Designing the Optimal Social Security Pension System

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Abstract

We extend a standard overlapping-generations general-equilibrium model with idiosyncratic working ability shocks to design the optimal social security pension system. There are two main features in our approach. First, we keep track of individual social security wealth explicitly so that we can evaluate a wide range of policies, including "private accounts," seamlessly. Second, we express our social security benefit function with two key parameters—one for intragenerational income redistribution and another for intergenerational income transfers. We find that the system would be optimal if benefits were proportional to individual social security wealth but those were on average *less than* actuarially fair.

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Key Words: social security reform; optimal policy design; heterogeneous-agent economy

1 Introduction

In the present paper, we construct an overlapping-generations general-equilibrium model with uninsurable idiosyncratic working ability shocks and calibrate the model to the U.S. economy to design the optimal social security pension system. There are two main features in our model. First, we keep track of individual social security wealth explicitly, instead of replicating the average indexed monthly earnings (AIME) in the U.S. system, so that we can analyze a traditional defined-benefit pension system and a defined contribution system, or "private accounts," seamlessly. This treatment also makes us analyze the effect of prefunding of social security more clearly. Second, we define the social security benefit function with two key parameters, yet the function can generate a wide range of social security systems with respect to intragenerational income redistribution and intergenerational income transfers.

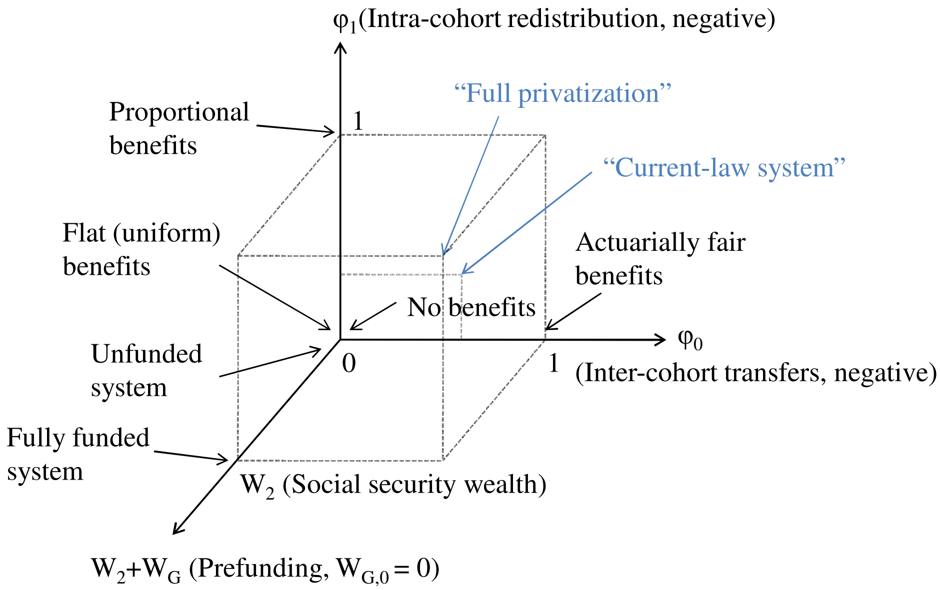

Figure 1: The three axes of social security pension systems

We describe a social security pension system as of the beginning of period t by

$$\{\tau_{P,s}, \varphi_{0,s}, \varphi_{1,s}, W_{2,s+1}, W_{G,s+1}\}_{s=t}^{\infty},$$

where $\tau_{P,t}$ is a flat payroll tax rate for the social security system, $\varphi_{0,t}$ and $\varphi_{1,t}$ are the two parameters of the benefit function, $W_{2,t+1}$ is the social security wealth, and $W_{G,t+1}$ is the rest of government wealth. For simplicity, we assume $\tau_{P,t} = 0.1$ or 10% throughout the paper; then $W_{2,t+1}$ is determined endogenously. The remaining control policy variables of the government are $\varphi_{0,t}$, $\varphi_{1,t}$, and $W_{G,t+1}$, where the first parameter determines the average actuarially fairness (the degree of intergenerational transfers), the second parameter determines the progressiveness (the degree of intragenerational redistribution), and the net government wealth determines whether the pension system is funded or unfunded.[1]

Figure 1 summarizes the three axes of social security pension systems. When $\varphi_{0,t} = \varphi_{1,t} = 1.0$, for example, the social security pension system is "fully privatized." If the system is also fully funded, it becomes similar to a mandatory version of Roth individual retirement accounts. Pay-as-you-go social security systems are described by $\varphi_{0,t}$ less than 1, where the parameter is determined so that payroll tax revenue is equal to benefit expenditure. For simplicity, we assume the government wealth, $W_{G,0}$, before the

[1]We abstract from the investment policy and the ownership of social security wealth, because the model economy does not have multiple assets such as risky assets and safe assets, and we assume social security wealth is not inheritable. Thus, it does not matter whether social security wealth is *owned* by the government or individual households, and there is no distinction between a defined benefit system and a defined contribution system.

introduction or reform of a social security pension system to be zero. When the government introduces a new social security pension system, social security wealth, $W_{2,t}$, is accumulated. If the rest of the government wealth, $W_{G,t}$, stays at the same level as that of the baseline economy, *i.e.*, $W_{2,t} + W_{G,t} = W_{2,t}$, the new system is said to be *fully funded* in our model. If the government increases its debt as social security wealth increases, *i.e.*, $W_{2,t}+W_{G,t} = 0$, the new system is said to be *unfunded*. Fully funded social security systems (or reforms) are budget neutral in the sense that the rest of the government budget is balanced.[2]

The main findings of the present paper are as follows: First, when the payroll tax rate was set at 10%, it would be optimal for social security benefits to be proportional to individual social security wealth, *i.e.*, $\varphi_{1,t} = 1$, in our main calibration. Although the result is fairly robust, the qualitative result would differ if we changed the parameters of the household's working ability process and the progressive income tax function at the same time. Second, it would be optimal for social security benefits to be on average less than actuarially fair, *i.e.*, $\varphi_{0,t} < 1$. When social security benefits are less than actuarially fair, the social security payroll tax generates labor supply distortion. When the social security system is budget neutral (fully funded), however, actual benefits less than actuarially fair benefits would generate additional revenue for the government, and the government could reduce individual income tax rates and, thus, labor supply and saving distortions.

A large amount of the literature have analyzed the effects of reforming the current social security pension system from an unfunded defined benefit (pay-as-you-go) system to a funded defined contribution system; for example, İmrohoroğlu, İmrohoroğlu, and Joines (1995), Kotlikoff, Smetters, and Walliser (1999), Conesa and Krueger (1999), and Nishiyama and Smetters (2007).[3] However, Geanakoplos, Mitchell, and Zeldes (1998) explain that privatization, diversification, and prefunding could be implemented separately without the other two. We could assume the government debt (including its unfunded liability) is kept at the same level while reforming the social security system. The welfare or efficiency effect of reducing the government's unfunded liability is the same as that of reducing debt in the rest of the government budget, and depends on the financing assumption of "transition costs," which would make the discussion on the op-

[2]When social security benefits are less than actuarially fair, we assume that the government captures the difference between the actual benefits and the actuarially fair benefits. There are three possible timings of this *taxation*: taxing social security pension contributions, taxing investment income in social security wealth, and taxing social security benefits. In the present paper, we assume the last mentioned option so that we can uses the present value of the individual payroll tax payments as a state variable. Keeping track of the value of the payroll tax payments instead of AIME, for example, makes generational accounting and intergenerational income redistribution clearer. In our assumption, when $\varphi_{0,t} < 1$, income is transferred from retired households to working-age households. However, the government can change the direction of intergenerational income transfers by increasing its debt (to make the social security system partially funded or unfunded) at the same time.

[3]Most previous papers assume that wealth in a fully funded defined contribution system is a perfect substitute of regular household wealth and analyze the *privatization* by partially or fully eliminating the current-law (pay-as-you-go, defined benefit) social security system. In the present paper, however, we have social security wealth explicitly as a state variable in our model and assume that the social security *private* accounts are similar to mandatory Roth IRAs when there is no income redistribution.

timal social security system unclear. Thus, we propose the optimal government debt policy to be analyzed separately from social security reform.

Other literature has investigated the optimal social security system with different methods. For example, Conesa and Garriga (2008) construct an representative-agent OLG economy with no idiosyncratic wage shocks, and they solve a Ramsey problem to find the optimal social security reform plans in the absence of annuities markets. The present paper focuses on lifetime income inequality rather than lifetime uncertainty. Huggett and Parra (2008) use a partial-equilibrium life-cycle model and compare the welfare gains from optimal social insurance reform policies and the gains from social planners solutions with incentive compatibility constraints. The present paper will contribute to this field by complementing the existing literature.

The rest of the paper is laid out as follows: Section 2 describes the model economy, Section 3 explains the specific functions and calibrations of the model, Section 4 shows the long-run (steady-state) analyses of social security systems, Section 5 shows the transition analyses of the selected social security systems, and Section 6 concludes the paper. The Appendix explains the computational algorithms used in this paper.

2 The Model Economy

The economy consists of a large number of households, a perfectly competitive representative firm with constant-returns-to-scale technology, and a government with commitment technology. Households are heterogeneous with respect to their ages, beginning-of-period regular wealth and social security wealth holdings, and individual working abilities. Households receive idiosyncratic working ability shocks in each period and choose consumption, leisure (or equivalently, working hours), and end-of-period regular wealth holdings to maximize their expected remaining lifetime utilities.

The Household's Optimization Problem. Let (a_1, a_2, e) be the individual state vector, where $a_1 \in A_1 = [0, \infty)$ is beginning-of-period regular wealth, $a_2 \in A_2 = [0, \infty)$ is beginning-of-period social security wealth, and $e \in E = [0, e_{\max}]$ is individual working ability. Let Ω_t denote a time series of factor prices and government policy variables that describe the state of the aggregate economy,

$$\Omega_t = \{r_s, w_s, \tau_{P,s}, \varphi_s, \psi_s, tr_s, C_{G,s}, W_{G,s}\}_{s=t}^{\infty},$$

where r_t is the interest rate, w_t is the average wage rate, $\tau_{P,t}$ is the social security payroll tax rate, $\varphi_t = (\varphi_{0,t}, \varphi_{1,t})$ are the parameters of social security benefit function, $\psi_t = (\psi_{0,t}, \psi_{1,t}, \psi_{2,t})$ are the parameters of progressive income tax function, tr_t is the lump-sum transfer from the government to households, $C_{G,t}$ is the government consumption expenditure, and $W_{G,t}$ is the beginning-of-period government wealth.[4] Let $v_i(a_1, a_2, e; \Omega_t)$ denote the household's value function at age i in period t. The household's optimization problem is

$$(1) \quad v_i(a_1, a_2, e; \Omega_t) = \max_{c,l,a_1'} \{u(c,l) + \tilde{\beta}\phi_i E[v_{i+1}(a_1', a_2', e'; \Omega_{t+1})|e]\}$$

subject to

$$(2) \quad a_1' = \frac{1}{(1+\mu)\phi_i}[(1+r_t)a_1 + (1-\tau_{P,t})w_t e(1-l) - \tau_I(r_t a + w_t e(1-l); \psi_t) + b_i(a_2; \Omega_t) + tr_t - c],$$

$$(3) \quad a_2' = \frac{1}{(1+\mu)\phi_i}[(1+r_t)a_2 + \tau_{P,t} w_t e(1-l) - \tilde{b}_i(a_2; \Omega_t)],$$

$$(4) \quad c \in (0, \infty), \quad l \in (0, 1], \quad a_1' \in [0, \infty),$$

where c is consumption, l is leisure, a_1' is regular wealth holding at the beginning of the next period, a_2' is social security wealth at the beginning of the next period, e' is working ability in the next period, $\tilde{\beta}$ is the growth-adjusted time discount factor, and ϕ_i is the survival probability at the end of age i. In the budget constraint, equation (2), μ is the labor-augmenting productivity growth rate, $w_t e$ is the individual wage rate, $1-l$ is working hours, $\tau_I(.; \psi_t)$ is the individual income tax function, and $b_i(a_2; \Omega_t)$ is the social security benefit function. In the state transition function of social security wealth, equation (3), $\tilde{b}_i(a_2; \Omega_t)$ is the actuarially fair social security benefit function explained below.[5] All of the individual variables except for leisure and working hours are growth adjusted, and we assume a perfect annuities market in the model economy. Thus, the wealth holdings at the beginning of the next period, a_1' and a_2', are both adjusted by the growth rate μ and the survival rate ϕ_i.

Let $c_i(a_1, a_2, e; \Omega_t)$, $l_i(a_1, a_2, e; \Omega_t)$, and $a_{1,i}'(a_1, a_2, e; \Omega_t)$ be the household decision rules, and let's

[4]The aggregate state of the economy is usually expressed as the distribution of households and government wealth, for example, $\Phi_t = (x_t(i, a_1, a_2, e), W_{G,t})$, where $x_t(.)$ is the population density function explained below. When the economy has no aggregate shocks, however, this infinite dimensional state vector can be replaced with a series of factor prices and government policy variables, which are perfectly foreseeable by households.

[5]When social security benefits are not actuarially fair, individual social security wealth, a_2, is a virtual account for bookkeeping purposes. We subtract an actuarially fair benefit, $\tilde{b}_i(a_2; \Omega_t)$, instead of a real benefit, $b_i(a_2; \Omega_t)$, from the social security account so that the benefit function can make a consistent income redistribution within and across age cohorts.

define $h_i(a_1, a_2, e; \Omega_t)$ and $a'_{2,i}(a_1, a_2, e; \Omega_t)$ as

$$h_i(a_1, a_2, e; \Omega_t) \equiv 1 - l_i(a_1, a_2, e; \Omega_t),$$
$$a'_{2,i}(a_1, a_2, e; \Omega_t) \equiv \frac{1}{(1+\mu)\phi_i}[(1+r_t)a_2 + \tau_{P,t} w_t e h_i(a_1, a_2, e; \Omega_t) - \tilde{b}_i(a_2; \Omega_t)].$$

The Distribution of Households. Let $x_{i,t}(a_1, a_2, e)$ be the growth-adjusted population density at age i in period t, and let $X_{i,t}(a_1, a_2, e)$ be the corresponding cumulative distribution. The growth-adjusted population of the "newborn" households, which enter the economy without any wealth holding, is normalized to unity, *i.e.*,

$$\int_{A_1 \times A_2 \times E} \mathrm{d}X_{1,t}(a_1, a_2, e) = \int_E \mathrm{d}X_{1,t}(0, 0, e) = 1.$$

Let $\pi_i(e'|e)$ be the transition probability of working ability from e at age i to e' at age $i+1$, and let ν be the population growth rate. Then, the law of motion of growth-adjusted population distribution is

$$x_{i+1,t+1}(a'_1, a'_2, e') = \frac{\phi_i}{1+\nu} \int_{A_1 \times A_2 \times E} \mathbf{1}_{[a'_1 = a'_{1,i}(a_1, a_2, e; \Omega_t), a'_2 = a'_{2,i}(a_1, a_2, e; \Omega_t)]} \pi_i(e'|e) \mathrm{d}X_{i,t}(a_1, a_2, e),$$

where $\mathbf{1}_{[y=f(x)]}$ is an indicator function that returns 1 if $y = f(x)$ and 0 otherwise.

The Firm's Problem. The representative firm chooses capital input, $\tilde{K}_t$, and labor input, $\tilde{L}_t$, to maximize its profit,

$$\max_{\tilde{K}_t, \tilde{L}_t} F(\tilde{K}_t, \tilde{L}_t) - (r_t + \delta)\tilde{K}_t - w_t \tilde{L}_t \tag{5}$$

for all t, taking factor prices, r_t and w_t, as given, where $F(.)$ is a constant-returns-to-scale production function, and δ is the depreciation rate of capital stock. The profit maximizing condition is

$$F_K(\tilde{K}_t, \tilde{L}_t) = r_t + \delta, \quad F_L(\tilde{K}_t, \tilde{L}_t) = w_t. \tag{6}$$

Let I be the highest possible age of households, *i.e.*, $\phi_I = 0$, in the model economy. Beginning-of-period growth-adjusted regular wealth, $W_{1,t}$, and social security wealth, $W_{2,t}$, are

$$(7) \quad W_{1,t} = \sum_{i=1}^{I} \int_{A_1 \times A_2 \times E} a_1 \, \mathrm{d}X_{i,t}(a_1, a_2, e),$$

$$(8) \quad W_{2,t} = \sum_{i=1}^{I} \int_{A_1 \times A_2 \times E} a_2 \, \mathrm{d}X_{i,t}(a_1, a_2, e).$$

In a closed economy, capital stock, K_t, is equal to national wealth, which is the sum of private wealth and government wealth,

$$(9) \quad K_t = W_{1,t} + W_{2,t} + W_{G,t},$$

and the labor supply in efficiency units, L_t, is

$$(10) \quad L_t = \sum_{i=1}^{I} \int_{A_1 \times A_2 \times E} e \, h_i(a_1, a_2, e; \Omega_t) \mathrm{d}X_{i,t}(a_1, a_2, e).$$

The factor markets clear when

$$(11) \quad K_t = \tilde{K}_t, \quad L_t = \tilde{L}_t.$$

The Social Security System. When social security benefits are actuarially fair, the expected discounted sum of the remaining lifetime benefits is equal to the current social security wealth, *i.e.*,

$$\sum_{j=i}^{I} \left(\prod_{k=i+1}^{j} \frac{\phi_{k-1}}{1 + r_{t+k-i}} \right) \bar{b} = (1 + r_t) a_2$$

for $i = I_R, ..., I$, where $\bar{b}$ is a constant annual benefit, and I_R is the starting age of receiving benefits. Thus, the actuarially fair social security benefit function is defined as

$$\tilde{b}_i(a_2; \Omega_t) \equiv \left[\sum_{j=i}^{I} \left(\prod_{k=i+1}^{j} \frac{\phi_{k-1}}{1 + r_{t+k-i}} \right) \right]^{-1} (1 + r_t) a_2 \equiv \tilde{b}_i'(a_2; \Omega_t) a_2$$

for $i = I_R, ..., I$, and 0 otherwise, where $\tilde{b}_i'(a_2; \Omega_t)$ is the marginal actuarially fair benefit function, which is the inverse of the annuity factor and independent of a_2.

Now we define the general social security benefit function used in the model economy as

$$b_i(a_2;\Omega_t) \equiv \varphi_{0,t}[\varphi_{1,t}\tilde{b}_i(a_2;\Omega_t) + (1-\varphi_{1,t})\tilde{b}_i(\bar{a}_{2,i};\Omega_t)] = \tilde{b}_i'(a_2;\Omega_t)\varphi_{0,t}[\varphi_{1,t}a_2 + (1-\varphi_{1,t})\bar{a}_{2,i}],$$

where $\bar{a}_{2,i}$ is the age-cohort average social security wealth at age i in period t,

$$\bar{a}_{2,i} = \frac{\int_{A_1\times A_2\times E} a_2 \, \mathrm{d}X_{i,t}(a_1,a_2,e)}{\int_{A_1\times A_2\times E} \mathrm{d}X_{i,t}(a_1,a_2,e)},$$

$1-\varphi_{0,t}$ is the degree of inter-cohort income transfers, and $1-\varphi_{1,t}$ is the degree of intra-cohort income redistribution. The social security system is on average actuarially fair when $\varphi_{0,t}=1$, and the individual benefits are proportional to their own social security wealth when $\varphi_{1,t}=1$. In the steady-state equilibrium, $\varphi_{0,t}\varphi_{1,t}$ shows the share of the pension contribution in payroll tax payments, and $1-\varphi_{0,t}\varphi_{1,t}$ shows the share of the pure tax portion.

The Government Budget Constraint. The government individual income tax revenue is calculated as

$$(12)\quad T_{I,t} = \sum_{i=1}^{I}\int_{A_1\times A_2\times E} \tau_I(r_t a + w_t e\, h_i(a_1,a_2,e;\Omega_t);\varphi_t)\mathrm{d}X_{i,t}(a_1,a_2,e),$$

the lump-sum transfer expenditure is

$$(13)\quad Tr_t = tr_t \sum_{i=1}^{I}\int_{A_1\times A_2\times E} \mathrm{d}X_{i,t}(a_1,a_2,e),$$

the social security payroll tax revenue is

$$(14)\quad T_{P,t} = \tau_{P,t} w_t L_t,$$

and the social security benefit expenditure is

$$\begin{aligned}(15)\quad B_t &= \sum_{i=I_R}^{I}\int_{A_1\times A_2\times E} b_i(a_2;\Omega_t)\mathrm{d}X_{i,t}(a_1,a_2,e)\\ &= \sum_{i=I_R}^{I}\int_{A_1\times A_2\times E} \varphi_{0,t}\tilde{b}_i(a_2;\Omega_t)\mathrm{d}X_{i,t}(a_1,a_2,e) \equiv \varphi_{0,t}\tilde{B}_t,\end{aligned}$$

where $\tilde{B}_t$ is the actuarially fair benefit expenditure. From the law of motion of a_2 in the household's problem, aggregate social security wealth follows

$$(16)\quad W_{2,t+1} = \frac{1}{(1+\mu)(1+\nu)}[(1+r_t)W_{2,t} + T_{P,t} - \tilde{B}_t].$$

The government inter-temporal budget constraint is, accordingly,

$$(17)\quad W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)}[(1+r_t)W_{G,t} + T_{I,t} - Tr_t - C_{G,t} + (1-\varphi_{0,t})\tilde{B}_t],$$

where $(1-\varphi_{0,t})\tilde{B}_t$ is the difference between actuarially fair benefits and actual benefits. The transversality condition is

$$(18)\quad \lim_{t\to\infty}\left(\prod_{s=1}^{t}\frac{(1+\mu)(1+\nu)}{1+r_s}\right)W_{G,t+1} = 0.$$

DEFINITION Recursive Competitive Equilibrium: Let (i, a_1, a_2, e) be the individual state of the households. A time series of factor prices and government policy variables,

$$\Omega_t = \{r_s, w_s, \tau_{P,s}, \varphi_s, \psi_s, tr_s, C_{G,s}, W_{G,s}\}_{s=t}^{\infty},$$

the value functions of households, $\{v_i(a_1, a_2, e; \Omega_s)\}_{s=t}^{\infty}$, the decision rules of households,

$$\{\mathbf{d}_i(a_1, a_2, e; \Omega_s)\}_{s=t}^{\infty} \equiv \{c_i(a_1, a_2, e; \Omega_s), l_i(a_1, a_2, e; \Omega_s), a'_{1,i}(a_1, a_2, e; \Omega_s)\}_{s=t}^{\infty},$$

and the distribution of households, $\{x_{i,s}(a_1, a_2, e)\}_{s=t}^{\infty}$, are in a recursive competitive equilibrium if, for all $s = t, ..., \infty$, each household solves the optimization problem (1)-(4), taking Ω_s as given; the firm solves its profit maximization problem (5)-(6), taking Ω_s as given; the government policy schedule satisfies its inter-temporal budget constraint, (12)-(18); and the goods market and factor markets clear, *i.e.*, (7)-(11) hold. The economy is in a steady-state equilibrium (on the balanced growth path) if in addition $\Omega_{s+1} = \Omega_s$ and

$$x_{i,s+1}(a_1, a_2, e) = x_{i,s}(a_1, a_2, e)$$

for all $s = t, ..., \infty$.

Table 1: Main Parameters and Factor Prices

α	γ	θ	δ	μ	υ	ρ	σ_ε	β	r	w	K/Y
0.36	2.0	0.30	0.048	0.018	0.010	0.95	0.20	0.9694	0.0520	1.0	3.0

3 Calibration

We first construct a baseline economy, which is on the balanced growth path, without a social security pension system as a steady-state equilibrium. Table 1 summarizes the main parameter values and target values of the main calibration explained below. In Section 4.3, we will change some of these parameter values to examine the robustness of our numerical results.

Household's Preferences. The period utility function is one of Cobb-Douglas and constant relative risk aversion,

$$u(c,l) = \frac{(c^{\alpha} l^{1-\alpha})^{1-\gamma}}{1-\gamma} = \frac{[c^{\alpha}(1-h)^{1-\alpha}]^{1-\gamma}}{1-\gamma},$$

which is compatible with the growth economy. The share parameter of consumption, α, is 0.36, following Cooley and Prescott (1995). The coefficient of relative risk aversion, γ, is 2.0, which is between the number in the real business cycle literature and the one in Auerbach and Kotlikoff (1987). The labor-augmenting productivity growth rate, μ, is set at 0.018 or 1.8%. The subjective time discount factor, β, is chosen to be 0.9694 so that the capital-output ratio, K/Y, of the baseline economy is equal to 3.0. The growth-adjusted time discount factor is calculated as $\tilde{\beta} = \beta(1+\mu)^{\alpha(1-\gamma)}$.

Household's Demographics. We assume households enter the economy at the beginning of actual age 21 ($i = 1$) and possibly live up to the age of 100 ($I = 80$). The population growth rate, ν, is set at 0.01 or 1.0%. Table 2 shows the end-of-period survival rates, ϕ_i, for actual ages between 21 and 100. The numbers are from the 2003 male period life table in Social Security Administration (2008). The survival rate at the end of age 100 is replaced with 0. The total population of the model economy is 41.9308 when the population of age-21 households is normalized to unity.

Worker's Age-Wage Profiles. We assume households stop working at actual age 65 ($I_R = 45$) and start receiving social security pension benefits. The individual working ability, e_i, at age i before retirement is

Table 2: End-of-Period Survival Rates

Age	Survival rate	Age	Survival rate	Age	Survival rate	Age	Survival rate
21	0.998611	41	0.997236	61	0.986593	81	0.921207
22	0.998551	42	0.996991	62	0.985383	82	0.913187
23	0.998546	43	0.996721	63	0.984117	83	0.904158
24	0.998579	44	0.996429	64	0.982764	84	0.894091
25	0.998626	45	0.996110	65	0.981249	85	0.882983
26	0.998664	46	0.995770	66	0.979553	86	0.870830
27	0.998687	47	0.995422	67	0.977713	87	0.857617
28	0.998684	48	0.995070	68	0.975722	88	0.843320
29	0.998659	49	0.994707	69	0.973546	89	0.827908
30	0.998625	50	0.994304	70	0.971096	90	0.811356
31	0.998586	51	0.993862	71	0.968359	91	0.793646
32	0.998532	52	0.993406	72	0.965378	92	0.774775
33	0.998461	53	0.992937	73	0.962145	93	0.754751
34	0.998373	54	0.992439	74	0.958603	94	0.733598
35	0.998268	55	0.991892	75	0.954557	95	0.712552
36	0.998147	56	0.991269	76	0.950027	96	0.691927
37	0.998006	57	0.990551	77	0.945195	97	0.672057
38	0.997843	58	0.989723	78	0.940088	98	0.653282
39	0.997660	59	0.988788	79	0.934543	99	0.635946
40	0.997458	60	0.987737	80	0.928313	100	0.000000

Source: Table 4.C6 in Social Security Administration (2008).

defined as

$$\ln e_i = \ln \bar{e}_i + \ln z_i,$$

where $\bar{e}_i$ is the average working ability of age i households, and the persistent shock, z_i, follows an AR(1) process,

$$\ln z_i = \rho \ln z_{i-1} + \varepsilon_i,$$

where $\varepsilon_i \sim N(\mu_\varepsilon, \sigma_\varepsilon^2)$. The unconditional expected value of z_i is normalized to unity and $z_0 = 1$. The auto-correlation parameter, ρ, is assumed to be 0.95 in the main calibration, and the standard deviation of shock, σ_ε, is 0.2. The log deviation from the mean, $\ln z_i$, is also normally distributed and

$$\mu_{\ln z_i} = -\frac{1}{2}\sigma_{\ln z_i}^2,$$
$$\sigma_{\ln z_i}^2 = \sum_{j=1}^{i} \rho^{2(i-1)}\sigma_\varepsilon^2 = \frac{1-\rho^{2i}}{1-\rho^2}\sigma_\varepsilon^2.$$

We construct the average working ability, $\bar{e}_i$, from the 2005 median earnings of male workers by age group in Social Security Administration (2008). Because the median earnings are not shown for all ages between 21 and 64, we smooth out the raw data by taking the 5-year moving average and an additional 3-year moving average. We discretize the log deviation, $\ln z_i$, into five levels by using the Gauss-Hermite quadrature. The Hermite weights of five nodes are $\pi = (0.011257, 0.222076, 0.533333, 0.222076, 0.011257)^{\top}$. Table 3 shows the age-working ability profile of the main calibration.[6] We also calculate the Markov transition matrix, by using the bivariate normal distribution function with correlation $\rho = 0.95$, as

$$\Gamma = \begin{pmatrix} 0.674670 & 0.325330 & 0.000000 & 0.000000 & 0.000000 \\ 0.016492 & 0.809283 & 0.174225 & 0.000000 & 0.000000 \\ 0.000000 & 0.072546 & 0.854908 & 0.072546 & 0.000000 \\ 0.000000 & 0.000000 & 0.174225 & 0.809283 & 0.016491 \\ 0.000000 & 0.000000 & 0.000000 & 0.325328 & 0.674662 \end{pmatrix}.$$

Firm's Production Technology. The production function is also one of Cobb-Douglas with constant-returns-to-scale technology,

$$F(K_t, L_t) = AK_t^{\theta} L_t^{1-\theta}.$$

The share parameter of capital, θ, is 0.30, and the depreciation rate of capital stock, δ, is 0.048. The total factor productivity is calculated as $A = (K/Y)^{-\theta}(1-\theta)^{\theta-1}$ in the baseline economy, so that the average wage rate w is normalized to unity. When $w = 1.0$, the interest rate, r, is equal to 0.0520 or 5.20%.

Government Policies. The progressive income tax function is the one specified in Gouveia and Strauss (1994),

$$\tau_I(y; \psi_t) = \psi_{0,t}[y - (y^{-\psi_{1,t}} + \psi_{2,t})^{-1/\psi_{1,t}}],$$

where y is taxable income, $r_t a_1 + w_t e(1-l)$, with a unit adjustment. The parameters $\psi_{1,t}$ and $\psi_{2,t}$ of the function are 0.839 and 0.029, respectively, which are the simple averages of their estimated parameters in the years between 1979 and 89. The parameter $\psi_{0,t}$ outside the bracket is the limit of the effective marginal income tax rate as taxable income goes to infinity, and it is set at 0.30 in the baseline economy. The unit

[6] See Judd (1998) for the Gauss-Hermite quadrature. The Fortran program used to calculate the individual working ability profile and the Markov transition matrix will be provided upon request.

Table 3: Individual Working Abilities by Age

Age $i+20$	$\bar{e}_i$	e_i^1	e_i^2	e_i^3	e_i^4	e_i^5
21	0.3186	0.1764	0.2381	0.3123	0.4096	0.5530
22	0.3825	0.1674	0.2533	0.3682	0.5352	0.8098
23	0.4523	0.1670	0.2740	0.4284	0.6698	1.0987
24	0.5281	0.1705	0.2978	0.4929	0.8157	1.4249
25	0.6039	0.1745	0.3209	0.5562	0.9640	1.7727
26	0.6797	0.1789	0.3433	0.6185	1.1144	2.1388
27	0.7471	0.1815	0.3613	0.6726	1.2521	2.4922
28	0.8062	0.1828	0.3753	0.7187	1.3763	2.8263
29	0.8570	0.1829	0.3859	0.7573	1.4861	3.1355
30	0.9077	0.1838	0.3970	0.7958	1.5952	3.4461
31	0.9585	0.1852	0.4085	0.8343	1.7040	3.7581
32	1.0012	0.1857	0.4170	0.8659	1.7978	4.0379
33	1.0361	0.1853	0.4228	0.8908	1.8768	4.2836
34	1.0629	0.1840	0.4260	0.9091	1.9400	4.4917
35	1.0897	0.1833	0.4297	0.9276	2.0023	4.6950
36	1.1166	0.1830	0.4340	0.9464	2.0638	4.8942
37	1.1388	0.1824	0.4369	0.9615	2.1159	5.0683
38	1.1565	0.1815	0.4386	0.9730	2.1587	5.2174
39	1.1696	0.1802	0.4390	0.9809	2.1921	5.3408
40	1.1827	0.1792	0.4397	0.9891	2.2247	5.4594
41	1.1958	0.1785	0.4409	0.9975	2.2566	5.5735
42	1.2077	0.1779	0.4420	1.0051	2.2857	5.6778
43	1.2184	0.1774	0.4429	1.0119	2.3119	5.7727
44	1.2278	0.1768	0.4436	1.0177	2.3351	5.8577
45	1.2373	0.1765	0.4446	1.0239	2.3580	5.9400
46	1.2467	0.1763	0.4458	1.0301	2.3803	6.0186
47	1.2544	0.1760	0.4465	1.0350	2.3990	6.0863
48	1.2603	0.1756	0.4468	1.0385	2.4138	6.1426
49	1.2644	0.1751	0.4467	1.0407	2.4249	6.1876
50	1.2685	0.1746	0.4467	1.0431	2.4357	6.2304
51	1.2727	0.1743	0.4469	1.0455	2.4463	6.2715
52	1.2708	0.1732	0.4450	1.0431	2.4450	6.2807
53	1.2630	0.1715	0.4412	1.0359	2.4321	6.2588
54	1.2493	0.1690	0.4355	1.0240	2.4076	6.2057
55	1.2356	0.1666	0.4299	1.0122	2.3829	6.1509
56	1.2218	0.1642	0.4244	1.0003	2.3578	6.0940
57	1.2035	0.1613	0.4174	0.9848	2.3238	6.0133
58	1.1807	0.1578	0.4089	0.9657	2.2809	5.9086
59	1.1383	0.1518	0.3937	0.9307	2.2000	5.7046
60	1.0809	0.1439	0.3734	0.8834	2.0900	5.4238
61	1.0086	0.1340	0.3481	0.8241	1.9509	5.0669
62	0.9206	0.1221	0.3174	0.7519	1.7813	4.6296
63	0.8169	0.1082	0.2814	0.6671	1.5811	4.1120
64	0.7124	0.0942	0.2452	0.5816	1.3792	3.5890

Source: Author's calculation from Table 4.B6 in Social Security Administration (2008). The population weighted average of working abilities is normalized to 1.0.

of income y is adjusted to \$1,000 by multiplying 150. In the baseline economy of our main calibration, the average labor income of working-age households is 0.3680 or \$55,209 with this adjustment, which is roughly equal to the 2007 estimate of median income of households under age 65, which is \$56,545, according to the U.S. Census Bureau.

We assume the government net wealth, W_G, to be 0 for simplicity in the baseline economy. We also assume a uniform lump-sum transfer, tr, to be 0.01, for computational convenience, so that households can barely survive without income and wealth, but they never choose that state by themselves. With equation (17), we calculate government consumption expenditure endogenously as

$$C_{G,t} = (1 + r_t)W_{G,t} + T_t - Tr_t - (1 + \mu)(1 + \nu)W_{G,t+1} = T_t - Tr_t$$

in the baseline economy, and we keep it and the lump-sum transfer expenditure at the same levels in the policy experiments shown below.

4 Long-Run Effects of Social Security Pensions

In the present paper, we define the terms of the social security pension system as follows:

- The social security pension system is *fully funded* if social security wealth does not affect the level of government net wealth, *i.e.*, $W_{G,t} = 0$ when $W_{2,t} > 0$; the system is *unfunded* if the increase in social security wealth is equal to the increase in government debt each period, *i.e.*, $W_{2,t} + W_{G,t} = 0$; and the system is *partially funded* if $-W_{2,t} < W_{G,t} < 0$;

- The social security benefit schedule is *flat* if all households in the same age cohort receive the same benefit, *i.e.*, $\varphi_{1,t} = 0$; and the schedule is *proportional* if individual benefits are proportional to their social security wealth, *i.e.*, $\varphi_{1,t} = 1$;

- The social security benefit schedule is *on average actuarially fair* if the aggregate benefits are actuarially fair for each age cohort, *i.e.*, $\varphi_{0,t} = 1$; and the schedule is *actuarially fair* if $\varphi_{0,t} = \varphi_{1,t} = 1$;

- The social security pension system is *pay-as-you-go* if payroll tax revenue is equal to social security benefit expenditure, *i.e.*, $T_{P,t} = B_t$; and the system is *fully privatized* if it is fully funded, and the benefit schedule is actuarially fair, *i.e.*, $W_{G,t} = 0$ and $\varphi_{0,t} = \varphi_{1,t} = 1$.

We calculate the long-run effects of introducing social security pension systems under balanced budget assumptions. Note that net government wealth, W_G, is normalized to zero in the baseline economy. If the government budget is balanced without including social security wealth, then we call the system a fully funded social security system. If the government budget is balanced by including social security wealth, we call the system an unfunded social security system.[7] The definition of a fully funded system is stricter in the present paper than the usual definition—the system is fully funded if the government has no unfunded liability—because the liability is smaller than social security wealth when the promised benefits are less than actuarially fair.

In our definition, a flat benefit schedule provides a uniform benefit within an age cohort but not across age cohorts in a growth economy. If the social security system is pay-as-you-go, parameter $\varphi_{0,t}$ is determined endogenously to satisfy $T_{P,t} = \varphi_{0,t}\tilde{B}_t$. A pay-as-you-go social security system is not necessarily an unfunded system. If the social security system is pay-as-you-go and fully funded, for example, the government receives an investment return on social security wealth. If the social security system is unfunded and pay-as-you-go, the government budget constraint, equation (17), is simplified to $T_{I,t} = C_{G,t} + Tr_t$.

The baseline economy is in a steady-state equilibrium without social security pensions. In this section we analyze the long-run effects of introducing fully-funded social security pension systems with a 10% flat payroll tax. We examine the following four polar cases:

- **Run (a)** an, on average, actuarially fair system with a flat benefit schedule ($\varphi_0 = 1$ and $\varphi_1 = 0$);
- **Run (b)** an, on average, actuarially fair system with a proportional benefit schedule ($\varphi_0 = \varphi_1 = 1$);
- **Run (c)** a pay-as-you-go system with a flat benefit schedule ($\varphi_0 = T_P/\tilde{B}$ and $\varphi_1 = 0$);
- **Run (d)** a pay-as-you-go system with a proportional benefit schedule ($\varphi_0 = T_P/\tilde{B}$ and $\varphi_1 = 1$).

We consider only fully funded cases in the long-run analyses, because the long-run welfare implication of a less-than-fully-funded social security system is obvious. We will analyze the effects of introducing partially funded social security systems by solving the model for equilibrium transition paths later.

In all four experiments, we fix government spending, C_G and tr, and government net wealth, W_G, at baseline levels, and adjust marginal individual income tax rates proportionally to satisfy the government budget constraint. Since we assumed $W_G = 0$ in the baseline economy, the budget constraint, equation

[7] If the government accumulates social security wealth in "trust funds," but the government increases its debt in the rest of the government budget at the same time, then the social security system is said to be "unfunded" in our definition. Given a benefit schedule, the allocation between social security wealth and rest of the government's wealth is irrelevant to the economy. We assume that a social security (reform) policy is always fully funded and that an unfunded social security (reform) policy is a combination of a social security policy and an additional debt-financed income redistribution policy.

(17), becomes

$$T_I = Tr + C_G - (1-\varphi_0)\tilde{B},$$

and we change one of the parameters of the income tax function, ψ_0, to balance the government budget in the long run. We measure the long-run welfare change due to the introduction of a social security system by the percent change of the expected total lifetime resource of "newborn" households, *i.e.*,

$$\left[\left(\frac{Ev_1(a_1, a_2, e; \Omega_\infty)}{Ev_1(a_1, a_2, e; \Omega_0)}\right)^{1/(1-\gamma)} - 1\right] \times 100,$$

where $v_1(.;\Omega_0)$ is the average value of age-21 households in the baseline economy, and $v_1(.;\Omega_\infty)$ is the average value of age-21 households in the alternative economy with social security. Since $a_1 = a_2 = 0$ for age-21 households,

$$\begin{aligned} Ev_1(a_1, a_2, e; \Omega_0) &= \int_E v_1(0,0,e;\Omega_0)\mathrm{d}X_{1,0}(0,0,e) = \sum_j v_1(0,0,e_j;\Omega_0)\pi(e_j), \\ Ev_1(a_1, a_2, e; \Omega_\infty) &= \int_E v_1(0,0,e;\Omega_\infty)\mathrm{d}X_{1,\infty}(0,0,e) = \sum_j v_1(0,0,e_j;\Omega_\infty)\pi(e_j). \end{aligned}$$

4.1 On Average Actuarially Fair Systems

We first assume that the new social security pension systems are on average actuarially fair, *i.e.*, there are no inter-cohort income transfers, by setting $\varphi_0 = 1$. When $\varphi_1 = 0$, the social security benefits are uniform within each age cohort but not necessarily across cohorts. When $\varphi_1 = 1$, the social security benefits are proportional to social security wealth holdings, and there is no intra-cohort income redistribution at all. In the latter, the social security pension system is similar to a mandatory version of Roth individual retirement accounts (IRAs), in which contributions are income taxable but investment returns and distributions are not taxable.

Table 4 shows the long-run effects of introducing social security pensions. The first column, Run 1 (a), assumes $\varphi_0 = 1$ and $\varphi_1 = 0$. Since the product $\varphi_0\varphi_1$ is zero, the social security benefits are completely independent of the payroll tax payments each household has made. Households consider the payroll tax as a pure labor income tax.

If a social security pension system of this kind were introduced, the number of working hours would decrease by 4.7%, and labor supply in efficiency units would decrease by 7.1% from the baseline economy without social security pensions. Because social security wealth is fully funded by assumption, national

Table 4: The Long-Run Effects of Social Security Pensions

	Run 1 (a)	Run 1 (b)	Run 1 (c)	Run 1 (d)
φ_0	1.000	1.000	0.811	0.815
φ_1	0.000	1.000	0.000	1.000
% changes from the baseline				
National wealth	16.3	24.8	24.4	32.2
Labor supply	-7.1	-0.5	-4.6	1.1
Total output (GDP)	-0.6	6.5	3.3	9.6
Private consumption	-6.7	1.3	-3.6	3.5
Working hours	-4.7	1.0	-2.9	2.3
Interest rate	-27.9	-28.3	-32.7	-32.9
Wage rate	7.0	7.0	8.3	8.4
Income tax rate (limit)	17.9	7.2	-0.9	-8.9
Welfare of age-21 households	-1.26	-0.75	-0.22	0.11
Changes as a % of baseline GDP				
Income tax revenue	0.0	0.0	-1.7	-1.7
Payroll tax revenue	7.0	7.5	7.2	7.7
Benefit expenditure	9.3	9.9	7.2	7.7
Actuarially fair benefit expenditure	9.3	9.9	8.9	9.4
% shares in total wealth				
Regular wealth	29.0	29.7	34.0	34.6
Social security wealth	71.0	70.3	66.0	65.4

wealth would increase by 16.3%. Total output (GDP) would decrease by 0.6%. The interest rate would fall by 27.9% from 5.20% to 3.75%, and the average wage rate would rise by 7.0%. The share of regular wealth in total private wealth would fall to 29.0%, and the share of social security wealth would be 70.4%. Because labor income and taxable capital income would decrease, to balance the government budget, the government would have to raise marginal income tax rates by 17.9% from $\psi_0 = 0.30$ to $\psi_0 = 0.3537$. The disposable income of households would decrease, the saving rate (including social security wealth) would increase, and private consumption would decrease by 6.7%. With the reductions in consumption and working hours, the expected lifetime value of age-21 households would fall by 1.26%.

The second column, Run 1 (b), assumes $\varphi_1 = 1$ instead. As explained above, under this assumption, the social security pension system is similar to a mandatory version of Roth IRAs and, since the product $\varphi_0\varphi_1$ is one, households consider payroll tax payments as pure pension contributions. If a social security system of this kind were introduced, working hours would increase by 1.0%, though the labor supply in efficiency units would decrease by 0.5%. National wealth would increase by 24.8% from the baseline economy. Total output would also increase by 6.5%. Because regular taxable wealth would be replaced by social security wealth, in which capital income was not taxed by assumption, the government would have to raise marginal income tax rates by 7.2% from $\psi_0 = 0.30$ to $\psi_0 = 0.3216$. Although private consumption would increase by 1.3%, the welfare level of age-21 households would decline by 0.75%.

We also solve the model for several other steady-state equilibria with φ_1 between 0 and 1. The average welfare level of age-21 households is concave but strictly increasing in φ_1. Thus, $\varphi_1 = 1$ is optimal when $\varphi_0 = 1$ in our main calibration of the model.

Figure 2 shows selected individual variables before and after the introduction of social security pensions. The numbers are the population-weighted average of each age. If social security pension systems with $\varphi_0 = 1$ were introduced, because the interest rate would fall significantly, the downward slopes of consumption after retirement would be steeper compared to those in the baseline economy. Working hours would decrease for all working ages below 65. Individual income tax payments would shift from middle-age and elderly households to young households. A large share of regular taxable wealth would shift to nontaxable social security wealth. Social security benefits would be flat, by construction, in time series for each retired household, but those would be downward sloping in the cross section, since older households had paid less than younger households in a growth economy.

4.2 Pay-As-You-Go Systems

When the social security pension systems are pay-as-you-go, one of the parameters of the social security benefit function is determined endogenously so that the total benefit expenditure is equal to the payroll tax revenue. We calculate the parameter, φ_0, as the ratio of the payroll tax revenue, T_P, to the actuarially fair benefit expenditure, $\tilde{B}$. To satisfy the aggregate consistency of the economy, the difference between actuarially fair benefits and the actual benefits retired households receive, $\tilde{B} - B = (1 - \varphi_0)\tilde{B}$, is included in the government revenue as a "tax" on benefits. Thus, other things being equal, the government can reduce individual income tax rates, keeping its expenditure at the same level.

The third column, Run 1 (c), of Table 4 assumes $\varphi_1 = 0$. If a social security system of this kind were introduced, in the equilibrium, parameter φ_0 would be equal to 0.811 to satisfy the pay-as-you-go condition. The government could lower individual income tax revenue by 11.0%, or 1.7% as a percentage of GDP, from the baseline economy. However, the marginal income tax rates would fall only by 0.9% from $\psi_0 = 0.30$ to $\psi_0 = 0.2972$, because labor income and taxable capital income would decrease significantly. The number of working hours would decrease by 2.9%, and labor supply in efficiency units would decrease by 4.6%. National wealth would increase by 24.4%, which was larger than the increase in Run 1 (a), because the reduced social security benefits would make households accumulate more regular taxable wealth for their retirement. The share of regular wealth in total private wealth would be 34.0%. Total output would increase by 3.3% from the baseline economy. The interest rate would fall by 32.7%, and the average wage rate would rise by 8.3%. Payroll tax revenue and social security benefit expenditure would be balanced at 7.2% as a percentage of baseline GDP. Private consumption would decrease by 3.6%, and the welfare level of age-21 households would decline by 0.22%.

The pay-as-you-go assumption would make social security pensions less than actuarially fair and would generate labor supply distortion through the payroll tax. At the same time, the "tax" on benefits would allow the government to lower individual income tax rates, which would reduce the distortions on labor supply and capital accumulation. Overall, we find that the average welfare loss would be smaller if we introduced pay-as-you-go pensions rather than those that are, on average, actuarially fair.

The fourth column, Run 1 (d), assumes $\varphi_1 = 1$. In the equilibrium, parameter φ_0 would be equal to 0.815 to balance the social security budget. Because the product of parameters, $\varphi_0\varphi_1$, would relatively be high, the negative impact of the payroll tax on the labor supply would be smaller. The number of working hours would increase by 2.3%, and the labor supply in efficiency units would increase by 1.1%. The government could reduce marginal income tax rates proportionally by 8.9% from $\psi_0 = 0.30$ to $\psi_0 = 0.2732$. Higher labor income with the fixed payroll tax rate and lower income tax rates would generate larger capital accumulation. National wealth would increase by 32.2% from the baseline economy. Total output would increase by 9.6%. Private consumption would increase by 3.5%, and the average welfare level of age-21 households would improve slightly by 0.11%.

We again solve the model for several steady-state equilibria by changing φ_1 between 0 and 1 and find that $\varphi_1 = 1$ is optimal when we assume the pay-as-you-go social security system. Figure 3 shows selected individual variables before and after the introduction of pay-as-you-go social security pensions. The increase in private consumption from the baseline economy would be clearer when $\varphi_1 = 1$, and individual income tax payments would decrease for all ages.

4.3 Systems with Alternative Parameter Assumptions

We have so far found that the average welfare level of age-21 households would be the highest if the social security pension system was pay-as-you-go and benefits were proportional to social security wealth holdings, *i.e.*, $\varphi_0 < 1$ and $\varphi_1 = 1$. In this section, we check the robustness of our finding by changing the parameter values of the model. In all experiments below, we first recreate a baseline economy with the same target values, *e.g.*, the capital-output ratio is equal at 3.0 and the wage rate is normalized to 1.0, so that we can make a fair comparison of the models.

Table 5: The Long-Run Effects with Alternative Parameters

	Run (a)	Run (b)	Run (c)	Run (d)
φ_0	1.000	1.000		
φ_1	0.000	1.000	0.000	1.000
Run 1. $\gamma = 2.0, \psi_0 = 0.30, \rho = 0.95$				
φ_0			0.811	0.815
Total output (GDP)	-0.6	6.5	3.3	9.6
Welfare of age-21 households	-1.26	-0.75	-0.22	0.11
Run 2. $\gamma = 4.0$				
φ_0			0.842	0.835
Total output (GDP)	0.7	7.2	4.8	10.5
Welfare of age-21 households	-1.33	-0.89	-0.22	-0.04
Run 3. $\psi_0 = 0.25$				
φ_0			0.818	0.818
Total output (GDP)	0.9	6.8	4.1	9.6
Welfare of age-21 households	-0.98	-0.72	-0.09	0.09
Run 4. $\rho = 0.98$				
φ_0			0.838	0.826
Total output (GDP)	1.8	7.0	4.7	9.7
Welfare of age-21 households	-0.74	-0.72	0.05	0.06
Run 5. $\psi_0 = 0.25, \rho = 0.98$				
φ_0			0.847	0.829
Total output (GDP)	3.1	7.3	5.6	9.7
Welfare of age-21 households	-0.57	-0.68	0.14	0.04

The first panel (Run 1) of Table 5 shows the results with our main calibration of the model, explained in the previous two sections. The second panel (Run 2) shows the effects when the coefficient of relative risk aversion, γ, is increased to 4.0 from 2.0. If households were more risk averse, we could expect more positive welfare effects from social security systems with flat benefits rather than proportional benefits. Total output would be larger in all four runs, Runs 2 (a)-(d), due to the precautionary labor supply and savings. However, the welfare effects would be worse in all four runs compared to those in the main calibration. The third panel (Run 3) assumes the limit of marginal income tax rates, ψ_0, to be 0.25 instead of 0.30. If the after-tax income distribution is more unequal, we could expect better welfare effects from social security systems with a flat benefit schedule. Indeed, the welfare losses in Runs 3 (a) and 3 (c) would be smaller. Yet, the pay-as-you-go pension system with proportional benefits, Run 3 (d), would still generate the best result.

The fourth panel (Run 4) assumes a more persistent working ability process. Parameter ρ is increased to 0.98 from 0.95, keeping σ_ε at the same level as our main calibration. The income disparity of households near retirement would increase by about 50%. With a larger inequality in lifetime income, the welfare effects of introducing social security pension systems would improve when we assume flat benefits. The welfare losses and gains would be about the same levels between $\varphi_1 = 0$ and $\varphi_1 = 1$. Finally, the fifth panel (Run 5) shows the combined effects of Run 3 and Run 4. When the after-tax income inequality is significantly large in the model economy, the qualitative implication for the optimal social security pension system would be reversed. Social security pensions with the flat benefit schedule ($\varphi_1 = 0$) would generate higher welfare effects than pensions with the proportional benefit schedule ($\varphi_1 = 1$).

5 Transition Effects of Social Security Pensions

In our main calibration of the model, social security systems with the proportional benefit schedule, $\varphi_1 = 1$, are better than those with the flat benefit schedule, $\varphi_1 = 0$, in the long run. Also, pay-as-you-go social security systems, $\varphi_0 < 1$, work better than on average actuarially fair social security systems, $\varphi_0 = 1$. We also observed a modest long-run welfare gain with a pay-as-you-go social security system with proportional benefits. Is that long-run welfare gain large enough that the government can make all age cohorts (including the current households) on average better off? In this section, to evaluate the overall effects of introducing social security pension systems, we solve the same model for the following two equilibrium transition paths:

- introducing an on average actuarially fair social security system with the proportional benefit schedule ($\varphi_{0,t} = \varphi_{1,t} = 1$) at the beginning of period 1;
- introducing a social security system with modest inter-cohort income transfers and the proportional benefit schedule ($\varphi_{0,t} = 0.8$ and $\varphi_{1,t} = 1$) at the beginning of period 1.

In the second transition path, we assume the parameter $\varphi_{0,t}$ to be 0.8, which is roughly equal to the value calculated in the economy with the pay-as-you-go social security system in the long-run analysis. We fix the parameter, $\varphi_{0,t}$, at the same level throughout the transition path instead of calculating it endogenously by $\varphi_{0,t} = T_{P,t}/\tilde{B}_t$ each period. It is because the total actuarially fair benefit, $\tilde{B}_t$, is very small at the beginning, and that makes $\varphi_{0,t}$ unrealistically high for the first several decades of the transition path. For example, $\varphi_{0,t} > 20$ for the first 10 years, and $\varphi_{0,t} > 2.0$ for the first 27 years.

We also assume that the government inter-temporal budget constraint in the transition path is satisfied by both a one-time change in marginal income tax rates at the introduction of the social security system and period-by-period changes in government net wealth. The government chooses the rate of one-time proportional change in marginal income tax rates so that the economy with the social security system will return to the balanced growth path with non-zero government wealth in the long run. In the model economy, we choose a time-invariant value of ψ_0 and, thus, $\{T_{I,t}(\psi_0)\}_{t=1}^{\infty}$ such that

$$\sum_{t=1}^{\infty}\left(\prod_{s=1}^{t}\frac{(1+\mu)(1+\nu)}{1+r_s}\right)[T_{I,t}(\psi_0)-C_{G,t}-Tr_t+(1-\varphi_{0,t})\tilde{B}_t]=0,$$

and growth-adjusted net government wealth, $W_{G,t}$, follows

$$W_{G,t+1}=\frac{1}{(1+\mu)(1+\nu)}[(1+r_t)W_{G,t}+T_{I,t}(\psi_0)-Tr_t-C_{G,t}+(1-\varphi_{0,t})\tilde{B}_t],$$

with $W_{G,1}=0$ and $\lim_{t\to\infty}W_{G,t}=W_G$ is finite.

5.1 On Average Actuarially Fair System with Proportional Benefits

Figure 4 shows selected variables in an equilibrium transition path when a social security pension system with $\varphi_{0,t}=\varphi_{1,t}=1.0$ was introduced at the beginning of period 1. In the long-run steady-state analysis, the government would have to raise marginal income tax rates proportionally by 7.2% or 2.2 percentage points. In the transition analysis, the government would have to raise the tax rates by 6.1% or 1.8 percentage points at the beginning of period 1 to make the government budget sustainable. The government net wealth would increase by 7.5% in the long run as a percentage of baseline GDP.

Capital stock would increase by 26.8%, labor supply would decrease by 0.3%, and total output (GDP) would increase by 7.2% in the long run from the baseline balanced growth path. The interest rate would fall by 29.8% from 5.20% to 3.65%, and the average wage rate would rise by 7.5% in the long run. Income tax revenue would increase by 1.3% in the first year and decrease by 0.1% in the long run. Payroll tax revenue would increase immediately, and social security benefit expenditure would increase gradually for the first 60 years. In the long run, payroll tax revenue and benefit expenditure would be 7.5% and 9.2% as percentages of baseline GDP. The difference would be the interest income from social security wealth. National income would increase by 3.9% in the long run. Household disposable income would decline by 12.0% in the first year by the introduction of a 10% payroll tax, but it would increase by 7.3% in the long run. Accordingly, private consumption would also decline at the beginning by up to 2.8% and then it would increase by 1.6% in the long run.

5.2 Less than Actuarially Fair System with Proportional Benefits

Figure 5 shows an equilibrium transition path when a social security pension system with $\varphi_{0,t} = 0.8$ and $\varphi_{1,t} = 1.0$ was introduced. In the long-run analysis, when the pay-as-you-go social security pension system with proportional benefits was introduced, the parameter $\varphi_{0,t}$ would be 0.815, and the government could reduce marginal income tax rates proportionally by 8.9% or 2.7 percentage points. In the transition analysis, when $\varphi_{0,t}$ was assumed to be 0.80, the government could lower the marginal tax rates by 5.0% or 1.5 percentage points and would make the government budget sustainable. The government net wealth would decrease by 45.2% as a percentage of baseline GDP. In this experiment, the social security system is partially funded, because part of the social security wealth would be accompanied by government debt, and there would be some income transfers to the current households through the individual income tax cut.

Capital stock would increase by 21.0%, labor supply would increase slightly by 0.2%, and total output would increase by 6.1% in the long run from the baseline balanced growth path. The interest rate would rise by 1.1% at the time of the policy change, but it would fall in the long run by 23.8% from 5.20% to 3.96%. The average wage rate would rise by 5.8% in the long run. Individual income tax revenue would decrease by 1.6% in the long run. Payroll tax revenue would increase immediately in period 1, and social security benefit expenditure would increase gradually. In the long run, payroll tax revenue and benefit expenditure would increase by 7.4% and 8.4%, respectively, as percentages of baseline GDP. In the long run, social security benefit expenditure would be larger than payroll tax revenue, even though we assumed $\varphi_{0,t}$ to be 0.8, which is less than 0.815, because the interest rate would be higher in the transition analysis and the wage rate would be lower. National income would increase by 3.5% in the long run. However, household disposable income would decrease by 8.9% at the beginning, and it would increase gradually by 10.2% in the long run. Private consumption would also decrease in the short run but increase in the long run by 2.0%.

5.3 Welfare Effects of Social Security Pension Systems

We calculate percent changes in the welfare of households already in the economy at the time of the policy change by

$$\left[\left(\frac{\int_{A_1\times A_2\times E} v_i(a_1,a_2,e;\Omega_1)\mathrm{d}X_{i,1}(a_1,a_2,e)}{\int_{A_1\times A_2\times E} v_i(a_1,a_2,e;\Omega_0)\mathrm{d}X_{i,0}(a_1,a_2,e)}\right)^{1/(1-\gamma)} - 1\right]\times 100$$

for $i = 2, ..., 80$, where the distribution of households at the beginning of $t = 1$ is equal to the distribution in the baseline economy ($t = 0$). We also calculate percent changed in the welfare of age-21 households that enter the economy in period 1 and later by

$$\left[\left(\frac{\int_E v_1(0,0,e;\Omega_t)\mathrm{d}X_{1,t}(0,0,e)}{\int_E v_1(0,0,e;\Omega_0)\mathrm{d}X_{1,0}(0,0,e)}\right)^{1/(1-\gamma)} - 1\right] \times 100$$

for $t = 1, ..., \infty$.

Figure 6 shows the welfare changes by age cohort when social security pension systems with $(\varphi_{0,t}, \varphi_{1,t}) = (1.0, 1.0)$ and $(\varphi_{0,t}, \varphi_{1,t}) = (0.8, 1.0)$ were introduced at the beginning of period 1. The horizontal axis shows the period when households enter the economy at age 21, and the vertical axis shows the percent changes in welfare measured in terms of the remaining lifetime resources.

In both of the two transition paths, except for a few households that are very old at the time of the policy change, all households would be worse off when social security systems were introduced. Current households aged 65 or older would be mostly worse off because the interest rate would fall, except for the first period, gradually throughout the transition paths. When the system was on average actuarially fair, *i.e.*, $\varphi_{0,t} = 1.0$, only age-100 households would be better off, because the interest rate would rise in period 1. When the system is less than actuarially fair, $\varphi_{0,t} = 0.8$, current households aged 93 or older would be better off, because benefits from the income tax cut would exceed losses from the lowered interest rate. As the remaining lifetime got longer, the welfare losses of current retired households would become larger.

The welfare losses of current working-age households would become smaller for ages between 55 and 40, because the marginal welfare gains from the wage rate increase and the income tax cut would exceed the marginal losses from the falling interest rate. The welfare loss of current working households aged below 40 would increase again as the households get younger, partly because more households would be binding their liquidity constraints. In both cases, $\varphi_{0,t} = 1.0$ and $\varphi_{0,t} = 0.8$, age-21 households in period 1 would receive the largest welfare losses. Those households would be worse off by 1.80% and 1.49% when $\varphi_{0,t} = 1.0$ and $\varphi_{0,t} = 0.8$, respectively. As the average wage rate would go up in the transition paths, the welfare losses of age-21 "newborn" households would decrease gradually, and the losses would be 0.61% and 0.56% in the long run.

Overall, almost all households (both current generations and future generations) would be worse off when the government introduces the social security pension systems examined in the present paper. However, the welfare losses would be uniformly smaller if the social security pension system were less than actuarially fair, *e.g.*, $\varphi_{0,t} = 0.8$, because that would allow the government to lower the marginal income tax rates in exchange for a higher effective payroll tax rate. The intuitions behind this finding are as follows: First, the tax base of the effective payroll tax is a household's lifetime earnings and capital income on those rather than labor and capital income in each period. Taxing based on lifetime income would somehow reduce labor supply and saving distortions when part of the income tax burden was transferred to the effective payroll tax through a less than actuarially fair social security pension system. Second, the welfare losses from the social security system due to the liquidity constraint for young working-age households would be reduced if part of the tax burden was shifted from earlier working ages to retirement ages.

6 Concluding Remarks

In the present paper, we provide a model to analyze a wide range of social security pension systems that include a flat benefit pension system and a fully privatized pension system. According to the policy experiments, the benefit schedule without intra-cohort income redistribution would likely be optimal, but on average less than actuarially fair social security pensions would work better if the lowered benefits were accompanied by lowered individual income tax rates. This is probably because taxing lifetime income is less distortive than taxing labor income each year. However, to make this conclusion, we need to take this fiscal policy effect further. The model economy described in the present paper is highly simplified, and the social security systems are very much stylized. Also, due to space limits, the number of policies examined is small. We can extend the model for future research in the following aspects:

Introducing Risky Assets. When capital income is stochastic, the uncertainty and inequality of lifetime income of households will be larger, and the optimal progressivity of social security pension systems will be different. If the social security system is fully or partially funded and the asset returns are risky, however, we have to specify who will manage the social security wealth. If the benefit schedule is proportional, *i.e.*, if there is no intra-cohort income redistribution, we can allow households to choose their desirable portfolios of their own social security wealth. If the benefit schedule is progressive, for example, if $\varphi_1 = 0.5$, then we can consider the system as a two-tier social security system with flat benefits and proportional benefits, which was numerically analyzed by Huggett and Ventura (1999). In this case, to avoid moral hazard, we should allow households to manage only 50% of their social security wealth, and the rest of the social security wealth should be managed by the government with safe assets.

Prefunding vs. Underfunding. Whether the social security benefits are actuarially fair or pay-as-you-go, or proportional or flat has nothing to do with whether the social security pension system should be prefunded or not. The funding issue is a separate question in the rest of the government budget. If debt-financed government spending was desirable, for example, if the current elderly households unfairly suffered without government assistance, or if the economy was dynamically inefficient with a new social security system, then the social security pension system should be partially funded or unfunded, *i.e.*, it should be accompanied by an increase in government debt. Otherwise, a social security system with a onetime change in marginal income tax rates would likely be an optimal policy. We can apply this discussion to social security pension reform plans from the current-law social security system.

Liquidity Constraints and Moral Hazard. In the present paper, the artificial borrowing limit is set at zero. This borrowing constraint would affect the welfare effects of a new social security system. If unsecured borrowing were allowed, the welfare effects would probably be better for current young households and future households. Alternatively, if the payroll tax rate were age dependent and assumed to be increasing in age, then the welfare losses caused by the liquidity constraint would be reduced. If we assume a means-tested government assistance program for elderly households and an incentive problem caused by that, then a mandatory saving mechanism such as a social security pension system would likely improve the efficiency of the economy and would probably reverse the qualitative welfare implication, depending on the size of government assistance.

Appendix

The computational algorithms of solving the model for a steady-state equilibrium and an equilibrium transition path are similar to those described in Conesa and Krueger (1998). In the main calibration, we discretize the continuous asset spaces, $A_1 \times A_2$, into 60×50 nodes. We increase the number of nodes in alternative baseline economies as necessary. As explained in Section 3, we discretize the working ability space, E, into 5 nodes for each working age by using the Gauss-Hermite quadrature. To calculate an equilibrium transition path, we assume that the economy is in the initial steady-state equilibrium in period 0, a new social security pension system is introduced at the beginning of period 1, and that the economy reaches its final steady-state equilibrium within $T = 150$ periods.

In the following, we explain the specific algorithms to solve the household problem at each node, given the aggregate state of the economy, and to find a one-time marginal income tax rate adjustment that is sustainable in the transition analyses. See Miranda and Fackler (2002) for a further explanation on a complementarity problem, and see Judd (1998) for the logarithmic transformation and the Gauss-Hermite quadrature.

A Solving the Household Optimality Problem

The first-order conditions of the household problem, equations (1)-(4), are

$$u_c(c,l) \geq \frac{\tilde{\beta}}{1+\mu} E[v_{i+1,1}(a_1', a_2', e'; \Omega_{t+1})|e],$$

holding with equality if $a_1' > 0$, and

$$\begin{aligned} u_l(c,l) \geq & u_c(c,l)[1-\tau_{P,t}-\tau_I'(r_t a_1 + w_t e(1-l);\psi_t)]w_t e \\ & + \frac{\tilde{\beta}}{1+\mu} E[v_{i+1,2}(a_1', a_2', e'; \Omega_{t+1})|e]\tau_{P,t} w_t e, \end{aligned}$$

holding with equality if $l < 1$, where the state transition equations and simple constraints are

$$\begin{aligned} a_1' &= \frac{1}{(1+\mu)\phi_i}[(1+r_t)a_1 + (1-\tau_{P,t})w_t e(1-l) - \tau_I(r_t a + w_t e(1-l);\psi_t) + b_i(a_2;\Omega_t) + tr_t - c], \\ a_2' &= \frac{1}{(1+\mu)\phi_i}[(1+r_t)a_2 + \tau_{P,t} w_t e(1-l) - \tilde{b}_i(a_2;\Omega_t)], \\ & c \in (0,\infty), \quad l \in (0,1], \quad a_1' \in [0,\infty). \end{aligned}$$

Functions $v_{i+1,1}(.)$ and $v_{i+1,2}(.)$ are the marginal values with respect to the first and the second arguments, respectively, and these are obtained recursively as

$$\begin{aligned} v_{i,1}(a_1,a_2,e;\Omega_t) =& u_{c,i}(a_1,a_2,e;\Omega_t)\{1+[1-\tau_I'(r_t a_1 + w_t e(1-l_i(a_1,a_2,e;\Omega_t));\psi_t)]r_t\}, \\ v_{i,2}(a_1,a_2,e;\Omega_t) =& u_{c,i}(a_1,a_2,e;\Omega_t)b_i'(a_2;\Omega_t) \\ & + \frac{\tilde{\beta}}{1+\mu} E[v_{i+1,2}(a_1', a_2', e'; \Omega_{t+1})|e](1+r_t-\tilde{b}_i'(a_2;\Omega_t)), \end{aligned}$$

where $u_{c,i}(a_1,a_2,e;\Omega_t) \equiv u_c(c_i(a_1,a_2,e;\Omega_t), l_i(a_1,a_2,e;\Omega_t))$, and

$$v_{I+1,1}(a_1', a_2', e'; \Omega_{t+1}) = v_{I+1,2}(a_1', a_2', e'; \Omega_{t+1}) = 0.$$

For $i = I, ..., 1$, we solve the above household problem for $c_i(a_1,a_2,e;\Omega_t)$ and $l_i(a_1,a_2,e;\Omega_t)$ by using bilinear interpolation and calculating $v_{i,1}(a_1,a_2,e;\Omega_t)$ and $v_{i,2}(a_1,a_2,e;\Omega_t)$ recursively. To solve the household problem for c and l at each node, (a_1,a_2,e), given Ω_t, we construct the following complementarity problem:

$$g(c,l)=\min\left[\begin{pmatrix} f^1(c,l) \\ f^2(c,l) \end{pmatrix}, \begin{pmatrix} a_1' \\ 1-l \end{pmatrix}\right]=0,$$

where

$$\begin{aligned} f^1(c,l)=&u_c(c,l)-\frac{\tilde{\beta}}{1+\mu}E[v_{i+1,1}(a_1',a_2',e';\Omega_{t+1})|e], \\ f^2(c,l)=&u_l(c,l)-u_c(c,l)[1-\tau_{P,t}-\tau_I'(r_ta_1+w_te(1-l);\psi_t)]w_te \\ &-\frac{\tilde{\beta}}{1+\mu}E[v_{i+1,2}(a_1',a_2',e';\Omega_{t+1})|e]\tau_{P,t}w_te. \end{aligned}$$

Since c and l are both strictly positive when the utility function is Cobb-Douglas, we use a non-linear equation solver NEQNF (the Powell hybrid algorithm) in Fortran IMSL to solve $g(c,l)=0$ with the log transformation, *i.e.*, we solve $\mathsf{g}(\mathsf{c},\mathsf{l})\equiv g(e^{\mathsf{c}},e^{\mathsf{l}})=0$, where $\mathsf{c}=\ln c\in R$ and $\mathsf{l}=\ln l\in R$.

B Finding a Sustainable One-Time Income Tax Adjustment

In this paper, we calculate a one-time proportional change in the marginal income tax rates at the beginning of period 1 as follows:

1. Set the initial time series of aggregate variables and government policy variables,

   $$\Omega_1^0=\{K_t^0/L_t^0,\bar{a}_{2,t}^0,\psi_{0,t}^0,W_{G,t}^0\}_{t=1}^T,$$

   where $\psi_{0,t}^0=\psi_0^0$ for all t, $\bar{a}_{2,t}^0=\{\bar{a}_{2,i,t}^0\}_{i=1}^I$, and $W_{G,1}^0=W_{G,0}$.

2. Compute the final steady-state equilibrium in period T, assuming that $W_{G,T}$ is endogenous, *i.e.*,

   $$W_{G,T}^0=\frac{C_{G,T}+Tr_T-T_{I,T}(\psi_0^0)-(1-\varphi_{0,T})\tilde{B}_T}{(1+r_T)-(1+\mu)(1+\nu)}.$$

3. For $t=T-1,T-2,...,1$, compute backward the household decision rules, $\{\mathbf{d}_i(a_1,a_2,e;\Omega_t^0)\}_{i=1}^I$, and the marginal value functions, $\{v_{1,i}(a_1,a_2,e;\Omega_t^0)\}_{i=1}^I$ and $\{v_{2,i}(a_1,a_2,e;\Omega_t^0)\}_{i=1}^I$.

4. For $t=1,2,...,T-1$, compute forward the aggregate variables, $\{K_t^1/L_t^1,\bar{a}_{2,t}^1,W_{G,t+1}^1\}$, and the distributions of households, $\{x_{i,t+1}(a_1,a_2,e)\}_{i=1}^I$, using the decision rules, $\{\mathbf{d}_i(a_1,a_2,e;\Omega_t^0)\}_{i=1}^I$.

5. If $W_{G,T}^1$ and $W_{G,T}^0$ are close enough, let $\psi_0^1 = \psi_0^0$ and go to Step 6; otherwise, update ψ_0^0 by

$$\psi_0^0 \longleftarrow \eta \frac{\sum_{t=1}^{\infty}\left(\prod_{s=1}^{t}\frac{(1+\mu)(1+\nu)}{1+r_s}\right)[C_{G,t} + Tr_t - (1-\varphi_{0,t})\tilde{B}_t] - (1+\mu)(1+\nu)W_{G,1}^0}{\sum_{t=1}^{\infty}\left(\prod_{s=1}^{t}\frac{(1+\mu)(1+\nu)}{1+r_s}\right)T_{I,t}(\psi_0^0)}\psi_0^0 \\ + (1-\eta)\psi_0^0,$$

where $\eta \in (0, 1]$, and return to Step 4.

6. If $\Omega_1^1 = \{K_t^1/L_t^1, \bar{a}_{2,t}^1, \psi_{0,t}^1, W_{G,t}^1\}_{t=1}^T$ and Ω_1^0 are close enough, stop; otherwise, update Ω_1^0 by

$$\Omega_1^1 \longleftarrow \kappa\Omega_1^1 + (1-\kappa)\Omega_1^0,$$

where $\kappa \in (0, 1]$, and return to Step 2.

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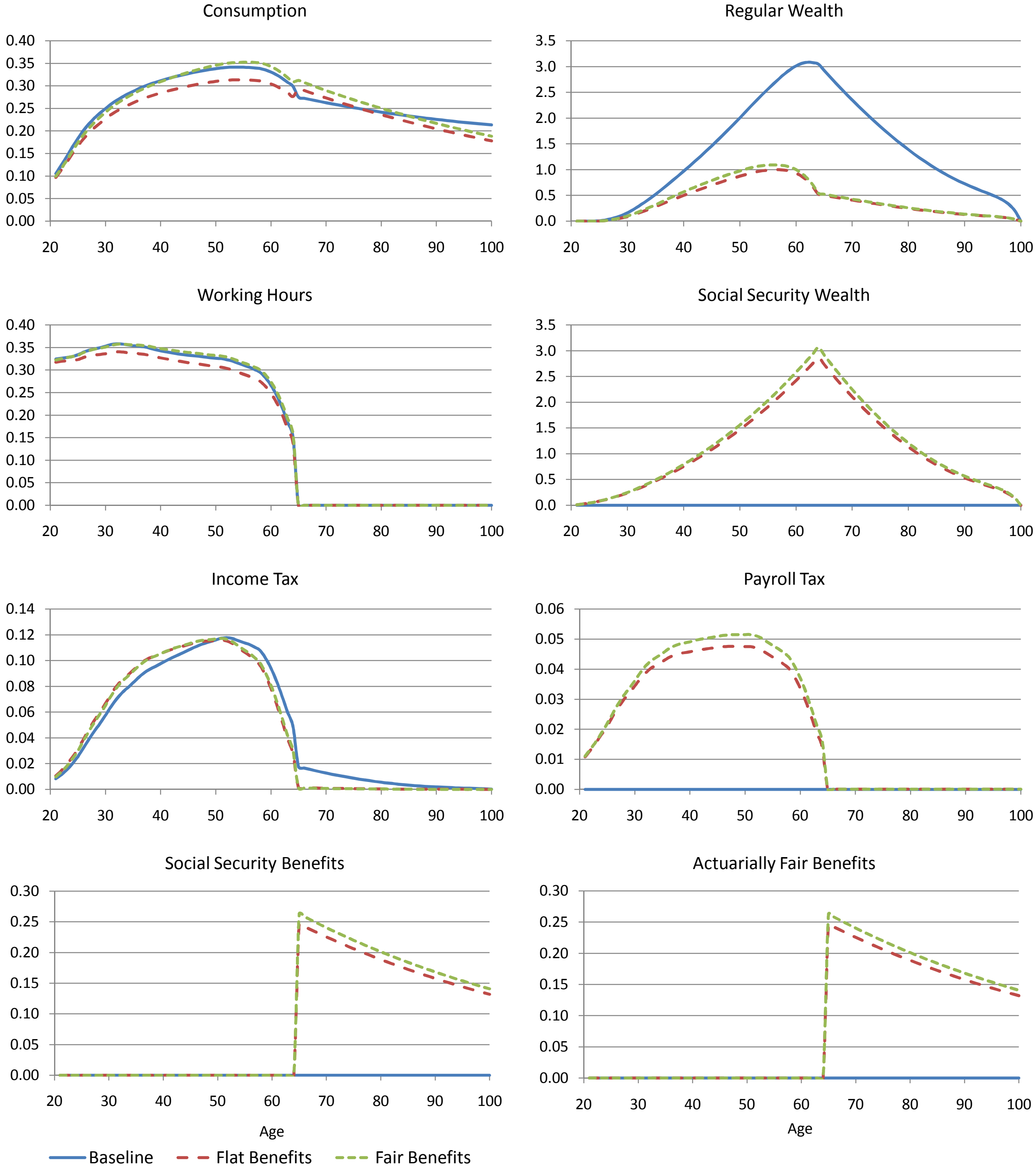

Figure 2: The long-run effects of social security pensions with $\varphi_0 = 1.0$ by age

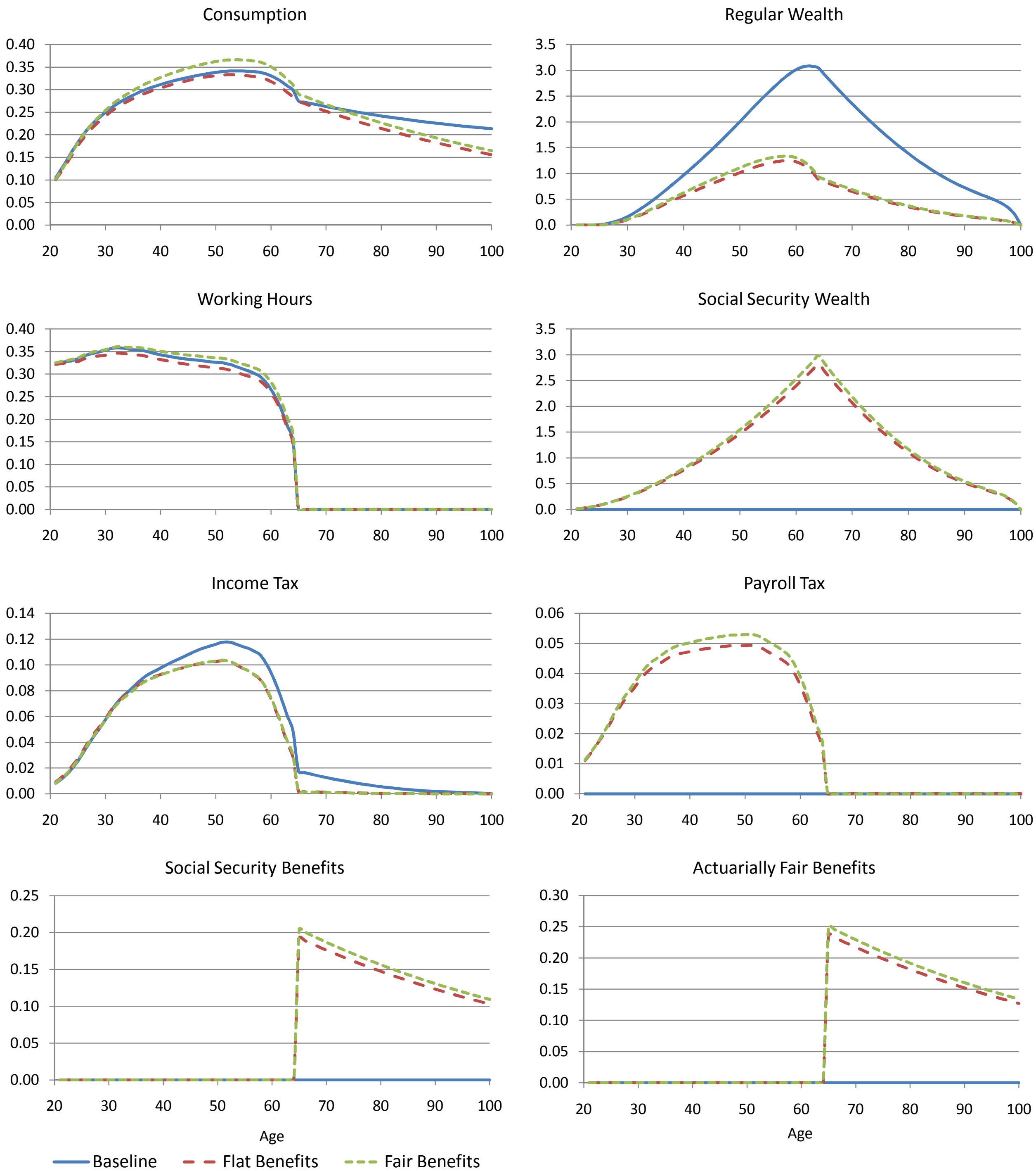

Figure 3: The long-run effects of social security pensions with $\varphi_0 < 1.0$ by age

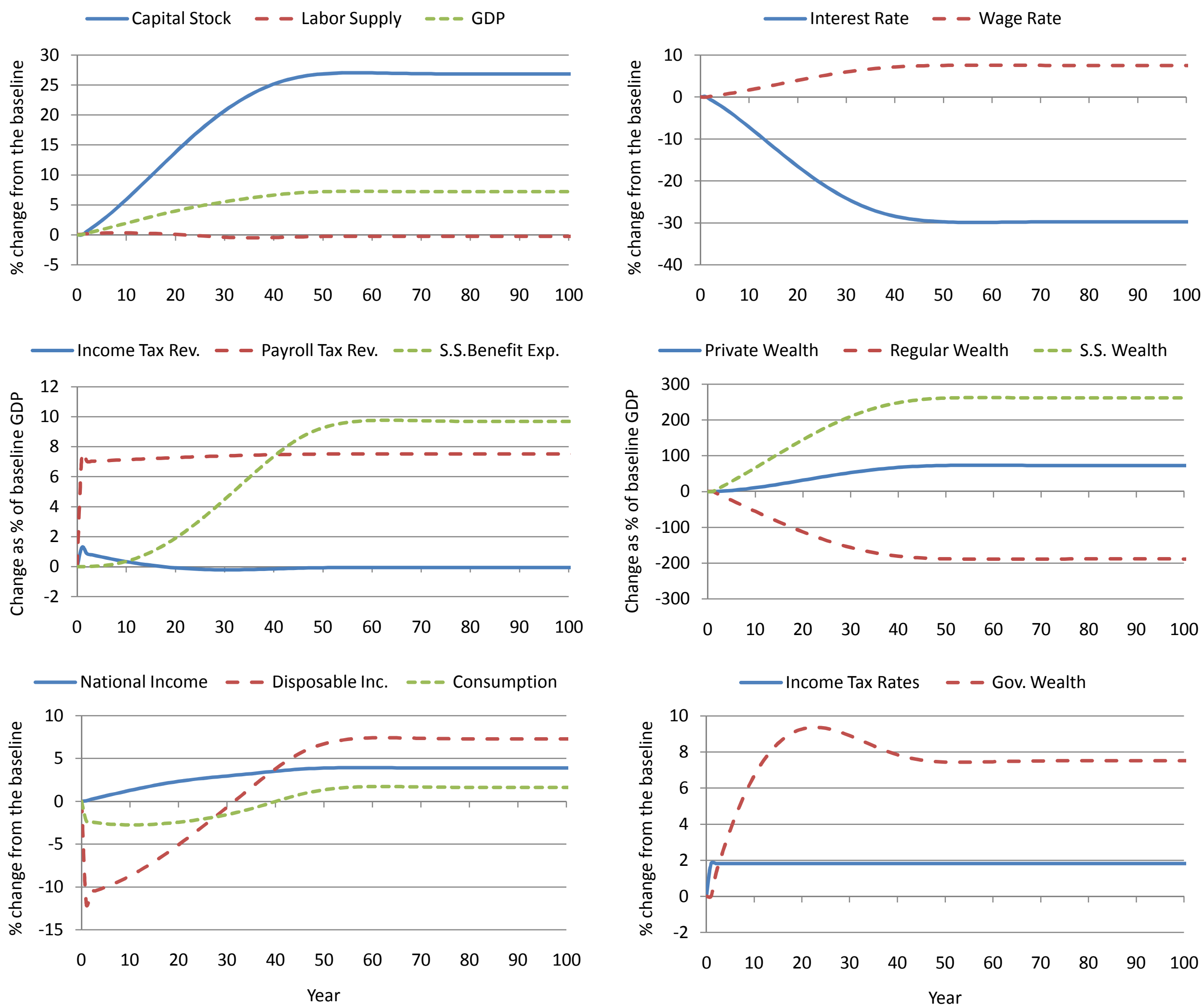

Figure 4: The equilibrium transition path when a social security pension system with $\varphi_0 = \varphi_1 = 1.0$ was introduced at $t = 1$

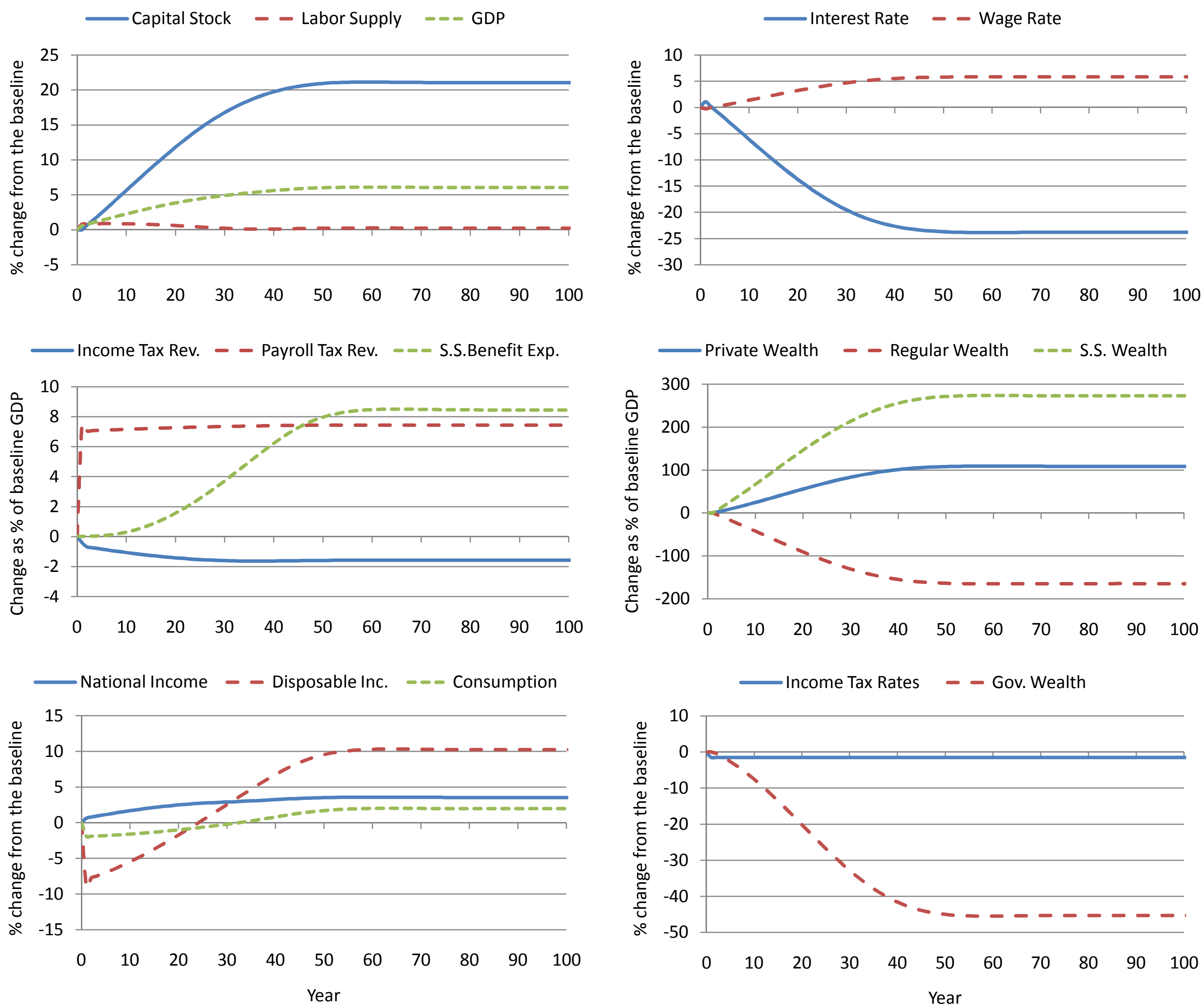

Figure 5: The equilibrium transition path when a social security pension system with $\varphi_0 = 0.8$ and $\varphi_1 = 1.0$ was introduced at $t = 1$

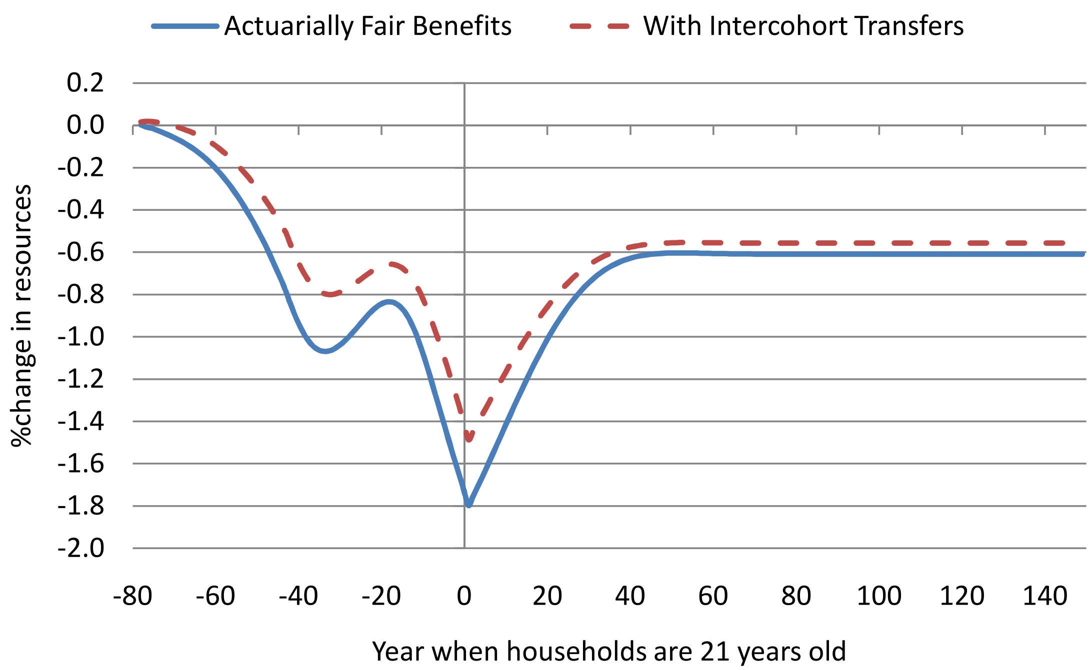

Figure 6: The welfare effects by age cohort when social security pension systems were introduced at $t = 1$