A Price-Differentiation Model of the Interbank Market and Its Empirical Application

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Abstract

We build a model of short-term interbank loans. The variation across different banks in their cost from handling an excess or a deficit of liquidity ('liquidity cost') drives the variation in interest rates. We characterize the shape of the interest rate curve as a function of loan size and find that a small bank that trades with a large bank tends to get better interest rates for larger loans. The model is consistent with the following observations on the Mexican interbank market, obtained from a unique dataset of interbank loan transactions: (i) the variation in the interest rates on the loans between two large banks is small; (ii) small banks lending to (borrowing from) large banks receive lower (pay higher) interest rates than large banks lending to (borrowing from) other large banks; and (iii) a small bank trading with a large bank gets more favorable interest rates for larger loans. Finally, as an application of the model, we discuss how banking environment changes during a financial crisis. In particular, we estimate the shape of the liquidity cost function and use that information to measure the shift in the liquidity cost that banks faced during the 2008 financial crisis. We find that the increased disadvantage that small banks experienced in the interbank market during the crisis can largely be explained by a shift in the liquidity cost, rather than by changes in loan supply and demand.

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1 Introduction

In this paper, we build a model of bank behavior in the overnight interbank market and develop an empirical framework to relate changes in the interbank market to changes in the liquidity condition of the financial system. We focus on the interaction between a large and central bank and a relatively small bank[1] because it is where most of the interesting variations in interest rates occur in the data. We abstract away from the fact that these banks form a trading network[2] and study the interaction between a large bank and a small bank in isolation.

The importance of liquidity in the financial system has been studied extensively in the economics and finance literature. However, there are relatively few studies that develop an empirically motivated framework to understand the behavior of a bank in a marketplace of liquidity.[3] One of the reasons is that an interbank market of short-term loans is typically an over-the-counter market, and the transactions in the market are not usually reported to the public. We take advantage of a unique dataset of interbank market transactions from Bank of Mexico which is constructed from the reports from individual banks.[4]

Existing empirical papers on interbank markets study various aspects of the market. One set of these papers (Furfine (2001), Cocco, Gomes and Martins (2009) and Ashcraft and Duffie (2007a and 2007b))[5] typically estimate regression models to measure how interest rates are affected by various factors, such as default risk and 'relationship strength,' which

[1]An important question is how we can classify a bank as large or small, and we will discuss the exact classification in the main section of the paper. The idea that banks show different behavior based on their size is not new, and has been employed in empirical studies of interbank markets, such as Furfine (1999) and Afonso, Kovner and Schoar (2011). These papers used different quantiles from bank asset size distribution to classify a bank as large or small. Our approach is different and based on distinctive behaviors of a few largest banks in the system.

[2]There is a growing literature on trading networks. Acemoglu, Ozdaglar and Tahbaz-Salehi (2014) is a recent example of a theoretical study. On the empirical side, Cocco, Gomes and Martins (2009) and Aram and Christophersen (2010) employ common network characteristics of each bank such as centrality to explain some of the variation in the interest rates in the interbank market. Bech and Atalay (2008) computes various common network metrics for the observed trading network in the Federal Funds Market, an interbank loan market in the United States.

[3]There are models of bank behavior in the interbank market at a more abstract level; Ho and Saunders (1985), Coleman, Gilles and Labadi (1996), Gofman (2013) and Afonso and Lagos (2012) are examples. In addition, there are models that are concerned with over-the-counter markets in general, such as Duffie, Gârleanu and Pederson (2005) and Atkeson, Eisfeldt and Weill (2013).

[4]Existing empirical studies of interbank loan markets mostly reconstruct the interbank market transactions from the records of large-value payments between banks. Furfine (1999) invented the procedure. However, there are debates on how reliable it is; Armantier and Copeland (2012) argues that the procedure is highly unreliable.

[5]See Furfine (2001) and Cocco, Gomes and Martins (2009) for a conventional empirical setup in the literature. Ashcraft and Duffie (2007a and 2007b) refine earlier studies on the Federal Funds Market by using more detailed information on individual banks' liquidity state.

is typically defined as the frequency of past interaction for a given lender-borrower pair. Another set of papers (Afonso, Kovner and Schoar (2011) and Allen et al. (2012))[6] study how the interbank market outcomes reflect the state of the financial system. The unique contribution of our approach is to build a model that can rationalize and explain observed empirical regularities with simple assumptions on the primitives of a bank, more specifically, its cost in dealing with an excess or a deficit of liquidity.

In our model of the interbank market, the central object is the cost of handling an excess or a deficit of liquidity ('liquidity cost'). A bank may need more liquidity after a period of activity due to, for example, transfer instructions from customers[7]. The bank faces a marginal cost in securing the necessary liquidity, which is increasing with the size of the shortage. Similarly, a bank that has accumulated unnecessarily large amount of liquidity wants to spend it to generate returns and faces a marginal cost of doing so, which is again increasing with the size of the excess. In the view of the model, the interbank market is an alternative to facing this increasing liquidity cost. This cost structure determines the interest rate on a loan in the model.

In the model, a large bank is a bank that has zero liquidity cost. Then, if there is any trade between two such large banks, the interest rate should show little variation, as the lender would not accept a low interest rate and the borrower would not accept a high interest rate. In the data, we map this observation into the fact that there is a small group of largest banks, between which the variation in interest rates is extremely small. Moreover, it turns out that most of the loans are either between two banks in that group of largest banks or between a bank in the group and a bank outside the group. This observation motivates the focus on the trade between a large bank and a small bank.

The large bank acts as a monopolist and offers a schedule of interest rates as a function of the size of the loan that the small bank lends to or borrows from the large bank. This approach to loan pricing in the context of the interbank market is also a novel contribution of our model. In practice, a small bank trades with multiple banks but it typically trades a majority of loans with a certain large bank. The model implies that a small bank gets a 'better rate' for a larger loan under broad assumptions. For example, when a small bank lends to a large bank, it tends to receive higher interest rates for larger loans. We confirm

[6]Afonso, Kovner and Schoar (2011) studies the Federal Funds Market during the 2008 financial crisis and finds that interest rates on loans become more sensitive to borrowers' characteristics. Allen et al. (2012) defines a market-wide measure of bargaining power between lenders and borrowers and shows how it is related to the state of the financial market.

[7]Aside from anecdotal evidence, there is an empirical support for the view that the interbank market exists to offset cash excess/deficit that is created by payments to/from other banks. Sokolov et al. (2012) studies the network structure of the interbank market in Australia and shows that overnight interbank loan flows largely offset interbank large-value payment flows.

that the data support the conclusion of our model.

As an application of the model, we consider the 2008 financial crisis. At the peak of the financial crisis, we find that (i) the average value (size) of the loans that small banks lend to large banks increased significantly, and (ii) the interest rates that small banks receive for the loan they lend to large banks, relative to the central bank target rate, fall at the peak of the financial crisis. We may interpret the increase in the value of loans as the small banks' increased precautionary savings; they maintain a higher level of cash holdings, which they lend to the large banks. The fall in the interest rate may be explained by a combination of two reasons: (i) The increased supply of lending by small banks lowers the interest rate they receive on the loans they lend, or (ii) due to the worsening of the financial conditions, the cost of liquidity increases, so small banks accept lower interest rates on the loans that they lend. We can compare the impact of these two different factors by estimating the parameters in our model. We find that the second factor, the increase in the cost of liquidity, can explain a large part of the fall in the interest rates that small banks receive from the large banks.

Section 2 describes the Mexican interbank market and presents empirical observations that motivate our modeling approach. Section 3 presents the model setup and discusses its implications. Section 4 tests some of the implications of the model, develops a framework to estimate parameters of the model, and discusses the implication of the estimation result on the impact of the 2008 financial crisis on the banking sector. Section 5 concludes.

2 Data Description and Empirical Patterns

2.1 Data Description

The dataset that we use is a record of all transactions in the interbank call money market in Mexico. A call money operation is an uncollateralized loan that can be recalled by the lender before it is due; if the lender recalls a loan, the lender receives back the principal of the loan immediately but earns zero interest on the loan. As far as we know[8], recalling is rare, so we treat these loans as simple uncollateralized loans. These call money operations can have a maturity of 1, 2 or 3 banking days, but an absolute majority of them are overnight loans and we use only overnight loans in our dataset.

With the repo market, overnight interbank market is a primary source of overnight loans for banks. In the years 2008 and 2009, the total value of loans traded in the interbank market was about half of the total value of loans traded in the repo market.

[8]This knowledge is based on informal conversations with staff at the Bank of Mexico. There is no official statistic to back up this claim. The lender at least has an incentive to plan its lending properly so that it does not have to recall too often, as recalled loans earn zero interest rate.

The market is an over-the-counter market with no centralized exchange[9] Therefore, a loan is made in the market through a mutual agreement between two banks.

The time span of our dataset is from 01/21/2008 to 12/31/2009 and the total number of transactions is 21,449. The number of banks with at least one transaction during this period is 38 and after removing banks without reliable balance sheet information, for example, new entrants in the market, we are left with 30 banks. In this process, we do not lose many observations because the banks that are removed are typically unimportant in the market, with only a small number of transactions.

The mean principal value of the loan is 536 million Mexican pesos, which is roughly 40 million US dollars based on exchange rates that prevailed in the years 2008 and 2009. The cross-sectional standard deviation of interest rates observed on a single day, averaged over all the banking days from 01/21/2008 to 12/31/2009, is 12 basis points, or 0.12 percentage points, in terms of annualized interest rates (all the interest rates that appear in this paper are in terms of annualized rates).

2.2 Variation in Interest Rates and Bank Size

We observe that among the few largest banks in the system, the variation in the interest rates on the loans between them is very small. To see this result, we compute, for each number n, the standard deviation of interest rates on the loans between the n largest banks in terms of total assets. To control for the change in the interest rate over time, we actually compute the standard deviation of the difference between the interest rate and the central bank target rate.

Figure 1 shows that for $n \leq 4$, the standard deviation is very small and there is little variation in interest rates. As n grows beyond 4, however, the standard deviation tends to increase with n.

Another way to look at this pattern is to define the four largest banks as 'large banks' and the remaining banks as 'small banks.' As we will show soon, most of the loans, in terms of total loan values, are either (i) between two large banks or (ii) between a large bank and a small bank. If we compare the mean interest rates (as before, we use the difference from the target rate to control for changes over time) on the loans based on (i) whether the lender of the loan is a large bank or a small bank; and (ii) whether the borrower of the loan is a large bank or a small bank, we find that (i) the mean of the interest rates between two large banks is very close to zero; (ii) the mean of the interest rates that small banks pay to large banks

[9] We do not know why there does not exist an exchange for this market. One reason may be that the number of transactions is small, so there is not a strong incentive to establish an exchange. More generally, many financial assets are traded outside exchanges.

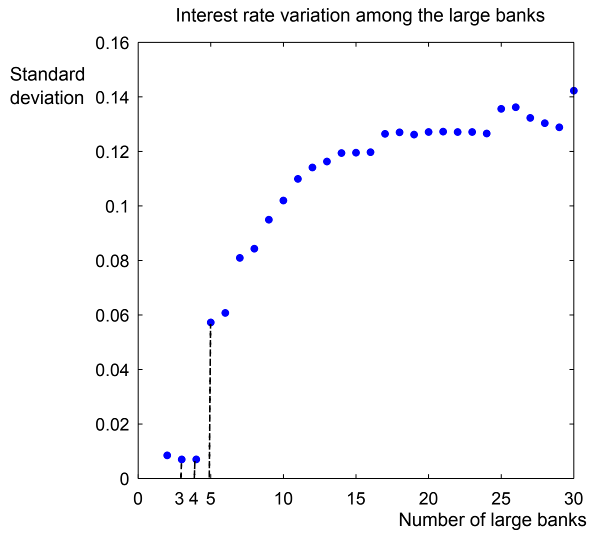

Figure 1: Variation in Interest Rates within Different Subgroups of Banks

		Borrower is:	
		Large	Small
Lender is:	Large	0.000 (0.007)	0.019 (0.118)
	Small	-0.150 (0.124)	-0.007 (0.137)

* Numbers in () are standard deviations, not standard errors of the mean.

Table 1: Mean of Interest Rates for Different Bank Size

when small banks borrow from large banks is significantly positive[10]; and (iii) the mean of the interest rates that small banks receive from large banks when small banks lend to large banks is significantly negative.

This result does not concern only the means of interest rates. Figure 2 plots the distribution of interest rates, separately for different size categories that the lender and the borrower belong to. It shows that in most cases, the interest rate that small banks receive from (pay to) large banks when small banks lend to (borrow from) large banks is negative (positive).

2.3 Counterparty Choice of Small Banks

Generally, in the data, the relatively small banks mostly trade, in total loan value terms, with some of the largest banks in the system rather than with other small banks. Therefore, if we divide the banks into two groups, large and small, as we did above, most of the loans will be either between two large banks or between a large bank and a small bank.

[10]The number of such cases is small, however, because small banks mostly lend to large banks, rather than borrowing from them

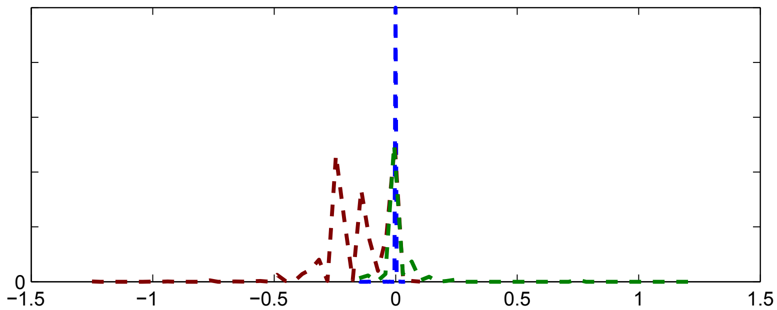

Figure 2: Densities of Interest Rates for Different Bank Size

To see how this observation depends on how we define the large banks, we compute the total value of the loans between small banks divided by the total value of all the loans. We compute the ratio for each number n, which represents the number of largest banks that are classified as 'large.' When this ratio is small, most of the loans are between two large banks or between a large bank and a small bank.

In figure 3, as we increase the number of large banks, starting from two, the total value of loans between two small banks decreases rapidly until we reach four, then it slowly decreases afterward. The four largest banks seem to act as counterparties to many other banks in the system, while the others mostly trade with one of these largest banks. With $n = 4$, in value terms, 47.9% of total loans are between two large banks and 43.6% of the loans are between a large bank and a small bank. Loans between two small banks only account for 8.5% of the loans.

If we define large banks to be the four largest banks in the system, a typical small bank is observed to trade mostly with one of the large banks. Even though a small bank trades with more than one large banks in practice, it is very easy to identify which large bank it principally trades with. On average, 61% of a small bank's trade with large banks is concentrated on the most frequent trading counterparty. In contrast, the second most frequent trading counterparty accounts for only 20% of trades.

2.4 Linking the Observations to Modeling Assumptions

In this subsection, we discuss how the modeling approach that we will lay out in detail in the next section is consistent with the empirical patterns described and can rationalize some of the patterns in a straightforward manner. First, we model the interaction between a large bank and a small bank, in isolation from all the other banks. Also, we assume that the large bank acts as a 'monopolist,' in the sense that it offers its profit-maximizing schedule

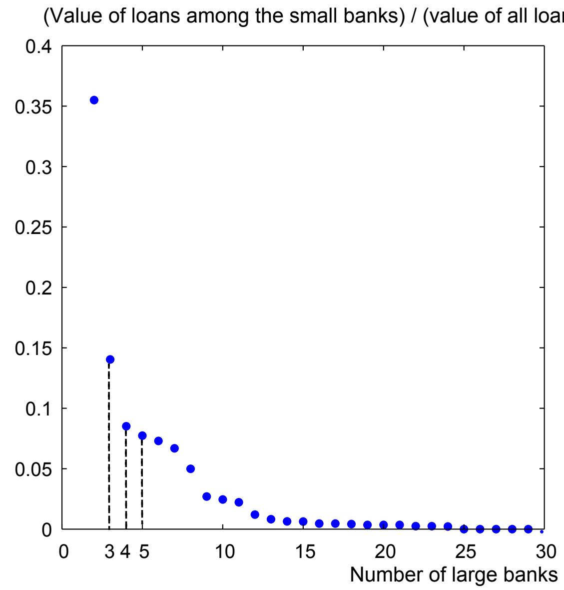

Figure 3: Total Value of Loans between Small Banks

of interest rates as a function of loan value (size) to the small bank.

Since (i) most of the loans are either between two large banks or between a large bank and a small bank; and (ii) there is little variation in interest rates on the loans between two large banks, most of the interesting variations in interest rates are observed on the loans between a large bank and a small bank. Also, since a small bank typically has a single large bank with which it trades a majority of its loans, the assumption that the large bank acts as a type of monopolist toward the small bank is a reasonable simplification.[11]

Then, we assume that faced with an excess of liquidity (cash) or a deficit of liquidity, a large bank can handle it costlessly, while a small bank faces some cost (we denote this cost as the 'cost of liquidity') to handle the excess or deficit. For example, if a large bank has some excess cash, it can costlessly find an opportunity to invest the cash and generate returns. However, a small bank with excess cash will face some cost in finding and investing the cash to generate returns. This cost, in practice, may represent differential access to potential trading opportunities.

Under these assumptions, it is very easy to rationalize the observations on interest rates. To begin with, let us assume that all the banks face the same price of liquidity, which is the central bank target rate, apart from the cost of liquidity defined above. Since large banks face zero cost of liquidity, their marginal valuation of liquidity is the central bank target rate, and they will lend and borrow at the target rate irrespective of their positions in the

[11] In addition, the interest rate that the most preferred trading counterparty offers is not better than what other banks offer. Therefore, it seems that the choice of counterparty is not mainly motivated by price.

market.

However, small banks face a positive cost of liquidity, so their marginal valuation of liquidity is lower than the central bank target rate when they have an excess of liquidity. A profit-increasing trade can be made when small banks lend some of this excess liquidity to a large bank because the large bank faces no cost in handling the excess liquidity. Then, the interest rate that a small bank receives for the loan will be between the central bank target rate and the small bank's own valuation, and will generally be below the target rate due to the positive cost of liquidity. The same argument can rationalize the observation that when a small bank is borrowing from a large bank, it pays an interest rate higher than the central bank target rate[12].

2.5 Relation to the Existing Literature

The observation that large banks have a relative advantage in interest rates against small banks is consistent with the results from existing empirical literature, for example, Furfine (2001) and Cocco, Gomes and Martins (2009). Also, the analysis on the network structure of transactions in the interbank market in Bech and Atalay (2008) is consistent with our picture of the interbank market, in which a small group of large banks are involved in a large fraction of transactions in the market. Furfine (1999) also finds the same pattern with data on the Federal Funds Market.

3 Model

3.1 Setup

There are two banks in the model, a large bank (bank L) and a small bank (bank S). Let x be the amount of excess liquidity that bank S holds and let l be the amount of loan that the small bank lends to the large bank; $x < 0$ means that bank S has $-x$ amount of liquidity deficit and $l < 0$ means that the small bank borrows $-l$ from the large bank.

First, when $l = 0$, the bank S's profit function π_S is

$$\pi_S = \int_0^x (p - c(y))dy. \tag{1}$$

$p - c(y)$ is the marginal value of liquidity, where p is a constant and $c(\cdot)$ is a strictly increasing

[12]The number of such cases is small, however, because small banks mostly lend to large banks, rather than borrowing from them.

function such that $c(0) = 0$. In this setup, p is the value of liquidity under no cost and $c(\cdot)$ is the marginal cost of liquidity, in the sense that $\int_0^x (p - c(y))dy < px$; $c(y)$ always works against the small bank because its sign is the same as that of y.

With $l \neq 0$, bank S transfers l amount of liquidity to bank L, so the profit function of bank S is

$$\pi_S = \int_0^{x-l} (p - c(y))dy + rl. \tag{2}$$

r is the net interest rate on the loan of value l. Compared to equation (1), bank S's liquidity position changes from x to $x - l$ because it transfers l amount of liquidity to bank L. In return, bank S receives interest payment rl.

Bank L's profit function π_L has the same form as π_S, except that it has no marginal cost term. Therefore, by assumption, bank L has zero cost of liquidity:

$$\pi_L = pl - rl. \tag{3}$$

Bank L may have its own excess liquidity, but we do not need to consider it here; since the marginal value of liquidity is a constant for bank L, its own excess liquidity does not affect the problem of determining r and l.

In this setup, bank L 'absorbs' some of the liquidity excess or deficit of bank S because it can then deal with that absorbed position at a lower cost. The interest rate r has to be lower (higher) than p when bank S is lending (borrowing) so that bank L does not make a loss from the trade.

3.2 Trading Mechanism

x, the liquidity position of bank S, is a random variable. Bank S knows its exact realization, but bank L only knows its probability distribution. Bank L's problem is to offer a schedule or menu of interest rates as a function of the loan value to maximize its expected profit, taking into account the fact that bank S will choose the point on the interest rate schedule that maximizes its own profit.

Formally, let $r(l)$ be the interest rate schedule offered by bank L. l can be either positive or negative; as mentioned before, a negative l means that bank S is borrowing from bank L. The problem of bank S is to maximize its own profit given x and the interest rate schedule $r(l)$:

$$\max_{l} \int_{0}^{x-l} (p - c(y))dy + r(l)l. \tag{4}$$

The outcome of this optimization problem will characterize the amount of liquidity (cash) that bank S lends to bank L. The outcome, l, depends on the interest rate schedule offered, $r(\cdot)$, and on x. Therefore, we can write l as $l(x|r(\cdot))$.

With this new notation, we can formally write the expected profit maximization problem of bank L:

$$\max_{r(\cdot)} \int_{-\infty}^{\infty} (p - r(l(x|r(\cdot))))(l(x|r(\cdot)))f(x)dx, \tag{5}$$

where $f(x)$ is the probability density function of x. A full characterization, with necessary computational steps, is presented in the appendix. Below, we will omit computational steps and discuss only essential characteristics of the solution.

3.3 Characterization of the Solution

In solving bank L's problem, the case for $x > 0$ can be solved separately from $x < 0$. The reason is that bank L will always offer $r \leq p$ for $l > 0$ and offer $r \geq p$ for $l < 0$ to avoid making a loss. Given this fact and the increasing cost function $c(\cdot)$, bank S has no incentive to choose $l < 0$ when $x > 0$ or to choose $l > 0$ when $x < 0$. Therefore, from this point on, we assume $x > 0$. This case is more relevant because small banks mostly lend to large banks rather than borrow from them, and once we solve the bank L's maximization for this problem, we can solve the problem for $x < 0$ in the same way.

Another result is that given some $r(\cdot)$, $l(x|r(\cdot)) \geq l(x'|r(\cdot))$ if $x > x'$. Since the marginal cost function $c(x)$ is increasing in x, bank S benefits more from increasing its lending when its liquidity position x is larger. This result lets us write $l(x) = l(x|r(\cdot))$ as a weakly increasing function of x.

Furthermore, for $r(l)$ and $l(x)$ to satisfy the incentive compatibility constraint for bank S, bank S should be indifferent to choosing between $(r(l(x)), l(x))$ and $(r(l(x + dx)), l(x + dx))$ when its liquidity position is x, which produces the following condition[13]:

$$l'(x)[r(l(x)) + r'(l(x))l(x) - p + c(x - l(x))] = 0. \text{ (IC)} \tag{6}$$

Finally, to characterize the solution, we use a the first-order condition from differentiating

[13]This condition makes sense only if x is a continuous random variable. In this section, we are not concerned with presenting exact technical conditions. They are discussed in the appendix.

the objective function with respect to $l(x)$. Roughly speaking, the first-order condition is a balance between (i) the profit from increasing $l(x)$ and $r(l(x))$ for some x in such a way to leave bank S's profit the same but increase bank L's profit, and (ii) the cost of increasing $r(l)$ for all $l > l(x)$ to conserve the incentive compatibility of bank S. The resulting expression is

$$f(x)c(x - l(x)) - (1 - F(x))c'(x - l(x)) + \lambda(x) = 0, \text{ (FOC)} \tag{7}$$

where $F(x)$ is the cumulative distribution function of x and $\lambda(x)$ is a shadow cost of the constraints (i) $l(x) \geq 0$ for all x, and (ii) $l(x)$ is a weakly increasing function of x.

The specific functional form of the solutions $r(l)$ and $l(x)$ depend on the functional form of c and the distribution of x. However, there is a general tendency for the interest rate to increase as l increases, at least for large values of l, as long as the distribution of x does not have a heavy tail in the sense that either the support of x is bounded or the inverse of hazard function $(\frac{1-F(x)}{f(x)})$ becomes small for a large x. Then, the first-order condition $f(x)c(x - l(x)) - (1 - F(x))c'(x - l(x)) = 0$ implies that $x - l(x)$ should be close to 0. Intuitively, when the distribution of x is bounded or does not have a heavy tail, the large bank wants to lend as much as possible for large values of x. The reason is that the cost to conserve the incentive compatibility of bank S, $(1 - F(x))c'(x - l(x))$ becomes small relative to the profit from lending more when x is large.

Then, when $c(x - l(x))$ gets small, $r'(l(x)) = \frac{p-c(x-l(x))-r(l(x))}{l(x)}$ tends to be positive, given the equation (IC).

3.4 Additional Assumptions

We assume that the marginal cost of liquidity, $c(y)$, takes the form of a power function for $y \geq 0$, $c(y) = \alpha y^\theta$, for positive constants α and θ[14]. If the hazard rate of x, $\frac{f(x)}{1-F(x)}$ is monotonically weakly increasing in x[15], the solution to the optimization problem of bank L has a simple solution: $l(x)$, the amount of loan that bank S lends to bank L is

$$l(x) = [x - \theta \frac{1 - F(x)}{f(x)}]^+ \tag{8}$$

where the notation $[]^+$ denotes the maximum of the expression inside the brackets and 0. The condition (IC) can be rewritten as:

[14]Given that we are now considering the case of bank S lending, we do not care about the exact form of $c(y)$ when $y < 0$. We only need that $c(y)$ is an increasing function of y.

[15]A normal distribution, a uniform distribution and an exponential distribution are examples of such a distribution.

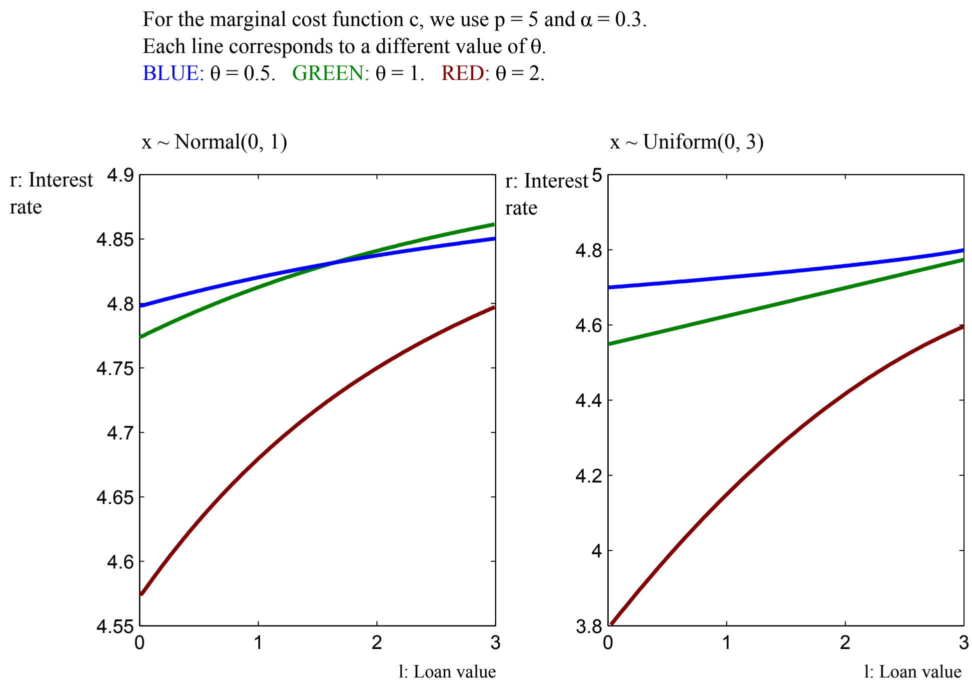

Figure 4: Interest Rate Schedule $r(l)$ as a Function of Loan Value

$$r(l(x)) + r'(l(x))l(x) - p + \alpha(\theta(x - l(x)))^{\theta} = 0. \tag{9}$$

More conveniently,

$$r(l) = p - \frac{1}{l}\int_{x_0}^{l^{-1}(l)} \alpha(\theta\frac{1 - F(l^{-1}(l))}{f(l^{-1}(l))})^{\theta} dl, \tag{10}$$

where l denotes both the loan size as a variable in the argument of $r(l)$ and the loan size $l(x)$ as a function of x at the same time, in a slight abuse of notation. Also, $x_0 = sup(\{x|l(x) = 0\})$. Since $\frac{1-F(x)}{f(x)}$ is monotonically decreasing in x, $r(l)$ is a monotonically increasing function of l.

Figure 4 shows the solutions for some chosen values of θ and some chosen distributions of x.

3.5 Discussion

A unique approach of our model is to let bank L determine the optimal interest rate schedule as a function of loan value. This approach is well known in the industrial organization literature as a screening problem, but it had not been previously applied in the context of interbank loan market. With some additional assumptions, we showed that the interest rate that bank S receives is an increasing function of the value of the loan.

This result contrasts with that from a profit-sharing approach, which has been used in many studies of interbank market[16] and over-the-counter markets in general. Under the same assumptions on the cost function as we made above, if interest rate is set to share the profit generated from the transfer of liquidity between bank S and bank L at the ratio of β and $1-\beta$, the interest rate $r(l)$ would be

$$r(l(x)) = p - (1-\beta) \int_{x-l(x)}^{x} c(x-y)dy. \tag{11}$$

In contrast with the model developed in this section, $r(l(x))$ is a decreasing function of l as long as $l(x)$ is an increasing function of x. Since the marginal cost of liquidity c is increasing in its argument, larger loans generally correspond to more average cost per unit, which, in turn, corresponds to low interest rates according to the profit-sharing rule. As we will show in the next section, we indeed observe that the interest rate tends to be higher for a larger loan.

In developing a model of interest rate, we have not considered the risk level of the borrower. For a risk-neutral lender, the premium (additive) on the interest rate due to a default risk will be approximately the default rate itself. When a large bank is borrowing from a small bank, the default risk of the borrower is typically very small; a large bank, in our model, corresponds to a few largest banks in the system.[17] The fact that there is very little variance in interest rates also indirectly confirms this assessment because if there is any significant variation in default risk across the large banks, it should be reflected in the interest rate.

However, default risk premium may not be ignored when we consider a small bank borrowing from a large bank. In that case, the observation that small banks pay an interest rate higher than the central bank target rate to large banks may be explained by concerns about default. Even then, the small banks still pay higher interest rates when they borrow from large banks rather than from other small banks, so default risk alone cannot explain the data.

In the empirical analysis in the next chapter, we exclusively use the loans that small banks

[16]For example, in Allen et al. (2012), the profit is shared according to a fixed ratio, which represents relative bargaining powers of the lender and the borrower. Please note that, however, the paper does not study the relationship between interest rate and loan size in particular; it mainly studies the relative bargaining power between borrowers and lenders.

[17]These largest banks typically are assessed to have a strong ability to repay short-term debt obligations. For example, for the four largest banks in the Mexican system, their most recent ratings by Moody's are P-2; the historical default probability within three months for corporations rated in that caterogy is 0.00 percent. Therefore, annualized, the contribution to the interest rate from default risk should be around or less than 1 basis point.

lend to large banks because small banks mostly lend to large banks rather than borrow from them. Therefore, we do not have to worry much about the role that default risk may play[18].

4 Empirical Application of the Model

4.1 Relationship between the Interest Rate and Loan Size

Here, we briefly confirm that interest and loan size, controlling for the identify of the lender and the borrower, have a positive correlation. We estimate a linear regression of the form

$$r_i - p_t = \alpha + \beta l_i + \epsilon_i \tag{12}$$

on the loans between a small bank and its most frequent trading partner (a large bank), separately for each small bank and for each 60-business-day time window to control for any significant changes in the relationship over time. i is simply an index for observations, r_i is the interest rate, p_t is the average interest rate between the large banks on day t (which is practically identical to the central bank target rate), ϵ_i is the error term, and α and β are linear regression coefficients. The cofficient β turns out to be mostly positive; it is positive in about 75% of the regressions.

4.2 Application to 2008 Financial Crisis

Casual observations indicate that near the peak of the 2008 financial crisis (measured by the peak in the implied stock market volatility index[19] in Mexico) both (i) the total value of loans in the interbank market increased[20] and (ii) the interest rate 'discount' (the absolute value of the difference between individual interest rates and the central bank target rate) on the loans that small banks lend to large banks increased. In this section, we only consider

[18]There still can be other variables that matter. For example, there are papers that study the effect on the interest rate of the time of the day when the loan is made; Hamilton (1996) is an example. Unfortunately, we do not have any information on this factor, so we need to assume that the time effect is not large enough to invalidate our analysis.

[19]The precise definition of implied stock market volatility is discussed, for example, in Bollerslev, Tauchen and Zhou (2009). The precised definition of crisis period in our paper is discussed in appendix with a description of the empirical procedure.

[20]This observation may surprise some readers who think that an uncollaterized market such as the overnight interbank market would not function well when the financial market is in distress. In fact, overnight interbank market has been functioning well through different crises: Furfine (2002) documents that the Federal Funds Market worked well during the Russian debt crisis, and Afonso, Kovner and Schoar (2011) documents that there was at most a small drop in the total value of loans traded in the Federal Funds Market after the default of Lehman Brothers in 2008.

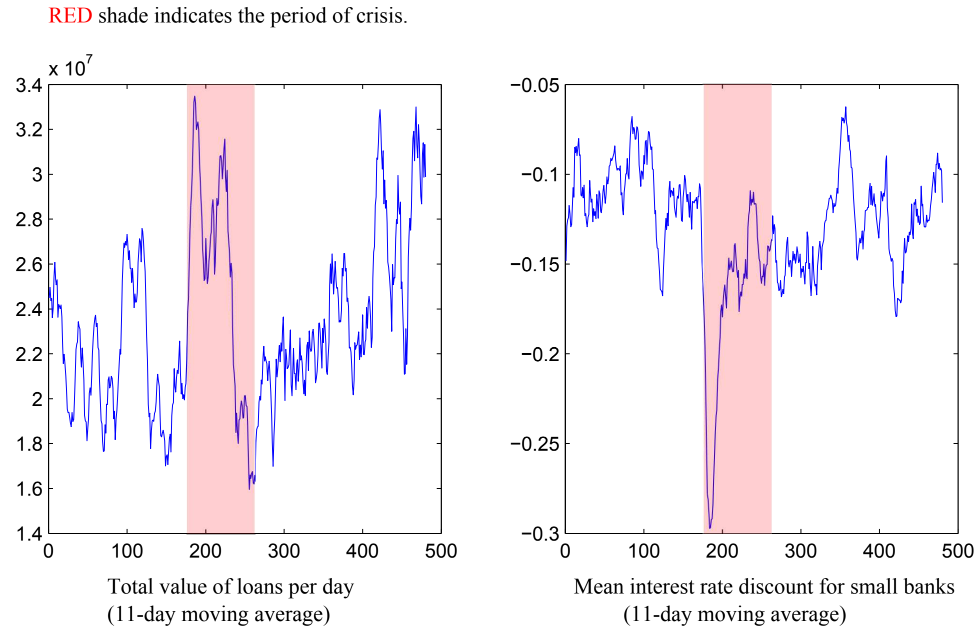

Figure 5: Behavior of the Interbank Market at the Time of Crisis

the loans that small banks lend to large banks, not the loans that small banks borrow from large banks.

Since both the total value of loans traded and the interest rate discount seem to have been significantly affected by the financial crisis, at least initially, it makes sense to consider a change in underlying parameters characterizing banks' liquidity position and liquidity cost during the crisis.

Given the power function parametrization of the cost function, $c(y) = \alpha y^{\theta}$, an increase in the interest rate discount can be explained by two changes: (i) an increase in α ('cost change') and (ii) a change in the distribution of x, the liquidity position of the small bank. (Usually a shift in the distribution farther away from $x = 0$ causes the interest rate schedule $r(l)$ to shift down, increasing the average interest rate discount, given the increasing cost function.)

Since both the total value of loans and the mean value of loans have increased at the peak of the 2008 financial crisis, part of the observed increase in the interest rate discount should be explained by a shift in the distribution of x to the right. (In the direction of increasing x.) The amount by which such a shift affects the interest rate discount is determined by the shape parameter of the cost function, θ.

For example, if we change α to $(1+\Delta)\alpha$, the interest rate schedule offered by the large bank will shift down from $r(l)$ to $p + (1+\Delta)(r(l) - p)$. Alternatively, if we shift the distribution of x by using $x' = (1+\Delta)$ in place of x, the interest rate schedule will shift down to $p + (1+\Delta)^{\theta}(r(l) - p)$.[21] Intuitively, a larger θ implies that the marginal cost of liquidity grows faster when the liquidity position of the small bank is large, so a shift in the distribution of x must have a larger effect with a larger θ.

Therefore, by measuring θ from the data, we can estimate whether the data are consistent with an increase in α at the peak of the crisis. In addition, we quantitatively compare the effects of the two sources of the increased interest rate discount.

In summary, by examining the shape of the interest rate schedule $r(l)$, we can estimate how a shift in the distribution in x should affect the interest rate discount; as the formulas derived in the previous section and figure 4 suggests, a larger θ is associated with a steeper $r(l)$. This information, in turn, lets us estimate how much of the change in interest rate discount is not explained by a shift in the distribution of x. This 'residual' maps into a change in α.

4.3 Parameter Estimation Procedure

In this subsection, we describe the estimation procedure briefly. A full description and discussion of alternative specifications are presented in the appendix.

For each small bank, we assume that x follows a distribution with monotonically weakly increasing hazard rate. In particular, we use a linear failure rate distribution,[22] which is characterized by two shape parameters a and b and whose probability density function is $(a+bx)exp(-ax-\frac{b^2x^2}{2})$ for $x > 0$. In principle, we can use any other distribution with reasonable shapes, but we have chosen this particular distribution for computational convenience; its inverse hazard rate is simply given by $a + bx$.

At the peak of the crisis (which we call simply 'crisis period'), we assume that the distribution of x is shifted to the right so that its distribution becomes that of Cx. It means that the parameters of the distribution a and b should be changed to a' and b' as a function of C.

We assume that θ is a fundamental parameter that does not change over time. Instead, the cost function shifts due to a shift in α: We use two parameters, α and α', to parametrize the cost function outside the crisis period and within the crisis period, respectively.

In summary, we estimate the six parameters $(a, b, C, \theta, \alpha, \alpha')$ for each bank. We use the following moment conditions to estimate their values. First, we index individual observations by the day t[23]; l_t denotes the value of the loan and r_t denotes the interest rate. D_t is the

[21] The mathematics involved in computing these results are simple and presented in the appendix.

[22] For a description of this distribution, see Sarhan and Kundu (2009)

[23] In most cases, the number of loans between a small bank and its most frequent trading partner (a large bank) is one. Even when there are more than one loans, the interest rate on the multiple loans made on the same day tend to be the same. Therefore, there is very little ambiguity in this definition of variables.

dummy variable that takes the value of 1 if and only if t is inside the crisis period, and p_t is the average interest rate on the loans between large banks, which is practically identical to the central bank target rate.

The three moment conditions that relate to the shape of the distribution of x are:

$$
\begin{aligned}
E[(l_t - E_{(a,b,\theta)}[l])(1 - D_t)] &= 0, && (13)\\
E[(l_t - E_{(a_c,b_c,\theta)}[l])D_t] &= 0, && (14)\\
E[(l_t^2 - E_{(a,b,\theta)}[l^2])(1 - D_t) + (l_t^2 - E_{(a_c,b_c,\theta)}[l^2])D_t] &= 0, && (15)
\end{aligned}
$$

where $E_{(a,b,\theta)}$ denotes the theoretically expected value of functions of l given the parameters a, b and θ. Equations (13) and (14) are conditions on the mean of l and equation (15) is a condition on the second moment of l.

We obtain four additional moment conditions from the (IC) condition derived in the last section, $r + \frac{dr}{dl}l - p + \alpha\theta^\theta(l^{-1}(l) - l)^\theta = 0$. Redefining $\alpha = \alpha\theta^\theta$, we have

$$
\begin{aligned}
E[(r_t + \frac{dr}{dl}l_t - p_t + \alpha(l^{-1}_{(a,b,\theta)}(l_t) - l_t)^\theta)(1 - D_t)] &= 0, && (16)\\
E[(r_t + \frac{dr}{dl}l_t - p_t + \alpha'(l^{-1}_{(a',b',\theta)}(l_t) - l_t)^\theta)D_t] &= 0, && (17)\\
E[(r_t + \frac{dr}{dl}l_t - p_t + \alpha(l^{-1}_{(a,b,\theta)}(l_t) - l_t)^\theta)(1 - D_t)l_t] &= 0, && (18)\\
E[(r_t + \frac{dr}{dl}l_t - p_t + \alpha'(l^{-1}_{(a',b',\theta)}(l_t) - l_t)^\theta)D_t l_t] &= 0, && (19)
\end{aligned}
$$

where $\frac{dr}{dl}$ is the empirically estimated derivative and $l^{-1}_{(a,b,\theta)}$ is the mapping from l to liquidity position x which depends on the parameters a, b and θ. Roughly, θ corresponds to how steep the curve $r(l)$ is.

These last moments are the (FOC) condition derived in the last section, interacted with the dummy variable D_t and the loan size l_t.

4.4 Estimation Results

Table 2 shows the summary of estimated θ for individual banks (Results for individual banks are reported in the appendix). For several banks, θ is very close to zero because when there is a negative relationship between the interest rate and the loan size in the data, $\theta = 0$ tends to produce the best fit[24]. The mean and median of estimated θ across the small banks is 0.90 and 0.43, respectively, which seem to be reasonable (compared to values such as $\theta = 5$, which implies that the marginal cost of liquidity $c(x)$ grows extremely fast with x).

[24] $\theta = 0$ implies a flat $r(l)$.

	For all small banks ($n = 24$):	For small banks with θ not close to 0 ($\theta > 0$):
Mean	0.90	1.44
Median	0.43	0.90

*There are a few banks for which the estimated θ is unusually large, at around 5. These few observations drive up the mean relative to the median.

Table 2: Summary of Estimated θ

Using the estimated parameters, we can ask how much of the increased discount on the interest rate that small banks receive can be attributed to an increased cost of liquidity (change in α) rather than to an increased need to lend by small banks. The estimation result suggests that a large part (about 87%) of the increased discount can be attributed to the increased cost of liquidity rather than simply to the increased need to lend by small banks.

4.5 Discussion

Figure 6 is the plot of model-generated (with fitted parameters) change in the interest rate discount versus the observed interest rate discount. The figure shows that the estimation process can reasonably fit the model to the data.

The behavior of the interbank market at the peak of the financial crisis looks much like that of precautionary saving. The small banks lend more to large banks because they maintain a higher level of cash, even though the benefit of doing so in terms of interest rate is smaller in the crisis period.

A regulator monitoring this market may want to know whether the observed increase in the discount on the interest rate that small banks receive is either (i) an indication of an increased supply of lending by small banks, or (ii) an indication of a higher cost of liquidity for small banks. Case (ii) may indicate a worsening of the financial health of the system, while case (i) may simply be regarded as a shift in supply/demand with no strong implication on the financial health of the system.

5 Conclusion

In this paper, we have developed a model of bank behavior in the overnight interbank market. The primitive characteristic responsible for determining the interest rate is the cost of liquidity in the model. The model describes how a large and central bank in the system

(Average across banks)	Data	Generated by the fitted model
Percentage rise in the loan value during the crisis ($\times 100$)	0.072	0.118
Average interest rate discount outside the crisis (pp)	-0.152	-0.132
Average interest rate discount during the crisis (pp)	-0.201	-0.197
Change in the discount, (Row 3) - (Row 2)	-0.0485	-0.0655

Contributions to the model-generated change in discount:

Total	Increased cost (change in α)	Increased demand to lend
-0.0655	-0.0571	-0.0084
(100%)	(87%)	(13%)

Table 3: Different Contributions to the Increased Discount during the Crisis

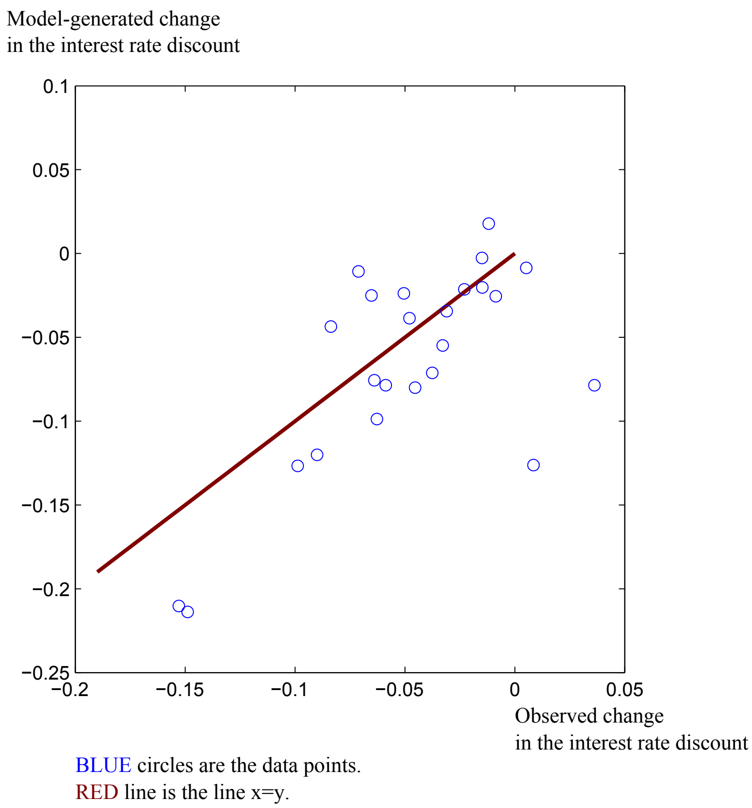

Figure 6: Model Fit of Interest Rate Discount

can absorb the liquidity excess and deficit of smaller banks. Our contribution is to build a model that can rationalize important features of the data in an intuitive manner.

In addition, our model, under some broad assumptions, shows that the interest rate disadvantage that a small bank experiences against a large bank decreases with the size of the loan. For example, when a small bank is lending to a large bank, the interest rate tends to be higher for larger loans. This result is also a unique contribution of our model in the context of the interbank market; for example, under the same assumptions, a model in which the price is determined according to a profit-sharing rule in fixed ratio would predict that the interest rate decreased with the loan value. The empirical results are consistent with the prediction of our model.

Finally, we estimate the parameters of our model under the context of 2008 financial crisis. In particular, we ask whether the drop near the peak of the financial crisis in the interest rates on the loans that small banks lend to large banks can be explained by an increased need to lend by small banks, supposedly due to the precautionary increase in cash holdings by small banks. Our estimation suggests that it is only a small part of the story, and a large part of the drop has been caused by a general increase in the cost of liquidity.

Our paper leads to different avenues for future research. First, we simplified the setup and focused on the one-to-one interaction between a large bank and a small bank. However, in practice, trading occurs on a network of interconnected banks. It will be interesting to expand our setup so that we can also characterize and empirically observe the effect of the shape of the trading network on the observed variation in the interest rates.[25] Another possible extension is to apply our framework to other markets which have a small group of large players that participate in the majority of transactions. A repo market, for example, can be a natural candidate, even though the market analysis will be more complicated with variation in the type of collateral, and so on.

[25] There are papers that study interbank exposure networks. However, as far as we know, there is no study that builds an analytical framework to study the interest rate variation in an interbank market of liquidity. Existing empirical literature that studies the effect of common network metrics such as centrality typically does not have a comprehensive model of how they should affect the interest rates. Instead, the studies tend to rely on general arguments on market power, outside opportunities, and so on.

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7 Appendix

Currently, a web version of the appendix is located at `http://web.mit.edu/kkim1/www/kk_app1.pdf`.