Knowledge Linkages and Multi-Sector Firm Innovations*

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Abstract

Using cross-sector patent citations, we develop a novel measure characterizing the *generality* or *applicability* of different technologies. We find scale dependence in firms' innovation and sectoral entry decisions; that is, the new technologies that larger firms innovate tend to be pioneering, younger, and less general, while smaller firms self-select into more general and established sectors. Controlling for firm size, firms with a product mix of higher generality innovate more quickly in both existing and new sectors. We then develop a multi-sector firm innovation model which allows for heterogeneous cross-sector knowledge spillovers and fixed sectoral R&D costs. The model generates *sequential* sectoral entry, which helps to explain many empirical regularities regarding innovation and firm dynamics. It also explains the skewed concentration of firms across sectors and the sector size distribution observed in the data. Fixed R&D costs function as entry barriers and lead to incomplete knowledge circulation across sectors, disproportionally decreasing the size of sectors with more applicable knowledge. This R&D resource misallocation has a large negative impact on growth.

Keywords: innovations; multi-sector firms; cross-sector knowledge spillovers; sequential sectoral entry; technology space; resource allocation

JEL Classification: L16, L25, L60, O33, O41.

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1 Introduction

Firms grow by innovating and expanding the set of goods they produce. The knowledge accumulated from different inventions varies greatly in its applicability to future innovations. Some inventions, such as the transistor, create general purpose knowledge that can be applied in a vast range of sectors. Other inventions, such as the space pen, introduce technologies that are limited in their scope of application. Naturally, different types of innovation have vastly different impacts on technological and economic growth; therefore, understanding the effects of cross-sector knowledge spillovers on firm innovation and entry decisions is important for analyzing the efficiency of R&D policies and resource allocation in the economy. It also sheds light on path-dependent growth.

Existing theories of firm innovations, however, tend to treat different forms of technical change as being isolated from each other and having equal importance. In this paper, we first take an important step forward by constructing a quantitative measure that captures the *generality* or *applicability* of different technologies. We explore empirically how firms make their innovation decisions based on this measure and how their decisions, in turn, affect a firm's growth.

Motivated by the empirical observations, we develop a novel model of multi-sector firm innovation in which knowledge linkages across sectors are heterogenous. Our model has several tractable implications. Most importantly, it captures the rich dynamics of firm innovation and sequential entry into multiple sectors, which has not previously been explored. The resulting sequential sectoral entry also explains scale dependent firm growth and volatility of firm growth, sector size distribution, heterogeneous R&D intensities in different sectors and the skewed concentration of firms across sectors. In addition, the model implies that fixed R&D costs block knowledge circulation across sectors and distort the R&D resource allocation by pushing research effort away from more applicable sectors to less applicable ones. This R&D resource misallocation has a large negative impact on growth. A growth-enhancing R&D policy thus not only encourages a firm's overall R&D investment, but also concentrates firm investment in sectors with highly applicable knowledge.

Based on the network formed by cross-sector patent citations, we employ Kleinberg's (1998) iterative algorithm to develop a measure, called the *authority weight*, which values a sector's importance as a knowledge contributor to the economy (i.e. knowledge generality or applicability).[1] Employing this measure, we find that, in U.S. patent data, different-sized firms enter different sectors depending on the intrinsic knowledge utilization linkages among sectors. Specifically, small firms tend to enter more general and more established sectors in

[1] A dual measure of the authority weight is the hub weight, which values a sector's ability to learn from other sectors.

which the knowledge stock is large and the potential for future expansion into new sectors is high. In contrast, large firms with a broader product scope enter younger, more pioneering and less general sectors at the periphery of the technology space, where firms can enjoy larger market shares. In addition, new firms self-select over a firm's life cycle into sectors at the center of the technology space and then gradually venture towards the fringe. Sequential sectoral entrance appears to be a natural process of technological development.

More importantly, the knowledge authority weight of a firm's existing product mix matters for its subsequent growth. After controlling for firm size, we find that firms who patent in sectors with higher applicability are able to innovate faster in new sectors (extensive margin) and in their existing product set (intensive margin). While productivity and factor inputs are important for a firm's growth, the knowledge generality of its products matters greatly as well.

Although a great deal of theoretical work has been done in recent years on innovation and firm dynamics (e.g., Aghion and Howitt, 1992; Klette and Kortum, 2004; Luttmer, 2007; Lentz and Mortensen, 2008; Bernard, Redding and Schott, 2009a, 2009b; Acemoglu and Cao, 2010), most of these studies assume that a firm's innovation applies to a product or a sector that is randomly drawn from a potential pool.[2] There are no explicit interactions between different sectors or distinctions between innovations with different degrees of generality, hence there is no room to discuss explicitly the sequentiality of innovations among different sectors and the relationship between R&D investment allocations across sectors and economic growth.

Our second contribution is to develop a dynamic general equilibrium model of multi-sector firm innovation and sectoral entry and exit that incorporates cross-sector knowledge diffusion. Although only forty-one percent of U.S. manufacturing firms operate multiple product lines, these firms account for ninety-one percent of total sales (Bernard, Redding and Schott, 2006). Therefore, understanding how firms expand their product range adds important insights to aggregate production. Our approach extends previous literature by allowing for heterogeneous cross-sector knowledge spillovers, fixed sectoral R&D costs and imitation (i.e. access to public knowledge). To provide economic incentives for developing specific technologies, especially ones that could be widely applied to many sectors throughout the economy, we assume that cross-firm knowledge spillover is incomplete and that firms can accumulate private knowledge through previous innovations. At any given time, a multi-sector firm makes sectoral entry, exit selections and R&D investment decisions, taking into consideration the attributes of its current product mix and the exogenous knowledge linkages across sectors. For a given sector, only firms that have accumulated enough knowledge capital

[2]Products and sectors are interchangable both in this paper and the papers listed.

in related sectors self-select to enter.

The model provides a coherent framework to explain many observations at the firm and sector levels and implies that fixed R&D costs distort the cross-sector R&D resource allocation and generate a large negative effect on aggregate growth.

First, at the firm level, small firms and new firms grow faster because they self-select into established sectors with general purpose knowledge, where the public knowledge stock is abundant, allowing smaller firms to learn easily from other peers (intensive growth), and the potential of applying the knowledge to innovating in other sectors is high (extensive growth). By contrast, large multi-sector firms with significant private knowledge can afford to enter less general sectors and enjoy larger market shares in these less developed frontier sectors. Thus, after allowing for heterogeneous knowledge linkages between different sector pairs, the two well-documented counteracting effects of R&D – the inter-firm knowledge spillover effects and the rivalry effect – affect firms of varying sizes differently. Small firms focus more on how much they can learn from public knowledge and extensive growth potential, while large firms with ample knowledge are concerned more with their rivals and market shares.

Because knowledge in different sectors is related, firms can expand through the product space by developing goods close to their current product mix. When the scope and generality of a firm's knowledge increase, so do the opportunities to innovate, profit and grow in related sectors. As a feedback effect, existing sectors also benefit from the growth in the new innovating sectors as a consequence of knowledge spillover in the opposite direction.

The fact that small firms self-select to enter center sectors also explains why small firms exhibit more volatile growth relative to large firms. The central location on the product network endows small firms with more open routes to new sectors and, on average, the extensive margin contributes more to a smaller firm's growth than that of a large firm. The difficulty of overcoming entry costs when entering new sectors generates higher growth rate turbulence. In addition, the model also endogenously gives rise to the Pareto firm size distribution both within sectors and across all sectors, which is consistent with firm-level evidence.

Second, at the sector level, our model provides a micro-founded explanation for heterogenous sector sizes and R&D intensities. The cross-sector knowledge diffusion implies that the value of the knowledge associated with a specific sector is not limited to the discounted stream of future profits it generates in that sector, but more importantly, is determined by the contribution of this knowledge to future innovations in related sectors. In general, the knowledge of a better connected sector in the product space is more valuable. Hence, such sectors attract a larger share of R&D investment from firms. In the patent data, we find that both the numbers of firms and sector sizes (measured by the number of patents) are larger for sectors with higher applicability. These observations can be explained by firms' sequential sectoral entry behavior. Every firm starts from highly connected center sectors, but only large, highly knowledgable firms venture to the fringe of the product space. Thus, center sectors accommodate a larger number of firms than peripheral sectors.

Third, our paper points out a new type of aggregate-level efficiency loss that occurs because of sectoral entry barriers. Besides reducing the varieties available to consumers and deterring extensive growth by small but fast growing firms, entry costs also cause a misallocation of resources across sectors. An ideal resource allocation across sectors pushes more resources towards sectors with high applicability, because these sectors are the centers of knowledge circulation and impose a prominent, positive impact on economic growth. In reality, only large firms can afford a series of fixed entry costs and reach the periphery of the product space; small firms are excluded from cross-sector knowledge applications between center sectors and peripheral sectors. The incomplete knowledge circulation across the product space slows down growth in all sectors and reduces those of center sectors more than the sizes of peripheral sectors. As a result, highly applicable sectors are smaller than is justified by their knowledge contribution to the economy.

Our paper shows that innovation success does not arrive randomly at *every* sector and that not all innovations in different sectors are equally valuable. The positive externalities of knowledge spillovers from sectors with general purpose knowledge indicate that policies should subsidize sectors that utilize and develop highly applicable, relevant and influential technology. Since small firms tend to self-select into such sectors, this policy suggestion is consistent with existing R&D subsidy programs that target small business growth.

To assess the success of the model and the growth effects of sectoral fixed R&D costs, we estimate the model parameters using existing information of 42 SIC 2-3 digit sectors. Specifically, we estimate the exogenous cross-sector knowledge diffusion matrix from patent citation data and feed it into the model. For the scenario without fixed costs, we analytically solve for general equilibrium. In the general case with sectoral entry costs, we simulate an economy with a large number of multi-sector firms and set the sectoral fixed cost to match the number of sectors for an average firm in the patent data. The simulation results correspond to the empirical findings about scale-dependent sectoral entry decisions, the positive correlation between a firm's growth and initial product mix authority weight, and the skewed concentration of firms, R&D investment and sector size distribution. We then compare the growth rate of varieties and resource allocations under these two scenarios. In the simulation, we find that when moving to the extreme case with no fixed R&D costs in any sector, the growth of the total product varieties increases from 17 percent to 1200 percent; firms, R&D spending and varieties allocations become much less concentrated on the center sectors.

Our work contributes to the earlier literature in development economics that emphasizes the role of sectoral linkages and complementarity in explaining growth (Leontief, 1936; Hirschman, 1958). Prior work in this area typically focuses on input-output relationships between sectors (Jones, 2010a)[3] or export-based measures of sectoral relatedness (Hidalgo, Klinger and Hausmann, 2007). In our paper, we focus on sectoral linkages implied by knowledge content. Our new measure exhibits several advantages. First, it is more suitable for studying innovation and growth, as it reveals the intrinsic knowledge utilization between sectors. Second, it is comparable across all sectors, instead of being sector-pairwise. Third, it captures not only the direct impact of a given technology to the connected sectors but also ranks its importance in the whole technology space.[4]

The observations in our paper add to the large empirical literature on knowledge spillovers and externalities. Using firm-level R&D investment data in five high-tech industries and a different methodology, Bernstein and Nadiri (1988) find that inter-sectoral knowledge spillovers are heterogenous and highly significant. In his survey paper, Wieser (2005) claims that spillovers between sectors are more important than those within sectors, when considering both the social and private return of R&D.

Our paper is also related to the expanding literature on misallocation and economic growth.[5] It is most closely related to Jones (2010b), who suggests that misallocation effects can be amplified through the input-output structure of the economy. In the context of knowledge spillovers, the misallocation of research resources affects growth because highly applicable knowledge is not sufficiently internalized and utilized by innovating firms. In the area of entry costs and growth, the paper also is related to Barseghyan (2008), Barseghyan and DiCecio (2010) and Boedo and Mukoyama (2009), who study how entry and firing costs affect productivity and output across countries through their impacts on firm size distribution and the average productivity of producing firms.

The rest of the paper is organized as follows. Section 2 describes the construction of our measure of authority weight and the empirical findings using patent citation data. Section 3 introduces the model and Section 4 discusses characteristics of the general equilibrium. Section 5 simulates the economy with fixed R&D costs and tests the model's implications. Section 6 discusses welfare and policy implications. Section 7 concludes.

[3]Other research studies the role of input-output relationship in understanding sectoral co-movements and the transmission of shocks over the business cycle, such as Lucas, 1981; Long and Plosser, 1993; Basu, 1995; Horvath, 1998; Conley and Dupor, 2003; Carvalho, 2009.

[4]In the Appendix, we make a comparison between our measures of sectoral linkages and previous ones based on input-output table and export data.

[5]For example, Ciccone (2002), Restuccia and Rogerson (2008), Hsieh and Klenow (2009).

2 Empirical Evidence

2.1 Data Sources

We use patent applications in the 2006 edition of the NBER Patent Citation Data[6] to characterize firms' innovation activities and their citations to trace the direction and intensity of knowledge flows and to construct indices of knowledge linkages among sectors. The data provides detailed information of every patent granted by the United States Patent and Trade Office (USPTO) from 1976 to 2006.We summarize each firm's patent stock in each disaggregated technological class (intensive margin of innovation) and the number of categories (extensive margin of innovation) for each year.[7]

Each patent corresponds to one of the 476 3-digit United States Patent Classification System (USPCS) technological classes and also one of more than 800 7-digit International Patent Classification (IPC) classes. We mostly report the results based on USPCS codes, but we check for robustness using the IPC classes. We also present some evidence based on industrial sector classification, as the model is estimated based on this categorization. To translate the data into the industrial classifications, we use the 2005 edition of the concordance table provided by the USPTO to map USPCS into SIC72 (Standard Industrial Classification in 1972) codes, which constructs 42 industrial sectors.[8] We summarize citations made to patents that belong to the same technological class to form the inter-sectoral knowledge spillover network.

2.2 Construction of Sectoral Knowledge Authority Weight

This network structure formed by cross-sector patent citations contains rich information about the knowledge linkages between sectors. Some sectors contain general purpose knowledge that is widely applicable in other sectors. These sectors act as knowledge *authorities* in the network. Other sectors rely on knowledge from many other sectors and serve as important knowledge *hubs*. These sectors resemble focused hubs that direct users to the recommended authorities in the network.

We apply an algorithm (Kleinberg, 1998) which extracts information from hyperlinked

[6]See Hall, Jaffe and Trajtenberg, 2001 for details.

[7]When firms accumulate more patents over time, they not only increase the number of patents in existing patent categories, but also expand into new categories. These two measures of firm size are highly correlated.

[8]The patents are classified according to either the intrinsic nature of the invention or the function for which the invention is used or applied. It is inherently difficult to allocate the technological category to economically relevant industries in a differentiation finer than 42 sectors, even with detailed firm level information. First, most of the patents are issued by multi-product firms that are present in multiple SIC-4 industries. Second, in the best scenario, one only has industry information about the origin of the patents but not the industry to which the patent is actually applied.

environments to the cross-sector patent citation network. We use an index, the authority weight, to capture the intuitive notions of the relevance, applicability and importance of knowledge in different sectors. Sector i's authority weight is proportional to the sum of the hub weights of the sectors that utilize knowledge from sector i. Sector i's hub weight is proportional to the sum of the authority weights of the sectors that provide knowledge to sector i.

Formally, let aw^i denote the authority weight and hw^i denote the hub weight of sector i. They are calculated according to the following iterative algorithm:

$$\begin{aligned} aw^i &= \lambda \sum_j W^{ij} hw^j \\ hw^i &= \mu \sum_j W^{ji} aw^j \end{aligned}$$

where λ and μ are the inverse of the norms of vectors $\boldsymbol{aw}$ and $\boldsymbol{hw}$, respectively. W^{ij} is the weight of the link, corresponding to the strength of citations made by sector j (second superscript) to sector i (the first superscript). We assume a simple form for that weight[9]

$$W^{ij} = \begin{cases} 1, & \text{if } j \text{ is citing } i \\ 0, & \text{otherwise} \end{cases}$$

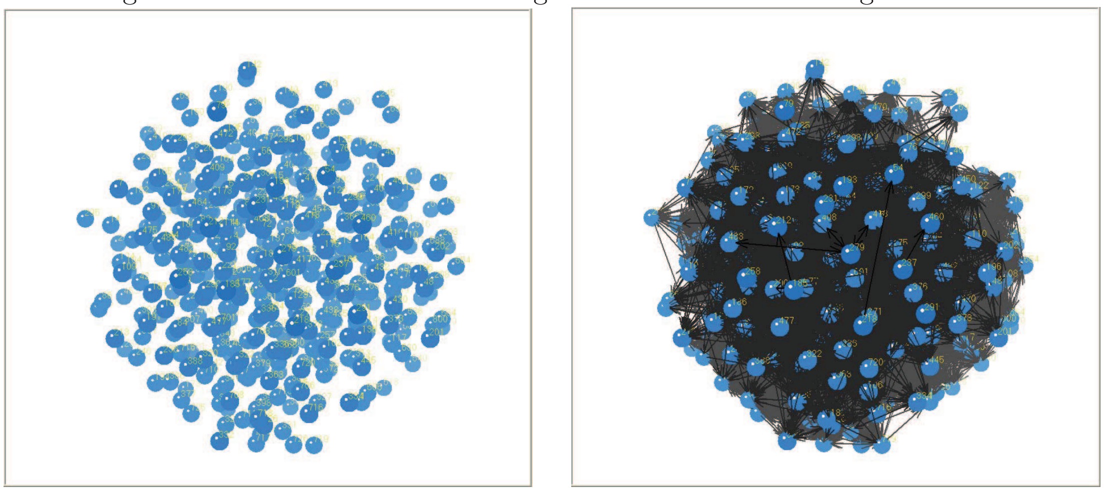
Generally speaking, a sector with a high authority weight gives large knowledge flows to sectors with highly ranked hub weights, and a sector with a high hub weight utilizes large knowledge flows from sectors with highly ranked authority weights. This measure of authority weight is more suitable for our purposes than a simple citation count (i.e. Garfield's impact factor) because not all citations are equally important. For example, when two sectors receive the same number of citations, it is desirable to rank the sector that receives citations from more important sectors higher than the other sector.

Figure 1 presents a three-dimensional network of inter-sectoral knowledge flows using patent citation data. Each vertex corresponds to a sector defined at the 3-digit USPCS level of disaggregation. Each directed link represents inter-sectoral citations, in which the arrow points to the citing sector or the direction of the knowledge flow. These inter-sectoral citations link all patent categories to form a globe-shaped network. The sectors located at the center (fringe) of the globe have the highest (lowest) authority weights among all categories.

In the data, the sectoral authority weight is positively correlated with the size of patent

[9]We also calculated aw and hw using the total number of citations from sector j to sector i as W^{ij} and find the results are robust to this alternative construction.

Figure 1: Network of the Knowledge Flows Across Technological Classes



(a) Without Citation Links (b) With Citation Links

stocks and the total number of firms in that sector. Our model in Section 3 endogenously generates this observation. In addition, authority weights and hub weights are highly correlated across sectors. Going forward, we only use the authority weight to measure the applicability of a given sector to other sectors.

2.2.1 Properties of the Authority Weight

We find that the authority weight is greatly heterogenous across sectors, even using the most disaggregated sector classification. The distribution of authority weights is highly skewed, close to a log normal distribution. In addition, a sector's authority weight ranking changes over time. Table 1 presents the five technological categories with the highest authority weight for the three years 1979, 1989 and 1999. In 1979, for example, the "Internal combustion engines" technology is the most widely applicable, but it drops out of the top five in 1989 and in 1999. In 1999, "Semiconductor device manufacturing" becomes the most applicable category.

We also translate the technological class into 42 industry sectors (at SIC 2-3 digit level) and compute the authority weight using the same method as described.[10] The variance of log authority weight is smaller at the 42 sector level compared to the more disaggregated classification. Table 2 shows the complete list of all 42 sectors sorted by authority weight in 2000. As one would expect, in general, the more complicated product classes such as "Electronics" and "Professional and scientific instruments" have higher authority weights, implying

[10] We use the total number of citations from sector j to sector i as W^{ij} to calculate aw and hw. Because most sectors cite each other at the less disaggregated 3-digit industrial level, using the previous weights reduces significant heterogeneity in sectors' authority weights and hub weights.

Table 1: List of the Top Five Technological Categories in Terms of Authority Weight for 1979, 1989, and 1999

year	category	names	authority weight
1999	438	Semiconductor device manufacturing: process	0.85278
1999	257	Active solid-state devices (e.g., transistors, solid-state diodes)	0.46518
1999	370	Multiplex communications	0.17430
1999	361	Electricity: electrical systems and devices	0.06167
1999	365	Static information storage and retrieval	0.04785
1989	514	Drug, bio-affecting and body treating compositions	0.93968
1989	424	Drug, bio-affecting and body treating compositions	0.29949
1989	428	Stock material or miscellaneous articles	0.05668
1989	604	Surgery	0.05495
1989	435	Chemistry: molecular biology and microbiology	0.05422
1979	123	Internal-combustion engines	0.99720
1979	514	Drug, bio-affecting and body treating compositions	0.04112
1979	60	Power plants	0.03811
1979	261	Gas and liquid contact apparatus	0.02636
1979	73	Measuring and testing	0.02058

that these sectors are more likely to be located at the center of the technology network. In contrast, more primary products tend to be in the periphery. There are a few exceptions; for example, "Transportation equipment" and "Aircraft and parts" both have low authority weights, but this is not surprising given that the technologies in these sectors are likely to be specialized.

2.3 Empirical Findings

We measure firm size by firm patent stock in all sectors.[11] In the patent data, many firms innovate in multiple sectors. Table 3 shows that more than half of firms innovate in more than one sector, and larger firms innovate in many areas. For example, a firm that has registered more than 10,000 patents by 2005 has inventions in some 260 different technological sector categories on average. Even a small firm is likely to innovate in more than one sector

[11] We can use name-matching procedures provided by Hall, et al. (2005) to link the NBER patent data to Compustat firm data; however, only 15% of the patenting firms are in Compustat. Based on this limited information, we find that the patent stock is positively correlated with standard measures of firm size (correlation coefficient is 0.6): sales and employment.

Table 2: List of 42 Sectors Ranked According to the Authority Weight

Field	Sector Name	Authority Weight	Hub Weight
36	Railroad equipment	0.00017	0.00021
38	Miscellaneous transportation equipment	0.00022	0.00027
37	Motorcycles, bicycles, and parts	0.00024	0.00022
35	Ship and boat building and repairing	0.00033	0.00029
28	Household appliances	0.00041	0.00070
25	Miscellaneous machinery, except electrical	0.00045	0.00033
14	Primary ferrous products	0.00059	0.00090
34	Guided missiles and space vehicles and parts	0.00069	0.00040
1	Food and kindred products	0.00093	0.00078
40	Aircraft and parts	0.00125	0.00108
39	Ordinance except missiles	0.00133	0.00102
7	Soaps, detergents, cleaners, perfumes, cosmetics and toiletries	0.00189	0.00158
11	Petroleum and natural gas extraction	0.00190	0.00170
3	Industrial inorganic chemistry	0.00232	0.00291
17	Engines and turbines	0.00268	0.00303
8	Paints, varnishes, lacquers, enamels, and allied products	0.00273	0.00346
24	Refrigeration and service industry machinery	0.00284	0.00304
15	Primary and secondary non-ferrous metals	0.00329	0.00358
9	Miscellaneous chemical products	0.00429	0.00428
5	Plastics materials and synthetic resins	0.00466	0.00657
18	Farm and garden machinery and equipment	0.00528	0.00593
19	Construction, mining and material handling machinery and equipment	0.00575	0.00614
13	Stone, clay, glass and concrete products	0.00670	0.00740
33	Motor vehicles and other motor vehicle equipment	0.00712	0.00693
2	Textile mill products	0.00776	0.00829
4	Industrial organic chemistry	0.00834	0.00898
6	Agricultural chemicals	0.00865	0.00651
20	Metal working machinery and equipment	0.00942	0.01143
10	Drugs and medicines	0.00982	0.00737
29	Electrical lighting and wiring equipment	0.01623	0.01278
30	Miscellaneous electrical machinery, equipment and supplies	0.01861	0.02048
22	Special industry machinery, except metal working	0.02046	0.02034
27	Electrical industrial apparatus	0.02110	0.02267
23	General industrial machinery and equipment	0.02431	0.02592
16	Fabricated metal products	0.02988	0.03529
31	Radio and television receiving equipment except communication types	0.03663	0.04815
42	All Other Sectors	0.03800	0.03936
12	Rubber and miscellaneous plastics products	0.04078	0.04329
26	Electrical transmission and distribution equipment	0.04212	0.05120
21	Office computing and accounting machines	0.32458	0.29495
41	Professional and scientific instruments	0.56854	0.56551
32	Electronic components and accessories and communications equipment	0.74939	0.76206

Table 3: Number of Product Classes of Firm Innovation by Firm Size, 2005

number of patents	average number of patents	average number of patent categories	median number of patent categories	percentage
1-10	3.1	1.7	1	46.4%
10-100	35.4	9.0	7	32.0%
100-1,000	320.7	42.1	35	16.7%
1,000-10,000	2733.9	128.3	119	4.3%
10,000-100,000	18410.9	259.4	254	0.6%

category.[12]

2.3.1 Firm Sizes, Innovation and Sectoral Entry

We first study how innovating firms expand across technological categories given the heterogenous authority weights of different patent classes. Our key finding is that the sectoral entry decision is scale-dependent. Compared to small firms, large firms enter younger sectors with lower authority weights, and the new sectors of large firms tend to be farther away from their existing sectors in the technology space. Compared to existing sectors, the new sectors of all firms tend to be younger, closer to those firms' current product mixes, and have lower authority weights. During a firm's life cycle, a firm starts at the center of the technology space and gradually ventures towards the periphery of the technology space. Our paper is motivated by this finding and we will explain later that sequential sectoral entry is key to understanding many other firm and sectoral level observations.

As a firm accumulates more patent stock and enters more sectors, its cross-sector distribution of patents also changes. In order to summarize the position of a firm in the overall technology space, we define a multi-product firm's overall product authority weight, $faw_{f,t}$, as the patent stock-weighted average of the individual product's authority weight. That is

$$faw_{f,t} = \sum_i aw_{i,t}\left(\frac{ps^i_{f,t}}{\sum_i ps^i_{f,t}}\right),$$

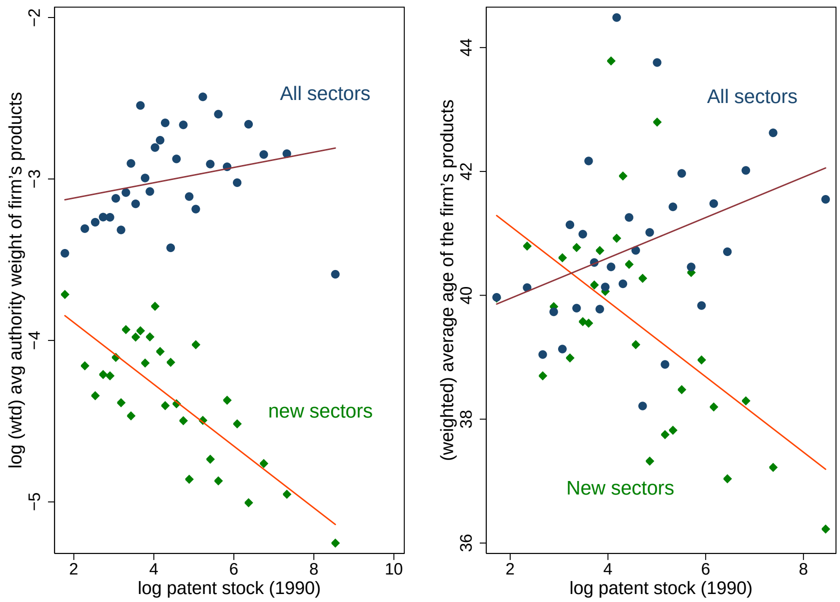
where $ps^i_{f,t}$ is the size of patent stock in technological category i owned by firm f in year t and $aw_{i,t}$ denotes the authority weight of the category i in year t.

The left panel in Figure 2 illustrates the scale dependence in the weighted average knowledge applicability of the current product mix by distinguishing sectors new to a firm from other sectors in which the firm innovates.[13] A low authority weight of a given sector could

[12] Although our dataset is different, this observation is also consistent with the findings in Broda and Weinstein (2007) that firms with higher sales also sell a greater variety of goods and sell in more sectors.

[13] We define a sector as new to a firm if the firm has not innovated in that sector in the past 10 years. In

Figure 2: Firm-level Observation: Scale Dependence of Authority Weight and Age of Firm's Patents



either imply that a sector is old and the knowledge is obsolete or that a sector is young and pioneering, such that not many firms have entered it and produced patents. To distinguish the former from the latter, we study the age of the sectors in which different-sized firms innovate in the right panel of Figure 2.[14] Firm sizes (i.e. patent stocks) are divided into 30 bins. Each graph presents the variable of interest according to the size bin of patent stocks in 1990. To complement our findings, we also construct a sector-pair knowledge distance measure in the Appendix which captures the shortest distance between two sectors in the technology network formed by patent citations.

Two observations stand out. First, a larger firm's product mix tends to be more applicable and older; however, this observation is sharply reversed when focusing on the flow of newly entered patent classes. Compared to small firms, large firms enter new sectors that have a lower authority weight and are younger. Second, independent of firm size, the new sectors entered by a given firm tend to be lower in applicability and younger relative to the existing sectors.

To further investigate the innovation patterns over time, we run the following two fixed

the patent data, we find that the time gap between two innovations in the same sector by the same firm is on average 2.2 years and, in rare cases, can be as high as 31 years. 95% of firms have a gap smaller than 7 years.

[14] The product age is defined as the prevailing year minus the first year in which the product exists in the patent dataset. The NBER patent data (1963-99 version) includes patents back to 1901.

effect regressions, controlling for firm fixed effects in each case.

$$\begin{aligned}\ln x_{if,t} &= \beta_0 + \beta_1 \ln ps_{f,t} + \beta_2 I_{if,t} + \mu_f + t + \varepsilon_{if,t} \\ \ln x_{if,t} &= \beta_0' + \beta_1' \ln ps_{f,t} + \beta_2' I_{if,t} \cdot \ln ps_{f,t} + \mu_f + t + \varepsilon_{if,t}\end{aligned}$$

where $ps_{f,t}$ is firm f's patent stocks over all sectors, $I_{if,t}$ is a dummy variable equal to one if firm f is a new entrant in sector i at time t and $x_{if,t}$ is the outcome variable, such as authority weight, age and median distance from other sectors to sector i in which firm f innovates at time t. The results shown in Table 4 are consistent with the cross-sectional findings. The first regression results suggest that the new sectors that a firm enters are younger, and closer to their existing products, but farther away from the center of the technology space than the existing sectors. When we compare the new sectors that different firms choose to enter, larger firms tend to enter less applicable, younger sectors that are also more technologically isolated from the existing product mix, since $\beta_1' + \beta_2' < 0$.[15]

2.3.2 Firm Innovation Rates

We find that the applicability of a firm's current product mix predicts its subsequent innovation rate. The left panel of Figure 3 plots the growth rates of patent stock by firm size over a one year interval. The figure shows that the difference in innovation rates between small and large firms can be as high as 40 percent within a year. In the right panel of Figure 3, we plot the innovation rates against the initial firm product applicability levels (defined below). We find that firms that initially produce highly applicable products innovate 30 percent faster than firms that produce poorly applicable ones.

We formally study this observation in a group of panel regressions, in which we regress the average innovation rate, $g_{f,t}$ over the past three years on the initial firm size (i.e. patent stock) and the initial authority weight of the firm's product mix.[16]

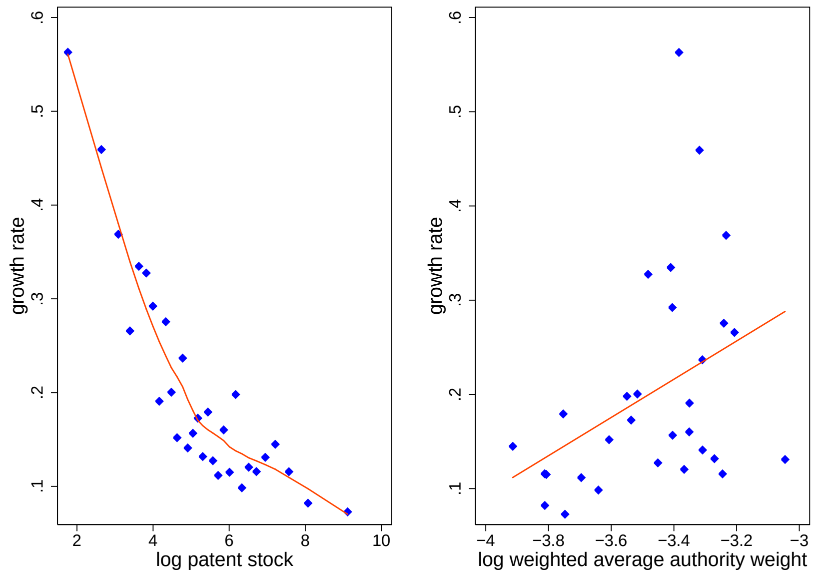
$$g_{f,t} = \beta_0 + \beta_1 \ln ps_{f,t-3} + \beta_2 \ln faw_{f,t-3} + \mu_f + \varepsilon_{f,t}$$

It is also informative to study the extensive margin and the intensive margin of firm growth rate separately. Let g_{ext} be the extensive growth rate attributable to patent applications in

[15] There is also a common exit pattern. We define that a firm exits a patent class if it stops applying for patents in that class for 10 years. We also find that large firms give up center sectors (sectors with a high authority weight) as they expand towards the fringe of the product space. This suggests there is an increasing maintenance cost as firms carry a larger scope of products. We conjecture that large firms exit center products to avoid intense competition from small firms, who enter center products for the prospects of future extensive growth and the large public knowledge pool.

[16] Varying the time lag does not change the results.

Figure 3: Firm-level Observation: Firm Innovation Rates, Firm Sizes and Initial Product Applicability



new technological classes and g_{int} be the intensive growth rate coming from patent applications in existing classes. Define ps_t^{New} as the number of patent applications in *new* classes at time t. Therefore,

$$
\begin{aligned}
g_{ext,t} &= \frac{ps_{f,t}^{New}}{ps_{f,t-3}} \\
g_{int,t} &= \frac{(ps_{f,t} - ps_{f,t}^{New}) - ps_{f,t-3}}{ps_{f,t-3}}
\end{aligned}
$$

The panel regression results are recorded in the lower panel of Table 4. All three innovation rates decrease with firm size but increase with the initial firm's product authority weight. There are two explanations for the decreasing intensive growth margin with respect to firm size. First, products from the same firm may be closer substitutes than products of different firms. Since a firm's new products become closer competitors of its previous products in the same category, the return from one more product in the existing category decreases as a firm accumulates more products in the same class. Second, the gain from learning is smaller for firms with significant knowledge in a particular class. It is worth noting that the initial authority weight of the product mix counts more for a firm's extensive growth than its intensive growth. Intuitively, given the knowledge stock, firms that start from higher applicable initial product mixes can effectively apply their knowledge to innovate in many other related sectors. Simultaneously, the accumulated knowledge in the related sectors contributes to

Table 4: Firm Entry, Innovation Rates and Knowledge Linkages

Dependent	Independent variables			
	$\log ps$	Dummy(new sector)	$logps$	$\log ps$*Dummy
$\log aw$	-0.039	-0.300	0.029	-0.111
	(0.013)***	(0.010)***	(0.013)***	(-0.003)***
$\log dist$	0.0129	0.027	0.0066	0.0084
	(0.0008)***	(0.0009)***	(0.0008)***	(0.0002)***
$\log age$	0.111	-0.532	-0.256	-1.603
	(0.047)***	(0.073)***	(0.042)***	(0.016)***
	$\log ps$ (t-3)	$\log faw$ (t-3)		
g	-16.71	0.538		
	(0.292)***	(0.049)***		
g_{ext}	-14.32	0.421		
	(0.287)***	(0.051)***		
g_{int}	-2.38	0.117		
	(0.062)***	(0.022)***		

Note: We also control for year, firm fixed effect and clusters *** significance at 1% level. Robust standard errors are reported in parentheses.

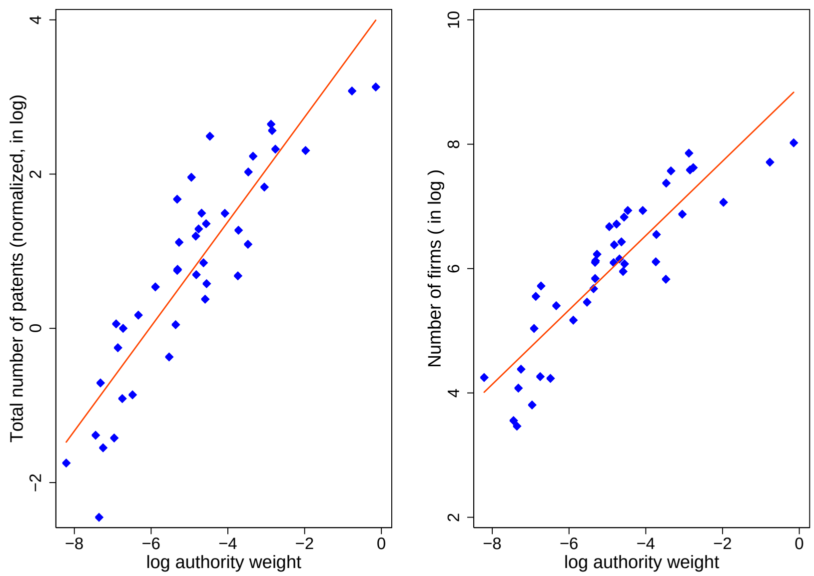
innovation in existing sectors, driving growth along the intensive margin.[17]

2.3.3 Sectoral Sizes, Number of Firms and Sectoral Authority Weight

Figure 4 presents two observations at the sector level. We plot the total number of patents and the number of innovating firms in various sectors against the sectoral authority weight. It shows that there are significantly more patents and larger numbers of firms in center sectors. Because the vertical axis is in logarithm, this implies that sectoral patent stocks and the number of firms are highly skewed toward center sectors. These observations imply that important obstacles exist that prevent small firms from reaching the far end of the technology space. We model these obstacles as fixed sectoral R&D costs in the model.

[17] We also investigate quality-adjusted growth rates, which are measured by the growth rates of the forward-citation-weighted number of patents. When adjusted by the number of inward citations, larger firms' growth rates drop even faster, because the number of inward citations per patent decreases with firm size in both the extensive margin (number of classes) and the intensive margin (number of patents within the class). The results above are also robust to different levels of disaggregation of the technology classes (800 categories according to IPC or 42 industry sectors).

Figure 4: Sector-level Observation: Sectoral Sizes, Number of Firms and Sector Applicability



3 The Model

Our model focuses on firms' innovation behavior. Understanding how innovation takes place at the firm level is important because patenting firms account for a highly disproportionate share of economic activity and play an important role in the aggregate production (Balasubramanian and Sivadasan, 2008). We regard product innovation as a process of generating new varieties in different sectors and knowledge as a particular type of know-how or technology embodied in a sector that can be used to innovate in another sector by spillovers.[18]

There are three key features of the model: first, a firm conducting R&D in one specific sector can apply knowledge accumulated in *all* related sectors; second, a firm has to pay a fixed cost, F^i, to rent a license or research facilities for conducting research in sector i, and third, firms can learn from each other, even though not completely.

These elements are motivated by the following observations. First, 37 percent of citations in NBER patent data are inter-sector citations (when sectors are defined at the 3-digit US SIC level, and the percentage becomes higher when using more disaggregated classifications). This highlights the important role of inter-sector spillovers in individual firms' innovation behavior. The percentage of inter-sector citations in total citations varies between 27 percent

[18] Balasubramanian and Sivadasan (2008) provide evidence showing that firms patenting significantly increases firms' product scope rather than reduce the cost of producing existing goods, consistent with our interpretation of innovation in the model. Earlier evidence cited by Scherer (1980) also shows that firms allocate 87% of their research outlays to product improvement and developing new products and the rest to developing new processes. In our paper, we interpret "product improvement" as developing new varieties in existing sectors and "new products" as varieties in sectors new to the firm.

and 70 percent in different sectors. Second, firms develop products in multiple but not all U.S. patent classifications. In our data, an average firm applies for patents in 3.8 out of 42 industry categories. In a model without sectoral entry barriers, all firms would enter all sectors because of the positive inter-sector spillovers, and firms would grow at rates independent of their product mixes. In this case, we would also not observe a concentration of firms and varieties in center sectors. Third, 85 percent of citations are given to patents owned by other institutions, which suggests that public information and imitation are important knowledge sources for R&D. The public knowledge in the model helps to explain why a newborn firm can still enter the economy while paying a fixed entry cost and why they initially choose to enter highly applicable sectors. Access to public knowledge also prevents firms from getting too small, which helps to ensure a stationary firm size distribution.

There are three types of goods: a final consumption good, sectoral goods and sectoral-differentiated goods. Sectoral goods are aggregated over differentiated goods that are produced by individual monopolistically competitive firms. We interpret varieties within a sector in our model as corresponding to the lowest level of disaggregation of commodities and each variety maps into a patent used in the patent data. Sectors, however, map into technology categories in the patent data. Although the model corresponds well with analysis of sectors with limited numbers of competitors, to make the analysis more tractable, we follow Hopenhayn (1992) and Klette and Kortum (2004) by assuming each firm is relatively small compared to the entire sector.

3.1 Demand

The representative infinitely lived consumer's optimization problem is:

$$U = \max_{\{C_t\}} \sum_{t=0}^{\infty} \beta^t \log C_t$$

where final goods consumption is a Cobb-Douglas combination of the sectoral consumption Q_t^i. There are K sectors in the economy. Let s^i capture the share of income spent in sector i. $\beta < 1$ is the intertemporal discount rate. We have

$$\log C_t = \sum_{i=1}^{K} s^i \log\left(Q_t^i\right), \tag{1}$$

where Q_t^i and P_t^i are the consumption and the price index of differentiated goods in sector i. The consumption and price of good k in sector i is given by $x_{k,t}^i$ and $p_{k,t}^i$.

$$Q_t^i = \left(\int_0^{I_t^i} \left(x_{k,t}^i \right)^{\frac{\sigma^i - 1}{\sigma^i}} dk \right)^{\frac{\sigma^i}{\sigma^i - 1}}, \; i = 1, 2, ..., K,$$

where I_t^i denotes (the measure of) the total number of varieties in sector i and $\sigma^i > 1$ is the elasticity of substitution between differentiated goods of the same sector i. The demand function for goods within a sector is given by:

$$x_{k,t}^i = \left(\frac{p_{k,t}^i}{P_t^i} \right)^{-\sigma^i} Q_t^i. \tag{2}$$

3.2 Firm Production

Each firm produces a set of goods in various sectors. In the model, every patent turns into a product. Firm size $\boldsymbol{z}_{f,t}$ is a K-dimension vector, where $z_{f,t}^i$ is the number of differentiated sector i goods produced by firm f at time t.

Firms hire one unit of labor to produce one unit of a differentiated good. All goods in the same sector are charged at the same price and are sold in the same quantity. We normalize the wage to one and assume perfect competition in the final good sector. The final good price is $P_t = B \prod_i^K (P_t^i)^{s^i}$, where B is some constant consistent with the Cobb-Douglas specification in Equation (1) and

$$P_t^i = (I_t^i)^{\frac{1}{1-\sigma^i}} \frac{\sigma^i}{\sigma^i - 1}.$$

Because $\sigma^i > 1$, the sectoral price decreases with the total number of varieties in that sector.

Given the demand equation (2), the *aggregate* profit from producing in sector i is time-invariant:

$$\pi^i = \frac{s^i Y}{\sigma^i},$$

where the total income Y is the same as total expenditure, PC, in the closed economy. Per-unit profit of a differentiated good in sector i is π^i / I_t^i. When a sector expands, the profit per variety decreases. Firm f's profit in sector i with current number of products $z_{f,t}^i$ is given by $\pi^i z_{f,t}^i / I_t^i$.

3.3 Knowledge Diffusion Matrix

In the model, firms understand the magnitude of knowledge linkages between any two sectors. We use matrix $\boldsymbol{A}$ to describe the knowledge diffusion network. The $\{i,j\}^{th}$ element, A^{ij}, measures the innovation productivity when using knowledge in sector j to conduct R&D in sector i. Sectors i and j are viewed as connected with a directed link pointing from j to i when $A^{ij} > 0$. Generally speaking, sector i is an important knowledge contributor if there are many large positive elements in the i^{th} column of matrix $\boldsymbol{A}$; sector i is good at utilizing knowledge if there are many large positive elements in the i^{th} row of $\boldsymbol{A}$.

3.4 Firm's R&D Decision

We denote the set of sectors in which firm f is producing at time t as $S_{f,t}$, which includes all sectors i such that $z^i_{f,t} > 0$. The notation i represents the knowledge receiving sector and j represents a knowledge providing sector. As an incumbent of sector i, firm f invents $\Delta z^i_{f,t}$ number of new goods by applying its private knowledge stock, $z^j_{f,t}$, from sectors $j \in S_{f,t}$, and public knowledge stock $\bar{z}^j_{ft}$ from every sector j. Here, the private knowledge stock is measured by the number of varieties produced by the firm, which equals its past innovations $z_{f,t}$. $\bar{z}^j_t$ is the average number of goods per firm in sector j, which captures the size of public knowledge pool generated by all firms in that sector.[19] A sizable sector (i.e. high I^i) can have a small public knowledge pool if there are a large number of firms in the sector (i.e. high M^i). We allow firms to not only draw upon the internal sources for expansion (as in Klette and Kortum, 2004) but also on external sources such as imitation. $\boldsymbol{z}_{f,t}$, $\bar{\boldsymbol{z}}_t$ and $\Delta\boldsymbol{z}_{f,t}$ are K-dimensional vectors. The firm chooses the optimal investment in R&D, given the accessible knowledge stock in each sector and the knowledge diffusion matrix $\boldsymbol{A}$.

In a stationary equilibrium there always exist many large firms that have already entered every sector. The knowledge capital market is efficient, and as a result, the value of one variety (or patent) in every sector is equivalent to the price that an all-sector firm is willing to pay. Thus, we derive the marginal value of one unit of sectoral knowledge from an all-sector firm's optimal R&D decision. In this case, the value of a small firm that has entered only a subset of sectors is the total value of all its varieties. That is, if a small firm cannot apply its sector j knowledge to sector i because it cannot afford the entry cost, it can still sell its patents to large firms that have already entered sector i. As long as there are many such

[19] This assumption ensures that the sectoral growth rate is independent of the number of firms and population in the general equilibrium. When learning is costly, each firm is too small to access all existing knowledge in the sector. When firms randomly meet and imitate a limited number of peers, the average number of patents per firm is a better proxy for public knowledge pool than the total number of patents in that sector.

large potential buyers, the market price of knowledge capital will be bid up to its marginal value for an all-sector firm.[20]

The per period net profit of an all-sector firm is given by the total profit from production in all sectors minus the cost of hiring R&D researchers. We do not distinguish the production labor from the researchers, and we assume that they are equally paid. Let $R^{ij}_{f,t}$ denote firm f's R&D input when utilizing sector j's knowledge to invent new goods in sector i. We assume the research workers' efficiency increases at the same rate as the average number of innovations per firm in the innovating sector i, $\bar{z}^i_t$, thus the effective R&D is given by $\bar{z}^i_t R^i_{f,t}$. This assumption is important in removing the "scale effect" from the model.

The all-sector firm f solves the following problem:

$$\max_{\left\{R^{ij}_{f,t}\right\}_{i,j\in\{1,2,\ldots,K\}}} V(\boldsymbol{z}_{f,t}) = \sum_{j=1}^{K} \pi^j \frac{z^j_{f,t}}{I^j_t} - \sum_{i=1}^{K}\sum_{j=1}^{K} R^{ij}_{f,t} + \frac{1}{1+r} E[V(\boldsymbol{z}_{f,t+1})] \tag{3}$$

subject to the knowledge capital accumulation condition in every sector

$$z^i_{f,t+1} = z^i_{f,t} + \Delta z^i_{f,t}, \tag{4}$$

where the incremental innovation is

$$\Delta z^i_{f,t} = \sum_{j=1}^{K} \left[A^{ij} \left(\bar{z}^i_t R^{ij}_{f,t}\right)^{\alpha} \left(z^j_{f,t} + \gamma^j \bar{z}^j_t\right)^{1-\alpha} + \varepsilon^{ij}_{f,t}(z^i_{f,t} + \gamma^j \bar{z}^j_t) \right] \tag{5}$$

Here, the innovation production is a Cobb-Douglas combination of innovation productivity, a firm's R&D investment and accessible knowledge stock. γ governs the relative effectiveness of public knowledge to private knowledge in innovations. By its nature, innovation includes the discovery of the unknown; therefore, the success of a research project can be uncertain. We assume that in each period the success rate of innovation is subject to a zero mean innovation shock $\varepsilon^{ij}_{f,t}$ that is firm- and sector-pair specific. One important assumption is that these shocks are identical and independently distributed across firm, sector pairs and time. In addition, household time preferences pin down the discount factor $\frac{1}{1+r} = \beta \frac{u'(C_{t+1})/P_{t+1}}{u'(C_t)/P_t} = \beta \frac{C_t P_t}{C_{t+1}P_{t+1}} = \beta$.

The solutions (derived formally in the Appendix) are described by the following equations. On the balanced growth path, the firm's value is a linear combination of the value of its

[20] Without an efficient knowledge capital market, small firms might be unmotivated to conduct R&D, since they could not internalize cross-sector knowledge spillovers when there is a fixed cost to enter every related sector.

knowledge in all sectors,

$$V(\boldsymbol{z}_{f,t}) = \sum_{j=1}^{K} v^j \frac{z^j_{f,t}}{I^j_t} + u^j,$$

where v^j is the market value of sector j's knowledge stock,

$$v^j = (1-\rho)^{-1}(\pi^j + \sum_{i=1}^{K} \omega^{ij}), \tag{6}$$

and u^j captures the rent from public knowledge equally shared by every firm in sector j,

$$u^j = (1-\rho)^{-1} \sum_{i=1}^{K} \omega^{ij} \frac{\gamma^j}{M^j}. \tag{7}$$

Here, ω^{ij} captures the marginal value (contribution) of applying sector j's knowledge to innovation in sector i,

$$\omega^{ij} = \frac{I^j}{I^i} \frac{1-\alpha}{\alpha} \left(A^{ij} \alpha \rho v^i\right)^{\frac{1}{1-\alpha}} (M^i)^{\frac{\alpha}{\alpha-1}}. \tag{8}$$

M^j is the number (mass) of firms in sector j. For simplicity, let $\rho \equiv \beta/g$ and $g = I_{t+1}/I_t$ for the rest of the paper. This marginal value increases with the relative size of sector j to i, the value of application sector i, the knowledge spillover strength from j to i and decreases with the number of firms in the application sector i.

The optimal investment in each type of R&D is proportional to the knowledge capital and positively related to the connectivity between sector i and j, ω^{ij}. A larger share of R&D input in innovation activity, α, also implies more R&D investment.

$$R^{ij}_{f,t} = \frac{\alpha}{1-\alpha} \omega^{ij} \frac{z^j_{f,t} + \gamma^j \bar{z}^j_t}{I^j_t} \tag{9}$$

Importantly, Equation (6) shows that the market value of knowledge in sector j, v^j, depends on not only the *direct* economic value, i.e. the present discounted value of the future profits in sector j, but also on its *indirect* technological value captured by its contribution to future innovations in all K sectors. When knowledge from different sectors is not connected, i.e. $A^{ij} = 0$ for $i \neq j$, the marginal contribution of specific knowledge is limited to the future innovation and production within its sector. In our paper, we emphasize the role of transferrable knowledge, i.e. $A^{ij} > 0$. A stronger knowledge contribution to other sectors A^{ij} and a higher marginal value of product from the knowledge-receiving sector v^i increases the marginal value of innovation in the knowledge contributing sector j. Fast sectoral growth dilutes the marginal firm value v^j. Similarly, Equation (7) implies that when public knowledge is easier to access or when knowledge in other sectors is more valuable, the rent from external knowledge is higher.

The value of firm f is thus given by

$$V\left(\boldsymbol{z}_{f,t}\right)=\sum_{j=1}^{K}(1-\rho)^{-1}[\pi^{j}\frac{z_{f,t}^{j}}{I_{t}^{j}}+\sum_{i=1}^{K}\omega^{ij}\frac{z_{f,t}^{j}+\gamma^{j}\bar{z}_{t}^{j}}{I_{t}^{j}}]. \tag{10}$$

3.5 Sectoral Entry and Exit

Firms need to rent a license or research facilities and devices to conduct research in a sector. Every period, the opportunity cost to stay in sector i is equal to the discounted interest rate loss associated with renting the license or facilities, $\frac{r}{1+r}F$, which is also the entry cost for a potential entrant. A firm chooses to enter or stay in sector i if its expected profit from innovating in sector i using available private and public knowledge is greater than the fixed cost of conducting R&D in that sector. The expected profit of entering or staying is the difference between the expected value of investing in R&D in sector i and not investing, liquidating the knowledge stock, and dismissing the research group in the next period. The price of the knowledge a firm can sell is given by the knowledge value, v^i as in Equation (6).

The fixed cost is the same for all firms innovating in a given sector, but the expected profit of continuation or entry is firm-specific. Formally, the firm would enter or continue its research in sector i if

$$\frac{r}{1+r}F\leq-\sum_{j=1}^{K}R_{f,t}^{ij}+\beta\frac{v^{i}E_{t}\Delta z_{f,t}^{i}}{I_{t+1}^{i}}=\sum_{j=1}^{K}\omega^{ij}\frac{\left(z_{f,t}^{j}+\gamma^{j}\bar{z}_{t}^{j}\right)}{I_{t}^{j}} \tag{11}$$

Although $z_{f,t}^{i}=0$, a firm can apply its previous private and public knowledge from other sectors to invent new goods and expect that the additional future value from innovating in sector i will exceed the rental cost.

A firm exits sector i if the license cost is higher than the expected benefit of staying (i.e. inequality (11) is reversed). The exiting firm sells its sector-i knowledge stock z_f^i to an all-sector firm immediately upon exit. This is because if the firm keeps the patents without continuing research in that sector, these patents' value, $v^i z_f^i/I^i$ would depreciate in the future (I^i increases over time but v^i stays constant).

Since firm innovation is subject to stochastic shocks, there are a number of fortunate firms in every period that accumulate just enough private knowledge capital in related sectors to enter a new sector, and a number of unfortunate firms who lose knowledge capital and have to exit. In this sense, entry and exit are both *self-selection* processes.

The series of sectoral entry and exit decisions govern the number of firms and the average firm size in each sector in the general equilibrium. Higher fixed costs cause fewer firms to enter but increase the average firm size in sector i, because potential entrants have to increase knowledge in related sectors before entry. In addition, more accessible public knowledge (i.e. a larger γ^j) induces more entrants and a smaller average firm size in every sector, because if the rent from public knowledge covers a larger share of entry cost, then potential entrants can enter with less related private knowledge.

The entry and exit conditions also explain why, empirically, there are more firms in general and more small firms in particular in highly applicable sectors compared to isolated sectors. Operating in frontier sectors requires a minimum stock of knowledge in all related sectors on a firm's extensive growth path.

As shown in Equation (11), many large positive elements in the i^{th} row of knowledge diffusion matrix $\boldsymbol{A}$ means there are more and wider open routes that allow potential entrants to enter sector i. An increase in the knowledge value in sector i, v^i, also attracts entry and deters exit. A larger number of existing goods, I^i, and more incumbent firms, M^i, deter entry.

3.6 Aggregate Conditions

In this economy, the household owns all the firms and finances all the potential entrants. In the stationary equilibrium, there are no net entrants. Therefore, the household's total income in the stationary equilibrium is

$$Y = wL + r \int_{i \in S_f} \sum_i v^i \frac{z_f^i}{I^i} df = L + r \sum_i v^i \tag{12}$$

In this economy, there are three types of labor: production workers, researchers and workers who are engaged in making entry licenses. Hence, the labor input required to make entries in sector i is $F^i \Delta M^i$. However, on the balanced growth path, the number of licenses is fixed. The number of entering firms is the same as exiting firms, i.e. the number of net entry $\Delta M^i = 0$. Formally, the labor market clearing condition is:

$$L = \sum_{j=1}^{K} \frac{\sigma^j - 1}{\sigma^j} s^j Y + \sum_{i=1}^{K} \sum_{j=1}^{K} \int_{i,j \in S_{f,t+1}} R_{f,t}^{ij} df + \sum_{i=1}^{K} F^i \Delta M_t^i. \tag{13}$$

4 General Equilibrium Analysis

4.1 Firm Size Dynamics

In a typical firm's life span, a firm starts from a relatively highly applicable sector. After accumulating enough background knowledge, a small firm with a sequence of good draws of innovation shocks can expand into related sectors along the knowledge diffusion network. After several rounds of entry selection, only a few large, multi-sector firms can reach the frontier sectors at the edge of the knowledge networks.

Putting (4), (5) and (9) together and normalizing the firm size in each sector by the total size of that sector, i.e. $\tilde{z}^i_{f,t} = z^i_{f,t}/I^i_t$[21], yields the following firm size dynamics:

$$\tilde{\boldsymbol{z}}_{f,t+1} = \boldsymbol{\Phi_{f,t}}\tilde{\boldsymbol{z}}_{f,t} + \boldsymbol{\Theta_{f,t}}\gamma\bar{\tilde{\boldsymbol{z}}}_t, \tag{14}$$

where the $\{i,j\}^{th}$ elements of the $K \times K$ matrices $\boldsymbol{\Phi_{f,t}}$ and $\boldsymbol{\Theta_{f,t}}$ are given by $\phi^{ij}_{f,t}$ and $\theta^{ij}_{f,t}$ respectively, defined as:

$$\begin{aligned}
\phi^{ij}_{f,t} &= \frac{1}{1+g}[1_{\{\text{if } i=j\}} + \left(A^{ij}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha\rho v^i}{M^i}\right)^{\frac{\alpha}{1-\alpha}} + \varepsilon^{ij}_{f,t}] \\
\theta^{ij}_{f,t} &= \frac{1}{1+g}\left(A^{ij}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha\rho v^i}{M^i}\right)^{\frac{\alpha}{1-\alpha}} + \varepsilon^{ij}_{f,t}
\end{aligned}$$

where $1_{\{\text{if } i=j\}}$ is one if $i = j$ and zero otherwise. In general, an incumbent firm innovates faster, if there are more large elements in the i^{th} row of matrix A, sectoral knowledge is more valuable, and the number of incumbents is smaller.

The existence of public knowledge plays an important role in attenuating the size dispersion generated by idiosyncratic innovation shocks and generating constant firm size heterogeneity over time. In fact, without public knowledge, Equation (14) becomes $\tilde{\boldsymbol{z}}_{f,t+1} = \boldsymbol{\Phi}_{f,t}\tilde{\boldsymbol{z}}_{f,t}$; this would cause the variance to grow explosively over time and the distribution to be lognormal without a steady state.

We assume that the shocks to the growth rate of firm size, $\varepsilon^{ij}_{f,t}$, are identically and independently distributed. According to Kesten (1973), equation (14) implies that firm size distribution converges to a Pareto distribution with shape coefficient b such that Champernowne's equation holds, i.e. $E\Phi^b_{f,t}(\varepsilon_{f,t}) = 1$.[22]

[21] In the stationary equilibrium, the number of firms is constant. Thus, this normalization is equivalent to normalization by average firm size, which means the normalized mean size is always one.

[22] See Gabaix (2009) and Cai (2009) for more detailed explanations about size distribution. The relative heterogeneity of firm sizes in a sector should negatively depend on the accessibility of public knowledge, γ^i and the relatedness of this sector with other sectors, and positively relates to innovation shock σ^i in that

4.2 Growth Rate and Fixed R&D Costs

The number of varieties in sector i evolves according to $I^i_{t+1} = (I^i_t + \int_{i\in S_{f,t+1}} \triangle z^i_{f,t} df)$. Define τ^{ij} as the fraction of private knowledge of sector j owned by firms who innovate in both sector i and j, i.e. $\tau^{ij} = \int_{i,j\in S_f} z^j_f df / I^j$. Using Equation (5) we derive the (gross) growth rate of the number of varieties in every sector, $g = I^i_{t+1}/I^i_t$, as

$$g = \sum_{j=1}^{K} \frac{(1+\gamma)\omega^{ij}}{(1-\alpha)\rho v^i} \tau^{ij}. \tag{15}$$

Combining (15) with (6) and (8), we obtain

$$g = (1-\beta)\left[\lambda \frac{\sum_i \sum_j \omega^{ij} + \sum_i \pi^i}{\sum_i \sum_j \omega^{ij}\tau^{ij}} - 1\right]^{-1}, \tag{16}$$

where $\lambda = \frac{(1-\alpha)\beta}{(1+\gamma)}$. Equation (16) shows that sectoral entry costs decrease the growth rate of the total number of varieties in the economy, because $\tau^{ij} < 1$.

Without license costs, every firm in the economy is an all-sector firm. Combining (15) with (6) and (8) allows us to express the growth rate of the number of products in the economy by the ratio of total production profit in the economy and the total contribution of all the knowledge from producing in every sector in Equation (17).

$$g = (1-\beta)\left[\lambda(1 + \frac{\sum_{i=1}^{K} \pi^i}{\sum_{i=1}^{K}\sum_{j=1}^{K} \omega^{ij}}) - 1\right]^{-1}. \tag{17}$$

Recall that the marginal value of an additional market share in sector i, v^i, depends on two elements: the profit flow, π^i, and the value associated with knowledge spillovers from sector i to all other sectors, $\sum_j^K \omega^{ij}$. Interestingly, Equation (17) implies that what is important for growth is the share of the firm value accounted by knowledge spillovers across sectors relative to the value generated by profit.

It is worth pointing out that by assuming that the efficiency of R&D workers is proportional to the average knowledge stock in that sector, we eliminate the "scale effects" of population on economic growth. This can be seen from equation (15). Both ω^{ij} and v^i are proportional to the total population in the economy; therefore, the growth rate of varieties is independent of the level of population.[23]

same sector.

[23]Jones (1990) first pointed out that the "scale effects" that plague many endogenous growth models are not consistent with empirical evidence. For a detailed discussion on this, also see Jones (1999).

4.2.1 The Resource Allocation Effects of Fixed R&D Costs

Combining Equations (9) and (15), we find that sectoral R&D resources are allocated according to the shadow values of the sectoral knowledge. With the same expenditure share, s^i, in every sector, we have the sectoral R&D intensity given by

$$R^i \equiv \frac{\sum_{j=1}^{K} R^{ij}}{sY} = \frac{\alpha\beta}{sY}\frac{g-1}{g}v^i. \tag{18}$$

Therefore, any policy that distorts $\{v^i\}$ also causes misallocation of research investment across sectors.

We identify another efficiency loss due to sectoral fixed R&D costs – a misallocation of research resources across sectors, which is an unexplored area in the literature. An ideal economy would allocate significantly more research resources to sectors with more applicable knowledge. However, sectoral fixed costs distort R&D resource allocation across sectors by pushing research effort away from center sectors toward peripheral sectors; hence, the relative sizes of center sectors are smaller than they would be without entry costs.

Later we show in our simulation that positive fixed R&D costs indeed lead to the flattening of the size distribution of sectors (see Figure 12) because firms enter center sectors before venturing into fringe sectors, knowledge generated by center sectors is fully utilized in innovation at fringe sectors, but since not all firms that innovate at the center are able to reach the fringe, knowledge does not completely flow back to the center from the fringe. Secondly, the lesser number of incumbent firms at fringe sectors means the market value per firm is larger. According to Equation (9), each incumbent firm therefore invests a disproportionally large research fund in these sectors.

At the aggregate, the sectoral fixed cost leads to the incomplete cross-sector knowledge circulation in the technology network and insufficient R&D resource in the GPT sectors, which constitute the knowledge engine of the economy. This reduces the innovation rate in all sectors and decreases the growth rate of the economy.

4.3 Special Case with No Fixed R&D Costs: Analytical Results

When there is no fixed cost to enter a new sector, every firm produces and conducts research in every sector in the stationary equilibrium. Hence $M^j = M$. We define the second component of the value of entry in (11) as $F^e \equiv \sum_{j=1}^{K} \omega^{ij}\gamma^j \bar{z}^j / I^j$, which can be interpreted as the registration cost of a new firm. Under the condition of free entry, the total number of firms that exist in the economy is determined by this registration cost. The model can be solved analytically based on the following equilibrium conditions:

$$
\begin{aligned}
v^j &= \frac{1}{1-\rho}(\pi^j + \sum_i \omega^{ij}) \\
\omega^{ij} &= \frac{I^j}{I^i}\frac{1-\alpha}{\alpha}(A^{ij}\alpha\rho v^i)^{\frac{1}{1-\alpha}}(M)^{\frac{\alpha}{\alpha-1}} \\
g &= \frac{\sum_j (1+\gamma^j)\omega^{ij}}{(1-\alpha)\rho v^i} \\
L &= \sum_i \frac{\sigma^i - 1}{\sigma^i} s^i Y + \frac{\alpha}{1-\alpha}\sum_i\sum_j \omega^{ij}(1+\gamma^j) \\
Y &= L + r\sum_i v^i \\
F^e &= \sum_i \omega^{ij}\gamma^j / M
\end{aligned}
$$

For a set of parameter values, we can solve for $K^2 + 2K + 2$ numbers of unknowns $\{v^j\}_j$, $\{\omega^{ij}\}_{ij}$, $\{I^j/I^i\}_{ij}$, M, g, Y, using the same number of equations above.

We use industrial classifications with $K = 42$ sectors listed in Table 2 in our estimation. The relevant patent citation data (1980-2000) is employed to discipline some of the parameters.[24] Each element of the knowledge diffusion matrix is estimated by the relative contribution of sector j's knowledge in sector i's R&D using the patent citation data. Specifically, it is the fraction of citations made to sector j by sector i, OC^{ij}/OC^i, adjusted by the relative importance of sector j, IC^j/IC, which is measured by the overall citations received by j as a ratio of total citations. This adjustment is supported by the observation that sectors with large patent stocks tend to be cited more frequently. Therefore,

$$A^{ij} = \frac{OC^{ij}}{OC^i} / \frac{IC^j}{IC}.$$

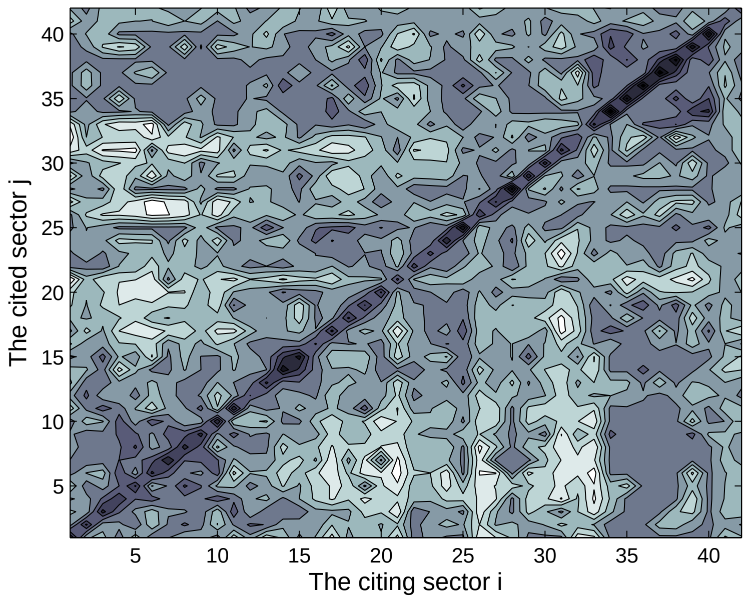
Figure 5 shows a contour graph of the knowledge diffusion matrix for these sectors. The darkest area on the diagonal reflects the fact that a large proportion of citations goes to patents in the same sector. This is not particularly surprising given that sectors in this case are not highly disaggregated; however, most sectors also allocate a fair amount of citations to patents from other sectors, reflecting the importance of cross-sector knowledge spillovers.

Our measures of the elasticities of substitution between varieties within a sector i, σ^i, are adapted from Broda and Weinstein (2003).[25] The parameter governing the relative

[24] The specific parameter values are available in our web Appendix.

[25] Their data use the SITC Rev. 2.3 industry classification. We match SITC 3 digit classifications to 3 digit SIC72 to be consistent with patent data.

Figure 5: Contour Graph of Knowledge Diffusion Across Sectors

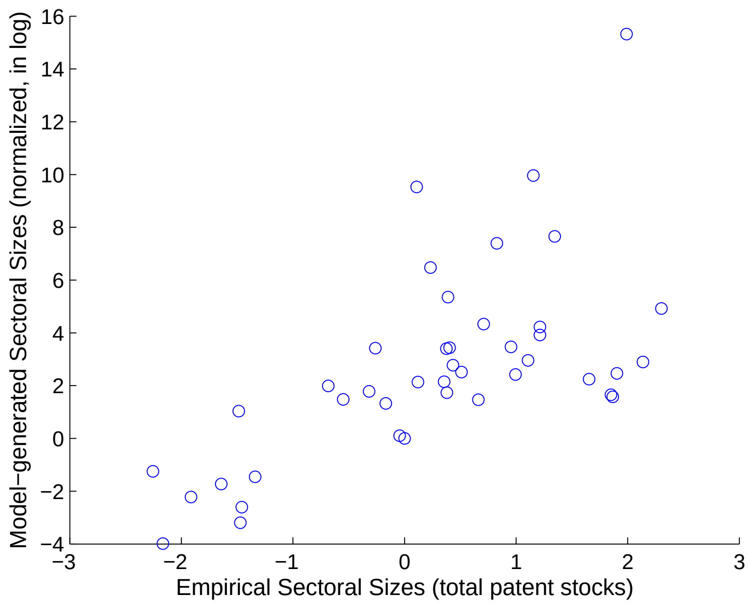


effectiveness of public knowledge, γ, is estimated by using the ratio between outside citations and inside citations for average sized patenting firms in 1990.[26] The discount factor β is set to be 0.99. The R&D labor input share in the innovation production function, α, is set to be 1/2. We assume the expenditure share s^i is equal to $1/K$. $F^e = 1.56 \times 10^{-9}$ is calculated to fit the empirical observation that the number of firms in the U.S. is 5.07 million relative to the 249 million population in 1990 (Axtell, 2001).

Our model predicts that sectors with high applicability attract more R&D investment and contain a larger number of varieties because the values of knowledge are higher in these sectors (the correlation between v^i and aw^i is 0.79). Figure 6 plots the model generated sector sizes (i.e. total number of products) against the real sector sizes in the patent data (i.e. total number of patents), both relative to a numeraire sector. These two series are highly positively correlated, with the main difference being that the simulated sector sizes are more dispersed than the real ones (the logged relative sizes vary between $[-4, 15]$ as opposed to $[-2.5, 2.5]$). In the next section, we will show that introducing fixed R&D costs decreases the dispersion of sectoral sizes, and disproportionally reduces the size of sectors with higher applicability.

[26]Since the public knowledge pool size is assumed to be equal to the average firm size, an average-sized firm faces the same size of public and private knowledge pools.

Figure 6: Model Generated Sector Sizes v.s. Empirical Sector Sizes ($F = 0$)



5 Simulation and Quantitative Analysis

We cannot analytically solve for the general equilibrium in the case with sectoral fixed R&D costs. We thus simulate the model by computing a large number of multi-product firms' optimal innovation decisions to test several of the model's key implications of firm dynamics, product entry and exit, and estimate the growth effect of the fixed costs.[27]

Lacking direct information on sectoral fixed R&D costs, we assume the entry cost is identical across sectors and calibrate it to match the observation that the average number of sectors per firm in 1990 is 3.8 sectors out of 42 (see Appendix A.4 for details).

There are no aggregate shocks in the economy. The idiosyncratic innovation shocks are specified as $\varepsilon_{f,t}^{ij} = (A^{ij})^{\frac{1}{1-\alpha}} (A^{ij}\alpha\rho v^{i})^{\frac{\alpha}{1-\alpha}} \left(\xi_{f,t}^{ij} - 1\right)$, where $\xi_{f,t}^{ij}$ is a random draw from $\log N(-\frac{(\bar{\sigma}^{ij})^2}{2}, \bar{\sigma}^{ij})$, $\forall f, t, i, j$, and $E\varepsilon_{f,t}^{ij} = 0$. We calculate $\bar{\sigma}^{ij}$ according to the standard deviation of $\frac{C_{ff}^{ij}}{C_{f}^{i}} \frac{\triangle ps_{f}^{i}}{ps_{f}^{j}}$, where $\frac{C_{ff}^{ij}}{C_{f}^{i}}$ gives the percentage of firm f's self-citations from sector i to sector j among its total number of citations made from sector i.

We run the simulation for 100 rounds and report the median values of relevant variables. The detailed description of the simulation process is available in Appendix A.4.

[27] A center sector attracts a large number of entrants due to its high knowledge value, v^i, but the number of firm stops growing to a certain level, because a high number of firms deters future entry. Similarly, a peripheral sector still accommodates a few incumbents, despite its low knowledge value. This mechanism guarantees a stationary distribution of firms across sectors in the simulation.

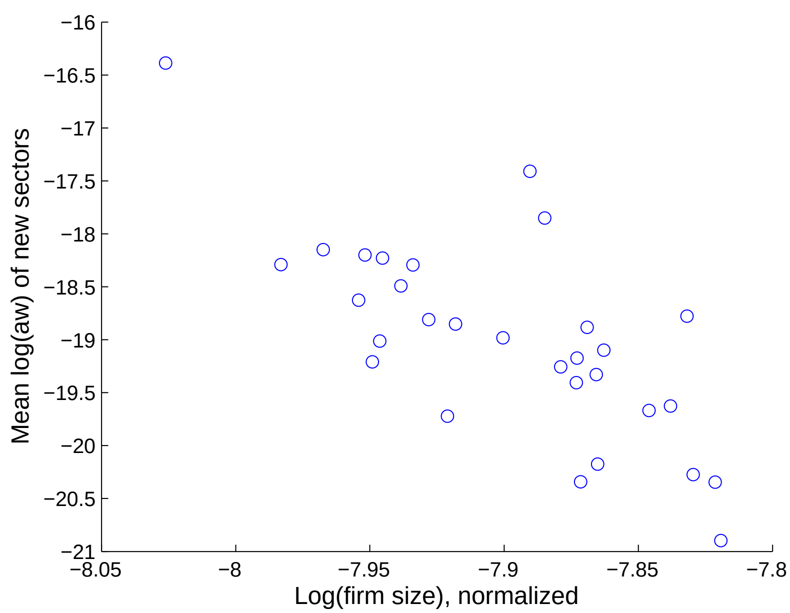
5.1 Simulation Results

5.1.1 Firm level

In this section, we focus on three main firm-level facts that our model can explain: sequential sectoral entry, the relationship between firm growth, firm growth volatilities and firm knowledge generality.

Sequential sectoral entry. The simulation of the model generates the same observations as in the data: the new sectors that small firms enter have high applicability (see Figure 2). Figure 7 shows that this negative relationship between firm size (measured by total number of varieties) and the weighted average authority weight of the new sectors is generated by the model. This is because a new firm self-selects into center sectors first and then sequentially enters other related sectors, venturing towards the periphery.

Figure 7: Scale Dependence of Applicability of Firms' New Sectors

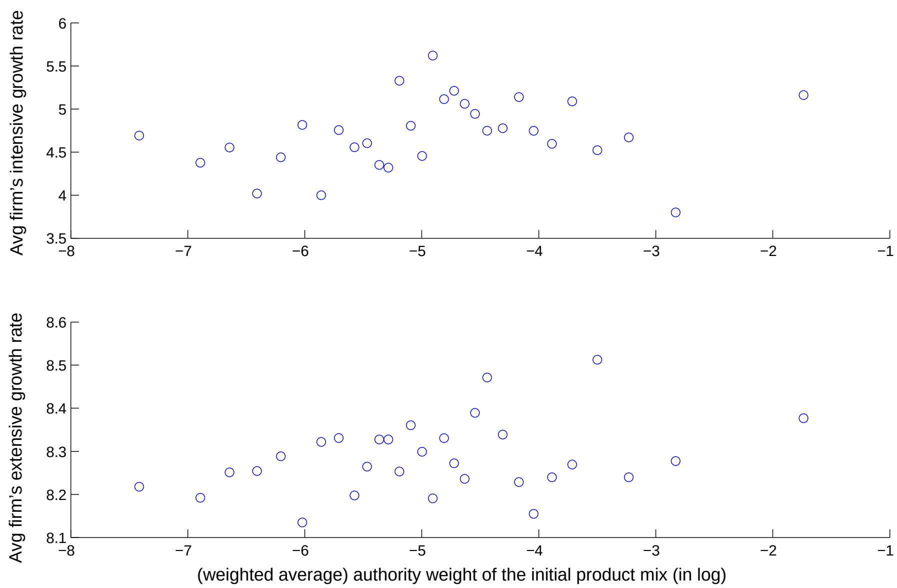


Firm growth. Substituting the optimal R&D from Equation (9) into the innovation rate from Equation (5) yields the firm's growth rate in expectation given by:

$$\mathbb{E}\frac{\Delta z_{f,t}}{z_{f,t}} = \underbrace{\sum_{j=1}^{K}\sum_{i=1}^{K} A^{ij}(\bar{z}^i\frac{\alpha}{1-\alpha}\omega^{ij})^{\alpha}\frac{z^j_{f,t}}{z_{f,t}}}_{\text{wtd avg } aw \text{ of product mix}} + \underbrace{\sum_{j=1}^{K}\sum_{i=1}^{K} A^{ij}(\bar{z}^i\frac{\alpha}{1-\alpha}\omega^{ij})^{\alpha}\gamma\frac{\bar{z}^j_t}{z_{f,t}}}_{\text{scale dependence}}$$

The first term resembles the initial (weighted) average applicability of the firm's product mix in our empirical section, and the second term indicates that larger firms tend to innovate more slowly. Figure 8 plots the simulated surviving firm's average intensive and extensive growth rate over 10 periods against its initial authority weight (one circle represents one firm). Firms are divided into 30 size bins and one circle corresponds to an average firm in each size bin. It is evident that both the extensive and intensive growth rates are positively related to the authority weight of a firm's starting product mix. A high product mix applicability opens more potential routes for a firm to expand across sectors, thus boosting the firm's extensive growth rate. At the same time, a high applicability also allows a firm to apply knowledge from many related sectors to innovating in the existing sector, which drives up the firms' intensive growth rate.

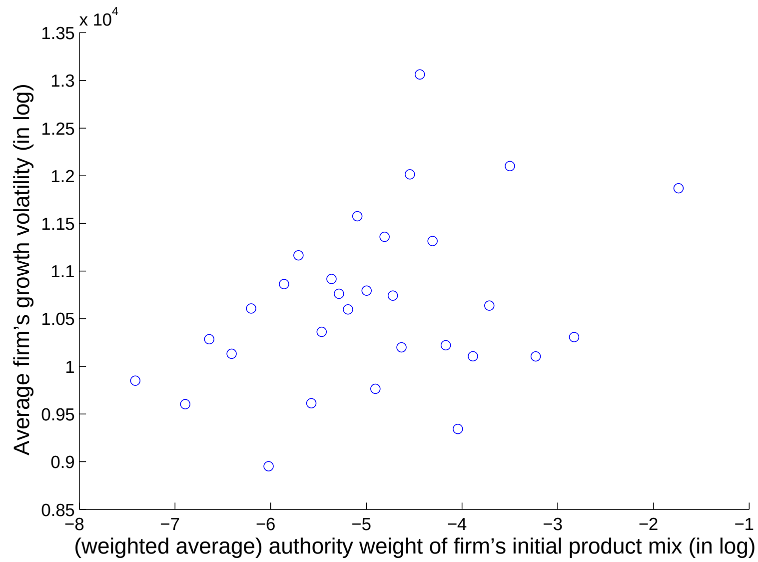
Figure 8: Applicability of Firm's Initial Product Mix and Firm Growth Rate



Firm growth volatility. It has been documented that the volatility of growth rates is higher for smaller firms (Klette and Kortum, 2004; Sutton 1997; Caves 1998). Our model also predicts a similar pattern and offers an explanation based on the importance of extensive growth. Extensive growth counts for a larger fraction of total growth for small firms, which self-select into sectors with high knowledge applicabilities. In Figure 9 and Figure 10, one circle corresponds to an average firm in each size bin. They show that the variance of a firm's growth rate is positively related to the firm-specific product mix applicability and negatively correlated with firm size (normalized by overall firm sizes) in the first ten periods.

To understand this relationship, we need to compare the importance of extensive growth versus intensive growth for firms of various sizes. A firm initially has a product mix with a higher knowledge applicability, extensive growth contributes a larger share to the firm's total growth rate. Since extensive growth involves fixed costs and a minimum stock of knowledge

Figure 9: Firm Growth Volatility and Applicability of Its Product Mix



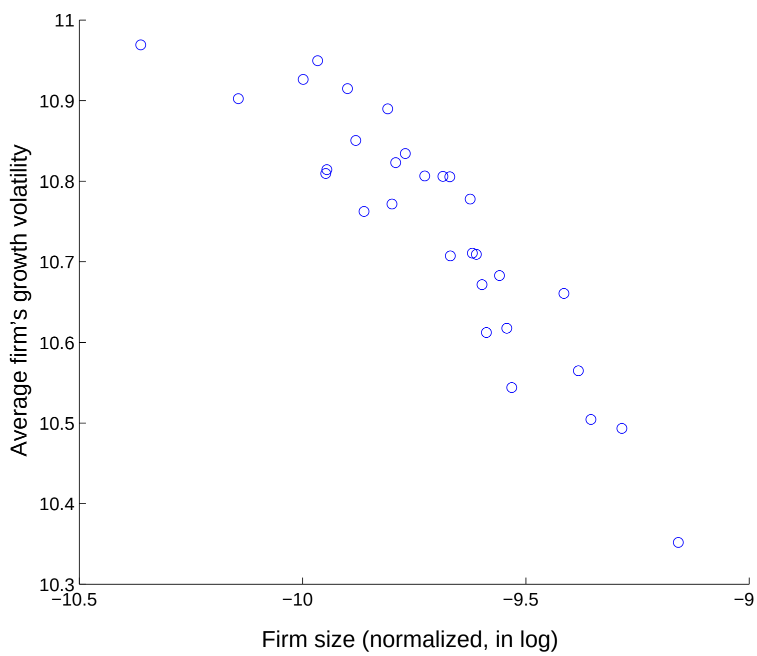


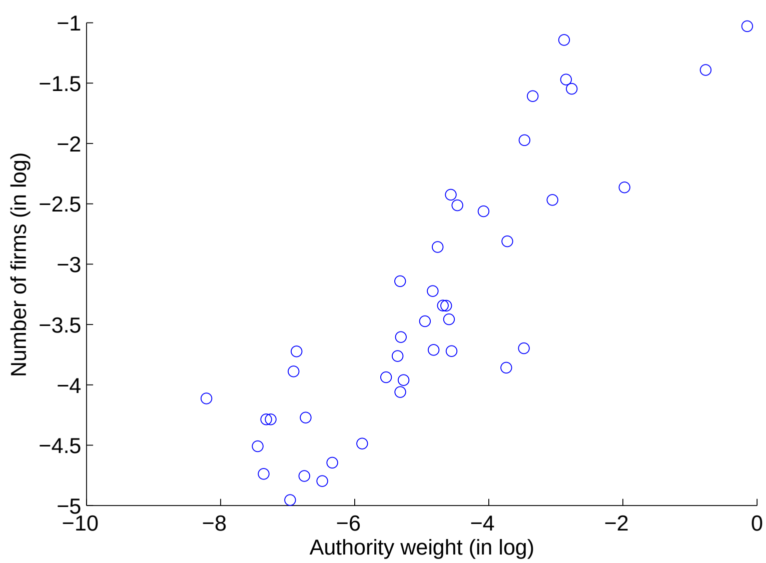
Figure 10: Firm Growth Volatility and Firm Sizes

capital upon entering every new sector, the fixed cost causes kinks and bumps in a firm's growth path. Once the firm has already entered a sector, intensive growth within the sector is relatively smoother. A firm's growth therefore exhibits more volatility when its product mix is highly authoritative and leads to more entries into new sectors.

5.1.2 Sector level

Number of firms. Although we start with the same number of firms across sectors in our simulation, Figure 11 shows that there are more firms in sectors with general knowledge than there are in isolated sectors at the end of the simulation. Center sectors are connected to more sectors with stronger knowledge linkages, allowing firms in related sectors to enter more easily. New firms choose to enter center sectors for their high potential of extensive growth and more available public knowledge; only firms that receive good shocks in incumbent sectors choose to expand into related sectors farther away from the center, where the market share per firm is larger. In the end, only very large firms can cover a large enough product scope to reach the fringe of the product space.

Figure 11: Sectoral Number of Firms and Applicability



Sectoral sizes and fixed R&D costs. With the estimated fixed R&D costs, it is still true that in the stationary equilibrium there are more varieties in sectors with highly applicable knowledge. Figure 12, however, shows that compared to the case with zero fixed costs, sectoral size distribution becomes much flatter when $F > 0$, bringing the sector size dispersion closer to the data. This is because sectoral fixed costs distort R&D resource allocation across sectors by pushing research effort away from highly applicable sectors toward less applicable sectors, as shown in Figure 13.

5.1.3 Aggregate level

As discussed before, Equation (16) implies that fixed R&D costs impede the growth of the number of varieties in the economy (because $\tau^{ij} < 1$ when $F > 0$). Our simulation shows that when fixed costs increase from zero to positive, the growth rate decreases from 1200 percent to 17 percent. Although it is unrealistic to remove all the fixed R&D costs, our experiment suggests that there is potential to greatly increase growth when fixed costs are reduced. Given the "love of variety" type of demand function, the real wage and consumer welfare fall when less innovations shrink the range of differentiated goods.

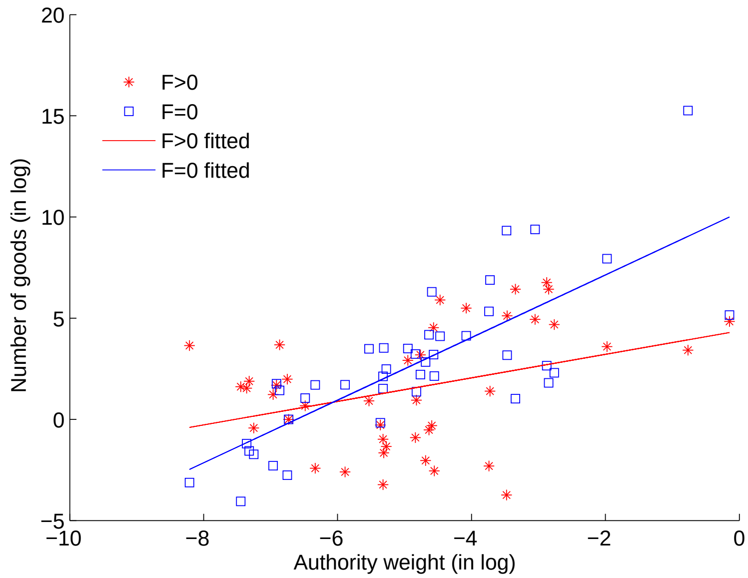
According to theories of industry structure (e.g., Hopenhayn 1992), higher entry costs lead to lower average firm productivity by protecting incumbent firms. Our model suggests another channel. As explained in section 4.2.1, fewer firms when facing entry barriers are able to use private and public knowledge in related sectors when innovating. This in turn implies that less aggregate knowledge is accumulated in the economy under the Pareto firm size distribution; therefore, the aggregate innovation rate and growth rate are reduced.

6 Final Remarks

Economic historians have emphasized the drastic impact of "technological prime movers" on growth (David, 1991; Rosenberg, 1982; Landes, 1969). Without formal models, this insight has not been incorporated in most theories of growth. In this paper, we explore the role of inter-sectoral knowledge spillovers on firm innovations and growth. We propose a new measure of sector relatedness based on a spillover network linking the knowledge receiving and sending sectors. We then incorporate it into a model with endogenous innovation, entry and exit decisions made by multiple-sector firms. We find that sectoral fixed R&D costs lower economic growth by blocking cross-sector knowledge circulation and prevents R&D resource from concentrating in the general purpose technology sectors.

In the patent data, we find that firms follow a common trend when they expand across sectors: firms start from highly applicable center sectors and gradually expand to related sectors towards the fringe of the product network. This sequential sectoral entry simultaneously explains many observations at firm and sector levels, including the dynamics of firm innovation, firm size distribution, scale dependent firm growth and volatility of firm growth, sector size distribution, heterogeneous R&D intensities in different sectors and the skewed concentration of firms towards center sectors.

Figure 12: Sectoral Sizes (With and Without Fixed Costs) and Applicability



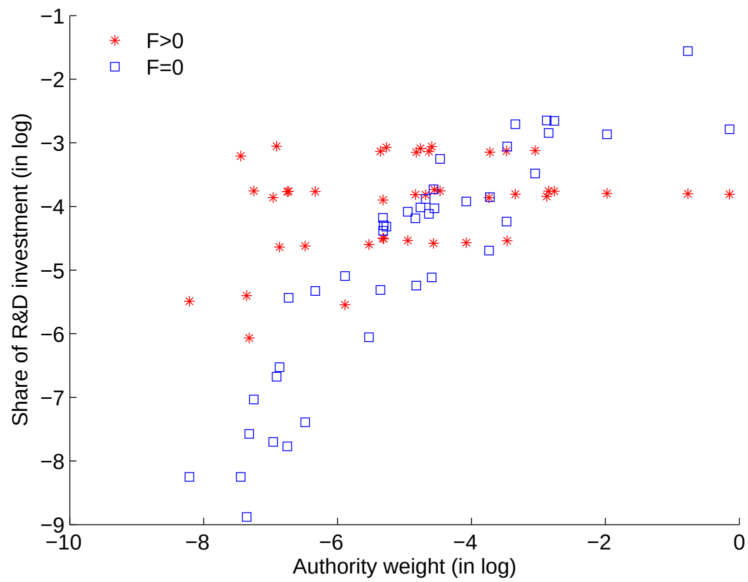


Figure 13: R&D Resource Allocation (With and Without Fixed Costs) and Applicability

Our study has important implications for economic growth and R&D policies. First, heterogenous sectoral knowledge spillovers suggest that industrial or R&D policies that favor highly applicable sectors boost economic growth. Second, institutional reforms that lower sectoral entry costs reinforce the effect of industry policies, because it can be challenging to shift to more advanced industry given the fixed cost of learning and adapting technology in new sectors. Third, competition policies that encourage joint R&D ventures in highly related sectors can benefit growth, because firms are better able to internalize knowledge spillovers. A successful example is China. Over the past two decades, China has significantly shifted its industrial structure from specializing in exporting low or medium knowledge applicabile (e.g. "Textile mill products" and "Food and kindred products") to exporting proportionally more highly applicable products (e.g. "Electronic components and communications equipment" and "Office computing and accounting machines"). The Chinese government has adopted a set of policies promoting structural transformation.

The sector relatedness implied by knowledge linkages could potentially help understand the non-random products co-production phenomenon documented by Bernard, Redding and Schott (2009a), in which some pairs of products (e.g., fabricated metal and industrial machinery) are systematically more likely to be produced by the same firms than other product pairs. Our analysis suggests that the knowledge incorporated in these product pairs is highly transferable between sectors. In addition, by emphasizing the future technological contribution from the innovating sectors to other using sectors, our model also predicts a positive relationship between a firm's market value and the authority weights of its patenting sectors.[28] Empirical investigation of these predictions could be interesting for future research.

Our empirical findings at the firm level also hold at the country level. In a related paper, we measure a country's product scope by its export product mix, and we find that in general countries with an export mix of higher applicability exhibit accelerated economic growth. Although it has long been recognized that industrialization creates spillover benefits that fuel subsequent growth (Rosenstein-Rodan 1943; Hirschman 1958), these ideas have been largely unstudied due to the lack of a concrete measure quantifying the spillover effects of the menu of technology and models that incorporate this measure. Our results should be of interest to researchers studying how a country's location on the product space affects its future economic performance.

[28]Hall, Thoma and Torrisi (2007) find that Tobin's q is significantly positively associated with a firm's R&D and patent stock, and modestly increases with the quality of patents.

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A Appendix

A.1 Comparison with Alternative Sectoral Linkages Measures

We compare our measures of sectoral linkages with those constructed using the U.S. Input-Output tables. A high authority weight using I-O tables implies the sector is an important buyer of materials produced by other sectors, and a high hub weight signals the importance of that sector as an input in the production of other sectors. We also compare our measure with the outcome-based measure of relatedness by Hidalgo et al. (2007). Since the classification of sectors varies for different measures, we simply report in Table 5 the top ten sectors from each measure to get a sense of the differences.

The top sectors in Hidalgo et al. (2007) appear to be the median sectors according to our measure of the authority weight. The reason is that in their paper, two products are related if they are exported by the same set of countries. The product space topology shows that the most related products lie in the center of the network, and the most isolated products are located at the fringe of the network; in between them, the products with median relatedness connect them in a complete network. If on average a country only covers a small set of connected products with various authority weights, the products with a median level authority weight are more likely to be exported, together with other products by more countries.

The top manufacturing sectors in the I-O tables are a mixture of important sectors in terms of knowledge linkages (for example, "Semiconductor and related devices") and important sectors in production linkages (for example, "Paperboard container"). As a result, neither of these two measures is highly correlated with ours.

A.2 Distance between Sectors

In addition to the sector specific authority weight, we also construct a pairwise knowledge distance measure to facilitate our studies. Define a K dimension distance matrix D, where the $\{i, j\}^{th}$ element, $D^{ij} = d$ if $(C\hat{\ }d)^{ij} > 0$ and $(C\hat{\ }(d-1))^{ij} = 0$. $C\hat{\ }d$ denotes the d^{th} power of the matrix C. D^{ij} is the shortest path distance between the nodes i and j. If $(C\hat{\ }d)^{ij} > 0$, there is at least one indirect route via other $d-1$ nodes between nodes i and j. If $(C\hat{\ }d)^{ij} > 0$, that means there exists at least one d-step route between i and j. If $(C\hat{\ }(d-1))^{ij} = 0$ is also true, then d is the shortest path distance between i and j.

The mean of D's i^{th} column is the average distance between product i and all other products. The average distance to other products is negatively correlated (-0.49) with our authority weight measure, since higher authority weight products are located closer to the

Table 5: Comparison with IO Table Constructed Relatedness Measure and Hidalgo et al. (2007)'s Relatedness Measures

Top 10 sectors with the highest authority weight using Patent data, 1995-2000	
Code (USPS)	Names
43	Electronic components and accessories and communications equipment
55	Professional and scientific instruments
27	Office computing and accounting machines
35	Electrical transmission and distribution equipment
42	Radio and television receiving equipment except communication types
16	Rubber and miscellaneous plastics products
56	All other SICs
21	Fabricated metal products
36	Electrical industrial apparatus
29	Special industry machinery, except metal working

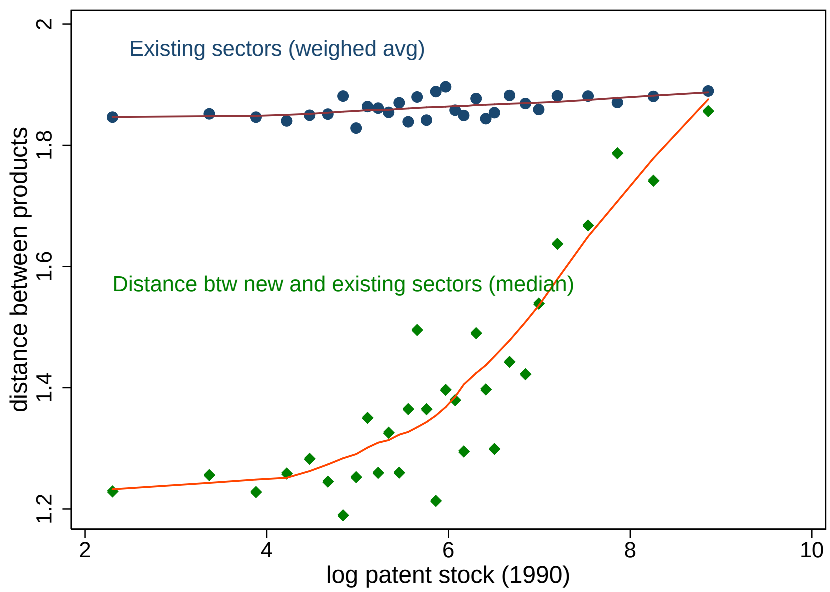
Top 10 sectors with the highest authority weight using US IO table, 2002	
code (NAICS)	Names
336300	Motor vehicle parts manufacturing
324110	Petroleum refineries
32619A	Other plastics product manufacturing
331110	Iron and steel mills and ferroalloy manufacturing
323110	Printing
325190	Other basic organic chemical manufacturing
334413	Semiconductor and related device manufacturing
325412	Pharmaceutical preparation manufacturing
322120	Paper mills
322210	Paperboard container manufacturing

Top 10 goods in the densest part of the product space 2000, Hausmann and Klinger (2007)	
code (SITC)	Names
6785	Tube and pipe fittings (e.g. joints, elbows) of iron steel
6996	Miscellaneous articles of base metal
6921	Reservoirs, tanks, vats and similar containers
6210	Materials of rubber (e.g., pastes, plates, sheets, etc.)
7849	Other parts and accessories of motor vehicles
8935	Art. of electric lighting of materials of Div. 58
8939	Miscellaneous art. of materials of Div. 58
7139	Parts of int. comb. piston engines of 713.2-/713.8-
7492	Taps, cocks, valves, etc. for pipes, tanks, vats, etc.
5822	Aminoplasts

center of the network, which are connected to more other products. The negative correlation is not perfect, because the average distance ignores the volume of knowledge flow between products and the importance of connected products. Nevertheless, the distance measure helps to understand the connectivity between products.

Since, by definition, large firms innovate in more sectors, their product distribution is more spread out in the technology space than small firms, and naturally these large firms have a higher average distance measure of their existing product. To avoid this bias, we investigate the median distance between the new product and firms' existing products instead of the mean distance. Figure 14 shows that larger firms make significantly bigger jumps across the technology space when they enter a new sector and, for firms of all sizes, the new sectors they enter tend to be relatively close to their current product mix on the technology network.

Figure 14: Distance Between Products in the Technology Space and Firm Sizes



A.3 Derivation of the Firm's Innovation Problem

We adopt the guess-and-verify method to solve the all-sector firm's problem. Guess that the value of a firm is a linear combination of its accessible knowledge capital in all the sectors in which it is producing:

$$V(z_{f,t}) = \sum_{j=1}^{K} \frac{v^j z^j_{f,t} + u^j}{I^j_t}$$

Substituting it back to the Bellman equation, we get

$$V(z_{f,t}) = \sum_{j=1}^{K} \pi^j \frac{z^j_{f,t}}{I^j_t} - \sum_{i=1}^{K}\sum_{j=1}^{K} R^{ij}_{f,t} + \frac{1}{1+r}[\sum_{j=1}^{K}(v^j \frac{z^j_{f,t} + \sum_{i=1}^{K}\left[A^{ji}\left(\bar{z}^j_t R^{ji}_{f,t}\right)^{\alpha}\left(z^i_{f,t}+\gamma^i \bar{z}^i_t\right)^{1-\alpha}\right]}{I^j_{t+1}} + \frac{u^j}{I^j_{t+1}}]. \tag{19}$$

Given that $M^i \bar{z}^i_t = I^i_t$, the first order condition with respect to $R^{ij}_{f,t}$ is:

$$R^{ij}_{f,t} = \frac{I^j_t}{I^i_t}\left(\frac{A^{ij}\alpha\rho^i_t v^i}{M^i}\right)^{\frac{1}{1-\alpha}} M^i \left(\frac{z^j_{f,t}+\gamma \bar{z}^j_t}{I^j_t}\right), \tag{20}$$

Substituting the optimal R&D in Equation (20) back to the Bellman equation (19) and comparing the coefficient of $z^i_{f,t}$ in the firm value function we get:

$$v^j = (1-\rho^j_t)^{-1}[\pi^j + \frac{1-\alpha}{\alpha}\sum_{i=1}^{K}\frac{I^j_t}{I^i_t}\left(A^{ij}\alpha\rho^i_t v^i\right)^{\frac{1}{1-\alpha}} M^{i\frac{\alpha}{\alpha-1}}].$$

Comparing the constant terms, we have

$$u^j = (1-\rho^j_t)^{-1}\frac{1-\alpha}{\alpha}\sum_{i=1}^{K}\frac{I^j_t}{I^i_t}\left(A^{ij}\alpha\rho^i_t v^i\right)^{\frac{1}{1-\alpha}} M^{i\frac{\alpha}{\alpha-1}}\gamma^j \bar{z}^j_t,$$

where $\rho^j_t = \frac{1}{1+r}\frac{I^j_t}{I^j_{t+1}}$. To simplify the notations, define the value of sector j's knowledge in contributing to innovations in sector i as

$$\omega^{ij}_t = \frac{1-\alpha}{\alpha}\frac{I^j_t}{I^i_t}\left(A^{ij}\alpha\rho^i_t v^i\right)^{\frac{1}{1-\alpha}} M^{i\frac{\alpha}{\alpha-1}},$$

Substituting it back, we have

$$\begin{aligned} v^j &= (1-\rho^j_t)^{-1}(\pi^j + \sum_{i=1}^{K}\omega^{ij}_t), \\ u^j &= (1-\rho^j_t)^{-1}\sum_{i=1}^{K}\omega^{ij}_t \frac{\gamma^j \bar{z}^j_t}{I^j_t} \end{aligned}$$

and

$$R^{ij}_{f,t} = \frac{\alpha}{1-\alpha}\omega^{ij}_t \frac{z^j_{f,t}+\gamma \bar{z}^j_t}{I^j_t}$$

The evolution of the number of differentiated goods in sector i is:

$$
\begin{aligned}
I_{t+1}^{i} &= I_t^i + \int\limits_{i\in S_{f,t}} \triangle z_{f,t}^{i} df \\
&= I_t^i + \int\limits_{i.j\in S_{f,t}} \sum_{j=1}^{K} \left[A^{ij\frac{1}{1-\alpha}} \left(\frac{\alpha\rho_t^i v^i}{M^i}\right)^{\frac{\alpha}{1-\alpha}} \left(z_{f,t}^{j} + \gamma^j \bar{z}_t^j\right) + \varepsilon_{f,t}^{ij}(z_{f,t}^{i} + \gamma^i \bar{z}_t^i) \right] df \\
&= I_t^i + \sum_{j=1}^{K} \left[A^{ij\frac{1}{1-\alpha}} \left(\frac{\alpha\beta v^i}{g_t^i}\right)^{\frac{\alpha}{1-\alpha}} \right] (1+\gamma^j) I_t^{ji}
\end{aligned}
$$

where I_t^{ji} represents the total number of goods that are produced by firms that produce in both i and j sectors and $g_t^i = I_{t+1}^i / I_t^i$. Rearranging the terms, we have

$$
(g_t^i - 1)(g_t^i)^{\frac{\alpha}{1-\alpha}} = (1+\gamma^j) \sum_{j=1}^{K} \left(A^{ij}\right)^{\frac{1}{1-\alpha}} \left(\alpha\beta v^i\right)^{\frac{\alpha}{1-\alpha}} \frac{I_t^{ji}}{I_t^i}, \tag{21}
$$

The number of goods in every sector grows at the same speed, because inter-sector knowledge spillovers keep all sectors on the same track. If one sector i had been growing slower than other sectors for a lengthy period, its number of goods would be extremely small relative to other sectors. Equation (21) implies that the cross-sector knowledge spillovers would increase g_t^i tremendously through a large ratio I_t^{ji}/I_t^i until g_t^i is equal to the common growth rate. Therefore, in the stationary equilibrium, I_t^j/I_t^i is a constant, determined by the initial relative sector sizes, and consequently, growth rate is a constant as well, i.e. $g_t^i = g_t^j = g$.

This result further implies that, $\rho_t^j = \beta/g \equiv \rho$ and ω_t^{ij} are both constants and independent of sector or sector-pair, consistent with our original guess. In a stationary equilibrium, the measure (number) of firms in sector j, M^j, is constant. Therefore, we have Equations (8), (6), (7) and (9). Now we can verify our previous guess that the firm's value is a linear constant-coefficient combination of its knowledge in all sectors.

$$
V\left(z_{f,t}\right) = \sum_{j=1}^{K} (1-\rho)^{-1} [\pi^j \frac{z_{f,t}^j}{I_t^j} + \sum_{i=1}^{K} \omega^{ij} \frac{z_{f,t}^j + \gamma^j \bar{z}_t^j}{I_t^j}].
$$

A.4 Simulation Procedure

We start the simulation with $N = 10^4$ firms and $K = 42$ sectors.

The simulation procedure involves the following steps:

1. At the initial period, randomly assign $N \times 3.6/42$ firms into each sector. The initial

firm size in each sector is also a random draw from a log-normal distribution[29] with parameters 0 and 2. Guess a starting value of g_0. Take I_0^j/I_0^i and M_0^i, $i, j = 1, 2, ..., K$ from the random generated initial firm size data.

2. At the beginning of period t, calculate the firm value vector $\boldsymbol{v}_t$ and matrix $\boldsymbol{\omega}_t$ using g_{t-1}, M_{t-1}^i, and I_{t-1}^j/I_{t-1}^i according to Equations (6) and (8). Calculate the expected profit $score_{f,t}^i = \sum_{j=1}^K \omega^{ij} \frac{\left(z_{f,t}^j + \gamma^j \bar{z}_t^j\right)}{I_t^j}$ of operating in sector i for each firm f according to the right hand side of Equation (11). Rank all (i, f) pairs according to their $score_{f,t}^i$. A potential entrant enters sector i, if its $score_{f,t}^i$ is larger than the top $10 \times 3.6/42/(1 - 3.6/42)$ percentile of (i, f) pairs among potential entrants. An incumbent firm f of sector i exits, if its $score_{f,t}^i$ is smaller than the bottom 10 percentile of (i, f) pairs among incumbents[30]. Also, we make sure the entry cutoff is always larger or equal to the exit cutoff. At the beginning, the exit cutoff level of expected profit is lower than the entry cutoff. At the end of the simulation, the exit and entry cutoffs converge to a common level $\bar{c}$. F is, thus, pinned down by $\bar{c}\frac{1+r}{r}$.

3. As a new entrant to sector i, firm f's size evolves according to Equation (14) with $z_{f,t}^i = 0$.

4. A surviving firm's size evolves according to Equation (14).

5. Calculate g_t, M_t^i, and I_t^j/I_t^i from the simulated data.

6. Take it to the next period.

7. Repeat the steps 2 to 6 until $|v_t - v_{t+1}| \leq 10^{-6}$.

We then repeat the same simulation procedure 100 times and report the median of the all simulations; mean of the last 10 periods.

[29] The randomness initialization ensures that each firm starts from a different product mix, so that we can compare a firm's future growth rates conditional on its initial sector's connectivity.

[30] Because the ratio between incumbents and potential entrants is $3.6/(42 - 3.6)$, the number of entrants is equal to the number of exits.