Deadline and welfare effects of scheduling information releases

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Abstract

How should institutions convey relevant information to the public? Should they schedule their communications or release information as it becomes available? What are the welfare effects of an unanticipated information release? We model a decentralized asset market and show that a credible schedule delays trade towards the information release date and unanticipated information arrivals entail a loss of insurance opportunities. We apply these findings to the scheduling of monetary policy decisions following the Federal Open Market Committee meetings from 1995 till 2010 and study the effects on the dynamics of trade on the 30-day Federal Funds Futures market. We use the model to empirically identify periods of credible (prior to 2001) and non-credible scheduling (after 2001). Finally we measure the loss in risk-trading activity due to off schedule announcements.

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1. Introduction

The value of public information is among the fundamental questions in economics and finance. A strictly related issue is how public and private agencies should convey information

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to the public. Should these agencies schedule their communications or release the information as it becomes available? Once a schedule is in place, how important is the credibility of the procedure? Today, many public institutions release information according to a schedule of announcements: this is the case of central banks announcing target interest rates at the end of the respective monetary policy committee meetings and of governmental agencies, like the Bureau of Labor Statistics, responsible for the official release of economic statistics on unemployment and inflation.

Little consideration has been given to the effects of such scheduling on the trade dynamics and on welfare. The theoretical contribution of this paper is to show that observables like trade volume dynamics can identify traders' preferences as well as welfare effects of credible scheduling. In particular we will address the following questions: 1) Does scheduling communications change the dynamics of trade and if so how? 2) Can the reliability of the schedule be inferred from the dynamics of trade? 3) What are the welfare costs of "spontaneous" deviations from the schedule *i.e.*, off schedule information releases?

These issues are first analyzed in a theoretical model. We then apply the model to identify the effects of the Federal Open Market Committee (FOMC) scheduled and off schedule monetary policy announcements on the trading volume of Chicago Board of Trade (CBOT) 30-Day Federal Funds Futures.

The theoretical model studies a simple, two-period economy under uncertainty where participating agents differ in their present wealth and in the distribution of their future wealth. To transfer wealth across time, they can hold a bank account where they can either deposit or borrow. To hedge uncertainty, they can hold a risky asset. The asset is traded in a non-centralized market by agents facing idiosyncratic risk and hence with different hedging needs. The asset market is operating according to a simple matching mechanism that can be briefly described as follows: at each trading session, buyers are randomly matched pairwise to sellers, each declares the reservation price and if the seller's asking price is lower than the buyer's bid they split the gains from trade equally; successful traders leave the market and unsuccessful ones can continue their search for a counterparty till the end of the first period.

Uncertainty is resolved by a public information release revealing the realisation of each agent's second period wealth and the asset's return. Agents are symmetrically informed about both the realization and the timing of the release.

In the first instance, the model analyzes the dynamics of trade when the information provider credibly commits to a schedule of releases; subsequently the model studies the case when the date of the information release is stochastic or equivalently the information provider does not credibly commit to a schedule.

We can summarize the results of the theoretical model as follows: 1) if (and only if) agents are risk averse, scheduling the communications changes trade dynamics by delaying a large volume of transactions towards the information release date. This is the deadline effect of credible scheduling; 2) when risk averse agents exchange the asset in order to hedge uncertainty, spontaneous deviations from the schedule might be welfare impairing as once uncertainty is resolved risk sharing opportunities are lost. This is as in Hirshleifer (1971)[1]. 3) with stochastic information releases (it suffices a small but positive probability of an off schedule intervention), both deadline and welfare effects vanish.

The key insight is that the perfectly anticipated future arrival of payoff-relevant public information acts as a trading deadline for risk averse investors. When the schedule is reliable, traders act *as if* they had a limited time to exchange the asset as once uncertainty is resolved the asset ceases to provide a hedging role and becomes redundant. Risk aversion in each period plays a different role in the characterization of the trade dynamics: if the concavity of the second period utility makes the agents willing to hold the asset as a hedging instrument, the concavity of the first period utility determines the trading dynamics in the asset market. Despite the absence of discounting and the infinitely frequent trading opportunities, the economy is not frictionless nor it approaches the competitive outcome.

We apply these theoretical findings to the FOMC monetary policy scheduling from January 1995 to July 2010 and look at the impact on the dynamics of trade of the CBOT 30-Day

[1]Notice that our model, for tractability reasons, will focus on the Hirshleifer effect only and will not account for the other benefits of receiving the information earlier rather than later (the "Blackwell effect" after Blackwell (1951)). On the comparison of the two effects see Gottardi and Rahi (2011).

Federal Funds Futures market. This is a decentralized market for interest rate futures where agents trade in order to hedge against changes in the target rate. We first study the deadline effect by checking if the volume of transactions is higher the days before a scheduled meeting. Our empirical analysis shows a statistically significant deadline effect for meetings prior to September 2001. We identify this split in the data set by employing rolling windows of 400 trading days with 60 days of overlapping gap. The two periods have a different monetary policy scheduling credibility: high till 2001 and less so afterwards. Unfortunately our model does not shed light on the reasons causing this shift in credibility.

We then turn to the evidence on the welfare effect of unscheduled announcements. If monetary policy scheduling is credible, an unanticipated resolution of uncertainty, due to an off schedule announcement, could be welfare impairing in the sense pointed out by Hirshleifer (1971). Although a substantial theoretical literature has extended and qualified Hirshleifer's result and identified sufficiently general conditions for the argument to hold [2], the empirical evidence is at best sparse[3], probably due to the difficulty in identifying instances where risk averse traders are "taken by surprise" by the earlier resolution of uncertainty. It seems natural to ask whether the welfare effect pointed out by Hirshleifer is a purely theoretical conjecture with little empirical relevance or, to the contrary, there are important instances where we observe its occurrence. If so what is the magnitude of the loss, either in terms of welfare or of trading volumes? The off schedule FOMC meetings, all occurring in the period identified of credible scheduling, provide the opportunity for running a natural experiment on the effects of monetary policy surprises and for quantifying the implied "missed" expected volume of trade following these surprises.

1.1 Related Literature

The literature on decentralized exchange mechanisms has extensively studied the conditions under which the equilibrium of an economy with frictions approaches the competitive equi-

[2] For a discussion of the literature see Schlee (2001) and Gottardi and Rahi (2011).

[3] The only pieces of evidence we are aware of come from medical studies, in particular Lerman *et al.* (1996) and Quaid and Morris (1993).

librium as the frictions become small. Among the different models of decentralized exchange, Gale's (1987) is probably the one we are closer to. Beyond the many important similarities, *e.g.,* a finite measure of agents on each side of the market trading one, indivisible unit of the good (an asset in our case) and the random matching of buyers and sellers, there are important differences between the models. First of all, in our economy the bargaining process plays a less relevant role and it is not explicitly modelled as gains from trade are equally split as in the Nash bargaining solution. This is not to deny the importance of the actual bargaining mechanism nor without loss of generality but it substantially simplifies the analysis, at least in this first approach to the problem. Also, in our model all potential traders are active and there are no costs in accessing the market. Most importantly, in order to identify the role of the endogenous frictions we are interested in, we analyze the economy where discounting as well as other sources of exogenous friction, *e.g.,* holding costs, are absent. As in Gale's paper and the subsequent literature, we analyse agents' optimal strategies as the number of trading sessions tends to infinity. One would expect that the absence of exogenous trading frictions coupled with infinite trading opportunities would result in flat trade where participants fix their bid or ask prices and wait for a successful meeting. This is not the case here. We will show that the solution to the problem of risk averse agents under credible scheduling is non-stationary and qualitatively different from the stochastic case where it is stationary. The increasing dynamics of the trading volume comes exactly from the non-stationarity of agents' strategies.

Modeling decentralized financial markets is not new. Starting with Duffie, Gârleanu and Pedersen (2005), the recent and growing literature of over-the-counter markets has successfully shown how trading frictions, namely search and negotiation costs as well as market makers' accessibility and the distribution of bargaining power, determine the equilibrium price. This strand of literature has also accounted for the rôle of search frictions in determining assets' liquidity (see Lagos and Rocheteau (2007) and (2009)) as well as the impact of risk aversion on prices (Duffie, Gârleanu and Pedersen (2007)).

The focus of the present paper is distinct. Our primary interest is to study how expectations on public information arrivals affect trade dynamics and, conversely, if the reliability of the schedule and agents' risk attitudes can be inferred from the observation of the dynamics of trade. In our case the stylized bargaining process plays little role and the trade's dynamics is characterised by the dynamics of the bids and ask prices independently of the resulting price settling the exchange at equilibrium. The effects we identify are present *despite* the absence of the frictions that play a central role in the over-the-counter markets literature. Finally, the presence of a deadline as well as the non-stationary dynamics of the game are the obvious reasons we do not analyse steady states. A constant trading volume is however obtained even in the presence of credible scheduling when agents are risk neutral.

Section 2 presents the theoretical model. Section 3 and 4 characterize the dynamics of trade under credible and non-credible scheduling. Section 5 compares the results under the alternative credibility regimes. Section 6 presents the empirical model and the analysis. Section 7 concludes. With the exception of Theorem 1, all proofs can be found in the appendix.

2. Description of the economy

We consider a one-good economy populated by a continuum of agent (or traders) and an information provider. Agents, denoted by a are partitioned into buyers, $a = b \in [0,1]$, and sellers, $a = s \in [0,1]$. The economy extends over time $t \in [0,1]$ under uncertainty. At $t = 0$ a strictly positive, agent-specific endowment ω_0^a of the non-storable consumption good is randomly distributed to each agent a with full support $\mathbb{R}_+$. Each agent knows her own endowment and the endowments' distribution for the rest of the economy. At $t = 0$, each sellers is also endowed with one unit of an indivisible asset[4]. At $t = 1$ each agent receives a stochastic, agent specific endowment $\omega_1^a \in \mathbb{R}_+$[5] and the asset pays a random, positive return ρ. Both the distribution of endowments at $t = 1$ and the distribution of asset's returns are known to the traders at $t = 0$. Due to the heterogeneity of the endowments' distribution,

[4]The one-asset indivisibility assumption is for tractability but not unrealistic in the case of trade of contracts of large size as interest rates futures.

[5]ω_0^a denotes the *realization* of the first period endowment but ω_1^a is a *random variable*. This is a convenient abuse of notation.

agents face idiosyncratic risk that they might be willing to trade away by selling the asset, if $a = s$, or buying it, if $a = b$. Between $t = 0$ and $t = 1$, active buyers and sellers are randomly matched. The matching is anonymous, *i.e.,* although each agent knows the counterparty's type (a seller knows she has been matched with a buyer and viceversa[6]), she does not know the exact identity, *i.e.,* the counterparty's first period endowment and the distribution of future endowments. Once matched, the buyer declares the price he is willing to pay and, contemporarily, the seller the price she is willing to accept. If the bid is greater than the ask price, exchange takes place. At any time $t \in (0, 1)$ agents have also access to a bank account, where they can either deposit or borrow the consumption good at a fix the interest rate r. Buyers can pay for the asset by holding a negative deposit at the same time[7]. Short selling of the asset is not allowed. In order to clearly identify the asset as an insurance instrument, in this paper we will focus only on the interior solution to each agent's problem, and hence we will assume that agents are not borrowing constrained[8].

Each agent consumes the good at a time $t \in (0, 1)$ and at $t = 1$. Agents' preferences are represented by the same utility function $u_0(\cdot) + Eu_1(\cdot)$, where $u_0(\cdot)$ and $u_1(\cdot)$ are assumed to be strictly increasing in consumption. We will allow the utility function to be time dependent and weakly concave in order to characterize the rôle that risk aversion plays in each period. There are no transaction costs, no costs of holding the asset or delaying consumption during $t \in (0, 1)$. The structure of the economy is common knowledge.

The information provider's only rôle is to resolve uncertainty by announcing the endowments' realization and the value of the asset's return ρ. One possible interpretation is that the information provider observes a signal or chooses a variable that in turn affects the endowments' allocation and the asset's return. He also chooses how to release information, *i.e.,*

[6]This is different than part of the literature (*e.g.,* Duffie *et al.* (2005)) where the match might occur with agents of the same type and in that case they both need to wait for the meeting of a counterparty of different type.

[7]We allow the agent to commit to pay the asset with funds that he/she might need to withdraw from the bank account after the trade has taken place: this is for convenience but not without loss of generality.

[8]This restriction is not particularly relevant for most of the characterization of the dynamics but it is relevant in some cases as in the proof or Lemma 2 and in the characterization of the welfare analysis. For the application we have in mind we do not think it as particularly restrictive, though it is not without loss of generality.

by committing or not to a schedule and, if not committed, when to release the information. Although we do not model his preferences, we characterize the consequences of different decisions.

Notice that once the information is conveyed to the markets, the asset ceases to be an insurance instrument and its rate of return is pinned down by the bank's rate. In fact, once the information reaches the market, no seller is going to ask less than $\frac{\rho}{r}$. To do otherwise would be dominated by the borrowing an amount $\frac{\rho}{r}$ and returning the amount ρ in the second period. Hence the ask price is at least $\frac{\rho}{r}$ for all $s \in [0, 1]$. Similarly, once the information is released no buyer would bid more than $\frac{\rho}{r}$ as to do otherwise would be dominated by a deposit of $\frac{\rho}{r}$ in the first period delivering ρ in the second. Hence the bids are at most $\frac{\rho}{r}$ for all $b \in [0, 1]$. As exchange takes place only when the bid is strictly greater than the ask price, the asset market becomes inactive after the information release. The following lemma states more formally this simple argument. Notice that neither this result nor the following Lemma u1-linear-iff-v1=0 depend on the credibility of the schedule.

Lemma 1. *Let the information be released at any $\tilde{t} < 1$. Then the asset market is inactive for any following $t > \tilde{t}$.*

Let now B_1^b be the highest bid buyer $b \in [0, 1]$ is willing to offer for any $t \in (0, 1)$, *i.e.*, the bid that makes him indifferent between trading the asset at price B_1^b or not buying it at all. Similarly, let S_1^s the lowest ask price seller $s \in [0, 1]$ is willing to accept for any $t \in (0, 1)$, *i.e.*, the price that makes her indifferent between selling at S_1^s or not selling at all[9]. The following lemma characterizes the probability of trade in the asset market if agents are risk neutral in the second period. This follows from an argument similar to the one used to prove Lemma 1 and in this case derived from the redundancy of the asset once risk aversion in the second period is removed.

Lemma 2. *If the second period utility u_1 is linear, then* $\Pr\{(b, s) : B_1^b > S_1^s\} = 0$.

Proof of Lemma 2: See Appendix.

[9]Section 3.1 will show that B_1^b and S_1^s are equal to the last trading session's prices for the case of credible scheduling with trading sessions frequent enough, hence the choice of notation here.

In order to proceed we will need to assume that the distribution on endowments and traders' preferences are such that for a positive measure of traders there is room for trade at some stage of the game, *i.e.,* endowments are not distributed in such a way that there is no positive mass of buyers willing to offer the ask price satisfying the demand of any positive mass the sellers.

Assumption 1. *There is a set of traders such that* $\Pr\{(b,s) : B_1^b > S_1^s\} > 0$.

Notice that an economy populated by second period risk neutral agents will never satisfy Assumption 1 irrespective of the distribution of endowments.

2.1 The trading process

We discretize the time interval [0,1] by partitioning it into $L+1$ subperiods each of equal length $\epsilon = (L+1)^{-1}$. Each $t = l\epsilon,\ l = 1, ..., L$, denotes a trading session in the asset market and $\mathcal{T}_\epsilon = \{\epsilon, .., l\epsilon, \ldots, L\epsilon\}$ the set of trading sessions. Trade in the asset does not take place at $t = 0$ and $t = 1$ and between trading sessions.

At any session $t \in \mathcal{T}_\epsilon$ each agent is characterized by the type $a \in \{b, s\}$, the agent-specific endowment ω_0^a, the history $h_t \in H_t = \{in, out\}$ and the future endowment's distribution. Since there are no costs for accessing the asset market, at the first trading session $t = \epsilon$, all histories h_ϵ take value *in*. Once the agent has successfully traded, she leaves the market, her history takes value *out* and she proceeds to choose the first period consumption and the value of the bank account[10]. The latter could be negative either in order to transfer consumption from the second period or for financing the purchase of the asset. Buyers and sellers that fail to exchange proceed to the next trading session where almost surely will meet a different counterparty. The agents not able to trade by the last trading session at time $t = L\epsilon$ choose the value of the bank account and first period consumption. For a given $\epsilon > 0$, the timeline can be described as follows:

[10]This sequence of the choices is optimal for the agent as there are no costs in delaying consumption nor losses of interest in deferring the deposit (or costs for an early withdrawal).

$t=0$	$t=\epsilon,\dots,l\epsilon,\dots L\epsilon$	$L\epsilon<t<1$	$t=1$
$\bullet$	$\bullet$	$\bullet$	$\bullet$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
first period endowments and asset distributed	trade, deposit, consumption	last deposit, last consumption, no trade	asset and deposit pay-off endowments distributed consumption

We now proceed by analyzing the game when the exact date for the information release is fixed, common knowledge and fully credible. In section 4 we will analyze the case where the information release date is stochastic, *i.e.,* where the provider can release the information at any $t \in (0,1)$ with positive probability.

3. Credible scheduling

Let us now assume that the information provider publicly commits to release the information at $t=1$. This choice of date is for convenience only and without loss of generality. As for Lemma 1, were the commitment taken for any other $t<1$ the asset market would stay open after the information release but would be inactive.

Fix now a trading frequency $\epsilon>0$. At any trading session $t \in \mathcal{T}_\epsilon$ a buyer b active in the market with a bid b_t, meeting a seller with ask price $x \in [0,b_t)$ acquires the asset at a price $\frac{b_t+x}{2}$, he then chooses the value of the bank account, denoted by y_t and obtains utility $u_0(\omega_0^b - \frac{b_t+x}{2} - y_t) + Eu_1(\omega_1^b + \rho + ry_t)$. Similarly, a seller s with an ask price s_t, meeting a buyer with bid $x \in (s_t,\infty)$ and choosing a value for the deposit of y_t obtains utility $u_0(\omega_0^s + \frac{s_t+x}{2} - y_t) + Eu_1(\omega_1^s + ry_t)$. As we assumed preferences and endowments' distribution to be common knowledge, at any trading session the agents can compute the distribution of bid and ask prices in order to maximize their expected utility.

At $t \in \mathcal{T}_\epsilon$ the ϵ-step optimization problem for agent of type $a=b,s$ active in the market is given by:

$$V^b_\epsilon(t,in) = \max_{b_t} \int_0^{b_t\text{-}} [u_0(\omega_0^b - \frac{b_t + x}{2} - Y^b_{\epsilon,t}(x)) + Eu_1(\omega_1^b + \rho + rY^b_{\epsilon,t})] dG_{\epsilon,t}(x)$$
$$+ (1 - G_{\epsilon,t}(b_t))V^b_\epsilon(t+\epsilon,\, in), \tag{1}$$
$$\text{where: } Y^b_{\epsilon,t}(x) \in \arg\max_{y_t} u_0(\omega_0^b - \frac{b_t + x}{2} - y_t) + Eu_1(\omega_1^b + \rho + ry_t) \text{ for all } x < b_t;$$

$$V^s_\epsilon(t,in) = \max_{s_t} \int_{s_t+}^{\infty} [u_0(\omega_0^s + \frac{s_t + x}{2} - Y^s_{\epsilon,t}(x)) + Eu_1(\omega_1^s + rY^s_{\epsilon,t})] dF_{\epsilon,t}(x)$$
$$+ F_{\epsilon,t}(s_t)V^s_\epsilon(t+\epsilon,\, in), \tag{2}$$
$$\text{where: } Y^s_{\epsilon,t}(x) \in \arg\max_{y_t} u_0(\omega_0^s + \frac{s_t + x}{2} - y_t) + Eu_1(\omega_1^s + ry_t) \text{ for all } x > s_t,$$

where $V^a_\epsilon(t,in)$ is the value function at t for the agent $a = b, s$, $G_{\epsilon,t}(x)$ is the proportion of sellers at t willing to sell for a price less or equal to x, $F_{\epsilon,t}(x)$ is the proportion of buyers at t willing to buy for a price less or equal to x. The problem reflects the fact that it is always optimal for the agent to decide his bank's deposit (positive or negative) and consumption after observing the outcome of the trade in the asset market. Between any two consecutive trading sessions t and $t+\epsilon \in \mathcal{T}_\epsilon$, *i.e.*, the interval $(t, t+\epsilon)$, the value function of agent $a = b, s$ is fixed at $V^a_\epsilon(t+\epsilon, in)$. The same holds for histories, bid and ask prices and their distribution, all being held constant in the interval $(t, t+\epsilon)$ at their $t+\epsilon$ value. The optimal trading strategy is given by the vector $(A^a_{\epsilon,t}, Y^a_{\epsilon,t}) : H_t \rightarrow \Re_+ \times \Re \cup \emptyset,\ t \in \mathcal{T}_\epsilon$ where $A^a_{\epsilon,t} = B^b_{\epsilon,t}$ and $A^a_{\epsilon,t} = S^s_{\epsilon,t}$ denote the optimal bid and ask price for buyer b and seller s, respectively. History taking value *out* is mapped into the empty set.

Notice that our results on the dynamics of the volume of trade will follow from the characterization of individuals' trading strategies and without looking at the (subgame-perfect) equilibrium of the game upon which our analysis can shed little light.

Once the last trading opportunity has elapsed and before the endowments and returns are distributed the value function for an agent that could not find a match is given by:

$$V^b_\epsilon(1, in) = \max_{y_{in}} u_0(\omega^b_0 - y_{in}) + Eu_1(\omega^b_1 + ry_{in}), \tag{3}$$

$$V^s_\epsilon(1, in) = \max_{y_{in}} u_0(\omega^s_0 - y_{in}) + Eu_1(\omega^s_1 + \rho + ry_{in}), \tag{4}$$

where y_{in} denotes the bank account variable in the last period and Y^a_{in} will denote the solution to agent $a = b, s$'s problem (this is to distinguish them from y_1 and Y^a_1 used later). Equations (3) and (4) give the terminal values preventing the value function from been unbounded.

3.1 The volume of trade is non-decreasing

We start by showing that the dynamics of both bids and ask prices weakly monotonic. Let us first define for each agent $a = b, s$, $\lim_{\epsilon\to 0} A^a_{\epsilon,t} \equiv A^a_t$, $A = B, S$ and $\lim_{\epsilon\to 0} Y^a_{\epsilon,t} \equiv Y^a_t$. These limits exist and are unique since preferences are continuous and monotone. Similarly, let $\lim_{\epsilon\to 0} V^a_\epsilon(t) \equiv V^a(t)$ and define $G_t(\cdot)$ and $F_t(\cdot)$ the distributions of the limiting bid and ask prices, respectively. The following result establishes the first fact on the dynamics in the limit of bid and ask prices: for ϵ small enough, bids are non-decreasing and ask prices are non-increasing.

Theorem 1. *Suppose the trading sessions are frequent enough,* i.e., *ϵ sufficiently small. Then for all continuity points*[11] *$t > t'$, buyer b has an optimal bid such that $B^b_t \geq B^b_{t'}$, and seller s has an optimal ask price such that $S^s_t \leq S^s_{t'}$.*

Proof of Theorem 1: See Appendix.

We can now give a first characterization of the dynamics of the volume of trade. Let $v_{\epsilon,t}$ denote the expected volume of trade been defined as the proportion of exchanges taking place at a given session $t \in \mathcal{T}_\epsilon$, *i.e.*,

$$v_{\epsilon,t} = \int\int 1\{(b, s) : B^b_{\epsilon,t} > S^s_{\epsilon,t}\} dF_{\epsilon,t}(S^s_{\epsilon,t}) dG_{\epsilon,t}(B^b_{\epsilon,t}),$$

[11] As we show in the appendix, the set of problems for which the continuation values are discontinuous is negligeable.

where $1\{\cdot\}$ is an indicator event.

Therefore the expected volume of trade is simply the probability of trading:

$$v_{\epsilon,t} = \Pr\{(b,s) : B^b_{\epsilon,t} > S^s_{\epsilon,t}\}.$$

Let now $v_t \equiv \lim_{\epsilon\to 0} v_{\epsilon,t}$ and $v_1 \equiv \lim_{\epsilon\to 0} v_{\epsilon,1-\epsilon}$. The following theorem shows that the volume of trade is non-decreasing in t.

Theorem 2. *For any two continuity points $t > t' \in (0,1)$, the volume of trade is non-decreasing in time, i.e., $v_1 \geq v_t \geq v_{t'}$.*

Proof of Theorem 2: See Appendix.

As an immediate and useful observation following Theorem 2 we have:

Corollary 1. *If $v_1 = 0$ then $v_t = 0$ for all $t \in (0,1)$.*

Proof of Corollary 1: See Appendix.

Corollary 1 has the convenient implication that in order to study the no-trade case it suffices to analyze the distribution of bid and ask prices at the last trading session. Notice that by the weak monotonicity in Theorem 1, B^b_1 and S^s_1 in Lemma 2 are equal to the last session's bid and ask prices for agent b and s, respectively. Moreover, given the definition of volume, $v_1 = \Pr\{(b,s) : B^b_1 > S^s_1\}$. Corollary 1 with Lemma 2 have the immediate implication that when almost all agents are risk neutral in the second period then the volume of trade must be nil at any time t irrespective of the trading frequency ϵ.

3.2 The deadline effect

Theorem 2 does not rule out a constant dynamics of trade. In this section we show that the volume of trade is indeed increasing if and only if (almost all) agents are risk averse in both periods. In Lemma 2 we already characterized the dynamics of trade when the agents are risk neutral in the second period. We will now characterize the trade dynamics when the first period utility is linear and finally complete the analysis with the case of risk averse agents in both periods.

Let us now assume that the second period utility function is concave and agents' first period preferences can be represented by a linear function. Our argument will proceed as follows: we first show that a positive, constant volume of trade is possible if and only if the bid and ask prices are stationary. We then show that the latter occurs if and only if the first period utility function is linear. As a corollary we obtain that an increasing trade must come from risk averse agents.

Lemma 3. *The bid and ask prices are stationary for almost all sellers and buyers if and only if the volume is constant and strictly positive,* i.e., $v_t = v_1 > 0$ *if and only if for almost all* b *and* $s \in [0, 1]$:

$$\Pr\{a : A_t^a = A_1^a,\ a \in [0,1]\} = 1,\ a = b, s \text{ and } A = B, S. \tag{5}$$

Proof of Lemma 3: See Appendix.

Notice that equation (5) implies that $G_t(\cdot)$ and $F_t(\cdot)$ are stationary distributions. We are now in a position to prove the following result:

Lemma 4. *Let* u_1 *concave. Then the volume of trade is positive and constant if and only if the first period utility function is linear for almost all buyers and sellers.*

Proof of Lemma 4: See Appendix.

We can now complete the characterization of the dynamics of the volume of trade when the schedule is fully credible. We are left to consider the case where (almost) all agents are risk averse.

Theorem 3. *There exists a* t' *such that for* $t > t'$, $v_t > v_{t'}$ *if and only if for almost all buyers and sellers both first and second period utility functions are concave.*

Proof of Theorem 3: See Appendix.

Let us summarize the results up to here. If agents are risk averse in both periods, Theorem 1 and 3 tell us that buyers start from a low bid and increase their offer as the deadline approaches. Sellers behave symmetrically. These results counter the argument that patient traders able to exchange infinitely often do set their bid or ask prices and wait for a successful meeting. The concavity of the utility function plays a different rôle in each period. The second period agents' risk aversion makes them trading the asset for hedging purposes; the concavity of the utility function in the trading period determines the dynamics of trade. In absence of first period concavity this reverts to the stationary case and is consistent with the benchmark models of the search and bargaining literature (*e.g.,* Gale (1986)). The result on the dynamics of trade when agents are risk neutral in the first period implies also that the presence of a deadline, though fully credible, is not sufficient for a deadline effect.

3.4 The welfare effect of unanticipated information releases

Suppose now that the information provider, although previously committed to release the information at $t = 1$, decides to act at $t' < 1$ taking the markets "by surprise." This unanticipated early release of information is equivalent to assigning to each active agent $a = b, s$ a utility level of $V^a(1)$ and might entail a welfare loss equivalent to the difference of the utility level at $t = 1$ and at $t = t'$, *i.e.:* $V^a(1) - V^a(t')$. If the value functions are stationary, unanticipated releases of information are welfare neutral. As proved earlier, these are stationary whenever agents are risk neutral, either in the first or in the second period. Conversely, if the value functions are non-stationary, *i.e.,* when risk averse in both periods, then a welfare loss can occur. This establishes a necessary relationship between deadline and welfare effects. We can summarize the results in the following statement.

Theorem 4. *There exists at t' such that an unanticipated early release of information at any $t < t'$ entails a welfare loss if and only if both first and second period utility functions are concave.*

Theorem 4 does not claim that a welfare loss will occur following an unanticipated release of information at *any* t'. This is simply follows from the fact that Theorem 3 does not claim that the dynamics of trade is strictly increasing at any t.

4. Non-credible scheduling, *i.e.,* stochastic information releases

Let us now assume that the information provider does not commit to a date for the information release or, equivalently, that any commitment to a schedule of releases is not credible, *i.e.,* traders assign a positive probability to the event that the information provider might act at any $t < 1$. Then for every $t \in (0,1)$ define $\{\tau_t : t \in (0,1)\}$ as:

$$1 - \tau_t = \Pr(\text{information release at } t).$$

Assumption 2. *The probability of an off schedule information release is strictly positive for all t,* i.e.,

$$\sup_{t\in(0,1)} \tau_t < 1.$$

Notice that we do not require τ_t to be constant over time. In the case τ_t is actually constant, the problem is equivalent to the case with discounting.

In this section we shall show that under Assumption 1 and 2 the volume of trade is positive and constant. We consider each agent ϵ-step problem where the agents failing to successfully exchange at $t \in \mathcal{T}_\epsilon$ proceed to the next trading session with probability $\tau_{t+\epsilon} < 1$ and with complementary probability are left with utility equal to $V^a(1, in),\ a = b, s$. The optimization problem for any $\epsilon > 0$ can now be written as:

$$\begin{aligned}
\widehat{V}^b_\epsilon(t, in) &= \max_{b_t} \int_0^{b_t\text{-}} [u_0(\omega^b_0 - \frac{b_t + x}{2} - \widehat{Y}^b_t) + Eu_1(\omega^b_1 + \rho + r\widehat{Y}^b_t)] d\widehat{G}_{\epsilon,t}(x) \\
&\quad + (1 - \widehat{G}_{\epsilon,t}(b_t))[\tau_{t+\epsilon}\widehat{V}^b_\epsilon(t+\epsilon, in) + (1 - \tau_{t+\epsilon})V^b(1, in)] \qquad (6) \\
\widehat{Y}^b_t &\in \arg\max_{y_t} u(\omega^b_0 - \frac{b_t + x}{2} - y_t) + Eu(\omega^b_1 + \rho + ry_t) \text{ for all } x < b_t;
\end{aligned}$$

$$\begin{aligned}
\widehat{V}^s_\epsilon(t, in) &= \max_{s_t} \int_{s_t+}^{\infty} [u_0(\omega^s_0 + \frac{s_t + x}{2} - \widehat{Y}^s_t) + Eu_1(\omega^s_1 + r\widehat{Y}^s_t)] d\widehat{F}_{\epsilon,t}(x) \\
&\quad + \widehat{F}_{\epsilon,t}(s_t)[\tau_{t+\epsilon}\widehat{V}^s_\epsilon(t+\epsilon, in) + (1 - \tau_{t+\epsilon})V^s(1, in)], \qquad (7) \\
\widehat{Y}^s_t &\in \arg\max_{y_t} u_0(\omega^s_0 + \frac{s_t + x}{2} - y_t) + Eu_1(\omega^s_1 + ry_t) \text{ for all } x < b_t.
\end{aligned}$$

The buyers' and sellers' problem at $t = 1$ are the same as in equations (3) and (4).

Denote by $\widehat{B}^b_{\epsilon,t}$ and $\widehat{S}^s_{\epsilon,t}$ the solutions to problem (6) and (7), respectively. The next result shows that bid and ask prices as well as the value of the bank account are independent of the time and the frequency but not agent-independent. This follows from the stationarity of the problem under stochastic deadlines.

Lemma 5. *For a given agent $a = b$ and $s \in [0, 1]$ and for any trading frequency $\epsilon > 0$:*

1. the value function $\widehat{V}^a_\epsilon(t, in) = V^a(1, in)$;

2. the value of the bank account, the bid and ask prices are stationary, ϵ-independent and equal to the last session's account and prices under credible scheduling, i.e. $\widehat{Y}^a_{\epsilon,t} = Y^a_1$, $\widehat{A}^a_{\epsilon,t} = A^a_1$.

Proof of Lemma 5: See Appendix.

The following theorem shows that risk averse agents facing an uncertain deadline behave at each instant as they would behave at the last trading opportunity in a credible scheduling regime, *i.e.*, they use their last period's bids and ask prices. It follows that the trading volume remains constant. Moreover this is equal to the volume of trade in the last trading session under credible scheduling for $\epsilon \to 0$

Theorem 5. *Under stochastic information releases the volume of trade is ϵ-independent, constant and equal to v_1.*

Proof of Theorem 5: See Appendix.

Notice that the result is independent of the value of the probability τ_t. The theorem implies that, once the probability of an information release is positive, the level of reliability of the schedule does not affect the volume and the dynamics of trade. This is a consequence of the contraction mapping theorem that holds in this case but not in the case of credible scheduling analyzed before. Moreover, Theorem 5 implies that the dynamics of the volume of transactions when risk averse agents trade under non-credible scheduling is undistinguishable from the the dynamics of trade under credible scheduling when almost all traders are risk neutral in the first period.

Welfare effect: By definition there are no "surprises" when the scheduling is non-credible and hence, no welfare effects.

5. Credible vs. non-credible scheduling

The above results show that alternative ways of releasing information have different implications in terms of trading dynamics and welfare. A direct comparison on the total volume of trade and on welfare between credible and non-credible scheduling of information releases can now be derived as a simple corollary.

Corollary 2. *Let the first and second period utilities concave. Then:*

1. the total volume of trade is higher under stochastic information releases than under credible scheduling.

2. The expected utility of a trader at any $t \in (0, 1)$ is higher under credible scheduling than under stochastic information releases.

Proof of Corollary 2: See Appendix.

Part 1 of this last corollary is particularly relevant in the present work as the empirical section will provide a test of the model based of this claim.

6. The Empirical Model

Most independent central banks, including the U.S. Fed, the Bank of England and the ECB, deliver their monetary policy decisions to the public by announcing interest rate levels at scheduled and publicly available dates. Scheduling monetary policy, it is usually argued, increases "transparency, accountability and the dialogue with the public" (Bank of Canada, (2000)). Monetary policy authorities retain the ability to act off schedule, though this might undermine their policy's credibility. When the schedule is credible, off schedule announcements are often said to "surprise" the markets. Other monetary authorities, *e.g.,* the Reserve Bank of India, prefer to exercise discretion by informing the markets about rate changes whenever considered appropriate.

In this section we apply our theoretical findings to the FOMC monetary policy scheduling from 3rd January 1995 to 31th July 2010 and look at the impact on the dynamics of trade of the CBOT 30-Day Federal Funds Futures market. These are contracts of \$5mil size on the daily federal funds overnight rate reported by the New York Fed[12]. Since there are 10 meetings of the FOMC a year, each 30-day contract covers at most one scheduled announcement. Throughout the 15 years' period included in our data set the FOMC had 128 scheduled meetings and 17 off schedule meetings[13]. Only 4 of these latter led to a change of rate: the quarter point reduction on 18/19 of October 1998 and the half point cuts on 3rd January 2001, 18th April and 17th September 2001.

By Theorem 3, under agents' risk aversion the reliability of the schedule of information releases is identifiable by the increasing trade dynamics. Conversely, a stationary dynamics is associated with unreliable scheduling or traders risk neutrality. We use this fact in order to identify the reliability of the schedule on interest rate decisions along with traders' attitudes towards risk in the period covered by our data set. We will show that the latter can be partitioned into two sub-periods of reliable and unreliable scheduling before and after October 2001, respectively. We then look at off schedule information releases followed by a change in interest rate. As they occurred before October 2001, all of them fall in the period of reliable scheduling and an absence of a deadline effect prior to those releases would indicate that they were unanticipated by the market. The implication is that off schedule announcements must have conveyed information to the active risk averse agents prior to risk sharing trading. We compute the average implicit lost trade due to an early information release.

6.1 Modeling the trading volume

We look at the trading volume of short term interest rate futures as a function of changes of the Federal Funds Target Rate (TR in the notation below) following the announcements

[12] See also http://www.cbot.com/cbot/pub/cont_detail/0,3206,1525+14446,00.html.

[13] When the off schedule meetings occur at non-trading days, we take the first trading day after the meeting as the effective announcement day.

of scheduled and off schedule meetings by analyzing the following model:

$$\begin{aligned} v_t = \; & c + \gamma_1 \left|\Delta_e \pi_t\right| + \gamma_2 \left|\Delta_e g_t\right| \\ & + \textstyle\sum_{j=-J}^{J} (\alpha_j^0 SD_{t-j}^0 + \beta_j UD_{t-j}^+ + \alpha_j SD_{t-j}^+) + \epsilon_t, \end{aligned} \tag{8}$$

where v_t is the daily volume traded of CBOT® 30-Day Federal Funds Futures at time t in the Chicago Board of Trade. The volume v_t depends on the absolute magnitude of changes in rate expectations. We model the changes in rate expectations as a function of absolute magnitude changes of the difference between the actual and median forecast values[14] of output growth $\Delta_e g_t$ and inflation $\Delta_e \pi_t$ as in Taylor (1999).

In order to capture the excess volume of trade when the interest rate changes (up and above the expected changes modelled), we differentiate between the announcements that lead to a change the federal fund rate from the ones that do not by introducing the following dummy variables:

$$SD_t^0 = I\left[\left|\Delta TR_t\right| = 0 \text{ and there is a scheduled meeting at } t\right],$$

and

$$SD_t^+ = I\left[\left|\Delta TR_t\right| \neq 0 \text{ and there is a scheduled meeting at } t\right],$$

where I is an indicator function. We also introduce a separate but similar set of variables, UD_t^+, in order to capture the effect of an off schedule rate change:

$$UD_t^+ = I\left[\left|\Delta TR_t\right| \neq 0 \text{ and there is a off schedule meeting at } t\right]$$

where r_t is the Federal Funds rate. The lag dummies SD_{t-j}^0, SD_{t-j}^+ and UD_{t-j}^+ $j = 1, .., J$ capture the effect of possible excess trading the day before scheduled announcement whereas to capture the increase in trade the day after the announcement we include the lead dummies SD_{t+j}^0, SD_{t+j} and UD_{t+j}^+, $j = 1, .., J$.

[14] The data on expectations are obtained from Datastream.

6.2 Estimation and results

Identifying scheduling reliability: We first start by identifying periods of different scheduling reliability (if any) by using rolling windows of 400 days with a 60 days overlapping gap. Our test identifies two sub-periods in our data set: the first, till September 2001, where there is statistically significant excess trade (*i.e.,* a deadline effect) two and one day before an announcement of a rate change, and the second after then. For this reason we split the plots and the table, pre and post October 2001. Figure 1 plots the t-values of α_1 and α_2 for the significance of excess trade one and two days before a scheduled announcement for the period January 1995-September 2001 and October 2001-July 2010, respectively (the t-values for $\alpha_j,\ j > 2$, are insignificant). Each bar on the graphs represents a 400-day window. If in any of the rolling periods one of the three events SD^+, SD^0 or UD^+ does not occur, we drop the respective dummy variable for that period and leave blank the corresponding window. The three horizontal lines indicate the significance at 1%, 5% and 10% levels.

[INSERT FIGURE 1]

The volume of trade under alternative scheduling reliability: We then test the model's prediction stating that the total volume of trade is higher during the periods of unreliable scheduling. Figure 2 represents the plots for the average volume of trade relative to the periods before and after October 2001. It reports the average volume of trade on the day of a scheduled meeting that is followed by an interest rate change, the volume of trade one and two days prior to that meeting and the average of all the other days. The figure clearly shows that there is a deadline effect in the first period but that the total volume is significantly higher in the second period.

[INSERT FIGURE 2]

Recall that the theoretical model does not allow us to distinguish the dynamics of a non credible schedule from the case of traders' linear first period utility function. Assuming that the change in the pattern of trade around October 2001 were not due to a shock in the traders' preferences but to a shift in agents' beliefs on the reliability of the monetary authority scheduling, an obvious question is to ask is why the credibility of monetary policy has been affected after October 2001. Several reasons can be attributed to shocks in traders' beliefs (including the events on September 11th) though our model is silent about the reasons affecting the credibility of the schedule.

Loss of trade after surprises: We can then turn to the evaluation of the loss of trading volume due to unanticipated information arrivals. Recall that the emphasis is on the *timing* of information release and not on its content. The relevant events in this case are the four off schedule announcements that led to a change in interest rate. Notice that all of them occurred before October 2001, the period with significant deadline effects of schedule announcements and hence of credible scheduling. We first show that these were indeed surprises to the market by verifying the absence of a deadline effect the days prior to these announcements. Table 1 summarizes the results of the tests[15].

[INSERT TABLE 1 AND TABLE 2 HERE]

We can now quantify the loss of trade due to unanticipated information releases. For this we look at the average excess trade one and two days before each off schedule announcement compared to the value of the intercept in the table, *i.e.*, $\left(\widehat{\alpha}_j - \widehat{\beta}_j\right)/\widehat{c}$, $j = -1, -2$. We find that an average excess trade of 49% and of 37% the day before and two days before the announcement, respectively.

7. Conclusion

In this paper we have argued that scheduling the communication of payoff relevant public information changes financial markets' behavior non-trivially by entailing a deadline effect. The theoretical contribution has shown that observables like trading volume dynamics can identify the reliability of the scheduling of information releases along with welfare effects

[15] Since in the second period there are no off schedule announcements followed by changes in interest rates we drop the variables UD^{+}_{t-j} during that period.

of credible scheduling. We applied the theoretical model to the FOMC monetary policy announcements and identified the periods of credible monetary policy. Finally we have shown that in those periods unscheduled announcements entail a loss of trade.

Some final observations are in order: 1) our welfare analysis focuses on the effect leading to a loss of insurance opportunities and abstracts from other potentially beneficial effects that might arise from an early release of information. As pointed out by Gottardi and Rahi (2011), if there is room for trade after the new information has reached the market, agents can achieve a larger set of state contingent payoffs by conditioning their portfolios on this information: if markets are sufficiently incomplete, the latter positive effect might overcome the welfare loss due to the Hirshleifer effect. This important point is beyond the aim of our analysis; 2) our paper is also silent about trade increases observed after FOMC announcements. We do not account for this effect though this clearly shows up in our data and it is well known that trade for many asset classes increases right after news are released; 3) the dynamics of trade in interest rates futures is substantially different than the one observed prior to scheduled corporate announcements where trade is depressed rather than increased. The financial economics literature has identified informational asymmetries as the main reason for the volume of trade to decrease as uninformed agents avoid the exchange with informed counterparties[16]. If trading volume before scheduled announcements is indeed correlated with the extent of information asymmetries then our empirical findings would imply that there are little informational asymmetries on monetary policy decisions.

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[16]For the theoretical literature see Admati and Pfeinderer (1988) and Forster and Viswanathan (1990). For the empirical studies see Chae (2005). For alternative explanations see George, Kaul, and Nimalendran (1994)).

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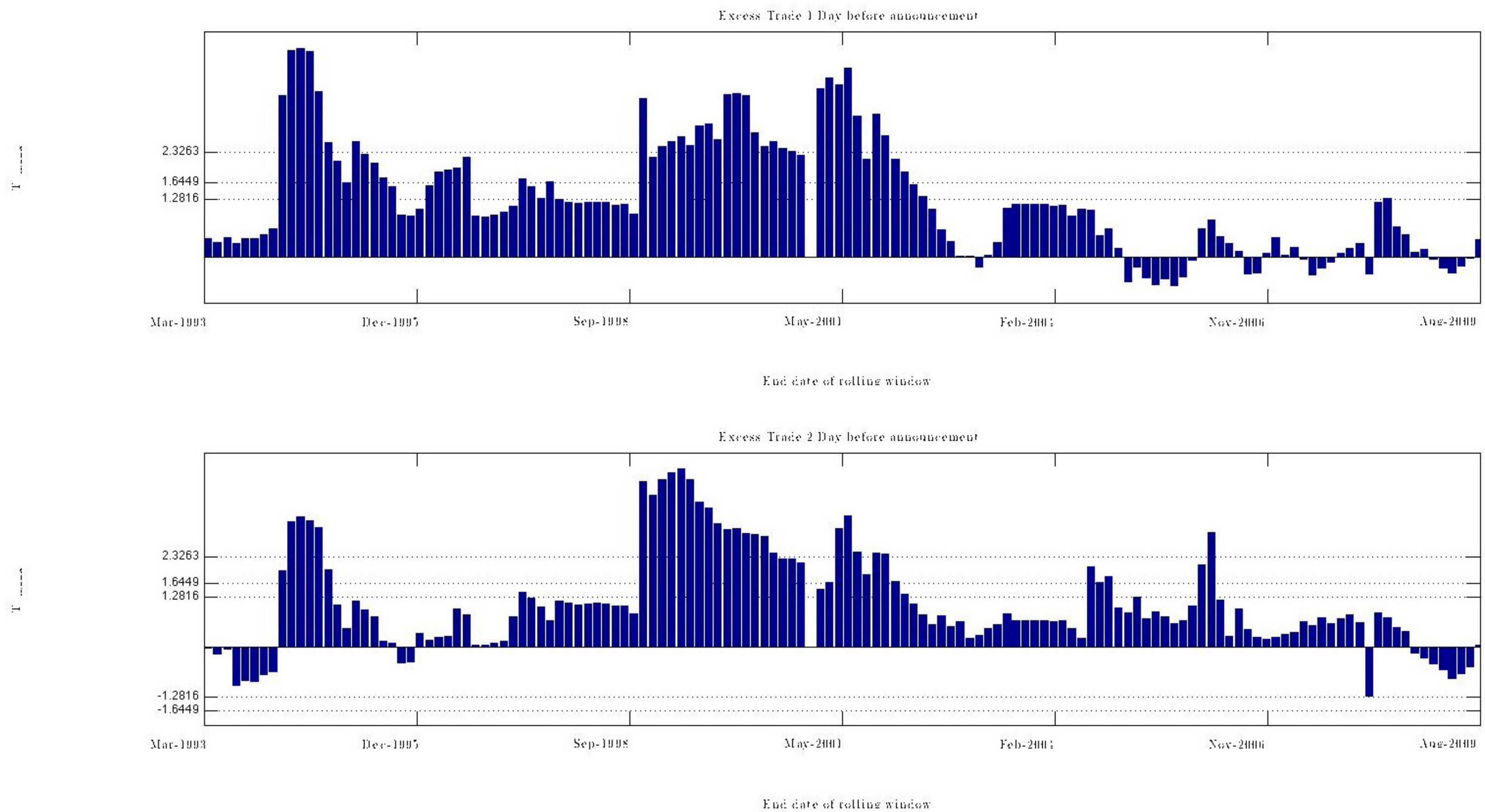

Figure 1: Rolling window estimation

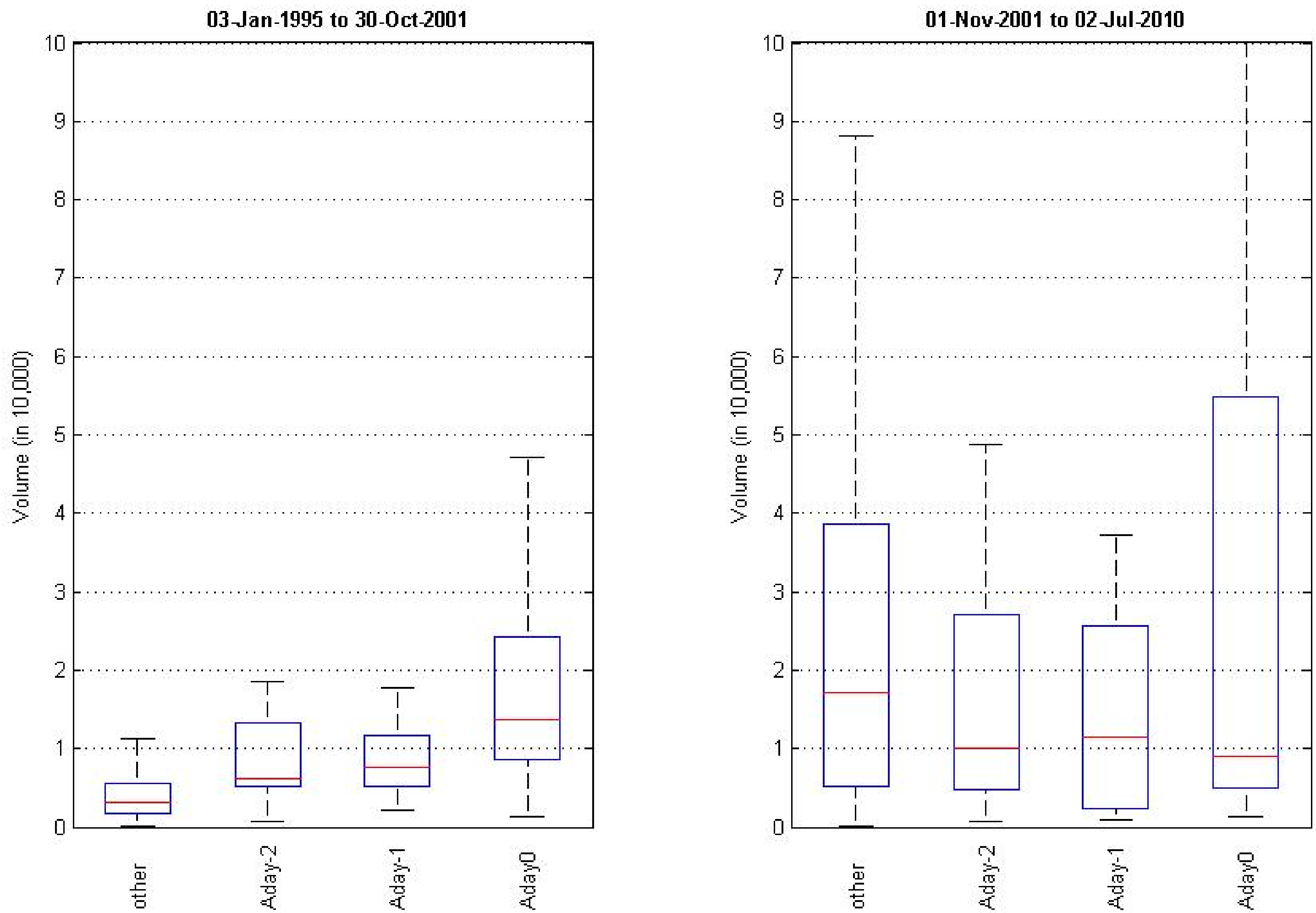

Figure 2: Excess Trade Before and After October-2001

Table 1: Estimation Results before Sept-2001
Dependent variable: volume

From : 03-Jan-1995 to 30-Sept-2001			
$\overline{R}^2$	0.16		
DW	0.66		
T	1673	K	18
Variable	Coefficient	t-statistic	t-probability
const	4201.94	33.30	0.00
Δg_t	95.45	0.10	0.92
$\Delta \pi_t$	4246.67	0.86	0.39
SD^+_{-2}	4985.58	4.48	0.00
SD^+_{-1}	5911.39	5.32	0.00
SD^+_0	13514.13	12.14	0.00
SD^+_1	8618.00	7.75	0.00
SD^+_2	4369.75	3.93	0.00
SD^0_{-2}	-810.41	-1.01	0.31
SD^0_{-1}	-610.00	-0.76	0.45
SD^0_0	1758.77	2.19	0.03
SD^0_1	-33.51	-0.04	0.97
SD^0_2	-1140.34	-1.42	0.16
UD^+_{-2}	3444.39	1.27	0.20
UD^+_{-1}	3865.61	1.42	0.15
UD^+_0	17781.72	6.56	0.00
UD^+_1	13043.72	4.81	0.00
UD^+_2	5596.06	2.07	0.04

Table 2: Estimation Results after Oct-2001
Dependent variable: volume

From : 01-Oct-2001 to 02-Jul-2010			
$\overline{R}^2$	0.02		
DW	0.37		
T	2176	K	13
Variable	Coefficient	t-statistic	t-probability
const	25837.46	35.76	0.00
Δg_t	4259.44	0.86	0.39
$\Delta \pi_t$	-4185.36	-0.35	0.73
SD^+_{-2}	-2913.41	-0.50	0.61
SD^+_{-1}	-4773.36	-0.83	0.41
SD^+_0	14986.61	2.59	0.01
SD^+_1	3983.38	0.69	0.49
SD^+_2	-5517.07	-0.96	0.34
SD^0_{-2}	8067.29	1.66	0.10
SD^0_{-1}	12530.17	2.57	0.01
SD^0_0	24585.18	5.04	0.00
SD^0_1	11644.65	2.39	0.02
SD^0_2	7321.35	1.49	0.14

Appendix

Proof of Lemma 2:[17] The linearity of u_1 implies that, for any t and ϵ no seller would accept an amount less than $r^{-1}E\rho$, *i.e.,* the expected return she would obtain by holding the asset instead of selling it. Similarly, no buyer would be willing to offer more than $r^{-1}E\rho$, *i.e.,* the amount he would obtain by choosing to deposit the equivalent amount of his bid instead of buying the asset. The result follows. □

Proof of Theorem 1: Substituting the solution $S^s_{\epsilon,t}$ in the seller's problem (2) obtain:

$$\begin{aligned} V^s_\epsilon(t) = &\int_{S^s_{\epsilon,t}+}^{\infty} [u_0(\omega^s_0 + \frac{S^s_{\epsilon,t}+x}{2} - Y^s_{\epsilon,t}) + Eu_1(\omega^s_1 + rY^s_{\epsilon,t})]dF_{\epsilon,t}(x) \\ &+ F_{\epsilon,t}(S^s_{\epsilon,t})V^s_\epsilon(t+\epsilon). \end{aligned} \tag{9}$$

This can be written as:

$$\begin{aligned} &V^s_\epsilon(t) - V^s_\epsilon(t+\epsilon) \\ &= \int_{S^s_{\epsilon,t}+}^{\infty} [u_0(\omega^s_0 + \frac{S^s_{\epsilon,t}+x}{2} - Y^s_{\epsilon,t}) + Eu_1(\omega^s_1 + rY^s_{\epsilon,t}) - V^s_\epsilon(t+\epsilon)]dF_{\epsilon,t}(x). \end{aligned} \tag{10}$$

Let us consider a seller s at time t with a positive probability of meeting a buyer with bid $x > S^s_{\epsilon,t}$. Seller s willingness to trade when receiving offer $x > S^s_{\epsilon,t}$ implies that:

$$u_0(\omega^s_0 + \frac{S^s_{\epsilon,t}+x}{2} - Y^s_{\epsilon,t}) + Eu_1(\omega^s_1 + rY^s_{\epsilon,t}) \geq V^s_\epsilon(t+\epsilon). \tag{11}$$

Equation (11) implies that $V^s_\epsilon(t)$ is a monotone decreasing function in t. Therefore by Lusin's Theorem (p. 230, Billingsley (1986)), $V^s_\epsilon(t)$ can be approximated arbitrarily close by a continuous function $V^s(t)$ in the L^p space as $\|V^s_\epsilon - V^s\|_p \to 0$. The function $V^s(t)$ is integrable with respect to t and is finite. It follows that the set of ϵ-step sellers' problems defined by (2) for which the value functions $V^s(t)$ are continuous in t are dense in the set of all ϵ-step

[17] Recall we are assuming an interior solution to the agent's problem and hence that agents are not borrowing constrained. An alternative proof based directly on the agent's maximization problem is also available.

sellers' problems. Equivalently, the set of problems for which the continuation values are discontinuous is negligible. Therefore $\lim_{\epsilon\to 0} V_\epsilon^s(t+\varepsilon) = \lim_{\epsilon\to 0} V_\epsilon^s(t) \equiv V^s(t)$. Hence, as the frequency of trading increases the value function becomes continuous function. A similar argument holds for the set of the buyer ϵ-step problems defined in (1). From (10) it follows that:

$$\begin{aligned}
&V_\epsilon^s(t) - V_\epsilon^s(t+\epsilon)\\
&= \int_{S_{\epsilon,t}^s+}^{\infty} [u_0(\omega_0^s + \frac{S_{\epsilon,t}^s + x}{2} - Y_{\epsilon,t}^s) + Eu_1(\omega_1^s + rY_{\epsilon,t}^s) - V_\epsilon^s(t+\epsilon)] dF_{\epsilon,t}(x)\\
&\geq \int_{S_{\epsilon,t}+}^{\infty} [u_0(\omega_0^s + S_{\epsilon,t} - Y_{\epsilon,t}^s) + Eu_1(\omega_1^s + rY_{\epsilon,t}^s) - V_\epsilon^s(t+\epsilon)] dF_{\epsilon,t}(x) \qquad (12)\\
&= [u_0(\omega_0^s + S_{\epsilon,t}^s - Y_{\epsilon,t}^s) + Eu_1(\omega_1^s + rY_{\epsilon,t}^s) - V_\epsilon^s(t+\epsilon)] \left(1 - F_{\epsilon,t}(S_{\epsilon,t}^s)\right).
\end{aligned}$$

From (12) it follows that for a given trading session $t \in \mathcal{T}_\epsilon$:

$$\begin{aligned}
&\lim_{\epsilon\to 0}[V_\epsilon^s(t) - V_\epsilon^s(t+\epsilon)]\\
&\geq \lim_{\epsilon\to 0}[u_0(\omega_0^s + S_{\epsilon,t}^s - Y_{\epsilon,t}^s) + Eu_1(\omega_1^s + rY_{\epsilon,t}^s) - V_\epsilon^s(t+\epsilon)] \left(1 - F_{\epsilon,t}(S_{\epsilon,t}^s)\right). \qquad (13)
\end{aligned}$$

It follows that:

$$\begin{aligned}
&0 \geq \lim_{\epsilon\to 0}[u_0(\omega_0^s + S_{\epsilon,t}^s - Y_{\epsilon,t}^s) + Eu_1(\omega_1^s + rY_{\epsilon,t}^s)] - \lim_{\epsilon\to 0} V_\epsilon^s(t+\epsilon), \text{ or}\\
&\lim_{\epsilon\to 0} V_\epsilon^s(t+\epsilon) \geq \lim_{\epsilon\to 0}[u_0(\omega_0^s + S_{\epsilon,t}^s - Y_{\epsilon,t}^s) + Eu_1(\omega_1^s + rY^s)]. \qquad (14)
\end{aligned}$$

However, since the seller is willing to trade at $S_{\epsilon,t}^s$ it also follows that:

$$\lim_{\epsilon\to 0}[u_0(\omega_0^s + S_{\epsilon,t}^s - Y_{\epsilon,t}^s) + Eu_1(\omega_1^s + rY_{\epsilon,t}^s)] \geq \lim_{\epsilon\to 0} V_\epsilon^s(t+\epsilon), \qquad (15)$$

and from (14) and (15) obtain:

$$\lim_{\epsilon\to 0} V_\epsilon^s(t+\epsilon) = \lim_{\epsilon\to 0}[u_0(\omega_0^s + S_{\epsilon,t}^s - Y_{\epsilon,t}^s) + Eu_1(\omega_1^s + rY_{\epsilon,t}^s)]. \qquad (16)$$

Substituting the appropriate notation for the limits, equation (16) becomes:

$$V^s(t) = u_0(\omega_0^s + S_t^s - Y_t^s) + Eu_1(\omega_1^s + rY_t^s). \tag{17}$$

Notice that,

$$\frac{\partial V^s(t)}{\partial S_t^s} = u_0'(\omega_0^s + S_t^s - Y_t^s)\left(1 - \frac{\partial Y_t^s}{\partial S_t^s}\right) + rEu_1'(\omega_1^s + rY_t^s)\frac{\partial Y_t^s}{\partial S_t^s}.$$

We know from (2) that Y_t^s satisfies

$$-u_0'(\omega_0^s + S_t^s - Y_t^s) + rEu_1'(\omega_1^s + rY_t^s) = 0,$$

therefore:

$$\frac{\partial V^s(t)}{\partial S_t^s} = u_0'(\omega_0^s + S_t^s - Y_t^s) > 0. \tag{18}$$

Since $V_\epsilon^s(t) \le V_\epsilon^s(t')$ for any two continuity points $t, t' \in (0,1)$ such that $t > t'$ computing the limits for $\epsilon \to 0$ obtain:

$$V^s(t) \le V^s(t'). \tag{19}$$

A similar argument proves the same monotonicity of the buyer's value function $V^b(t)$. Since $V^s(t)$ is monotonically increasing in S_t^s and $V^b(t)$ is monotonically decreasing in B_t^b, this proves the statement. □

Proof of Theorem 2: Consider two continuity points $t, t' \in (0,1)$ such that $t > t'$. Consider also a seller s and a buyer b with limit bid and ask prices such that $B_{t'}^b > S_{t'}^s$. Then from Theorem 1 we have $B_t^b \ge B_{t'}^b$ and $S_t^s \le S_{t'}^s$ and hence $B_t^b > S_t^s$.
Therefore,

$$\{(b,s) : B_t^b > S_t^s\} \supseteq \{(b,s) : B_{t'}^b > S_{t'}^s\}.$$

It follows that:

$$v_t = \Pr\{(b,s) : B_t^b > S_t^s\} \ge \Pr\{(b,s) : B_{t'}^b > S_{t'}^s\} = v_{t'}.$$

□

Proof of Corollary 1: This simply follows from the weakly increasing dynamics of volume given in Theorem 2. □

Proof of Lemma 3: Suppose equation (5) holds. Then for any pair $t \neq t' \in (0,1)$, $v_t = \Pr\left\{(b,s) : B_t^b > S_t^s\right\} = v_{t'}$.

Suppose now that $v_t = v > 0$ for all t but equation (5) does not hold $a = s$. Then by Theorem 1 there is a $\underline{t}$ such that for all $t > \underline{t}$, there exists a subset of sellers such that: $\Omega_{\underline{t}} = \{s : S_{\underline{t}}^s > S_t^s\}$ and $\Pr(\Omega_{\underline{t}}) > 0$. This implies:

$$\begin{aligned} v_{\underline{t}} &= \Pr\left\{(b,s) : B_t^b > S_{\underline{t}}^s\right\} \\ &= \Pr\left\{(b,s) : B_t^b > S_{\underline{t}}^s,\ s \in \Omega_{\underline{t}})\right\} + \Pr\left\{(b,s) : B_t^b > S_t^s,\ s \in \Omega_{\underline{t}}^c\right\} \\ &< \Pr\left\{(b,s) : B_t^b > S_t^s,\ s \in \Omega_{\underline{t}})\right\} + \Pr\left\{(b,s) : B_t^b > S_t^s,\ s \in \Omega_{\underline{t}}^c\right\} \\ &= \Pr\left\{(b,s) : B_t^b > S_t^s\right\} = v_t, \end{aligned}$$

a contradiction. A similar contradiction results if equation (5) does not hold for $a = b$. □

Proof of Lemma 4: We start by showing that if u_0 is linear then the volume is constant and positive. Let's assume that the first period preferences can be represented by a linear function, say $u_0(x) = k_1 + k_2 x$, where k_1 and $k_2 > 0$ are two constant terms.

Let $e_1^s = Eu_1(\omega_1^s + rY_1^s)$ and $e_{in}^s = Eu_1(\omega_1^s + \rho + rY_{in}^s)$. Then by equation (4) in the last trading session, the ask price S_1^s can be solved as:

$$V_\epsilon^s(1) = u_0(\omega_0^s + S_1^s - Y_1^s) + e_1^s = u_0(\omega_0^s - Y_{in}^s) + e_{in}^s. \tag{20}$$

Therefore since u_0 is linear:

$$\frac{\partial V_\epsilon^s(1)}{\partial \omega_0^s} = \frac{\partial\left[u_0(\omega_0^s - Y_{in}^s) + e_{in}^s\right]}{\partial \omega_0^s} = k_2.$$

Also $\frac{\partial V_\epsilon^s(1)}{\partial e_1^s} = \frac{\partial V_\epsilon^s(1)}{\partial e_{in}^s} = 1$ and $\frac{\partial e_1^s}{\partial e_{in}^s} = 1$.

Suppose for any $\epsilon > 0$, $\frac{\partial V_\epsilon^a(t+\epsilon)}{\partial \omega_0^a} = k_2$ and $\frac{\partial V_\epsilon^s(t+\epsilon)}{\partial e_{in}^s} = 1$. Since $S_{\epsilon,t}^s$ is the argmax of the problem in (2) and $Y_{\epsilon,t}^s$ satisfies $u_0'(\omega_0^s + \frac{S_{\epsilon,t}^s + x}{2} - Y_{\epsilon,t}^s) = rEu_1'(\omega_1^s + rY_{\epsilon,t}^s)$, it follows that:

$$
\begin{aligned}
\frac{\partial V_\epsilon^s(t)}{\partial \omega_0^s} &= \int_{S_{\epsilon,t}^s+}^{\infty} \frac{\partial}{\partial \omega_0^s}[u_0(\omega_0^s + \frac{S_{\epsilon,t}^s + x}{2} - Y_{\epsilon,t}^s) + Eu_1(\omega_1^s + rY_{\epsilon,t}^s)]dF_{\epsilon,t}(x) \\
&+ F_{\epsilon,t}(S_{\epsilon,t}) \frac{\partial V_\epsilon^s(t+\epsilon)}{\partial \omega_0^s} \\
&= k_2 \left[1 - F_{\epsilon,t}(S_{\epsilon,t}^s)\right] + k_2 F_{\epsilon,t}(S_{\epsilon,t}^s) = k_2,
\end{aligned}
\tag{21}
$$

and similarly

$$
\frac{\partial V_\epsilon^s(t)}{\partial e_2^s} = \frac{\partial e_1^s}{\partial e_{in}^s} \left[1 - F_{\epsilon,t}(S_{\epsilon,t}^s)\right] + F_{\epsilon,t}(S_{\epsilon,t}^s) = 1. \tag{22}
$$

Therefore solving the differential equations obtain $V_\epsilon^s(t) = V_\epsilon^s(t') = const + k_2 \omega_0^s + e_{in}^s$, for all t, t'. Comparing $V_\epsilon^s(t)$ with $V_\epsilon^s(1)$ we conclude that $const = k_1 - k_2 Y_{in}^s$. Therefore:

$$
V_\epsilon^s(t) = k_1 + k_2 \left(\omega_0^s - Y_{in}^s\right) + e_{in}^s
$$

Hence $V_\epsilon^s(t)$ is independent of t and equal to $V^s(1)$ implying from (18) that $S_{\epsilon,t}^s = S_{\epsilon,t'}^s = S_1^s$. A similar argument proves that $B_{\epsilon,t}^b = B_{\epsilon,t'}^b = B_1^b$. The result follows from Lemma 3.

Let now $v_t = v > 0$. Consider a seller $s \in [0,1]$. By Lemma 3 the prices and the distributions are time independent. Hence:

$$
\frac{\partial V^s(t)}{\partial \omega_0^s} = \frac{\partial V^s(1)}{\partial \omega_0^s} = u_0'(\omega_0^s + S_1^s - Y_1^s) \tag{23}
$$

for all t. Differentiating with respect to ω_0^s and using the envelope theorem and (23), for all $s \in [0,1]$ and $x > S_1^s$ obtain:

$$
\frac{\partial V^s(t)}{\partial \omega_0^s} = \int_{S_1^s+}^{\infty} \frac{\partial}{\partial \omega_0^s} \left[u_0(\omega_0^s + \frac{S_1^s + x}{2} - Y_1^s) + Eu_1(\omega_1^s + rY_1^s)\right] dF_\epsilon(x) + F_\epsilon(S_1^s) \frac{\partial}{\partial \omega_0^s} V^s(t+\epsilon),
$$

$$u_0'(\omega_0^s + S_1^s - Y_1^s) = \int_{S_1^s+}^{\infty} u_0'(\omega_0^s + \frac{S_1^s + x}{2} - Y_1^s) dF_{\epsilon,t}(x) + F_{\epsilon,t}(S_1^s) u_0'(\omega_0^s + S_1^s - Y_1^s)$$

$$0 = \int_{S_1^s+}^{\infty} \left[u_0'(\omega_0^s + S_1^s - Y_1^s) - u_0'(\omega_0^s + \frac{S_1^s + x}{2} - Y_1^s) \right] dF_\epsilon(x).$$

Since $u_0'' \leq 0$ we have $u_0'(\omega_0^s + S_1^s - Y_1^s) - u_0'(\omega_0^s + \frac{S_1^s+x}{2} - Y_1^s) \geq 0$ for all $x > S_1^s$. So the last equation holds if for all $x > S_1^s$ and for all $s \in [0,1]$:

$$u_0'(\omega_0^s + S_1^s - Y_1^s) = u_0'(\omega_0^s + \frac{S_1^s + x}{2} - Y_1^s). \tag{24}$$

It follows that the utility in the first period must linear.

Let now u_0 linear. The proof that v_t is constant and positive follows from Lemma 3. $\square$

Proof of Theorem 3: The result follows directly from Theorem 2 and Lemma 4. $\square$

Proof of Lemma 5: Consider a seller s and the corresponding Lagrangian function:

$$\mathcal{L}(\widehat{s}, x, y : \omega_0^s, V, \overline{V}) = [u_0(\omega_0^s + \frac{\hat{s} + x}{2} - y) + Eu_1(\omega_1^s + ry)] I\left[x > \hat{s}\right] + I\left[x \leq \hat{s}\right] \left[\tau V + (1-\tau)\overline{V}\right]$$
$$+ \lambda(x) \left[u_0'\left(\omega_0^s + \frac{\hat{s} + x}{2} - y\right) - Eu_1'(\omega_s + ry) \right].$$

Notice that $\widehat{V}_\epsilon^s(t) = \int_0^\infty \mathcal{L}(\hat{S}^s, x, Y_{\epsilon,t}^s : \omega_0^b, \widehat{V}_\epsilon^s(t+\epsilon), V_\epsilon^s(1, in)) d\widehat{F}_{\epsilon,t}(x)$. Then define a functional $\Phi : \mathcal{V} \to \mathcal{V}$, where $\mathcal{V}$ is the space of bounded continuous functions such that:

$$\Phi\left(\widehat{V}_\epsilon^s(t)\right) = \int_0^\infty \mathcal{L}(\widehat{S}_{\epsilon,t}^s, x, \widehat{Y}_{\epsilon,t} : \omega, \widehat{V}_\epsilon^s(t), V_\epsilon^s(1, in)) d\widehat{F}_{\epsilon,t}(x).$$

where $(\widehat{S}_{\epsilon,t}^s,\ \widehat{Y}_{\epsilon,t}^s)$ is the argmax of (7). We show that Φ is a contraction mapping. Let $\widehat{V}_\epsilon^s(t)$ and $\widehat{V}_\epsilon'^s(t)$ be two functions then:

$$\Phi(\widehat{V}_\epsilon^s(t)) - \Phi(\widehat{V}_\epsilon'^s(t)) = \tau_t \widehat{F}_{\epsilon,t}(\widehat{S}_{\epsilon,t}^s)(\widehat{V}_\epsilon^s(t) - \widehat{V}_\epsilon'^s(t)).$$

Since $\sup_{t\in(0,1)} \tau_t < 1$ and $\widehat{F}_{\epsilon,t}(\widehat{S}_{\epsilon,t}^s) \leq 1$, we can choose a $\delta < 1$ such that $\sup_{t\in(0,1)} \tau_t\, \widehat{F}_{\epsilon,t}(\widehat{S}_{\epsilon,t}^s) \leq \delta < 1$ therefore,

$$\left\|\Phi(\widehat{V}_\epsilon(t)) - \Phi(\widehat{V}_\epsilon'^s(t))\right\| \leq \delta \left\|\widehat{V}_\epsilon^s(t) - \widehat{V}_\epsilon'^s(t)\right\|. \tag{25}$$

Hence for any $\epsilon > 0$, by the contraction mapping theorem, it follows that: 1. $\widehat{V}_\epsilon^s(t) = \widehat{V}_\epsilon'^s(t) = V_\epsilon^s(1, in)$ and 2. The ask prices as well as bank account choices are stationary and independent of t. The proof for the bids is similar. □

Proof of Theorem 5: From Lemma 5.2, let $\widehat{A}^a_{\epsilon,t} = \widehat{A}^a$ and $\widehat{Y}^a_{\epsilon,t} = \widehat{Y}^a,\ a = b, s$ the stationary prices and bank account choices, respectively. Therefore $\widehat{F}_{\epsilon,t}(x) = \widehat{F}(x)$ and $\widehat{G}_{\epsilon,t}(x) = \widehat{G}(x)$ are stationary distributions and $\widehat{V}^a(t) = V^a(1, in),\ a = b, s$. Then for seller s:

$$\begin{aligned} V^s(1, in) &= \int_{\widehat{S}^s+}^{\infty} [u_0(\omega_0^s + \frac{\widehat{S}^s + x}{2} - \widehat{Y}^s) + Eu_1(\omega_1^s + r\widehat{Y}^s)] d\widehat{F}(x) \\ &+ \widehat{F}(\widehat{S}^s)\left[\tau_{t+\epsilon} V^s(1, in) + (1 - \tau_{t+\epsilon}) V^s(1, in)\right] \\ &= \int_{\widehat{S}^s+}^{\infty} [u_0(\omega_0^s + \frac{\widehat{S}^s + x}{2} - \widehat{Y}^s) + Eu_1(\omega_1^s + r\widehat{Y}^s)] d\widehat{F}(x) + \widehat{F}(\widehat{S}^s) V^s(1, in). \end{aligned}$$

Therefore:

$$\int_{\widehat{S}^s+}^{\infty} [u_0(\omega_0^s + \frac{\widehat{S}^s + x}{2} - \widehat{Y}^s) + Eu_1(\omega_1^s + r\widehat{Y}^s) - V^s(1, in)] d\widehat{F}(x) = 0,$$

implying that:

$$[u_0(\omega_0^s + \frac{\widehat{S}^s + x}{2} - \widehat{Y}^s) + Eu_1(\omega_1^s + r\widehat{Y}^s) - V^s(1, in)] d\widehat{F}(x) = 0 \text{ for almost all } x \in (\widehat{S}^s, \infty).$$

In particular, for $x > \widehat{S}^s$, $d\widehat{F}(x) = 0$ since u is strictly increasing it follows that:

$$u_0(\omega_0^s + \widehat{S}^s - \widehat{Y}^s) + Eu_1(\omega_1^s + r\widehat{Y}^s) = V^s(1, in),$$

implying that $\widehat{S}^s$ is the price of the last trading opportunity under credible scheduling. The same argument applies to the buyers. The expected volume at any $t(0,1)$ is given by:

$$\widehat{v} \equiv \Pr\{(b,s) : \widehat{B}^b > \widehat{S}^s\} = \Pr\{(b,s) : B_1^b > S_1^s\} = v_1,$$

since the bid and ask prices are stationary. $\square$

Proof of Corollary 2: 1. By Theorem 3 there exists a $t' \in \mathcal{T}_\epsilon$ such that for all $t \in \mathcal{T}_\epsilon$ greater than t', $v_t > v_{t'}$. By Theorem 5, $\widehat{v} = v_1$ for any $t \in (0,1)$. Therefore: $\int_0^1 \widehat{v}dt > \int_0^1 v_t dt$. 2. By equation (19) for $a = b, s$, $V^a(t)$ is decreasing in t and by Lemma 5.1 $\widehat{V}^a(t) = V^a(1, in)$. Therefore: $V^a(t) > \widehat{V}^a(t),\ t \in (0,1),\ a = b, s.$ $\square$