Toxic Asset Bubble & Imbalances

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This draft: September 2013
First draft: August 2013

Abstract

First, we show that welfare reducing asset bubbles, which we call toxic bubbles, can emerge in a standard framework of rational bubble. In our model, limited liability and defaultable debt contracts induce banks to shift the bubble's risk of collapse to their creditors. Risk shifting enables risky toxic bubbles to emerge in equilibrium, even if all agents are rational and forward looking. We then analyze welfare effects of an intervention policy that pricks an existing bubble, and of a preventative macroprudential banking regulation.

Second, we generalize the model to two large open economies. The global model shows that an emerging economy can "outsource" a toxic bubble to a developed economy through global financial intermediation. Hence, a toxic bubble can emerge in the developed economy, where there is *no* dynamic inefficiency. The model formalizes the idea that a "savings glut" coming from emerging economies to the United States fueled the boom of a subprime mortgage bubble. The model provides a tractable framework for bubble policy analysis in open economies.

Keywords: rational bubble; financial friction; global imbalances; financial crisis.

[1] The views expressed in this paper are those of the author and should not be interpreted as the official views of the Bank of Japan.

1 Introduction

The recent boom and bust of housing prices in the U.S. has revived a need to understand the role of *risky* asset bubbles[2] in macroeconomics. How do imperfections in the financial sector, especially imperfections in the incentive structure of financial intermediation, affect the riskiness of asset bubbles? Can there be a rational but "toxic asset bubble", i.e., a welfare reducing bubble in a financial market with rational and forward looking participants? Can capital inflows, especially the massive "savings glut" from emerging economies into the U.S., possibly "fuel" a toxic bubble in the U.S., as some have argued?[3] Should a government or central bank "prick" a risky bubble if they can detect one? How should they regulate banks if they cannot detect bubbles? These questions are at the heart of ongoing debates among not only economists but also policy makers[4] on regulating booms and busts in the financial market.

In this paper, we develop a tractable macroeconomic framework of risky asset bubbles to address these questions. Our model features characteristics that are particularly relevant to the recent crisis: imperfect financial intermediation, global capital flows (such as the savings glut) and large open economies (such as the U.S.). We proceed in two steps.

First, to analyze the relationship between financial market and risky bubbles, we embed an agency problem in financial intermediation into an otherwise standard infinite horizon, overlapping generation model of rational bubbles.[5] Specifically, households deposit their wealth in banks, and banks use these deposits to invest in either physical capital or in a bubbly asset that faces an exogenous risk of collapse. As usual, we assume banks have limited liability and issue only standard debt contracts.[6] This allows banks to "shift" the bubble's risk to the depositors. In other words, banks have an option to default on the depositors if the bubble bursts. Interestingly, this phenomenon of risk shifting allows *toxic bubbles* – excessively risky bubbles that reduce households' expected utility (even if households are risk-neutral)– to emerge in equilibrium, despite that agents in our model are rational and forward looking. In short, with risk shifting, toxic bubbles can emerges in an otherwise standard rational bubble framework. This model of toxic asset bubbles is our first contribution.

Second, to analyze the relationship between global capital flows (especially flows

[2] In this paper, we follow the bubble literature to define an *asset bubble* (or a *bubbly asset*) as an asset whose price is strictly larger than its "fundamental value" (the risk-adjusted total discounted sum of its future dividends). A bubble is risky if it can stochastically collapse.

[3] For example, see Bernanke (2005) and Rajan (2011).

[4] See Bernanke and Gertler (2001), Bernanke (2010), Greenspan (2009) and Mishkin (2008).

[5] For a standard model, see Tirole (1985), and also Samuelson (1958) or Diamond (1965).

[6] We provide a microfoundation for standard debt contracts in the appendix.

of savings in emerging economies into developed ones) and risky bubbles, we build a more sophisticated model with two large open economies: North and South. The structure of each economy is similar to that in the first model. Banks can borrow and lend to each other internationally. In this global setting, we establish a novel "global dynamic inefficiency" condition that is necessary and sufficient for a bubble to emerge in either economy. This condition is much weaker than imposing that both economies are dynamically inefficient (i.e., households in each country face a shortage of storage for value that yields sufficiently high return). In particular, it holds even if the North is dynamically *efficient* as long as the South is sufficiently dynamically inefficient. In equilibrium, savings flow out of the dynamically inefficient South to seek storage in the dynamically efficient North. This "savings glut" enable a bubble to emerge in the North (where households do not need a bubble to store their savings). And the bubble can be toxic, as both Northern and Southern banks do not fully internalize the risk of bubble collapse. This result is a contribution for two reasons.

On the one hand, dynamic inefficiency is essential for rational bubbles to emerge in the standard framework of Tirole (1985). Dynamic inefficiency, or shortage of storage, is an appropriate assumption for emerging economies, where capital markets are underdeveloped, but is *inappropriate* for financially developed economies such as the U.S. In fact, Abel et al. (1989) argues empirically that OECD nations are not dynamically inefficient. This is an empirical challenge for the standard model. Our result implies that, because of capital inflows, rational bubbles can still emerge in dynamically efficient but *open* economies (such as the US).

On the other hand, our model provides a formal argument to the view that the "savings glut" from emerging markets contributed to the creation of the subprime mortgage bubble in the U.S., especially with the help of the U.S. financial market. In our model, the South "outsources" a toxic bubble to the financially developed North, thanks to the agency problem of risk shifting in the globalized financial markets. This model of global toxic bubbles is our second contribution.

Both of our closed and open economy models serve as tractable frameworks for policy analysis. First, suppose that policy makers can observe bubbles and their risk of collapse Not surprisingly, an *anticipated* policy that commits to prick excessively risky bubbles improves welfare in the steady state. More interestingly, a carefully designed *unanticipated* policy where the authority pricks excessively risky bubbles, particularly by buying bubbly assets from the current old and refraining from re-selling them, improves the welfare of *all* generations. This is not an obvious result, since in our model as well as in standard bubble models, unanticipated pricking alone always reduces welfare of the current holders of the bubbly asset and thus cannot improve welfare.

Second, suppose policy makers can*not* observe bubbles or their risk. We show that a macroprudential policy in the form of a risk-adjusted capital requirement can *prevent* toxic asset bubbles and improves welfare. Basically, this capital requirement forces banks to internalize some of the loss into their equity holdings when the bubble collapses.

Regarding open economies, even though capital inflows can generate a toxic bubble, we show that open capital account still dominates closed capital account. Hence, our model suggests that prudential banking regulations are better than strict capital control in preventing emergence of excessively risky bubbles in a global economy.

Literature review: To our knowledge, our paper provides two contributions to the literature. We provide a first framework in which an unanticipated policy that involves pricking an existing bubble can be Pareto-improving.[7] (We are also not aware of other papers that explicitly analyze both expected and unexpected bubble policies.) And we provide a first general equilibrium model of asset bubbles and global imbalances with two large open economies.

We are indebted to insights from the finite-horizon model of risk shifting in Allen and Gale (2000).[8] Our methodological contribution is to embed risk shifting into an infinite horizon *and* open economy framework, which is closer to traditional macroeconomic analysis.

In the recent literature of rational bubbles,[9] our work is related to models in which rational bubbles are inefficient, but popping them is not Pareto-improving (Saint-Paul (1992), Grossman and Yanagawa (1993), and Farhi and Tirole (2012)). In these papers, bubbles are inefficient as they divert scarce resources away from productive investments, but not because of their risk of collapse.

Our work is also related to Martin and Ventura (2012), which inserts financial friction into a rational bubble model and shows that bubble can emerge even if the economy is dynamically efficient. In their model, friction in the credit market hinders reallocation of resources from inefficient to efficient entrepreneurs, and bubbles *increase* welfare by providing a channel for resource reallocation. In contrast, in our model, bubbles may reduce welfare, and bubbles emerge despite dynamic efficiency not because of credit friction, but because of capital inflows.

Our work bridges the bubble literature and the global imbalances literature,[10] and

[7] The survey of Barlevy (2012) has also made this observation.

[8] Other papers in this strand include Allen and Gale (2000), Barlevy and Fisher (2011) and Barlevy (2013).

[9] See Scherbina (2013) and Barlevy (2012) for recent surveys. Also see Santos and Woodford (1997) for a critique on earlier rational bubble models. For non-rational bubble approaches, see Allen et al. (1993), Abreu and Brunnermeier (2003) and Brunnermeier (2003).

[10] See Caballero et al. (2008a,b), Gourinchas and Jeanne (2013), Justiniano et al. (2013) and the

potentially provide new insights. On the positive side, we formalize how global imbalances (South-to-North capital flows) and asset bubbles are connected, as informally discussed in Bernanke (2005).

On the normative side, Obstfeld and Rogoff (2009) observes that the global imbalances literature often predicts global imbalances as an essentially benign phenomenon, where residents in emerging markets enjoy safety for their savings, while residents in developed countries benefit from cheap credit. This is because literature often assumes the developed countries' financial markets are perfect.

Our paper contributes a normative perspective, by formally showing that unregulated global capital flows can generate in the recipient country a toxic bubble that reduces global welfare, if banks can shift risks to their creditors.

Finally, our work builds on insights from Caballero and Krishnamurthy (2006, 2009). The first shows how risky bubbles can reduce social welfare in a *small* open economy with dynamic *inefficiency*. The second shows how capital inflows can make the U.S. financial market specialize in holding excessively risky assets in a *partial* equilibrium one open economy model with *no bubble*. None of these papers feature risk shifting or analyze pricking policies.

The plan for the rest of the paper is the following. Section 2 provides the workhorse model: a closed economy model of a toxic bubble. First, the section lays out the standard neoclassical growth model with overlapping generations and dynamic inefficiency. Then we add the paper's main ingredient: financial friction in the form of an agency problem in the banking sector. Then we establish results about the emergence and properties of a toxic bubble. Section 3 applies the results of toxic bubbles to a model of two large open economies. First, we derive the global dynamic inefficiency condition. Then we study the dynamics and consequences of global capital flows. The last section concludes and points out questions for future research.

2 Closed economy model

2.1 Benchmark: standard set-up with no bubble

Time is discrete, denoted by $t = 0, 1, 2, ...$ The economy is a closed economy with one homogeneous consumption good and inhabited by households with overlapping generations. Each household lives for two periods and has a constant unit population. The household has no initial asset and supplies one unit of labor inelastically. The household consumes only when it becomes old. For simplicity, households are assumed

citations within.

to be risk-neutral. (We will consider risk aversion later.)

Output Y_t is produced by a representative competitive firm, using physical capital and labor, according to production function:

$$Y_t = F_t(K_t, L_t) = K_t^{\alpha}(A_t L_t)^{1-\alpha}, \quad 0 < \alpha < 1.$$

where K_t and L_t denote the capital stock and labor employed by the firm. The labor-augmenting productivity A_t grows at a constant exogenous rate $g = A_{t+1}/A_t \geq 1$. For simplicity, we assume capital depreciates completely after one period.

Let r_t be the rental rate of capital and W_t be the wage rate. Also, for each variable X_t, denote x_t as the detrended variable: $x_t \equiv X_t/A_t$. For example, $y_t = Y_t/A_t$ and $w_t = W_t/A_t$. The profit maximization by the firm yields factor prices, given by:

$$w_t = (1-\alpha)(K_t/A_t)^{\alpha} L_t^{-\alpha} = (1-\alpha)k_t^{\alpha} \tag{2.1}$$

$$r_t = \alpha(K_t/A_t)^{\alpha-1} L_t^{1-\alpha} = \alpha k_t^{\alpha-1}, \tag{2.2}$$

where the labor market clearing condition, $L_t = 1$, is imposed in the final equality in each equation.

Each young household in period t receives a wage income of W_t for its labor. Since the household only consumes in old age, it saves all of the income. Capital investment K_{t+1} is the only way to save. Thus, $K_{t+1} = W_t$, or equivalently:

$$k_{t+1} = \frac{w_t}{g} = \frac{(1-\alpha)}{g} k_t^{\alpha}, \tag{2.3}$$

where w_t is substituted out using (2.1). Equation (2.3) governs the law of motion for capital. When she becomes old, she consumes all the capital income: $C_{t+1} = r_{t+1}K_{t+1}$, or equivalently:

$$c_{t+1} = r_{t+1}k_{t+1}. \tag{2.4}$$

The standard notion of competitive equilibrium is applied to this economy. Given an initial capital stock $k_0 > 0$, a *equilibrium for an economy with no bubble* consists of allocation $\{k_{t+1}, c_t\}_{t=0}^{\infty}$ and the rental rate $\{r_t\}_{t=0}^{\infty}$ that satisfy (2.2)-(2.4). It is straightforward that the equilibrium uniquely exists and the capital stock converges to the steady state k_{nb} where $k_t = k_{nb}$ for all t satisfies (2.3). Hence, the following result is immediate:

Lemma 1. *[No bubble steady state] The steady state equilibrium in an economy with* *no bubble is:*

$$\begin{aligned} k_{nb} &= \left(\frac{1-\alpha}{g}\right)^{\frac{1}{1-\alpha}}, \\ c_{nb} &= \alpha\left(\frac{1-\alpha}{g}\right)^{\frac{\alpha}{1-\alpha}}, \\ r_{nb} &= \frac{\alpha}{1-\alpha}g. \end{aligned}$$

In the steady state, the economy grows at the rate of g.

Golden rule

As standard in a neoclassical growth model, we consider a benevolent central planner who wants to maximize steady state consumption. The resource constraint in steady state is:

$$c = f(k) - g \cdot k.$$

Let k_{gold} be the detrended capital-labor ratio in the steady state that maximizes c. This ratio is known as the golden rule ratio. In the steady state, the consumption is given by the resource constraint: $c = k^{\alpha} - gk$. Maximizing c with respect to k yields:

$$k_{gold} = \left(\frac{\alpha}{g}\right)^{\frac{1}{1-\alpha}}.$$

Under the golden rule, the detrended consumption and the interest rate in steady state are:

$$\begin{aligned} c_{gold} &= (1-\alpha)\left(\frac{\alpha}{g}\right)^{\frac{\alpha}{1-\alpha}}. \\ r_{gold} &= g. \end{aligned}$$

Let us compare the competitive equilibrium in the steady state and the economy under the golden rule. Suppose that the following condition holds:[11]

$$\text{Dynamic inefficiency: } \alpha \quad < 1/2 \quad . \tag{2.5}$$

[11] If the utility function is $\log c_{young} + E\beta \log c_{old}$, then the economy is dynamic inefficient if

$$\frac{\beta}{1+\beta}\alpha < 1 - \alpha.$$

Then, $\alpha < 1 - \alpha$, and hence:

$$\begin{aligned} k_{nb} &> k_{gold}, \\ c_{nb} &< c_{gold}, \\ r_{nb} &< r_{gold} = g. \end{aligned}$$

In words, when there is insufficient storage of value in the economy (capital share α less than $1/2$), people will accumulate too much capital ($k_{nb} > k_{gold}$), leading to an interest rate that is smaller than the growth rate of the economy ($r_{nb} < g$), and consumption that is sub-optimally low ($c_{nb} < c_{gold}$).

Condition (2.5) captures the notion that many emerging economies, whose capital market is underdeveloped, suffer from a shortage of stores of value. It is a classic result that this condition, known as dynamic inefficiency, gives rise to a bubble (see Tirole (1985)). The next subsection recaptures this result for a stochastic bubble.

2.2 Benchmark: bubble but no financial friction

Consider an asset in fixed supply, normalized to unity, that has no fundamental value, i.e., it pays no dividend in any period.[12] Let $\{\tilde{P}_t\}_{t\geq 0}$ be its price process. The asset is called a bubble if the price has a positive value. We assume a bubble burst in each period with exogenous probability $\lambda \in [0, 1)$. A critical assumption here is that young households have a direct access to financial markets so that they invest in a safe asset (capital stock) and/or a bubble asset. This setup is equivalent to the one in which young households make a deposit in intermediaries that behave competitively on behalf of the households.

Suppose that there is a bubble in period t. Taking the bubble's price process as given, each young household chooses a portfolio, consisting of the capital investment, K_{t+1}, and the bubbly asset, a_t, to maximize the expected consumption in old age:

$$\max_{K_{t+1}\geq 0, a_t \geq 0} E_t[C_{t+1}(\tilde{P}_{t+1})]$$

[12] The assumption of zero dividend allows us to formalize bubbles in the simplest way possible. However, it is not necessary. The price of any asset that pays positive dividends can have a bubbly component. Formally, consider an asset that pays dividend d_t in every period. For simplicity assume the gross interest rate is a constant r. Let its price process be $Q_t = f_t + \tilde{P}_t$, where $f_t = \sum \frac{d_t}{r^t}$ is the fundamental component, and $\tilde{P}_t$ is the bubbly component, as defined in this section. See Ikeda (2013) for such an approach in a dynamic stochastic general equilibrium model with asset bubble (but with no risk shifting).

$$\begin{aligned} \text{s.t.} \quad C_{t+1}(\tilde{P}_{t+1}) &= \tilde{P}_{t+1}a_t + r_{t+1}K_{t+1} \\ W_t &= \tilde{P}_t a_t + K_{t+1}, \end{aligned}$$

where the stochastic process $\tilde{P}_{t+1} = 0$ with probability λ and $\tilde{P}_{t+1} = P_{t+1} > 0$ with probability $1 - \lambda$. We call this problem the "self-investment problem" of young people. The first-order conditions of the problem yield:

$$(1-\lambda)g\frac{p_{t+1}}{p_t} = r_{t+1}, \tag{2.6}$$

$$(1-\alpha)k_t^{\alpha} = w_t = p_t + gk_{t+1}, \tag{2.7}$$

where $p_t = P_t/A_t$ is the detrended price of bubble, and the bubble asset market clearing condition, $a_t = 1$, and (2.1) were used in deriving (2.7). An arbitrage condition, (2.6), implies that the expected return by investing in a bubble asset (LHS of (2.6)) has to be equated to the return by investing in capital stock (RHS of (2.6)). Conditional that the bubble sustained in period t, i.e., $p_t > 0$, the consumption is given by:

$$c_t = p_t + r_t k_t, \tag{2.8}$$

where the bubble asset market clearing condition, $a_t = 1$, is imposed.

To complete the set up, we assume that if either there is no bubble in period $t-1$ or the bubble has bursted in period $t-1$, then a new bubble emerges with exogenous probability $\rho \geq 0$ in period t. This is not an essential assumption, but when $\rho > 0$, the condition guarantees that the economy reaches a bubbly steady state (defined below) with a positive probability in the long run. For simplicity, assume that the new bubble also bursts with the same probability λ.

Suppose that there is a bubble in period $t = 0$, $p_0 > 0$. Suppose, for expositional purpose, that a bubble will not emerge after a bubble burst. Then, given an initial capital stock $k_0 > 0$, a *bubbly equilibrium with self-investment* consists of allocation $\{k_{t+1}(\tilde{p}_t), c_t(\tilde{p}_t)\}_{t=0}^{\infty}$ and prices $\{r_t(\tilde{p}_t), \tilde{p}_t\}_{t=0}^{\infty}$ such that (i) if $p_t > 0$, $\tilde{p}_{t+1} = 0$ with probability λ and $\tilde{p}_{t+1} = p_{t+1}$ with probability $1-\lambda$; (ii) if $p_t > 0$, the allocation and the prices satisfy (2.2), (2.6), (2.7) and (2.8); (iii) if $p_t = 0$, a bubble never emerges and the allocation and the rental rate coincide with those of an equilibrium for an economy with no bubble.

Conditional that a bubble does not burst, there exist a steady state in which all detrended variables and the rental rate stay constant. Formally, a *bubbly steady state with self-investment* consists of allocation $\{k, c\}$ and prices $\{r, p\}$ that satisfy the conditions

(i) and (ii) of a bubbly equilibrium in a self-investment economy where time subscript t is removed. If a bubble bursts, the return earned by old household consists only of return on capital, rk. Thus, in the steady state, the expected consumption, which is the expected utility of young household, is given by $c^e = (1-\lambda)c + \lambda(rk)$, where rk is the consumption when the bubble bursts.

The following Lemma summarizes the standard results of a bubbly steady state with self-investment. The Lemma states that: if (i) young people can invest directly in financial assets, (ii) the economy is dynamically inefficient *and* (iii) the probability of bubble burst is low enough, then there exits a unique bubbly steady state equilibrium, and the bubble improves social welfare.

Lemma 2. *[Self-investment bubbly steady state]*

1. *There exists a bubbly steady state equilibrium if and only if the economy is dynamically inefficient ($\alpha < 1/2$) and the probability of bubble burst is lower than a threshold $\underline{\lambda}$:*
$$\lambda < \underline{\lambda} \equiv \frac{1-2\alpha}{1-\alpha}$$
2. *This bubbly steady state equilibrium is unique, and satisfies:*
$$\begin{aligned} k_{self} &= \left(\frac{\alpha}{(1-\lambda)g}\right)^{\frac{1}{1-\alpha}} \\ r_{self} &= (1-\lambda)g \\ p_{self} &= (1-\alpha)(k_{self})^{\alpha} - g \cdot k_{self}, \end{aligned}$$
and the expected consumption in the next period is given as:
$$c^e_{self} = (1-\lambda)(1-\alpha)\left(\frac{\alpha}{(1-\lambda)g}\right)^{\frac{\alpha}{1-\alpha}}.$$
3. *The bubble improves welfare: the expected consumption is greater than that in the no bubble equilibrium, i.e., $c^e_{self} > c_{nb}$.*

Proof. Appendix. □

Lemma 2 states the standard result that a bubble is welfare improving. This is common in the literature of bubbles. We will overturn this result by modifying this standard model in a simplest possible manner in the next subsection.

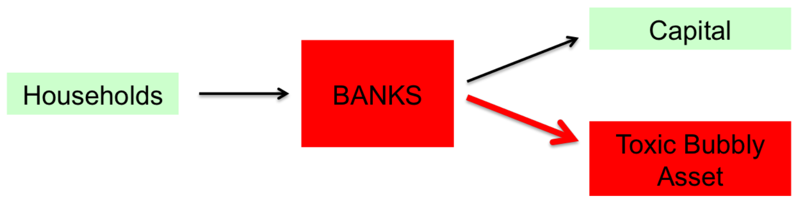

Fig. 2.1: Diagram of the closed economy model with financial friction

2.3 Bubble with financial friction

2.3.1 Set-up

We modify the self-investment benchmark by introducing financial friction. In particular, we assume that young people cannot directly invest in capital or bubbly asset. Instead, they save their income with financial intermediaries, which will invest for them.[13] We call these financial intermediaries "banks." We assume banks offer a debt contract and are protected by a limited liability law. We model limited liability in the simplest possibly way: that banks' profits are zero when they default on loans. Furthermore, as in Jensen and Meckling (1976), Stiglitz and Weiss (1981), Allen and Gorton (1993) and Allen and Gale (2000), we assume that households lend to banks through a standard debt contract. We provide a microfoundation for this assumption in the appendix, following Townsend (1979).

In every period t, a unit population of new banks are born. Newly born banks offer a debt contract to young households at the interest rate r^D_{t+1}, take deposits of young households, D_t, then use these deposits to invest in capital and the bubbly asset. They reap profits in period $t+1$, and exit. Each bank chooses its portfolio to maximize the expected profit:

$$\max_{a_t, K_{t+1} \geq 0} E_t[\Pi_{t+1}(\tilde{P}_{t+1})] = E_t \max\{\underbrace{\tilde{P}_{t+1}a_t + r_{t+1}K_{t+1} - r^D_{t+1}D_t}_{\text{no default}}, \underbrace{0}_{\text{default}}\}$$

subject to $D_t = \tilde{P}_t a_t + K_{t+1}$. The profit captures the *agency problem* of banks: if the return on investment,$\tilde{P}_{t+1}a_t + r_{t+1}K_{t+1}$, is smaller than the amount owed plus interest, $r^D_{t+1}D_t$, then the bank defaults. Hence the bank does not fully internalize the downside risk of its investment. This is a standard financial agency problem (see Jensen and Meckling (1976) and Stiglitz and Weiss (1981)). This is the only form of financial friction in our model. Figure 2.1 is a diagram that summarizes the model's set-up.

The banks' problem is solved by guessing and verifying that banks default on their promised payment only when a bubble bursts. Then, the banks expected profits are

[13] This is a realistic assumption. Most individuals and households do not directly manage their wealth. Instead, they delegate wealth management to financial institutions.

expressed as:

$$E_t[\Pi_{t+1}(\tilde{P}_{t+1})] = (1-\lambda)(P_{t+1}a_t + r_{t+1}K_{t+1} - r^D_{t+1}D_t) + \lambda \times 0.$$

Thus, the banks maximize the profits when a bubble sustains, ignoring the profits (loss) when a bubble bursts. Maximizing the profits yields the following arbitrage conditions: $r^D_{t+1} = r_{t+1}$ and

$$g\frac{p_{t+1}}{p_t} = r_{t+1}, \tag{2.9}$$

Compared with an economy with no agency problem, the only difference between (2.6) and (2.9) is the absence of the probability of bubble burst λ in (2.9). Because the banks pay attention only to the profits when a bubble sustains, they do not take into account the possibility of bubble burst, resulting in the absence of λ in (2.9).

The arbitrage conditions of the banks' problem imply that the banks earn zero profits even when a bubble sustains because of perfect competition. Then, it is verified that the profits would be negative when the bubble bursts if the promised interest rate on deposits were paid:

$$\begin{aligned} 0 \times a_t + r_{t+1}K_{t+1} - r^D_{t+1}D_t &= 0 \times a_t + r_{t+1}K_{t+1} - r_{t+1}(P_t a_t + K_{t+1}), \\ &= -P_t a_t = -P_t < 0. \end{aligned}$$

Therefore, the banks default on the deposits when the bubble bursts, as guessed in solving the banks' problem.

A *bubbly equilibrium with financial friction* and the steady state are defined exactly in the same manner as done for an bubbly equilibrium with self-investment, except that condition (2.6) is replaced with condition (2.9). In the equilibrium the amount of deposits and the interest rate on deposits are given by $D_t = D_t/A_t = w_t$ and $r^D_{t+1} = r_{t+1}$ respectively.

We first study the steady state properties of the bubbly equilibrium with financial friction and summarize the properties in the following Lemma.

Lemma 3. *[Bubbly steady state with financial friction]*

1. *There is a bubbly steady state equilibrium if and only if the economy is dynamically inefficient ($\alpha < 1/2$).*

2. *The bubbly steady state equilibrium is unique and coincides with an equilibrium under the golden rule: $k_b = k_{gold}$, $c_b = c_{gold}$ and $r_b = r_{gold}$. The bubble is given by*

$p_b = (1-\alpha)k_b^\alpha - g \cdot k_b$ and the expected consumption in the next period is given by

$$c_b^e = [(1-\lambda)(1-\alpha) + \lambda\alpha] \left(\frac{\alpha}{g}\right)^{\frac{\alpha}{1-\alpha}}.$$

Proof. Appendix. □

In the bubbly equilibrium with financial friction, the allocation coincides with that in the golden rule. Thus, in the bubbly steady state, the consumption is higher than that when there are no financial friction: $c_b > c_{self} > c_{nb}$. Interestingly, the size of bubble is greater in the bubbly steady state with financial friction than that with self-investment: $p^* > p_{self}$. To see this, in the steady state, the size of bubble is expressed as a function of capital as $p(k) = (1-\alpha)k^\alpha - g \cdot k$. Taking the first derivative, we obtain $p'(k) < 0$ for $k > k_{gold}$. Because $k_{self} > k_{gold} = k_b$, we obtain the result, $p_b > p_{self}$. Therefore, the financial friction in the economy increases the size of bubble and increase the consumption and thus improve welfare conditional on an event that the bubble persists. This result seems to support an intuition that a boom caused by a bubble is perceived as a good event when the bubble sustains. A bubble, however, can be collapsed eventually and thus may not be welfare-improving, which we will investigate below.

2.3.2 Toxic asset bubble

In this economy, the welfare is measured simply by the expected consumption of households. From Lemma 3, the expected consumption in the bubbly steady state with financial friction is expressed as $c_b^e = (1-\lambda)c_{gold} + \lambda\alpha/(1-\alpha) \cdot c_{gold}$. This expression implies that the consumption drops to $\alpha/(1-\alpha) \cdot c_{gold} < c_{nb}$ if the bubble is collapsed.[14] Thus, for given $\alpha < 1/2$, as the probability of bubble burst λ rises, the expected consumption becomes lower and can go under the consumption in the steady state with no bubble, c_{nb}. Put differently, the bubble becomes toxic and welfare-reducing in the bubbly steady state with financial friction if the probability of bubble burst is great enough. Formally, $c_b^e < c_{nb}$ if and only if $\lambda > \bar{\lambda}$, where:

$$\bar{\lambda} \equiv [1 - (\frac{\alpha}{1-\alpha})^{\frac{1-2\alpha}{1-\alpha}}] \cdot (\frac{1-\alpha}{1-2\alpha}). \tag{2.10}$$

Since $\alpha < 1-\alpha$, it follows that $\bar{\lambda} < 1$. The appendix shows that $\bar{\lambda} > \underline{\lambda}$.

We summarize the result in the following proposition:

[14] From Lemma 1, the consumption in the steady state with no bubble is expressed as $c_{nb} = [\alpha/(1-\alpha)]^{(1-2\alpha)/(1-\alpha)} \cdot c_{gold}$, which is greater than $\alpha/(1-\alpha) \cdot c_{gold}$ because $\alpha < 1-\alpha$.

Proposition 1. [Toxic asset bubble]

1. *Welfare in the bubbly steady state with financial friction is* ***worse*** *than in the steady state with no bubble, i.e.,* $c_b^e < c_{nb}$*, if and only if the probability of bubble burst* λ *is greater than* $\bar{\lambda}$ *defined by (2.10).*

2. *Suppose that young households can self-invest. When* $\lambda > \bar{\lambda}$*, there is no bubbly steady state with self-investment.*

Proof. Appendix. □

It is the agency problem that allows the banks to engage in unproductive investment by purchasing a bubble asset and makes the welfare worse. In the bubbly steady state with financial friction, the bubble grows at the rate of g if the bubble is sustained and bursts with probability λ. Thus, the expected return on bubble is given by $(1-\lambda)g$. The return on bubble becomes lower as the bubble is more likely to burst. Without the agency problem, the banks would take into account a low return on bubble as shown in the arbitrage condition (2.6). Thus, a bubble would not be sustained if the probability of bubble burst is great enough so that the return on bubble becomes too low, at which the banks' demand for the bubble asset drops to zero, as stated in Proposition 1(2). With the agency problem, however, the banks ignore a low return in case of a bubble bursts as clearly seen by the arbitrage condition (2.9). The banks behave as if the bubble would last forever and invest in the bubble asset even if the probability of bubble burst becomes greater. Ironically, the consumption attains its maximum, and thus coincides with the golden rule as long as the bubble is sustained. The consumption drops sharply, however, when the bubble is collapsed. Because the banks, with the agency problem, ignore the latter event, the welfare becomes worse than the steady state with no bubble if the probability of bubble burst is great enough.

It is worth noting that the banks, with the agency problem, take on excessive risk by investing in a risky bubble asset when $\lambda > \bar{\lambda}$. The banks can, instead, invest all of their money in safe capital investment, which will improve welfare. Though what matters for households is the expected return and not the risk itself under the assumption of risk-neutrality, the excessive risk-taking associated with unproductive investment in a bubble asset is a unique phenomenon this model can explain.

2.4 Policies

We showed that a bubble can be toxic, i.e., welfare-reducing in the steady state. The market failure caused by financial friction may rationalize policy interventions. In this subsection we explore three polices to tackle a toxic bubble: prudential banking policy, monetary policy and a pricking-bubble policy.

2.4.1 Leaning against the bubble

We have shown that the emergence of bubble can worsen the welfare in the steady state when banks have an agency problem:

Corollary 1. *An anticipated policy that pricks toxic bubbles ($\lambda > \overline{\lambda}$) improves the steady state expected life-time utility of all generations.*

From normative perspective, this result backs the view that risky asset bubble should be prevented from emergence.[15]

But, does the result support a view that a toxic bubble should be pricked when the bubble *already* exists? We now study the dynamics after a bubble burst and address the question. Suppose that an equilibrium is in a bubbly steady state with financial friction initially. We consider a case in which a bubble bursts in the beginning of period T and converges to the steady state with no bubble.

For simplicity, we make the following assumption for the rest of this subsection: once a bubble is pricked, new bubbles do not emerge ($\rho = 0$). This is for simplicity. It is straightforward to extend the analysis below if new bubbles can emerge.[16]

For $t \geq T$, the transitional dynamics of consumption and capital are characterized by (2.3) and (2.4). We rewrite the equations for convenience here:

$$\begin{aligned} k_{t+1} &= \left(\frac{1-\alpha}{g}\right) k_t^{\alpha}, \text{ with } k_T = k_b, \\ c_t &= \alpha k_t^{\alpha}, \end{aligned}$$

for all $t \geq T$. Comparing with the welfare in the bubbly steady state, it is obvious that the old household in period T suffers: $c_T = \alpha k_b^{\alpha} < c_b^e$. The welfare of the households born in period T is measured by $c_{T+1} = \alpha k_{T+1}^{\alpha}$ where $k_{T+1} = \left(\left(1-\alpha\right)/g\right) k_b^{\alpha}$. Some calculations show that the welfare of the households who are born just after the bubble burst is greater than the welfare in the bubbly steady state if and only if $\lambda > \check{\lambda}$, where:[17]

$$\check{\lambda} = \frac{1-\alpha}{1-2\alpha}\left[1-\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right] \in \left(1/2, 1\right). \tag{2.11}$$

Because the consumption increases as capital stock converges from k_b to $k_{nb} > k_b$, the

[15] In the dynamic path from the steady state with no bubble to the bubbly steady state with financial friction, it could be the case that some generations in the transition enjoy a higher expected consumption. We plan to consider the transition in our future work.

[16] In particular, the unanticipated policy not only pricks a toxic bubble, but it is also permanent. Hence, once the bubble is pricked, agents expect that future toxic bubbles will also be pricked. This effectively removes future toxic bubbles.

[17] To derive this condition, note that c_{T+1} is expressed as $c_{T+1} = \alpha \left(c_{gold}/g\right)^{\alpha}$ while c_b^e is expressed as $c_b^e = \left[1-\lambda+\lambda\alpha/\left(1-\alpha\right)\right] c_{gold}$. Solving the condition $c_{T+1} > c_b^e$ for λ leads to the condition.

welfare of generations born after T is greater than that of generations born in T.

Intuitively, if the probability burst is great enough, the expected consumption becomes close to the level of consumption when the bubble is collapsed. Because the banks, with the agency problem, ignore the possibility of bubble burst, the expected consumption in the bubbly steady state is so low that pricking the bubble is welfare-improving for all households except the old households at the time the bubble collapses.

This result raises an interesting question whether pricking a bubble can be Pareto improving, i.e., welfare improving for all households including, if pricking a bubble is associated with government transfers from the young households to the old households when a bubble is pricked. It turns out that pricking a bubble is Pareto improving if the probability of bubble burst is great enough.

Let θ denote lump-sum transfers from the young households to the old households when a bubble is pricked. Then, the old households prefer to prick a bubble if and only if the consumption when the bubble is picked is equal or greater than that when the bubble is sustained: $c_T = \alpha k_b^\alpha + \theta \geq c_b^e$. Setting θ such that the old households are indifferent between the two events, $c_T = c_b^e$, yields $\theta = \alpha k_b^\alpha - c_e^D$. With such a transfer, the young households' net income changes to $w_T - \theta$ when a bubble is pricked. The young households' welfare is given by its consumption in the next period, or, $c_{T+1} = \alpha[(1/g)(k_b^\alpha - c_b^e)]^\alpha$. Solving $c_{T+1} \geq c_b^e$ for λ yields the following condition:

$$\lambda \geq \hat{\lambda} \equiv \frac{1-\alpha}{2-\alpha}\left[\frac{\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} - \frac{\alpha}{1-\alpha}}{1+\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}}\right] \in (0,1).$$

It can be shown that the threshold $\hat{\lambda}$ is greater than $\check{\lambda}$. Hence, if the probability of bubble burst is great enough, there exists a pricking bubble policy with one time transfers θ such that the policy is Pareto improving. We summarize the result in the following proposition.

Proposition 2. [Pricking bubble] *Consider the bubbly steady state with financial friction.*

1. *If the bubbly is sufficiently risky, $\lambda > \check{\lambda}$, then an unanticipated policy that pricks the existing bubble improves welfare for all generations except for the current old generation (who holds the bubble).*

2. *If $\lambda > \hat{\lambda}$, then unanticipatedly pricking the existing bubble combined with a one-time transfer for $\theta = \alpha k_b^\alpha - c_e^D$ from the current young's wage income to the current old is a Pareto improvement.*

Proof. See preceding text. □

Admittedly, if taken as a face value, the threshold $\check{\lambda} \in (1/2, 1)$ is too big to be true in practice. If the probability of bubble burst is greater than 50 percent, the bubble is not likely to be sustained. The model, however, is so stylized that the value of the threshold is at most suggestive. With a richer model with risk-aversion and consumption by young households, the threshold would become lower.

2.4.2 Macro-prudential Banking Policy

In the previous section, we assumed that the authority can observe not only a bubble but also its risk. This is a strong assumption. In practice, it is difficult to detect a bubble, and extremely difficult to detect its risk. What policies can improve welfare given these difficulties?

In this subsection, we show that a macro-prudential banking policy in the form of capital requirement can prevent toxic bubbles from emerging. This prevention policy does not require the authority to observe either the existence of a bubble or the bubble's risk.

Assume that a bank born in period t is endowed with a small amount of equity ε_t. We also assume ε_t grows at rate g: $\varepsilon_t = g^t \varepsilon$, where $\varepsilon > 0$ is a small constant. Consider a (risk-adjusted) regulation that limits banks' *risky* investment (but not non-risky investment). Formally, the regulation imposes that each bank's risky investment, captured by term $P_t a_t$, cannot exceed $\mu \cdot \varepsilon_t$, where $\mu \geq 0$ is a "capital requirement" parameter chosen by the authority. Each bank's maximization problem becomes:

$$\max_{a_t \geq 0, K_{t+1} \geq 0} E_t[\max\{\underbrace{\tilde{P}_{t+1} a_t + r_{t+1} K_{t+1} - r^D_{t+1} D_t}_{\text{no default profit}},\ 0\}]$$

subject to:

$$\begin{aligned} \varepsilon_t + D_t &= P_t a_t + K_{t+1} \\ P_t a_t &\leq \mu \varepsilon_t. \end{aligned} \tag{2.12}$$

We are only interested in the scenario where the capital requirement is sufficiently strong that constraint (2.12) binds. Then it is straightforward to show that the problem is equivalent to:

$$\max_{a_t \geq 0, K_{t+1} \geq 0} E_t[\max\{\tilde{P}_{t+1} a_t + r_{t+1} K_{t+1} - r^D_{t+1} \cdot (P_t a_t + K_{t+1}),\quad \underbrace{-\frac{1}{\mu} r^D_{t+1} P_t a_t}_{\text{equity loss in default}}\ \}]$$

subject to

$$P_t a_t = \mu \varepsilon_t.$$

With capital requirement, the first order condition with respect to a_t is:

$$(1-\lambda)\frac{\tilde{P}_{t+1}}{P_t} \geq [(1-\lambda) + \frac{\lambda}{\mu}] r^D_{t+1} \tag{2.13}$$

(and the first order condition with respect to K_{t+1} is as before: $r_{t+1} \geq r^D_{t+1}$, and it is straightforward to show that equality happens in equilibrium).

Note that when $\mu = 1$, the right hand side of (2.13) is exactly r^D_{t+1}. In steady state, the interest rate $r^D = r$ will be $r = (1-\lambda)g$ (see appendix for proof), exactly as in the self-investment bubble benchmark. This is intuitive: when $\mu = 1$, banks are forced to use only their own equity to invest in the risky bubble ($P_t a_t = \varepsilon$).

In general, a stricter capital requirement (smaller μ) makes banks internalize the downside risk of investment more (the loss term $\frac{1}{\mu} r^D_{t+1} P_t a_t$ is bigger). In fact, we can show the following proposition:

Proposition 3. [Capital requirement] *For each level of risk λ, there exists a capital requirement μ (imposed in all periods) such that* no *bubbles whose risk of collapse is larger than λ can emerge in steady state.*

Proof. Appendix. □

A direct corollary is the following:

Corollary 2. [Capital requirement] *There exists a capital requirement μ (imposed in all periods) such that no toxic bubble (i.e., whose risk of collapse exceeds $\overline{\lambda}$) can emerge in steady state. This capital requirement policy improves welfare.*

Proof. Appendix. □

2.5 Robustness

This section discusses some robustness checks. A general finding is that our results so far do not change qualitatively if households are risk averse, if households consume at young age, or if there is a wedge in the capital market.

2.5.1 Risk averse households

A toxic bubble reduces social welfare even more when households are risk-averse. Suppose households' utility of consumption in old age is $u(c_t)$, where $u(\cdot)$ is a strictly increasing and strictly concave utility function. We still maintain the simplifying assumption that households do not consume in young age. It is straightforward to show that the bubbly steady state with financial friction, and the no-bubbly steady state remain the same. Furthermore, since

$$c_b^e < c_{nb},$$

and households are risk averse, it is immediate that:

$$E[u(c_b)] < u(c_{nb}).$$

In fact, there are two reasons for this loss in utility. First, expected consumption in the bubbly steady state is strictly smaller than the that in the no-bubble steady state. Second, consumption in the bubbly steady state is more volatile than that in the no-bubble steady state. The more risk averse are the households in old age, the larger is the second effect.

2.5.2 Young consumption

Suppose people also consume in young age, and they have a standard utility function:

$$\log(c_t^y) + \beta \log(c_{t+1}^o)$$

where c_t^y is consumption when young and c_t^o is consumption when old. It is straightforward to show that all the results qualitatively carry through. The new condition for dynamic inefficiency is:

$$\alpha < (1-\alpha)\frac{1+\beta}{\beta}.$$

2.5.3 Financial under-development wedge

The model can also be easily extended to have a wedge between public and private return to capital. Following Gourinchas and Rey (2013), assume that the private return to capital investment is:

$$r_t = (1-\tau)f'(k_t)$$

where τ is a wedge that captures financial underdevelopment. It is straightforward to show that our results qualitatively carry through. The new condition for dynamic inefficiency is:

$$(1-\tau)\alpha < (1-\alpha)\frac{1+\beta}{\beta}.$$

In other words, a bubble emerges if the economy's capital market is under-developed:

$$\tau > 1 - \frac{1-\alpha}{\alpha}\frac{1+\beta}{\beta}.$$

3 Two open economy model: bubbly capital flows

Now we generalize the model studied in Section 2.3 to a two-country setting. We first establish a "global dynamic inefficiency" condition, under which there is a bubbly steady state equilibrium. This condition is weaker than assuming that both economies are dynamically inefficient. In particular, even if one economy – a developed economy – is dynamically efficient, a bubble can still emerge in the economy if the other economy – an emerging economy – is sufficiently dynamically inefficient. Furthermore, the bubble emerges because of global imbalances, i.e., because of capital flows from an emerging economy to a developed economy. Then, we show that the bubble can be toxic. Our model thus matches stylized features of the global imbalances leading up to the recent crisis, and the subsequent bursting of the housing bubble in developed economies.

3.1 Set-up

There are two large economies: North and South. As usual, all variables in the Southern economy have a star. An environment for each economy is similar to that of the one-country model with financial frictions in Section 2.3. In each economy, there are overlapping generations with constant unit population. The production functions are:

$$\begin{aligned} F_t(K_t, L_t) &= (K_t)^{\alpha}(g^t L_t)^{1-\alpha}, \\ F_t^*(K_t^*, L_t^*) &= (K_t)^{\alpha^*}(g^t L_t^*)^{1-\alpha^*}, \end{aligned}$$

where g is the growth rate of labor-augmenting technology. The capital income ratio satisfies $0 < \alpha, \alpha^* < 1$ and $\alpha \neq \alpha^*$. Note that we assume both economies have an identical technological growth rate g.[18]

In every period, there is a unit population of newly born competitive banks. Each bank exists for two periods. In the first period they take deposits, and in the second period they repay debt or default, earn profits, and exit. As in the one-country model in Section 2.3, young households can save only through banks. In addition, it is assumed that banks cannot directly invest in foreign assets, but banks can use international inter-bank borrowing/lending.[19] Then, the balance sheet of banks in the North is expressed

[18] If not, then one country's relative size is asymptotically zero, which is not interesting nor realistic.

[19] This assumption is in line with the fact of cross-border financial activities. Compared to cross-border bond investment, very little of global capital flows are in the form of cross-border equity

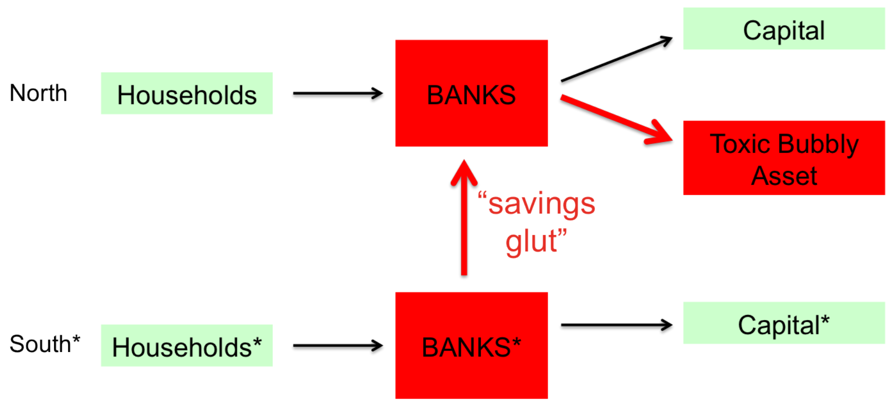

Fig. 3.1: Diagram of the open economy model with financial friction

as

$$K_{t+1} + P_t a_t + D_t = W_t + D_t^*, \tag{3.1}$$

where $D_t \geq 0$ ($D_t^* \geq 0$) denotes the amount of bonds issued by Southern banks (Northern banks). The left-hand-side of (3.1) is an asset side consisting of capital K_{t+1}, bubble assets $P_t a_t$ and bonds held by Northern banks D_t. The right-hand-side of (3.1) is a liability side consisting of deposit which is equal to wage income W_t, and bonds held by Southern banks D_t^*. A balance sheet for the South is symmetric to (3.1).

Finally we assume, for analytical simplicity, that if a domestic bank goes bankrupt,[20] domestic residents are the first claimers of the defaulted bank's assets, and foreign lenders are the second. In practice, international bankruptcy laws maybe harder to enforce than domestic bankruptcy laws. Furthermore, the assumption allows us to describe financial contagion from one country to the other easily. With this assumption, if banks in one country go bankrupt, then in equilibrium, banks in the other country will also go bankrupt. Figure 3.1 summarizes the open economy model.

3.2 Benchmarks

3.2.1 Two closed economies

If two economies are closed, then they are mere copies of the closed economy considered in section 2. If an economy is dynamically inefficient, then there is a unique bubbly steady state equilibrium with financial friction in that economy.

For the rest of the paper, we assume that capital can flow freely between the banks in the two economies.

investment or cross-border real estate investment.

[20] Note that we have ruled out strategic defaults by banks. Banks will always repay if they can.

3.2.2 Open economy: no bubble

We consider the benchmark open economy model where there is no bubble. It is then straightforward to see that there will be no default, and hence no agency problem. The aggregate global deposits or savings in each period is $W_t + W_t^*$. In equilibrium, market clearing of capital requires $K_{t+1} + K_{t+1}^* = W_t + W_t^*$. Detrending the condition by g^t yields:

$$gk_{t+1} + gk_{t+1}^* = w_t + w_t^* = (1-\alpha)k_t^\alpha + (1-\alpha^*)(k_t^*)^{\alpha^*}, \tag{3.2}$$

where equation (2.1) was used in the second equality.[21] The first order conditions of banks' maximization problem lead to an equation that plays the role of the "UIP" (uncovered interest parity) condition in this setting: $r_t = r_t^*$, or equivalently:

$$\alpha k_t^{\alpha-1} = \alpha^*(k_t^*)^{\alpha^*-1}, \tag{3.3}$$

where equation (2.2) was used.

The steady state for capital stock, k_{nb} and k_{nb}^*, can be solved from (3.2) and (3.3). The world interest rate is then given by $r_{nb} = r_{nb}^* = \alpha k_{nb}^{\alpha-1}$. We define that the world economy is globally dynamic inefficient when the world interest is less than the growth rate of the world economy: $r_{nb} = r_{nb}^* < g$. The following Lemma states a necessary and sufficient condition for global dynamic inefficiency.

Lemma 4. *[Global dynamic inefficiency] Consider a steady state with no bubble in the two-economy model. The global economy is dynamic inefficient, i.e., $r_{nb} = r_{nb}^* < g$, if and only if*

$$(1-2\alpha)\left(\frac{\alpha}{g}\right)^{\frac{\alpha}{1-\alpha}} + (1-2\alpha^*)\left(\frac{\alpha^*}{g}\right)^{\frac{\alpha^*}{1-\alpha^*}} > 0. \tag{3.4}$$

Proof. Appendix. □

The global dynamic inefficiency condition, (3.4), implies that the global economy becomes dynamic inefficient even if the North is dynamically efficient, i.e., $\alpha > 1/2$, as long as the degree of dynamic inefficiency in the South is strong enough, i.e., $\alpha^* < 1/2$ is low enough, to satisfy condition (3.4).

For later reference, we calculate welfare or consumption for the North and the South. Suppose that the North is dynamically efficient, $\alpha > 1/2$, but the world economy is globally dynamic inefficient so that (3.4) holds. For the North, the detrended consumption in the next period is given by:

$$c_{nb} = r_{nb}w_{nb}/g = \frac{\alpha(1-\alpha)}{g}k_{nb}^{2\alpha-1}.$$

[21] Here the equation for the South is symmetric to the North. We will not mention equations for the South if the equations are symmetric to the North.

Because the world economy is globally dynamic inefficient, the world interest rate, r_{nb}, is less than g, so that $k_{nb} > (\alpha/g)^{1/(1-\alpha)}$. Given the assumption of $\alpha > 1/2$, the consumption is greater than that under the golden rule in a closed economy:

$$c_{nb} > \frac{\alpha(1-\alpha)}{g}\left(\frac{\alpha}{g}\right)^{\frac{2\alpha-1}{1-\alpha}} = (1-\alpha)\left(\frac{\alpha}{g}\right)^{\frac{\alpha}{1-\alpha}} \equiv c_{gold}. \tag{3.5}$$

The inequality (3.6) shows that the North benefits from a free trade more than what the country can achieve by choosing a feasible allocation in a closed economy. For the South, the world interest rate, r^*_{nb}, is greater than the interest rate in a closed economy.[22] Then, the capital held by the South in an open economy is smaller than that in a closed economy: $k^*_{nb} < [(1-\alpha^*)/g]^{1/(1-\alpha^*)}$. Because the assumption of $\alpha > 1/2$ and the global dynamic inefficiency implies $\alpha^* < 1/2$, the consumption is greater than that in a closed economy:

$$c^*_{nb} = \frac{\alpha^*(1-\alpha^*)}{g}(k^*_{nb})^{2\alpha^*-1} > \underbrace{\alpha^*\left(\frac{1-\alpha^*}{g}\right)^{\frac{\alpha^*}{1-\alpha^*}}}_{c^*_{closed}}.$$

Similar to the North, the South also benefits from a free trade. But, unlike the North, the fact that $k^*_{nb} > k^*_{gold}$ implies that the consumption does not exceed that under the golden rule: $c^*_{nb} < c^*_{gold}$. To summarize,

$$c^*_{closed} < c^*_{nb} < c^*_{gold} \tag{3.6}$$

3.3 Global imbalances and toxic bubble

Now we study a two-country model with a bubble. Time begins in period $t = T$. We assume there is no bubble in South in $t \geq T$. We will see later that T represents the period when the bubble in the South bursts (think of the Asian crisis). Again, recall that interbank borrowing and lending flows freely between two economies. We will show that:

1. Despite that North is dynamically efficient, a bubble can emerge in equilibrium in the North. This bubble is made possible by a "saving glut" pouring from the South to the North.

[22] To confirm this, suppose contrarily that $r^*_{nb} = r_{nb}$ is equal or lower than the interest rate in a closed economy. Then, capital is so abundant that $gk^* - (1-\alpha)(k^*)^{\alpha^*} \geq 0$, where the equality holds when r_{nb} is equal to the interest rate in a closed economy. From the market clearing condition for capital, (3.2), it must be $gk - (1-\alpha)k^{\alpha} \leq 0$. This condition, however, contradicts to $k_{nb} > k^*_{nb}$, which is derived from $r^*_{nb} = r_{nb}$ and $\alpha^* < \alpha$. Therefore, it must be r_{nb} is strictly greater than the interest rate in a closed economy.

2. The bubble can be toxic: ex-ante welfare for both North and South in bubbly steady state can be worse than that in the steady state with no bubble.

3.3.1 Equilibrium with a Bubble

Without loss of generality, we assume $\alpha^* < \alpha$ so that given the amount of capital, saving or investment yields lower return in the South than in the North. We restrict our attention to an equilibrium with bubble in the North but not in the South. We will show that a bubble can be sustained in the steady state if and only if the condition of global dynamic inefficiency, (3.4), holds.

Portfolio problem of Southern banks

Under the assumption that there is no bubble in the South, southern banks have only two investment options: investing in domestic capital and lending to Northern banks. Let $\tilde{r}_t^D$ (and $\tilde{r}_t^{b*}$) be the state-contingent interest rate on deposits in Northern banks (Southern banks). Because of the international bankruptcy assumption, the interest rate is paid only if borrower banks do not default, thus

$$\tilde{r}_{t+1}^D = \begin{cases} r_{t+1}^D & \text{if Northern banks do not default} \\ 0 & \text{o.t.w.} \end{cases}$$

Southern banks solve an expected profit maximization problem:

$$\max_{K_{t+1}^* \geq 0, D_t^* \geq 0} E_t \Pi_{t+1}^*(\tilde{r}_{t+1}^D) \tag{3.7}$$

where

$$\Pi_{t+1}^*(\tilde{r}_{t+1}^D) \equiv \max\{\tilde{r}_{t+1}^D D_t^* + r_{t+1}^* K_{t+1}^* - r_{t+1}^{b*}(W_t^* + D_t),\ 0\},$$

and the banks' liability $W_t^* + D_t$ is equal to the asset $D_t^* + K_{t+1}^*$. Because the saving technology in the South yields lower return than that in the North, $\alpha^* < \alpha$, the saving flows from the South to the North. Thus, without loss of generality, we normalize the amount of Northern banks' deposits in Southern banks is zero: $D_t = 0$. We guess and verify that Southern banks default when Northern banks default. Under this guess, the first-order condition of the problem (3.7) yields

$$\begin{aligned} r_{t+1}^* &= \alpha \left(k_{t+1}^*\right)^{\alpha^*-1} = r_{t+1}^{b*} = r_{t+1}^D, &(3.8)\\ W_t^* &= D_t^* + K_{t+1}^* \text{ or } (1-\alpha^*)\left(k_t^*\right)^{\alpha^*} = D_t^* + g k_{t+1}^*, &(3.9)\end{aligned}$$

The firm's optimality conditions, (2.1) and (2.2), were applied to (3.7) and (3.9) to substitute out w_t^* and r_t^*.

Portfolio problem of Northern banks

Northern banks optimizes a portfolio consisting of holding of the Northern bubbly asset and Northern capital:

$$\max_{K_{t+1}\geq 0, D_t\geq 0, a_t\geq 0} E_t \max\{\tilde{P}_{t+1}a_t + r_{t+1}K_{t+1} - r_{t+1}^D (W_t + D_t^*),\ 0\}. \tag{3.10}$$

where the banks' liability $W_t + D_t^*$ is equal to the asset $P_t a_t + K_{t+1}$. Solving the problem (3.10) yields

$$r_{t+1} = \alpha k_{t+1}^{\alpha-1} = r_{t+1}^D = g\frac{p_{t+1}}{p_t}, \tag{3.11}$$

$$W_t + D_t^* = P_t a_t + K_{t+1} \quad \text{or} \quad (1-\alpha)\,k_t^\alpha + D_t^* = p_t + g k_{t+1}. \tag{3.12}$$

In deriving these conditions, the firm's optimality conditions, (2.1) and (2.2) were used to substitute w_t and r_{t+1}. Also, the market clearing condition for the bubble asset was imposed: $a_t = 1$.

When the bubble bursts, the Northern banks have $r_{t+1}K_{t+1}$ while they have obiligation to pay $r_{t+1}^D\left(P_t a_t + K_{t+1}\right) = r_{t+1}\left(P_t a_t + K_{t+1}\right) > r_{t+1}K_{t+1}$. Thus, the Northern banks go bankrupt when the bubble bursts.

3.3.2 Equilibrium and steady state

Combining equations (3.8), (3.9), (3.11), and (3.12), we obtain:

$$\alpha k_{t+1}^{\alpha-1} = \alpha\left(k_{t+1}^*\right)^{\alpha^*-1} = g\frac{p_{t+1}}{p_t}, \tag{3.13}$$

$$p_t + g k_{t+1} + g k_{t+1}^* = (1-\alpha)\,k_t^\alpha + (1-\alpha)\left(k_t^*\right)^{\alpha^*}. \tag{3.14}$$

The three conditions (equalities) govern the three variables, $\left\{k_{t+1}, k_{t+1}^*, p_t\right\}$, in an equilibrium with a bubble. Now we verify the guess made at the beginning of the analysis: Southern banks default if Northern banks default. When Northern banks default, Southern banks revenue is $r_{t+1}^* K_{t+1}^*$ while they promised to pay $r_{t+1}^{b*}\left(K_{t+1}^* + D_t^*\right)$. The revenue becomes less than the promised payment if $D_t^* > 0$ or $D_t^* = (1-\alpha^*)\,k_t^{*\alpha} - g k_{t+1}^* > 0$. We will verify the guess by showing that $D_t^* > 0$ in the steady state and focusing on dynamics near the steady state so that $D_t^* > 0$ holds.

In the steady state, condition (3.13) implies that $k = (\alpha/g)^{1/(1-\alpha)} \equiv k_{gold}$ and $k^* = (\alpha^*/g)^{1/(1-\alpha^*)} \equiv k_{gold}^*$. It is straightforward to see that $b^* = (1-2\alpha^*)(\alpha^*/g)^{\alpha^*/(1-\alpha^*)} > 0$

if and only if $\alpha^* < 1/2$. Combined the expression for k and k^* with condition (3.14) leads to the existence condition for bubble, summarized in the following Lemma.

Lemma 5. *Suppose that the North is dynamically efficient: $\alpha > 1/2$. Then, a bubbly steady state equilibrium exists, i.e., $p > 0$,* if and only if *the economy is globally dynamic inefficient, i.e., condition (3.4) holds.*

Proof. See preceding text. □

As we noted, the globally dynamic inefficiency condition (3.4) holds, and thus a bubble can emerge in the North even if the North is dynamically efficient, as long as the degree of dynamic inefficiency in the South is strong enough. Intuitively, in a closed Northern economy, because of dynamic efficiency, there is no excess savings to invest in a bubbly asset. But in a financially globalized world, some savings of the South flows to the North. Hence, with sufficient excess savings in the South, banks in the North have sufficient total deposit to invest in a bubble.

We shall calculate the welfare in the steady state of an equilibrium with a bubble for the North and the South. In the steady state, when the bubble sustains, the Northern households receive and invest $w = (1-\alpha)\,k_{gold}^{\alpha}$, earn return g per unit of investment, and consume gw in the next period. The same logic applies exactly to the South households. Thus, the detrended consumption in the next period when the bubble sustains is given by c_{gold} and c_{gold}^* for the North and the South respectively. When the bubble bursts, the Northern banks go bankrupt and the Northern households collect their claims first. From the Northern banks' balance sheet, (3.12), the banks have gk_{gold} and the Northern households attempt to collect $(1-\alpha)\,k_{gold}^{\alpha}$. Because $\alpha > 1/2$, the banks' remaining asset, gk_{gold}, exceed the household' claims, $(1-\alpha)\,k_{gold}^{\alpha}$, so that the households collect their claims perfectly. The Southern households, as the second claimers, receive the residual, $gk_{gold}-(1-\alpha)\,k_{gold}^{\alpha}$, from the Northern banks in addition to the return gk_{gold}^* from the Southern banks. To summarize, the welfare or the expected consumption for the North and the South is given respectively by:

$$c_b^e = (1-\lambda)\,c_{gold} + \lambda c_{gold} = c_{gold}, \tag{3.15}$$

$$c_b^{*e} = (1-\lambda)\,c_{gold}^* + \lambda\left[gk_{gold} - (1-\alpha)\,k_{gold}^{\alpha} + gk_{gold}^*\right]. \tag{3.16}$$

3.3.3 Toxic bubble

Now we compare the welfare between the bubble steady state and the no-bubble steady state. For the North, (3.5) and (3.15) imply that the bubble steady state is always worse than the no-bubble steady state, independent of the probability of bubble burst. [Add intuition]

For the South, the inequalities in (3.6) imply that the bubble steady state may become worse than the no-bubble steady state depending if the probability of bubble burst is higher than a certain threshold. A necessary and sufficient condition for the existence of such a threshold is that the consumption in the no-bubble steady state is strictly greater than the consumption in case of bubble burst in the bubble steady state: $c_{nb} > gk_{gold} - (1-\alpha) k_{gold}^{\alpha} + gk_{gold}^{*}$. Subtracting the consumption in case of bubble burst from the lower bound of c_{nb} in (3.6), we obtain:

$$\begin{aligned}
& \alpha^* \left(\frac{1-\alpha^*}{g}\right)^{\frac{\alpha^*}{1-\alpha^*}} - \left[gk_{gold} - (1-\alpha) k_{gold}^{\alpha} + gk_{gold}^{*}\right] \\
= & \left[\left(\frac{1-\alpha^*}{\alpha^*}\right)^{\frac{\alpha^*}{1-\alpha^*}} - 1\right] \alpha^* \left(\frac{\alpha^*}{g}\right)^{\frac{\alpha^*}{1-\alpha^*}} - (2\alpha - 1) \left(\frac{\alpha}{g}\right)^{\frac{\alpha}{1-\alpha}} \\
> & \left[\left(\frac{1-\alpha^*}{\alpha^*}\right)^{\frac{\alpha^*}{1-\alpha^*}} - 1\right] \alpha^* \left(\frac{\alpha^*}{g}\right)^{\frac{\alpha^*}{1-\alpha^*}} - (1 - 2\alpha^*) \left(\frac{\alpha^*}{g}\right)^{\frac{\alpha^*}{1-\alpha^*}} \\
= & \left\{\left(\frac{1-\alpha^*}{\alpha^*}\right)^{\frac{\alpha^*}{1-\alpha^*}} - \frac{1-\alpha^*}{\alpha^*}\right\} \alpha^* \left(\frac{\alpha^*}{g}\right)^{\frac{\alpha^*}{1-\alpha^*}} > 0,
\end{aligned}$$

where the condition of global dynamic inefficiency, (3.4), was used in the first inequality. The final inequality is sufficient for $c_{nb} > gk_{gold} - (1-\alpha) k_{gold}^{\alpha} + gk_{gold}^{*}$, and thus sufficient for the existence for a threshold above which the bubble becomes toxic for the South. We summarize the results in the following proposition.

Proposition 4. *Suppose that $\alpha > 1/2$ and the condition of global dynamic inefficiency, (3.4), holds. Then,*

- *The welfare of the North is always worth off in the bubble steady state than in the no-bubble steady state.*
- *Also, there exists a threshold such that if the probability of the bubble burst is greater than the threshold, then the South households become worse off in the bubbly steady state than in the no-bubble one.*

Proof. See preceding text. □

4 Conclusion

This paper attempts to make two contributions. First, to the bubble literature, the paper overturns the prediction in standard models that pricking a bubble cannot be a Pareto optimal policy. To this end, we build a tractable general equilibrium model of toxic bubbles – bubbles that reduce expected social welfare, by introducing financial friction into an otherwise standard bubble framework. Financial intermediation generates excessive investment in the risky bubbly asset, since with limited liability, banks do not fully internalize the consequence of the bubble's collapse. Excessive investment in the risky bubbly asset reduces the expected social welfare for two reasons: first, it crowds out capital investment and hence consumption when the bubble bursts, and second, it generates excessive volatility. Then we show that if the bubble is sufficiently risky, unanticipatedly pricking an existing bubble, combined with a one-time transfer from the living young to the living old, is a Pareto optimal policy.

Second, to the global imbalances literature, the paper overturns the prediction in many models that global imbalances are benign. We build a tractable, general equilibrium model of two large open economies with financial friction. Capital flows from the dynamically inefficient South to the dynamically efficient North can generate a toxic bubble in the North, despite the fact that Northern households do not have any need for a bubble (since their domestic economy is dynamically efficient). Our paper thus provides a formal framework to a notion held in policy circles that the "savings glut" from emerging markets helped create the bubble of subprime mortgage in the U.S.

We conduct preliminary analysis of prudential banking regulations and monetary policies. However, finding optimal policies to regulate risky asset bubbles still remains a promising area for future research. We also conduct a preliminary analysis of policies again bubbles in the presence of political friction. We show that any policy that commits to pricking a toxic bubble can be a Pareto dominated strategy, if there is any small but positive probability that the banking sector can politically prevent bubble pricking. Therefore, the authority should only prick toxic bubbles if they can *fully* commit to doing so in all states of nature, which is an unlikely assumption in reality. Optimal regulations of bubbles with political friction is an interesting avenue for future research. Another open question is policy coordination: how should central banks in emerging markets and in developed economies coordinate in regulating banks and bubbles? We believe our tractable framework of toxic bubble provides a natural framework for policy experiments and future research.

Appendix

A Proofs

Proof of Lemma 2. A bubbly steady state exists if and only if $p_{self} > 0$, or $p_{self} = (1 - \alpha)k_{self}^{\alpha} - g \cdot k_{self} > 0$. Equation (2.6) implies that $r_{self} = (1-\lambda)g$ so that it is straightforward to show that $k_{self} = [\alpha/((1-\lambda)g)]^{1/(1-\alpha)}$ from (2.2). Combining the expression for k_{self} and the condition of $p_{self} > 0$ yields: $p_{self} > 0$ if and only if $\lambda < (1-2\alpha)/(1-\alpha)$.

The consumption in the next period is $p_{self} + r_{self}k_{self}$ if the bubble sustains, while it is $r_{self}k_{self}$ if the bubble bursts. Thus, the expected consumption is given by $c^e_{self} = (1-\lambda)p_{self} + r_{self}k_{self}$. The exact expression for c^e_{self} is obtained after substituting out p_{self} and k_{self}:

$$c^e_{self} = (1-\lambda)(1-\alpha)\left(\frac{\alpha}{(1-\lambda)g}\right)^{\frac{\alpha}{1-\alpha}} = (1-\lambda)^{\frac{1-2\alpha}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}g^{-\frac{\alpha}{1-\alpha}}.$$

Using the existence condition, $\lambda < (1-2\alpha)/(1-\alpha)$, the following inequality is obtained:

$$c^e_{self} > \left(1-\frac{1-2\alpha}{1-\alpha}\right)^{\frac{1-2\alpha}{1-\alpha}}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}g^{-\frac{\alpha}{1-\alpha}} = \alpha\left(\frac{1-\alpha}{g}\right)^{\frac{\alpha}{1-\alpha}} = c_{nb}.$$

□

Proof of Lemma 3. The bubbly steady state exists if and only if $p^* = (1-\alpha)k^{*\alpha} - g\cdot k^* > 0$. The arbitrage condition (2.9) implies $r^* = g$. From (2.2), the capital stock is given by $k^* = (\alpha/g)^{1/(1-\alpha)} = k_{gold}$. Substituting the expression for k^* into the condition of $p^* > 0$ yields the result: $p^* > 0$ if and only if $\alpha < 1/2$.

The consumption in the next period is given by $(1-\lambda)p^* + r^*k^*$ as in Lemma 2. Substituting out p^*, k^* and r^*, we obtain

$$\begin{aligned} c^{e*} &= (1-\lambda)[(1-\alpha)k^{*\alpha} - g\cdot k^*] + r^*k^*, \\ &= [(1-\lambda)(1-\alpha) + \lambda\alpha]\left(\frac{\alpha}{g}\right)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

□

Proof of Proposition 1. First, we shall show $c^{e*} < c_{nb}$ if and only if $\lambda > \bar{\lambda}$. In the main text, we showed that $c^{e*} = (1-\lambda)c_{gold} + \lambda\alpha/(1-\alpha)\cdot c_{nb}$ and $c_{nb} = [\alpha/(1-\alpha)]^{(1-2\alpha)/(1-\alpha)}\cdot c_{gold}$. Therefore, calculating $c^{e*} > c_{nb}$ leads to the results, where $\bar{\lambda}$ is given by (2.10). The threshold $\bar{\lambda}$ is actually a probability satisfying $\bar{\lambda} \in (0,1)$ as follows:

$$\begin{aligned} \bar{\lambda} &= \frac{1-\alpha-\alpha^{\frac{1-2\alpha}{1-\alpha}}(1-\alpha)^{\frac{\alpha}{1-\alpha}}}{1-2\alpha} > \frac{1-\alpha-(1-\alpha)^{\frac{1-2\alpha}{1-\alpha}}(1-\alpha)^{\frac{\alpha}{1-\alpha}}}{1-2\alpha} = 0, \\ \bar{\lambda} &= \frac{1-\alpha-\alpha^{\frac{1-2\alpha}{1-\alpha}}(1-\alpha)^{\frac{\alpha}{1-\alpha}}}{1-2\alpha} < \frac{1-\alpha-\alpha^{\frac{1-2\alpha}{1-\alpha}}(\alpha)^{\frac{\alpha}{1-\alpha}}}{1-2\alpha} = 1. \end{aligned}$$

In the inequalities, the condition of dynamic inefficiency, $\alpha < 1-\alpha$, has been used.

Next, we shall show that $\bar{\lambda}$ is greater than $\underline{\lambda}$. The difference between $\bar{\lambda}$ and $\underline{\lambda}$ is calculated as

$$\begin{aligned}\bar{\lambda}-\underline{\lambda} &= \frac{1-\alpha-\alpha^{\frac{1-2\alpha}{1-\alpha}}(1-\alpha)^{\frac{\alpha}{1-\alpha}}}{1-2\alpha}-\frac{1-2\alpha}{1-\alpha} \\ &= \frac{(1-\alpha)^2-\alpha^{\frac{1-2\alpha}{1-\alpha}}(1-\alpha)^{\frac{1}{1-\alpha}}-(1-2\alpha)^2}{(1-2\alpha)(1-\alpha)}.\end{aligned}$$

The denominator is positive because $\alpha < 1/2$. The numerator is further calculated as

$$\begin{aligned}(1-\alpha)^2-\alpha^{\frac{1-2\alpha}{1-\alpha}}(1-\alpha)^{\frac{1}{1-\alpha}}-(1-2\alpha)^2 &= 2\alpha-3\alpha^2-\alpha^{\frac{1-2\alpha}{1-\alpha}}(1-\alpha)^{\frac{1}{1-\alpha}} \\ &= \alpha\left[2-3\alpha-\alpha^{-\frac{\alpha}{1-\alpha}}(1-\alpha)^{\frac{1}{1-\alpha}}\right] \\ &= \frac{\alpha}{1-\alpha}\left[2-\left(\frac{1-\alpha}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}-\frac{\alpha}{1-\alpha}\right]\end{aligned}$$

Let us denote $\delta \equiv \alpha/(1-\alpha)$. Because $0 < \alpha < 1/2$, we have $0 < \delta < 1$. The above expression now can be written as

$$\begin{aligned}\frac{\alpha}{1-\alpha}\left[2-\left(\frac{1-\alpha}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}-\frac{\alpha}{1-\alpha}\right] &= \delta\left(2-\delta^{-\delta}-\delta\right) \\ &= \delta\left[2-\delta\left(1+\delta^{1-\delta}\right)\right] > 0.\end{aligned}$$

The strict inequality follows from $\delta^{1-\delta} < 1$ because $0 < \delta < 1$. Therefore, $\bar{\lambda}-\underline{\lambda} > 0$. □

Proof of Lemma 4. Without loss of generality we assume $\alpha^* < \alpha$. Equation (3.3), with time subscript removed, can be solved for k^* as

$$k^* = \left(\frac{\alpha^*}{\alpha}\right)^{\frac{1}{1-\alpha^*}} k^{\frac{1-\alpha}{1-\alpha^*}}.$$

Substituting this into equations (3.2), with time subscript removed, yields

$$gk + g\left(\frac{\alpha^*}{\alpha}\right)^{\frac{1}{1-\alpha^*}} k^{\frac{1-\alpha}{1-\alpha^*}} = (1-\alpha)\,k^{\alpha} + (1-\alpha^*)\left(\frac{\alpha^*}{\alpha}\right)^{\frac{\alpha^*}{1-\alpha^*}} k^{\frac{1-\alpha}{1-\alpha^*}\alpha^*}.$$

Rearranging this equation yields

$$\frac{g}{1-\alpha}k^{1-\alpha} = \frac{k^{\frac{\alpha-\alpha^*}{1-\alpha^*}}+\frac{1-\alpha^*}{1-\alpha}\left(\frac{\alpha^*}{\alpha}\right)^{\frac{1}{1-\alpha^*}}}{k^{\frac{\alpha-\alpha^*}{1-\alpha^*}}+\left(\frac{\alpha^*}{\alpha}\right)^{\frac{1}{1-\alpha^*}}} \overset{\text{def}}{=} g\,(k)\,.$$

Function $g\,(k)$ satisfied $g'\,(k) < 0$, $g\,(0) > 1$ and $\lim_{k\to\infty} g\,(k) = 1$. Note that $r < g$ if and only if $k > k_{gold} = (\alpha/g)^{\frac{1}{1-\alpha}}$. Thus, $r < g$ if and only if

$$\frac{g}{1-\alpha}k_{gold}^{1-\alpha} < g\,(k_{gold})\,.$$

Because the left-hand-side is increasing and the right-hand-side is decreasing, the steady state must satisfy $k > k_{gold}$, and thus $r < g$. Substituting k_{gold} into the condition and rearranging it, we obtain condition (3.4). □

B Microfoundation for standard debt contract

We provide the micro-foundations of a debt contract assumed in the main text. The micro-foundations are based on asymmetric information and costly state verification à la Townsend (1979). In this setting, the environment of banks remains the same. In a bubbly equilibrium the banks invest in both capital and a bubble asset. The banks return is high or low depending on the event of bubble burst. The environment of a households sector differs from that in the main text. In particular, households do not observe the banks ex-post return without a cost. The households can observe the return only when they conduct costly auditing. No stochastic auditing is allowed. In addition, the households cannot make a contract which specifies the portfolio of capital and a bubble asset. The assumption of asymmetric information implies that the households do not observe the event of bubble burst when they receive the return.

In the model, there are only two states: h and l, where h denotes a high return (when a bubble sustains) and l denotes a low return (when a bubble bursts). The households do not observe the state without conducting costly auditing, but they know the probability of low return, λ. Without loss of generality, we restrict our attention to a truth-telling contract in which the banks truthfully reveal the state $s \in \{h, l\}$. In this setting, the households decide three objects which depend on state s. First, they make a auditing decision, $\delta(s) \in \{0, 1\}$, where 0 indicates no auditing and 1 indicates auditing. Second, they choose the amount of repayment per unit of deposit when they audited the banks, $r_a(s)$. Third, they choose the amount of repayment per unit of deposit when they did not audit the banks, $r(s)$. The households' objective is to maximize the expected repayment per unit of loan:

$$(1-\lambda)\{\delta(h)[r_a(h)-\epsilon]+[1-\delta(h)]r(h)\}+\lambda\{\delta(l)[r_a(l)-\epsilon]+[1-\delta(l)]r(l)\}, \tag{B.1}$$

where $\epsilon > 0$ denotes the auditing cost per unit of loan. The banks are competitive and protected by a limited liability law. The resulting participation constraints of banks are: for $s \in \{h, l\}$

$$s - r(s) \geq 0, \qquad s - r_a(s) \geq 0, \tag{B.2}$$

where the LHS in each equation denotes the profit of banks per unit of deposit in case of no-monitoring and monitoring respectively. Two incentive constraints are required to make the banks to reveal a state truthfully. First, if the households do not audit the banks in the both states, the repayment has to be the same:

$$r\,(h) = r\,(l) \;\text{ if } \delta\,(h) = \delta\,(l) = 0. \tag{B.3}$$

Otherwise, the banks will always report a state with lower repayment. Second, if the households audit the banks in a low state but not in a high state, the repayment in a low state is equal or less than that in a high state:

$$r_a\,(l) \leq r\,(h) \;\text{ if } \delta\,(l) = 1 \text{ and } \delta\,(h) = 0. \tag{B.4}$$

Otherwise, the banks would report a high state and pay less when they are in a low state.

A contact that maximizes the return received by the households has two features. First, the participation constraints (B.2) are binding: $r(s) = s$ if $\delta(s) = 0$ and $r_a(s) = s$ if $\delta(s) = 1$. Otherwise, the households can increase the return by raising the repayment. Second, the households audit only when the banks report a low state: $\delta(h) = 0$ and $\delta(l) = 1$. This auditing is enough to prevent the banks to fake a state. If the banks in a high state faked to be in a low state, the household would audit the banks and confiscate all the banks assets. Thus, the banks have no incentive to fake when they are in a high state. If the households did not audit when the banks report a low state, the banks in a high state would fake to be in a low state and thus the households return would be lower. Auditing in a high state as well would not change the repayment, but the return would be low because of an additional auditing cost.

From (B.1) the households expected return under the contract is given by

$$(1-\lambda)h + \lambda(l-\epsilon). \tag{B.5}$$

So far, the returns, h and l, have been taken as given. In the model, h and l, are endogenously determined by the banks portfolio choice. In particular, in the model, h and l are corresponding to:

$$h = \frac{P_{t+1}a_t + r_{t+1}K_{t+1}}{D_t}, \quad l = \frac{r_{t+1}K_{t+1}}{D_t}.$$

By assumption, the household cannot write a contract which depends on the banks portfolio choice between capital and a bubble asset, $\{K_{t+1}, a_t\}$, though the households observe the portfolio choice. The households would arrange the contact so as to make the banks to choose the portfolio to maximize (B.5). Under the contract such that the households confiscate all the banks assets in case of auditing, however, the banks do not take into account the earning in a low state. Thus, the households arrange the contract to maximize the banks earning in a high state. Given that the banks are competitive, one way to maximize h is to offer a debt contract with an interest rate r_{t+1}^d and let the interest rate determined by the competitive banks. The resulting financial arrangement is exactly the same as in the main text except the presence of auditing costs ϵ. The model in the main text corresponds to a limiting case where $\epsilon \to 0$.

References

Abel, A. B., Mankiw, N. G., Summers, L. H., and Zeckhauser, R. J. (1989). Assessing dynamic efficiency: Theory and evidence. *The Review of Economic Studies*, 56(1):1–19.

Abreu, D. and Brunnermeier, M. K. (2003). Bubbles and crashes. *Econometrica*, 71(1):173–204.

Allen, F. and Gale, D. (2000). Bubbles and crises. *The Economic Journal*, 110(460):236–255.

Allen, F. and Gorton, G. (1993). Churning bubbles. *The Review of Economic Studies*, 60(4):813–836.

Allen, F., Morris, S., and Postlewaite, A. (1993). Finite bubbles with short sale constraints and asymmetric information. *Journal of Economic Theory*, 61(2):206–229.

Barlevy, G. (2012). Rethinking theoretical models of bubbles. *New Perspectives on Asset Price Bubbles*, page 41.

Barlevy, G. (2013). A leverage-based model of speculative bubbles.

Barlevy, G. and Fisher, J. (2011). Mortgage choices and housing speculation.

Bernanke, B. S. (2005). The global saving glut and the us current account deficit. *Board of Governors of the Federal Reserve System (US) Speech*, (Mar 10).

Bernanke, B. S. (2010). Monetary policy and the housing bubble. In *Speech at the Annual Meeting of the American Economic Association*.

Bernanke, B. S. and Gertler, M. (2001). Should central banks respond to movements in asset prices? *The American Economic Review*, 91(2):253–257.

Brunnermeier, M. K. (2003). *Asset pricing under asymmetric information: Bubbles, crashes, technical analysis, and herding*. Oxford University Press.

Caballero, R. J., Farhi, E., and Gourinchas, P.-O. (2008a). An equilibrium model of global imbalances and low interest rates. *American Economic Review*, 98(1):358–393.

Caballero, R. J., Farhi, E., and Gourinchas, P.-O. (2008b). Financial crash, commodity prices, and global imbalances. *Brookings Papers on Economic Activity*, pages 1–55.

Caballero, R. J. and Krishnamurthy, A. (2006). Bubbles and capital flow volatility: Causes and risk management. *Journal of Monetary Economics*, 53(1):35–53.

Caballero, R. J. and Krishnamurthy, A. (2009). Global imbalances and financial fragility. *The American Economic Review*, 99(2):584–588.

Diamond, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review*, 55(5):1126–1150.

Farhi, E. and Tirole, J. (2012). Bubbly liquidity. *The Review of Economic Studies*, 79(2):678–706.

Gourinchas, P.-O. and Jeanne, O. (2013). Capital flows to developing countries: The allocation puzzle. *The Review of Economic Studies*.

Gourinchas, P.-O. and Rey, H. (2013). External adjustment, global imbalances, valuation effects. *Handbook of International Economics*, 4.

Greenspan, A. (2009). The fed didn't cause the housing bubble. *Wall Street Journal*, 11:A15.

Grossman, G. M. and Yanagawa, N. (1993). Asset bubbles and endogenous growth. *Journal of Monetary Economics*, 31(1):3–19.

Ikeda, D. (2013). Monetary policy and inflation dynamics in asset price bubbles. *Bank of Japan Working Paper Series*.

Jensen, M. C. and Meckling, W. H. (1976). Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics*, 3(4):305–360.

Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2013). The effects of the saving and banking glut on the us economy.

Martin, A. and Ventura, J. (2012). Economic growth with bubbles. *American Economic Review*, 102(6):3033–3058.

Mishkin, F. S. (2008). How should we respond to asset price bubbles? *Banque de France, Financial Stability Review*, 12:65–74.

Obstfeld, M. and Rogoff, K. (2009). Global imbalances and the financial crisis: Products of common causes. *CEPR Discussion Papers*.

Rajan, R. G. (2011). *Fault Lines: How Hidden Fractures Still Threaten the World Economy (New in Paper)*. Princeton University Press.

Saint-Paul, G. (1992). Fiscal policy in an endogenous growth model. *The Quarterly Journal of Economics*, 107(4):1243–1259.

Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *The Journal of Political Economy*, 66(6):467–482.

Santos, M. S. and Woodford, M. (1997). Rational asset pricing bubbles. *Econometrica: Journal of the Econometric Society*, pages 19–57.

Scherbina, A. (2013). *Asset Price Bubbles: A Selective Survey.* International Monetary Fund.

Stiglitz, J. E. and Weiss, A. (1981). Credit rationing in markets with imperfect information. *The American Economic Review*, 71(3):393–410.

Tirole, J. (1985). Asset bubbles and overlapping generations. *Econometrica: Journal of the Econometric Society*, pages 1499–1528.

Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic theory*, 21(2):265–293.