On Existence and Fragility of Repo Markets

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August 30, 2012

Abstract

This paper presents a model of an over-the-counter bond market in which bond dealers and cash investors arrange repurchase agreements (repos) endogenously. If cash investors buy bonds to store their cash, then they suffer an endogenous bond-liquidation cost because they must sell their bonds by the scheduled times of their cash payments. This cost provides incentive for both dealers and cash investors to arrange repos with endogenous margins. As part of multiple equilibria, the bond-liquidation cost also gives rise to another equilibrium in which cash investors stop entering into repos all at once. Credit market interventions block this equilibrium.

JEL: G24.
Keywords: Repo; Over-the-counter market; Securities broker-dealer; Cash investor; Market collapse.

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1 Introduction

Repurchase agreements, or repos, are one of the primary instruments in the money market. In a repo, a cash investor buys bonds with an agreement that the seller of the bonds, typically a bond dealer, will buy back the bonds at a later date. A question arises from this observation regarding why cash investors need such agreements when they can simply buy and resell bonds in a series of spot transactions. In this paper, I present a model to illustrate that cash investors suffer an endogenous bond-liquidation cost if the bond market is an over-the-counter (OTC) market. This cost provides incentive for both bond dealers and cash investors to combine initial spot sales and repurchases of bonds between the same trading parties by arranging repos. Furthermore, the bond-liquidation cost makes repos exist in tandem with a possibility of a repo-market collapse. This result provides an explanation as to why a repo market with default-free bonds, such as the U.S. tri-party repo market, can collapse, as concerned during the recent financial crisis.

In the model, cash investors buy long-term bonds to store cash, and resell bonds when they need to pay out cash. In each bond transaction, a cash investor trades with a bond dealer bilaterally in an OTC bond market. This feature of the model is based on the fact that most bond markets are OTC markets in practice (see Harris 2003). When a cash investor resells bonds, the buying dealer can negotiate down the bond price, because a cash investor must retrieve cash by the scheduled time of the investor's cash payment. This ex-post price discount on a cash investor's bonds discourages a cash investor from buying bonds in a spot transaction.

If a cash investor arranges a repo with a dealer, however, the dealer offers a sufficiently low ask price of bonds for the cash investor to buy bonds. A dealer can lower the ask price in this case because a repo allows a dealer to secure a chance to buy bonds from a cash investor later with the price discount described above. In equilibrium, the ask price of bonds with a repo becomes lower than the interdealer bond price. Because the interdealer bond price is the marginal bond acquisition cost for a dealer, a dealer must finance part of the bond acquisition cost by the dealer's own cash. Thus, a repo margin emerges endogenously.

This equilibrium with repos is part of multiple equilibria. There exists another equilibrium in which cash investors stop transacting with dealers all at once. In this equilibrium, dealers suffer aggregate cash shortage because cash investors holding cash stop entering into repos with dealers. As a result, dealers run short of cash to repurchase bonds from cash investors who entered into repos before and need cash now. In search of market liquidity, these cash investors try to sell their bonds directly to other cash investors in the OTC bond market. Cash investors holding cash buy these bonds, because they can negotiate down the bond price given the sellers' imminent need for cash. Thus, they stop spending their cash on entering into repos with dealers.

This result is consistent with the concern over a collapse of the U.S. tri-party repo market in the run-up to the Bear Stearns' collapse in March 2008. As will be described in Section 2, a perhaps puzzling feature of this concern was that most of the bonds in the market were Treasury securities and agency debt, which are default-free. The result of the model, however, indicates that a repo market with default-free bonds can collapse.

The model implies two credit market policies to prevent a repo-market collapse. One is a central-bank facility for lending cash to dealers like the Primary Dealer Credit Facility (PDCF), which was introduced by the Federal Reserve in March 2008. This policy works if dealers have a sufficiently high time discount factor. The other is a bond purchase program in which the central bank commits to buying bonds within a certain range of prices in the interdealer market. The effect of this policy does not depend on the time discount factor for dealers. Thus, a bond purchase program is a more robust policy than a dealer credit facility.

1.1 Related literature

This paper is related to several strands of the literature. Duffie, Gârleanu and Pedersen (2005) show that bid-ask spreads appear when asset dealers set their prices in light of their clients' outside options in an OTC market.[1] They derive this result without inventory risk to dealers or asymmetric information.[2] In this paper, similar bid-ask spreads arise, because the difficulty for cash investors to postpone their cash payments lowers the value of their outside options in OTC bond transactions. Based on this result, I show that bid-ask spreads provide incentive to arrange repos for both dealers and cash investors.

The analysis of fragility of a repo market also adds to Duffie, Gârleanu and Pedersen's work. In their model, dealers never suffer aggregate cash shortage because, in aggregate, dealers are always matched with a fixed number of investors through random matching at each time point. In this paper, aggregate cash shortage for dealers, and hence a repo-market collapse, can occur, because a cash investor can choose to transact with either a dealer or another investor. In this regard, this paper is related to Miao's (2006) model, in which investors choose between a decentralized market among investors and a centralized market intermediated by dealers. Miao focuses on spot asset trade and analyzes an equilibrium in which both markets are active.

Martin, Skeie and von Thadden (2010) present a model featuring the fact that, before the reform of the U.S. tri-party repo market in 2010, clearing banks used to return cash to cash investors in tri-party repos during each daytime. They show that such behaviour of clearing banks leads to a possibility of unexpected runs on dealers with repos, given an exogenous asset-liquidation cost. In this paper, I illustrate another mechanism of a repo-market collapse

[1] Lagos and Rocheteau (2010) extend Duffie, Gârleanu and Pedersen's model to introduce unrestricted asset holdings by dealers' clients. They show that the adjustments of asset holdings have important effects on trade volume, bid-ask spreads, and trading delays.

[2] This result contrasts with an earlier literature that explains bid-ask spreads by inventory risk to dealers (Garman 1976, Amihud and Mendelson 1980, and Ho and Stoll 1981) or asymmetric information (Bagehot 1971, Glosten and Milgrom 1985, and Kyle 1985).

by featuring an endogenous bond-liquidation cost in an OTC bond market. Also, Monnet and Narajabad (2011) present a generic model of repos to analyze co-existence of repos and spot trade of assets in an OTC market. They show that investors arrange repos between them if each investor has uncertainty about the future use of assets. This paper differs from their work in analyzing repos between bond dealers and cash investors. Relatedly, there is a strand of literature on special repo rates due to security lending to short-sellers, such as Duffie (1996) and Vayanos and Weill (2008). In this paper, I focus on general repos by considering homogeneous bonds in the model.

From a broader perspective, there is a vast literature on debt financing and asset prices. For example, Shleifer and Vishny (1997) and Gromb and Vayanos (2002) analyze the effects of arbitrage on asset markets when arbitragers have borrowing constraints. Geanakoplos (2009) derives endogenous collateral constraints based on Value-at-Risk and analyzes leverage cycles with heterogeneous investor beliefs in his framework. He and Xiong (2012) and Simsek (2012) introduce richer set-ups on shocks and beliefs into Geanakoplos' model. Brunnermeier and Pedersen (2009) also model collateral constraints based on Value-at-Risk and analyze the linkage between market liquidity and funding liquidity in asset markets. In this paper, I highlight the feature of OTC markets that trading parties in each transaction conduct bilateral bargaining. This feature of OTC bond markets causes a bond-liquidation cost for cash investors. This cost gives rise to repos with endogenous margins, which make repos resemble secured debt with down payments.

2 The concern over a collapse of the U.S. tri-party repo market during the recent financial crisis

In this section, I briefly summarize the concern over a collapse of the U.S. tri-party repo market during the recent financial crisis as an empirical motivation for this paper.

A repo is a combination of a spot sale of securities and an agreement to repurchase the securities later. In this transaction, the initial buyer of securities usually pays less than the market value of the securities received. The remaining value of the securities is called a margin, which must be financed by the initial seller's own cash. The initial buyer earns a return on the securities through a higher repurchase price paid by the initial seller than the initial spot-sale price that the initial buyer pays.

The main repo market in the U.S. is the tri-party repo market. In this market, the typical initial buyers are institutional cash investors, and the typical initial sellers are bond dealers.[3] Most of the bonds traded in the market are default-free. Analyzing data from the two clearing banks involved with tri-party repos (The Bank of New York Mellon and JPMorgan Chase), Copeland, Martin and Walker (2010) report that around 85% of the bonds traded in the market were Treasury securities and agency debt over the sample period between July 2008 and January 2010.[4]

While, as far as I know, there is no more publicly available data on OTC transactions in the tri-party repo market, anecdotal evidence suggests that policy makers had serious concern over a collapse of the market in the run-up to the Bear Stearns' collapse in March 14, 2008.[5] For example, the Financial Crisis Inquiry Commission (2011) notes the concern expressed by Federal Reserve Board Chairman Ben Bernanke, such that:

[3] The majority of dealers in the market are securities broker-dealers called primary dealers, who can trade directly with the Federal Reserve Bank of New York in its open market operations. The two large groups of cash investors in the market are money market funds (MMFs) and securities lenders, who receive cash collateral from short-sellers in exchange for lending securities. Municipalities and non-financial firms are also among cash investors in the market. See Copeland, Martin and Walker (2010) for more details.

[4] Also, Krishnamurthy, Nagel and Orlov (2011) find that Treasury securities and agency debt accounted for more than around 70% of the bonds with repos held by a sample of MMFs between the first quarters of 2007 and 2010. Note that agency debt was deemed safe between 2007 and 2010. Moody's long-term ratings for Fannie Mae and Freddie Mac, the two government-sponsored enterprises that guarantee agency mortgage-backed securities (agency MBS), remained at Aaa between 2007 and 2010 because of government guarantees. Agency MBS accounted for most of the agency debt in the tri-party repo market, as reported by Copeland, Martin and Walker (2010).

[5] For turmoils in repo markets in general, Gorton and Metrick (2012) provide empirical analysis on repo haircuts in a wide range of asset markets over 2007-08.

> The $2.8 trillion tri-party repo market had "really [begun] to break down," Bernanke said. "As the fear increased," short-term lenders began demanding more collateral, "which was making it more and more difficult for the financial firms to finance themselves and creating more and more liquidity pressure on them. And, it was heading sort of to a black hole." He saw the collapse of Bear Stearns as threatening to freeze the tri-party repo market, leaving the short-term lenders with collateral they would try to "dump on the market. You would have a big crunch in asset prices." (pp. 290-291)

Also, in accordance with Bernanke's comment, Adrian, Burke and McAndrews (2009, page 2) at the Federal Reserve Bank of New York note an increase in haircuts for repos backed by Treasury securities and agency debt before the Bear Stearns' collapse.[6]

These observations by the Fed officials are puzzling, given the fact that most of the bonds traded in the tri-party repo market were default-free. In the following, I illustrate that a repo market with default-free bonds can collapse, using a model in which bond dealers and cash investors choose to arrange repos endogenously.

3 A basic model of a bond market with cash investors

I start from a set-up without dealers. This set-up is the basic market structure in this paper, into which dealers will be introduced later.

3.1 The set-up

Time is discrete and its horizon is infinite. In each period, a $[0, 1]$ continuum of risk-neutral investors are born with a fixed amount e_I (> 0) of a cash endowment for each. They live for two periods and consume cash in the last period of their lives. I call investors in their first

[6] A haircut is the ratio of the margin to the value of securities underlying the repo.

period "young" and those in their last period "old". Each investor is indexed by $i \in \mathbb{Z} \times [0, 1]$, which is the pair of the integer denoting the period when the investor is born and the real number assigned to the investor on the unit continuum of the cohort.

Young investors can invest their cash in two instruments. One is safe short-term bills that return a fixed amount $1 + r$ (≥ 1) of cash in the next period for each unit of cash invested. I call this instrument "T-bills". The other instrument is safe long-term bonds which generate a fixed amount d (> 0) of cash dividends for the holders of bonds at the beginning of every period. I call this instrument "bonds". Bonds are divisible and their supply is fixed to unity. Thus, bonds are Lucas trees. An investor can store cash by buying bonds when young and reselling them when old.[7]

Investors can trade bonds in a brokered OTC market. In each period, each young investor is matched with an old investor, and vice versa, through pairwise random matching. Implicitly, the matching can be interpreted as arranged by brokers. The terms of bond trade in each match are determined by Nash bargaining with equal bargaining powers for both parties in the match. See Figure 1 for a summary of the bond market structure.

3.2 An endogenous bond-liquidation cost for investors

The finite time horizon for each investor's cash consumption represents the difficulty for cash investors in practice to postpone their cash payments when they need to pay out cash. Now I show that this difficulty leads to an endogenous bond-liquidation cost.

I consider the case in which investors are homogeneous. In this case, each old investor holds a unit of bonds at the beginning of each period. Each old investor sells the investor's whole bond to a young investor in the brokered market, because an old investor can consume cash only in the current period. Thus, the cash consumption of an old investor equals the

[7] If an old investor does not resell bonds, then that investor keeps holding the bonds even after exiting from the economy. Nobody gains utility from the cash dividends of the bonds after the investor's exit in this case. This case does not occur in equilibrium as shown below.

sum of gross returns on the investor's bond and T-bills:

$$c_{i,t} = d + p_{BR,t} + (1+r)(e_I - p_{BR,t-1}), \tag{1}$$

where i is the index for an old investor, t denotes the time period, $c_{i,t}$ is the investor's consumption, and $p_{BR,t}$ is the price of a unit of bonds in the brokered market. The value of $p_{BR,t}$ becomes identical for each investor as shown below.

Suppose that the cash endowment for each young investor, e_I, is arbitrarily large, so that young investors always have a residual of cash to invest in T-bills at the end of each period. Given Equation (1), the Nash-bargaining problem over bond trade between a young and an old investor can be written as:

$$\max_{p_{BR,t}} (p_{BR,t} - 0)^{0.5}[E_t(d + p_{BR,t+1}) + (1+r)(e_I - p_{BR,t}) - (1+r)e_I]^{0.5}, \tag{2}$$

where the left parenthesis and the right square bracket represent the gains from trade for an old and a young investor, respectively. In the left parenthesis, there appears a zero as the outside option value of keeping holding a unit of bonds for the old investor, because the old investor needs to consume cash now. In the right square bracket, $(1+r)e_I$ is the expected consumption of the young investor when the investor does not buy any bond, in case of which the investor invests all of the investor's cash in T-bills.

The solution for the bargaining problem is:

$$p_{BR,t} = \frac{0.5E_t(d + p_{BR,t+1})}{1+r}, \tag{3}$$

which implies that a young investor can buy a bond at a price lower than the indifference price for a young investor, $E_t(d + p_{BR,t+1})/(1+r)$. Intuitively speaking, a young investor can negotiate down the price of an old investor's bond because an old investor must obtain cash to consume by the end of the current period. This price discount is a bond-liquidation cost from an old investor's perspective.

4 A model of a bond market with dealers and cash investors

Now I introduce dealers into the basic model. In addition to investors described above, there exists a $[0, 1]$ continuum of infinite-lived risk-neutral dealers maximizing the expected discounted consumption of cash:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} c_{j,s}, \tag{4}$$

where β $(\in (0, 1))$ is the time discount factor for dealers, j $(\in [0, 1])$ is the index for a dealer, and $c_{j,t}$ is the consumption of cash.

An investor has two choices for the investor's bond trade in each period. One is to enter the brokered market to trade bonds with another investor as in the basic model. The other is to trade bonds with a dealer in a dealer market. An investor cannot choose both options within a period because it takes time to find a trading counterparty in each market. Dealers and investors take as given the matching probabilities in each market, which will be defined below. See Figure 2 for a summary of the bond market structure with dealers.

4.1 The brokered market for investors

I redefine the matching probabilities in the brokered market. As in the basic model, young investors entering the brokered market are matched with old investors in the market, and vice versa, through pairwise random matching. The matching probabilities are:

$$\mu_{BR,Y,t} \equiv \min \left\{ 1, \ \frac{\theta_{BR,O,t}}{\theta_{BR,Y,t}} \right\}, \quad \mu_{BR,O,t} \equiv \min \left\{ 1, \ \frac{\theta_{BR,Y,t}}{\theta_{BR,O,t}} \right\}, \tag{5}$$

where: $\mu_{BR,Y,t}$ and $\mu_{BR,O,t}$ denote the matching probabilities for a young and an old investor in the market, respectively; and $\theta_{BR,Y,t}$ and $\theta_{BR,O,t}$ are the fractions of young and old investors entering the market, respectively. Thus, the short side of the market matches with probability one.[8] If $\theta_{BR,Y,t} = \theta_{BR,O,t} = 0$, then $\mu_{BR,Y,t} = \mu_{BR,O,t} = 0$. Once matched, a young and an old investor trade bonds as in the basic model.

4.2 The dealer market for investors

Investors can enter a dealer market instead of the brokered market. The dealer market consists of two sub-markets; one for young investors and the other for old investors. Every dealer participates in both sub-markets. In each sub-market, investors and dealers are matched through pairwise random matching. The matching probability in each sub-market is similar to the brokered market, i.e., the short side matches with probability one.

More specifically, if a young investor enters the dealer market, then the investor is always randomly matched with one of the dealers. The probability for a dealer to meet with a young investor equals the fraction of young investors entering the dealer market, $1 - \theta_{BR,Y,t}$. A dealer cannot be matched with more than one young investor in each period. Note that the populations of young investors and dealers are unity in each period. Thus, $1 - \theta_{BR,Y,t}$ is the ratio of young investors to dealers in the dealer market. Because the ratio of dealers to young investors in the dealer market, $1/(1 - \theta_{BR,Y,t})$, is equal to, or greater than, one, the matching probability for a young investor is always one.

A matched pair of a young investor and a dealer bargain over bond trade. The terms of trade are: the price and the quantity of bonds that the young investor buys; and whether to arrange a repo or not. If a young investor arranges a repo with a dealer, then the investor will be matched with the same dealer again in the next period. In this case, the investor cannot

[8] In the basic model described above, every investor enters the brokered market in each period because there is no alternative way to trade bonds. Thus, $\theta_{BR,Y,t} = \theta_{BR,O,t} = 1$ for all t. Accordingly, every young investor meets with an old investor, and vice versa, in each period.

enter the brokered market when old. In each match between a dealer and an old investor, the pair negotiate the price and the quantity of bonds that the old investor sells. Thus, a repo combines a spot sale and a repurchase of bonds between the same trading parties.

In contrast, an old investor without a repo can choose between the brokered and the dealer market. If an old investor without a repo enters the dealer market, then the investor is always, but randomly, matched with one of the dealers. Here, note that a repo does not increase the matching probability for an old investor exogenously because it is always one. The probability for a dealer to be matched with an old investor without a repo equals the fraction of old investors entering the dealer market without repos.[9] A dealer cannot be matched with more than one old investor without a repo in each period.

Overall, a dealer can deal with three investors at most in each period: a young investor, an old investor with a repo, and an old investor without a repo. Without loss of generality, each dealer is matched with investors in this order in each period. The result of the model is insensitive to the order of the matches (see Appendix B). The outcome of each match is determined by Nash bargaining with equal bargaining powers for both parties in the match, as in the brokered market. See Figure 3 for a summary of the structure of the dealer market.

4.3 The interdealer markets and settlements

After meeting with investors, dealers can trade bonds in a competitive interdealer bond market. This assumption is based on the feature of the interdealer market for U.S. Treasury securities in practice, in which interdealer brokers allow dealers to trade in size anonymously and distribute the best bid and ask price to dealers. See Huang, Cai and Wang (2002) and Fleming and Mizrach (2009) for more details. Also, dealers can borrow and lend cash overnight at a competitive interest rate in an interdealer loan market. This assumption

[9] The fraction can be written as $1 - \theta_{BR,O,t} - \theta_{RP,O,t}$, where $\theta_{RP,O,t}$ denotes the fraction of old investors who return to the dealers for the investors' repos in period t.

makes it tractable to solve the Nash-bargaining problem for each match between an investor and a dealer.[10] Without loss of generality, I assume that dealers cannot short-sell bonds in the interdealer bond market, because short-selling is equivalent to taking interdealer loans.

The settlements of bond transactions take place only at the end of each period.[11] Hence dealers can settle transactions with investors after trading in the interdealer markets in the same period. After the settlements, young investors can invest the residual of their cash in T-bills, T-bills return cash to old investors, and dealers and old investors can consume cash.

Finally, dealers and investors take as given the competitive interdealer bond price and interest rate. An equilibrium is such that these two competitive interdealer prices clear the interdealer markets in each period. See Appendix A for an analytical definition of the market clearing conditions.

5 Existence of a repo market

In this section, I show the existence of a symmetric stationary equilibrium in which all investors transact with homogeneous dealers. I assume that:

$$\beta(1+r) < 1, \tag{6}$$

which implies that the rate of return that dealers require for their investments, β^{-1}, is higher than the rate of return on T-bills, $1+r$. Also, the cash endowment for each young investor, e_I, is arbitrarily large, as assumed in the basic model.

In this equilibrium, no entry of investors into the brokered market leads to a zero matching probability in the brokered market. Thus every investor enters the dealer market. Given no

[10]With this assumption, each dealer takes as given the marginal shadow values of bonds and cash for the dealer. See Appendix B for more details.

[11]This assumption reflects the fact that the settlement date of an asset transaction is typically set to a few days after the transaction date in practice.

entry of investors into the brokered market, I solve the events in each period backward. See Appendix B for the solution and its proof. Here, I sketch the solution with its intuition.

5.1 *The pay-off for each dealer at the end of each period*

At the end of each period, a dealer can trade bonds and arrange loans in the interdealer markets, and can also consume cash. The highest marginal return from these three options determines the shadow value of cash for a dealer at the end of each period:

$$1+\eta_t = \max\left\{1,\ \beta(1+rr_t)E_t(1+\eta_{t+1}),\ \frac{\beta E_t[(d+p_{ID,t+1})(1+\eta_{t+1})]}{p_{ID,t}}\right\}, \tag{7}$$

where $1+\eta_t$ denotes the shadow value of a unit of cash at the end of period t for each dealer, rr_t the competitive interdealer interest rate, and $p_{ID,t}$ the competitive interdealer bond price.[12] Note that a dealer takes as given all of the variables on the right-hand side of Equation (7). Hence the value of $1+\eta_t$ is exogenous for every dealer.

The shadow value of cash, $1+\eta_t$, can be used to summarize each dealer's expected discounted utility at the end of each period by the following value function, $V_{j,t}$:

$$\begin{aligned} V_{j,t} = (1+\eta_t)\,\big[&(d+p_{ID,t})a_{j,t-1} + b_{j,t-1} + (p_{Y,j,t}-p_{ID,t})q_{Y,j,t} + \left(p'_{Y,j,t}-p_{ID,t}\right)x_{j,t} \\ &+ (p_{ID,t}-p_{O,j,t})q_{O,j,t} + (p_{ID,t}-p_{RP,j,t})x_{j,t-1}\big] + \beta E_t V^*_{x,j,t+1}x_{j,t}, \end{aligned} \tag{8}$$

where j and t denote the indices for a dealer and the time period, respectively. The terms inside the square bracket on the right-hand side of Equation (8) are cash flows for the dealer during the period. The first two terms are from bonds ($a_{j,t-1}$) and interdealer loans ($b_{j,t-1}$) held at the beginning of the period. The other terms are cash flows due to bond transactions with investors in the period. See Table 1 for the notation of the variables. These cash flows

[12] The variable η_t is the Lagrange multiplier for a non-negativity constraint on a dealer's consumption. See Appendix B for more details.

are evaluated by $1 + \eta_t$.

The variable $x_{j,t}$ denotes the amount of bonds that the dealer sells to a young investor with a repo in the period. The value of $V^*_{x,j,t+1}$ equals the marginal return for the dealer from repurchasing bonds from the investor and reselling the bonds in the interdealer bond market in the next period:

$$V^*_{x,j,t+1} = (1 + \eta_{t+1})(p_{ID,t+1} - p_{RP,j,t+1}), \tag{9}$$

where $p_{RP,j,t+1}$ is the repurchase price of bonds. Thus, $E_t V^*_{x,j,t+1} x_{j,t}$ is the expected return from arranging a repo for the dealer.

5.2 An endogenous bond-liquidation cost in bilateral bargaining between a dealer and an old investor

Moving backward, I solve the Nash-bargaining problem for a match between a dealer and an old investor without a repo in the dealer market. As in Equation (1), the consumption of an old investor without a repo equals the gross returns on bonds and T-bills:

$$c_{i,t} = (d + p_{O,j,t}) q_{Y,j',t-1} + (1 + r)(e_I - p_{Y,j',t-1} q_{Y,j',t-1}), \tag{10}$$

where: i is the index for an old investor without a repo; j is the index for a dealer matched with the old investor in the current period; $p_{O,j,t}$ is the bid price of bonds offered by the dealer for the old investor; $q_{Y,j',t-1}$ is the amount of bonds that the old investor bought from some dealer j' without a repo in the previous period; and $p_{Y,j',t-1}$ is the ask price of bonds for the old investor then. Note that every investor buys bonds from a dealer when young, given no entry of investors into the brokered market.

Because an old investor needs to sell all of the investor's bonds to consume cash within the current period, the Nash-bargaining problem for a match between dealer j and an old investor without a repo can be written as:

$$\max_{p_{O,j,t}} \left(p_{O,j,t} q_{Y,j',t-1} - 0\right)^{0.5} \left[\left(p_{ID,j,t} - p_{O,j,t}\right) q_{Y,j',t-1}\right]^{0.5}, \tag{11}$$

where $p_{ID,t}$ is the interdealer bond price. In the left parenthesis, a zero appears as the outside option value of keeping holding bonds for the old investor, because the investor does not gain any utility from keeping holding bonds after the current period. In the right square bracket, $(p_{ID,j,t} - p_{O,j,t})q_{Y,j',t-1}$ is the net profit for dealer j from buying the old investor's bonds and reselling them in the interdealer bond market. The solution for the problem is:

$$p_{O,j,t} = 0.5 p_{ID,t}. \tag{12}$$

The Nash-bargaining problem for a match between a dealer and an old investor does not change even if the investor has a repo.[13] Thus, the price of bonds repurchased by dealer j from an old investor with a repo in period t, $p_{RP,j,t}$, satisfies:

$$p_{RP,j,t} = 0.5 p_{ID,t}. \tag{13}$$

Equations (12) and (13) imply that a dealer can make a profit by buying an old investor's bonds and reselling them in the interdealer bond market, because an old investor's imminent need for cash allows a dealer to negotiate down the price of an old investor's bonds. This price discount is of the same nature as the bond-liquidation cost described in Section 3.

[13] This is because the existence of a repo does not alter the events following a match between a dealer and an old investor.

5.3 Emergence of a repo in bilateral bargaining between a dealer and a young investor

Finally, I describe matches between dealers and young investors. Given no entry of investors into the brokered market, every young investor enters the dealer market. As a result, every dealer meets with a young investor with probability one, and vice versa, given the matching probabilities assumed above. The Nash-bargaining problem for each match between a dealer and a young investor can be written as:

$$\max_{\{p_{Y,j,t},\ q_{Y,j,t},\ p'_{Y,j,t},\ x_{j,t}\}} \left\{ (p_{Y,j,t} - p_{ID,t}) q_{Y,j,t} + \left[p'_{Y,j,t} - p_{ID,t} + \frac{\beta E_t V^*_{x,j,t+1}}{1+\eta_t} \right] x_{j,t} \right\}^{0.5}$$

$$\cdot \left\{ \left[d + E_t p_{O,j',t+1} - (1+r) p_{Y,j,t} \right] q_{Y,j,t} + \left[d + E_t p_{RP,j,t+1} - (1+r) p'_{Y,j,t} \right] x_{j,t} \right\}^{0.5}, \quad (14)$$

$$\text{s.t.} \quad p_{Y,j,t} q_{Y,j,t} + p'_{Y,j,t} x_{j,t} \le e_I,\ q_{Y,j,t} \ge 0,\ x_{j,t} \ge 0, \quad (15)$$

where: j is the index for the dealer; $(p_{Y,j,t},\ q_{Y,j,t})$ and $(p'_{Y,j,t},\ x_{j,t})$ are the pairs of the ask price and the quantity of bonds offered by the dealer for the young investor without and with a repo, respectively; and $p_{O,j',t+1}$ is the resale bond price for the young investor when the investor becomes old, if the investor buys bonds without a repo now. In this case, the index j' denotes a randomly matched dealer who buys bonds from the investor in the next period. Equation (15) contains the budget constraint and the non-negative constraints on the quantities of bonds for the young investor.[14]

The left and the right curly brackets in Equation (14) show the gains from trade for the dealer and the young investor, respectively. In the left curly bracket, the first term, $(p_{Y,j,t} - p_{ID,t}) q_{Y,j,t}$, is the dealer's net profit from selling bonds to the young investor without a repo. The competitive interdealer bond price, $p_{ID,t}$, appears as the marginal bond acquisition cost for the dealer, because a dealer can buy any amount of bonds at that price.

[14]Precisely speaking, $q_{Y,j,t}$ or $x_{j,t}$ must equal 0 given the assumption that a young investor chooses whether to arrange a repo for all of the investor's bonds or not. I omit this constraint here because it is satisfied endogenously as shown below.

The second term in the left curly bracket, $[p'_{Y,j,t} - p_{ID,t} + \beta E_t V^*_{x,j,t+1}/(1+\eta_t)]x_{j,t}$, is the dealer's expected net profit from selling bonds with a repo. The key difference between the first and the second term is in the presence of the expected discounted ex-post profit from a repo for the dealer, $\beta E_t V^*_{x,j,t+1}/(1+\eta_t)$.[15] This term appears only in the second term because the young investor will be randomly matched with one of the dealers in the next period if the dealer does not arrange a repo now. In this case, the dealer has a zero probability to meet again with that investor, because each dealer has a zero measure on the unit continuum of dealers.[16] Thus, the dealer needs to arrange a repo to secure a chance to buy bonds from the investor in the next period.

In the right curly bracket in Equation (14), the first term, $[d + E_t p_{O,j',t+1} - (1+r)p_{Y,j,t}]q_{Y,j,t}$, is the gain for the young investor from buying bonds without a repo. This term includes the opportunity cost of paying the bond price for the young investor, $(1+r)p_{Y,j,t}$. The second term, $\left[d + E_t p_{RP,j,t+1} - (1+r)p'_{Y,j,t}\right] x_{j,t}$, is the gain for the young investor from buying bonds with a repo. Because $p_{O,j',t+1} = p_{RP,j,t+1}$ as shown in Equations (12) and (13), the joint gain from trade for the dealer and the young investor is larger if they arrange a repo. The dealer can induce the young investor to enter into a repo by lowering the ask price of bonds for the investor, $p'_{Y,j,t}$. Hence, the young investor buys bonds with a repo, i.e., $q_{Y,j,t} = 0$.

Substituting $q_{Y,j,t} = 0$ into the Nash-bargaining problem (14) implies that, if there exist any gains from trade for the dealer and the young investor, then the investor would buy an arbitrarily large amount of bonds with a repo (i.e., $x_{j,t}$ would be arbitrarily large) with an arbitrarily large cash endowment, e_I. Since such large demand for bonds from every young

[15] This term is discounted by $\beta/(1+\eta_t)$, which is the effective time discount factor for dealers. This factor takes into account the shadow value of cash, $1+\eta_t$, in the current period.

[16] Note that every dealer has capacity to serve both old investors with and without a repo in each period, as assumed in the previous section. Taking as given the probability for a dealer to be matched with an old investor without a repo in the next period, a dealer just loses the chance to meet with an investor again if the dealer does not arrange a repo when the investor is young.

investor exceeds the fixed supply of bonds in the economy, the competitive interdealer bond price, $p_{ID,t}$, takes such a value that there are no gains from trade for dealers and young investors in equilibrium:

$$p_{ID,t} = \frac{d + E_t p_{RP,j,t+1}}{1+r} + \frac{\beta E_t V^*_{x,j,t+1}}{1+\eta_t}. \tag{16}$$

Given this value of $p_{ID,t}$, each pair of a dealer and a young investor choose:

$$p'_{Y,j,t} = \frac{d + E_t p_{RP,j,t+1}}{1+r}, \tag{17}$$

with which the values of the left and the right curly bracket in Equation (14) are zero.

I can show that the value of $p_{ID,t}$ is too high for dealers to buy bonds in the interdealer bond market, given a low time discount factor for dealers as assumed in Condition (6).[17] Thus, every bond is sold to a young investor with a repo in each period:[18]

$$x_{j,t} = 1. \tag{18}$$

5.4 An endogenous repo margin

Equations (16) and (17) imply that the ask price of bonds with a repo, $p'_{Y,j,t}$, is lower than the interdealer bond price, $p_{ID,t}$:

$$p_{ID,t} - p'_{Y,j,t} = \frac{\beta E_t V^*_{x,j,t+1}}{1+\eta_t} > 0, \tag{19}$$

where the last inequality follows from Equations (7), (9) and (13). The difference between $p'_{Y,j,t}$ and $p_{ID,t}$ is a repo margin, because the interdealer bond price, $p_{ID,t}$, is the marginal

[17]That is, $a_{j,t} = 0$ for all j and t.
[18]Note that both the supply of bonds and the population of young investors in each period are unity.

bond acquisition cost for dealers. Equation (19) implies that the expected discounted gain for a dealer from repurchasing bonds in the next period offsets the dealer's loss from financing the repo margin in the current period.

Each dealer finances the repo margin by the profit that the dealer earns from repurchasing bonds from an old investor and reselling the bonds in the interdealer bond market: $(p_{ID,t} - p_{RP,j,t})x_{j,t-1}$. Dealers consume the residual of the profit:

$$c_{j,t} = (p_{ID,t} - p_{RP,j,t})x_{j,t-1} - (p_{ID,t} - p'_{Y,j,t})x_{j,t} > 0, \tag{20}$$

where the last inequality holds in the stationary equilibrium.[19]

Equation (20) implies that dealers spend all of the profits on current consumption. Dealers buy no bond in the interdealer bond market for their own holding as described above. Also, homogeneous dealers do not take or provide interdealer loans in the symmetric stationary equilibrium, because otherwise the interdealer loan market would not clear.[20] Accordingly, the interdealer interest rate, rr_t, must satisfy:

$$\beta(1 + rr_t) = 1, \tag{21}$$

in the equilibrium.

5.5 Repos as secured debt

The model described so far features a repo as a combination of a spot sale and a repurchase of bonds between the same trading parties. It is also possible to interpret a repo in the

[19] To confirm the inequality, derive the stationary equilibrium value of $p_{ID,t}$ from Equations (9), (13) and (16). The stationary equilibrium value is shown in Equation (24). Then, substitute Equations (13) and (17) and $x_{j,t} = 1$ for all j and t into Equation (20). The inequality implies that $\eta_t = 0$, because the shadow value of cash equals the marginal utility from consumption, which is unity.

[20] That is, $b_{j,t} = 0$ for all j and t.

model as a secured debt contract, if I consider a possibility of renegotiations of contracts, following Hart and Moore (1994) and Kiyotaki and Moore (1997).[21]

Suppose that a dealer and a young investor arranging a repo can specify the repurchase price of bonds in the next period in advance. Also, suppose that the dealer and the investor can renegotiate the repurchase price when they meet again. Given this possibility of an ex-post renegotiation, the only pledgeable repurchase price in a repo is the expected value of the repurchase price after a renegotiation in the next period. This value is $E_t p_{RP,j,t+1}$ in which $p_{RP,j,t+1}$ satisfies Equation (13). This result holds because a renegotiation of a repo is equivalent to a bilateral bond transaction between a dealer and an old investor. Accordingly, $E_t p_{RP,j,t+1}$ becomes the contracted repurchase price in a repo between a dealer and a young investor. Thus, given that $p_{RP,j,t+1} = p_{O,j,t+1}$ as implied by Equations (12) and (13), a dealer's repayment to an old investor is anchored by the price of an old investor's bond in the dealer market. Note that a renegotiation of a repo does not occur on the equilibrium path because the contracted repurchase price already equals the outcome of a renegotiation. All of the other results remain the same as described above.

Even if a dealer could commit to paying an arbitrary repurchase price to an investor in the next period, there would be still a repo. In this case, a repo would allow a dealer to commit to not negotiating down the price of an old investor's bonds, so that the dealer can raise the ask price of bonds for a young investor. A dealer would prefer to increase the current revenue through a higher ask price because of a low time discount factor for dealers as assumed in Condition (6). But in this case, a repo would be effectively non-secured debt for a dealer.

[21] The two papers explain the existence of secured debt by considering renegotiations of debt by borrowers given their intangible human capital for production. Here, I consider a possibility of renegotiations of repos by dealers given old investors' need for cash.

6 Fragility of a repo market

In this section, I set two conditions on the cash endowment for each young investor, e_I, and the time discount factor for dealers, β:

$$e_I \in \left(p_{ID,SS},\ \frac{(d + 0.5p_{ID,SS})(d + 0.75p_{ID,SS})}{0.5(1 + r)p_{ID,SS}} \right], \tag{22}$$

$$\beta(1 + r) < \min \left\{ 1,\ \frac{4 + 6r + (1 + 2r)\sqrt{33 + 32r}}{5 + \sqrt{33 + 32r}} \right\}, \tag{23}$$

where $p_{ID,SS}$ denotes the value of the interdealer bond price, $p_{ID,t}$, in the stationary equilibrium with repos described in the previous section:[22]

$$p_{ID,SS} = \frac{d}{0.5[1 - \beta(1 + r)] + r}. \tag{24}$$

Condition (22) indicates that e_I is not extremely large, but large enough for each young investor to buy a bond at $p_{ID,SS}$. Thus, Condition (22) ensures the existence of the stationary equilibrium with repos described above.[23] Condition (23) incorporates Condition (6) assumed above and also ensures that the range for e_I in Condition (22) is a non-empty set.

Also, I assume that an old investor with a repo can enter the brokered market.[24] In this case, the investor causes a settlement fail.[25] For simplicity, I assume no punishment for a

[22]Equation (24) can be derived from Equations (9), (13) and (16).

[23]The condition $e_I > p_{ID,SS}$ ensures that $p_{ID,t} = p_{ID,SS}$ in the stationary equilibrium with repos. If $p_{ID,t} < p_{ID,SS}$, then young investors would put all of their cash on repos, as described above. In this case, dealers would need to obtain an amount $\int (e_I/p_{Y',j,t})dj$ of bonds in aggregate. Because $p_{Y',j,t} \leq p_{ID,t}$, the bond demand would exceed the bond supply in the economy. Hence the market clearing condition would be violated.

[24]It is just for simplicity to prohibit an old investor with a repo from entering the brokered market in the previous section. Even if an old investor with a repo can enter the brokered market, no old investor does so because there is no investor in the brokered market in the equilibrium described in the previous section. Also, without loss of generality, I assume that an old investor with a repo never meets with a dealer other than the dealer for the repo. This behaviour is weakly optimal for an old investor because a dealer offers the same terms of trade for an old investor's bonds regardless of existence of a repo, as implied by Equations (12) and (13).

[25]A settlement fail is a failure to deliver securities to the buyer. Here, the buyer is the dealer for the old

settlement fail because a settlement fail is not regarded as default immediately in practice. See Fleming and Garbade (2005) for more details on settlement fails.

Given Conditions (22) and (23) and the possibility of settlement fails by old investors, the stationary equilibrium with repos becomes part of multiple equilibria. There exists another equilibrium in which every investor enters the brokered market unexpectedly in period t, given the stationary equilibrium with repos in period $t-1$. Thus, the repo market can collapse.[26] The economy returns to the stationary equilibrium with repos from period $t+1$ onward in this equilibrium.[27] In this section, I guess and verify the existence of such an equilibrium.

6.1 The existence of the stationary equilibrium with repos from period $t+1$ onward

Suppose that every young investor buys a bond in the brokered market in period t, as verified below. Thus, each old investor in period $t+1$ has a bond without a repo at the beginning of the period. All of the results described in Section 5 hold for period $t+1$ and later, except that each old investor in period $t+1$ is randomly matched with a dealer, and vice versa.

6.2 Bilateral bargaining between a young and an old investor in the brokered market in period t

Now I describe events in period t. When every investor enters the brokered market in period t, every young investor is matched with an old investor, and vice versa, given the matching probabilities in the brokered market assumed above. Since each young investor in period

investor's repo.

[26]Regarding the interpretation of this result, note that MMFs, a main group of cash investors in the U.S. tri-party repo market, cannot hold long-term securities by regulation. Implicitly, a repo-market collapse shown below can be interpreted as ultimate cash investors, such as corporate treasuries, stop putting their cash on MMFs and instead buy bonds directly from other cash investors who liquidate bonds in the brokered market.

[27]If e_I and β satisfy certain conditions tighter than Conditions (22) and (23) and if r is sufficiently close to 0, then there exists an equilibrium in which every investor enters the brokered market from period t to period T for an arbitrarily large integer T. See Appendix D for more details.

$t-1$ buys a bond with a repo in the stationary equilibrium with repos (i.e., $x_{j,t-1} = 1$ for all j), each old investor in period t has a bond at the beginning of the period. Hence, each young investor buys a bond from an old investor in the brokered market in period t.

The Nash-bargaining problem for each match between a young and an old investor in the brokered market takes the same form as Equation (2), except that the bid price of bonds in the dealer market, $p_{O,j,t+1}$, replaces the bond price in the brokered market, $p_{BR,t+1}$, as the resale bond price for the young investor when the investor becomes old. This modification is necessary because the economy will return to the stationary equilibrium with repos from the next period onward. The solution for the Nash-bargaining problem yields:[28]

$$p_{BR,t} = \frac{0.5(d + E_t p_{O,j,t+1})}{1+r}. \tag{25}$$

Equation (25) implies that, as in the basic model, a young investor can buy an old investor's bonds at a price lower than the indifference price for a young investor, $(d+E_t p_{O,j,t+1})/(1+r)$, because the old investor's imminent need for cash allows the young investor to negotiate down the bond price.

6.3 The dominance of the brokered market for old investors in period t

For each old investor in period t, the alternative to entering the brokered market is returning to the dealer for the investor's repo. In this case, the repurchase price offered by the dealer, $p_{RP,j,t}$, would equal $0.5p_{ID,t}$ as shown in Equation (13).

Entering the brokered market becomes a dominant choice for an old investor in period t if the bond price in the brokered market is equal to, or higher than, the repurchase price

[28] Condition (22) ensures that each young investor has an enough cash endowment, e_I, to pay $p_{BR,t}$ for a bond in the brokered market in period t, given Equation (26) and that $p_{ID,t} < p_{ID,SS}$ in period t as shown below.

offered by the dealer for the investor's repo. Accordingly, suppose that:

$$p_{BR,t} = p_{RP,j,t} = 0.5p_{ID,t}, \tag{26}$$

in period t, so that old investors in the period are indifferent between entering the brokered market and returning to the dealers for their repos. Equation (26) holds if and only if the interdealer bond price, $p_{ID,t}$, takes the following value in period t:[29]

$$p_{ID,t} = \frac{(1-0.5\beta)d}{0.5[1-\beta(1+r)]+r}. \tag{27}$$

6.4 The dominance of the brokered market for young investors in period t

A young investor in period t is faced with the following trade-off. On one hand, the investor can buy an old investor's bond at a discounted price in the brokered market, as shown above. On the other hand, the investor can buy more than a unit of bonds in the dealer market, because a dealer has access to the competitive interdealer bond market.

If a young investor entered the dealer market in period t, then the investor would arrange a repo with a dealer. In this case, the investor would be able to commit to returning to the dealer in the next period, because the stationary equilibrium with repos resumes from the next period onward. The Nash-bargaining problem for a match between a young investor and a dealer in period t would yield:

$$(p'_{Y,j,t},\ x_{j,t}) = \left(0.5\left[p_{ID,t} - \frac{E_t V^*_{x,j,t+1}}{1+rr_t}\right] + 0.5\left(\frac{d + E_t p_{RP,j,t+1}}{1+r}\right),\ \frac{e_I}{p'_{Y,j,t}}\right). \tag{28}$$

See Appendix C for the proof.

Intuitively speaking, the ask price of bonds with a repo, $p'_{Y,j,t}$, would equal the average

[29] Equation (27) is derived from Equations (12), (24) and (25), given that the economy returns to the stationary equilibrium with repos from period $t+1$ onward, i.e., $p_{ID,t+1} = p_{ID,SS}$.

of the indifference price for the dealer, $p_{ID,t} - E_t V^*_{x,j,t+1}/(1+rr_t)$, and the one for the young investor, $(d + E_t p_{RP,j,t+1})/(1+r)$, given the equal bargaining powers for the dealer and the investor.[30] Because Equations (26) and (27) imply that the indifference price for the dealer is strictly less than the one for the young investor, there would be strictly positive gains from trade for the dealer and the young investor. As a result, the young investor would put all of the investor's cash on the repo (i.e., $x_{j,t} = e_I/p'_{Y,j,t}$) to earn a higher rate of return than the rate of return on T-bills, $1+r$.

Given Condition (22) and Equation (27), the cash endowment for a young investor, e_I, is small enough that the expected consumption of a young investor in period t is higher if the investor enters the brokered market rather than the dealer market:[31]

$$d + E_t p_{O,j,t+1} + (1+r)(e_I - p_{BR,t}) \geq (d + E_t p_{RP,j,t+1}) x_{j,t}. \tag{29}$$

Thus, young investors enter the brokered market in period t.

6.5 Aggregate cash shortage in the interdealer markets in period t

Now I only need to verify that Equation (27) clears the interdealer markets in period t. Comparison between Equations (24) and (27) implies that the interdealer bond price, $p_{ID,t}$, drops in period t.[32] As a result, the rate of return on bonds in the interdealer bond market rises in period t:[33]

$$\frac{d + E_t p_{ID,t+1}}{p_{ID,t}} = 1 + r + \frac{1}{2-\beta}. \tag{30}$$

[30] If $p'_{Y,j,t} = p_{ID,t} - E_t V^*_{x,j,t+1}/(1+rr_t)$, then the rate of return on the repo for the dealer, $E_t V^*_{x,j,t+1}/(p'_{Y,j,t} - p_{ID,t})$, equals the rate of return on interdealer loans, $1 + rr_t$. Similarly, if $p'_{Y,j,t} = (d + E_t p_{RP,j,t+1})/(1+r)$, then the rate of return on the repo for the young investor, $(d + E_t p_{RP,j,t+1})/p'_{Y,j,t}$, equals the rate of return on T-bills, $1+r$.

[31] Equation (29) holds with equality if and only if e_I equals the upper bound shown in Condition (22).

[32] Note that $p_{ID,t-1} = p_{ID,SS}$ because of the stationary equilibrium with repos in period $t-1$.

[33] Equation (30) can be derived from Equations (16) and (27) and $p_{ID,t+1} = p_{ID,SS}$.

Given Condition (23), the right-hand side of Equation (30) exceeds the intertemporal marginal rate of substitution for dealers, β^{-1}, if $\beta(1+r)$ is sufficiently close to 1. In this case, dealers would buy bonds in the interdealer bond market if they had their own cash or could borrow cash at a sufficiently low cost. However, dealers become cashless as no young investor buys bonds from dealers in period t.[34] Also, a resulting zero cash supply in the interdealer loan market makes the interdealer interest rate, rr_t, sufficiently high to discourage dealers from taking interdealer loans in equilibrium:[35]

$$1 + rr_t \geq \frac{d + E_t p_{ID,t+1}}{p_{ID,t}} = 1 + r + \frac{1}{2-\beta}. \tag{31}$$

If the right-hand side of Equation (30) is smaller than β^{-1}, then dealers would be unwilling to buy bonds even if they had their own cash.[36] In this case, $1 + rr_t$ remains equal to β^{-1} so that dealers do not take or provide interdealer loans. Overall, the interdealer markets in period t clear with Equation (27) and the interdealer interest rate, rr_t, satisfying:[37]

$$1 + rr_t = \max\left\{1 + r + \frac{1}{2-\beta}, \frac{1}{\beta}\right\}. \tag{32}$$

6.6 The effects of credit market interventions

Given the existence of multiple equilibria, I discuss the effects of credit market interventions on a repo-market collapse. Suppose that there exists a central bank which can commit to providing interdealer loans at a rate less than $r + (2-\beta)^{-1}$, but more than $\beta^{-1} - 1$. This

[34]Dealers own no bond in the stationary equilibrium with repos in period $t-1$ as described in Section 5. Thus, dealers do not receive any cash dividend from bonds at the beginning of period t.

[35]The interdealer loan market clears because dealers do not have cash to provide interdealer loans.

[36]This case can be consistent with Condition (23).

[37]Even though Equation (26) makes each old investor indifferent between the dealer and the brokered market, the aggregate cash shortage for dealers makes it necessary that every old investor enters the brokered market. If a positive measure of old investors entered the dealer market, then the zero cash supply in the interdealer markets would prevent dealers from obtaining cash to pay for old investors' bonds by selling the bonds in the interdealer bond market or taking interdealer loans.

dealer credit facility is akin to the PDCF, because it involves a penalty rate above the normal interdealer interest rate prevailing in the stationary equilibrium with repos, $\beta^{-1} - 1$.[38] The results described above imply that, if the intertemporal marginal rate of substitution for dealers, β^{-1}, is less than $1 + r + (2 - \beta)^{-1}$, then the dealer credit facility eliminates the equilibrium with a repo-market collapse. If there occurred the equilibrium with a repo-market collapse in this case, then dealers would borrow cash from the central bank to buy bonds in the interdealer bond market. A resulting rise in the interdealer bond price, $p_{ID,t}$, would enable dealers to repurchase bonds from old investors at a sufficiently high price. Accordingly, old investors stop leaving for the brokered market. No liquidation of bonds by old investors in the brokered market would induce young investors to enter the dealer market and thus arrange repos with dealers.

This result, however, does not hold if $\beta^{-1} \geq 1 + r + (2 - \beta)^{-1}$. In this case, dealers require too high a rate of return for their investments. Accordingly, they would not borrow from the central bank to buy bonds in the interdealer bond market, unless the interest rate offered by the central bank were below the normal interdealer interest rate, $\beta^{-1} - 1$. But such a policy would block both the stationary equilibrium with repos and the equilibrium with a repo-market collapse. To eliminate only the latter equilibrium, the central bank can commit to a direct bond purchase in the interdealer bond market at a price higher than the one prevailing during a repo-market collapse (see Equation 27), but not more than the one prevailing in the stationary equilibrium with repos (see Equation 24). This policy works regardless of the value of β.

Finally, note that preventing a repo-market collapse is not Pareto-improving. Young investors in period t are better off in the equilibrium with a repo-market collapse, because they can earn a higher rate of return on bonds than in the stationary equilibrium with repos. Thus, a repo-market collapse benefits investors holding cash. These investors' gains come

[38] See Equation (21).

from the losses for dealers and old investors in period t: the old investors, who hold bonds at the beginning of period t, suffer a drop in the price for their bonds; and dealers lose the profit from trading with investors in period t (see Equation 20). Investors born in period $t+1$ or later are indifferent.

7 Conclusions

I have presented a model featuring bond dealers and cash investors in an OTC bond market. The model illustrates that bilateral bargaining over bond trade leads to an endogenous bond-liquidation cost for cash investors. This cost explains both the existence of repos and the possibility of an unexpected repo-market collapse. Using this model, I have discussed the conditions under which a dealer credit facility and a bond purchase program by the central bank prevent a repo-market collapse.

In this paper, I take as given the OTC bond market structure. A question remains regarding the optimal market design, such as whether to introduce a centralized bond market or a set-up to ensure anonymity of cash investors. Also, the empirical implications of the model are yet to be tested. One of the testable implications is that a repo margin is increasing in the difference between the interdealer bond price and the repurchase bond price (Equation 19). Another implication is that spot transactions in a brokered bond market increase if a repo market collapses. Addressing these issues are left for future research.

References

[1] Adrian, Tobias, Christopher R. Burke, and James J. McAndrews, 2009. The Federal Reserve's Primary Dealer Credit Facility. Federal Reserve Bank of New York *Current Issues in Economics and Finance* 15, no. 4 (August).

[2] Amihud, Yakov, and Haim Mendelson, 1980. Dealership Markets: Market Making with Inventory. *Journal of Financial Economics* 8, 31-53.

[3] Bagehot, Walter, 1971. The Only Game in Town. *Financial Analysts Journal* 27, 12-14.

[4] Brunnermeier, Markus K., and Lasse H. Pedersen. 2009. Market Liquidity and Funding Liquidity. *Review of Financial Studies* 22, 2201-38.

[5] Copeland, Adam, Antoine Martin, and Michael Walker, 2010. The Tri-Party Repo Market before the 2010 Reforms. Federal Reserve Bank of New York Staff Report 477.

[6] Duffie, Darrell, 1996. Special Repo Rates. *Journal of Finance* 51, 493-526.

[7] Duffie, Darrell, Nicolae Gârleanu, and Lasse H. Pedersen, 2005. Over-the-Counter Markets. *Econometrica* 73, 1815-1847.

[8] Financial Crisis Inquiry Commission, 2011. *The Financial Crisis Inquiry Report.* `http://www.gpoaccess.gov/fcic/fcic.pdf`

[9] Fleming, Michael J., and Kenneth D. Garbade, 2005. Explaining Settlement Fails. Federal Reserve Bank of New York *Current Issues in Economics and Finance* 11, no. 9 (September).

[10] Fleming, Michael J., and Bruce Mizrach, 2009. The Microstructure of a U.S. Treasury ECN: The BrokerTec Platform. Federal Reserve Bank of New York Staff Report 381.

[11] Garman, Mark B., 1976. Market Microstructure. *Journal of Financial Economics* 3, 257-275.

[12] Geanakoplos, John, 2009. The Leverage Cycle, in Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, eds.: *NBER Macroeconomics Annual 2009* (University of Chicago Press).

[13] Glosten, Lawrence R., and Paul R. Milgrom, 1985. Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders. *Journal of Financial Economics* 14, 71-100.

[14] Gorton, Gary, and Andrew Metrick, 2012. Securitized Banking and the Run on Repo *Journal of Financial Economics* 104, 425-451.

[15] Gromb, Denis, and Dimitri Vayanos, 2002. Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs. *Journal of Financial Economics* 66, 361-407.

[16] Harris, Larry, 2003. *Trading and Exchanges: Market Microstructure for Practitioners* (Oxford University Press).

[17] Hart, Oliver and John Moore, 1994. A Theory of Debt Based on the Inalienability of Human Capital. *Quarterly Journal of Economics* 109, 841-879.

[18] He, Zhiguo, and Wei Xiong, 2012. Debt Financing in Asset Markets. *American Economic Review: Papers & Proceedings* 102, 88-94.

[19] Ho, Thomas, and Hans R. Stoll, 1981. Optimal Dealer Pricing under Transactions and Return Uncertainty. *Journal of Financial Economics* 9, 47-73.

[20] Huang, Roger D., Jun Cai, and Xiaozu Wang, 2002. Information-Based Trading in the Treasury Note Interdealer Broker Market. *Journal of Financial Intermediation* 11, 269-296.

[21] Kiyotaki, Nobuhiro, and John Moore, 1997. Credit Cycles. *Journal of Political Economy* 105, 211-248.

[22] Krishnamurthy, Arvind, Stefan Nagel, and Dmitry Orlov, 2011. Sizing Up Repo. Manuscript. Northwestern University.

[23] Kyle, Albert S., 1985. Continuous Auctions and Insider Trading. *Econometrica* 6, 1315-1335.

[24] Lagos, Ricardo, and Guillaume Rocheteau, 2010. Liquidity in Asset Markets with Search Frictions. *Econometrica* 77, 403-426.

[25] Martin, Antoine, David Skeie, and Ernst-Ludwig von Thadden, 2010. Repo Runs. Federal Reserve Bank of New York Staff Report 444.

[26] Miao, Jianjun, 2006. A Search Model of Centralized and Decentralized Trade. *Review of Economic Dynamics* 9, 68-92.

[27] Monnet, Cyril, and Borghan N. Narajabad, 2011. Repurchase Agreements or, Why Rent When You Can Buy? Manuscript. University of Bern.

[28] Shleifer, Andrei, and Robert W. Vishny, 1997. The Limit of Arbitrage. *Journal of Finance* 52, 35-55.

[29] Simsek, Alp. 2012. Belief Disagreements and Collateral Constraints. Manuscript. Harvard University.

[30] Vayanos, Dimitri, and Pierre-Olivier Weill, 2008. A Search-Based Theory of the On-the-Run Phenomenon. *Journal of Finance* 63, 1361-1398.

Appendices (not for publication)

A The market clearing conditions in the model of a bond market with dealers and cash investors

The market clearing conditions for the interdealer bond price, $p_{ID,t}$, and interest rate, rr_t, are:

$$\int a_{j,t} + x_{j,t} + q_{Y,j,t}\, dj + \int_{i \in I_t} q_{BR,i,t}\, di = 1, \tag{A.1}$$

$$\int b_{j,t}\, dj = 0, \tag{A.2}$$

where: $a_{j,t}$ is the amount of bonds owned by dealer j at the end of period t; $x_{j,t}$ and $q_{Y,j,t}$ are the amounts of bonds sold by dealer j to young investors with and without a repo, respectively; $q_{BR,i,t}$ is the amount of bonds that old investor i sells to a young investor in the brokered market; I_t is the set of the indices for old investors in period t; and $b_{j,t}$ is the net balance of interdealer loans for dealer j at the end of period t. The left-hand side of Equation (A.1) is the sum of bonds held by dealers and young investors at the end of period t. The right-hand side is the supply of bonds in the economy.

B The proof for the existence of a repo market in Section 5

In the following, I prove the existence of a symmetric stationary equilibrium in which every investor enters the dealer market. It is self-fulfilling that no investor enters the brokered market, as described in Section 5. Thus $q_{BR,i,t} = 0$ for all i and t. Given this result, I solve each event in each period backward.

B.1 Utility maximization by dealers in the interdealer markets

The maximization problem for dealer j in the interdealer markets can be written as:

$$V_t(S_{j,t-1},\ Z_{Y,j,t},\ Z_{O,j,t},\ Z_{RP,j,t}) =$$

$$\max_{\{c_{j,t},\ q_{ID,j,t},\ b_{j,t}\}} c_{j,t} + \beta E_t V_{t+1}(S_{j,t},\ Z^*_{Y,j,t+1}(S_{j,t}),\ Z^*_{O,j,t+1}(S_{j,t}),\ Z^*_{RP,j,t+1}(S_{j,t})), \tag{B.1}$$

$$\text{s.t. } c_{j,t} + p_{ID,t}q_{ID,j,t} + \frac{b_{j,t}}{1+rr_t}$$

$$= da_{j,t-1} + b_{j,t-1} + p_{Y,j,t}q_{Y,j,t} + p'_{Y,j,t}x_{j,t} - p_{O,j,t}q_{O,j,t} - p_{RP,j,t}q_{RP,j,t}, \tag{B.2}$$

$$a_{j,t} = q_{ID,j,t} + q_{O,j,t} + q_{RP,j,t} - q_{Y,j,t} - x_{j,t} + a_{j,t-1}, \tag{B.3}$$

$$c_{j,t},\ a_{j,t} \geq 0, \tag{B.4}$$

where V_t is the value function for each dealer's expected discounted consumption of cash. See Table 1 for the notation of variables.

In this problem, a dealer chooses the current consumption, $c_{j,t}$, the net balance of bonds to trade in the interdealer bond market, $q_{ID,j,t}$, and the net balance of interdealer loans, $b_{j,t}$, given the competitive interdealer bond price, $p_{ID,j,t}$ and interest rate, rr_t. Equation (B.2) is the flow of funds constraint, the right-hand side of which records the cash flows before the interdealer markets and the dealer's consumption in the period. Equation (B.3) is the law of motion for bonds owned by the dealer at the end of the period, $a_{j,t}$. Equation (B.4) contains the non-negativity constraint on $c_{j,t}$ and the no short-sale constraint on $a_{j,t}$.

The vector $S_{j,t-1}$ stores the state variables for dealer j at the beginning of period t: the amount of bonds owned by the dealer at the end of the previous period, $a_{j,t-1}$; the amount of bonds sold to a young investor with a repo in the previous period, $x_{j,t-1}$; and the balance of interdealer loans at the end of the previous period, $b_{j,t-1}$. The other pre-determined variables for the dealer in the interdealer markets are $Z_{Y,j,t}$, $Z_{O,j,t}$, and $Z_{RP,j,t}$, which denote the terms of bond trade in the same period with: a young investor; an old investor without a repo; and an old investor with a repo, in order. The functions $Z^*_{Y,j,t+1}(S_{j,t})$, $Z^*_{O,j,t+1}(S_{j,t})$ and $Z^*_{RP,j,t+1}(S_{j,t})$ return the values of $Z_{Y,j,t+1}$, $Z_{O,j,t+1}$ and $Z_{RP,j,t+1}$, in order, conditional on $S_{j,t}$.

I denote the derivatives of the value function, V_{t+1}, with respect to the state variables stored in $S_{j,t}$ ($\equiv [a_{j,t},\ b_{j,t},\ x_{j,t}]$) by:

$$\begin{bmatrix} V^*_{a,t+1} & V^*_{b,t+1} & V^*_{x,t+1} \end{bmatrix} \equiv \frac{dV_{t+1}(S_{j,t}, Z^*_{Y,j,t+1}(S_{j,t}), Z^*_{O,j,t+1}(S_{j,t}), Z^*_{RP,j,t+1}(S_{j,t}))}{dS^\top_{j,t}}. \tag{B.5}$$

I guess and verify that dealers and investors take as given the values of $V^*_{a,t+1}$, $V^*_{b,t+1}$ and $V^*_{x,t+1}$.

Given this conjecture, the solution for the maximization problem yields:

$$1 + \eta_t = \beta(1 + rr_t)E_t V^*_{b,t+1} \geq 1, \tag{B.6}$$

$$\eta_t c_{j,t} = 0, \tag{B.7}$$

$$[(1 + \eta_t)p_{ID,t} - \beta E_t V^*_{a,t+1}]a_{j,t} = 0, \tag{B.8}$$

where η_t is the Lagrange multiplier for $c_{j,t} \geq 0$ in Equation (B.4). Equation (B.6) implies that the shadow value of a unit of cash for a dealer at the end of the period, $1 + \eta_t$, is pinned down by the discounted rate of return on interdealer loans, $\beta(1+rr_t)E_t V^*_{b,t+1}$. Thus, dealers and investors take the value of η_t as given. This result holds because a dealer can use the interdealer loan market as a buffer for the dealer's excess cash or cash shortage at a competitive interest rate, rr_t. In equilibrium, the cost of interdealer loans (i.e., $\beta(1 + rr_t)E_t V^*_{b,t+1}$) must be equal to, or greater than, the marginal utility from consumption (i.e., 1). Otherwise every dealer would take interdealer loans, which would violate the market clearing condition (A.2).

To confirm Equation (B.7), note that $c_{j,t} = 0$ if $\beta(1 + rr_t)E_t V^*_{b,t+1} > 1$, because in this case dealers are better off by postponing consumption. Equation (B.8) follows from the first-order condition with respect to $q_{ID,j,t}$, which implies that:

$$a_{j,t} \begin{cases} = 0, & \text{if } (1+\eta_t)p_{ID,t} > \beta E_t V^*_{a,t+1}, \\ = \infty, & \text{if } (1+\eta_t)p_{ID,t} < \beta E_t V^*_{a,t+1}, \\ \in [0,\infty), & \text{if } (1+\eta_t)p_{ID,t} = \beta E_t V^*_{a,t+1}. \end{cases} \tag{B.9}$$

Since $a_{j,t} = \infty$ for all j would violate the market clearing condition (A.1), Equation (B.8) must hold.

Substituting Equations (B.2), (B.3) and (B.6)-(B.8) into Equation (B.1) yields:

$$\begin{aligned} &V_t(S_{j,t-1},\ Z_{Y,j,t},\ Z_{O,j,t},\ Z_{RP,j,t}) \\ &\quad = (1+\eta_t)\left[(d+p_{ID,t})a_{j,t-1} + b_{j,t-1} + (p_{Y,j,t} - p_{ID,t})q_{Y,j,t} + \left(p'_{Y,j,t} - p_{ID,t}\right)x_{j,t}\right. \\ &\qquad \left. + (p_{ID,t} - p_{O,j,t})q_{O,j,t} + (p_{ID,t} - p_{RP,j,t})q_{RP,j,t}\right] + \beta E_t\left(V^*_{x,t+1}x_{j,t} + f_{j,t+1}\right), \end{aligned} \tag{B.10}$$

where:

$$f_{j,t+1} \equiv V_{t+1}(S_{j,t}, Z^*_{Y,j,t+1}, Z^*_{O,j,t+1}, Z^*_{RP,j,t+1}) - V^*_{a,t+1}a_{j,t} - V^*_{b,t+1}b_{j,t} - V^*_{x,t+1}x_{j,t}. \tag{B.11}$$

Given the conjecture that dealers and investors take as given the values of $V^*_{a,t+1}$, $V^*_{b,t+1}$ and $V^*_{x,t+1}$, $f_{j,t+1}$ is the residual component of V_{t+1} that does not depend on the state variables for dealer j. Thus, dealers and investors take $f_{j,t}$ as given for all j and t.

B.2 Bilateral bargaining between a dealer and an old investor

Given no entry of investors into the brokered market, the consumption of old investor i without a repo equals:

$$dq_{Y,j',t-1} + p_{O,j,t}q_{O,j,t} + (1+r)s_{i,t-1}, \tag{B.12}$$

where: $q_{Y,j',t-1}$ is the amount of bonds that the investor bought from a randomly matched dealer j' without a repo in the previous period; $(p_{O,j,t},\ q_{O,j,t})$ is the pair of the price and the quantity of bonds that the investor sells to a randomly matched dealer j in the current period; and $s_{i,t-1}$ is the amount of cash invested in T-bills by the investor in the previous period.

Given Equation (B.12), the Nash-bargaining problem for a match between dealer j and old investor i without a repo can be written as:

$$\max_{Z_{O,j,t}} \left[V_t(S_{j,t-1},\ Z_{Y,j,t},\ Z_{O,j,t},\ Z_{RP,j,t}) - V_t(S_{j,t-1},\ Z_{Y,j,t},\ \mathbf{0}, Z_{RP,j,t})\right]^{0.5}$$
$$\cdot \left\{dq_{Y,j',t-1} + p_{O,j,t}q_{O,j,t} + (1+r)s_{i,t-1} - [dq_{Y,j',t-1} + (1+r)s_{i,t-1}]\right\}^{0.5}, \tag{B.13}$$
$$\text{s.t. } q_{O,j,t} \leq q_{Y,j',t-1}. \tag{B.14}$$

The solution is $Z_{O,j,t} = [0.5p_{ID,t},\ q_{Y,j',t-1}]$. Thus:

$$Z^*_{O,j,t} = \begin{cases} [0.5p_{ID,t},\ q_{Y,j',t-1}], & \text{if dealer } j \text{ meets with old investor } i \text{ without a repo,} \\ \mathbf{0}, & \text{if dealer } j \text{ meets no old investor without a repo.} \end{cases} \tag{B.15}$$

The Nash-bargaining problem for a match between a dealer and an old investor with a repo is similar to the bargaining problem described above, except that: $(p_{O,j,t},\ q_{O,j,t})$ is replaced with the pair of the price and the quantity of bonds repurchased by the dealer from the investor with a repo, $(p_{RP,j,t},\ q_{RP,j,t})$; $q_{Y,j',t-1}$ is replaced with $x_{j,t-1}$; and $Z^*_{O,j,t}$ is substituted into $Z_{O,j,t}$. Because the variables contained in $Z^*_{O,j,t}$ are exogenous to dealers and investors, solving the Nash-bargaining problem for $Z_{RP,j,t}$ $(\equiv [p_{RP,j,t},\ q_{RP,j,t}])$ yields:

$$Z^*_{RP,j,t} = [0.5 p_{ID,t},\ x_{j,t-1}]. \tag{B.16}$$

B.3 Bilateral bargaining between a dealer and a young investor

Given no entry of investors into the brokered market, every young investor enters the dealer market. Thus, each dealer meets with a young investor, and vice versa, given the matching probabilities assumed in Section 4. The Nash-bargaining problem for a match between dealer j and young investor i can be written as:

$$\max_{Z_{Y,j,t}} \left[V_t(S_{j,t-1},\ Z_{Y,j,t},\ Z^*_{O,j,t},\ Z^*_{RP,j,t}) - V_t(S_{j,t-1},\ \mathbf{0},\ Z^*_{O,j,t},\ Z^*_{RP,j,t})\right]^{0.5}$$

$$\cdot \{E_t[(d + p_{O,j',t+1})q_{Y,j,t} + (d + p_{RP,j,t+1})x_{j,t}] + (1+r)s_{i,t} - (1+r)e_I\}^{0.5}, \tag{B.17}$$

$$\text{s.t. } s_{i,t} = e_I - (p_{Y,j,t}q_{Y,j,t} + p'_{Y,j,t}x_{j,t}) \geq 0, \tag{B.18}$$

$$q_{Y,j,t}, x_{j,t} \geq 0. \tag{B.19}$$

The subscript j' denotes the index for a dealer randomly matched with the young investor in the next period in case that the investor buys bonds without a repo in the current period. Also, $(p_{Y,j,t},\ q_{Y,j,t})$ and $(p'_{Y,j,t},\ x_{j,t})$ are the pairs of the price and the quantity of bonds when dealer j sells the bonds without and with a repo, respectively. Formally speaking, there should be such a constraint that $q_{Y,j,t}$ or $x_{j,t}$ must equal 0, because of the assumption that a young investor must choose whether to buy all of their bonds with a repo or not. This constraint is satisfied endogenously, as shown below.

Now suppose that the terms of trade between a dealer and a young investor in the next period, $Z^*_{Y,j,t+1}$, is independent of the dealer's state variables, $S_{j,t}$, as will be verified later. Given this conjecture, Equations (B.10), (B.15) and (B.16) imply that:

$$V^*_{a,t+1} = (1+\eta_{t+1})(d + p_{ID,t+1}), \tag{B.20}$$

$$V^*_{b,t+1} = (1+\eta_{t+1}), \tag{B.21}$$

$$V^*_{x,t+1} = (1+\eta_{t+1})(p_{ID,t+1} - p_{RP,j,t+1}) = (1+\eta_{t+1})0.5p_{ID,t+1}, \tag{B.22}$$

which verify the initial conjecture that dealers and investors take as given the values of $V^*_{a,t+1}$, $V^*_{b,t+1}$, and $V^*_{x,t+1}$.

The value of $V^*_{x,t+1}$ is positive because the interdealer bond price, $p_{ID,t+1}$, must be positive in equilibrium. Also, Equations (B.15) and (B.16) imply that $E_t p_{O,j',t+1} = E_t p_{RP,t+1}$. Given $V^*_{x,t+1} > 0$ and $E_t p_{O,j',t+1} = E_t p_{RP,t+1}$, the Nash-bargaining problem implies that $p_{Y,j,t} = q_{Y,j,t} = 0$, that is, the young investor and the dealer arrange a repo.

Substituting $p_{Y,j,t} = q_{Y,j,t} = 0$ into the Nash-bargaining problem (B.17)-(B.19) reduces the problem to:

$$\max_{\{p'_{Y,j,t},\, x_{j,t}\}} \left(p'_{Y,j,t} - p_{ID,t} + \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t}\right)^{0.5} \left[d + E_t p_{RP,j,t+1} - (1+r)p'_{Y,j,t}\right]^{0.5} x_{j,t}, \tag{B.23}$$

$$\text{s.t. } e_I - p'_{Y,j,t} x_{j,t} \geq 0, \tag{B.24}$$

$$x_{j,t} \geq 0. \tag{B.25}$$

Thus:

$$x_{j,t} \begin{cases} = 0, & \text{if } p_{ID,t} - \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t} > \frac{d + E_t p_{RP,j,t+1}}{1+r}, \\ \in \left[0, \frac{e_I}{p'_{Y,j,t}}\right], & \text{if } p_{ID,t} - \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t} = \frac{d + E_t p_{RP,j,t+1}}{1+r}, \\ = \frac{e_I}{p'_{Y,j,t}}, & \text{if } p_{ID,t} - \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t} < \frac{d + E_t p_{RP,j,t+1}}{1+r}. \end{cases} \tag{B.26}$$

Suppose that:

$$\eta_t = 0, \tag{B.27}$$

for all t, as will be verified at the end of the section. If $p_{ID,t} - \beta E_t V^*_{x,t+1}/(1+\eta_t) > (d + E_t p_{RP,j,t+1})/(1+r)$, then $p_{ID,t} > \beta(d + p_{ID,t+1}) = \beta V^*_{a,t+1}$, given Condition (6) and Equations (B.20) and (B.27). Thus, $a_{j,t} = 0$ for all j as implied by Equation (B.9). This result, however, would violate the market clearing condition (A.1), because $x_{j,t} = q_{Y,j,t} = 0$ for all j and $q_{BR,i,t} = 0$ for all i. Similarly, if $p_{ID,t} - \beta E_t V^*_{x,t+1}/(1+\eta_t) < (d + E_t p_{RP,j,t+1})/(1+r)$, then an arbitrary large value of $x_{j,t}$ for all j given an arbitrarily large value of e_I would violate the market clearing condition (A.1). Hence:

$$p_{ID,t} = \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t} + \frac{d + E_t p_{RP,j,t+1}}{1+r}. \tag{B.28}$$

Accordingly, the ask price of bonds with a repo for a young investor, $p'_{Y,j,t}$, satisfies:

$$p'_{Y,j,t} = \frac{d + E_t p_{RP,j,t+1}}{1+r}, \tag{B.29}$$

for any positive value of $x_{j,t}$.

Equations (B.20), (B.22) and (B.28) and $\eta_{t+1} = 0$ given Equation (B.27) jointly imply that:

$$p_{ID,t} = \frac{d}{0.5[1-\beta(1+r)] + r}, \tag{B.30}$$

$$\frac{E_t V^*_{a,t+1}}{p_{ID,t}} = 1.5 + r - 0.5\beta(1+r) < \beta^{-1}, \tag{B.31}$$

in the stationary equilibrium, in which $p_{ID,t}$ is constant for all t. The last inequality in Equation (B.31) follows from Condition (6). Given Equation (B.9), Equation (B.31) implies that dealers do not own bonds at the end of each period, i.e., $a_{j,t} = 0$. Thus, given that $a_{j,t} = q_{Y,j,t} = 0$ for all j and $q_{BR,i,t} = 0$ for all i, the market clearing condition (A.1) implies that $x_{j,t} = 1$ in the symmetric stationary equilibrium, in which $x_{j,t}$ takes the same value for all j.

Overall, the solution for the Nash-bargaining problem (B.17) is:

$$Z^*_{Y,j,t} = \left[0,\ 0,\ \frac{d + 0.5E_t p_{ID,t+1}}{1+r},\ 1\right]. \tag{B.32}$$

Backward induction with Equation (B.32) confirms the conjecture that $Z^*_{Y,j,t+1}$ is independent of $S_{j,t}$. Note that $Z^*_{O,j,t}$, $Z^*_{RP,j,t}$ and $Z^*_{Y,j,t}$ are independent of one another in equilibrium. Thus, the result of the model is insensitive to the order of the matches.

The result that $p_{Y,j,t} = q_{Y,j,t} = 0$ for all j (i.e., dealers and young investors arrange repos) implies that $Z^*_{O,j,t} = \mathbf{0}$ for all j because there is no old investor without a repo in each period. This result, in turn, leads to $f_{j,t} = 0$, because substituting $q_{Y,j,t} = q_{O,j,t} = 0$ and $q_{RP,j,t} = x_{j,t-1}$ into Equation (B.10) implies that all terms of the value function, V_t, are linear to the state variables, $a_{j,t-1}$, $b_{j,t-1}$ and $x_{j,t-1}$.

B.4 Dealers' consumption of cash

Equation (B.6) implies that dealers are indifferent to interdealer loans. Thus, $b_{j,t} = 0$ for all j to satisfy the market clearing condition (A.2) in the symmetric stationary equilibrium in which dealers are homogeneous. Substituting Equations (B.15), (B.16) and (B.32) and $a_{j,t} = b_{j,t} = 0$ into Equation (B.2) yields:

$$c_{j,t} = (p_{ID,t} - p_{RP,j,t})x_{j,t-1} - (p_{ID,t} - p'_{Y,j,t})x_{j,t} = \frac{0.5(1-\beta)d}{0.5[1-\beta(1+r)]+r} > 0, \tag{B.33}$$

in the stationary equilibrium, where the last inequality follows from Condition (6). Hence Equation (B.27) is confirmed as conjectured, given Equation (B.7). Equations (B.6), (B.21) and (B.27) imply that:

$$\beta(1 + rr_t) = 1. \tag{B.34}$$

C The Nash-bargaining problem between a dealer and a young investor in the equilibrium with a repo-market collapse in Section 6

To incorporate the possibility that an old investor with a repo does not return to the dealer for the repo, I introduce the following shock to $x_{j,t-1}$: substitute zero into $x_{j,t-1}$ for dealer j at the beginning of period t if an investor who arranges a repo with the dealer in period $t-1$ does not return to the dealer in period t. Given this shock to $x_{j,t-1}$ in period t, Equations (B.6)-(B.9) remain correct. A dealer's value function at the end of the period, $V_t(S_{j,t-1},\ Z_{Y,j,t},\ Z_{O,j,t},\ Z_{RP,j,t})$, is also the same as in Equation (B.10). I can guess and verify Equations (B.20)-(B.22) as shown in Appendix B.3.

Accordingly, the Nash-bargaining problem for a match between a dealer and a young investor is identical to the one defined by Equations (B.17)-(B.19). Then, the Nash-bargaining problem is reduced to the one defined by Equations (B.23)-(B.25), because the dealer and the young investor arrange a repo (i.e., $p_{Y,j,t} = q_{Y,j,t} = 0$) as described in Appendix B.3. The Nash-bargaining problem implies that the price of bonds with a repo, $p'_{Y,j,t}$, falls between the following range:

$$p'_{Y,j,t} \in \left[p_{ID,t} - \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t},\ \frac{d + E_t p_{RP,j,t+1}}{1+r} \right]. \tag{C.1}$$

If Equation (27) holds, then:

$$p_{ID,t} - \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t} < \frac{d + E_t p_{RP,j,t+1}}{1+r}. \tag{C.2}$$

Thus, there exists such a value of $p'_{Y,j,t}$ that both the young investor and the dealer are strictly better off than their outside options. Accordingly, the young investor puts all of their cash, e_I, on the repo that the investor arranges with the dealer. Hence:

$$x_{j,t} = \frac{e_I}{p'_{Y,j,t}}. \tag{C.3}$$

Given this value of $x_{j,t}$, the Nash-bargaining problem yields:

$$p'_{Y,j,t} = 0.5\left(p_{ID,t} - \frac{\beta E_t V^*_{x,t+1}}{1+\eta_t}\right) + 0.5\left(\frac{d + E_t p_{RP,j,t+1}}{1+r}\right). \tag{C.4}$$

D An equilibrium with a prolonged repo-market collapse

Suppose that the economy was in the symmetric stationary equilibrium with repos in period $t' - 1$. Given Condition (6), I describe a sufficient condition for the existence of such an equilibrium that every investor enters the brokered market unexpectedly from period t' to period T and that the economy returns to the stationary equilibrium with repos from period $T + 1$ onward.

From period t' to period T, each investor buys a bond from an old investor in the previous cohort when young, and resells the bond to a young investor in the next cohort when old. Thus each old investor in each period between t' and T has a unit of bonds at the beginning of the period, as in the case described in Section 6. Accordingly, the condition for each investor to enter the brokered market in period T is Equation (29) given Equations (27) and (32) for $t = T$.

Periods between t' and $T-1$ differ from period T only in that every investor enters the brokered market in the next period. For $t = t', t'+1, ..., T-1$, Equation (3) holds for the bond price in the brokered market, $p_{BR,t}$, rather than Equation (25). Equation (3), however, is equivalent to Equation (25), because $p_{BR,j,t} = p_{O,j,t} = 0.5p_{ID,t}$ for $t = t'+1, ..., T$ given Equation (26). Also, each young investor in each period between t' and $T-1$ would be able to commit to a repo if the investor entered the dealer market, because Equation (26) holds in the next period. Accordingly, Equation (28) is unchanged. Hence Equation (29) must hold for $t = t', t'+1, ..., T-1$, given that young investors enter the brokered market in each period between t' and $T-1$.

For $t = t', t'+1, ..., T-1$, Equation (26) holds if and only if:

$$p_{ID,t} = \frac{d + 0.5E_t p_{ID,t+1}}{1+r}, \tag{D.1}$$

as implied by Equation (3). Then, Equation (D.1) holds if:

$$1 + rr_t = \max\left\{\frac{d + E_t p_{ID,t+1}}{p_{ID,t}}, \ \frac{1}{\beta}\right\}. \tag{D.2}$$

The reason is the same as described for Equation (32) in Section 6.

Now I derive the conditions with which Equation (29) holds for $t = t', t'+1, ..., T$, given Equations (D.1) and (D.2) for $t = t', t'+1, ..., T-1$ and Equations (27) and (32) for $t = T$. For $t = t', t'+1, ..., T$, Equation (29) can be rewritten as:

$$e_I \leq f(t) \equiv \frac{(d + 0.5E_t P_{ID,t+1})(d + 0.75E_t P_{ID,t+1})}{0.5(1+r)E_t P_{ID,t+1}}. \tag{D.3}$$

Thus, the cash endowment for each young investor, e_I, must satisfies:

$$e_I \in \left(p_{ID,SS}, \min_{t=t',t'+1,...,T} f(t) \right], \tag{D.4}$$

to ensure the existence of the stationary equilibrium with repos as well as Equation (29) for $t = t', t' + 1, ..., T$.

If the T-bill rate, r, satisfies:

$$8r^2 + 8r - 1 < 0, \tag{D.5}$$

then I can show that:

$$f(t) < f(t + 1), \tag{D.6}$$

for $t = t', t' + 1, ..., T - 1$. Then, Equation (D.1) implies that:

$$\inf_{T=t',t'+1,...} \min_{t=t',t'+1,...,T} f(t) = \frac{(d + 0.5\hat{P})(d + 0.75\hat{P})}{0.5(1 + r)\hat{P}}, \tag{D.7}$$

where $\hat{P}$ is the limit of $p_{ID,t'}$ as $T \to \infty$:

$$\hat{P} \equiv \frac{d}{0.5 + r}. \tag{D.8}$$

Substituting Equations (D.7) and (D.8) into Condition (D.4) yields that, if r and e_I satisfy Condition (D.5) and:

$$e_I \in \left(p_{ID,SS}, \frac{(d + 0.5\hat{P})(d + 0.75\hat{P})}{0.5(1 + r)\hat{P}} \right], \tag{D.9}$$

then T can be an arbitrary integer not less than t'. The range for e_I in Condition (D.9) is non-empty if:

$$\beta(1+r) < \frac{(1+2r)(3+4r)}{5+4r}. \tag{D.10}$$

To satisfy this condition as well as $\beta(1+r) < 1$ as assumed in Condition (6), β must satisfy:

$$\beta(1+r) < \min\left\{1,\ \frac{(1+2r)(3+4r)}{5+4r}\right\}. \tag{D.11}$$

Figure 1: The bond market structure in the basic model

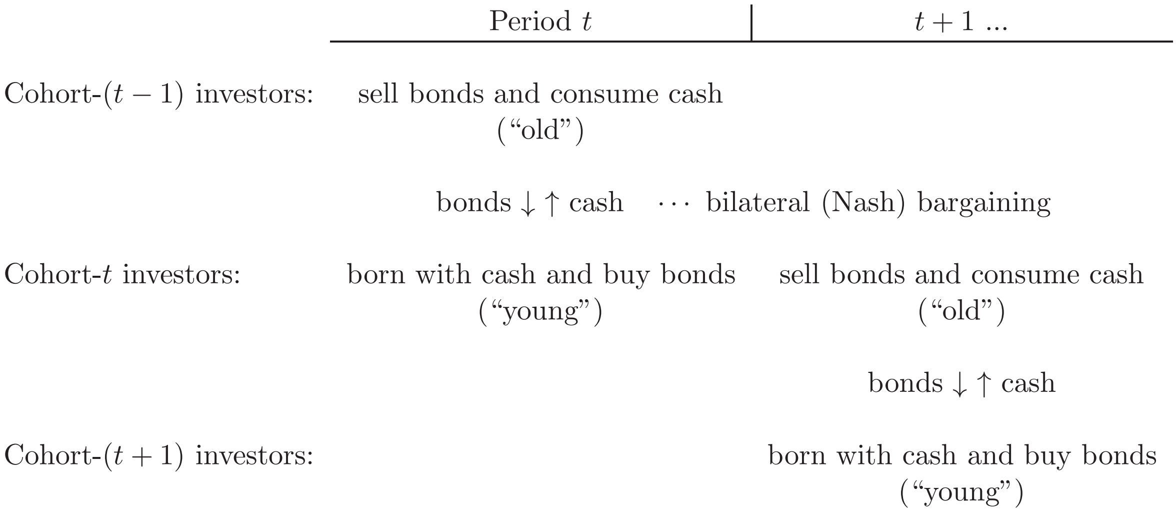

Figure 2: The bond market structure in the extended model

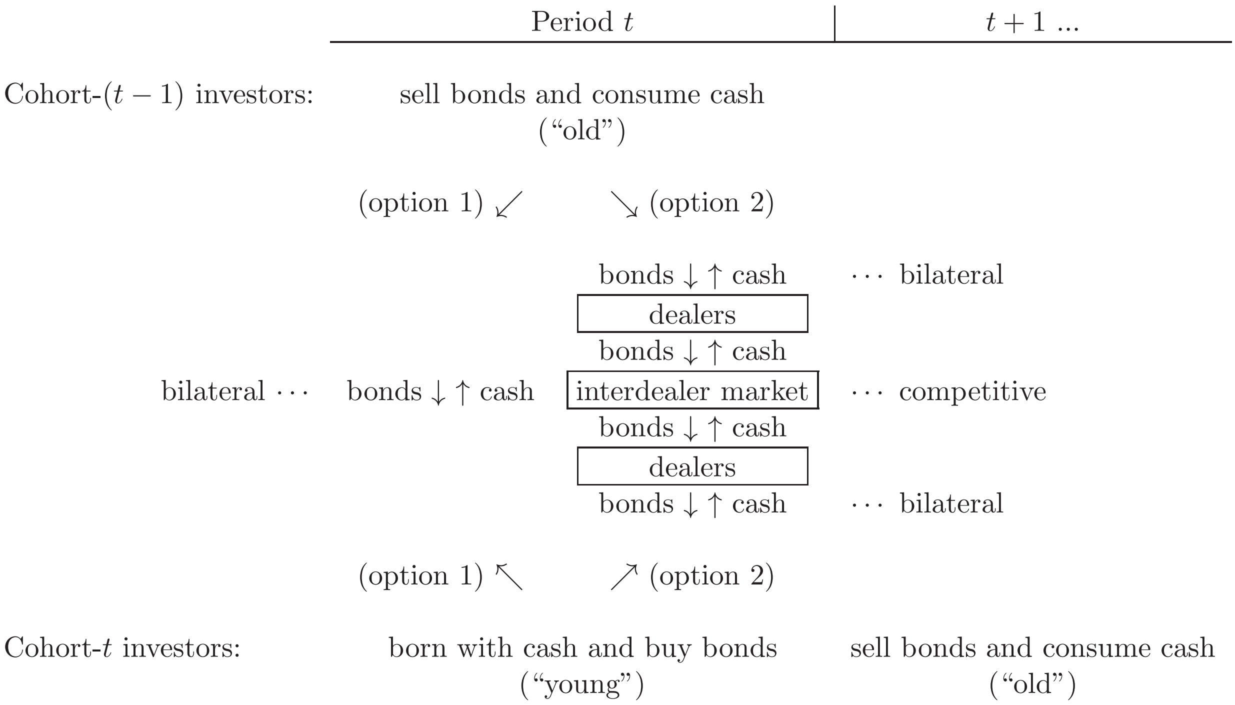

Figure 3: Details on bilateral matches between dealers and investors in Figure 2

Young investors choosing option 2

young investors
↑
(pairwise random matching in the dealer market)
↓
all dealers

Terms of trade in each match: the price and the quantity of bonds that the young investor buys, and whether to arrange a repo.

Old investors choosing option 2

old investors with repos	old investors without repos
↑	↑
(pre-specified matching between repo counterparties)	(pairwise random matching in the dealer market)
↓	↓
dealers with repos	all dealers

Term of trade in each match: the price and the quantity of bonds that the old investor sells.

Table 1: The notation of variables in period t

Variables	Definitions
	(Parameters)
β	the time discount factor for dealers
r	the interest rate on T-bills
d	dividends per bond every period
e_I	the cash endowment for each young investor
	(State variables for dealer j)
$a_{j,t}$	the amount of bonds the dealer owns at the end of period t
$b_{j,t}$	the net balance of interdealer loans at the end of period t
$x_{j,t}$	the amount of bonds sold to a young investor with a repo in period t
	(Terms of trade between dealer j and an investor)
$(p_{Y,j,t},\ q_{Y,j,t})$	the price-quantity pair for bonds sold to a young investor without a repo
$(p'_{Y,j,t},\ x_{j,t})$	the price-quantity pair for bonds sold to a young investor with a repo
$(p_{O,j,t},\ q_{O,j,t})$	the price-quantity pair for bonds bought from an old investor without a repo
$(p_{RP,j,t}, q_{RP,j,t})$	the price-quantity pair for bonds repurchased from an old investor with a repo
	(Other variables for dealer j)
$q_{ID,j,t}$	the net balance of bonds to trade in the interdealer market
$c_{j,t}$	consumption of cash
	(Competitive interdealer prices)
$p_{ID,t}$	the interdealer bond price
rr_t	the interdealer interest rate
	(Variables for investor i)
$p_{BR,t}$	the bond price in the brokered market (endogenously identical for all i in t)
$q_{BR,i,t}$	the quantity of bonds sold in the brokered market when old
$s_{i,t}$	the amount of cash invested in T-bills when young
$a_{i,t}$	the amount of bonds held at the end of period t when young
$c_{i,t}$	consumption of cash when old
	(Pre-determined variables for dealer j in the interdealer markets)
$S_{j,t}$	$S_{j,t} \equiv [a_{j,t},\ b_{j,t},\ x_{j,t}]$ (the endogenous state variables)
$Z_{Y,j,t}$	$Z_{Y,j,t} \equiv [p_{Y,j,t},\ q_{Y,j,t},\ p'_{Y,j,t},\ x_{j,t}]$ (the terms of trade with a young investor)
$Z_{O,j,t}$	$Z_{O,j,t} \equiv [p_{O,j,t},\ q_{O,j,t}]$ (the terms of trade with an old investor without a repo)
$Z_{RP,j,t}$	$Z_{RP,j,t} \equiv [p_{RP,j,t},\ q_{RP,j,t}]$ (the terms of trade with an old investor with a repo)
	(Functions for dealer j)
$Z^*_{k,j,t+1}(S_{j,t})$ for $k = Y, O, RP$	the value of $Z_{k,j,t+1}$ given $S_{j,t}$