

Because $\bar{l}$ is constant and is independent of the state and policy variables, we can apply the analysis procedure in the previous sections by redefining $x_t \equiv (1-\alpha)Ak_t - Rb_t/(1-\bar{l})$. Therefore, the analysis and results are qualitatively unchanged when we keep the assumption of a logarithmic utility function. However, if we adopt a more generalized utility function, the leisure l_t would depend on the state and policy variables; the analysis and results would essentially change. Furthermore, an analytical solution would be unavailable because of a labor-leisure interaction.

B.3 A Constant Intertemporal Elasticity of Substitution Utility Function

At this point, we have conducted an analysis by assuming a logarithmic utility function. This specification makes an analysis tractable, but results in a saving function which is independent of the interest rate. This subsection introduces a constant intertemporal elasticity of substitution utility function to resolve this problem. The main result of this subsection is that an analytical solution is still available for this generalization of the utility function.

For the purpose of analysis, we assume the following lifetime utility function:

$$U_t^y = \frac{(c_t^y)^{1-\sigma} - 1}{1-\sigma} + \theta\frac{(g_t)^{1-\sigma} - 1}{1-\sigma} + p\beta \cdot \left[\frac{\left(c_{t+1}^o\right)^{1-\sigma} - 1}{1-\sigma} + \theta\frac{(g_{t+1})^{1-\sigma} - 1}{1-\sigma}\right],$$

where $\sigma > 0$. If $\sigma = 1$, then the above expression is reduced to a logarithmic utility function in the benchmark model. A young individual at time t chooses saving to maximize this utility subject to the lifetime budget constraint.

Following the same procedure as in Section 3, we can derive the political objective function given by:

$$\Omega_t = \frac{1+n}{1-\sigma}\phi(p)\left\{A(x_t - g_t) - (1+n)x_{t+1}\right\}^{1-\sigma} + (p+(1+n))\theta\frac{(g_t)^{1-\sigma}}{1-\sigma} + (1+n)p\beta\theta\frac{(g_{t+1})^{1-\sigma}}{1-\sigma}, \tag{30}$$

where:

$$\phi(p) \equiv \left[\left(\frac{R/p}{R/p + (R\beta)^{1/\sigma}}\right)^{\sigma} \cdot ((1-\alpha)A)^{1-\sigma} \cdot \left\{\frac{R}{(1-\alpha)A} + \frac{R/p}{R/p + (R\beta)^{1/\sigma}}\right\}^{1-\sigma}\right]^{-1},$$

and the terms unrelated to political decision are omitted from the expression. The function is reduced to the one obtained in the logarithmic utility function case if $\sigma \to 1$. The process of deriving the above objective function is given in Appendix B.4.

In order to obtain a solution, we guess that $g_{t+1} = G_3 \cdot x_{t+1}$ where $G_3 \in (0, \infty)$ is a constant parameter. Under this conjecture, we solve the problem of maximizing Ω_t and obtain the following policy functions:

$$g_t = A \cdot \left[\left\{\frac{(1+n)\phi(p)A}{(p+(1+n))\theta}\right\}^{1/\sigma} + A + (1+n)\left\{\frac{Ap\beta\,(G_3)^{1-\sigma}}{p+(1+n)}\right\}^{1/\sigma}\right]^{-1} \cdot x_t,$$

$$x_{t+1} = A\left\{\frac{Ap\beta\,(G_3)^{1-\sigma}}{p+(1+n)}\right\}^{1/\sigma} \cdot \left[\left\{\frac{(1+n)\phi(p)A}{(p+(1+n))\theta}\right\}^{1/\sigma} + A + (1+n)\left\{\frac{Ap\beta\,(G_3)^{1-\sigma}}{p+(1+n)}\right\}^{1/\sigma}\right]^{-1} \cdot x_t.$$

These functions constitute a Markov-perfect political equilibrium as long as G_3 satisfies the following equation:

$$G_3 = A \cdot \left[\left\{\frac{(1+n)\phi(p)A}{(p+(1+n))\theta}\right\}^{1/\sigma} + A + (1+n)\left\{\frac{Ap\beta\,(G_3)^{1-\sigma}}{p+(1+n)}\right\}^{1/\sigma}\right]^{-1},$$

or:

$$\left[\left\{\frac{(1+n)\phi(p)A}{(p+(1+n))\theta}\right\}^{1/\sigma} + A\right] \cdot G_3 + (1+n)\left\{\frac{Ap\beta}{p+(1+n)}\right\}^{1/\sigma}(G_3)^{1/\sigma} = A. \tag{31}$$

The left-hand side of (31), denoted by LHS, satisfies the following properties:

$$\partial LHS/\partial G_3 > 0,\ \lim_{G_3 \to 0} G_3 = 0,\ \lim_{G_3 \to \infty} G_3 = \infty.$$

Therefore, there exists a unique $G_3 \in (0, \infty)$ satisfying (31).

The derived policy functions indicate that there are new effects of a population growth rate n and longevity p, which are not observed in the benchmark model. However, it is difficult to explore their qualitative implications because of the nonlinearity of the equation (31) that determines G_3. We need to rely on a quantitative method, which will be left for future work.

B.4 Derivation of the Political Objective Function

First, we solve an individual utility maximization problem and obtain the following optimal solutions of consumption and savings:

$$c_t^y = \frac{\tilde{R}_{t+1}}{\tilde{R}_{t+1} + \left(\tilde{R}_{t+1} p\beta\right)^{1/\sigma}}(1-\tau_t)w_t; c_{t+1}^o = \frac{\tilde{R}_{t+1}\left(\tilde{R}_{t+1} p\beta\right)^{1/\sigma}}{\tilde{R}_{t+1} + \left(\tilde{R}_{t+1} p\beta\right)^{1/\sigma}}(1-\tau_t)w_t;$$

$$s_t = \frac{\left(\tilde{R}_{t+1} p\beta\right)^{1/\sigma}}{\tilde{R}_{t+1} + \left(\tilde{R}_{t+1} p\beta\right)^{1/\sigma}}(1-\tau_t)w_t.$$

With factor markets clearing conditions, $w_t = (1-\alpha)Ak_t$ and $R_t = R \equiv \alpha A$, and an arbitrage condition, $\tilde{R}_t = R/p$, these solutions are rewritten as:

$$c_t^y = \frac{R/p}{(R/p) + (R\beta)^{1/\sigma}}(1-\tau_t)(1-\alpha)Ak_t; c_{t+1}^o = \frac{(R/p)(R\beta)^{1/\sigma}}{(R/p) + (R\beta)^{1/\sigma}}(1-\tau_t)(1-\alpha)Ak_t;$$

$$s_t = \frac{(R\beta)^{1/\sigma}}{(R/p) + (R\beta)^{1/\sigma}}(1-\tau_t)(1-\alpha)Ak_t.$$

Second, let us consider the capital market clearing condition. Recall that the government budget constraint is $(1+n)b_{t+1} + \tau_t(1-\alpha)Ak_t = g_t + Rb_t$. Plugging this into the capital market clearing condition $(1+n)(k_{t+1} + b_{t+1}) = s_t$, we obtain:

$$(1+n)(k_{t+1} + b_{t+1}) = \frac{(R\beta)^{1/\sigma}}{(R/p) + (R\beta)^{1/\sigma}}\left[(1-\alpha)Ak_t - g_t - Rb_t + (1+n)b_{t+1}\right].$$

With the definition of $x_t \equiv (1-\alpha)Ak_t - Rb_t$, we can rearrange the above expression as:

$$\begin{aligned}(1+n)b_{t+1} = &\left[\frac{R}{(1-\alpha)A} + \frac{R/p}{(R/p) + (R\beta)^{1/\sigma}}\right]^{-1} \\ &\times \left[\frac{(R\beta)^{1/\sigma}}{(R/p) + (R\beta)^{1/\sigma}}(x_t - g_t) - \frac{1+n}{(1-\alpha)A}x_{t+1}\right]. \end{aligned} \tag{32}$$

Third, we write down the political objective function. For this purpose, we first calculate the indirect utility functions of the young and the old as follows:

$$V_t^y = \frac{1}{1-\sigma}\left(\frac{R/p}{(R/p)+(R\beta)^{1/\sigma}}\right)^{-\sigma}((1-\tau_t)w_t)^{1-\sigma} + \theta\frac{(g_t)^{1-\sigma}}{1-\sigma} + p\beta\theta\frac{(g_{t+1})^{1-\sigma}}{1-\sigma} - \frac{(1+p\beta)(1+\theta)}{1-\sigma};$$

$$V_t^o = \frac{1}{1-\sigma}\left(\frac{R}{p}(1+n)\right)^{1-\sigma}(k_t+b_t)^{1-\sigma} + \theta\frac{(g_t)^{1-\sigma}}{1-\sigma} - \frac{(1+\theta)}{1-\sigma}.$$

Plugging these functions into the political objective function $\Omega_t = pV_t^o + (1+n)V_t^y$ and rearranging the terms, we obtain:

$$\begin{aligned}\Omega_t = &\frac{1+n}{1-\sigma}\left(\frac{R/p}{(R/p)+(R\beta)^{1/\sigma}}\right)^{-\sigma}[(1-\alpha)Ak_t - g_t - Rb_t + (1+n)b_{t+1}] \\ &+ (p+(1+n))\theta\frac{(g_t)^{1-\sigma}}{1-\sigma} + (1+n)p\beta\theta\frac{(g_{t+1})^{1-\sigma}}{1-\sigma} \qquad (33)\end{aligned}$$

where the terms unrelated to political decisions are omitted from the expression. Substitution of (32) into (33), we obtain (30).

References

[1] Acemoglu, D., and Robinson, J.A.,2005. *Economic Origins of Dictatorship and Democracy,* Cambridge University Press.

[2] Azariadis, C., and Galasso, V., 2002. Fiscal constitutions. *Journal of Economic Theory* 103, 255–281.

[3] Bassetto, M., 2008. Political economy of taxation in an overlapping-generations economy. *Review of Economic Dynamics* 11, 18–43.

[4] Beauchemin, K.R., 1998. Intergenerational politics, fiscal policy and productivity. *Review of Economic Dynamics* 1, 835–858.

[5] Cukierman, A., and Meltzer, A.H., 1989. A political theory of government debt and deficits in a neo-Ricardian framework. *American Economic Review* 79, 713–732.

[6] Forni, L., 2005. Social security as Markov equilibrium in OLG models. *Review of Economic Dynamics* 8, 178-194.

[7] Gonzalez-Eiras, M., and Niepelt, D., 2008. The future of social security. *Journal of Monetary Economics* 55, 197-218.

[8] Gonzalez-Eiras, M., 2011. Social security as Markov equilibrium in OLG models: a note. *Review of Economic Dynamics* 14, 549–552.

[9] Gradstein, M., and Kaganovich, M., 2004. Aging population and education finance. *Journal of Public Economics*, 88, 2469–2485.

[10] Gonzalez-Eiras, M., and Niepelt, D.,2011. Ageing, government budgets, retirement, and growth. European Economic Review 56, 97–115.

[11] Grossman, G.M., and Helpman, E., 1998. Intergenerational redistribution with short-lived governments. *Economic Journal* 108, 1299–1329.

[12] Hassler, J., Storesletten, K., Rodriguez Mora, J.V., and Zilibotti, F., 2003. The survival of the welfare state. *American Economic Review* 93, 87–112.

[13] Hassler, J., Krusell, P., Storesletten, K., and Zilibotti, F., 2005. The dynamics of government. *Journal of Monetary Economics* 52, 1331–1358.

[14] Hassler, J., Storesletten, K., and Zilibotti, F., 2007. Democratic public good provision. *Journal of Economic Theory* 133, 127–151.

[15] Holtz-Eakin, D., Lovely, M.E., and Tosun, M.S., 2004. Generational conflict, fiscal policy, and economic growth. *Journal of Macroeconomics* 26, 1–23.

[16] Iturbe-Ormaetxe, I., and Valera, G., 2012. Social security reform and the support for public education. *Journal of Population Economics* 25, 609–634.

[17] Kaganovich, M., and Meier, V., 2012. Social security systems, human capital, and growth in a small open economy. *Journal of Public Economic Theory* 14, 573–600.

[18] Krusell, P., Quadrini, V., and Rios-Rull, J-V.„ 1997. Politico-economic equilibrium and economic growth. *Journal of Economic Dynamics and Control* 21, 243–272.

[19] Kuehnel , J., 2011. Population aging, the composition of government spending, and endogenous growth in politico-economic equilibrium. Department of Economics, University of Heidelberg, Discussion Paper Series No. 510.

[20] Lindbeck, A., and J. Weibull. 1987. Balanced budget redistribution and the outcome of political competition. *Public Choice* 52, 273–297.

[21] Lucas, R. Jr., 1988. On the mechanics of economic development. *Journal of Monetary Economics* 22, 3–42.

[22] OECD, 2011. *OECD Factbook 2011-2012: Economic, Environmental and Social Statistics*, OECD Publishing.

[23] Persson, T., Tabellini, G., 2000. *Political Economics: Explaining Economic Policy.* MIT Press.

[24] Razin, A., Sadka, E., and Swagel, P., 2002. The aging population and the size of the welfare state. *Journal of Political Economy* 110, 900–918.

[25] Röhrs, S., 2010. Public debt in a political economy. Available at http://www.econ.uzh.ch/faculty/roehrs/research/Paper2_20100706.pdf

[26] Romer, Paul M., 1986. Increasing returns and long run growth. *Journal of Political Economy* 94, 1002—1037.

[27] Song, Z., 2011. The dynamics of inequality and social security in general equilibrium. *Review of Economic Dynamics* 14, 613–635.

[28] Song, Z., Storesletten, K., and Zilibotti, F., 2012. Rotten parents and disciplined children: a politico-economic theory of public expenditure and debt. *Econometrica* 80, 2785–2803.

[29] Tosun, M.S., 2008. Endogenous fiscal policy and capital market transmissions in the presence of demographic shocks. *Journal of Economic Dynamics and Control* 32, 2031–2060.