# Equilibrium Default and Temptation

Makoto Nakajima
University of Illinois at Urbana-Champaign*

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**Very Preliminary**

**Abstract**

In this paper I quantitatively investigate macroeconomic and welfare implications of the recent consumer bankruptcy law reform using a general equilibrium life-cycle model with unsecured debt and equilibrium default where agents have preferences featuring temptation and self-control problems. The preference used here includes quasi-hyperbolic discounting as the extreme case where temptation is infinitely strong. The key components of the U.S. bankruptcy law reform which was enacted in 2005 are *(i)* subjecting filers to means-testing, and *(ii)* increased cost of filing for bankruptcy. I find that, both the standard model with exponential discounting and the model with temptation and self-control (or quasi-hyperbolic discounting) do well in replicating the responses of the U.S. economy after the bankruptcy reform. Both models correctly predict that the number of bankruptcy filings decrease, and the amount of loans and the average interest rate of loans do not change substantially. However, the macroeconomic implications of the recent bankruptcy reform will crucially depend on what type of shocks is dominant. In particular, if defaults are not mainly due to expenditure shocks, but rather due to series of unfavorable income realizations, models with exponential discounting predict an *increase* in the number of bankruptcy filings, while models with temptation and self-control still predict a *decrease* in the number of bankruptcy filings in response to the recent bankruptcy reform. Regarding the welfare implications of the bankruptcy law reform, the implications from different models are similar; both models with exponential discounting and those with temptation and self-control imply *welfare loss* from the bankruptcy reform, mainly because of the loss of welfare of those who cannot file even if they want. I also find that, if the same level of punishment for bankruptcy is used, models with temptation and self-control problem generates a larger debt and more bankruptcy filings than the model with exponential discounting.

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*Department of Economics, University of Illinois at Urbana-Champaign. 1206 South 6th Street, Champaign, IL 61820. E-mail: `makoto@uiuc.edu`.

# 1 Introduction

There are two main goals in this paper. First, I investigate the properties of the model with equilibrium default and preference which features temptation and self-control problem. The preference that I use include the standard exponential discounting as one extreme case and the quasi-hyperbolic discounting as the other extreme case. Second, I ask whether the model with temptation and self-control problem is a better model than the model with the standard exponential discounting in replicating the response of the U.S. economy when the bankruptcy law reform was introduced in 2005. The paper is motivated by the popular belief that a model with preferences featuring temptation and self-control is a better model for capturing borrowing and defaulting behavior.

Build on earlier studies such as Strotz (1956) and Pollak (1968), Laibson (1997) Laibson (1996) study macroeconomic models which feature variable rate of time preference, in particular, quasi-hyperbolic discounting, and consequently multiple-selves framework. Since one of the key implications of the models with quasi-hyperbolic discounting is the under-saving, or over-consumption, these models are considered to have a potential to better capture the borrowing and defaulting behavior. For example, Laibson et al. (2003) show that quasi-hyperbolic discounting model can explain why majority of households with credit cards pay interest on the cards even if they have asset as well. White (2007) argues that hyperbolic discounting preference is an important feature in constructing a model of bankruptcies for policy evaluation.

One of the problems with the quasi-hyperbolic discounting model is that it is not straightforward how to conduct welfare analysis, because there are multiple selves within one agent. Krusell et al. (2005) introduces the preference which features temptation and self-control problem which not only can be understood as a generalization of the quasi-hyperbolic discounting model but also enables welfare analysis in a more natural way. In their framework, agents are *tempted* to choose current consumption using a higher discount factor and thus consume more. At the same time, agents use *self-control* to fight against such temptation. When the strength of temptation goes to infinity, agents completely succumb to temptation, and virtually makes the model the same as the multiple-selves model. In this sense, the model by Krusell et al. (2005) includes the model with quasi-hyperbolic discounting as the extreme case. On the other hand, when the strength of temptation goes to zero, then the model goes back to the standard model with exponential discounting, because there is no temptation. Naturally, when temptation exists, the problem of an agent involves the problem of tempted decision as well as problem of self-control.

İmrohoroğlu et al. (2003) studies the macroeconomic and welfare implications of unfunded social security when agents have quasi-hyperbolic discounting and therefore face time inconsistency problem. Bucciol (2007) solves a life-cycle model with temptation preference, to investigate how the temptation affects the portfolio choice between risky and risk-free assets over the life-cycle.

On the other hand, quantitative general equilibrium model with equilibrium bankruptcies has been developed recently. Pioneer work are Livshits et al. (2007b), Chatterjee et al. (2007), and Athreya (2002). One of the key questions in this paper is whether and how the model with equilibrium bankruptcy performs better with the preference that features temptation and self-control.

Another key question is of normative nature. I ask whether and how there is a difference in terms of welfare effect of some policy changes. As Krusell et al. (2005) find, even though the model with exponential discounting and the model with temptation and self-control problems might be observationally equivalent, they might have different welfare implications. If that is the case, the optimal policy can be different for these models. In the case of Krusell et al. (2005), even though Barro (1999) find that the two neoclassical growth models with different preference are almost observationally equivalent, they have different implications about optimal capital income tax rate. This is because agents with temptation can benefit from negative capital income tax (subsidy to saving). The current paper shares the spirit with the paper by Krusell et al. (2005) in the sense that the welfare effect of bankruptcy law reform in the economies with different assumptions on preference is investigated.

There are five main findings. First, models with different preference specifications exhibit very similar average life-cycle profiles of asset, but models with temptation (including quasi-hyperbolic discounting model as the extreme case) show a drop in consumption at the time of retirement and a second hump in the average consumption profile after retirement. Second, conditional on the same level of punishment for defaults, models with temptation generates a larger amount of debt and a larger number of defaults. This finding is consistent with the over-borrowing and over-consumption story usually associated with hyperbolic discounting preference. Third, under the baseline calibration, both the standard exponential discounting model and the model with temptation and self-control replicate the reaction of the U.S. economy against the recent bankruptcy law reform equally well. Both models correctly predict a decline in the number of bankruptcies, and no significant change in the amount of loans and the average loan interest rate. Fourth, however, the result crucially depends on what type of shocks is dominant. In particular, if defaults are not mainly due to expenditure shocks, but rather due to series of unfavorable income realizations, models with exponential discounting predict an *increase* in the number of bankruptcy filings, which is a counterfactual implication, while models with temptation and self-control still predict a *decrease* in the number of bankruptcy filings in response to the recent bankruptcy reform. Fifth, the welfare implications of the two class of models in response to the recent bankruptcy law reform are similar; both imply a mild *welfare loss* from the reform, mainly due to the welfare loss of those who cannot file when it is optimal to do so. In sum, under the baseline calibration, in studying the macroeconomic and welfare implications of the recent bankruptcy law reform, using the model with temptation and self-control does not give a clear advantage over the standard model with exponential discounting. The properties of the models become very different depending on the major cause of bankruptcy filings.

The remaining parts of the paper are organized as follows. Section 2 gives overview of the U.S. bankruptcy law, and description of the recent bankruptcy law reform. The section also includes description of the data around the time of the reform that are related to debt and bankruptcy. Section 3 sets up the model. Section 4 describes how the model is calibrated. Section 5 comments on how the model is numerically solved. Section 6 compares the properties of the calibrated models with different preference specifications. Section 7 investigates how models with different preference specifications react differently to the artificial bankruptcy law reform. In Section 8, welfare implications of bankruptcy law reform are explored. Section 9 concludes.

## 2 Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA)

In this section, I will first overview the bankruptcy scheme in the U.S. in general.[1] Then I will describe the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA), many of the provisions of which were enacted in October 2005, and how data related to debt and defaults change around the time when BAPCPA was introduced.[2]

In the background of BAPCPA was a concern of the sharp increase in the number of consumer bankruptcies since the early 1980s.[3] For example, the number of consumer bankruptcies increased more than five-fold between 1980 to 2002, from $287,570$ to $1,539,111$. The number of bankruptcies over the total population (of age 18 and above) was 0.18% in 1980 but rose to 0.72% by 2002. The main concern behind the bankruptcy law reform was that there are many people who were "abusing" the bankruptcy law, or generally the moral hazard problem. Naturally, the reform is intended to make the bankruptcy scheme from a debtor-friendly one to a more creditor-friendly one.

There are two major types of consumer bankruptcies; Chapter 7 and Chapter 13. Chapter 7, which is also called *liquidation*, allows debtors to clean up the debt, after paying back a part of the existing debt using the asset which are non-exempt, and get a "fresh start" in the sense that, once the Chapter 7 bankruptcy was in place, there is no future obligation to pay back the debt. The other major bankruptcy option is Chapter 13. It is an option of *individual debt adjustment*. Under Chapter 13, bankrupts can draw their own repayment plan, and, upon acceptance by the judge, reschedule the repayment plan according to the proposed repayment plan. The asset at the time of bankruptcy filing need not be used for immediate repayment as in Chapter 7, but bankrupts have to use their future income for repayment. Historically, the proportion of Chapter 7 bankruptcies remains stable at about 70% of the total consumer bankruptcies. There is also a study which reports that many who filed for bankruptcy under Chapter 13 end up filing for the Chapter 7 bankruptcy (Chatterjee et al. (2007)). The focus of the paper is Chapter 7 bankruptcy.[4]

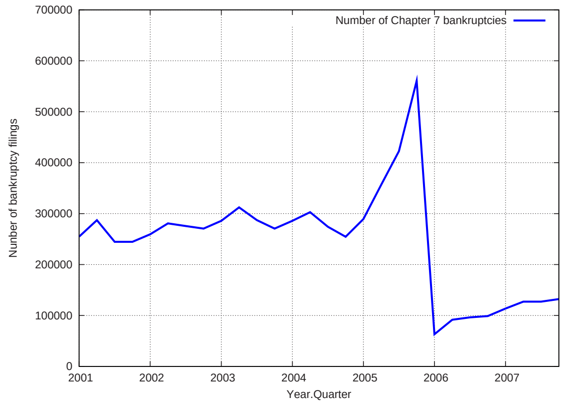
There are three key elements in the Bankruptcy Abuse Prevention and Consumer Protection Act (White (2007)). First, filers cannot choose which chapter to use under BAPCPA. Instead, only the filers who pass the means-test can file for Chapter 7 bankruptcies. Simply put, in order to be qualified to file under Chapter 7 bankruptcy, the recent income of the filer must be below the median income level. If a filer cannot pass the means-test, Chapter 13 is the only option.

[1] see Chatterjee et al. (2007) for more details.

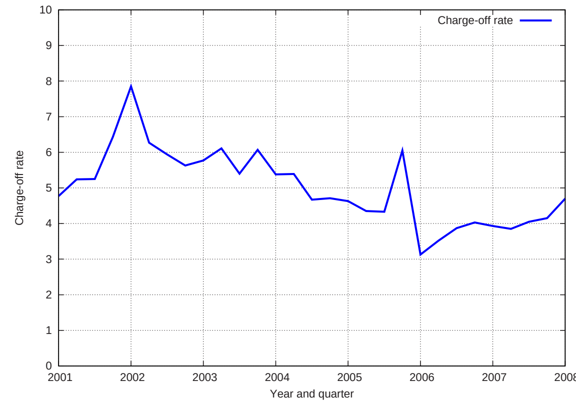
[2] see White (2007) for more details.

[3] Livshits et al. (2007a) investigate the reasons for the rise using quantitative macroeconomic model similar to the model used in this paper.

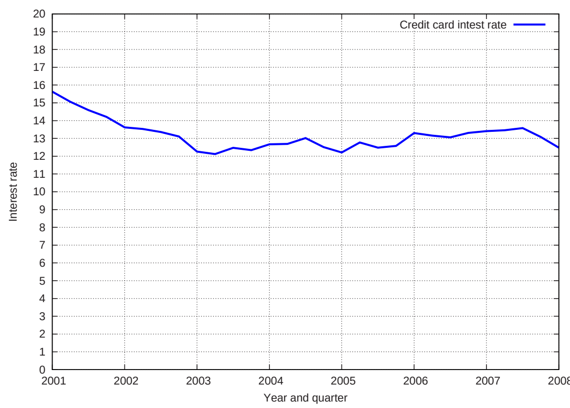
[4] Li and Sarte (2006) investigates the model with both chapters of bankruptcy.



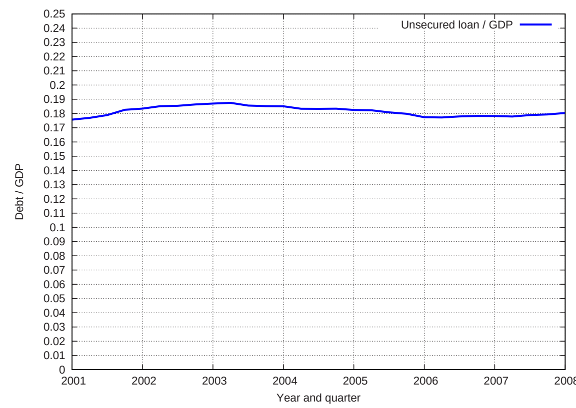

(a) Number of Chapter 7 bankruptcy filings




(b) Charge-off rate on credit card loans




(c) Interest rate on credit card loans




(d) Balance of consumer credit / Nominal GDP

**Figure 1: Changes around the introduction of BAPCPA: 2001-2008**

Second, filers of Chapter 13 can no longer make their own repayment plan under BAPCPA. Instead, they have to keep repaying, using all of their income above the essential living expenses. Third, the cost of filing went up. According to White (2007), typical out-of-pocket expenses of filing for Chapter 7 bankruptcy increased from 600 dollars to 2,500 dollars, and costs for Chapter 13 bankruptcy increased from 1,600 dollars to 3,500 dollars. Since only Chapter 7 is considered in this paper, the first and the last changes are explicitly introduced in the model section of the paper.

What happened to the number of bankruptcies and consumer credit market around the introduction of BAPCPA? Figure 1 summarizes changes in the data related to debt and bankruptcies that occurred around the introduction of BAPCPA in October 2005. Figure 1(a) shows the changes in the number of Chapter 7 bankruptcy filings. After the sharp increase since the 1980s, the number has been stable in the early 2000s until before the BAPCPA was enacted . Many debtors rushed to file for bankruptcies in the last quarter of 2005, right before the bankruptcy reform took effect. Right after BAPCPA was enacted, the number of bankruptcy filings plumed, partly because many potential filers filed before 2006. Since the first quarter of 2006, there has

Table 1: **Macroeconomic effect of BAPCPA: U.S. economy**

| Period | 2000-2004 | 2006-2007 | Change[1] |
|---|---|---|---|
| Proportion of defaulters[2] | 0.526 | 0.203 | −0.62 |
| Consumer credit interest rate (%) | 11.26 | 10.72 | −0.54 |
| Charge-off rate (%) | 5.46 | 3.82 | −1.64 |
| Unsecured debt / GDP[3] (%) | 7.79 | 7.90 | +0.01 |

[1] Percentage change for proportion of defaulters and unsecured debt / GDP, and change in percentage points for others.

[2] Among 22 years old and above.

[3] Balance of unsecured credit as defined by Livshits et al. (2007a).

been a gradual increase, but it does not seem that the number quickly goes back to the level before BAPCPA was enacted. In this sense, the reform achieved what was intended to achieve.

Figure 1(b) shows the trend of the charge-off rate of all the credit card loans. Corresponding the spike of the number of bankruptcies, there is a relatively small spike of the charge-off rate in the last quarter of 2005. Again in parallel to the drop in the number of bankruptcy filings, the charge-off rate dropped as well in the first quarter of 2006, but the rate seem to be almost recovering to the pre-BAPCPA level.

Figure 1(c) shows the trend of the average interest rate of credit card loans. It has been stable throughout the period, including the period around late 2005. The same trend can be seen about the balance of consumer credit relative to GDP, which is shown in Figure 1(d).

In sum, at this point, it is natural to assume that the effect of the bankruptcy law reform in 2005 was a decline in the number of bankruptcies, and a mild decline in the charge-off rate, but no substantial effect for other related data. Later in this paper, the performance of models with different specifications will be evaluated based on how well the models can replicate the observed changed that have been described in this section. Table 1 summarizes the data before and after the introduction of BAPCPA. The number of Chapter 7 bankruptcy filings is normalized by the population size of age 22 and above. Real interest rate of credit card loans is computed by subtracting the 1-year ahead CPI inflation rate from the nominal rate. Balance of consumer credit is defined as the sum of revolving and non-revolving loans. A problem of the data is that the balance of non-revolving loans includes the balance of auto loans. I remove the non-revolving auto-loans following Livshits et al. (2007a).

# 3 Model

## 3.1 Demographics

Time is discrete. In each period, the economy is populated by $I$ overlapping generations of agents. In time $t$, a measure $(1+\pi)^t$ of agents are born. $\pi$ is the population growth rate. Each generation is populated by a mass of measure-zero agents. agents are born at age 1 and could live up to age $I$. There is a probability of early death. Specifically, $s_i$ is the probability with which an age-$i$ agent survives to age $i+1$. With probability $(1-s_i)$, an age-$i$ agent does not survive to age $i+1$. $I$ is the maximum possible age, which implies $s_I = 0$.

Agents retire at age $1 < I_R < I$. Agents with age $i \leq I_R$ are called *workers*, and those with age $i > I_R$ are called *retirees*. $I_R$ is a parameter, implying that retirement is mandatory.

## 3.2 Preference

I use the preference that features temptation and self-controlling, which is developed by Gul and Pesendorfer (2001, 2004a). In particular, I use the formulation of the preference with long but finite horizon, developed by Krusell et al. (2005). The preference is characterized by an period utility function $u(c)$, and three parameters; $\gamma$, $\beta$, and $\delta$. I assume that $u(c)$ is strictly increasing and strictly concave.

$\delta$ is the standard discount factor. In order to distinguish from $\beta$, $\delta$ is also called the *long-term* discount factor. $\gamma$ represents the *strength* of the temptation. $\beta$ is the *short-term* discount factor, or the *nature* of temptation, following the terminology of Krusell et al. (2005).

The preference is both general and interesting in the sense that the preference includes both the standard exponential discounting as well as the (quasi-)hyperbolic discounting as two extreme cases. In one extreme case where $\gamma = 0$, an agent does not feel tempted, and the preference becomes the standard preference with exponential discounting factor $\delta$. With $\gamma = 0$ (no temptation), $\beta$, which is the nature of temptation, does not matter. The preference of an agent becomes time-consistent.

On the other extreme, if the strength of the temptation is infinitely strong, or, in other words, the agent is *succumbed to temptation*, the preference becomes the quasi-hyperbolic discounting preference. In some period $t$, the agent discounts the utility by $\beta\delta$ in period $t+1$ but by $\delta$ from period $t+2$ on. It is known that the preference exhibits time-inconsistency, because the discount factor applied between period $t+1$ and $t+2$ is $\delta$ from the perspective of period $t$, but the discount factor changes to $\beta\delta$ when period $t+1$ is reached. it is not straightforward to implement a welfare analysis using the quasi-hyperbolic discounting framework, because the preference changes within the same agent over time. Or, in other words, there are multiple selves within an agent. One very important benefit of using the preference with temptation and self-control is that the preference and thus welfare is defined naturally (Krusell et al. (2005)).

In the intermediate case where $\gamma$ is strictly positive but finite, and $\beta$ is less than one, the agent is *tempted* to some extent to consume more in the current period. In other words, the agent is tempted to make the decision by discounting the future with the discount factor $\beta\delta$ instead of using the standard exponential discount factor $\delta$. How much the agent is tempted is represented by $\gamma$. If $\gamma$ is higher, the agent is more strongly tempted. I present the formal representation of the preference when the recursive problem of an agent is presented.

## 3.3 Technology

There is a representative firm which has an access to a constant returns to the following constant scale production technology

$$Y = ZF(K, L) \tag{1}$$

where $Y$ is output, $Z$ is the level of total factor productivity, $K$ is capital stock, and $L$ is labor supply. Capital depreciates at a constant rate $\nu$ per period.

When a credit card company makes a loan to an agents, it is assumed that there is a transaction cost $\iota$ that is proportional to the size of the loan. There is no transaction cost for saving.

## 3.4 Endowment

Agents are born with zero asset. Each agent in endowed with one unit of time each period, but agents inelastically supply labor since leisure is not valued. Labor productivity of an agent $e$ takes the following form:

$$e(i, p, t) = e_i \exp(p_i + t_i) \tag{2}$$

where $e_i$ is the average profile of labor productivity, and is common across all age-i agents. $p_i$ is the persistent shock to productivity. $p_i$ is drawn from an i.i.d. normal distribution when an agent is born, and follows an AR(1) process with normally distributed innovation term. $t_i$ is the transitory shock to labor productivity. $t_i$ is drawn from an i.i.d. normal distribution.

An agent also faces shocks to mandatory expenditure $x \geq 0$. $x$ is independent and identically distributed, but the distribution can depend on the type, in particular, age, agents.

## 3.5 Bankruptcy

I allow agents to default on their debt or bills associated with expenditure shocks. The default option is modeled as in Chatterjee et al. (2007). The default option in the model resembles in procedure and consequences a Chapter 7 bankruptcy filing, in particular, before the reform of the Bankruptcy Law in 2005.

Suppose an agent has a negative amount of asset (debt) or receives an expenditure shock with which the asset position becomes negative, and the agent decides to file for a bankruptcy, the following things happen:

1. The debt and the expenditure shock (think a hospital bill) is wiped out and the agent does not have an obligation to pay back the debt or the expenditure in the future (the *fresh start*).
2. The agent cannot save during the current period. If the agent tries to save, the saving will be completely garnished.
3. The agent has to pay the proportion $\xi$ of the current income as cost of filing for bankruptcy.
4. Proportion $\eta$ of the current labor income is garnished. Social security benefit is not subject to this garnishment. This is intended to capture the effort of agents to replay until they decide to file for a bankruptcy.
5. The credit history of the agent turns bad. I use $h = 0$ and $h = 1$ to denote a good and bad credit history, respectively.
6. While the credit history is bad ($h = 1$), the agent is excluded from the loan market. In other words, the borrowing constraint is zero.
7. With probability $\lambda$, the agent's bad credit history is wiped out, or, $h$ turns from one to zero.

The benefit of using the default option is to get away from debt or expenditures. The default option is a means of partial insurance. The costs are *(i)* filing cost, *(ii)* the income garnishment in the period of default and *(iii)* temporary exclusion from the loan market. Agents in debt or with an expenditure shock weigh the benefit and the cost of filing for a bankruptcy, and files if it is optimal to do so or there is no other option. The former is called *voluntary default* and the latter is called *involuntary default.*

It is possible that an agent with a bad credit history cannot consume a positive consumption when the agent is hit by an expenditure shock. Only in this case (*involuntary default*), default by agents with a bad credit history is allowed. An agent with a bad credit history cannot choose voluntary default.

## 3.6 Annuity Market

There is a perfect annuity market which allows agents to insure against uncertain lifetime. Agents of the same type with the same positive amount of asset will optimally sign a contract among themselves so that the total wealth is distributed by the survivors in the next period. Practically, for agents of age $i$ who face the survival probability of $s_i$, they only need to save $as_i$ to receive $a$ in the next period.

For agents with a negative amount of asset, they are not willing to sign an annuity contract among themselves.[5] Debt of the deceased will be completely imposed on the credit card company that extended a loan to the deceased. However, in this case, the credit card company pool agents of the same type so that the risk of death will be shared by all the borrowers of the same type. At the end, even for borrowers, pooling of mortality risks by credit card companies virtually work as a working annuity market.

## 3.7 Government

The government runs a simple pay-as-you-go social security program. The government imposes a flat payroll tax rate $\tau_S$ to all workers, and use the proceeds to finance social security benefits $b_i$ of the current retirees. It is assumed that all retirees receive the same amount of benefits, and the government budget associated with the social security program balances each period. Naturally, $b_i = 0$ for $i \leq O_R$ and $b_i = \bar{b}$ for $i > I_R$. $\bar{b}$ is the constant amount of benefit.

## 3.8 Agent's Problem

The problem of an agent is defined recursively. The individual state variables are $(i, h, p, t, x, a)$, where $i$ s age, $h$ is credit history, $p$ and $t$ are persistent and transitory components of shocks to individual productivity, $x$ is the mandatory expenditure shock, and $a$ is asset position. I will present the problem of an agent separating two parts, *temptation* problem and *self-control* problem.

Let's start with the *tempting* problem. An agent with individual state $(i, h, p, t, x, a)$ with $h = 0$ (good credit history) solves the following *tempting* problem:

$$W^*(i,0,p,t,x,a) = \max\{W_0^*(i,0,p,t,x,a), W_1^*(i,0,p,t,x,a)\} \tag{3}$$

$$W_0^*(i,0,p,t,x,a) = \begin{cases} -\infty & \text{if } B(i,0,p,t,x,a) = \emptyset \\ \max_{a' \in B(i,0,p,t,x,a)} W_0(i,0,p,t,x,a|a') & \text{if } B(i,0,p,t,x,a) \neq \emptyset \end{cases} \tag{4}$$

$$W_0(i,0,p,t,x,a|a') = \gamma\{u(c_0) + \beta\delta s_i \mathbb{E}V(i+1,0,p',t',x',a')\} \tag{5}$$

$$W_1^*(i,0,p,t,x,a) = \gamma\{u(c_1) + \beta\delta s_i \mathbb{E}V(i+1,1,p',t',x',0)\} \tag{6}$$

where

$$B(i,0,p,t,x,a) = \\ \{a' \in \mathbb{R} | we(i,p,t)(1-\tau_S) + b_i + a = c + x + q(i,0,p,t,x,a')a', c \geq 0\} \tag{7}$$

[5] In case a deceased held both asset and debt, the common thing to happen in many states in the U.S. is that the person who inherits the asset also inherits the debt. On the other hand, being in debt in the model means the negative net asset position, and thus there is no reason that, unless the debt is guaranteed by somebody else, the debt is inherited by another person.

$$c_0 = we(i,p,t)(1-\tau_S) + b_i + a - x - q(i,h,p,t,x,a')a' \tag{8}$$
$$c_1 = we(i,p,t)(1-\tau_S)(1-\eta-\xi) + b_i(1-\xi) \tag{9}$$

Equation (3) states that the agent can choose between defaulting and non-defaulting when the agent has a good credit history ($h = 0$). $W_0^*$ and $W_1^*$ correspond to the value conditional on not-defaulting and defaulting, respectively. Equation (4) states that the value is negative infinity when the choice set, defined by equation (7), is empty. This case corresponds to *involuntary* default. Otherwise, the agent solves the consumption-savings problem conditional on not defaulting. The maximand is defined by the equation (5), with the consumption defined by equation (8). The income of the agent consists of labor income net of payroll tax ($we(i,p,t)(1-\tau_S)$), and social security benefit ($b_i$). $w$ is wage rate. $a$ is the asset (debt is $a$ is negative) carried over from the previous period. $x$ is the mandatory expenditure. $c$ is consumption. $a'$ is the asset (or debt) carried over to the next period. $q(i,h,p,t,x,a')$ is the discount price for an agent of type $(i,h,p,t,x)$ choosing the asset position $a'$. Notice that the current asset holding of an agent, $a$, does not matter for the price of loans. To ease the notation, $q(i,h,p,t,x,a')$ is used to capture the annuity contract used by agents with a positive $a'$.

The value conditional on defaulting is defined by the equation (6). A fraction $\eta$ is garnished away from labor income, a fraction $\zeta$ of labor income as well as social security benefit is paid as the cost of bankruptcy, and the agent will start the next period with zero saving. But the agent is free from the debt and the mandatory expenditure upon default. That can be observed by the definition of consumption, defined by equation (9) If $B \neq \emptyset$ and the agent chooses to default, it is called *voluntary* default.

Notice that the future value is discounted by $\beta\delta$. Think that $\delta$ is the standard long-term discount factor. When $\beta < 1$, $\beta$ shifts the weight to the current utility relative to the future value. Or, loosely speaking, the agent is tempted to consume more rather than saving. The *nature* of temptation for the agent is that the agent is tempted to discount future value more. The *strength* of temptation is characterized by $\gamma$. When $\gamma$ is larger, the agent is more strongly tempted to discount future at a higher discount rate.

In case an agent has a bad credit history ($h = 1$), the *tempting* problem for the agent is formalized as follows:

$$W^*(i,1,p,t,x,a) = \begin{cases} W_1^*(i,1,p,t,x,a) & \text{if } B(i,1,p,t,x,a) = \emptyset \\ \max_{a' \in B(i,1,p,t,x,a)} W_0(i,1,p,t,x,a|a') & \text{if } B(i,1,p,t,x,a) \neq \emptyset \end{cases} \tag{10}$$

$$W_0(i,1,p,t,x,a|a') = \gamma\{u(c_0) + \beta\delta s_i \mathbb{E}(\lambda V(i+1,0,p',t',x',a') + (1-\lambda)V(i+1,1,p',t',x',a'))\} \tag{11}$$

$$W_1^*(i,1,p,t,x,a) = \gamma\{u(c_1) + \beta\delta s_i \mathbb{E}V(i+1,1,p',t',x',0)\} \tag{12}$$

where $B(i,h,p,t,x,a)$ is defined by equation (13) below, and $c_0$ and $c_1$ are defined by equations (8) and (9).

$$B(i,1,p,t,x,a) = \\ \{a' \in \mathbb{R} | we(i,p,t)(1-\tau_S) + b_i + a = c + x + q(i,1,p,t,x,a')a', c \geq 0, a' \geq 0\} \tag{13}$$

Notice three things. First, an agent with a bad credit history does not have a choice with respect to whether to default or not. Only involuntary defaults occurs, when $B(i, 1, p, t, x, a) = \emptyset$. See equation (10) above. Second, When an agent had a bad credit history, and does not (involuntarily) default in the current period, the agent's credit history is cleaned up in the next period with a probability $\lambda$, and the bad credit history remains with a probability $(1 - \lambda)$. You can see this in equation (11) above. $\lambda$ will be calibrated later to make sure that the average duration for which the bad credit history is kept matches the same statistics in the U.S. economy. This is a way to reduce the size of the state space and simplify an already complex model slightly. Finally, the budget set $B(i, 1, p, t, x, a)$ is almost the same as in the case for the agent with a good credit history, but there is one additional constraint; $a' \geq 0$. Basically, this constraint excludes the agent with a bad credit history from the credit market.

Now that we defined the temptation problem, we are ready to define the self-control problem. The temptation problem will be a part of the self-control that an agent solves since a key of the self-control problem is how successfully an agent can resist the temptation.

$$V^*(i, 0, p, t, x, a) = \max\{V_0^*(i, 0, p, t, x, a), V_1^*(i, 0, p, t, x, a)\} \tag{14}$$

$$V_0^*(i, 0, p, t, x, a) = \begin{cases} -\infty & \text{if } B(i, 0, p, t, x, a) = \emptyset \\ \max_{a' \in B(i,0,p,t,x,a)} V_0(i, 0, p, t, x, a|a') & \text{if } B(i, 0, p, t, x, a) \neq \emptyset \end{cases} \tag{15}$$

$$V_0(i, 0, p, t, x, a|a') = u(c_0) + \delta s_i \mathbb{E} V(i + 1, 0, p', t', x', a') + W_0(i, 0, p, t, x, a|a') - W^*(i, 0, p, t, x, a) \tag{16}$$

$$V_1^*(i, 0, p, t, x, a) = u(c_1) + \delta s_i \mathbb{E} V(i + 1, 1, p', t', x', 0) + W_1(i, 0, p, t, x, a|0) - W^*(i, 0, p, t, x, a) \tag{17}$$

subject to equations (7), (8), and (9).

Equation (14), (15) are almost identical with the corresponding equations in the temptation problem. An agent chooses between defaulting and non-defaulting, and the value of non-defaulting is zero if the feasible set is empty.

Equation (16) is where the two problems become apparently different. The agent discounts future utility only with the long-term discount factor $\delta$. However, there are two additional terms in the maximand. $W_0(i, 0, p, t, x, a|a')$ is the value associated with the *temptation* problem conditional on the agent's current decision. $W^*(i, 0, p, t, x, a)$ is the value of the *temptation* problem with the optimal decision associated with the tempting problem.

If there is no temptation ($\gamma = 0$), the temptation problem doesn't matter, because the last two terms of the maximand disappear. As a result, the problem goes back to the standard Bellman equation with exponential discounting. Similarly, if $\beta = 1$, the discount factor used for the tempting problem is the same as the problem here. Therefore, temptation doesn't need to be controlled, and the optimal decision associated with the current problem is turns out to coincide with the optimal decision associated with the tempting problem. In short, the current problem goes back to the standard problem without temptation if either $\gamma = 0$ (strength of temptation is zero) or $\beta = 1$ (short-term discount factor plays no role). In case neither holds, the agent's problem has two dimensions. First, the agent wants to solve the standard problem with long-term discount factor $\delta$. On the other hand, the agent wants to choose the action that is close to the one that would be chosen under the *temptation* problem so that $W_0(i,0,p,t,x,a|a') - W^*(i,0,p,t,x,a)$ is brought to close to zero.

The relative strength of the two considerations, or the *strength* of the temptation, is controlled by the parameter $\gamma$. In case $\gamma = \infty$, the agent chooses the action as if the agent is solving the tempting problem. But the optimal value is based on the standard long-term discounting. This is exactly what is achieved in the so-called quasi-hyperbolic discounting model in Laibson (1997) and Angeletos et al. (2001). The current approach with temptation not only includes the quasi-hyperbolic discounting model as an extreme case, but has a very important advantage over the quasi-hyperbolic discounting model, as argued by Krusell et al. (2005). How? Since the utility changes over time, the same agent in different periods can be naturally seen as different selves in the quasi-hyperbolic discounting model. This feature makes it non-trivial to define the welfare of agents. On the other hand, in the temptation model, utility of an agent does not change, and thus it is natural to define the welfare of agents.

Equation (17) is similar to Equation (16) in the sense that the future value is discounted only with $\delta$ and there are two additional terms associated with temptation. However, since (17) represents the value conditional on defaulting, there is no choice in terms of savings, since the agent is not allowed to save in the defaulting period.

Finally, the problem of an agent with a bad credit history ($h = 1$) can be characterized as follows:

$$V^*(i,1,p,t,x,a) = \begin{cases} V_1^*(i,1,p,t,x,a) & \text{if } B(i,1,p,t,x,a) = \emptyset \\ \max_{a' \in B(i,1,p,t,x,a)} V_0(i,1,p,t,x,a|a') & \text{if } B(i,1,p,t,x,a) \neq \emptyset \end{cases} \tag{18}$$

$$V_0(i,1,p,t,x,a|a') = u(c_0) + \delta s_i \mathbb{E} V(i+1,1,p',t',x',a') + W_0(i,1,p,t,x,a|a') - W^*(i,1,p,t,x,a) \tag{19}$$

$$V_1^*(i,1,p,t,x,a) = u(c_1) + \delta s_i \mathbb{E} V(i+1,1,p',t',x',0) + W_1(i,1,p,t,x,a|0) - W^*(i,1,p,t,x,a) \tag{20}$$

subject to equations (8), (9), and (13).

The optimal value function associated with the problem defined above is $V^*(i,h,p,t,x,a)$. The optimal saving function is denoted as $a' = g_a(i,h,p,t,x,a)$. The optimal policy rule for default decision is denoted as $h' = g_h(i,h,p,t,x,a)$, where $g_h(i,h,p,t,x,a) = 1$ and $g_h(i,h,p,t,x,a) = 0$ denote defaulting and non-defaulting, respectively.

### 3.9 Credit Card Companies

The only loans available in the model are unsecured loans. The unsecured loans are provided by competitive credit sector that consists of a large number of credit card companies. Free entry is assumed. Credit card companies can target to one type of agents with one level of debt. Since the credit sector is competitive, free entry is assumed, and each credit card company can target one specific level of asset, it is impossible in equilibrium to *cross-subsidize*, that is, offering one type of agent an interest rate which implies a negative profit while offering another type of agent an interest rate which implies a positive profit, so that, in sum, the credit card company makes a positive total profit. In this case, there is always an incentive for another credit card company to offer a lower interest rate for the second type of agents and steal the profitable customers away. In equilibrium, any loans to any type of agents and any level of debt make zero profit.

Suppose that a credit card company makes loans to type-$(i, 0, p, t, x, a)$ agents who borrow $a'$ each.[6] By making loans to a mass of agents of the same type, the credit card company can insure away the default risk, even if the loans are unsecured. In other words, credit sector provides a partial insurance, by pooling risk of default. Now, assume the credit card company makes loans to measure $m$ agents of the same type. Zero profit condition associated with the loans made to type-$(i, 0, p, t, x, a)$ agents whose measure is $m$ and who borrow $a'$ each can be expressed as follows:

$$\begin{aligned} & ms_i(-a') \int_{p'} \int_{t'} \int_{x'} \mathcal{I}_{g_h(i+1,0,p',t',x',a')=0} f_x(x') f_t(t') f_p(p'|p) dx'dt'dp' \\ & + ms_i \int_{p'} \int_{t'} \int_{x'} \mathcal{I}_{g_h(i+1,0,p',t',x',a')=1} we(i,p,t)(1-\tau_S) \frac{\eta(-a')}{x'-a'} f_x(x') f_t(t') f_p(p'|p) dx'dt'dp' \\ & \qquad = m(-a'q(i,0,p,t,x,a'))(1+r+\iota) \end{aligned} \tag{21}$$

where $\mathcal{I}$ is an indicator function which takes the value of one if the logical statement attached to it is true, and zero otherwise. $f_x$, $f_t$ and $f_p$ are density functions associated with the three types of shocks. The first term on the left hand side is the sum of the income of the credit card company for the agents of type $(i + 1, 0, p', t', x', a')$ who repay. The second term represents the sum of the income of the company when the agent of type $(i + 1, 0, p', t', x', a')$ defaults. When an agent defaults, the fraction $\eta$ of the labor income of the agent is garnished. If there is no mandatory expenditure shock ($x = 0$), all of the garnished amount is received by the credit card company as income. If the agent also receives a bill of a positive amount ($x > 0$), the garnished income is proportionally allocated between the credit card company and the issuer of the bill. $\frac{-a'}{x'-a'}$ represents the fraction that the credit card company receives. The right hand side of the equation is the total cost of loans. Notice that there is a transaction cost for loans $\iota$ in addition to the risk-free interest rate $r$. If the equation (21) is solved for q(i,0,p,t,x,a'), we can obtain the formula for the equilibrium discount price of loans, as follows:

$$q(i,0,p,t,x,a') = \frac{s_i \int_{p'} \int_{t'} \int_{x'} \mathcal{I}_{g_h=0} + \mathcal{I}_{g_h=1} \frac{\eta we(i,p,t)(1-\tau_S)}{x'-a'} f_x(x') f_t(t') f_p(p'|p) dx'dt'dp'}{1+r+\iota} \tag{22}$$

where $g_h$ is a short-hand notation for $g_h(i + 1, 0, p', t', x', a')$. Notice that, in case there is no default for the loan, the price of loan will be:

$$q(i,0,p,t,x,a') = \frac{s_i}{1+r+\iota} \tag{23}$$

When there is no transaction cost ($\iota = 0$), this is the equilibrium loan price for $a' \geq 0$ and the only loan price available for those with a bad credit history. Notice that there is a survival

[6] Notice that $h = 0$. We only need to consider $h = 0$ as agents with a bad credit history ($h = 1$) cannot borrow.

probability in the numerator. The credit card company is basically providing annuity among debtors. The way survival probability is in the formula implies that the loan price (interest rate) is lower (higher) for older agents, as they tend to have lower survival probabilities. This feature can explain why older agents cannot borrow much.

In case all agents default on the debt in the next period, the price of loans will be:

$$q(i,0,p,t,x,a') = \frac{s_i \int_{p'} \int_{t'} \int_{x'} \frac{\eta w e(i,p,t)(1-\tau_S)}{x'-a'} f_x(x') f_t(t') f_p(p'|p) dx' dt' dp'}{1+r+\iota} \tag{24}$$

Consider the special case where $\eta = 0$. In case the loan is defaulted with probability one:

$$q(i,0,p,t,x,a') = 0 \tag{25}$$

This is because, when $\eta = 0$, credit card companies cannot garnish anything from a defaulted customer. In this case, we can define $\underline{a}(i,0,p,t,x,a')$ which satisfies:

$$\underline{a}(i,0,p,t,x,a') = \max_{a'} q(i,0,p,t,x,a') = 0 \tag{26}$$

$\underline{a}(i,0,p,t,x,a')$ is the endogenous borrowing constraint for the agent of type $(i,0,p,t,x,a')$. For an agent with a bad credit history, $\underline{a}(i,1,p,t,x,a') = 0$. The model with bankruptcy generates nontrivial endogenous borrowing constraint. By construction, the constraint is less strict than the natural borrowing limit of Aiyagari (1994), and less strict than the not-too-tight borrowing constraint by Alvarez and Jermann (2000). This is because both borrowing constraints are associated with no default in equilibrium, while the constraint here allows default in equilibrium.[7] See Chatterjee et al. (2007) for further characterization of the equilibrium loan price function.

Finally, for agents with a positive $a'$, or a bad credit history ($h = 1$), we can define the pricing function as follows:

$$q(i,h,p,t,x,a') = \frac{s_i}{1+r} \quad \forall h = 1 \text{ or } a' \geq 0 \tag{27}$$

This equations takes into account the annuity contract signed among the agents with a positive asset.

## 3.10 Equilibrium

I will define below the recursive competitive equilibrium where the demographic structure is stationary, even though the size of population is growing at a constant rate $\pi$. In the equilibrium with stationary demographic structure, prices $\{r, w, q(i,h,p,t,x,a')\}$ are constant over time.

[7] Since the loan price $q$ (interest rate) goes down (up) as the size of the debt increases, typically no agent borrows as much as $\underline{a}$. Actually, in Chatterjee et al. (2007), the largest size of debt held by an agent in the baseline equilibrium is smaller than the natural borrowing limit applied to the model. In this sense, the *effective* borrowing constraint in the model with bankruptcy can be more strict than the natural borrowing limit even if $\underline{a}$ is less strict than the natural borrowing limit. A high interest rate (low loan price) effectively works as a borrowing constraint.

Let $\mathbf{M}$ be the space of individual state. $(i, h, p, t, x, a) \in \mathbf{M}$. Let $\mathcal{M}$ be the Borel $\sigma - algebra$ generated by $\mathbf{M}$, and $\mu$ the probability measure defined over $\mathcal{M}$. I will use a probability space $(\mathbf{M}, \mathcal{M}, \mu)$ to represent a type distribution of agents.

**Definition 1 (Stationary Recursive competitive equilibrium)**
*A stationary recursive competitive equilibrium is a set of prices $\{r, w, q(i, h, p, t, x, a')\}$, government policy variable $\{\tau_S, b_i\}$, aggregate capital stock $K$, labor supply $L$, value function $V^*(i, h, p, t, x, a)$, optimal decision rules $g_a(i, h, p, t, x, a)$, $g_h(i, h, p, t, x, a)$, and the stationary measure after normalization $\mu$, such that:*

1. *Given the prices, and policy variables, $V^*(i, h, p, t, x, a)$ is a solution to the agent's optimization problem defined in Section 3.8, and $g_a(i, h, p, t, x, a)$ and $g_h(i, h, p, t, x, a)$ are the associated optimal decision rules.*

2. *The prices $r$ and $w$ are determined competitively, i.e.,*

$$r = ZF_K(K, L) - \nu \tag{28}$$
$$w = ZF_L(K, L) \tag{29}$$

3. *Loan price function $q(i, h, p, t, x, a')$ satisfies the zero profit conditions for all types. Specifically, the loan price functions are characterized as (22) and (27).*

4. *Measure of agents $\mu$ is consistent with the demographic transition, stochastic process of shocks, and optimal decision rules, after normalization.*

5. *Aggregate capital and labor are consistent with the individual optimal decisions, i.e.:*

$$K = \frac{1}{1+\pi} \int_{\mathcal{M}} g_a(i, h, p, t, x, a) q(i, h, p, t, x, g_a(i, h, p, t, x, a)) d\mu \tag{30}$$
$$L = \int_{\mathcal{M}} e(i, p, t) d\mu \tag{31}$$

6. *Government satisfies period-by-period budget balance with respect to the social security program, i.e.,*

$$\int_{\mathcal{M}} b_i \; d\mu = \int_{\mathcal{M}} e(i, p, t) w \tau_S d\mu \tag{32}$$

# 4 Calibration

## 4.1 Demographics

One period is set as one year in the model. Age 1 in the model corresponds to the actual age of 22. $I$ is set at 79, meaning that the maximum actual age is 100. $I_R$ is set at 43, implying that

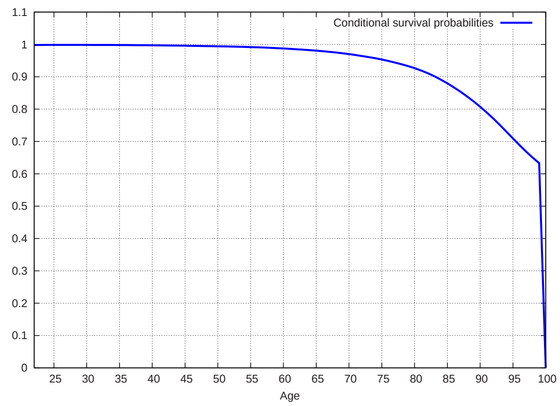



**Figure 2: Conditional survival probabilities**

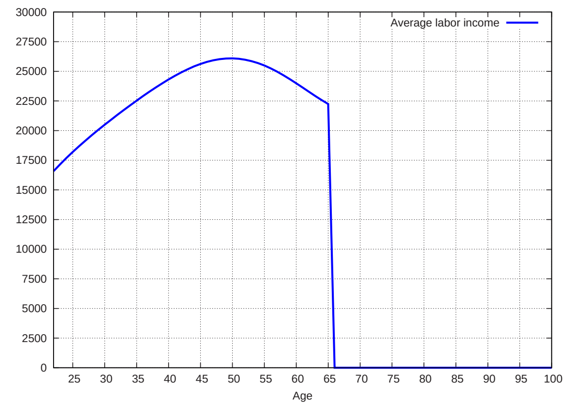



**Figure 3: Average life-cycle profile of labor productivity**

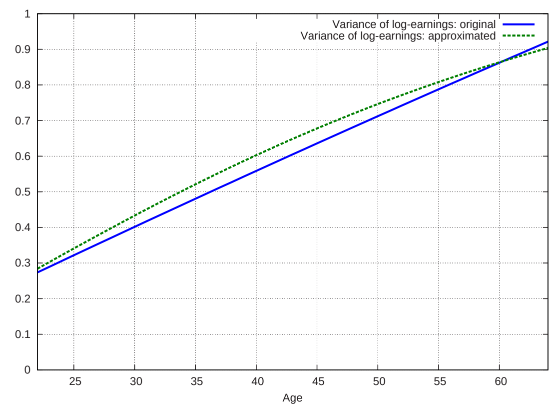



**Figure 4: Life-cycle profile of log-earnings variances**

the agents become retired at the actual age of 65. The population growth rate, $\pi$, is set at 1.2% annually. This growth rate corresponds to the average annual population growth rate of the U.S. over the last 50 years. The survival probabilities $\{s_i\}_{i=1}^{I}$ are taken from the life table in Social Security Administration (2007).[8] Figure 2 shows the conditional survival probabilities used.

## 4.2 Preference

For the period utility function, the following constant relative risk aversion (CRRA) functional form is used:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{33}$$

$\sigma$ is set at 2.0, which is the commonly used value in the literature.

[8] Table 4.C6 of Social Security Administration (2007).

Discount factors $\beta$ and $\delta$ and the parameter controlling the strength of temptation $\gamma$ are calibrated differently for different economies. For the baseline model economy with exponential discounting consumers, $\gamma$ is set to zero, and $\delta$ is calibrated mainly to match the aggregate balance of financial assets in the steady state, which is 1.47 of the aggregate output.[9] This choice of the aggregate saving makes the shape of the average life-cycle profile of consumption similar to the empirical counterpart, provided by Gourinchas and Parker (2002).

For economies with agents who face temptation and self-control problem, I use the short-term discount factor $\beta$ of 0.70 and 0.55. The short-term discount factor of 0.7 corresponds to the discount rate of 40% which is estimated by Laibson et al. (2007). Discount factor of 0.55 corresponds to the 80% discount rate, which is twice the baseline value. I use $\beta$ of 0.55 for robustness check. As for the strength of temptation $\gamma$, I also use variety of values. In particular, I try $\gamma$ of 1, 10, and $\infty$. $\gamma = \infty$ implies the quasi-hyperbolic discounting preference.

In all cases with temptation and self-control problem, the remaining parameter $\delta$ is calibrated to match the same target for the aggregate savings. Of course, $\delta$ will be different for different economies, but all the models are calibrated to match the same set of targets so that all models with different preference parameters are observationally equivalent with respect to the chosen targets.

## 4.3 Technology

The following standard Cobb-Douglas production function is assumed:

$$Y = ZK^{\theta}L^{1-\theta} \tag{34}$$

$Z$ is pinned down such that, in the baseline steady state, the output is normalized to one. $\theta$ is set at 0.247. Capital depreciated at the constant rate of $\nu = 0.107$ per year. These values are consistent with the economy only with financial assets.

The transaction cost for loans $\iota$ is set at 4%, which is the value used by Livshits et al. (2007b) and reflects the average cost of loans in the U.S. economy.

## 4.4 Bankruptcy

There are four parameters associated with the bankruptcy scheme; $\lambda$, which controls the average length of punishment, $\eta$, which defines the amount of labor income garnished during the period of filing, $\xi$, which controls the cost of filing for a bankruptcy, and $\overline{r}$, which is the ceiling of the interest rate charged for debt.[10] $\lambda$ is set at 0.1, implying that, on average, defaulters cannot obtain new debt for 10 years after filing for a bankruptcy. This average punishment period corresponds to a 10 year period during which a bankruptcy filing stays on a person's credit record according to

[9] With $\gamma = 0$, the short-term discount factor $\beta$ does not matter.

[10] Practically, $\overline{r}$ is converted into $\underline{q} = \frac{1}{1+\overline{r}}$, which is the lower bound of the price of debt in the model.

the Fair Credit Reporting Act. $\eta$ is chosen such that the number of bankruptcies in the model matches the same number in the U.S. economy (0.526% of adult (age 22 and above) population per year). However, notice that the parameter will be chosen jointly with other parameters. According to White (2007), the average cost of filing for a Chapter 7 bankruptcy was 600 dollars before the BAPCPA was introduced. $\xi$ is pinned down by converting 600 dollars into the unit in the model. I obtain $\xi = 0.0135$ Finally, in the baseline specification, $\overline{r}$ is set at 100%. In the baseline model with exponential discounting agents, the bound does not bind. I will later change $\overline{r}$ to see the effect of imposing binding interest rate ceiling on macroeconomic aggregates and welfare.

## 4.5 Government

The payroll tax rate for the social security contribution $\tau_S$ is set at 0.074. The tax rate is chosen such that the ratio of the average social security benefit to the average labor income in the model matches the counterpart in the U.S. economy. In the U.S. economy the ratio is 33.7%.

## 4.6 Labor Productivity

The average life-cycle profile of the earnings $\{e_i\}_{i=1}^{I}$ is taken from the estimates of Gourinchas and Parker (2002). Figure 3 shows the life-cycle profile of the average labor productivity used in the model. Since mandatory retirement at the model age of $I_R$, $e_i = 0$ for $i > I_R$.

As for the shock component of the individual labor productivity, I use the empirical results of Storesletten et al. (2004). Using Panel Study on Income Dynamics (PSID), they estimate the following stochastic process for individual labor productivity:

$$y_i = y^{perm} + y_i^{pers} + y_i^{tran} \tag{35}$$

$$y_{i+1}^{pers} = \rho_{pers} y_i^{pers} + \epsilon_i^{pers} \tag{36}$$

where $y_i$ is the deviation of log-earnings from the average log-earnings at age i. $y_i$ consists of three components; permanent component $y_i^{perm}$, persistent component $y_i^{pers}$, and transitory component $y_i^{tran}$. The permanent component is drawn at the beginning of agents' life from $N(0, \sigma_{perm}^2)$. The persistent component is initially zero and follows an AR(1) process with the persistence parameter $\rho_{pers}$. The shock $\epsilon^{pers}$ is drawn from $N(0, \sigma_{pers}^2)$. The transitory components is i.i.d. drawn from $N(0, \sigma_{tran}^2)$. Using PSID, they estimate $\sigma_{perm}^2 = 0.2105$, $\rho_{pers} = 0.9989$, $\sigma_{pers}^2 = 0.0166$, and $\sigma_{tran}^2 = 0.063$. Since the persistence parameter is close to one, the permanent component is combined with the persistence component, by assuming that the distribution of the permanent component is the initial distribution of the persistent component $y^{pers}$.[11] The persistent component is approximated by first order Markov process with $N^{pers} = 10$ abscissas. I use the method developed by Tauchen (1986). The transitory component is also approximated by

[11] Separating permanent and persistent component did not change the main results of the paper in a sizable manner.

**Table 2: Expenditure shock**

| State | Probability | Magnitude (dollars) |
|---|---|---|
| No shock | 0.97109 | 0 |
| Larger medical expense shock | 0.00100 | 125,000 |
| Smaller medical expense shock | 0.01000 | 43,500 |
| Divorce + unwanted birth shock | 0.01791 | 7,950 |

discrete distribution, with $N^{tran} = 10$ abscissas. The original normal distribution is approximated by the method proposed by Ada and Cooper (2003), which allocates probability of exactly $\frac{1}{N^{tran}}$ to each of the $N^{tran}$ abscissas. The approximated stochastic process captures the life-cycle profile of the original stochastic process well; the process estimated by Storesletten et al. (2004) generates cross-sectional variances of log earnings of 0.2735 for age 22 (age 1 in the model) agents and 0.8624 for age 60 (age 39 in the model), and the discretized stochastic process used in the model generates variances of log earnings of 0.2840 for age 22 agents and 0.8636 for age 60 agents. Figure 4 compares the variances of log-earnings of the original process estimated by Storesletten et al. (2004) and the approximated process used in the model.[12]

## 4.7 Expenditure Shocks

Expenditure shock is intended to mainly capture the defaults due to marital disruption, health-care bills, and unwanted births. Marital disruption and health-care bills account for 14% and 16% of the reasons for bankruptcy, respectively (Chakravarty and Rhee (1999)). Livshits et al. (2007b) argue the importance of expenditures related to unwanted births.

The size and probability of expenditure shocks are calibrated following Livshits et al. (2007b).[13] I consider three types of expenditure shocks, which are *(i)* catastrophic out-of-pocket medical expenses, *(ii)* divorce and *(iii)* unplanned births.

Regarding the out-of-pocket medical expenses, French and Jones (2004) estimate the process for the out-of-pocket health care costs and find that the process has a long tail, appropriately captured by adding catastrophic heal care costs shock. They find that, with probability of 1%, individuals receive a health care cost shock of at least 43,500 dollars, and with 0.1%, individuals receive health care bills of at least 125,000 dollars. I use these two as the small and the large catastrophic medical expense shocks in the model.

As for the divorce shock, in the recent U.S. data, the annual national divorce rate is 3.9 per 1,000

[12] The approximation method by Tauchen (1986) can be controlled by a choice of the parameter which controls the size of the domain of approximating discrete stochastic process. I choose the parameter such that the approximating stochastic process generates a life-cycle profile of log earnings variances that is close to the data counterpart.

[13] There is some adjustment needed as they use three years as one period while one period is one year in the current model.

persons. If the number is converted to per-person basis and using population of age between 22 and 64 as the numerator, the divorce probability is 0.0136 per year.[14] Since a typical cost of divorce is 15,000, I use 7500 which is the half of the total cost at the size of the divorce shock.

Finally, regarding the unwanted births, the annual national birth rate is 14.1 per 1000 in the recent U.S. economy. In addition, I assume that the cost is shared among the average number of adults in a household (which is 1.92). The shock also affects only the population of age between 22 and 64. Finally, according to Livshits et al. (2007b), the proportion of births which are self-reported as unwanted is 0.091. Taking them into account, the adjusted probability of having unwanted births is 0.0043.[15] AS for the magnitude of the shock, typical annual cost of baby is 18,000 dollars. Since the cost is shared by the average number of adults in a household, the per-person size of unplanned birth shock turns out to be 9,375 dollars.

In order to reduce the computational cost, and because the probability of having an unwanted kid is small compared with the probabilities of other types of expenditure shocks, I merge the shock of having an unwanted birth into the divorce shock. The probability of getting this merged shock is 0.01791 per period. The magnitude of the shock is the weighted average of the two shocks, which is 7,950 dollars. Table 2 summarizes the expenditure shock.

### 4.8 Simultaneously Calibrated Parameters

As I mentioned, there are two parameters, $\delta$ and $\eta$, which cannot be pinned down independently from the model. In order to calibrate the two parameters, I find the value of the two parameters such that two closely related targets are achieved. The targets are $\frac{K}{Y} = 1.47$ and the proportion of defaulters each year is 0.526%. I find the value of $\delta$ and $\eta$ such that, in the stationary equilibrium of the model, the two targets are achieved. Notice two things. First, in order to find such parameter values, it is necessary to run the model many times trying different combination of $(\delta, \eta)$. Second, the values of $(\delta, \eta)$ are different depending on the model specification. For example, $(\delta, \eta)$ are different between the model with exponential discounting agents, and the model with quasi-hyperbolic discounting agents. However, the targets are the same across different versions of the model. Same can be said to all the other models with different preference specifications.

Table 3 summarizes calibrated parameters for two versions of the model, one with exponential discounting agents, and the other with quasi-hyperbolic discounting agents. In the model with exponential discounting agents, the long-term discount factor $\delta$ is calibrated to be 0.9191. As for the model with quasi-hyperbolic discounting agents, Laibson et al. (2007) estimate $\delta$ to be 0.9588, which is slightly higher but close to the calibrated value of 0.9478.

The garnishment parameter $\eta$ is calibrated to be 0.4319 for the model with exponential discounting agents, and 0.5803 for the model with hyperbolic discounting agents. In order to match the

[14] 0.0136 is computed by 0.0039 multiplied by 2 (two persons involved in a divorce) and divided by 0.573 (proportion of persons of age between 22 and 64).

[15] 0.0043 is obtained by dividing 0.0141 by 0.573 (proportion of persons of age between 22 and 64) and multiplied by 1.92 (average number of adults in a household) and 0.091 (probability of unwanted births).

number of defaults in the data, it is necessary to assume a high garnishment rate for the economy with hyperbolic discounting agents, since agents tend to default more often with hyperbolic discounting.

# 5 Computation

Since the model cannot be solved analytically, numerical methods are employed. I solve individual agent's problem backward, starting from the last period of life, with discretized state space. There are two features worth pointing out.

First, since the problem of an agent of each type involves two optimization problems (associated with temptation and self-control problem, respectively), agent's problem for each individual type needs to be solved twice, first time for the temptation problem, and second time for the self-control problem.

Second, equilibrium price of debt, $q(i, h, p, t, x, a')$ is solved simultaneously with the agent's optimization problem. Once the optimal decision rules for agents of age i is obtained, the price of debt for age i-1 agents, $q(i-1, h, p, t, x, a')$, can be computed, using the optimal default policy $g_d(i, h, p, t, x, a)$. $q(i-1, h, p, t, x, a')$ in turn is used to solve the optimization problem of agents of age i-1. In short, there no need to use iteration to find an equilibrium loan price $q(i, h, p, t, x, a')$.

More details about the solution method employed can be found in Appendix **??**.

# 6 Comparison of Baseline Models

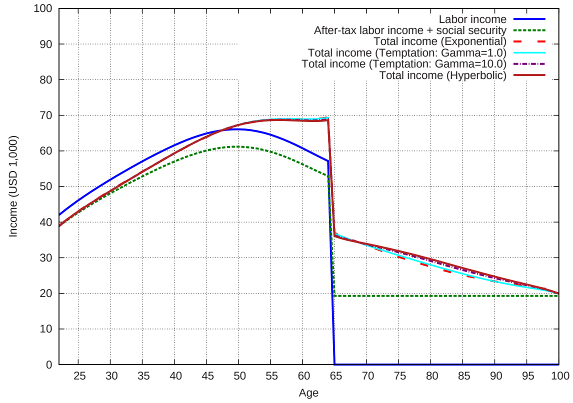
In this section, I will compare properties of the standard model with exponential discounting, with models with short-term discount factor ($\beta$) of 0.7 and varying degree of strength of temptation. Specifically, I will compare the following four models: *(i)* model with standard exponential discounting ($\gamma = 0$), *(ii)* model with weaker temptation ($\gamma = 1$), *(iii)* model with stronger temptation ($\gamma 10$), and *(iv)* model with infinitely strong temptation, or quasi-hyperbolic discounting ($\gamma = \infty$). Notice that all four models are calibrated independently, to match the same set of targets. In particular, the models have different long-term discount factor ($\delta$) and garnishment parameter ($\eta$).

Figure 5 compares the average life-cycle profile of models with varying strength of temptation. Figure 5(a) shows the labor income, after-tax (social security contribution) labor income, and total income, which includes the capital income, for the four economies. Since labor supply is inelastic, before tax and after-tax labor income are the same across four model economies. Moreover, total tax profile is also close to identical since the life-cycle profile of asset accumulation (see Figure 5(c)) are very similar across four model economies.

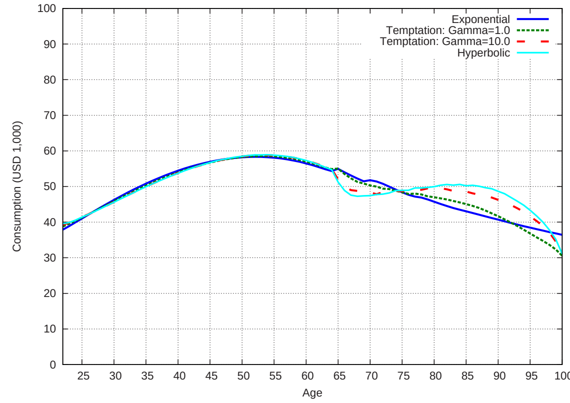
Figure 5(b) compares the average life-cycle profile of consumption. There is one noticeable difference. Models with temptation show *dual-humps*; consumption drops at the retirement age and

**Table 3: Summary of Calibration**

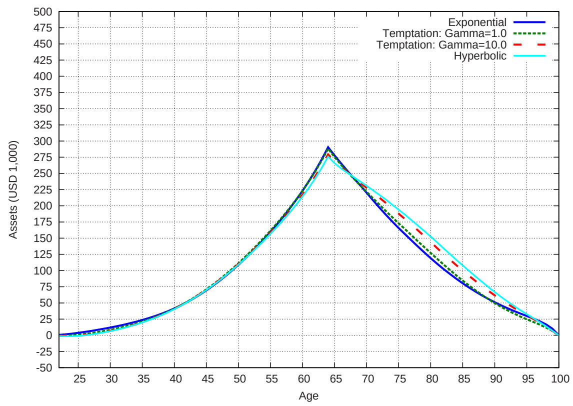
| Parameter | Value | Remark |
|---|---|---|
| **Common parameters** | | |
| $I$ | 79 | Maximum age (corresponding to 100 years old). |
| $I_R$ | 43 | Last working age (corresponding to 64 years old). |
| $\pi$ | 0.012 | Annual population growth rate. |
| $\{s_i\}$ | Fig 2 | Survival probabilities. |
| $\sigma$ | 2.000 | Coefficient of relative risk aversion. |
| Z | 0.1259 | Normalization to achieve $Y = 1.0$. |
| $\theta$ | 0.2470 | Capital share of income. |
| $\nu$ | 0.1090 | Annual depreciation rate. |
| $\iota$ | 0.0400 | Transaction cost of loans. |
| $\lambda$ | 0.1000 | On average 10 years of exclusion from loan market upon default. |
| $\xi$ | 0.0135 | Cost of bankruptcy is 600 dollars. |
| $\overline{r}$ | 1.0000 | Ceiling for interest rate is 100% per year. |
| $\tau_S$ | 0.0740 | Replacement ratio of social security benefit is 33.7%. |
| $\{e_i\}$ | Fig 3 | Average labor income profile. Following Gourinchas and Parker (2002). |
| $\sigma^2_{perm}$ | 0.2105 | Variance of permanent shock to earnings. From Storesletten et al. (2004) |
| $\sigma^2_{tran}$ | 0.0630 | Variance of transitory shock to earnings. From Storesletten et al. (2004) |
| $\sigma^2_{pers}$ | 0.0166 | Variance for persistent shocks to earnings. From Storesletten et al. (2004) |
| $\rho_{pers}$ | 0.9989 | Persistence of persistent shocks to earnings. From Storesletten et al. (2004) |
| $x_1$ | 1.9909 | Magnitude of expenditure shock: larger medical bills |
| $x_2$ | 0.6945 | Magnitude of expenditure shock: smaller medical bills |
| $x_3$ | 0.1269 | Magnitude of expenditure shock: divorce and unwanted births |
| $p^{x_1}$ | 0.0010 | Probability of expenditure shock: larger medical bills |
| $p^{x_2}$ | 0.0100 | Probability of expenditure shock: smaller medical bills |
| $p^{x_3}$ | 0.0179 | Probability of expenditure shock: divorce and unwanted births |
| **Exponential discounting model** | | |
| $\delta$ | 0.9191 | Long-run discount factor. Chosen to match $\frac{K}{Y} = 1.47$ |
| $\beta$ | – | Nature of temptation (Short-run discount factor). |
| $\gamma$ | 0.0000 | Strength of temptation. |
| $\eta$ | 0.4319 | Garnishment ratio. Chosen to match number of bankruptcies=0.526% |
| **Baseline Hyperbolic discounting model** | | |
| $\delta$ | 0.9478 | Long-run discount factor. Chosen to match $\frac{K}{Y} = 1.47$ |
| $\beta$ | 0.7000 | Nature of temptation (Short-run discount factor). |
| $\gamma$ | $\infty$ | Strength of temptation. |
| $\eta$ | 0.5803 | Garnishment ratio. Chosen to match number of bankruptcies=0.526% |



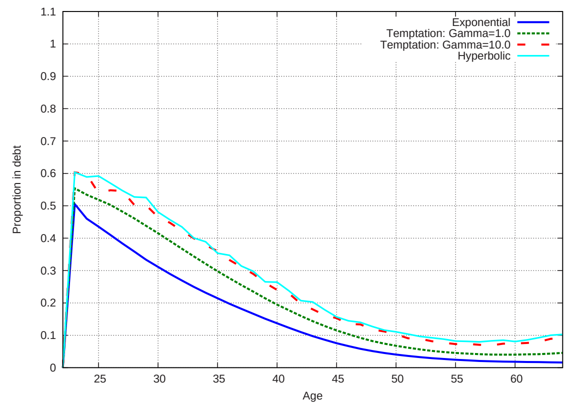

(a) Income



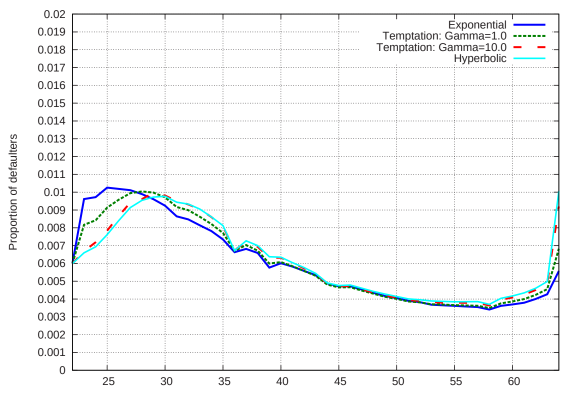

(b) Consumption



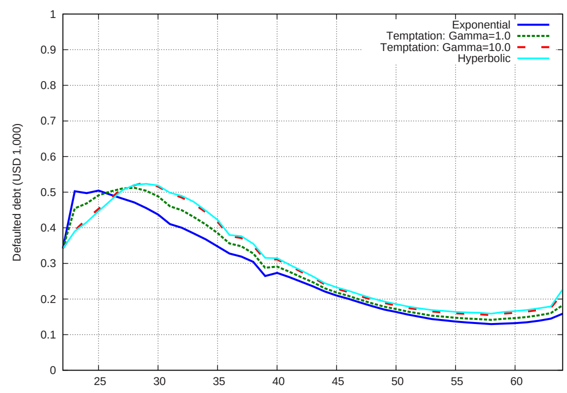

(c) Asset allocation




(d) Proportion of debtors




(e) Proportion of defaulters




(f) Defaulted amount

**Figure 5: Average life-cycle profiles of baseline models with varying strength of temptation**

Table 4: **Debt and defaults in models with quasi-hyperbolic discounting**

| Model | $\delta$ | $\eta$ | Defaults | Debt[1] | Capital[1] |
|---|---|---|---|---|---|
| Exponential: Baseline | 0.9191 | 0.4319 | 0.5260 | 0.0173 | 1.4700 |
| Hyperbolic: Baseline | 0.9478 | 0.5803 | 0.5261 | 0.0398 | 1.4700 |
| Hyperbolic: Fixed $\eta$ | 0.9479 | 0.4319 | 0.8910 | 0.0359 | 1.4699 |
| Hyperbolic: Fixed $\eta$ and $\delta$ | 0.9191 | 0.4319 | 1.0273 | 0.0397 | 1.1010 |

[1] Ratio over GDP.

there is another consumption hump of consumption during the retirement period. İmrohoroğlu et al. (2003) also found this hump in their model with quasi-hyperbolic discounting and argue that this feature can be an advantage of the models with hyperbolic discounting in the sense that the model can replicate the observed sudden drop in consumption at the time of retirement without relying on nonseparable utility from leisure. Notice that the size of the second hump increases with the strength of temptation.

Figure 5(c) compares the average life-cycle profile of asset/debt level. All look similar in general. Figure 5(d) shows the fraction of agents in debt for each age. The number of debtors is higher for all age groups when the temptation is stronger. But the downward-sloping shape is common across all economies. Figure 5(e) shows the default rate for each age group. Since all models are calibrated such that the total number of defaults is the same as in the U.S. economy, the total number of defaults is the same across all the models in the figure. Finally, Figure 5(f) shows that average amount of defaulted debt over the life-cycle. For both the number of bankruptcy filings and the amount of defaulted debt, the profile is skewed to the old when the temptation is strong.

In the calibrated models above, the number of bankruptcy filings in the stationary equilibrium is the same across models with different preference specifications because the garnishment parameter $\eta$ is adjusted to achieve the same target. What is the garnish parameter is not controlled? Table 4 answers the question. The table shows the number of filings, debt over GDP, and asset over GDP in the models with quasi-hyperbolic discounting, when the garnishment parameter $\eta$ is not controlled. The first and the second row of the table show the results from the baseline calibrations where the number of filings is matched to 0.526% per year. The third row shows properties of the economy with quasi-hyperbolic discounting where $\eta$ is set at the same level as in the exponential discounting model but the long-term discount rate $\delta$ is calibrated to match the same target regarding the aggregate saving (capital-output ratio of 1.47). Now the number of defaults is substantially higher at 0.89%. The total amount of debt is also higher compared with the exponential discounting model at 3.59% but lower than the baseline quasi-hyperbolic discounting model (3.97%). It is not surprising because, conditional on the preference specification, the amount of debt negatively depends on the strength of the severity of the punishment. The last row shows properties of the model with quasi-hyperbolic discounting where both $\eta$ and $\delta$

Table 5: **Macroeconomic effect of BAPCPA: Comparison of models**

| Model | Exponential | Temptation | Temptation | Hyperbolic |
|---|---|---|---|---|
| **Parameters** | | | | |
| $\beta$ (Short-run discount factor) | 1.0000 | 0.7000 | 0.7000 | 0.7000 |
| $\gamma$ (Strength of temptation) | 0.0000 | 1.0000 | 10.000 | $\infty$ |
| $\delta^1$ (Long-term discount factor) | 0.9191 | 0.9337 | 0.9456 | 0.9478 |
| $\eta^1$ (Wage garnishment rate) | 0.4319 | 0.5128 | 0.5702 | 0.5803 |
| **Before the reform** | | | | |
| % Defaults | 0.5260 | 0.5260 | 0.5259 | 0.5261 |
| With expenditure shocks | 0.4878 | 0.4606 | 0.4322 | 0.4273 |
| Without expenditure shocks | 0.0382 | 0.0654 | 0.0936 | 0.0988 |
| Proportion in debt | 0.1483 | 0.1939 | 0.2282 | 0.2368 |
| # Defaults / # Debtors | 0.0355 | 0.0271 | 0.0230 | 0.0222 |
| Debt / Debtor | 0.1232 | 0.1504 | 0.1697 | 0.1725 |
| Asset/GDP | 1.4700 | 1.4700 | 1.4700 | 1.4700 |
| Debt/GDP | 0.0173 | 0.0281 | 0.0376 | 0.0398 |
| Charge-off rate[2] | 0.0990 | 0.0717 | 0.0582 | 0.0561 |
| Risk-free interest rate | 0.0590 | 0.0590 | 0.0590 | 0.0590 |
| Avg loan rate | 0.1152 | 0.1161 | 0.1170 | 0.1171 |
| **After the reform** | | | | |
| % Defaults | 0.3936 | 0.3874 | 0.3873 | 0.3868 |
| With expenditure shocks | 0.3548 | 0.3383 | 0.3271 | 0.3235 |
| Without expenditure shocks | 0.0388 | 0.0491 | 0.0602 | 0.0632 |
| Proportion in debt | 0.1537 | 0.2414 | 0.2723 | 0.2779 |
| # Defaults / # Debtors | 0.0256 | 0.0161 | 0.0142 | 0.0139 |
| Debt / Debtor | 0.1401 | 0.1316 | 0.1520 | 0.1548 |
| Asset/GDP | 1.4721 | 1.4740 | 1.4716 | 1.4718 |
| Debt/GDP | 0.0205 | 0.0308 | 0.0404 | 0.0421 |
| Charge-off rate[2] | 0.0727 | 0.0545 | 0.0452 | 0.0439 |
| Risk-free interest rate | 0.0588 | 0.0587 | 0.0589 | 0.0589 |
| Avg loan rate | 0.1177 | 0.1160 | 0.1151 | 0.1151 |

[1] Jointly calibrated to match *(i)* K/Y ratio and *(ii)* number of bankruptcy filings.

[2] Includes both unsecured debt and expenditure shocks.

are fixed at the level in the baseline exponential discounting model. As easily expected, both the amount of debt, and the number of filings are higher, and the aggregate saving is substantially lower than in the calibrated model.

# 7 Macroeconomic Effect of BAPCPA

In this section, I will investigate how different models respond to the bankruptcy law reform which resembles the one which was enacted in the U.S. in 2005. I will compare the model predictions with the actual data after 2005 to judge which model does a better job in replicating what happened in response to the U.S. bankruptcy law reform in 2005.

According to White (2007), the two key elements of BAPCPA are *(i)* means-testing requirement, and *(ii)* higher cost of filing a bankruptcy. In the model experiment, I will introduce both elements. As for *(i)* , I assume that, under the new bankruptcy regime, only the agents whose total income is above median income in the model can file for a bankruptcy. In addition, in case the feasible set is empty conditional on not filing for bankruptcy, those agents are also allowed to file. This case can happen mainly because of the expenditure shocks. As for *(ii)* , White (2007) state that the debtors' out-of-pocket expenses of filing for Chapter 7 bankruptcy increased from around 600 dollars to around 2500 dollars due the new bankruptcy law. In order to accommodate the change, the bankruptcy cost parameter $\xi$ is adjusted upwards from the baseline level of 0.0135 to 0.0562. With the two changes, the new stationary equilibrium is computed for each model economies with different specification of preferences.

Table 5 summarizes the changes between the original stationary equilibrium, which corresponds to the U.S. economy before the BAPCPA, and the new stationary equilibrium, which corresponds to the U.S. economy after the bankruptcy reform. The four columns correspond to the models with *(i)* exponential discounting agents, *(ii)* agents with weaker temptation ($\gamma = 1$), *(iii)* agents with stronger temptation ($\gamma = 10$), and *(iv)* quasi-hyperbolic discounting agents ($\gamma = \infty$), respectively.

The first column of Table 5 shows the macroeconomic effect of the bankruptcy law reform in the model with exponential discounting. Most importantly, the model predict a decrease in the number of bankruptcy flings, from 0.526% to 0.394%. However, if the change in the number of bankruptcy filings is disaggregated, there is an interesting difference between the defaults due to expense shocks and those due to earnings shocks; the number of defaults due to expense shocks *decreases* (from 0.49% to 0.35%) while those due to earnings shocks *increases*, albeit slightly (from 0.0382% to 0.0388%).

In order to understand the reason behind the difference, it is important to understand the direct and indirect effect of creditor-friendly bankruptcy law reform. The direct effect of a tougher bankruptcy law is that some agents cannot file for bankruptcy even if they want. It happens if an agent is earning above median income in the current period. It happens more often with the expenditure shocks, because, if an agent wants to file for bankruptcy without getting hit by an expenditure shock, it is like that the agent is drawing a series of unfavorable income shocks. Therefore, a decline in the number of filings due to expenditure shocks is the result of the direct effect of tougher bankruptcy law.

At the same time, a creditor-friendly bankruptcy law gives agents stronger commitment to replay, and thus credit card companies will offer lower default premium for unsecured loans (a higher discount price of debt). This will encourage more agents to borrow, and borrow more. This is the indirect effect of a creditor-friendly bankruptcy law and is emphasized by Chatterjee et al. (2007). In the model, the proportion of debtors increases from 14.8% to 15.4%. The average size of debt increases as well, from 12.3% of per-capita output to 14.0%. Even if the overall proportion of agents who end up defaulting over the number of agents in debt decreases from 3.6% to 2.6%, the number of bankruptcy filings due to unfavorable income shocks increases at the end. For the case of defaults due to unfavorable income shocks, the indirect effect dominates the direct effect, because agents who default because of unfavorable income shocks tend to be affected less by the direct effect.

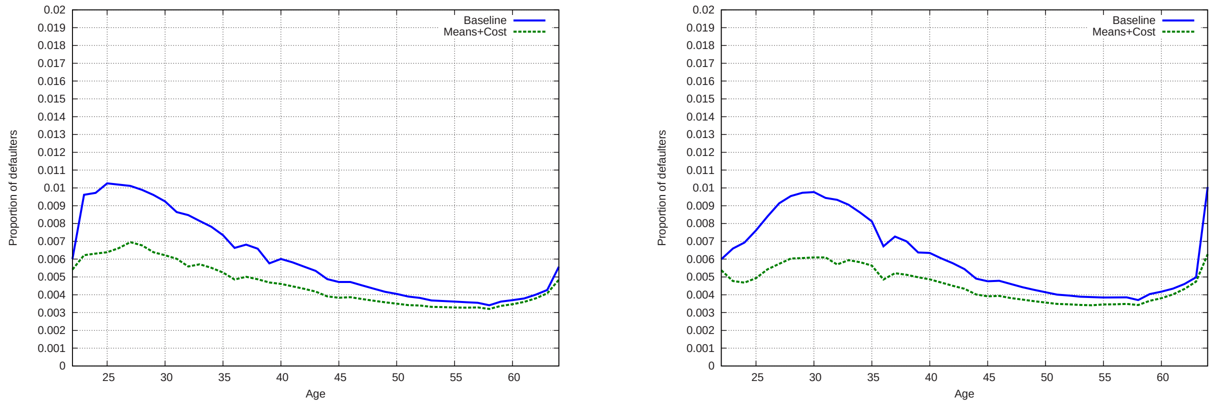
Charge off rate declines in response to the reform, from 9.9% to 7.3%, because more bills (expenditure shocks) will end up repaid. However, the average interest rate for loans increase, from 11.5% to 11.8%, because agents borrow more when the interest rate drops for the same size of debt.

Regarding the general equilibrium effect, there are two interesting effects going on. First, as the borrowing constraint becomes less strict, the total mount of debt *increases*, from 1.7% of GDP to 2.1%. Second, however, the total amount of capital *increases*, even if the amount of debt increases. There are two opposite effects. First effect is the negative effect to the aggregate saving due to the relaxed borrowing constraint. This channel is emphasized by Li and Sarte (2006). They argue that the welfare loss due to the general equilibrium effect in response to the introduction of the creditor-friendly bankruptcy law is important. The second effect is the increased precautionary motive saving because of the limited availability of bankruptcy filings. In case an agent cannot file for bankruptcy even if she wants, she might suffer a large consumption drop. Therefore, naturally, there will be a stronger saving motive to prepare against such situation. A higher total capital after the reform is introduced implies that the second effect dominates the first effect at the aggregate level. However, the magnitude of the general equilibrium effect is limited in the sense that the size of the changes in the number of defaults and debt is not significantly affected by the general equilibrium effect. Finally, risk-free interest rate drops, but very slightly (5.90% to 5.88%).

The last column of Table 5 shows the same statistics from the model economy populated with quasi-hyperbolic discounting agents. Similar to the model with exponential discounting agents, the number of bankruptcy filings *decreases* in response to the bankruptcy reform in 2005, from the initial level of 0.526% to 0.387%. In terms of the reasons of bankruptcies, both bankruptcies due to expense shocks and those due to earnings shocks decline by a similar proportion.

Also similar to the exponential discounting model, the number of agents in debt (23.7% to 27.8%), and total mount of debt (4.0% of GDP to 4,2%) increase while the charge-off rate (5.6% to 4.4%) and proportion of filers among debtors (2.2% to 1.4%) drop. However, the average size of debt per debtor decreases from 17.3% of per-capita output to 15.5%. Correspondingly, average interest rate of loans also drops from 11.7% to 11.5%. Contrary to a naive perception, agents borrow aggressively in the exponential discounting model rather than in the quasi-hyperbolic discounting model.

The general equilibrium effect is similar between the two opposite models; the total mount of




(a) Changes in the number of bankruptcies (Exponential)

(b) Changes in the number of bankruptcies (Hyperbolic)

**Figure 6: Effect of the BAPCPA on the average life-cycle profile: Comparison between exponential (left panels) and hyperbolic (right panels) discounting agents**

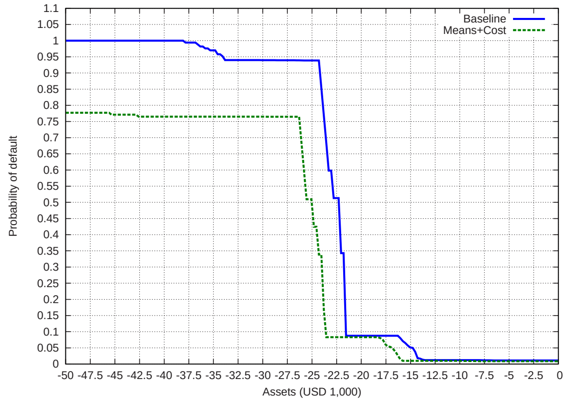
debt increases, while the total capital stock increases as well. Naturally, risk-free interest rate drops, but very slightly (5.90% to 5.89%).

The intermediate cases with the strength of temptation being less than infinity can be by largely located between the two extreme cases discussed above. Notice two things. First, even $\gamma = 1$ is sufficient to change the model predictions from the ones closer to exponential discounting case to the hyperbolic discounting case. The number of bankruptcies due to both expenditure shocks and unfavorable income shocks *decline*. Second, $\gamma = 10$ is already large enough such that the model behaves like the one with quasi-hyperbolic discounting agents, or agents who succumb completely to temptation.

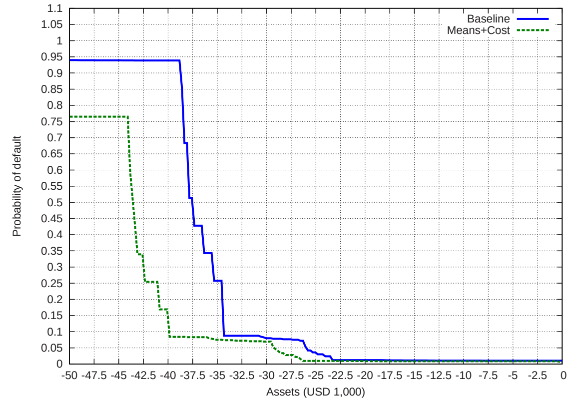
Figure 6 compares how the life-cycle profile of filings changes in response to the bankruptcy law reform, between model economies with exponential discounting agents (left panel) and with quasi-hyperbolic discounting agents (right panel). You can see that the pattern of the change in the number of filings is similar between the two models.

Figure 7 compares the default probability and the offered loan prices for a 30-year old agents with median income in the cohort, for both the model economy of exponential discounting agents and that of hyperbolic discounting agents. 30-year old is chosen as this is an age where agents borrow a lot and default frequently. In both model economies, default probability drops conditional on debt level, in response to the bankruptcy reform (see Figure 7(a) and Figure 7(b)). Correspondingly, the discount prices of loans become lower in response to the bankruptcy reform, in both model economies (see Figure 7(c) and Figure 7(d)). The borrowing constraints, which is endogenously determined in the model, are less strict in the model with quasi-hyperbolic discounting agents. This feature is basically due to the higher value of garnishment rate upon default.

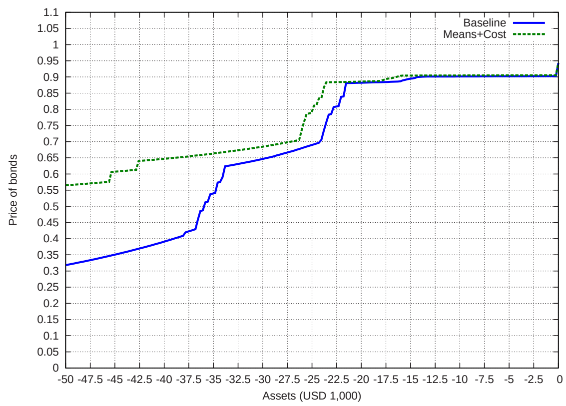
Table 6 compares the observed changes around the bankruptcy reform in the U.S, with the



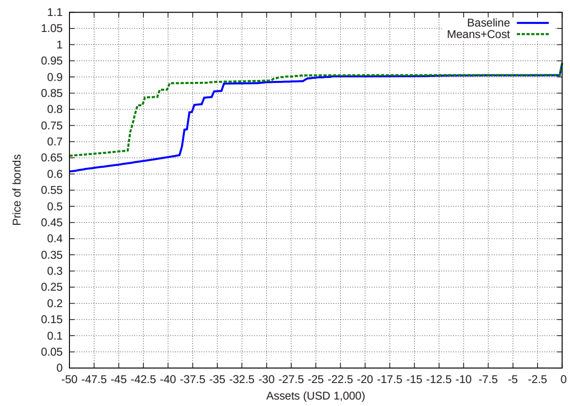

(a) Default probability for a 30-year old (Exponential)




(b) Default probability for a 30-year old (Hyperbolic)




(c) Loan prices for a 30 years old (Exponential)




(d) Loan prices for a 30 years old (Hyperbolic)

**Figure 7: Effect of the BAPCPA on the default probability and loans prices offered to a 30-year old agents with median income in the cohort: Comparison between exponential (left panels) and hyperbolic (right panels) discounting agents**

prediction of the models with exponential and quasi-hyperbolic discounting agents. The basic message is that both models can replicate the observed changes to a certain degree. In both models, the number of bankruptcy filings decline, although the size of the drop is smaller than in the data. Both models generate a small change in the average loan interest rate, but the direction is different; the model with exponential discounting predicts a small increase in the loan interest rate, while the model with quasi-hyperbolic discounting predicts a small decline in the loan interest rate as in the data. Charge-off rate decreases in both models as in the data, although the level is too high in the model with exponential discounting. Debt over GDP increases slightly in both models like in the data. Again the problem is that the level of debt over GDP in both models is lower compared with the data. This problem is related to what Laibson et al. (2003) call the *debt puzzle*, where individuals carry balance in the credit card debt by paying a high interest even if they also have a large amount of retirement wealth whose amount is sufficient to repay (at least a part of) the credit card debt. Telyukova (2008) also investigate a related

**Table 6: Macroeconomic effect of BAPCPA: Model vs Data**

| Period | 2000-2004 | 2006-2007 | Change[1] |
|---|---|---|---|
| **U.S. Economy** | | | |
| Proportion of defaulters[2] | 0.526 | 0.203 | −0.62 |
| Consumer credit interest rate (%) | 11.26 | 10.72 | −0.54 |
| Charge-off rate (%) | 5.46 | 3.82 | −1.64 |
| Unsecured debt / GDP[3] (%) | 7.79 | 7.90 | +0.01 |
| **Model with exponential discounting agents** | | | |
| Proportion of defaulters | 0.526 | 0.394 | −0.25 |
| Consumer credit interest rate (%) | 11.52 | 11.77 | +0.25 |
| Charge-off rate (%) | 9.90 | 7.27 | −2.63 |
| Unsecured debt / GDP (%) | 1.73 | 2.05 | +0.18 |
| **Model with hyperbolic discounting agents** | | | |
| Proportion of defaulters | 0.526 | 0.389 | −0.26 |
| Consumer credit interest rate (%) | 11.71 | 11.51 | −0.20 |
| Charge-off rate (%) | 5.61 | 4.39 | −1.22 |
| Unsecured debt / GDP (%) | 3.98 | 4.21 | +0.06 |

[1] Percentage change for proportion of defaulters and unsecured debt / GDP, and change in percentage points for others.

[2] Among 22 years old and above.

[3] Balance of unsecured credit as defined by Livshits et al. (2007a).

puzzle, the co-existence in the household portfolio of credit card debt and cash and other liquid assets, by carefully distinguishing the goods that can be purchased by cash and those that can be purchased by credit cards.

One striking difference between the model with exponential discounting and the model with preference featuring temptation and self-control is that the number of default due to unfavorable income shocks increase in the former while it decreases in the latter. To investigate the issue further, I shut down the expenditure shocks and investigate how the model economies react differently to the bankruptcy law reform. Table 7 summarizes the results. The models are re-calibrated such that the capital output ratio of 1.47 and the number of bankruptcy filings of 0.14% per year (one-third of the total filings) are achieved. It is clear that the exponential discounting model suffers dramatically; the number of bankruptcy filings *increases*. The average loan interest rate and charge off-rate increase accordingly. All of them are counterfactual. On the other hand, the model with quasi-hyperbolic discounting still performs decently. The bottom line is that, for exponential discounting model, the nature of shock which induces bankruptcies has a crucial effect on how the model reacts to the bankruptcy law reform, while it is not the case for the models with temptation and self-control (or quasi-hyperbolic discounting). In the baseline experiments, the exponential discounting model can be said to perform as well as the models with temptation and self-control, because the significant proportion of defaults is due to expenditure shocks. If that is not the case, the exponential discounting model performs very

**Table 7: Macroeconomic effect of BAPCPA: Models without expenditure shock**

| Period | 2000-2004 | 2006-2007 | Change[1] |
|---|---|---|---|
| **Exponential discounting without expenditure shock** | | | |
| Proportion of defaulters | 0.144 | 0.170 | +0.18 |
| Consumer credit interest rate (%) | 11.68 | 13.53 | +1.85 |
| Charge-off rate (%) | 1.45 | 2.94 | +1.49 |
| Unsecured debt / GDP (%) | 0.53 | 1.48 | +1.79 |
| **Hyperbolic discounting without expenditure shock** | | | |
| Proportion of defaulters | 0.141 | 0.074 | −0.48 |
| Consumer credit interest rate (%) | 11.02 | 10.73 | −0.29 |
| Charge-off rate (%) | 0.84 | 0.49 | −0.35 |
| Unsecured debt / GDP (%) | 3.72 | 3.88 | +0.04 |

[1] Percentage change for proportion of defaulters and unsecured debt / GDP, and change in percentage points for others.

poorly in replicating the observed response of the U.S. economy against BAPCPA.

# 8 Welfare Effect of BAPCPA

In comparing the welfare of agents in economies before and after the bankruptcy law reform, I use the ex-ante expected utility of newborns in the stationary equilibrium. This criterion is the one used by Conesa et al. (2007). Specifically, average welfare is computed by integrating the value of the newborns in the stationary equilibrium with respect to the initial shock to earnings. Moreover, changes in welfare is measure by the percentage changes in the flow consumption in all periods and nodes, $\epsilon$. Thanks to the homotheticity of the period utility function, $\epsilon$ can be computed as follows:

$$\epsilon = \left(\frac{EV_{new}}{EV_{old}}\right)^{\frac{1}{1-\sigma}} - 1 \tag{37}$$

where $EV_{old}$ and $EV_{new}$ are the ex-ante expected utility of newborns in the initial and the new stationary equilibrium, respectively. Moreover, notice that, in the case of preference featuring temptation and self-control problem, temptation part of utility is also affected by $\epsilon$.

Table 8 summarizes the welfare effect of introducing the bankruptcy law reform similar to BAPCPA into the models with different preference assumptions. No matter whether the general equilibrium effect from adjusted priced wages and interest rates are taken into account or not, the welfare effect is *small negative* for all cases. Moreover, the general equilibrium effect induces a small welfare gain in all cases, through a small increase in the aggregate saving.

**Table 8: Welfare effect of BAPCPA: Comparison of model**

| Model | Parameter | | Change in welfare (%)[1] | |
|---|---|---|---|---|
| | $\beta$ | $\gamma$ | With GE | Without GE |
| Exponential discounting | 1.00 | 0.00 | −0.17 | −0.23 |
| Temptation | 0.70 | 1.00 | −0.18 | −0.20 |
| Temptation | 0.70 | 10.0 | −0.16 | −0.18 |
| Hyperbolic discounting | 0.70 | $\infty$ | −0.15 | −0.18 |

[1] Measured by the percentage change in the flow consumption in all periods and nodes by introducing the bankruptcy law reform.

There are four effects that are working. First, there is a negative effect because some agents cannot file for bankruptcy if it is the best option under the new bankruptcy law. Second, relaxed borrowing constraints mitigate the negative effect of the creditor-friendly bankruptcy law. Third, the relaxed borrowing constraints could enable agents to better smooth out life-cycle consumption profile. This channel is emphasized by Livshits et al. (2007b) as the benefit of not having an option of filing for bankruptcy and consequently the relaxed borrowing constraints. Fourth, there is a general equilibrium effect. The welfare effect can be positive or negative, but it turns out to be positive in all cases, as the aggregate saving increases in all models. Judging from the total effect, the first effect seems to dominate the second and the third effect. The general equilibrium effect is positive, but not large enough to overturn the partial equilibrium effect.

# 9 Conclusion

In this paper, I first investigate the properties of the model with equilibrium default and preference which features temptation and self-control problem. The properties are compared with those of the model with the standard exponential discounting preference. Second, I compare the macroeconomic and welfare implications of the bankruptcy law reform which is similar to BAPCPA enacted in the U.S. in 2005, using models with different preference specifications.

There are five main findings. First, models with different preference specifications exhibit very similar average life-cycle profiles of asset, but models with temptation and self-control show a drop in consumption at the time of retirement and a second hump in the average consumption profile after retirement. Second, conditional on the same level of punishment for defaults, models with temptation generates a larger amount of debt and a larger number of defaults. Third, under the baseline calibration, both the standard exponential discounting model and the model with temptation and self-control replicate the reaction of the U.S. economy against the recent bankruptcy law reform equally well. Both models correctly predict a decline in the number of bankruptcies, and less significant change in the amount of loans and the average loan interest rate. Fourth, however, for exponential discounting model, the result crucially depends on what type of shocks is dominant. In particular, if defaults are not mainly due to expenditure shocks, but rather due to series of unfavorable income realizations, models with exponential discounting predict an *increase* in the number of bankruptcy filings, which is a counterfactual implication, while models with temptation and self-control still predict a *decrease* in the number of bankruptcy filings in response to the recent bankruptcy reform. Fifth, the welfare implications of the two class of models in response to the recent bankruptcy law reform are similar and small negative, with and without the general equilibrium effect. In sum, under the baseline calibration, in studying the macroeconomic and welfare implications of the recent bankruptcy law reform, using the model with temptation and self-control does not give a clear advantage over the standard model with exponential discounting. The properties of the models become very different depending on the major cause of bankruptcy filings.

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