Globalization and Volatility under Alternative Trade Structures*

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Abstract

This paper explores a dynamic two-country model with production externalities in which capital goods are not traded and international lending and borrowing are allowed. Unlike the integrated world economy model based on the Heckscher-Ohlin setting, our model yields indeterminacy of equilibrium under a wider set of parameter values than in the corresponding closed economy model. Our finding demonstrates that the assumption on trade structure would be a relevant determinant in considering the relation between globalization and economic volatility.

Keywords: two-country model, non-traded goods, equilibrium indeterminacy, social constant returns

JEL classification: F43, O41

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1 Introduction

Does globalization enhance economic volatility? The equilibrium business cycle theory based on indeterminacy and sunspots has presented two different answers to this question. On the one hand, authors such as Meng (2003), Meng and Velasco (2003 and 2004) and Weder (2001) show that small-open economies with production externalities produce indeterminacy of equilibrium under a wider set of parameter values than in the corresponding closed economy model. Hence, according to these studies, internationalization of an economy may increase economic volatility.[1] Nishimura and Shimomura (2002a), on the other hand, reveal that a world economy consisting of two symmetric countries with production externalities holds the same stability conditions as those for a closed economy counterpart. In addition, Sim and Ho (2007a) find that if one of the two counties has no production externalities in Nishimura and Shimomura's model, then the equilibrium path of the world economy would be determinate even though the country with production externalities exhibits autarkic indeterminacy. These studies indicate that opening up international trade does not necessarily enhance economic fluctuations.

These opposite results seemingly stem from the difference in the analytical frameworks used by the foregoing studies. The small-open economy models studied by Meng (2003) and others are based on the partial equilibrium analysis in which behavior of the rest of the world is exogenously given. In contrast, the models of world economy employ the general equilibrium approach that treats the world economic system as a closed economy consisting of multiple countries. The world economy models thus consider more complex interdependency between the countries than that assumed in the small-open economy models. One may conjecture that such a difference would generate the contrasting views as to the destabilizing effect of globalization.

The purpose of this paper is to reveal that the difference in conclusions mentioned above mainly comes from the assumptions on trade structures rather than from the modelling strategies. To confirm this fact, we modify the model studied by Nishimura and Shimomura (2002a) by introducing non-traded goods and international financial transactions. Nishimura and Shimomura (2002a) use the standard Heckscher-Ohlin framework where both investment and consumption goods are freely traded but there is no intertemporal trade between the two countries. We assume that consumption goods are internationally traded but investment goods are non-tradables. Instead, it is assumed that international

[1] Lahiri (2001) also examines indeterminacy in a small-open economy model. Since he uses a somewhat non-standard framework, the model needs a high degree of external increasing returns to yield indeterminacy. Yong and Meng (2004) and Zhang (2008) also discuss equilibrium indeterminacy in small-open economies.

lending and borrowing are possible. Unlike the Heckscher-Ohlin setting, in the presence of non-traded goods, the factor price equalization fails to hold in our model. As a result, in our modified framework the factor intensities of production sectors in the home and foreign countries may differ from each other. This means that the dynamic behavior of our model out of the steady state will not be the same as that of a corresponding closed economy. Such a difference in transition dynamics generates the divergence of determinacy conditions between the closed economy and the integrated world economy consisting of symmetric countries.

Our main finding is that the equilibrium determinacy/indeterminacy conditions for the world economy with non-traded goods and financial transactions are similar to the stability conditions for the small-open economy models. More specifically, we show that our model may exhibit indeterminacy regardless of the restrictions on the preference structure. The closed-economy version of our model, which is essentially the same as the integrated world economy model of Nishimura and Shimomura (2002a), needs a high elasticity of intertemporal substitution in consumption to hold indeterminacy. It is to be noted that most of the small-open economy models with equilibrium indeterminacy assume the presence of international lending and borrowing.[2] Our study, therefore, demonstrates that even though the countries in the world economy have identical technologies and preferences, the presence of non-traded final goods and financial capital mobility would generate a divergence in dynamic behavior of the integrated world economy and a closed, single country. In this sense, the structure of international trade would be a relevant determinant for the relation between globalization and economic volatility.

The rest of the paper is organized as follows. The next section presents the basic assumptions for the following discussion. Section 3 reformulates the model of Nishimura and Shimomura (2002a) as a pseudo-planning problem and summarizes their conclusions. Section 4 modifies the planning problem in Section 3 in order to consider the presence of non-traded capital goods and intertemporal trade. This section displays our main findings. Section 5 gives economic implications of our finding and Section 6 presents concluding remarks.

2 Baseline Setting

Consider a world economy consisting of two countries, home and foreign. Both countries have the same production technologies. In each country there is a continuum of identical, infinitely-lived households. All the agents in both countries have an identical time discount rate and the same form of instantaneous felic-

[2] This is not the case for Nishimura and Shimomura (2002b) who explore the small-country version of the dynamic Heckscher-Ohlin model.

ity functions. The consumption-saving decision is made by the representative agent whose objective is to select the sequences of consumption to maximize a discounted sum of utilities over an infinite horizon. We assume that labor supply is fixed and each household supplies one unit of labor in each moment.

Here, we focus on the home country. As for the production side of the model, we assume that there are two production sectors in each country. The first sector ($i = 1$) produces investment goods and the second sector ($i = 2$) produces pure consumption goods. The production function of i-th sector in the home country is specified as

$$Y_i = A_i K_i^{a_i} L_i^{b_i} \bar{X}_i, \qquad a_i > 0,\ b_i > 0,\ \ 0 < a_i + b_i < 1, \qquad i = 1, 2$$

where Y_i, K_i and L_i are i-th sector's output, capital and labor input, respectively. Here $\bar{X}_i$ denotes the sector and country-specific production externalities. We define:

$$\bar{X}_i = \bar{K}_i^{\alpha_i - a_i} \bar{L}_i^{1-\alpha_1 - b_i}, \quad \alpha_i > a_i, \quad 1 - \alpha_i > b_i \quad i = 1, 2.$$

If we normalizes the number of producers to one, then it holds that $\bar{K}_i = K_i$ and $\bar{L}_i = L_i$ ($i = 1, 2$) in equilibrium.[3] This means that the i-th sector's social production technologies that internalize the external effects are:

$$Y_i = A_i K_i^{\alpha_i} L_i^{1-\alpha_i}, \qquad i = 1, 2.$$

Hence, the social technology satisfies constant returns to scale, while the private technology exhibits decreasing returns to scale.[4]

The production factor and commodity markets are competitive. Thus the real factor price equals the private marginal product of the factor in each production sector:

$$r = pa_1 \frac{Y_1}{K_1} = a_2 \frac{Y_2}{K_2}, \quad w = pb_1 \frac{Y_1}{L_1} = b_2 \frac{Y_2}{L_2},$$

where r, w and p respectively denote the rate of return to capital, the real wage and the price of investment good in terms of consumption good.

[3] As shown by Mino (2001), the main argument of this paper holds for a more general production function specified as

$$Y_i = f^i(K_i, L_i) E^i(\bar{K}_i, \bar{L}_i), \quad i = 1, 2,$$

where function $f^i(.)$ is homogeneous of degree $\gamma \in (0, 1)$ in K_i and L_i, while function $E^i(.)$ is homogeneous of degree $1 - \gamma$.

[4] Since the private technologies exhibit decreasing returns to scale, there exist positive profits in both production sectors. According to Benhabib and Nishimura (1998), we implicitly assume that there is an entry barrier in each industry to generate positive profits in each production sector.

We also assume that capital and labor are perfectly shiftable between the production sectors within a country, but they cannot move across the border. Therefore, the full-employment conditions for production factors are given by

$$K = K_1 + K_2, \qquad 1 = L_1 + L_2,$$

where the total labor force is assumed to be unity.

As was assumed, the foreign country has the same production technologies as those of the home country. It is also assumed that the labor force in the foreign country is normalized to unity as well. Thus the home and foreign countries differ only in their initial holdings of capital stocks.

3 A Dynamic Heckscher-Ohlin Model

To emphasize the role of trade structure in dynamic world economy models, we first summarize the dynamic properties of the Hecksher-Ohlin model of the integrated world economy with sector as well as country specific production externalities. For this purpose, we consider a pseudo-planning problem whose solution mimics the competitive equilibrium of the world economy. This approach simplifies model manipulation and helps to clarify the difference between the Heckscher-Ohlin setting and our model with non-traded goods and international financial transactions. The market economy version of the model in this section is discussed in detail by Nishimura and Shimomura (2002a) and Sim and Ho (2007a).[5]

3.1 A Pseudo-Planning Problem

In the standard Heckscher-Ohlin framework, it is assumed that both consumption and investment goods are tradables, but international lending and borrowing are impossible. In this setting, the representative agent in the home country solves the following problem:

$$\max \int_0^{\infty} e^{-\rho t} \frac{C^{1-\sigma} - 1}{1-\sigma} dt, \quad \sigma > 0, \quad \rho > 0$$

subject to

$$\dot{K} = Y_1 + Y_2/p - C/p - \delta K, \quad K_0 = \text{given } (> 0),$$

where C denotes consumption. Similarly, the the representative household of the foreign country solves

$$\max \int_0^{\infty} e^{-\rho t} \frac{C^{*1-\sigma} - 1}{1-\sigma} dt$$

[5] Nishimura and Shimomura's study is based on the dynamic Hechscher-Ohlin models examined by, for example, Chen (1992) and Stiglitz (1970).

subject to

$$\dot{K}^* = Y_1^* + Y_2^*/p - C^*/p - \delta K^*, \quad K_0^* = \text{given } (> 0),$$

where asterisks denote corresponding foreign variables. The world market equilibrium conditions for investment and consumption goods are respectively given by

$$\dot{K} + \dot{K}^* = Y_1 + Y_1^* + \delta K + \delta K^*, \tag{1}$$

$$C + C^* = Y_2 + Y_2^*. \tag{2}$$

In the pseudo-planning formulation that corresponds to the market economy described above, the planner is assumed to solve the following problem:

$$\max \int_0^\infty \left[\frac{C^{1-\sigma} - 1}{1-\sigma} + \mu^* \frac{C^{*1-\sigma} - 1}{1-\sigma} \right] e^{-\rho t} dt$$

subject to

$$\dot{K}_w = A_1 K_1^{a_1} L_1^{b_1} \bar{X}_1 + A_1 K_1^{*a_1} L_1^{*b_1} \bar{X}_1^* - \delta K_w, \tag{3}$$

$$C + C^* = A_2 K_2^{a_2} L_2^{b_2} \bar{X}_2 + A_2 K_2^{*a_2} L_2^{*b_2} \bar{X}_2^*, \tag{4}$$

$$K = K_1 + K_2, \quad K^* = K_1^* + K_2^*, \tag{5}$$

$$1 = L_1 + L_2, \quad 1 = L_1^* + L_2^*, \tag{6}$$

$$K_w = K + K^*, \tag{7}$$

together with the given initial levels of capital stocks, K_0 and K_0^*. Here, K_w stands for the aggregate capital stock in the world economy at large. In addition, μ^* in the objective function denotes a constant welfare weight on the instantaneous felicity of the foreign agents relative to the felicity in the home country. This value should be selected to make the planning solution equivalent to the competitive equilibrium. Constraints (3) and (4) are the equilibrium conditions for investment and consumption goods, respectively. Equations (5) and (6) represent the resource constraints in each country. Following Kehoe et al. (1992), we assume that in solving this problem the planner takes the sequences of external effects, $\{\bar{X}_i(t)\}_{t=0}^\infty$ and $\{\bar{X}_i^*(t)\}_{t=0}^\infty$ $(i = 1, 2)$, as given.

In what follows, we focus on an interior solution. Set up the current value Hamiltonian function:

$$\begin{aligned} \mathcal{H} \;=\; & \frac{C^{1-\sigma} - 1}{1-\sigma} + \mu^* \frac{C^{*1-\sigma} - 1}{1-\sigma} + q(A_1 K_1^{a_1} L_1^{b_1} \bar{X}_1 + A_1 K_1^{*a_1} L_1^{*b_1} \bar{X}_1^* - \delta K_w) \\ & + \lambda \Big[A_2 (K - K_1)^{a_2} (1 - L_1)^{b_2} \bar{X}_2 + A_2 (K^* - K_1^*)^{a_2} (1 - L_1^*)^{b_2} \bar{X}_2^* \\ & - C - C^* \Big] + \phi (K_w - K - K^*). \end{aligned}$$

In the above, q denotes the implicit price the aggregate capital, K_w, and λ and ϕ are Lagrangian multipliers. It is easy to see that q/λ corresponds to $1/\hat{p}$, that is, the price of investment good in terms of consumption good in the decentralized world economy.[6] The necessary conditions for an optimum include the following:

$$C^{-\sigma} = \lambda, \qquad \mu^* C^{*-\sigma} = \lambda, \tag{8a}$$

$$q a_1 A_1 K_1^{a_1 - 1} L_1^{b_1} \bar{X}_1 - \lambda a_2 A_2 K_2^{a_2 - 1} L_2^{b_2} \bar{X}_2 = 0, \tag{8b}$$

$$q b_1 A_1 K_1^{a_1} L_1^{b_1 - 1} \bar{X}_1 - \lambda b_2 A_2 K_2^{a} L_2^{b_2 - 1} \bar{X}_2 = 0, \tag{8c}$$

$$q a_1 A_1 K_1^{*a_1 - 1} L_1^{*b_1} \bar{X}_1^* - \lambda a_2 A_2 K_2^{*a_2 - 1} L_2^{*b_2} \bar{X}_2^* = 0, \tag{8d}$$

$$q b_1 A_1 K_1^{*a_1} L_1^{*b_1 - 1} \bar{X}_1^* - \lambda b_2 A_2 K_2^{*a_2} L_2^{*b_2 - 1} \bar{X}_2^* = 0, \tag{8e}$$

$$\lambda a_2 A_2 K_2{}^{a_2 - 1} L_2^{b_2} \bar{X}_2 - \phi = 0, \qquad \lambda a_2 A_2 K_2^{*a_2 - 1} L_2^{*b_2} \bar{X}_2^* - \phi = 0, \tag{8f}$$

$$\dot{q} = q\left(\rho + \delta\right) - \phi, \tag{8g}$$

$$\lim_{t \to \infty} q e^{-\rho t} K_w = 0. \tag{8h}$$

Equations $(8a)$ through $(8f)$ display the first-order conditions for maximizing the Hamiltonian function with respect to the control variables, C, C^*, K_1, L_1, K_1^*, L_1^*, K and K^* under given levels of $\bar{X}_i$ and $\bar{X}_i^*$ $(i = 1, 2)$. Equation $(8g)$ is the canonical equation of the costate variable for the aggregate capital, K_w, and $(8h)$ is the transversality condition.

3.2 Equilibrium Dynamics of the Integrated Economy

First, by use of $(8b)$ through $(8e)$, we obtain the following relations:

$$\frac{K_2}{L_2} = \left(\frac{a_2 b_1}{a_1 b_2}\right) \frac{K_1}{L_1}, \qquad \frac{K_2^*}{L_2^*} = \left(\frac{a_2 b_1}{a_1 b_2}\right) \frac{K_1^*}{L_1^*}. \tag{9}$$

From the equilibrium conditions, $\bar{K}_i = K_i$, $\bar{L}_i = L_i$, $\bar{K}_i^* = K_i^*$,and $\bar{L}_i^* = L_i^*$, we find that $(8b)$ and $(8d)$ present:

$$q a_1 A_1 K_1^{\alpha_1 - 1} L_1^{1 - \alpha_1} = \lambda a_2 A_2 K_2^{\alpha_2 - 1} L_2^{1 - \alpha_2}, \tag{10}$$

$$q a_1 A_1 K_1^{*\alpha_1 - 1} L_1^{*1 - \alpha_1} = \lambda a_2 A_2 K_2^{*\alpha_2 - 1} L_2^{*1 - \alpha_2}. \tag{11}$$

[6]Notice that λ equals the marginal utility of consumption and q equals the marginal value of capital in terms of utility. Therefore, q/λ denotes the value of investment good in terms of consumption good.

Using (9), (10) and (11), we obtain

$$\begin{aligned}\frac{q}{\lambda} &= \frac{A_2}{A_1}\left(\frac{a_2}{a_1}\right)^{\alpha_2}\left(\frac{b_2}{b_1}\right)^{\alpha_2-1}\left(\frac{K_1}{L_1}\right)^{\alpha_2-\alpha_1} \\ &= \frac{A_2}{A_1}\left(\frac{a_2}{a_1}\right)^{\alpha_2}\left(\frac{b_2}{b_1}\right)^{\alpha_2-1}\left(\frac{K_1^*}{L_1^*}\right)^{\alpha_2-\alpha_1}.\end{aligned} \tag{12}$$

As shown by the above conditions, because of the symmetry of the two countries, the factor intensities of the social technology in both countries are the same: $K_i/L_i = K_i^*/L_i^*$ $(i = 1, 2)$. Denoting $q/\lambda \equiv p$, from (12) we can express the capital intensities in the following manner:

$$K_i/L_i = K_i^*/L_i^* = k_i(p), \quad i = 1, 2.$$

The full-employment conditions in each country (5) and (6) are respectively summarized as

$$L_1 k_1(p) + (1 - L_1) k_2(p) = K,$$
$$L_1^* k_1(p) + (1 - L_1^*) k_2(p) = K^*.$$

In view of these full-employment conditions, we may express the social level of investment good output in each country as follows:

$$Y_1 = L_1 A_1 k_1(p)^{\alpha_1} = \frac{K - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1}, \tag{13a}$$

$$Y_1^* = L_1^* A_1 k_1(p)^{\alpha_1} = \frac{K^* - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1}. \tag{13b}$$

From (1), $(13a)$ and $(13b)$, we see that the dynamic equation for the aggregate capital of the world economy is given by

$$\dot{K}_w = \frac{K_w - 2k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1} - \delta K_w. \tag{14}$$

Equations $(8b)$, $(8f)$ and $(8g)$ yield the dynamic behavior of the shadow value of K_w:

$$\dot{q} = q\left[\rho + \delta - a_1 A_1 k_1(p)^{\alpha_1 - 1}\right]. \tag{15}$$

Equations in $(8a)$ mean that $C^*/C = \mu^{*-1/\sigma} \equiv \bar{m}$ for all $t \geq 0$. Since the households in both counties have an identical form of homothetic utility function, the relative level of optimal consumption stays constant over time. Thus, considering that $Y_2 = (1 - L_1) A_2 k_2^{\alpha_2}$ and $Y_2^* = (1 - L_1^*) A_2 k_2^{\alpha}$, the world market equilibrium condition for consumption goods (4) is expressed as

$$(1 + \bar{m})\lambda^{-\frac{1}{\sigma}} = \frac{2k_1(p) - K_w}{k_1(p) - k_2(p)} A_2 k_2(p)^{\alpha_2}. \tag{16}$$

This equation shows that the equilibrium level of λ can be expressed as $\lambda = \lambda(K_w, p;\ \bar{m})$. As a result, we obtain the following equation:

$$p = \frac{q}{\lambda(K_w, p, \bar{m})} \equiv \pi(K_w, q;\ \bar{m}). \tag{17}$$

Plugging (17) into (14) and (15) yields a complete dynamics system of the integrated world economy with respect to K_w and q.

Inspecting dynamic system (14) and (15), Nishimura and Shimomura (2002a) confirm that the steady state of the world economy where both countries imperfectly specialize is uniquely given under weak restrictions on parameter values. Then they show the following proposition:

Proposition 1 *(Nishimura and Shimomura 2002a) The steady-state equilibrium of the world is locally indeterminate if (i) the investment good sector is more capital intensive than the consumption good sector from the social perspective but it is less capital intensive from the private perspective, and (ii) the elasticity of intertemproal substitution in consumption,* $1/\sigma$*, is sufficiently high.*[7]

Given the conditions shown above, the steady state of the aggregate dynamic system is a sink so that there is a continuum of converging paths towards the steady-state equilibrium. Either if the social and private factor intensity rankings are the same or if the elasticity of intertemporal substitution in consumption is low enough, the dynamic system of the integrated world economy exhibits saddlepoint stability and, hence, the competitive equilibrium is at least locally determinate. As pointed out by Sim and Ho (2007b), the Heckscher-Ohlin model of two symmetric countries with constant-returns-to-scale technologies and homothetic preferences has the same dynamic properties as those of the corresponding closed economy. Therefore, the sufficient conditions for holding equilibrium indeterminacy shown above are essentially the same conditions for the closed economy with sector-specific externalities and social constant returns examined by Benhabib and Nishimura (1998).[8] This result demonstrates that in the standard

[7]More precisely, σ should satisfy

$$\frac{1}{\sigma} > \max\left\{1,\ \frac{(1-\alpha_1)a_2 b_1(\rho+\delta) + \alpha_1 a_1\left[\rho b_2 + \delta b_1 a_2 + (1-a_1)b_2\delta\right]}{(a_2 b_1 - a_1 b_2)(\alpha_1 - \alpha_2)\left[\rho + \delta(1-a_1)\right]}\right\}$$

to establish local indeterminacy in the steady-state equilibrium.

[8]In discussing two-sector closed economy model, Benhabib and Nishimura (1998) assume that the instantaneous utility function is linear in consumption (i.e. $\sigma = 0$). Hence, their model exhibits indeterminacy if condition (i) in Proposition 1 is satisfied. In the two-sector endogenous growth model with physical and human capital, condition (i) in Proposition 1 is sufficient for establishing indeterminacy: see Benhabib et al. (2000) and Mino (2001).

Heckscher-Ohlin world with symmetric countries, opening up international trade neither enhances nor diminishes economic volatility of each country.

When we consider the distributional dynamics between the two countries, it should be noted that the equilibrium trajectory of the world economy depends on $\bar{m}$: see equation (17). Nishimura and Shimomura (2002a) show that if the competitive equilibrium is indeterminate, the value of $\bar{m}\left(=\mu^{*-1/\sigma}\right)$ cannot be pinned down by the initial distribution of capital stocks, K_0 and K_0^*, alone. In the dynamic Heckscher-Ohlin world, the steady-state levels of K and K^* are path dependent and they are determined by the initial values of K_0 and K_0^*, if the converging path is determinate. If indeterminacy holds, then the level of $\bar{m}$ is indeterminate as well, and thus the terminal distribution of capital stocks in the steady state equilibrium is also indeterminate. As a result, the steady-state levels of relative factor endowment (so the steady-state patterns of international trade) may be affected by sunspot-driven fluctuations.[9]

4 A Model with Non-Tradable Capital Goods

We now assume that consumption goods are internationally traded but investment goods are non-tradables. Instead, we assume that international lending and borrowing are allowed. Therefore, while installed physical capital nor final goods for new investment can cross the borders, the households in both countries conduct intertemporal trade by trading financial assets.[10] Although such an assumption is restrictive one, it helps to elucidate the effect of the presence of non-traded goods in comparison with the case of free trade of final goods in the Heckscher-Ohlin model discussed in the previous section. Additionally, since a large portion of investment goods includes construction and structures, the investment goods sector shares a larger part of nontradables than the consumption good sector.[11]

[9] See also Nishimura and Shimomura (2006) for further investigation on equilibrium indeterminacy in the dynamic Heckscher-Ohlin model.

[10] The structure of our model is one of the dependent economy models discussed in open-economy macroeconomics literature. Sen and Turnovsky (1995) treat a small-open economy model with non-tradable capital and Turnovsky (1996, Chapter 7) studies a neoclassical two-country, two-sector model in which capital goods are not traded. See also Chapter 5 in Turnousky (2009) for a brief review of dependent economy models.

[11] Bems (2008) finds that the share of investment expenditure on non-traded goods is about 60%. and that this figure has been considerably stable over the last 50 years both in developed and developing countries.

4.1 A Decentralized Economy

We assume that the households in the home country can access to the international financial market where foreign bonds are freely traded. By trading bonds, the households in the home country can borrow from or lend to the foreign households. The representative household in the home country maximizes

$$U = \int_0^\infty \frac{C^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \qquad \delta > 0, \quad \rho > 0$$

subject to the flow budget constraint

$$\dot{\Omega} = R\Omega + w + \pi_1 + \pi_2 - C, \tag{18}$$

together with the no-Ponzi-game condition

$$\lim_{t\to\infty} \exp\left(-\int_0^t R_s ds\right)\Omega_t \geq 0.$$

and the initial value of Ω_0. Here, C is consumption, R denotes interest rate, π_i is the excess profits in the i-th sector[12] and Ω is the net wealth (in terms of the consumption goods). The net wealth of held by the household consists of domestic capital and foreign bonds:

$$\Omega = pK + B,$$

where B denotes the stock of foreign bonds (in terms of the consumption goods). When selecting its optimal consumption plan, the household take the sequences of $\{R_t, w_t,\ \pi_{1,t}, \pi_{2,t}, p_t\}_{t=0}^\infty$ are given.

The definition of net wealth yields $\dot{\Omega} = p\dot{K} + \dot{p}K + \dot{B}$. Thus, the flow budget constraint (18) can be rewritten as

$$\dot{B} = RB + \left(R - \frac{\dot{p}}{p}\right) pK + w + \pi_1 + \pi_2 - C - p\dot{K}$$

We also assume that the financial markets are perfect in the sense that domestic capital and foreign bonds are perfectly substitute each other. This means that the nonexistence of arbitrage holds in each moment, so that

$$r - \delta = R - \frac{\dot{p}}{p}. \tag{19}$$

[12] Remember that the private technology exhinits decreasing returns to scale with respect to capital and labor.

As a consequence, the optimization problem for the representative household in the home country is to maxize U by controlling C and I subject to the following constraints:

$$\dot{B} = RB + (r - \delta) pK + w + \pi_1 + \pi_2 - C - p(I - \delta K) \tag{20}$$

$$\dot{K} = I - \delta K \tag{21}$$

together with the initial holdings of K_0 and B_0. In this reformulation, the no-Ponzi-game condition is given by

$$\lim_{t\to\infty} \exp\left(-\int_0^t R_s ds\right) B_t \geq 0, \tag{22}$$

implying that the domestic households cannot borrow from the foreign households forever.

Since investment goods are traded in the domestic markets alone and consumption goods are internationally traded, the market equilibrium conditions for investment and consumption goods are respectively given by

$$Y_1 = I, \qquad Y_1^* = I^*, \tag{23}$$

$$Y_2 + Y_2 = C + C^*, \tag{24}$$

where I and I^* are gross investment in the home and foreign countries, respectively. Physical capital in each country accumulates according to

$$\dot{K} = I - \delta K, \qquad \dot{K}^* = I^* - \delta K^*. \tag{25}$$

Finally, the international bonds market clears in each moment and thus it holds that

$$B + B^* = 0, \tag{26}$$

implying that $\Omega + \Omega^* = K + K^*$. Bonds are IOUs between the home and foreign households, so that the aggregate stock of bonds is zero in the world market at large.

Finally, the production side of the economy is the same as that of the Hechscher-Ohlin model in the previous section. Profit maximization of both sectors equates the private marginal productivity of each factor and the factor prices, so that we obtain the following conditions:

$$r = pa_1 A_1 k_1^{\alpha_1 - 1} = a_2 A_2 k_2^{\alpha_2 - 1}, \tag{27}$$

$$w = pb_1 A_1 k_1^{\alpha_1} = b_2 A_2 k_2^{\alpha_2}, \tag{28}$$

$$r^* = p^* a_1 A_1 k_1^{*\alpha_1 - 1} = a_2 A_2 k_2^{*\alpha_2 - 1}, \tag{29}$$

$$w^* = p^* b_1 A_1 k_1^{*\alpha_1} = b_2 A_2 k_2^{*\alpha_2} \tag{30}$$

Again, we assume that both factor inputs are not traded so that the full employment conditions in both countries are:

$$K = K_1 + K_2, \qquad K^* = K_1^* + K_2^*$$

$$1 = L_1 + L_2, \qquad 1 = L_1^* + L_2^*.$$

4.2 A Pseudo-Planning Problem

As shown in the Appendix of the main text, the competitive equilibrium of the world economy can be characterized by the solution of the following pseudo-planning problem. In this problem the planner is assumed to solve the following:

$$\max \int_0^\infty \left[\frac{C^{1-\sigma} - 1}{1-\sigma} + \mu^* \frac{C^{*1-\sigma} - 1}{1-\sigma} \right] e^{-\rho t} dt$$

subject to

$$\dot{K} = A_1 K_1^{a_1} L_1^{b_1} \bar{X}_1 - \delta K,$$

$$\dot{K}^* = A_1 K_1^{*a_1} L_1^{*b_1} \bar{X}_1^* - \delta K^*$$

$$C + C^* = A_2 K_2^{a_2} L_2^{b_2} \bar{X}_2 + A_2 K_2^{*a_2} L_2^{*b_2} \bar{X}_2^*,$$

$$K = K_1 + K_2, \qquad K^* = K_1^* + K_2^*,$$

$$1 = L_1 + L_2, \qquad 1 = L_1^* + L_2^*,$$

as well as to the initial levels of K_0 and K_0^*. The difference between the planning problem given above and one discussed in the previous section is that in the present regime each country has its own capital accumulation equation due to the assumption that investment goods are not internationally traded.

The current-value Hamiltonian function is given by

$$\begin{aligned} \mathcal{H} \;=\; & \frac{C^{1-\sigma} - 1}{1-\sigma} + \mu^* \frac{C^{*1-\sigma} - 1}{1-\sigma} \\ & + q(A_1 K_1^{a_1} L_1^{b_1} \bar{X}_1 - \delta K) + q^* \left(A_1 K_1^{*a_1} L_1^{*b_1} \bar{X}_1^* - \delta K^* \right) \\ & + \lambda \left[A_2 \left(K - K_1 \right)^{a_2} \left(1 - L_1 \right)^{b_2} \bar{X}_2 + A_2 \left(K^* - K_1^* \right)^{a_2} \left(1 - L_1^* \right)^{b_2} \bar{X}_2^* - C - C^* \right], \end{aligned}$$

where q and q^* are the shadow values of capital stock in the home and foreign country, respectively. In what follows, we focus on the interior solution in which both countries imperfectly specialize in producing consumption and investment goods. The control variables in this problem are C, C^*, K_1, L_1, K_1^* and L_1^*, while the state variables are K and K^*. In parallel with the optimization in the previous section, we find that the necessary conditions for au optimum include the following :

$$C^{-\sigma} = \lambda, \qquad \mu^* C^{*-\sigma} = \lambda, \tag{31a}$$

$$qa_1 A_1 K_1^{a_1 - 1} L_1^{b_1} \bar{X}_1 - \lambda a_2 A_2 K_2^{a_2 - 1} L_2^{b_2} \bar{X}_2 = 0, \tag{31b}$$

$$q^* b_1 A_1 K_1^{a_1} L_1^{b_1 - 1} \bar{X}_1 - \lambda b_2 A_2 K_2^{a} L_2^{b_2 - 1} \bar{X}_2 = 0, \tag{31c}$$

$$qa_1 A_1 K_1^{*a_1 - 1} L_1^{*b_1} \bar{X}_1^* - \lambda a_2 A_2 K_2^{*a_2 - 1} L_2^{*b_2} \bar{X}_2^* = 0, \tag{31d}$$

$$q^* b_1 A_1 K_1^{*a_1} L_1^{*b_1 - 1} \bar{X}_1^* - \lambda b_2 A_2 K_2^{*a} L_2^{*b_2 - 1} \bar{X}_2^* = 0, \tag{31e}$$

$$\dot{q} = q(\rho + \delta) - \lambda a_2 A_2 K_2^{a_2 - 1} L_2^{b_2} \bar{X}_2, \tag{31f}$$

$$\dot{q}^* = q^*(\rho + \delta) - \lambda a_2 A_2 K_2^{*a_2 - 1} L_2^{*b_2} \bar{X}_2^*, \tag{31g}$$

$$\lim_{t \to \infty} e^{-\rho t} qK = 0, \qquad \lim_{t \to \infty} e^{-\rho t} q^* K^* = 0. \tag{31h}$$

4.3 Dynamic System

Again, we define $q/\lambda \equiv p$ and $q^*/\lambda \equiv p^*$, which represent the prices of consumption goods in terms of investment goods in the home and foreign countries, respectively. Then we replace (12) in the Heckscher-Ohlin model with the following:

$$\frac{A_2}{A_1}\left(\frac{a_2}{a_1}\right)^{\alpha_2}\left(\frac{b_2}{b_1}\right)^{1-\alpha_2}\left(\frac{K_1}{L_1}\right)^{\alpha_2-\alpha_1} = p,$$

$$\frac{A_2}{A_1}\left(\frac{a_2}{a_1}\right)^{\alpha_2}\left(\frac{b_2}{b_1}\right)^{1-\alpha_2}\left(\frac{K_1^*}{L_1^*}\right)^{\alpha_2-\alpha_1} = p^*.$$

These conditions, together with (9) , yield the following:

$$\frac{K_1}{L_1} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}}\left(\frac{a_1}{a_2}\right)^{\frac{\alpha_2}{\alpha_2-\alpha_1}}\left(\frac{b_1}{b_2}\right)^{\frac{\alpha_2-1}{\alpha_1-\alpha_2}} p^{\frac{1}{\alpha_2-\alpha_1}} \equiv k_1(p), \tag{32a}$$

$$\frac{K_1^*}{L_1^*} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}}\left(\frac{a_1}{a_2}\right)^{\frac{\alpha_2}{\alpha_2-\alpha_1}}\left(\frac{b_1}{b_2}\right)^{\frac{\alpha_2-1}{\alpha_1-\alpha_2}} p^{*\frac{1}{\alpha_2-\alpha_1}} \equiv k_1(p^*). \tag{32b}$$

Hence, from (9) the capital intensity in the consumption good sectors are given by:

$$\frac{K_2}{L_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}}\left(\frac{a_1}{a_2}\right)^{\frac{\alpha_1}{\alpha_2-\alpha_1}}\left(\frac{b_1}{b_2}\right)^{\frac{\alpha_1-1}{\alpha_1-\alpha_2}} p^{\frac{1}{\alpha_2-\alpha_1}} \equiv k_2(p),$$

$$\frac{K_2^*}{L_2^*} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}}\left(\frac{a_1}{a_2}\right)^{\frac{\alpha_1}{\alpha_2-\alpha_1}}\left(\frac{b_1}{b_2}\right)^{\frac{\alpha_1-1}{\alpha_1-\alpha_2}} p^{*\frac{1}{\alpha_2-\alpha_1}} \equiv k_2(p^*).$$

These expressions show that

$$\text{sign } k_i'(p) = \text{sign } k_i'(p^*) = \text{sign } (\alpha_2 - \alpha_1), \quad i = 1, 2. \tag{33}$$

Here, the sign of

$$\Delta_p = \alpha_1 - \alpha_2$$

represents the factor intensity ranking from the social perspective. When Δ_p is positive (negative), the aggregate technology of investment good sector is more (less) capital intensive than that of the consumption good sector.

Note that we have restricted our attention to the interior equilibrium in which both countries imperfectly specialize in producing consumption and investment goods. To ensure this restriction, we assume that relative price in each country satisfies the following condition:

$$0 < L_1 = \frac{K - k_2(p)}{k_1(p) - k_2(p)} < 1, \tag{34a}$$

$$0 < L_1^* = \frac{K^* - k_2(p^*)}{k_1(p^*) - k_2(p^*)} < 1. \tag{34b}$$

Using functions $k_1(p)$ and $k_2(p)$. we see that capital accumulation equation in each country is written as

$$\dot{K} = y^1(K, p) - \delta K, \tag{35}$$

$$\dot{K}^* = y^1(K^*, p^*) - \delta K^*, \tag{36}$$

where

$$y^1(K, p) \equiv \frac{K - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1(p)^{\alpha_1}, \tag{37a}$$

$$y^1(K^*, p^*) \equiv \frac{K^* - k_2(p^*)}{k_1(p^*) - k_2(p^*)} A_1 k_1(p^*)^{\alpha_1}. \tag{37b}$$

It is easy to see that these supply functions of investment goods satisfy:

$$\text{sign } y_K^1(K, p) = \text{sign } y_{K^*}^1(K^*, p^*) = \text{sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right), \tag{38a}$$

$$\text{sing } y_p^1(K, p) = \text{sing } y_{p^*}^1(K^*, p^*) = \text{sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(\alpha_1 - \alpha_2) \tag{38b}$$

Notice that the sign of

$$\Delta_s = \frac{a_1}{b_1} - \frac{a_2}{b_2}$$

shows the factor intensity ranking from the private perspective.

The shadow values of capital in both countries change according to

$$\dot{q} = q[\rho + \delta - r(p)], \tag{39}$$

$$\dot{q}^* = q^*[\rho + \delta - r(p^*)], \tag{40}$$

where $r(p) \equiv a_1 A_1 k_1(p)^{\alpha_1 - 1}$ and $r(p^*) \equiv a_1 A_1 k_1(p^*)^{\alpha_1 - 1}$. Dynamic equations (35), (36), (39) and (40) depict behaviors of capital stocks and implicit prices of capital in the home and foreign countries.

To derive a complete dynamic system, we should relate p and p^* to K, K^*, q and q^*. The world market equilibrium condition for the consumption good in the Heckscher-Ohlin world (equation (16)) is now replaced with

$$(1 + \bar{m})\lambda^{-\frac{1}{\sigma}} = y^2(K, p) + y^2(K^*, p^*), \tag{41}$$

where $\bar{m} = \mu^{*-1/\sigma}$ and

$$y^2(K, p) = \frac{k_1(p) - K}{k_1(p) - k_2(p)} A_2 k_2(p)^{\alpha_2}, \tag{42a}$$

$$y^2(K^*, p^*) = \frac{k_1(p^*) - K^*}{k_1(p^*) - k_2(p^*)} A_2 k_2(p^*)^{\alpha_2}. \tag{42b}$$

The supply functions of consumption goods satisfy the following:

$$\text{sign } y_K^2(K, p) = \text{sign } y_{K^*}^2(K^*, p^*) = -\text{sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right), \tag{43a}$$

$$\text{sign } y_p^2(K, p) = \text{sign } y_{p^*}^2(K^*, p^*) = -\text{sign } \left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(\alpha_1 - \alpha_2). \tag{43b}$$

In view of (41), we see that λ is expressed as a function of capital stocks, prices and $\bar{m}$:

$$\begin{aligned} \lambda &= (1 + \bar{m})^{\sigma} \left[y^2(K, p) + y^2(K^*, p^*)\right]^{\sigma} \\ &\equiv \lambda(K, K^*, p, p^*; \bar{m}). \end{aligned} \tag{44}$$

Thus by the definitions of p and p we obtain

$$p = \frac{q}{\lambda(K, K^*, p, p^*; \bar{m})},$$

$$p^* = \frac{q^*}{\lambda(K, K^*, p, p^*; \bar{m})}.$$

Solving these equations with respect to p and p^* yields the following expressions:

$$p = \pi(K, K^*, q, q^*; \bar{m}), \tag{45a}$$

$$p^* = \pi^*(K, K^*, q, q^*; \bar{m}). \tag{45b}$$

Substituting $(45a)$ and $(45b)$ into (35), (36), (39) and (40), we obtain a complete dynamic system that depicts the behaviors of K, K^*, q and q^*.

4.4 Equilibrium Indeterminacy

First, let us characterize the stationary equilibrium of the world economy. The steady state of the dynamic system derived above is established when $\dot{K} = \dot{K}^* = \dot{q} = \dot{q}^* = 0$. From (45$a$) and (45$b$) the relative price in the home and foreign countries, p and p^*, also stay constant in the steady-state equilibrium. As for the existence of a feasible steady state, we can confirm the following:

Proposition 2 *There exists a unique steady state in which both countries imperfectly specialize.*

Proof. When $\dot{q} = \dot{q}^* = 0$ in (39) and (40), it holds that

$$a_1 A_1 k_1 (p)^{\alpha_1 - 1} = a_1 A_1 k_1 (p^*)^{\alpha_1 - 1} = \rho + \delta.$$

Thus by use of (32a) and (32b), we find that

$$p = p^* = \left(\frac{A_2}{A_1}\right)\left(\frac{a_2}{a_1}\right)^{\alpha_2}\left(\frac{b_2}{b_1}\right)^{1-\alpha_2}\left(\frac{\rho+\delta}{a_1 A_1}\right)^{\frac{\alpha_2-\alpha_1}{\alpha_1-1}}.$$

Thus the steady-state levels of p and p^* are uniquely given and it holds that $p = p^*$ in the steady state. The steady-state levels of capital stocks satisfying $\dot{K} = \dot{K}^* = 0$ in (35) and (36) are determined by the following conditions:

$$\frac{K - k_2(p)}{k_1(p) - k_2(p)} A_1 k_1 (p)^{\alpha_1} = \delta K,$$

$$\frac{K^* - k_2(p^*)}{k_1(p^*) - k_2(p^*)} A_1 k_1 (p^*)^{\alpha_1} = \delta K^*$$

Using the conditions for $\dot{p} = \dot{p}^* = 0$ and the fact that $p = p^*$ holds in the steady state, we find that the steady-state level of capital stock in each county has the same value, which is given by

$$K = K^* = \frac{(aA_1)^{\frac{1}{1-\alpha_1}} (\rho+\delta)^{\frac{\alpha_1}{\alpha_1-1}}}{\rho + \delta\left(1 - \delta + \frac{a_2 b_1}{b_2}\right)}\left(\frac{a_2 b_1}{a_1 b_2}\right),$$

which has a positive value. In view of the steady-state levels of p and K derived above, the steady-state values of labor allocation to the investment good sector are:

$$L_1 = L_1^* = \frac{a_1 \delta\left(\frac{a_2 b_1}{a_1 b_2}\right)}{\rho + (1-a_1)\delta + a_1\delta\left(\frac{a_2 b_1}{a_1 b_2}\right)} \in (0,1).$$

Hence, (**??**) is fulfilled so that both countries imperfectly specialize. In addition, when p, p^*, K and K^* are given, from (41) the steady-state value of λ is uniquely determined as well, implying that $q = p\lambda$ and $q^* = p^*\lambda$ are also uniquely given in the steady state equilibrium. ■

In order to inspect local stability of the steady state, the following facts are useful:

Lemma 1 *In the symmetric steady state where $K = K^*$ and $q = q^*$, it holds the following relations:*

$$y_K^i(K,p) = y_{K^*}^i(K^*,p^*), \quad i = 1,2,$$

$$y_p^i(K,p) = y_{p^*}^i(K^*,p^*), \quad i = 1,2,$$

$$\pi_K(K,K^*,q,q^*) = \pi_K^*(K,K^*,q,q^*) = \pi_{K^*}(K,K^*,q,q^*) = \pi_{K^*}^*(K,K^*,q,q^*),$$

$$\pi_q(K,K^*,q,q^*) = \pi_{q^*}^*(K,K^*,q,q^*),$$

$$\pi_{q^*}(K,K^*,q,q^*) = \pi_q^*(K,K^*,q,q^*).$$

Proof. By the functional forms of $y_j^i(\cdot)$ ($i = 1,2$, $j = K, K^*, p, p^*$), it is easy to see that $y_K^i(K,p) = y_{K^*}^i(K^*,p^*)$ and $y_p^i(K,p) = y_{p^*}^i(K^*,p^*)$ are established when $p = p^*$ and $K = K^*$.As for the rest of the results, we may use $p\lambda(\cdot) = q$ and $p^*\lambda(\cdot) = q^*$ to drive the following:

$$\frac{\partial p}{\partial K} = \pi_K = \frac{\lambda_K}{\lambda + p\lambda_P}, \qquad \frac{\partial p}{\partial K^*} = \pi_{H^*} = \frac{\lambda_{K^*}}{\lambda + p\lambda_P}, \tag{46a}$$

$$\frac{\partial p^*}{\partial K} = \pi_K^* = \frac{\lambda_K}{\lambda + p^*\lambda_{P^*}}, \qquad \frac{\partial p^*}{\partial K^*} = \pi_{K^*}^* = \frac{\lambda_{K^*}}{\lambda + p^*\lambda_{P^*}}, \tag{46b}$$

$$\frac{\partial p}{\partial q} = \pi_q = \frac{\lambda + p\lambda_p}{\lambda(\lambda + 2p\lambda_p)}, \quad \frac{\partial p}{\partial q^*} = \pi_{q^*} = -\frac{p\lambda_p}{\lambda(\lambda + 2p\lambda_p)}, \tag{46c}$$

$$\frac{\partial p^*}{\partial q} = \pi_q^* = -\frac{p^*\lambda_{p^*}}{\lambda(\lambda + 2p^*\lambda_{p^*})}, \quad \frac{\partial p^*}{\partial q^*} = \pi_{q^*}^* = \frac{\lambda + p^*\lambda_{p^*}}{\lambda(\lambda + 2p^*\lambda_{p^*})}. \tag{46d}$$

Since $\lambda_K(\cdot) = \lambda_{K^*}(\cdot)$ and $\lambda_p(\cdot) = \lambda_{p^*}(\cdot)$ in the steady state where $K = K^*$ and $p = p^*$, we obtain $\pi_K = \pi_K^* = \pi_{K^*} = \pi^*$, $\pi_q = \pi_{q^*}^*$ and $\pi_{q^*} = \pi_q^*$. ■

We now inspect the dynamic behavior of our economy. As for local determinacy of the steady state, we find the following:

Proposition 3 *The steady-state equilibrium in the model with non-tradable capital is locally indeterminate, if the investment good sector is more capital intensive than the consumption good sector from the social perspective but it is less capital intensive from the private perspective.*

Proof. Let us linealize the dynamic system of (35), (36), (39) and (40) at the steady state. The coefficient matrix of the linealized system is given by

$$J = \begin{bmatrix} y_K^1 - \delta + y_p^1 \pi_K & y_p^1 \pi_{K^*} & y_p^1 \pi_q & y_p^1 \pi_{q^*} \\ y_{p^*}^1 \pi_K^* & y_{K^*}^1 - \delta + y_{p^*}^1 \pi_{*K^*} & y_{p^*}^1 \pi_q^* & y_{p^*}^1 \pi_{q^*}^* \\ -qr' \pi_K & -qr' \pi_{K^*} & -qr' \pi_q & -qr' \pi_{q^*} \\ -qr' \pi_K^* & -qr' \pi_{K^*}^* & -qr' \pi_q^* & -qr' \pi_{q^*}^* \end{bmatrix}.$$

In view of Lemma 1, the characteristic equation of J is written as

$$\begin{aligned} \Gamma(\eta) &= \det[\eta I - J] \\ &= \det \begin{bmatrix} \eta - (y_K^1 - \delta + y_p^1 \pi_K) & -y_p^1 \pi_K & -y_p^1 \pi_q & -y_p^1 \pi_{q^*} \\ -y_p^1 \pi_K & \eta - (y_K^1 - \delta + y_p^1 \pi_K) & -y_p^1 \pi_{q^*} & -y_p^1 \pi_q \\ qr' \pi_K & qr' \pi_K & \eta + qr' \pi_q & qr' \pi_{q^*} \\ qr' \pi_K & qr' \pi_K & qr' \pi_q & \eta + qr' \pi_q \end{bmatrix} \\ &= \det \begin{bmatrix} \eta - (y_K^1 - \delta) & 0 & \eta & 0 \\ 0 & \eta - (y_K^1 - \delta) & 0 & \eta \\ qr' \pi_K & qr' \pi_K & \eta + qr' \pi_q & qr' \pi_{q^*} \\ qr' \pi_K & qr' \pi_K & qr' \pi_{q^*} & \eta + qr' \pi_q \end{bmatrix} \\ &= \left[\eta - \left(y_K^1 - \delta\right)\right] \left[\eta + qr'(\pi_q - \pi_{q^*})\right] \xi(\eta). \end{aligned}$$

where η denotes the characteristic root of J and

$$\xi(\eta) \equiv \eta^2 + \left[qr'(\pi_q + \pi_{q^*}) - \left(y_K^1 - \delta\right) - 2y_p^1 \pi_K\right] \eta - qr' \left(y_K^1 - \delta\right)(\pi_q + \pi_{q^*}).$$

Our assumptions mean that $\frac{a_1}{b_1} - \frac{a_2}{b_2} < 0$ and $\alpha_1 - \alpha_2 > 0$. Thus from (43a) we see that $y_K^1 - \delta < 0$. In addition, note that from (46c) it holds that $\pi_q - \pi_{q^*} = 1/\lambda\,(> 0)$. Hence, using $r(p) \equiv a_1 A_1 k_1(p)^{\alpha_1 - 1}$, we obtain:

$$r'(\pi_q - \pi_{q^*}) = a_1 (a_1 - 1) A_1 (k_1(p))^{a_1 - 2} \frac{k_1'(p)}{\lambda} > 0.$$

As a consequence, at least two roots of $\Gamma(\eta) = 0$ have negative real parts. Equations in (46c) also show

$$\pi_q + \pi_{q^*} = \frac{1}{\lambda + 2p\lambda_p},$$

where

$$\begin{aligned} \lambda_p &= \frac{\partial}{\partial p}(1 + \bar{m})^{\frac{1}{\sigma}} \left[y^2(K, p) + y^2(K^*, p^*)\right]^{-\frac{1}{\sigma}} \\ &= -\frac{y_p^2}{\sigma}(1 + \bar{m})^{\frac{1}{\sigma}} \left[y^2(K, p) + y^2(K^*, p^*)\right]^{-\frac{1}{\sigma} - 1} < 0. \end{aligned}$$

Therefore, in the steady state equilibrium. the following holds:

$$\lambda + 2p\lambda_p = \frac{1}{\sigma}\left[\sigma - \frac{py_p^2(K,p)}{y^2(K,p)}\right]$$

Notice that under our assumptions, it holds that $y_p^2(K,p) > 0$. Suppose that σ is small enough to satisfy $\sigma < py_p^2/y^2$. Then $\lambda_p + 2p\lambda_p > 0$ so that $\pi_q + \pi_{q^*} < 0$, which leads to

$$-qr'\left(y_K^1 - \delta\right)(\pi_q + \pi_{q^*}) < 0.$$

This means that $\xi(\eta) = 0$ has one positive and one negative roots. As a result, $\Gamma(\eta) = 0$ has three stable roots. Hence, if σ is smaller than the price elasticity of supply function of consumption goods, then there locally exists a continuum of equilibrium paths converging to the steady state.

Now suppose that σ is larger than py_p^2/y^2. Then we obtain $\pi_q + \pi_{q^*} > 0$. Furthermore, it holds that

$$\begin{aligned} -2y_p^1\pi_K &= -2y_p^1\left(-\frac{p\lambda_K}{\lambda + 2p\lambda_p}\right) \\ &= -\frac{2py_p^1}{\lambda + 2p\lambda_p}y_K^2\left[\frac{(1+\bar{m})^{\sigma^{-1}}}{\sigma}\right]\left(2y^2\right)^{-\sigma^{-1}-1} > 0, \end{aligned}$$

because $y_p^1 < 0$ and $y_K^2 > 0$ under our assumptions. Consequently, the following inequalities are established:

$$-qr'\left(y_K^1 - \delta\right)(\pi_q + \pi_{q^*}) > 0,$$

$$qr'(\pi_q + \pi_{q^*}) - \left(y_K^1 - \delta\right) - 2y_K^1\pi_K > 0.$$

These conditions mean that $\xi(\eta) = 0$ has two roots with negative real parts and, hence, all the roots of $\Gamma(\eta) = 0$ are stable ones. In sum, if $\Delta_p = \frac{a_1}{b_1} - \frac{a_2}{b_2} < 0$ and $\Delta_s = \alpha_1 - \alpha_2 > 0$, then the characteristic equation of the linearlized system involves at least three stable roots, meaning that the converging path towards the steady state is locally indeterminate. ■

It is to be emphasized that, as the above proposition shows, in our setting indeterminacy may emerge regardless of the magnitude of σ. This is in contrast to the conclusion in Nishimura and Shimomura (2002a) showing that the indeterminacy conditions involve a high elasticity of substitution in consumption, $1/\sigma$. Since the closed economy version of our model is the same as that of Nishimura and Shimomura (2002a), we need the same condition for holding indeterminacy if our model economy is closed. Hence, our result shows that the financially integrated world with non-tradable capital goods tends to produce indeterminacy under a wider range of parameter spaces than in the closed economy counterpart. In this sense, our model claims that internationalization may enhance the possibility of sunspot-deriven economic fluctuations.

Finally, to complete our stability analysis, we summarize the findings for the other cases.

Proposition 4 *(i) If the private and the social factor-intensity rankings are the same, then the steady-state equilibrium is locally determinate, and (ii) if the capital good sector is more capital intensive than the consumption good sector from the private perspective but it is less capital intensive from the social perspective, then the steady state is unstable.*

Proof. (i) Note that $\text{sign } r'(p) = \text{sign } \left[(a_1 - 1) A_1 k_1^{a_1-2} k_1'(p)\right] = \text{sign } (\alpha_1 - \alpha_2)$. Thus if $\left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(\alpha_1 - \alpha_2) > 0$, then

$$\text{sign } \left(y_K^1 - \delta\right)\left[-qr'(\pi_q - \pi_{q^*})\right] < 0.$$

In addition, when $\left(\frac{a_1}{b_1} - \frac{a_2}{b_2}\right)(\alpha_1 - \alpha_2) > 0$,we obtain

$$\text{sign } [-qr'(y_K^1 - \delta)(\pi_q + \pi_q *)] = \text{sign } -\frac{r'(y_K^1 - \delta)}{\lambda + p\lambda_p} < 0,$$

because $\text{sign } \lambda_p = -\text{sign } y_p^2 > 0$ and $r'(y_K^1 - \delta) > 0$. As a results, $\Gamma(\eta) = 0$ has two stable and two unstable roots, so that there is a unique converging path around the steady state.

(ii) If $\frac{a_1}{b_1} - \frac{a_2}{b_2} > 0$ and $\alpha_1 - \alpha_2 < 0$, then $\lambda_p < 0$. Hence, in this case the sign of $\pi_q + \pi_{q^*}$ is not determined without imposing further restrictions. In the case of $\pi_q + \pi_{q^*} > 0$, we have two positive eigenvalues, $r'(\pi_q + \pi_{q^*}) > 0$ and $y_K^1 - \delta > 0$. On the other hand, if it holds that

$$-qr'\left(y_K^1 - \delta\right)(\pi_q + \pi_{q^*}) < 0,$$

then $\xi(\eta) = 0$ has one positive and one negative root. If $\pi_q + \pi_{q^*} > 0$, we see that $r'(\pi_q + \pi_{q^*}) < 0$ and $y_K^1 - \delta > 0$. In addition, if

$$\begin{aligned} -qr'\left(y_K^1 - \delta\right)(\pi_q + \pi_{q^*}) &< 0, \quad qr'(\pi_q + \pi_{q^*}) < 0, \\ -(y_K^1 - \delta) &< 0, \quad -2y_p^1 \pi_K < 0, \end{aligned}$$

then $\xi(\eta) = 0$ has two positive roots. Therefore, regardless of the sign of $\pi_q + \pi_{q^*}$, $\Gamma(\eta) = 0$ has only one stable root and thus the steady state equilibrium is locally unstable.If $\frac{a_1}{b_1} - \frac{a_2}{b_2} > 0$ and $\alpha_1 - \alpha_2 < 0$, then $r'(\pi_q + \pi_{q^*}) > 0$ and $y_K^1 - \delta > 0$.

Additionally, it is seen that

$$-qr'\left(y_K^1-\delta\right)\left(\pi_q+\pi_{q^*}\right)<0,$$

so that $\xi(\eta) = 0$ has one positive and one negative root. This reveals that $\Gamma(\eta) = 0$ has only one stable root and thus the steady-state equilibrium is locally unstable. ■

These results are also close to the stability conditions for the small open economy models with capital mobility examined by Meng and Velasco (2003 and 2004). This proposition gain emphasizes that the dynamic behavior of the financially integrated world economy with symmetric countries and non-traded capital goods is closer to the behavior of corresponding small-open economy rather than to the closed economy counterpart.

5 Discussion

5.1 Implications of Equilibrium Indeterminacy

As emphasized above, the result (i) in Proposition 2 claims that in our model equilibrium indeterminacy may emerge regardless of the magnitude of σ. We should note that the absence of the intertemporal trade in the Nishimura-Shimomura (NS) model is not the key for the difference in the indeterminacy conditions between the two models. From view point of intetermporal consumption decision of households, our discussion and the NS model share the similar properties: in our model households in the home country smooth their consumption by borrowing from or lending to the foreign households, while in the NS model households smooth consumption by trading investment goods with the foreign country. In fact, if we introduce international bond market into the NS model, the households can hold two completely substitutable means for consumption smoothing, so that the instantaneous equilibrium becomes indeterminate.[13] Therefore, the central reason for generating different indeterminacy conditions is the presence of non-tradable investment goods under which the factor-price equalization fails to hold out of the steady state.

We now consider the long-run consequence of equilibrium indeterminacy in our model. To see this, we should consider how to determine $\bar{m}$. In so doing, it is rather easy to use the market equilibrium conditions than the optimization conditions for the planning problem. Using the market equilibrium condition for the investment goods in (23) and the factor income distribution relation such that

[13] This is a reconfirmation of Mundel's (1957) well-known conclution: see, for example, Bajona. and Kehoe (2006).

$pY_1 + Y_2 = rpK + w + \pi_1 + \pi_2$ and $p^*Y_1^* + Y_2^* = p_1^*K^* + w^* + \pi_1^* + \pi_2^*$, we find that the dynamic equation of foreign bonds are expressed as

$$\dot{B} = RB + Y_2 - C,$$
$$\dot{B}^* = RB^* + Y_2^* - C^*.$$

These equations represents the current account of the each country. In view of the no-Ponzi game as well as the transversality conditions, the intertemporal constraint for the bond holdings in each country respectively given by the following equations:

$$\int_0^\infty \exp\left(-\int_0^t R_s ds\right) C_t dt = \int_0^\infty \exp\left(-\int_0^t R_s ds\right) y^2\left(K_t, p_t\right) dt + B_0,$$
$$\int_0^\infty \exp\left(-\int_0^t R_s ds\right) C_t^* dt = \int_0^\infty \exp\left(-\int_0^t R_s ds\right) y^2\left(K_t^*, p_t^*\right) dt + B_0^*.$$

Since it holds that $C_t^* = \bar{m}C_t$ for all $t \geq 0$, the above equations yield

$$\bar{m} = \frac{\int_0^\infty \exp\left(-\int_0^t R_s ds\right) y^2\left(K_t^*, p_t^*\right) dt + B_0^*}{\int_0^\infty \exp\left(-\int_0^t R_s ds\right) y^2\left(K_t, p_t\right) dt + B_0}. \tag{47}$$

Equation (47) demonstrates that $\bar{m}$ depends on the initial holdings of bonds, B_0 and B_0^*, as well as on the discounted present value of consumption goods produced in each country. It is to be noticed that the discounted present values of consumption goods do not depend on $\bar{m}$. To show this, we first differentiate both sides of (**??**) logarithmically with respect to time, which leads to

$$\frac{\dot{\lambda}}{\lambda} = -\sigma\left[\frac{Y_K^2 K}{Y^2}\frac{\dot{K}}{K} + \frac{Y_{K^*}^2 K^*}{Y^2}\frac{\dot{K}^*}{K^*} + \frac{Y_p^2 p}{Y^2}\frac{\dot{p}}{p} + \frac{Y^2 p^*}{Y^2}\frac{\dot{p}^*}{p^*}\right],$$

where $Y^2 \equiv y^2\left(K, p\right) + y^2\left(K^*, p^*\right)$ denotes the aggregate supply of consumption goods in the world market. Substituting (**??**), (**??**), $(45a)$, and $(45b)$ into the above, we obtain

$$\begin{aligned}\rho - R &= -\sigma\left[\frac{Y_K^2 K}{Y^2}\left(\frac{y^1\left(K, p\right) - \delta K}{K}\right) + \frac{Y_{K^*}^2 K^*}{Y^2}\left(\frac{y^2\left(K^*, p\right) - \delta K^*}{K^*}\right)\right. \\ &\quad \left. + \frac{Y_p^2 p}{Y^2}\left(R - r\left(p\right) + \delta\right) + \frac{Y_{p^*}^2 p^*}{Y^2}\left(R - r\left(p^*\right) + \delta\right)\right].\end{aligned}$$

Observe that both sides of the above equation does not involve $\bar{m}$. Solving the above with respect to R, we find that the equilibrium level of the world interest rate can be expressed as a function of K, K^*, p and p^* :

$$R_t = R\left(K_t, K_t^*, p_t, p_t^*\right). \tag{48}$$

Consequently, a complete dynamic system can be rewritten as

$$\left.\begin{array}{r}\dot{K}=y^{1}\left(K,p\right)-\delta K,\\ K^{*}=y^{1}\left(K^{*},p^{*}\right)-\delta K^{*},\\ \dot{p}=p\left[R\left(K,K^{*},p,p^{*}\right)+\delta-r\left(p\right)\right],\\ \dot{p}^{*}=p\left[R\left(K,K^{*},p,p^{*}\right)+\delta-r\left(p^{*}\right)\right].\end{array}\right\}\tag{49}$$

From (**??**) the steady state level of interest rate satisfies $R=\rho$.[14] Since the dynamic system (49) does not involve $\bar{m}$, if the steady state is locally determinate (i.e. the linearized system has two stable roots), then the equilibrium path of p_t and p_t^* are expressed as functions of K_t and K_t^* alone on the two-dimensional stable manifold. This means that from (48) the equilibrium level of interest rate, R_t, is also expressed by a function of K_t and K_t^*. Hence, the equilibrium path is determinate, under a given set of K_0 and K_0^*, the value of $\bar{m}$ is fixed if we specify B_0 and B_0^*.

In contrast, if the converging path of (49) is indeterminate, the given initial levels of K_0 and K_0^* cannot pin down the equilibrium paths of p_t and p_t^*: we cannot select a particular equilibrium path without specifying expectations formation of the households in both countries. Once we specify a particular trajectory of the world economy with self-fulfilling expectations, then we may determine the value of $\bar{m}$ in the same manner mentioned above.

In the steady state it holds that $\dot{B}=\dot{B}^*=0$ and $R=\rho$. Hence, we the steady-state level of bond holdings in both countries are

$$B=\frac{y^{2}\left(K,p\right)-C}{\rho}=\frac{\bar{m}-1}{\rho(1+\bar{m})}y^{2}\left(K,p\right),$$

$$B^{*}=\frac{y^{2}\left(K,p\right)-\bar{m}C}{\rho}=\frac{1-\bar{m}}{\rho(1+\bar{m})}y^{2}\left(K,p\right).$$

Therefore, when $\bar{m}$ is selected, the long-run asset position of each country is also determined. It is obvious that whether the home country becomes a creditor or a debtor in the long run depends solely on whether or not $\bar{m}$ exceeds one. As (47) indicates, the larger its initial capital and bond holdings are larger, the higher the possibility that the home country plays as a creditor in the long-run equilibrium. If there is a continuum of converging path around the steady state, the value of $\bar{m}$ determined by (47) is affected by the expectations formation of agents. This implies that sunspot-deriven expectations changes fluctuate the equilibrium path of the world economy and they may affect the steady state asset distribution of wealth (so that the long-run asset position of each country).

To sum up the above discussion, we have shown:

[14]We can show that dynamic analysis of (49) yields the same conclusion as that stated in Proposition 2: see Hu and Mino (2009). However, since function (48) is rather complex, stability analysis is more cumbersome than that shown in Appendix 2.

Proposition 5 *If the steady-state equilibrium of the world economy is locally determinate (indeterminate), then the steady-state level of asset position of each country is determinate (indeterminate).*

5.2 Small-Open Economy

If the home country is a small-open economy with non-traded investment goods and financial capital mobility, then the world interest rate is exogenously given. Thus the dynamic motion of the small-open economy are given by (35) and $\dot{\lambda} = \lambda\left(\bar{R} - \rho\right)$, where $\bar{R}$ is a given world interest rate. The conventional assumption is that the time discount rate is set to satisfy $\rho = \bar{R}$ to obtain a feasible steady state, which means that the shadow value of foreign bonds, λ, stays constant over time. As a consequence, in view of $p = q/\lambda$, the shadow value of capital is proportional to the relative price even out of the steady state. Thus the price dynamics is given by

$$\dot{p} = p\left(\bar{R} + \delta - r(p)\right) = \rho\left(\rho + \delta - r(p)\right), \tag{51}$$

and thus dynamic behaviors of capital and relative price are not affected by the value of σ. It is shown the determinacy/indeterminacy conditions given in Proposition 2 also applies to the dynamic system consisting of (35) and (51).

It is now obvious that the dynamic behavior of our model that assumes non-traded capital goods and financial transactions is close to the small-open economy counterpart with the same trade structure rather than to the NS model based on the Heckscher-Ohlin assumptions. Since in our integrated world economy model the world interest rate is an endogenous variable, the dynamic behavior of the integrated world economy during the transition is not exactly the same as that of the small-open economy. However, due to our assumption of the symmetric technologies and preference structures between the two countries, dynamic properties of the world economy near the steady state equilibrium is close to the dynamics of the corresponding small-open economy in which the world interest rate is taken as given. Such a close connection of dynamic property give rise to our main conclusion that the determinacy/indeterminacy conditions for the world economy are essentially the same as those for the small country counterpart at least near the steady state.

5.3 Non-Tradable Consumption Goods

Finally, consider the case where investment goods are tradables, while the consumption goods are non traded. In this case, the constraints for the planning problem are given by

$$I + I^* = A_1 K_1^{a_1} L_1^{b_1} \bar{X}_1 + A_1 K_1^{*a_1} L_1^{*b_1} \bar{X}_1^*,$$

$$C = A_2 (K - K_1)^{a_2} (1 - L_1)^{b_2} \bar{X}_2, \qquad C^* = A_2 (K - K_2^*)^{a_2} (1 - L_2^*)^{b_2} \bar{X},$$
$$\dot{K} = I - \delta K, \quad \dot{K}^* = I^* - \delta K^*.$$

The Hamiltonian function for this problem is:

$$\begin{aligned}\mathcal{H} &= \frac{C^{1-\sigma} - 1}{1-\sigma} + \mu^* \frac{C^{*1-\sigma} - 1}{1-\sigma} \\ &+\phi(A_1 K_1^{a_1} L_1^{b_1} \bar{X}_1 + A_1 K_1^{*a_1} L_1^{*b_1} \bar{X}_1^* - I - I^*) \\ &+\lambda \left[A_2 (K - K_1)^{a_2} (1 - L_1)^{b_2} \bar{X}_2 - C\right] + \lambda^* [A_2 (K^* - K_1^*)^{a_2} (1 - L_1^*)^{b_2} \bar{X}_2^* - C^*\Big] \\ &+q (I - \delta K) + q^* (I^* - \delta K^*).\end{aligned}$$

Maximizing the Hamiltonian function with respect to I and I^* gives $q = q^* = \phi$. In view of these conditions, we see that on the equilibrium path the shadow value of capital stocks change according to

$$\dot{q} = q \left(\rho + \delta - a_1 A_1 k_1 (p)^{\alpha_1 - 1}\right),$$
$$\dot{q}^* = q^* \left(\rho + \delta - a_1 A_1 k_1 (p^*)^{\alpha_1 - 1}\right),$$

where $p = q/\lambda$ and $p^* = q/\lambda^*$. These conditions mean that $k_1 (p) = k_1 (p^*)$, leading to $p = p^*$ $(\lambda = \lambda^*)$. Namely, the agents in both countries face with the same relative price, and thus the factor price equalization hot always holds even out of the steady state.

Consequelty, the dynamics of the world economy is described by the following aggretate equations that also holds in the Heckscher-Ohlin setting:

$$\dot{K}_w = \frac{2k_1 (p) - K_w}{k_1 (p) - k_2 (p)} A k_1 (p)^{\alpha_1} - \delta K_w,$$
$$\dot{q} = q \left(\rho + \delta - a_1 A k_1 (p)^{\alpha_1 - 1}\right).$$

where $K_w = K + K^*$. In order to derive the relation between q and λ, we can use the following maket equilibrium conditions for consumption goods in each country:

$$C = \lambda^{-1/\sigma} = \frac{K - k_2 (p)}{k_1 (p) - k_2 (p)} A_2 k_2 (p)^{\alpha_2},$$
$$C^* = (\lambda/\mu^*)^{-1/\sigma} = \frac{K^* - k_2 (p)}{k_1 (p) - k_2 (p)} A_2 k_2 (p)^{\alpha_2}.$$

We add these equations to obtain

$$\lambda^{-1/\sigma} \left(1 + \mu^{*1/\sigma}\right) = \frac{K_w - 2k_2 (p)}{k_1 (p) - k_2 (p)} A_2 k_2 (p)^{\alpha_2}$$

This condition is the same as (44), so that the equilibrium dynamics of the economy with non-tradable consumption goods and financial capital mobility is identical to that ot the Heckscher-Ohlin model.

6 Conclusion

This paper has investigated the relation between trade structure and equilibrium indeterminacy in a two country world. We have introduced non-traded capital goods and international financial transactions into the dynamic Heckscher-Ohlin model with production externalities examined by Nishimura and Shimomura (2002a). Our extension has demonstrated that the introduction of non-traded goods and financial asset mobility enhances the range of parameter values under which the perfect-foresight competitive equilibrium of the world economy is indeterminate. Since the standard Heckscher-Ohlin setting used by Nishimura and Shimomura (2002a) establishes the same stability conditions as these held in the corresponding closed economy, our finding indicates that the assumptions of trade structure of the world economy would be a critical determinant in considering relation between globalization and economic volatility.

The world economy as a whole is a closed economy in which there are heterogenous countries. Therefore, its model structure is similar to that of a closed, single economy model with heterogenous agents. In particular, if consumption and saving decisions are made by the representative household in each country, the behavior of the world economy model is closely connected to that of the closed economy model with heterogenous households. There is, however, a key difference between the world economy and the single country settings: when dealing with the world economy model, we should specify the transaction structure between the countries. Both of the Heckscher-Ohlin theory and the discussion in this paper assume specific structures of international trade. It is worth investigating how our conclusion would be modified under alternative forms of international trade. For example, Ono and Shibata (2010) demonstrate that if investment is associated with adjustment costs, the Heckscher-Ohlin model may have a meaningful equilibrium even if there are international lending and borrowing. They show that introducing financial transaction may affect the long-run trade patterns. This suggests that the presence of investment costs may affect our conclusions as well.[15]

In the literature on indeterminacy and sunspots, some authors have explored how the presence of heterogenous households may alter the indeterminacy conditions in the real business cycle models with market distortions. These studies have shown that the heterogeneity of agents often affects stability condition in a critical manner.[16] As mentioned in Section 1, Sim and Ho (2007a) reveal that the

[15] In the trade theory literature, the relation between equilibrium characterization of the world economy and trade structures have been discussed extensively: see, for example, Ethier and Svensson (1986) and Cremers (1997). We may use the results obtained in those studies to extend our argument.

[16] See, for example, see Ghiglino and Olszak-Duquenne (2005).

introduction of technological heterogeneity into the Nishimura-Shimomura model may produce a substantial change in equilibrium indeterminacy results. Those existing findings suggest that it is worth extending our model by considering further heterogeneity between the two countries in order to consider the impact of globalization on aggregate stability in a more general framework than the present paper.

Appendix

In this appendix we show that the pseudo-planning problem discussed in the main text characterizes the competitive equilibrium of the decentralized world economy.[17] For this purpose, we first derive the optimization conditions of the households and firms in both countries.

Set up the Hamiltonian function for the households in the home country in such a way that

$$\mathcal{H}=\frac{C^{1-\sigma}-1}{1-\sigma}+\zeta\left(R\Omega+w+\pi_1+\pi_2-C\right),$$

where ζ denotes the implicit value of net wealth. The necessary conditions for an optimum include the following:

$$C^{-\sigma}=\zeta, \tag{A1}$$

$$\dot{\zeta}=\zeta\left(\rho-R\right), \tag{A2}$$

together with the transversality conditions: $\lim_{t\to\infty} e^{-\rho t}\zeta_t\Omega_t=0$. Note that the transversality condition means that the non-Ponzi-game restriction holds with an equality. Profit maximization conditions in (**??**) yield

$$r=pa_1A_1K_1^{a_1-1}L_1^{b_1}\bar{X}_1-\delta=a_2A_2K_2^{a_2-1}L_2^{b_2}\bar{X}_2-\delta, \tag{A3}$$

$$w=pb_1A_1K_1^{a_1}L_1^{b_1-1}\bar{X}_1=b_2A_2K_2^{a_2}L_2^{b_2-1}\bar{X}_2. \tag{A4}$$

From the non-arbitrage condition (**??**) we obtain

$$R=r+\frac{\dot{p}}{p}=a_1A_1K_1^{\alpha_1-1}L_1^{1-\alpha_1}-\delta+\frac{\dot{p}}{p}. \tag{A5}$$

In the same vein, we obtain the conditions for the foreign country corresponding to the above as follows:

$$C^{*-\sigma}=\zeta^* \tag{A6}$$

[17] See Hu and Mino (2009) for a detailed analysis of the market economy version of this model.

$$\dot{\zeta}^* = \zeta^* (\rho - R) \tag{A7}$$

$$r^* = p^* a_1 A_1 K_1^{*a_1 - 1} L_1^{*b_1} \bar{X}_1^* = a_2 A_2 K_2^{*a_2 - 1} L_2^{*b_2} \bar{X}_2^* \tag{A8}$$

$$w^* = p^* b_1 A_1 K_1^{*a_1} L_1^{*b_1 - 1} \bar{X}_1^* = b_2 A_2 K_2^{*a_2} L_2^{*b_2 - 1} \bar{X}_2^* \tag{A9}$$

$$R = r^* + \frac{\dot{p}^*}{p^*} = a_1 A_1 K_1^{*\alpha_1 - 1} L_1^{*1-\alpha_1} - \delta + \frac{\dot{p}^*}{p^*}. \tag{A10}$$

It is seen that if we set $p = q/\lambda$ and $p^* = q^*/\lambda$, then (A3), (A4), (A8) and (A9) respectively correspond to (31b) through (31e) in the planning problem. Furthermore, by use of (A5), (A10), $\bar{X}_i = K_i^{\alpha_i - a_i} L_i^{1-\alpha_i - b_i}$ and $\bar{X}_i^* = K_i^{*\alpha_i - a_i} L_i^{*1-\alpha_i - b_i}$, we find

$$\frac{\dot{p}^*}{p^*} - \frac{\dot{p}}{p} = \frac{\dot{q}^*}{q} - \frac{\dot{q}}{q} = a_1 A_1 K_1^{\alpha_1 - 1} L_1^{1-\alpha_1} - a_1 A_1 K_1^{*\alpha_1 - 1} L_1^{*1-\alpha_1}.$$

This relation can be obtained from

$$\frac{\dot{q}}{q} = \rho + \delta - a_1 A_1 K_1^{\alpha_1 - 1} L_1^{1-\alpha_1},$$

$$\frac{\dot{q}^*}{q^*} = \rho + \delta - a_1 A_1 K_1^{*\alpha_1 - 1} L_1^{*1-\alpha_1},$$

which respectively correspond to (31f) and (31g).

To examine the relation between the transversality conditions for the market economy and those in the planning problem, it is to be noted that (A2) and (A7) mean that $\dot{\zeta}/\zeta = \dot{\zeta}^*/\zeta^* = \rho - R$. Therefore, in view of (A1) and (A6), we see that $C^*/C = (\zeta^*/\zeta)^{-1/\sigma}$ stays constant over time. Thus we may set $\zeta^*/\zeta = \mu^*$, i.e. the relative welfare weight on the foreign households in the planning problem. In addition, (A2) gives $\zeta_t = \zeta_0 \exp\left(\int_t^\infty (\rho - R_s) ds\right)$. Therefore, from the definitions of $\Omega_t = B + pK$ and $p = q/\lambda$, the non-Ponzi game condition, together with the transversality condition, for the household in the home country becomes

$$\lim_t \Omega_t \exp\left(-\int_0^t R_s ds\right) = \zeta_0 \lim e^{-\rho t} \zeta_t \left(B_t + \frac{q_t}{\lambda_t} K_t\right) = 0.$$

Hence, the non-Ponzi game scheme the economy as a whole (condition (**??**)) implies that $\lim e^{-\rho t} \zeta_t \frac{q_t}{\lambda_t} K_t = 0$, so that the transversality condition for the planing problem, $\lim_{t\to\infty} e^{-\rho t} q_t K_t = 0$, is established by setting $\zeta_t = \lambda_t$. Since $\zeta_t^* = \mu^* \zeta_t$, the non-Ponzi game conditions for the foreign households yields

$$\lim_t \Omega_t \exp\left(-\int_0^t R_s ds\right) = \mu^* \zeta_0 \lim e^{-\rho t} \zeta_t \left(B_t^* + \frac{q_t^*}{\lambda_t} K_t^*\right) = 0.$$

This and (**??**) ensure the transversality, condition $\lim_{t\to\infty} e^{-\rho t} q_t^* K_t^* = 0$, in the planning problem.

Finally, let us check the Walras law in the market economy. First, note that $\dot{\Omega} + \dot{\Omega}^* = p\dot{K} + p^*\dot{K}^* + \dot{p}K + \dot{p}^*K^*$. Thus adding up the flow budget constraint for the households in each country gives

$$\begin{aligned} & p\dot{K} + p^*\dot{K}^* + \dot{p}K + \dot{p}^*K^* \\ = \; & R\,(pK + p^*K) + w + w^* - C - C^* \\ = \; & \left(r + \frac{\dot{p}}{p}\right) pK + \left(r + \frac{\dot{p}^*}{p^*}\right) p^*K^* + w + w^* - C - C^*. \end{aligned} \tag{A11}$$

By use of the full-employment conditions, $K = K_1 + K_2$ and $1 = L_1 + L_2$, we obtain

$$\begin{aligned} rpK + w + \pi_1 + \pi_2 &= p\left(rK_1 + \frac{w}{p}L_1\right) + \pi_1 + prK_2 + wL_2 + \pi_2 \\ &= pY_1 + Y_2 - \delta K. \end{aligned} \tag{A12}$$

Similarly, it holds that

$$r^*p^*K^* + w^* + \pi_1^* + \pi_2^* = p^*Y_1^* + Y_2^* - \delta K^*. \tag{A13}$$

Substituting (A12) and (A13) into (A11) and using $Y_1 = \dot{K} + \delta K$ and $Y_1^* = \dot{K}^* + \delta K^*$, we obtain the world market equilibrium condition of the consumption goods: $Y_2 + Y_2^* = C + C^*$.

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