The Diamond-Rajan Bank Runs in a Production Economy*

(Incomplete and preliminary)

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Abstract

To analyze the macroeconomic consequences of a systemic bank run, we integrate the banking model à la Diamond and Rajan (2001a) into a simplified version of an infinite-horizon neoclassical growth model. The banking sector intermediates the collateral-secured loans from households to entrepreneurs. The entrepreneurs also deposit their working capital in the banks. The systemic bank run, which is a sunspot phenomenon in this model, results in a deep recession through causing a sudden shortage of the working capital. We show that an increase in the probability of occurrence of the systemic run can persistently lower output, consumption, labor, capital and the asset price, even if the systemic run does not actually occur. This result implies that the slowdown of economic growth after the financial crises may be caused by the increased fragility of the banking system or the raised fears of recurrence of the systemic runs.

1 Introduction

We have experienced a severe systemic crisis in the global financial market in 2008–2009 and the vulnerable and slow economic recovery in the US and Europe afterwards. There

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are several ways to understand these events and formulate them in a formal economic model. For example, Gertler and Kiyotaki (2010) formulate the crisis as a large shock to the capital depreciation in the economy where banks have limited commitment ability; and Kurlat (2009) and Bigio (2010) model the crisis as a breakdown of the market for financial assets due to the adverse selection à la Akerlof's (1970) lemon problem. In this paper we model the crisis as a systemic bank run, and we hypothesize that the stagnant economic performance after the crisis is caused by the widespread fears of the recurrence of the systemic bank run. To analyze the macroeconomic consequences of the financial crises, we integrate the banking model à la Diamond and Rajan (2001a) into a variant of the Kiyotaki-Moore (1997) model. The banking sector intermediates the collateral-secured loans from households to entrepreneurs. The entrepreneurs also deposits working capital in the banks. The systemic bank run, which is a sunspot phenomenon in this model, results in a deep recession through causing a shortage of the working capital. We show in a version of our model where entrepreneurs accumulate capital that an increase in the probability of a systemic bank run causes a persistent recession and lowers the asset price, even if the systemic run does not actually occur. The contributions of this paper are as follows.

- We incorporate the Diamond-Rajan banks into an infinite horizon business cycle model in an essential way: that is, we translate the "demand for liquidity" in the Diamond-Rajan models into the "demand for working capital for production" in the business cycle models. The liquidity shortage in the banking models should represent disruptions in the payment activities in various economic transactions; and the frictions on payment is naturally modeled as financial constraints on the working capital for wage payment and/or purchase of intermediate goods in the macroeconomic models.[1] The view we put forward in this paper is that *the systemic bank run can cause a sudden shortage of working capital that leads to a severe*

[1] It is well known that if the working capital is subject to a borrowing constraint, financial frictions that tighten the constraint amplifies the economic downturn. See, for example, Jermann and Quadrini (2007), Kobayashi and Nutahara (2008), Kobayashi, Nakajima, and Inaba (2010), and Mendoza (2010).

declines of output.

- Our model with capital accumulation implies that the fragility of the financial system, which is translated into an increase in the probability (θ) of recurrence of another systemic bank run in the model, can be a primal cause of the slow economic growth, which has been observed in 2009–2010 in the US and Europe, and in the 1990s in Japan.

Literature: Uhlig (2009) models the 2008 global financial crisis as a systemic bank run. He constructs a two-period model based on Diamond and Dybvig (1983), which is quite different from the Diamond-Rajan framework. Angelloni and Faia (2010) is close to our model. They integrate the Diamond-Rajan banking sector in a standard DSGE model. The major difference is that the bank run in their model is idiosyncratic and there is no systemic runs, while we focus on the systemic banking crisis in which all banks are run on.

The organization of the paper is as follows. In the next section, we present and analyze the basic model, in which land works as a factor of production and collateral for bank loans. In Section 3, we analyze the model with both land and capital accumulation. In Section 4, we analyze the model with capital only. Section 5 provides concluding remarks.

2 The Model with Land

We first describe the financial contract (demand deposits) between banks and depositors, and then embed it in the general equilibrium model.

2.1 Demand Deposit

There are three agents in this economy: households, entrepreneurs, and banks. In this subsection, we consider a one-period financial contract between these agents. The banks raise funds from depositors (i.e., households and entrepreneurs) and lend the funds to the entrepreneurs at the end of each period $t-1$. The entrepreneurs borrow from the banks and also deposits the working capital in the banks. The bank loan and deposits are paid off at the end of next period t. We assume the following assumptions for a bank, a depositor (a household or an entrepreneur), and a borrowing entrepreneur.

Assumption 1 *An entrepreneur pledges her own land, a_{t-1}, as collateral when she borrows from other agents at the end of period $t-1$. If the lender is a bank, the bank has a relation-specific loan-collection skill that enables it to seize $\{r_t^a + q_t\}a_{t-1}$ units of the consumer goods from the borrower at the end of period t, where r_t^a is the return from the land and q_t is the land price. If the lender is a household or another entrepreneur, the lender can seize $z\{r_t^a + q_t\}a_{t-1}$ with $0 < z < 1$. The bank's loan-collection skill is relation-specific in that only the bank that originated the loan can collect $\{r_t^a + q_t\}a_{t-1}$ from the borrower, while the other banks can collect only $z\{r_t^a + q_t\}a_{t-1}$. The borrowing entrepreneur cannot commit to repay a predetermined amount to the lender and can walk away without any penalty except for seizure of the above mentioned amounts. The banks have no funds to lend and they need to borrow from the depositors (households and entrepreneurs) in order to lend funds to the entrepreneurs. The banks cannot commit to use their relation-specific skill on behalf of their depositors and the banks can walk away from the depositors in the middle of period t without any penalty, leaving the loan assets to the depositors. When a bank walks away the depositors (households and entrepreneurs) become the collective owner of the bank loans to the borrowing entrepreneurs.*

The banks are the sole lenders to the entrepreneurs: Under this assumption, a borrowing entrepreneur cannot commit to repay a prespecified amount but can pledge a collateral, a_{t-1}, for the debt. Therefore, the entrepreneur is subject to the collateral constraint. We assume and justify later in the general equilibrium model that the collateral constraint is binding in equilibrium. Given that the collateral constraint is binding, the entrepreneur wants to borrow as much as possible. The banks can offer a strictly greater amount of funds to lend to an entrepreneur who has a_{t-1} than the other agents (households and other entrepreneurs) can, because the banks have superior loan-collection skill: if a household or an entrepreneur offers to lend B to the borrowing entrepreneur, a bank can offer to lend $z^{-1}B$ ($> B$) to the same borrower. Given that the borrower's collateral constraint binds, the borrower always choose to borrow from the banks, not from the other agents. Therefore, the banks become sole lenders in this economy as a result of lending competition among the banks and the other agents (households and entrepreneurs).

The banks cannot raise funds without issuing demandable debt: Since the banks have no funds to lend at the end of period $t-1$, they need to raise funds from the households and the entrepreneurs. It is shown, however, as follows that it is impossible for a bank to raise funds unless it issues demandable debts. Suppose that the bank raises a debt B, which is not demandable, from households and entrepreneurs and the bank lends it to an entrepreneur. Suppose also that the bank can collect C from the borrower using the relation-specific loan-collection skill. We assume that

$$\frac{C}{B} \geq 1 + r_t^m > \{z + (1-z)x\}\frac{C}{B}, \tag{1}$$

where x ($0 < x < 1$) is the parameter that represent the depositors' bargaining power (see below) and r_t^m is the risk-free rate of interest.[2] We assume that x is sufficiently small. By Assumption 1, the bank depositors (households and entrepreneurs) can collect at most zC if the bank walks away without collecting on the loan and they recover the loan by themselves. Since the bank cannot precommit to use the relation-specific loan collection skill on behalf of the depositors, it is *ex post* rational for the bank after making the loan to initiate a following renegotiation with the depositors: the bank offers to pay $\{z + (1-z)x\}C$ ($< C$) to the depositors and says that he will walk away leaving the loan assets to the depositors if they do not accept this offer. Since the bank can collect C and the depositors can collect zC, this offer means to split the surplus $(1-z)C$ between the bank and the depositors, according to the Nash bargaining between them,

[2] The risk-free rate is determined in the general equilibrium. Although we do not explicitly consider it in the following sections, we can introduce the market for the real government bonds in our model and define the risk-free rate as the interest rate for the government bonds.

while the depositors' bargaining power is x and the bank's bargaining power is $1 - x$. The depositors have no other choice than to accept the bank's offer $\{z + (1 - z)x\}C$, because the bank's relation-specific skill is necessary to generate the surplus $(1 - z)C$. Anticipating that the bank will initiate the renegotiation in the middle of period t, the households and the entrepreneurs do not deposits their funds in the bank at the end of period $t-1$, because (1) implies that the return on the bank deposit, $\{z+(1-z)x\}\frac{C}{B}$, is lower than the risk-free rate of interest, $1+r_t^m$. By issuing demand deposit, the banks can credibly commit to use their loan-collection skill on behalf of depositors and successfully raise funds.

Demand deposit as a commitment device: The demand deposit contract is a contract that gives the depositor who deposits the fund at the end of period $t - 1$ the unilateral right to withdraw a predetermined amount, C, at anytime in period t. The demand deposit contract has the following features:

- One bank issues the demand deposits to many depositors simultaneously.
- If a depositor withdraws at the end of period t, the bank pays the depositor C units of the consumer goods.
- If a depositor withdraws in period t before the consumer goods are produced, the bank gives C/κ units of the loan asset to the withdrawer as long as the bank asset remains, where κ is the recovery rate for the depositor in the case where the depositor directly recover the loan from the borrower. κ is defined as $\kappa = z+(1-z)x$ and therefore $0 < \kappa < 1$.[3]
- If many depositors withdraw before production of the consumer goods and the bank runs out of the loan asset, the remaining depositors get nothing. This is the *first-come, first-served principle.*

[3]See the bargaining process described in the proof of Lemma 1. It is shown that the depositors obtain κC, not zC, from the borrowing entrepreneur.

Demand deposit makes the banks credibly commit to pay the promised amount of the deposits and not to renegotiate down the payment. This is because if a bank tries to renegotiate with the depositors they immediately run on the bank and the bank ends up getting zero as a result of the bank run. We show this result by similar argument as Diamond and Rajan (2001a, 2001b).

Lemma 1 *If a bank tries to renegotiate with the depositors, they all run on the bank immediately and the bank ends up getting zero as a result of the bank run.*

(Proof) If a bank initiates the renegotiation with the depositors to reduce the payment, the dominant strategy for the depositors is to unilaterally withdraw the predetermined amount of the deposit. (See pp. 309–313 of Diamond and Rajan 2001a.) When a bank run occurs, the ownership of the bank loan is transferred to the depositors. The depositors who successfully withdrew become a collective owner of the bank loan. The depositors can collect $z\{r_t^a + q_t\}a_{t-1}$ by themselves if they directly collect the loan from the borrowing entrepreneur, while the bank can collect $\{r_t^a + q_t\}a_{t-1}$. After the bank run, the depositors collectively decide whether they directly collect the loan from the entrepreneur or they hire the original bank again and make him collect the loan on behalf of the depositors. It is easily shown as follows that the depositors decide not to hire the bank and that the rent that the bank can get is zero.

- Suppose that the depositors hire the original bank. The bank takes $\{r_t^a + q_t\}a_{t-1}$ from the entrepreneur. Since the surplus $(1-z)\{r_t^a + q_t\}a_{t-1}$ must be divided between the bank and the depositors with the Nash bargaining, the bank offers the depositors the payment of $\{z + (1-z)x\}\{r_t^a + q_t\}a_{t-1}$.

- In order to prevent the depositors from hiring the bank, the borrowing entrepreneur offers to pay $\{z + (1-z)x\}\{r_t^a + q_t\}a_{t-1} + \varepsilon$ directly to the depositors, where ε (> 0) is an infinitesimally small amount. If this offer is accepted by the depositors the entrepreneur pays $\{z + (1-z)x\}\{r_t^a + q_t\}a_{t-1} + \varepsilon$ to the depositors, while if the bank is hired by the depositors the entrepreneur must pay $\{r_t^a + q_t\}a_{t-1}$. Obviously the entrepreneur is better off by preventing the depositors from hiring the bank.

- The depositors accept the entrepreneur's offer and never hire the bank again after the bank run.[4]

[4]This result does not depend on the protocol of the Nash bargaining between the bank and the

Therefore, the bank can get no rent after the bank run. (End of Proof)

The demand deposit contract enables banks to credibly commit to use their human capital on behalf of the depositors, and it enables the banks to act as the intermediary between the households and the entrepreneurs. In the meanwhile, the demand deposit makes the banking system susceptible to the systemic bank run, because the depositors run on the banks and withdraw the deposits unilaterally in response to an adverse macroeconomic shock or a sunspot shock as we see in the following sections.

Simplification of the first-come, first-served principle: Before describing the general equilibrium model, we make the following assumption to simplify the analysis of the equilibrium. The first-come, first-served (FCFS) principle is essential in Diamond and Rajan (2001a) to derive the result that the unconditional withdrawal is the dominant strategy for the depositors when the bank initiates the renegotiation. The FCFS principle divides the depositors into two groups, i.e., the successful withdrawer and the unsuccessful withdrawer, when the bank run occurs. The heterogeneity of the depositors makes the analysis complicated. To avoid the complication, we adopt Allen and Gale's (1998) simplifying assumption for the depositors' payoff in the bank run.

Assumption 2 *All depositors divides the bank assets pro rata basis when the bank run occurs. Therefore, a depositor who has the right to withdraw D_i can seize ξD_i during the bank run, where ξ is determined as an equilibrium outcome and identical for all depositors in the bank.*

The pro rata payoff is realized if, for example, the depositors in one bank form a fair insurance contingent on the bank run.[5] Due to this assumption, we can analyze the macroeconomic variables assuming that the households and the entrepreneurs are iden-

depositors, which may take place after the bank run. Given any division of the surplus between the bank and the depositors, the depositors and the borrowing entrepreneurs can better off by dividing the bank's surplus between themselves. Therefore, the entrepreneur can always make an offer to the depositors that leads them to decide not to hire the original bank.

[5] The insurance contract should be such that a depositor is eligible for the insurance only if he does not make concession to the bank and tries to withdraw his deposit unilaterally.

tical, respectively, after the bank run. Although we assume this simplifying assumption, which is not rigorously consistent with Lemma 1, we are confident that the following analysis of the general equilibrium model does not change qualitatively even without Assumption 2.

2.2 The Environment

The economy is closed and time is discrete: $t = 0, 1, 2, \cdots$. There are three agents: households; entrepreneurs; and banks. The measures of these agents are normalized to one, respectively. Households and entrepreneurs live for infinite periods. Banks are one-period lived. At the end of period $t-1$, the banks are born, accept deposits from households and entrepreneurs, and make loans to entrepreneurs. If there is no bank run in period t, they collect repayment of loans from the borrowing entrepreneurs at the end of period t, then payout depositors, and die. If a bank run occurs in period t, the banks just walk away from the market leaving the loan assets to the depositors, and die at the end of the period. There are three goods traded:

- Land, a_t. Only entrepreneurs can own and operate land. Land is pledgeable as collateral for bank loans. Land is nondepletable and productive. Total supply of land is fixed: $a_t = 1$, $\forall t$. The holding of land a_t incurs the maintenance cost $\chi(a_t)$ to the owner-entrepreneur in period t.

- Labor, l_t. Only households can provide labor input to the entrepreneurs. The labor supply l_t incurs the disutility $\gamma(l_t)$ to the households in period t.

- Consumer goods, y_t. Only entrepreneurs can produce the consumer goods from land and labor, by the Cobb-Douglas technology.

$$y_t = A a_{t-1}^{\alpha} l_t^{1-\alpha}.$$

The households' utility is

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \{c_t - h_t - \gamma(l_t)\} \right],$$

where c_t is the consumption and h_t is the disutility due to the following backyard production: We assume for simplicity of the analysis that the households and entrepreneurs can produce h_t units of the consumer goods in their backyards, incurring h_t units of disutility contemporaneously. The entrepreneurs' utility is

$$E_0\left[\sum_{t=0}^{\infty}(\beta')^t\{c_t^E - h_t^E\}\right], \tag{2}$$

where c_t^E is the consumption and h_t^E is the disutility due to the backyard production. We assume that the households' discount factor, β, is larger than the entrepreneurs' one, β'. (Thus households are patient and the entrepreneurs are impatient.)

A bank makes loan to an entrepreneur taking the land as collateral. If the borrowing entrepreneur repudiates the repayment in period t, the bank can seize $\{r_t^a(s_t) + q_t(s_t)\}a_{t-1}$, where a_{t-1} is the land that the borrowing entrepreneur owns, r_t^a is the return from the land, and q_t is the land price. Note that r_t^a and q_t may vary depending on the realization of the sunspot variable, s_t, which is defined below.[6] If this loan is transferred to some other agent, the agent who acquires the loan can collect (at most) $\kappa\{r_t^a(s_t) + q_t(s_t)\}a_{t-1}$, where

$$0 < \kappa = z + (1-z)x < 1.$$

A borrowing entrepreneur cannot precommit to repay the debt. When the entrepreneur repudiates the debt, the collateral is (partially) seized by the creditor but there is no additional penalty for the repudiation. The value seized by the creditor is $\{r_t^a(s_t) + q_t(s_t)\}a_{t-1}$ if the creditor is the bank that originated the loan and $\kappa\{r_t^a(s_t) + q_t(s_t)\}a_{t-1}$ otherwise. We assume that there are a sunspot variable s_t, where

$$s_t = \begin{cases} e \;\; \text{(emergency)} & \text{with probability } \;\theta, \\ n \;\; \text{(normal time)} & \text{with probability } \;1-\theta. \end{cases}$$

[6]In this paper we focus on the stationary equilibrium path in which the macroeconomic variables, e.g., r_t^a and q_t, are time-invariant and depend only on the realization of s_t. Nevertheless, we put the subscript t on the variables in the following analysis in order to distinguish the variables at date t from those at other dates. Therefore, we denote a macroeconomic variable x in period t as $x_t(s_t)$.

The variable s_t is revealed at the beginning of period t. The macroeconomic variables, e.g., $r_t^a(s_t)$, depend on s_t.

Assumption 3 *The agents in this economy expect that $r_t^a(e)$ is much smaller than $r_t^a(n)$*

Due to this expectation, the systemic bank run occurs in equilibrium in the state where $s_t = e$. As a result of the systemic bank run, the expectation that $r_t^a = r_t^a(e)$ is smaller than $r_t^a(n)$ is justified in equilibrium. See Section 2.6 for the details.

Working capital for wage payment: The entrepreneurs need to buy labor input from the households in order to produce the consumer goods. If the entrepreneurs could commit to pay wages, they could have used the labor input just by promising to pay the wages afterwards and they could have actually paid wages in the form of the consumer goods after the production. We assume, however, the following assumption.

Assumption 4 *The worker-households cannot impose any penalty after production of the consumer goods to the entrepreneurs who break their promise to pay wages.*

This assumption makes the entrepreneurs unable to commit beforehand to pay wages to the worker-households after production Because of this lack of commitment, the entrepreneurs must pay wages before production in the form of credible claims, which are the bank deposits or the collateral-secured loans to (other) entrepreneurs. At the end of period $t-1$, the entrepreneurs choose to hold a certain amount of bank deposits for the wage payment in period t. If the bank run does not occur at the beginning of period t, the entrepreneurs pay wages in the form of bank deposits. If the bank run occurs, the banks walk away and the depositor-entrepreneurs are left with the loan assets that the banks originated at the end of period $t-1$. As the banks walk away, the value of the loan assets decreases to $\kappa\{r_t^a(e) + q_t(e)\}a_{t-1}$, which is the value that the depositors can recover after the bank run; and the depositor-entrepreneurs pay wages by transferring the loan assets, the values of which are less than the original bank deposits, directly to the worker-households.

2.3 Timing of events

The timing of events during the representative period t is as follows.

- At the beginning of period t:

  The households carry over the bank deposit, $(1 + r_{t-1})d^H_{t-1}$, where d^H_{t-1} is the amount of deposit made at the end of period $t-1$ and r_{t-1} is the deposit rate from $t-1$ to t. The entrepreneurs carry over the bank deposit, $(1 + r_{t-1})d^E_{t-1}$, and the land, a_{t-1}, as their assets, while a_{t-1} is pledged as collateral for the bank loan, b_{t-1}, that they borrowed at the end of the previous period $t-1$. The banks carry over the deposits, $(1+r_{t-1})d_{t-1}$, as their liabilities, where $d_{t-1} = d^H_{t-1} + d^E_{t-1}$. The sunspot variable, $s_t \in \{n, e\}$, is revealed. If $s_t = n$, the agents expect that $r^a_t = r^a_t(n)$ is large and there is no bank run. If $s_t = e$, the agents expect that $r^a_t = r^a_t(e)$ is strictly smaller than $r^a_t(n)$. In this case, the systemic bank run occurs as an equilibrium outcome (see Section 2.6) and the banks walk away from the market leaving the loan assets to the depositors. As a result of the bank run, the value of the loan assets becomes $\kappa\{r^a_t + q_t\}a_{t-1}$, which is the amount that the depositors can recover from the borrowers without the banker's help; and the bank deposits become direct claims of the loans to entrepreneurs, the value of which are $\xi_t(1+r_{t-1})d^H_{t-1}$ for the households and $\xi_t(1+r_{t-1})d^E_{t-1}$ for the entrepreneurs, where ξ_t (< 1) is the recovery rate of the bank deposits. Note that ξ_t is identical for all depositors due to Assumption 2.

- In the middle of period t:

  The households choose the labor supply, l_t. They sell l_t to the entrepreneurs at the wage rate w_t. If $s_t = n$, the payment of $w_t l_t$ is done by transfer of the bank deposits ($\{1 + r_{t-1}\}d^E_{t-1}$) from the entrepreneurs to the households. If $s_t = e$, the payment of $w_t l_t$ is done by transfer of the loan assets ($\xi_t\{1 + r_{t-1}\}d^E_{t-1}$) from the entrepreneurs to the households. The entrepreneurs produce the consumer goods ($Aa^\alpha_{t-1} l_t(s_t)^{1-\alpha}$) from the land a_{t-1} and the labor $l_t(s_t)$.

- At the end of period t:

The consumer goods market and the asset market open. The households choose the consumption, c_t , and the backyard production, h_t. They withdraw the bank deposits $(1+r_{t-1})(d^H_{t-1}+d^E_{t-1})$ if $s_t = n$, or collect the loans $\xi_t(1+r_{t-1})(d^H_{t-1}+d^E_{t-1})$ from the borrowing entrepreneurs directly if $s_t = e$.[7] They make new deposits, d^H_t, that they carry over to the next period. The entrepreneurs choose the consumption, c^E_t, and the backyard production, h^E_t. They sell land a_{t-1}, and repay $\{r^a_t + q_t\}a_{t-1}$ if $s_t = n$, or they repay a small part of the bank loans $\kappa\{r^a_t + q_t\}a_{t-1}$ if $s_t = e$. The entrepreneurs make new deposits, d^E_t, borrow new bank debts, b_t, and buy the land, a_t, that they carry over to the next period. The banks collect the loans $(r^a_t + q_t)a_{t-1}$, payout the deposits $(1 + r_{t-1})d_{t-1}$, eat any remaining profit, and die if $s_t = n$. (If $s_t = e$, the banks just die.) The new banks are born and they accept deposits, $d_t = d^H_t + d^E_t$, from the households and the entrepreneurs, and make loans b_t to the entrepreneurs.

2.4 Optimization Problems

There is only one stochastic variable, $s_t \in \{n, e\}$, which is a sunspot variable.

Household:

$$\max_{c_t,h_t,d^H_t,l_t} E_0\left[\sum_{t=0}^{\infty}\beta^t\{c_t - h_t - \gamma(l_t)\}\right],$$
$$\text{s.t.}\quad c_t(s_t) + d^H_t(s_t) = \xi_t(s_t)(1 + r_{t-1})d^H_{t-1} + w_t(s_t)l_t(s_t) + h_t(s_t),$$

where $\xi_t(s_t)$ is the recovery rate of the deposits, which is

$$\xi_t(s_t) = \begin{cases} 1 & \text{if the bank run does not occur,} \\ \xi_t\ (< 1) & \text{if the bank run occurs,} \end{cases}$$

where the value of ξ_t is determined in equilibrium. It is shown in Section 2.6 that the bank run occurs if $s_t = e$ and it does not occur if $s_t = n$. The first-order conditions

[7] Note that in the middle of period t the households obtain $(1 + r_{t-1})d^E_{t-1}$ units of the bank deposits if $s_t = n$ or the same amount of the loans originated by the banks if $s_t = e$, as the wage payment. The loan with the face value of $(1 + r_{t-1})d^E_{t-1}$ has the market value of $\xi_t(1 + r_{t-1})d^E_{t-1}$ for the households.

(FOCs) for the household's problem imply

$$\gamma'(l_t(s_t)) = w_t(s_t), \tag{3}$$

$$1 = \beta\{(1-\theta)\xi_{t+1}(n) + \theta\xi_{t+1}(e)\}(1 + r_t(s_t)). \tag{4}$$

Obviously, (4) implies that the interest rate does not depend on $s_t \in \{n, e\}$[8]:

$$r_t(n) = r_t(e) = r_t. \tag{5}$$

Entrepreneur: Given that $0 < \beta' < \beta < 1$, the entrepreneurs solve the following problem.

$$\max_{c_t^E, h_t^E, d_t^E, l_t, a_t, b_t} E_0 \left[\sum_{t=0}^{\infty} (\beta')^t \{c_t^E - h_t^E\} \right], \tag{6}$$

$$\begin{aligned} \text{s.t.} \quad & q_t(s_t)a_t(s_t) + d_t^E(s_t) - b_t(s_t) + c_t^E(s_t) \\ & \quad = A a_{t-1}^{\alpha} l_t(s_t)^{1-\alpha} + q_t(s_t)a_{t-1} - w_t(s_t)l_t(s_t) + \xi_t(s_t)(1 + r_{t-1})d_{t-1}^E \\ & \qquad - \kappa_t(s_t)\{r_t^a(s_t) + q_t(s_t)\}a_{t-1} - \chi(a_t(s_t)) + h_t^E(s_t), \qquad (7) \\ & w_t(s_t)l_t(s_t) \leq \xi_t(s_t)(1 + r_{t-1})d_{t-1}^E, \qquad (8) \\ & b_t(s_t) \leq B_t(a_t, s_t), \qquad (9) \end{aligned}$$

where $\kappa_t(s_t)$ is the recovery rate of the bank loan, $r_t^a(s_t) = A a_{t-1}^{\alpha} l_t(s_t)^{1-\alpha} - w_t(s_t)l_t(s_t)$ and in equilibrium

$$r_t^a(s_t) = \alpha A \left(\frac{l_t(s_t)}{a_{t-1}} \right)^{1-\alpha}, \tag{10}$$

and $B_t(a_t, s_t)$ is the debt capacity for the entrepreneur, which is determined as a solution to the bank's optimization problem. The recovery rate $\kappa_t(s_t)$ takes on the following values:

$$\kappa_t(s_t) = \begin{cases} 1 & \text{if the bank run does not occur,} \\ \kappa & \text{if the bank run occurs,} \end{cases}$$

It is shown in Section 2.6 that the bank run occurs if $s_t = e$ and it does not occur if $s_t = n$.

[8]Note that $\xi_{t+1}(s_{t+1}) \in \{\xi_{t+1}(n), \xi_{t+1}(e)\}$ does not depend on the realization of s_t.

Bank: Given the deposit rate r_t and the amount of collateral a_t of a borrowing entrepreneur, a bank maximizes the expected profit from a loan to the entrepreneur. Note that r_t is fixed at t and do not depend on the realization of s_{t+1}.

$$\begin{aligned}&\max_{b_t, d_t} E_t \pi^B_{t+1} \\ &= (1-\theta)\max\{[r^a_{t+1}(n) + q_{t+1}(n)]a_t - (1+r_t)d_t,\ 0\} \\ &\ + \theta \max\{[r^a_{t+1}(e) + q_{t+1}(e)]a_t - (1+r_t)d_t,\ 0\}, \end{aligned} \tag{11}$$

subject to

$$b_t = d_t, \tag{12}$$

$$b_t \geq B_t(a_t, s_t), \tag{13}$$

where $B_t(a_t, s_t)$ is the lower limit of the bank loans to a borrower who pledges a_t as collateral. The value of $B_t(a_t, s_t)$ is determined in equilibrium as a result of the competition among banks. The competition among banks drives $E_t\pi^B_{t+1}$ to zero, which implies that $B_t(a_t, s_t)$ does not depend on s_t and

$$B_t(a_t, s_t) = B_t(a_t) \equiv \frac{1}{1+r_t}\max\{r^a_{t+1}(n) + q_{t+1}(n),\ r^a_{t+1}(e) + q_{t+1}(e)\}. \tag{14}$$

2.5 Equilibrium conditions

The recovery rate of deposits during the bank run, ξ_t, is determined by

$$\xi_t(s_t) = \min\left\{1,\ \kappa_t(s_t)\frac{\{r^a_t(s_t) + q_t(s_t)\}a_t}{(1+r_{t-1})d_{t-1}}\right\}, \tag{15}$$

where d_{t-1} is the total amount of deposits in a bank and a_t is the total amount of collateral assets for the bank. The market clearing conditions are

$$a_t(s_t) = 1, \tag{16}$$

$$c_t(s_t) - h_t(s_t) + c^E_t(s_t) - h^E_t(s_t) = A a^\alpha_{t-1} l_t(s_t)^{1-\alpha}, \tag{17}$$

$$b_t(s_t) = d_t(s_t)\ (\equiv d^H_t(s_t) + d^E_t(s_t)). \tag{18}$$

2.6 Dynamics

Definition of the Competitive Equilibrium: *The competitive equilibrium is a set of prices,* $\{r_t(s_t), \kappa_t(s_t), \xi_t(s_t), q_t(s_t), w_t(s_t), r_t^a(s_t)\}$*, and quantities,* $\{a_t(s_t), c_t(s_t), c_t^E(s_t), h_t(s_t), h_t^E(s_t), l_t(s_t)\}$*, such that (i) given the prices, the quantities are the solution to the optimization problems of households, entrepreneurs, and banks; and (ii) the market clearing conditions are satisfied.*

To analyze the dynamics, we first check the FOCs for the entrepreneur's problem:

$$(1-\alpha)A\left(\frac{a_{t-1}}{l_t(s_t)}\right)^{\alpha} = w_t(s_t)(1+\mu_t(s_t)), \tag{19}$$

$$1 = \eta_t(s_t), \tag{20}$$

$$1 = \beta'[(1-\theta)\xi_{t+1}(n)\{1+\mu_{t+1}(n)\} + \theta\xi_{t+1}(e)\{1+\mu_{t+1}(e)\}](1+r_t), \tag{21}$$

$$\chi'(a_t(s_t)) + q_t(s_t) = (1-\kappa)\beta'\theta[r_{t+1}^a(e) + q_{t+1}(e)] + \eta_t(s_t)B_t'(a_t), \tag{22}$$

where $\mu_t(s_t)$ and $\eta_t(s_t)$ are the Lagrange multipliers for (8) and (9), respectively. Then, (20) implies that $\eta_t(n) = \eta_t(e) = 1$. Similarly, (22) and $a_t(s_t) = 1$ imply that the equilibrium asset price does not depend on the sunspot variable: $q_t(n) = q_t(e) \equiv q_t$. Summarizing the above arguments, we obtain the following lemma:

Lemma 2 *The variables* $\{r_t, q_t, \eta_t\}$ *do not depend on the realization of* s_t*.*

Since we assumed that $r_{t+1}^a(n) > r_{t+1}^a(e)$ in Assumption 3, this lemma and (14) implies that

$$B_t(a_t) = \frac{r_{t+1}^a(n) + q_{t+1}}{1+r_t}a_t. \tag{23}$$

Therefore, $d_t = b_t = B_t(a_t)$ and $B_t'(a_t) = \{r_{t+1}^a(n) + q_{t+1}\}/(1+r_t)$.

Condition for the Bank Run: The above results imply that in the state where $s_t = n$, the value of the bank asset is $\{r_t^a(n) + q_t\}a_{t-1}$ and the value of the bank liability (i.e., deposits) is $(1+r_{t-1})d_{t-1} = (1+r_{t-1})B_{t-1}(a_{t-1}) = \{r_t^a(n)+q_t\}a_{t-1}$. Therefore, the bank is solvent in the state $s_t = n$, and the bank run does not occur in this state. On the other hand, in the state where $s_t = e$, the value of the bank asset is $\{r_t^a(e) + q_t\}a_{t-1}$ and the value of the bank liability is $\{r_t^a(n) + q_t\}a_{t-1}$. Assumption 3 implies that the bank is insolvent in this state, and therefore the bank run occurs in the state where $s_t = e$. The recovery rate of the bank loan is thus given by

$$\kappa_t(s_t) = \begin{cases} 1 & \text{if } s_t = n, \\ \kappa & \text{if } s_t = e, \end{cases}$$

and the recovery rate of the bank deposit is given by

$$\xi_t(s_t) = \begin{cases} 1 & \text{if } s_t = n, \\ \xi_t & \text{if } s_t = e, \end{cases}$$

where

$$\xi_t = \kappa \frac{r_t^a(e) + q_t}{r_t^a(n) + q_t}. \tag{24}$$

The liquidity constraint (8) implies that

$$\xi_t w_t(n) l_t(n) = w_t(e) l_t(e). \tag{25}$$

The equilibrium path is determined as a sequence $\{a_t, r_t, q_t, w_t(s_t), r_t^a(s_t), l_t(s_t), \xi_t, \mu_t(s_t)\}_{t=0}^{\infty}$,[9] which satisfies (3), (10), (16), (19), (24), (25), and

$$\begin{aligned} 1 &= \beta\{1 - \theta + \xi_{t+1}\theta\}(1 + r_t), \\ 1 &= \beta'[(1 - \theta)\{1 + \mu_{t+1}(n)\} + \theta\xi_{t+1}\{1 + \mu_{t+1}(e)\}](1 + r_t), \\ \chi'(1) + q_t &= (1 - \kappa)\beta'\theta[r_{t+1}^a(e) + q_{t+1}] + \frac{r_{t+1}^a(n) + q_{t+1}}{1 + r_t}. \end{aligned}$$

The equilibrium path must also satisfy the transversality condition:

$$\lim_{t \to \infty} (\beta')^t q_t = 0.$$

2.7 Stationary Equilibrium

Since the state variable in this model is a_{t-1} and it is time-invariant, there exists an equilibrium path, along which the prices and quantities are all time-invariant. We focus on this stationary equilibrium.[10] In the stationary equilibrium, the macroeconomic variables

[9] The remaining variables $\{c_t(s_t), c_t^E(s_t), h_t(s_t), h_t^E(s_t)\}$ are determined by the resource constraints and the non-negativity constraints of respective variables.

[10] The macroeconomic variables in the equilibrium path may vary over time if the initial value of q_t is different from its value in the stationary equilibrium. We do not consider these cases in this paper.

are time-invariant and depend only on the realization of $s_t \in \{n, e\}$. Given θ, the stationary equilibrium is specified by the set of variables $\{r, q, w(n), w(e), r^a(n), r^a(e), l(n), l(e), \xi, \mu(n), \mu(e)\}$, which solves the following system of equations.

$$1 = \beta\{1 - (1 - \xi)\theta\}(1 + r), \tag{26}$$

$$1 = \beta'[(1 - \theta)\{1 + \mu(n)\} + \theta\xi\{1 + \mu(e)\}](1 + r), \tag{27}$$

$$(1 - \alpha)Al(n)^{-\alpha} = w(n)\{1 + \mu(n)\}, \tag{28}$$

$$(1 - \alpha)Al(e)^{-\alpha} = w(e)\{1 + \mu(e)\}, \tag{29}$$

$$\xi = \kappa\frac{r^a(e) + q}{r^a(n) + q}, \tag{30}$$

$$\chi'(1) + q = (1 - \kappa)\beta'\theta[r^a(e) + q] + \frac{r^a(n) + q}{1 + r}, \tag{31}$$

$$r^a(n) = \alpha Al(n)^{1-\alpha}, \tag{32}$$

$$r^a(e) = \alpha Al(e)^{1-\alpha}, \tag{33}$$

$$\xi w(n)l(n) = w(e)l(e), \tag{34}$$

$$\gamma'(l(n)) = w(n), \tag{35}$$

$$\gamma'(l(e)) = w(e). \tag{36}$$

We show the existence of the steady-state equilibrium by solving the system of equations (26)–(36) numerically. For numerical calculation, we specify the functional forms as follows:

$$\gamma(l_t) = -\psi\frac{(1 - l_t)^\sigma}{1 - \sigma},$$
$$\chi(a_t) = \frac{\phi}{2}a_t^2.$$

Figure 1 plots the variables corresponding to each value of θ. It is confirmed that output, labor and consumption are smaller when $s_t = e$ than when $s_t = n$. The economy falls in the severe recession when the bank run occurs ($s_t = e$), because of the shortage of the liquidity for wage payment. A counterintuitive feature of Figure 1 is that in the state where $s_t = n$, the output, labor and consumption are slightly increasing in θ.[11] As we

[11]It is analytically proven that $l(n)$ is increasing in θ at $\theta = 0$. See Section 3 for the details.

show in the next section, these variables become decreasing in θ in the modified model in which the entrepreneurs accumulate capital stocks.

3 The Model with Land and Capital

In this section we modify our basic model such that the entrepreneurs accumulate capital stocks, k_t, in each period. The consumer goods are produced from land, capital and labor by the following Cobb-Douglas technology:

$$y_t = Aa_{t-1}^{\nu}k_{t-1}^{\alpha}l_t^{1-\alpha-\nu}.$$

The entrepreneur can transform the consumer goods to the capital at one-to-one basis, and vice versa. The capital k_{t-1} depreciates to $(1-\delta)k_{t-1}$ at the end of period t. We assume that k_t is not pledgeable as collateral when the entrepreneurs borrow in period t and that the pledeable asset is only land. If a borrower repudiates the repayment of debt in period t, the bank can collect $\{r_t^a(s_t)+q_t\}a_{t-1}$, where

$$r_t^a(s_t) = \nu A\left(\frac{k_{t-1}}{a_{t-1}}\right)^{\alpha}\left(\frac{l_t(s_t)}{a_{t-1}}\right)^{1-\alpha-\nu},$$

and the depositors can collect $\kappa\{r_t^a(s_t)+q_t\}a_{t-1}$. The entrepreneur's optimization problem in the modified model is

$$\max_{c_t^E,h_t^E,d_t^E,k_t,l_t,a_t,b_t} E_0\left[\sum_{t=0}^{\infty}(\beta')^t\{c_t^F-h_t^F\}\right],$$

$$\begin{aligned}
\text{s.t.}\quad & q_t(s_t)a_t(s_t)+k_t(s_t)+d_t^E(s_t)-b_t(s_t)+c_t^E(s_t)\\
&\quad = Aa_{t-1}^{\nu}k_{t-1}^{\alpha}l_t(s_t)^{1-\alpha-\nu}+q_t(s_t)a_{t-1}+(1-\delta)k_{t-1}-w_t(s_t)l_t(s_t)\\
&\qquad +\xi_t(s_t)(1+r_{t-1})d_{t-1}^E-\kappa(s_t)\{r_t^a(s_t)+q_t(s_t)\}-\chi(a_t(s_t))+h_t^E(s_t),\\
& w_t(s_t)l_t(s_t)\le \xi_t(s_t)(1+r_{t-1})d_{t-1}^E,\\
& b_t(s_t)\le B_t(a_t).
\end{aligned}$$

The resource constraint for the consumer goods is

$$c_t(s_t)+c^E(s_t)+k_t(s_t) = Aa_{t-1}^{\nu}k_{t-1}^{\alpha}l_t(s_t)^{1-\alpha-\nu}+h_t(s_t)+h_t^E(s_t)+(1-\delta)k_{t-1}.$$

The FOC with respect to k_t is

$$1 = \beta'\{(1-\theta)\alpha A a_t^{\nu} k_t^{\alpha-1} l_{t+1}(n)^{1-\alpha-\nu} + \theta\alpha A a_t^{\nu} k_t^{\alpha-1} l_{t+1}(e)^{1-\alpha-\nu} + 1 - \delta\}, \tag{37}$$

which, together with $a_t = 1$, implies that k_t does not depend on the realization of s_t.

Stationary Equilibrium: Equation (37) implies that there exists a stationary equilibrium in which k_t is time-invariant. Capital stock can be time-invariant along the equilibrium path because the backyard production is available for the households and the entrepreneurs and this technology enables them to make any amount of investment. Therefore, the amount of investment at every period is chosen such that the capital stock is kept at the steady-state level. The stationary equilibrium is specified by the set of variables $\{r, q, w(n), w(e), r^a(n), r^a(e), k, l(n), l(e), \xi,\ \mu(n), \mu(e)\}$, which solves the following system of equations:

$$\begin{aligned}
&1 = \beta\{1-(1-\xi)\theta\}(1+r), \\
&1 = \beta'\{(1-\theta)\alpha A k^{\alpha-1} l(n)^{1-\alpha-\nu} + \theta\alpha A k^{\alpha-1} l(e)^{1-\alpha-\nu} + 1 - \delta\}, && (38) \\
&1 = \beta'[(1-\theta)\{1+\mu(n)\} + \theta\xi\{1+\mu(e)\}](1+r), \\
&(1-\alpha-\nu) A k^{\alpha} l(n)^{-\alpha-\nu} = w(n)\{1+\mu(n)\}, && (39) \\
&(1-\alpha-\nu) A k^{\alpha} l(e)^{-\alpha-\nu} = w(e)\{1+\mu(e)\}, && (40) \\
&\xi = \kappa \frac{r^a(e)+q}{r^a(n)+q}, \\
&\chi'(1) + q = (1-\kappa)\beta'\theta[r^a(e)+q] + \frac{r^a(n)+q}{1+r}, \\
&r^a(n) = \nu A k^{\alpha} l(n)^{1-\alpha-\nu}, \\
&r^a(e) = \nu A k^{\alpha} l(e)^{1-\alpha-\nu}, \\
&\xi w(n) l(n) = w(e) l(e), \\
&\gamma'(l(n)) = w(n), \\
&\gamma'(l(e)) = w(e).
\end{aligned}$$

We solve this system of equations numerically and show the result in Figure 2. The functional forms of $\gamma(l_t)$ and $\chi(a_t)$ are the same as those in the previous section. As Figure

2 shows, the output, labor and consumption in the state where $s_t = n$ are all decreasing in θ. The capital stock and the land price are also decreasing in θ. Although this result may depend on the parameter values, we are confident that this result qualitatively holds for a standard range of parameters. We compare the FOCs in the two models at $\theta = 0$ to see why k and l are decreasing in θ. In the basic model without capital, differentiation of (28) with respect to θ implies

$$\left[\gamma''(l(n))\frac{\beta}{\beta'} + (1-\alpha)\alpha A l(n)^{-1-\alpha}\right]\frac{dl(n)}{d\theta} = -\gamma'(l(n))\frac{d\mu(n)}{d\theta},$$

while in the model with capital, differentiation of (39) implies

$$\begin{aligned}&\left[\gamma''(l(n))\frac{\beta}{\beta'} + (1-\alpha-\nu)(\alpha+\nu)Ak^{\alpha}l(n)^{-1-\alpha-\nu}\right]\frac{dl(n)}{d\theta}\\ &\quad = -\gamma'(l(n))\frac{d\mu(n)}{d\theta} + (1-\alpha-\nu)\alpha A k^{\alpha-1}l(n)^{-\alpha-\nu}\frac{dk}{d\theta}.\end{aligned} \tag{41}$$

In both models it is easily shown that $\frac{d\mu(n)}{d\theta} < 0$ at $\theta = 0$ (See Appendix for the proof). Therefore, $\frac{dl(n)}{d\theta} > 0$ at $\theta = 0$ in the model without capital. The intuition is as follows: if θ increases, the entrepreneurs increase d^E to hold liquidity in case of bank run; the increase in d^E loosens the liquidity constraint on the wage payment in the state of $s_t = n$; and as a result of loosening of the constraint on wage payment the labor input increases when $s_t = n$. In the model with capital, if the sign of $\frac{dk}{d\theta}$ is negative and its absolute value is sufficiently large, $\frac{dl(n)}{d\theta}$ is negative at $\theta = 0$. The intuition is as follows: if θ increases, the loosening of the constraint on wage payment has an effect to increase $l(n)$, while the decrease in k lowers the marginal product of labor and has an effect to decrease $l(n)$; therefore, if k decreases to a sufficient extent in response to an increase in θ, the negative effect overwhelms and $\frac{dl(n)}{d\theta}$ become negative. On the other hand, a decrease in $l(n)$ directly reduces the (expected) marginal product of capital (MPK), and the decrease in the MPK leads to a decrease in k in equilibrium. This relationship is demonstrated as follows: Differentiating (38) with respect to θ, we obtain

$$(1-\alpha)\frac{l(n)^{1-\alpha-\nu}}{k}\frac{dk}{d\theta} = -l(n)^{1-\alpha-\nu} + l(e)^{1-\alpha-\nu} + (1-\alpha-\nu)l(n)^{-\alpha-\nu}\frac{dl(n)}{d\theta}, \tag{42}$$

which implies that if $\frac{dl(n)}{d\theta} < 0$ then $\frac{dk}{d\theta} < 0$, because $l(n) > l(e)$. We can derive the following lemma from equations (41) and (42).

Lemma 3 *The necessary and sufficient condition for* $\frac{dl(n)}{d\theta} < 0$ *and* $\frac{dk}{d\theta} < 0$ *at* $\theta = 0$ *in the model with land and capital is that the parameter values are set such that the following inequality holds at* $\theta = 0$*:*

$$l(e) < \{(1-\alpha)\xi + \alpha\}^{\frac{1}{1-\alpha-\nu}} l(n). \quad (43)$$

See Appendix for the proof. The sufficient condition for $\frac{dl(n)}{d\theta} < 0$ and $\frac{dk}{d\theta} < 0$ at $\theta = 0$ is given by setting $\xi = 0$ in (43):

$$l(e) < \alpha^{\frac{1}{1-\alpha-\nu}} l(n).$$

For example, for $\alpha = 0.3$ and $\nu = 0.05$, this condition is $l(e) < 0.156 \times l(n)$, which implies that κ (and $l(e)$) should be considerably small to make k and l decreasing in θ at $\theta = 0$.

The exogenous parameter θ in this model can be regarded a parameter that represents the fragility of the financial system, such as the deterioration in the capital ratio and/or the increase in the ratio of nonperforming assets in the balance sheets of the financial institutions. It may be interpreted as representing the loss of confidence in the market or the increased fears of the recurrence of the systemic bank run. The result of our simulation implies that the economic activities shrink and the asset prices decrease as the financial fragility (θ) increases, even if the systemic run does not actually occur ($s_t = n$). The mechanism in our model that the increase in θ worsens the macroeconomic variables in the state where $s_t = n$ may explain the slowdown of economic growth observed after the episodes of systemic crises.

4 Model with Capital only (without Land)

In this section, we consider a model in which land does not exist and capital works as collateral in bank lending. We distinguish the consumer goods and the capital goods, and introduce production technology for capital goods. The entrepreneurs' problem becomes

$$
\begin{aligned}
\max_{c_t^E, h_t^E, i_t, d_t^E, b_t, k_t, k_t', l_t} \quad & E_0 \left[\sum_t^{\infty} (\beta')^t \left\{ c_t^E - h_t^E \right\} \right] \\
\text{s.t.} \quad & c_t^E + i_t + d_t^E - b_t + q_t k_t - q_t k_t' \\
& = A k_{t-1}^{\alpha} \ell_t^{1-\alpha} - w_t \ell_t + (1-\delta) k_{t-1} + \tilde{\xi}(s_t)(1 + r_{t-1}) d_{t-1}^E \\
& \quad - \kappa_t(s_t) \{ r_t^k(n) + (1-\delta) q_t \} k_{t-1} + h_t^E, \\
& k_t' = (1-\delta) k_{t-1} + \Phi \left(\frac{i_t}{k_{t-1}} \right) k_{t-1}, \\
& w_t \ell_t \leq \tilde{\xi}(s_t)(1 + r_{t-1}) d_{t-1}^E, \\
& b_t \leq \frac{r_{t+1}^k(n) + (1-\delta) q_{t+1}}{1 + r_t} k_t,
\end{aligned}
$$

where $\Phi(\frac{i_t}{k_{t-1}})k_{t-1}$ is the installation function of capital stock and

$$
\Phi \left(\frac{i_t}{k_{t-1}} \right) k_{t-1} = B k_{t-1}^{\omega} i_t^{1-\omega}.
$$

The value of B is set so that $\Phi(\delta) = \delta$. Note that $k_t = k_t'$ in equilibrium, and q_t is the price of capital. The resource constraint for the consumer goods is

$$
c_t(s_t) + c_t^E(s_t) + i_t(s_t) = A k_{t-1}^{\alpha} l_t(s_t)^{1-\alpha} + h_t(s_t) + h_t^E(s_t).
$$

Stationary Equilibrium: The stationary equilibrium is determined by the following system of equations.

$$
\begin{aligned}
&1 = \beta\{1-(1-\xi)\theta\}(1+r), \\
&1 = \beta'[(1-\theta)\{1+\mu(n)\}+\theta\xi\{1+\mu(e)\}](1+r), \\
&q = (1-\kappa)\beta'\theta\{r^k(e)+(1-\delta)q\}+\beta' r^i + \frac{1}{1+r}\{r^k(n)+(1-\delta)q\}, && (44)\\
&(1-\alpha)Ak^{\alpha}l(n)^{-\alpha} = w(n)\{1+\mu(n)\}, && (45)\\
&(1-\alpha)Ak^{\alpha}l(e)^{-\alpha} = w(e)\{1+\mu(e)\}, \\
&\xi = \kappa\frac{r^k(e)+q}{r^k(n)+q}, \\
&r^k(n) = \alpha A k^{\alpha-1}l(n)^{1-\alpha}, \\
&r^k(e) = \alpha A k^{\alpha-1}l(e)^{1-\alpha}, \\
&q = \left\{\Phi'(\delta)\right\}^{-1}, \\
&r^i = \frac{\Phi(\delta)}{\Phi'(\delta)} - \delta = (q-1)\delta, \\
&\xi w(n)l(n) = w(e)l(e), \\
&\gamma'(l(n)) = w(n), \\
&\gamma'(l(e)) = w(e).
\end{aligned}
$$

We solve these equations numerically and show the result in Figure 3. This figure shows that the price of capital does not change as θ changes because q_t is a function of i_t/k_{t-1} and $i_t/k_{t-1} = \delta$ always holds in our model where the backyard production is available. It is shown in the model with capital only that the economic activities (output, labor, and consumption) all decrease in the state $s_t = n$ as θ increases. As in the previous section, we can see the relationship between $\frac{dk}{d\theta}$ and $\frac{dl}{d\theta}$ at $\theta = 0$. Differentiation of (45) implies that

$$
\begin{aligned}
&\left[\gamma''(l(n))\frac{\beta}{\beta'} + (1-\alpha)\alpha A k^{\alpha} l(n)^{-1-\alpha}\right]\frac{dl(n)}{d\theta} \\
&\qquad = -\gamma'(l(n))\frac{d\mu(n)}{d\theta} + (1-\alpha)\alpha A k^{\alpha-1} l(n)^{-\alpha}\frac{dk}{d\theta}.
\end{aligned}
$$

This equation shows as its counterpart in the previous section that if the sign of $\frac{dk}{d\theta}$ is negative and its absolute value is sufficiently large, $\frac{dl(n)}{d\theta}$ is negative at $\theta = 0$. On the other hand, it is shown as follows that $\frac{dl(n)}{d\theta} < 0$ leads to $\frac{dk}{d\theta} < 0$ through $\frac{dMPK}{dl(n)} > 0$. Differentiating (44) by θ at $\theta = 0$, given q and r^i are parameters, we have

$$
\begin{aligned}
&(1-\alpha)\alpha\beta A k^{\alpha-2} l(n)^{1-\alpha} \frac{dk}{d\theta} \\
&= (1-\alpha)\alpha\beta A k^{\alpha-1} l(n)^{-\alpha} \frac{dl(n)}{d\theta} \\
&+ (1-\kappa)\beta' \{\alpha A k^{\alpha-1} l(e)^{1-\alpha} + (1-\delta) q\} - (1-\xi)\beta\{\alpha A k^{\alpha-1} l(n)^{1-\alpha} + (1-\delta) q\}.
\end{aligned}
$$

Since $l(n) > l(e)$ and $\kappa > \xi$, it is easily shown that $\frac{dl(n)}{d\theta} < 0$ implies $\frac{dk}{d\theta} < 0$.

5 Conclusion

We present a model of systemic runs on a fragile banking system à la Diamond and Rajan (2000, 2001a, 2001b, 2005) in an infinite-horizon production economy, where the borrowers are subject to collateral constraint à la Kiyotaki and Moore (1997). The mechanism of the bank runs in our model is as follows:

- When the sunspot variable turns out to be bad ($s_t = e$), all agents expect that the value of bank asset becomes $\{r^a_t(e)+q_t\}a_t$, while the bank liability is $(1+r_{t-1})d_{t-1} = \{r^a_t(n) + q_t\}a_t$, where $r^a_t(e) < r^a_t(n)$.
- Since the depositors expect that the bank cannot payout the full amount of deposits, the depositors run on the bank.
- In the bank run, the depositors (households and entrepreneurs) can withdraw their deposits only partially.
- Since the entrepreneurs must pay the wages by withdrawn deposits, the bank run reduces the funds that the entrepreneurs can use for wage payments.
- The decrease in the wage payment reduces the aggregate labor input. The decrease in the labor input lowers the marginal product of land and thus decreases the return on the land from $r^a_t(n)$ to $r^a_t(e)$.

- In this way, the prophecy that $r^a_t = r^a_t(e)$ is self-fulfilled.

The bank run leads the economy into a severe recession due to the shortage of liquidity for the wage payment. The bank run in our model is not caused by the coordination failure among depositors in one bank as in Diamond and Dybvig (1983), but is caused by an economy-wide coordination failure, which changes the market value of the collateral asset. The key is that we translate the demand for liquidity in the Diamond-Rajan models into the demand for working capital, i.e., the borrowing for factor payments, in the business cycle models. The modified models with capital accumulation shows that an increase in the financial fragility (θ) causes a shrinkage of the economic activities, even if the systemic bank run does not actually occur. In the model with capital and land, it is shown that land price is lowered as the financial fragility increases. These results indicate that the typical slowdown of economic growth and stagnant asset prices after the financial crises may be caused by the increase in the financial fragility or the raised fears of recurrence of another systemic run.

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Appendix

Proof of $\frac{d\mu(n)}{d\theta} < 0$ at $\theta = 0$: We consider the basic model presented in Section 2. (The same arguments hold for the other models.)

Equation (26) implies that $1 + r = 1/\beta$ and $\beta \frac{dr}{d\theta} = 1 - \xi$ at $\theta = 0$. Equation (27) implies that $1 + \mu(n) = \beta/\beta'$ at $\theta = 0$. Together with these results, differentiation of (27) with respect to θ at $\theta = 0$ implies

$$\frac{d\mu(n)}{d\theta} = \frac{\beta\xi}{\beta'}\left[1 - \frac{\beta'}{\beta}(1 + \mu(e))\right] = \frac{\beta\xi}{\beta'}\left[1 - \frac{1 + \mu(e)}{1 + \mu(n)}\right]. \tag{46}$$

Equations (34), (35), and (36) imply that $l(e) < l(n)$. Equations (28) and (29) imply that $1 + \mu$ is a decreasing function of l. Therefore, $\frac{1+\mu(e)}{1+\mu(n)} > 1$, since $l(e) < l(n)$. This result and equation (46) imply that $\frac{d\mu(n)}{d\theta} < 0$.

Proof of Lemma 3: Equations (41) and (42) imply

$$\begin{aligned}
&\left[\gamma'(l(n))\frac{\beta}{\beta'} + \frac{(1-\alpha-\nu)\nu}{1-\alpha}Ak^{\alpha}l(n)^{-1-\alpha-\nu}\right]\frac{dl(n)}{d\theta} \\
&\qquad = -\gamma'(l(n))\frac{d\mu}{d\theta} - \{l(n)^{1-\alpha-\nu} - l(e)^{1-\alpha-\nu}\}\frac{(1-\alpha-\nu)\alpha}{1-\alpha}\frac{Ak^{\alpha}}{l(n)}.
\end{aligned} \tag{47}$$

The right-hand side of (47) is rewritten as follows using (46), (39) and (40):

$$\frac{(1-\alpha-\nu)\alpha}{1-\alpha}\frac{Ak^{\alpha}}{l(n)}\left[l(e)^{1-\alpha-\nu} - \{(1-\alpha)\xi + \alpha\}l(n)^{1-\alpha-\nu}\right]. \tag{48}$$

Therefore, $\frac{dl(n)}{d\theta} < 0$ at $\theta = 0$ if and only if (43) holds. Equation (42) implies that $\frac{dk}{d\theta} < 0$ if $\frac{dl(n)}{d\theta} < 0$.

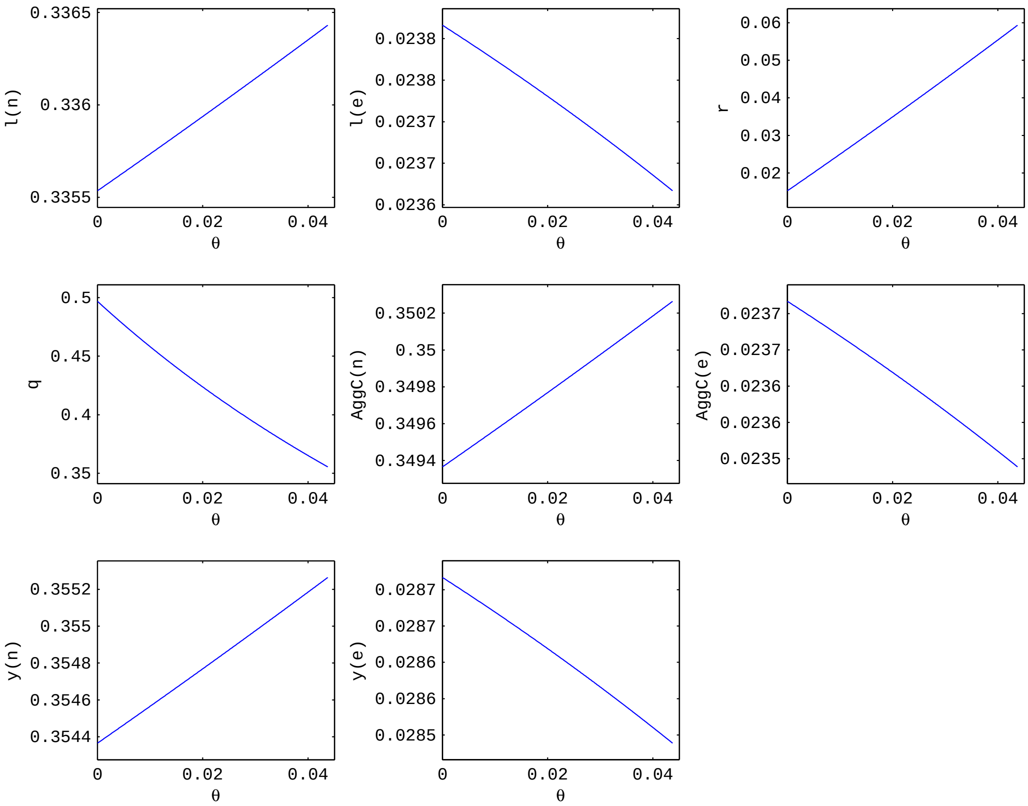

Figure 1: Stationary Equilibrium in the Model with Land

Parameters: $A = 1$, $\alpha = 0.3$, $\beta = 0.985$, $\beta' = 0.9 \times \beta$, $\delta = 0.04$, $\sigma = 1$, $\psi = 0.6$, $\phi = 0.01$, $\kappa = 0.05$.

Note: $AggC(s) = c(s) + c^E(s) - h(s) - h^E(s)$.

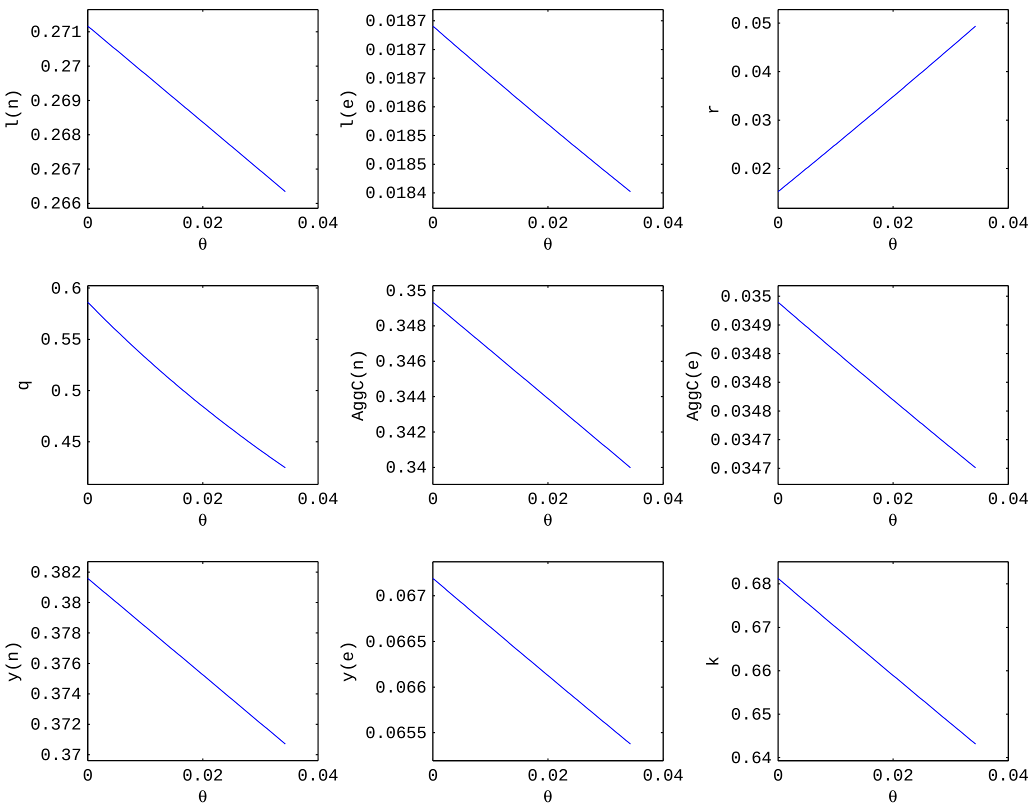

Figure 2: Stationary Equilibrium in the Model with Land and Capital

Parameters: $A = 1$, $\alpha = 0.3$, $\beta = 0.985$, $\beta' = 0.9 \times \beta$, $\delta = 0.04$, $\sigma = 1$, $\psi = 0.6$, $\phi = 0.01$, $\kappa = 0.05$, $\nu = 0.05$.

Note: $AggC(s) = c(s) + c^E(s) - h(s) - h^E(s)$.

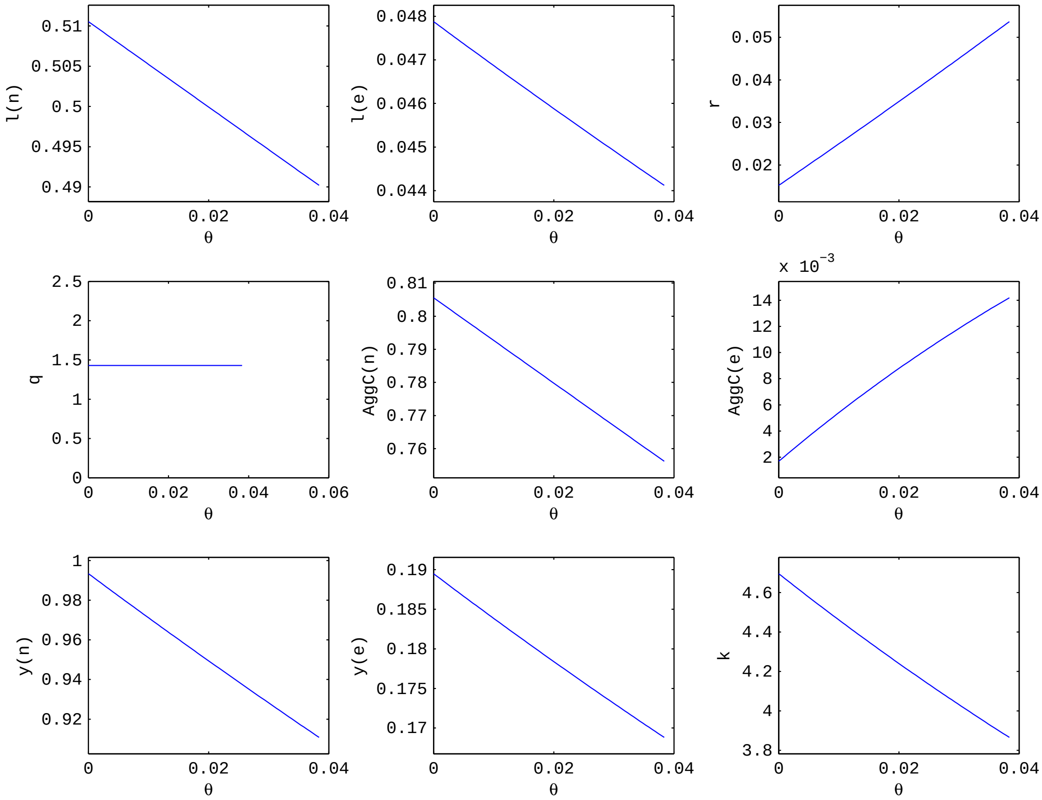

Figure 3: Stationary Equilibrium in the Model with Capital only

Parameters: $A = 1$, $\alpha = 0.3$, $\beta = 0.985$, $\beta' = 0.9\beta$, $\delta = 0.04$, $\sigma = 1$, $\psi = 0.6$, $\phi = 0.01$, $\omega = 0.3$, $\kappa = 0.05$, $B = 0.38073$.