Infrequent Changes of Policy Target: Robust Optimal Monetary Policy under Ambiguity[*]

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Abstract

In many countries, the monetary policy instrument sometimes remains unchanged for a long period and shows infrequent responses to exogenous shocks. The purpose of this paper is to provide a new explanation on why the central bank's policy instrument remains unchanged. In the following analysis, we explore how uncertainty on the private agents' expectations affects robust optimal monetary policy. We apply the Choquet expected decision theory to a new Keynesian model. A main result is that the policymaker may frequently keep the interest rate unchanged even when exogenous shocks change output gaps and inflation rates. This happens because a change of the interest rate increases uncertainty for the policymaker when how the private agents' expectations are formed is not well known. To the extent that the policymaker has uncertainty aversion, it can therefore be optimal for the policymaker to maintain an unchanged policy stance for some significant periods and to make discontinuous changes of the target rate. Our analysis departs from previous studies in that we determine an optimal monetary policy rule that allows time-variant feedback parameters in a Taylor rule. We show that if the policymaker has small uncertainty aversion, the calibrated optimal stop-go policy rule can predict actual target rates of FRB and ECB reasonably well.

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1. Introduction

In monetary economics, it has widely been discussed what policy rules central banks follow. A growing number of studies advocate a variety of monetary policy rules that can lead to good performance. In particular, many argue that macroeconomic stabilization should be implemented through a "Taylor rule" in which interest rates are adjusted in response to output gap and inflation rate. However, when we look at high-frequency data, the policy instrument sometimes remains unchanged for a long period and shows infrequent responses to frequent exogenous shocks. Figure 1 plots daily data of targeted federal fund (FF) rates from January 2001 to December 2007. It is easy to see that the changes of the targeted FF rates were rare from January 2002 to June 2004. Since the Federal Reserve's Trading Desk keeps the FF rate near a target set by the Federal Open Market Committee (FOMC), this implies that the baseline of the U.S. short-term interest rate changed infrequently.[1]

One of the reasons why the changes of the targeted FF rates were infrequent is that the FOMC meeting is usually held only eight times a year. It is the FOMC that decides some discontinuous jumps of the targeted rates. However, except in 2001 and 2005, the FOMC decided not to change the target rate in most of the meetings (see Table 1). Infrequent FOMC meetings would not be enough to explain less frequent changes of the targeted rates. More infrequent policy changes can be observed for the other central banks that face different environments. For example, Table 2 summarizes the number of monetary policy decisions and the number of decisions with no policy change in the Bank of Japan, the European Central Bank, and the Bank of England from 1999 to 2007. It is easy to see that these central banks changed the targeted policy instruments less frequently than the Federal Reserve Board throughout the period.

Why do the central banks decide not to change the policy targets frequently? The purpose of this paper is to provide a new explanation on why the central bank's policy instrument remains unchanged under uncertainty. In general, the policymaker faces various types of uncertainty when making the policy decision. This includes uncertainty on exogenous shocks and on structural parameters. However, uncertainty on the private agents' expectations is another uncertainty that the central bank usually faces. Since the expectations affect output gap and inflation rate, it is important to identify how the private agents' expectations are formed. However, as recent contribution of behavioral economics suggests, it is far from easy to predict what expectations the private agents will form.

[1] The realized federal fund rates that are called "effective federal fund rates" show some daily fluctuations over time. However, they only show small fluctuations around the targeted rates.

In the following analysis, we explore how uncertainty on the private agents' expectations affects optimal monetary policy in a new Keynesian model. The decision-making theory we use in the analysis is that of expected utility under a nonadditive probability measure, that is, the Choquet expected model, developed by Gilboa (1987) and Schmeidler (1989).[2] We apply the Choquet expected decision theory to a new Keynesian model. A main result is that the policymaker may frequently keep the interest rate unchanged even when exogenous shocks change output gaps and inflation rates. This happens because a change of the interest rate increases uncertainty for the policymaker when the private agents' expectations are not well known. To the extent that the policymaker has uncertainty aversion, it can therefore be optimal for the policymaker to maintain an unchanged policy stance for some significant periods and to make discontinuous changes of the target rate.

In previous literature, there are a large number of studies that focused on model uncertainty and the performance of policy rules across different models. Brainard (1967) is a seminal study that explored how the policymaker's optimal rule is altered when faced with parameter uncertainty. McCallum (1988) has argued for evaluating policy proposals in a variety of economic models as a means of assessing their robustness. Using five macroeconomic models, Levin, Wieland, and Williams (2003) identify the robust rules that respond to the inflation forecast and the output gap but that incorporate a substantial degree of policy inertia. Using a new Keynesian model, Giannoni and Woodford (2003a, 2003b) have analyzed policy rules that are robust to misspecification of the disturbance process of a known model, while Kimura and Kurozumi (2003) and Levin and Williams (2003b) have focused on whether parameter uncertainty leads to more cautious or more aggressive policy responses to shocks when the effects of structural parameters on the loss function are taken into account. However, since none of these studies considered Knightian uncertainty, the model uncertainty has never lead to the conclusion that it is optimal for the policymaker to keep the interest rate unchanged without responding to inflation and output gaps.

Several recent studies explored "robust optimal policy rules" under a version of Knightian uncertainty, which are designed to be robust in the sense of minimizing the worst case scenario when the policymaker believes that the true model is in a neighborhood of a given reference model. These studies include Hansen and Sargent (2003), Onatski and Stock (2002), Tetlow and von zur Muehlen (2001), and Giannoni (2006). Walsh (2004) has argued that optimal monetary policy under Hansen-Sargent

[2] Based on the Gilboa-Schmeidler's axioms, studies such as Epstein and Wang (1994), Mukerji and Tallon (2004), and Fukuda (2008) incorporate Knightian uncertainty in economic models.

framework is equivalent to that of Giannoni and Woodford where the optimal policy rule becomes less aggressive under uncertainty. In contrast, Onatski and Stock argued that the max-min approach of robust control provides robust monetary policies that are more aggressive than the optimal policies absent model uncertainty.[3] However, unlike ours, none of these studies has reached a conclusion that the optimal policy is to keep the policy instrument unchanged for some periods.

Our analysis departs from these previous studies in four important ways. First, instead of restricting ourselves to time-invariant feedback parameters, we determine a robust optimal monetary policy rule that allows time-variant feedback parameters. This leads to an optimal stop-go policy rule that sometimes responds to output and inflation gaps but sometimes does not. Second, we show that the calibrated optimal stop-go policy rules can predict actual target rates of FRB and ECB well if the policymaker has small uncertainty aversion. In literature, Hamilton and Jorà (2002) showed that statistical tool for forecasting a discrete-valued time series is useful in forecasting the federal fund rates. Our optimal stop-go policy rule not only supports their proposition but also provides theoretical background for the discrete-valued time series. Third, we consider the case where the private agents' expectations are uncertain for the policymaker before making the policy decision. Previous studies widely discussed what happens when exogenous shocks or/and structural parameters are uncertain. But few studies discussed when the private agents' expectations are uncertain. Our result suggests uncertainty on the private agents' expectations is another important uncertainty in a robust control framework. Fourth, we derive robust policy rules based on the Choquet expected utility model rather than on the max-min utility model. In literature, it is known that the two models are essentially the same. But in macroeconomic policy analysis, previous studies used the max-min utility model almost exclusively. Our analysis suggests that the Choquet expected utility model is an alternative useful framework to derive robust policy rules.

Our result is similar to that of Dow and Werlang (1992) in that a player chooses the status quo under Knightian uncertainty. Dow and Werlang provide a simple example where the optimal portfolio choice can be the status quo under Knightian uncertainty. However, given that the policy changes are rare, it deserves to pay a special attention to see why the central banks prefer the status quo under Knightian uncertainty. Central bankers have multiple objectives and confront a variety of economic circumstances. They know that their actions have significant impacts on the economy, but the timing, magnitude, and channels of those impacts are not fully understood. They, in contrast,

[3] Giannoni (2002, 2006) supports this under more general environments.

have a concern that their reputation would deteriorate dramatically if their actions have wrong impacts on the economy. Under the circumstances, it may become desirable for the central banks not to change the policy targets when the parameter uncertainty makes the impacts uncertain enough.

In macroeconomics, it has been a conventional wisdom that central banks implement monetary policy in a gradual fashion (see, for example, Blinder [1997]). Many researchers claim that this gradualism is due to 'optimal cautiousness', although some others suggest alternative interpretations (see, for example, Rudebusch [2005]). Interest-rate smoothing or monetary policy inertia is, however, different from monetary policy with infrequent changes and some discontinuous jumps. When using low frequency data, the two types of monetary policies may be observationally equivalent. But their macroeconomic implications will be different at least in the short-run and may be so even in the long-run. It is practically very important to pay a special attention to macroeconomic consequences of the stop-go policy that changes the policy instrument infrequently.

The paper proceeds as follows. Section 2 sets up the basic model and section 3 explains the policy objectives. Sections 4 derives the optimal monetary rules in a general framework and section 5 extends them in the case where the nature takes two states. Section 6 shows that the calibrated optimal stop-go policy rules can predict actual federal fund rates well. and section 7 checks their robustness. Section 8 shows that the calibrated optimal stop-go policy rules can predict actual rates of MRO set by ECB well. Section 9 summarizes our main results and refers to their implications.

2. The Basic Model

Our basic model follows a simple new Keynesian model:

(1) $$x_t = x_{t+1}^e - \alpha (i_t - \pi_{t+1}^e) + u_t,$$

(2) $$\pi_t = \beta \pi_{t+1}^e + k x_t + w_t,$$

where x_t = the gap between actual output and the flexible-price equilibrium output level, i_t = the nominal interest rate, π_t = the inflation rate, u_t = a demand disturbance, and w_t = a supply shock. The variable with superscript *e*, such as x_{t+1}^e and π_{t+1}^e, denotes the private agents' expectations.

Equation (1) is the Euler condition from the representative household's consumption decision, while equation (2) is a new Keynesian Phillips curve. Subscript *t* denotes time period. All variables are expressed as log deviations from the steady state. Although there is some arbitrariness in the information structure, the model is standard. We can show that essential results in the following analysis remains the same even when equations (1) and (2) include a variety of lagged variables, x_{t-1}, x_{t-2}, …, π_{t-1}, π_{t-2}, …, in their right-hand sides. We, however, impose an additional assumption that there is uncertainty on how the private agents will change their expectations when the interest rate changes.

In our model, we assume information structure in period *t* as follows. At the beginning of period *t*, innovations to u_t and w_t are realized. The policymaker observes the realized innovations without noises. However, the policymaker cannot see how the private agents will change x_{t+1}^e and π_{t+1}^e when it changes the nominal interest rate. Define $x_{t+1}^{e0} \equiv x_{t+1}^e$ when $i_t = i_{t-1}$ and $\pi_{t+1}^{e0} \equiv \pi_{t+1}^e$ when $i_t = i_{t-1}$. For analytical simplicity, we assume that the policymaker can observe x_{t+1}^{e0} and π_{t+1}^{e0} without uncertainty. Uncertainty thus arises for x_{t+1}^e and π_{t+1}^e if and only if $i_t \neq i_{t-1}$. We suppose that the private agents update their expectations as follows.

(3) $x_{t+1}^e - x_{t+1}^{e0} = -\phi_t \Delta i_t,$

(4) $\pi_{t+1}^e - \pi_{t+1}^{e0} = -\phi_t \delta \Delta i_t.$

where $\Delta i_t \equiv i_t - i_{t-1}$.

Equations (3) and (4) state that how x_{t+1}^e and π_{t+1}^e will change depends on how i_t changes and that there exists uncertainty on the elasticity. A straightforward justification is that the agents who follow a rule of thumb respond to the policy change sometimes aggressively but sometimes less aggressively in forming the expectation. However, the assumption might be consistent with behavior of rational agents who sometimes overestimate and sometimes underestimate parameters in a Taylor rule. The changeable behavior by the private agents, which is reflected in ϕ_t, is the source of uncertainty in the following model.

Under (3) and (4), the policymaker needs to decide the nominal interest rate i_t knowing that the private agents' behavior is highly volatile. Because of uncertainty on x_{t+1}^e and π_{t+1}^e, the policymaker will face uncertainty on what value will be realized for x_t and π_t when the nominal interest rate is changed in period *t*. To distinct the states before and after the policy change, we define x^0_t and π^0_t as the realized values of x_t and π_t when the nominal interest rate remains unchanged. By definition, it holds that

(5) $x^0_t = x_{t+1}^{e0} - \alpha\,(i_{t-1} - \pi_{t+1}^{e0}) + u_t,$

(6) $\pi^0_t = \beta\, \pi_{t+1}^{e0} + k\, x^0_t + w_t$.

Define $\eta \equiv 1+\alpha\delta$ and $\mu \equiv k + (\alpha k + \beta)\delta$. Equations (1)-(4) then lead to:

(7) $x_t = x^0_t - (\eta\phi_t + \alpha)\, \Delta i_t$,

(8) $\pi_t = \pi^0_t - (\mu\phi_t + \alpha k)\, \Delta i_t$.

where x_t and π_t denote the realized values after the nominal interest rate is determined.

Equations (7) and (8) determine the equilibrium values of x_t and π_t in our model. Because of uncertainty on ϕ_t, the policymaker cannot see exact values of x_t and π_t in period t unless $\Delta i_t = 0$. Our simplified assumption, in contrast, suggests that the policymaker can see x^0_t and π^0_t without uncertainty in period t. However, the simplified assumption of information on x^0_t and π^0_t is not crucial in the following analysis. What is crucial in the following analysis is that the change of the nominal interest rate induces additional model uncertainty for the policymaker. In Appendix 1, we will show that our main proposition (Proposition 1) can be extended even if x^0_t and π^0_t are uncertain for the policymaker.

3. The Policy Objectives

The policymaker chooses its policy instrument so as to achieve the policy objective. We suppose that the objective of the policymaker is to set the nominal interest rate at each point of time so as to minimize the "expected" value of the following loss function:

(9) $L_t = \lambda(x_t - x^*)^2 + (\pi_t - \pi^*)^2 + \omega(i_t - i_{t-1})^2$.

In the loss function, loss in period t depends on deviations of output gap and inflation from their targets x^* and π^* as well as interest rate changes in period t. Exogenous parameter λ and ω are greater than or equal to zero and are treated as independent of the specification of the structural equations.

What makes the following analysis distinctive from the standard minimization problem is that we characterize the expected loss minimization of the policymaker by the Choquet expectation. To distinguish it from standard expectation operator E, we defined the Choquet expectation operator by E^Q. Having aversion to Knightian uncertainty, the policymaker chooses its policy instrument Δi_t so as to minimize $E^Q\, L_t$.

More general representation of the Choquet expectation is extensively discussed in Schmeidler (1989).

Suppose that the random variable ϕ_t takes *n* alternative values, $\phi_1, \phi_2, ..., \phi_n$. Define $L_{j,t} \equiv L_t$ when $\phi_t = \phi_j$ (j = 1, 2, 3, …, n). Then, if $0 \le L_{1,t} \le L_{2,t} \le L_{3,t} \le \ldots \le L_{n,t}$, the Choquet expectation of the loss function is written as

$$(10) \qquad E^Q L_t = \sum_{i=1}^{n-1}\left[(L_{i,t} - L_{i+1,t})\theta(\cup_{j=1}^{i}\phi_j)\right] + L_{n,t},$$

where $\theta(\cdot)$ is a convex probability capacity (or a convex non-additive probability function).[4]

If $\theta(\cdot)$ is a probability measure, the problem is degenerated to the traditional expected loss minimization problem. In this case, substituting (7) and (8) into (9), the first-order condition $\partial E^Q L_t/\partial \Delta i_t = \partial EL_t/\partial \Delta i_t = 0$ leads to the time-invariant policy rule such that

$$(11) \qquad \Delta i_t = \frac{\lambda(x_t^0 - x^*)(\alpha + \eta E_t\phi_t) + (\pi_t^0 - \pi^*)(\alpha k + \mu E_t\phi_t)}{\omega + \lambda E_t(\alpha + \eta\phi_t)^2 + E_t(\alpha k + \mu\phi_t)^2}.$$

This simple monetary policy rule is similar to a Taylor rule in the sense that the nominal interest rate is adjusted in response to "output gap" and "inflation". Because of uncertainty in ϕ_t, the second moments of ϕ_t appear in the denominator of the feedback rule. This reflects a version of Brainard's effect where the policymaker's optimal rule becomes less aggressive under parameter uncertainty. It is noteworthy that the rule does not depend on how expectations are formed nor what stochastic processes the exogenous shocks follow. However, "output gap" and "inflation" in (11) are those before the central bank sets the new interest rate. In addition, unlike standard Taylor rules, the coefficient of lagged inflation is always equal to unity.[5]

4. Robust Optimal Policy Rules

[4] Let Ω be a state space and let $\Gamma(\Omega)$ denote the set of all subsets of Ω. Then, a convex probability capacity (or a convex non-additive probability function) is defined as function θ: $\Gamma(\Omega) \to [0, 1]$ that satisfies $\theta(\phi) = 0$, $\theta(\Omega) = 1$, $F \subseteq G \Rightarrow \theta(F) \le \theta(G)$ for all $F, G \subseteq \Omega$, and $\theta(F \cup G) + \theta(F \cap G) \ge \theta(F) + \theta(G)$ for all F, and $G \subseteq \Omega$. Since it is additive, it is not a probability measure unless the last inequality is always satisfied as an equality.

[5] In previous literature, Levin, Wieland, and Williams (1999) provides strong support for rules in which the first-difference of the federal funds rate responds to output and inflation gaps.

The policy rule (11) is no longer optimal when the policymaker has some aversion to uncertainty. One technical problem in deriving the optimal rule under uncertainty is that the "expected" loss function $E^Q L_t$ is not differentiable. However, since $E^Q L_t$ is convex in Δi_t, it is optimal for the central bank to set $\Delta i_t = \Phi$ if and only if $\partial E^Q L_t/\partial\Delta i_t \geq 0$ when Δi_t approaches to Φ from above and $\partial E^Q L_t/\partial\Delta i_t \leq 0$ when Δi_t approaches to Φ from below. This implies that the central bank decides not to change the nominal interest rate if and only if $\partial E^Q L_t/\Delta\partial i_t \geq 0$ as $\Delta i_t \to +0$ and $\partial E^Q L_t/\Delta\partial i_t \leq 0$ as $\Delta i_t \to -0$. We therefore obtain the following proposition.

Proposition 1: Suppose that the random variable ϕ_t takes n alternative values such that $\phi_1 \leq \phi_2 \leq \phi_3 \leq \ldots \leq \phi_n$. Then, the central bank decides not to change the nominal interest rate if and only if

$$(12)\quad \left\{\sum_{i=1}^{n-1}(\phi_{n+1-i}-\phi_{n-i})\theta_{II}(\cup_{j=1}^{i}\phi_{n+1-j})\right\}+\phi_1 \leq \frac{-\alpha\left\{\lambda(x_t^0-x^*)+k(\pi_t^0-\pi^*)\right\}}{\lambda\eta(x_t^0-x^*)+\mu(\pi_t^0-\pi^*)}$$
$$\leq \left\{\sum_{i=1}^{n-1}(\phi_i-\phi_{i+1})\theta_I(\cup_{j=1}^{i}\phi_j)\right\}+\phi_n.$$

where $\theta_I(\cdot)$ is a convex probability capacity in the case where $0 \leq L_{1,t} \leq L_{2,t} \leq L_{3,t} \leq \ldots \leq L_{n,t}$ and so is $\theta_{II}(\cdot)$ in the case where $L_{1,t} \geq L_{2,t} \geq L_{3,t} \geq \ldots \geq L_{n,t} \geq 0$.

Proof: Since $L_{j,t} \equiv \lambda\{x^0_t - (\alpha+\eta\phi_j)\Delta i_t - x^*\}^2 + \{\pi^0_t - (\mu\phi_j+\alpha k)\Delta i_t - \pi^*\}^2 + \omega\Delta i_t^2$ when $\phi_t = \phi_j$ ($j = 1, 2, 3, \ldots, n$), it holds that $L_{l,t} \leq L_{m,t}$ if and only if $[\lambda\{x^0_t - (\alpha+\eta\phi^*_{l,m})\Delta i_t - x^*\} + \{\pi^0_t - (\mu\phi^*_{l,m}+\alpha k)\Delta i_t - \pi^*\}]\,(\phi_l-\phi_m)\Delta i_t \geq 0$ where $\phi^*_{l,m} \equiv (\phi_l+\phi_m)/2$. When $\lambda\eta(x^0_t - x^*) + \mu(\pi^0_t - \pi^*) > 0$, this implies that $L_{l,t} \leq L_{m,t}$ as $\Delta i_t \to 0$ if and only if $(\phi_l-\phi_m)\Delta i_t \geq 0$. Since $\phi_1 \leq \phi_2 \leq \phi_3 \leq \ldots \leq \phi_n$, this shows that $0 \leq L_{1,t} \leq L_{2,t} \leq L_{3,t} \leq \ldots \leq L_{n,t}$ as $\Delta i_t \to -0$ and that $L_{1,t} \geq L_{2,t} \geq L_{3,t} \geq \ldots \geq L_{n,t} \geq 0$ as $\Delta i_t \to +0$ when $\lambda\eta(x^0_t - x^*) + \mu(\pi^0_t - \pi^*) > 0$.

When $\lambda\eta(x^0_t - x^*) + \mu(\pi^0_t - \pi^*) > 0$, the Choquet expectation (10) therefore implies that $E^Q L_t = \sum_{i=1}^{n-1}(L_{i,t}(\iota)-L_{i+1,t}(\iota))\theta_I(\cup_{j=1}^{i}\phi_j) + L_{n,t}$ as $\Delta i_t \to -0$ and $E^Q L_t = \sum_{i=1}^{n-1}(L_{n+1-i,t}-L_{n-i,t})\theta_{II}(\cup_{j=1}^{i}\phi_{n+1-j}) + L_{1,t}$ as $\Delta i_t \to +0$. Since $\left.\left(\frac{\partial L_{l,t}}{\partial\Delta i_t}-\frac{\partial L_{m,t}}{\partial\Delta i_t}\right)\right|_{\Delta i_t\to 0} = -2\{\lambda\eta(x^0_t - x^*) + \mu(\pi^0_t - \pi^*)\}(\phi_l - \phi_m)$, we obtain

$$\left.\frac{\partial E^Q L_t}{\partial \Delta i_t}\right|_{\Delta i_t=-0} = -2\{\lambda\eta(x^0_{\rm t} - x^*) + \mu\,(\pi^0_{\rm t} - \pi^*)\}\left[\left\{\sum_{i=1}^{n-1}(\phi_i - \phi_{i+1})\theta_I(\cup_{j=1}^{i}\phi_j)\right\} + \phi_n\right]$$

$$- 2\alpha\{\lambda\,(x^0_{\rm t} - x^*) + k(\pi^0_{\rm t} - \pi^*)\},$$

$$\left.\frac{\partial E^Q L_t}{\partial \Delta i_t}\right|_{\Delta i_t=+0} = -2\{\lambda\eta(x^0_{\rm t} - x^*) + \mu\,(\pi^0_{\rm t} - \pi^*)\}\left[\left\{\sum_{i=1}^{n-1}(\phi_{n+1-i} - \phi_{n-i})\theta_{II}(\cup_{j=1}^{i}\phi_{n+1-j})\right\} + \phi_1\right]$$

$$- 2\alpha\{\lambda\,(x^0_{\rm t} - x^*) + k(\pi^0_{\rm t} - \pi^*)\}.$$

These equations imply that when $\lambda\eta(x^0_{\rm t} - x^*) + \mu\,(\pi^0_{\rm t} - \pi^*) > 0$, the condition (12) holds if and only if $\partial E^Q L_{\rm t} /\Delta\partial i_{\rm t} \geq 0$ as $\Delta i_{\rm t} \to +0$ and $\partial E^Q L_{\rm t} /\Delta\partial i_{\rm t} \leq 0$ as $\Delta i_{\rm t} \to -0$.

Similarly, we can show that when $\lambda\eta(x^0_{\rm t} - x^*) + \mu\,(\pi^0_{\rm t} - \pi^*) < 0$, the condition (12) holds if and only if $\partial E^Q L_{\rm t} /\Delta\partial i_{\rm t} \geq 0$ as $\Delta i_{\rm t} \to +0$ and $\partial E^Q L_{\rm t} /\Delta\partial i_{\rm t} \leq 0$ as $\Delta i_{\rm t} \to -0$. This proves the proposition. [Q.E.D.]

The above result suggests that the policymaker may keep the policy instrument unchanged even if the exogenous shocks change output gaps and inflation rates. In the absence of uncertainty aversion, that is, if $\theta_I(\cdot)$and $\theta_{II}(\cdot)$ are the same probability measure, $\left\{\sum_{i=1}^{n-1}(\phi_i - \phi_{i+1})\theta_I(\cup_{j=1}^{i}\phi_j)\right\} + \phi_n = \left\{\sum_{i=1}^{n-1}(\phi_{n+1-i} - \phi_{n-i})\theta_{II}(\cup_{j=1}^{i}\phi_{n+1-j})\right\} + \phi_1$, so that the necessary and sufficient conditions in the above proposition are satisfied for no measurable parameter set. However, to the extent that the policymaker has uncertainty aversion, that is, when $\theta_I(\cdot)$and $\theta_{II}(\cdot)$ are a convex probability capacities, the conditions hold for some measurable parameter set. The reason why the policymaker may choose $\Delta i_{\rm t} = 0$ is that the policymaker faces additional uncertainty unless $\Delta i_{\rm t} = 0$. To the extent that the policymaker has uncertainty aversion, it can therefore be optimal to set $\Delta i_{\rm t} = 0$ for some measurable range.

If $\phi_1 \geq 0$, all terms in the condition (12) needs to be positive, so that the condition (12) does not hold unless $(x^0_{\rm t} - x^*)(\pi^0_{\rm t} - \pi^*) < 0$. This implies that some conflict between output stability and inflation stability is an important source for the policymaker to keep the interest rate unchanged. For example, when $x^0_{\rm t} > x^*$ and $\pi^0_{\rm t} < \pi^*$, lowering the interest rate achieves output stability but sacrifices inflation rate stability. The tradeoff is a source of infrequent changes of the interest rate under uncertainty in our model.

Unless the condition (12) holds, the central bank changes its interest rate based on a feedback rule. The rule is, however, time-variant in the sense that the feedback parameters vary depending on realized exogenous shocks and parameters. For example, note that $L_{l,t} = L_{m,t}$ as $\Delta i_t \to z_{l,m}$ for all l and m ($l \neq m$), where $z_{l,m} \equiv \frac{\lambda(x_t^0 - x^*) + (\pi_t^0 - \pi^*)}{\alpha(\lambda + k) + (\lambda\eta + \mu)(\phi_l + \phi_m)/2}$. It is then optimal for the central bank to set $\Delta i_t = z_{l,m}$ if and only if $\partial E^Q L_t/\partial \Delta i_t \geq 0$ as $\Delta i_t \to z_{l,m}+0$ and $\partial E^Q L_t/\partial \Delta i_t \leq 0$ as $\Delta i_t \to z_{l,m}-0$. In contrast, when $E^Q L_t/\partial \Delta i_t$ is differentiable around $\Delta i_t = z^*$, it is optimal for the central bank to set $\Delta i_t = z^*$ if and only if $\partial E^Q L_t/\partial \Delta i_t = 0$ as $\Delta i_t = z^*$. The general time-invariant feedback rule , however, takes highly complicated forms.

5. The Case of Two States

Our proposition can be understood more explicitly when the nature takes only two states: state A and state B. Suppose that the parameter ϕ_t follows a binomial distribution that takes either ϕ_A or ϕ_B, where $\phi_A > \phi_B$. Noting that $L_{j,t} \equiv \lambda\{x^0_t - (\alpha + \eta\phi_j)\Delta i_t - x^*\}^2 + \{\pi^0_t - (\mu\phi_j + \alpha k)\Delta i_t - \pi^*\}^2 + \omega \Delta i_t^2$ when $\phi_t = \phi_j$ (j = A, B), it holds that $L_{A,t} < L_{B,t}$ if and only if $[\lambda\{x^0_t - (\alpha + \eta\phi^*)\Delta i_t - x^*\} + \{\pi^0_t - (\mu\phi^* + \alpha k)\Delta i_t - \pi^*\}]\, \Delta i_t > 0$ where $\phi^* \equiv (\phi_A + \phi_B)/2$ and $\phi_A + \phi_B \geq 0$.

For the two states, we denote the convex probability capacity as follows: $\theta(\phi_A) = \nu(1-\varepsilon)$ and $\theta(\phi_B) = 1 - \theta(\phi_A)$ when $L_{A,t} > L_{B,t}$, $\theta(\phi_B) = (1-\nu)(1-\varepsilon)$ and $\theta(\phi_A) = 1 - \theta(\phi_B)$ when $L_{A,t} < L_{B,t}$, and $\theta(\phi_A \cup \phi_B) = 1$. A parameter ε (> 0) denotes the degree of ε-contamination in the Choquet expectation. Since the Choquet expectation puts more weight on the worst outcome, ν is contaminated to be smaller when $L_{A,t} < L_{B,t}$, so is $1-\nu$ when $L_{A,t} > L_{B,t}$ in the Choquet expectation.

The loss function is then written as

$$
\begin{aligned}
(13)\quad E^Q L_t &= \nu(1-\varepsilon)\,[\lambda\{x^0_t - (\alpha + \eta\phi_A)\Delta i_t - x^*\}^2 + \{\pi^0_t - (\mu\phi_A + \alpha k)\Delta i_t - \pi^*\}^2] \\
&\quad + \{1-\nu(1-\varepsilon)\}\,[\lambda\{x^0_t - (\alpha + \eta\phi_B)\Delta i_t - x^*\}^2 + \{\pi^0_t - (\mu\phi_B + \alpha k)\Delta i_t - \pi^*\}^2] \\
&\quad + \omega\Delta i_t^2, \qquad \text{when } L_{A,t} < L_{B,t}, \\
&= \{1-(1-\nu)(1-\varepsilon)\}[\lambda\{x^0_t - (\alpha + \eta\phi_A)\Delta i_t - x^*\}^2 + \{\pi^0_t - (\mu\phi_A + \alpha k)\Delta i_t - \pi^*\}^2] \\
&\quad + (1-\nu)(1-\varepsilon)\,[\lambda\{x^0_t - (\alpha + \eta\phi_B)\Delta i_t - x^*\}^2 + \{\pi^0_t - (\mu\phi_B + \alpha k)\Delta i_t - \pi^*\}^2] \\
&\quad + \omega\Delta i_t^2, \qquad \text{when } L_{A,t} > L_{B,t}.
\end{aligned}
$$

When $\varepsilon = 0$, the problem is degenerated to the traditional expected loss minimization problem. When $\varepsilon = 1$, the problem is degenerated to the classical mini-max problem where the policymaker minimizes only the worst case scenario. An increase in ε implies that the policymaker becomes less certain that the subjective distribution is true distribution. Thus, an increase in ε can be interpreted as an increase in Knightian uncertainty.

Unless ε = 0, the "expected" loss function $E^Q L_t$ is not differentiable. However, the proposition in the last section leads to the following corollary.

Corollary: In the case where the nature takes the two states, the central bank decides not to change the nominal interest rate if and only if

$$(14) \quad \phi_I \leq \frac{-\alpha\left\{\lambda(x_t^0 - x^*) + k(\pi_t^0 - \pi^*)\right\}}{\lambda\eta(x_t^0 - x^*) + \mu(\pi_t^0 - \pi^*)} \leq \phi_{II}.$$

where $\phi_I \equiv \nu(1\text{-}\varepsilon)\phi_A + \{1\text{-}\nu(1\text{-}\varepsilon)\}\phi_B$ and $\phi_{II} \equiv \{1\text{-}(1\text{-}\nu)(1\text{-}\varepsilon)\}\phi_A + (1\text{-}\nu)(1\text{-}\varepsilon)\,\phi_B$.

Proof: Recall that $\theta(\phi_A) = \nu(1\text{-}\varepsilon)$ and $\theta(\phi_B) = 1 - \theta(\phi_A)$ when $L_{A,t} > L_{B,t}$, $\theta(\phi_B) = (1\text{-}\nu)(1\text{-}\varepsilon)$ and $\theta(\phi_A) = 1 - \theta(\phi_B)$ when $L_{A,t} < L_{B,t}$, and $\theta(\phi_A \cup \phi_B) = 1$. For the probability capacity, $\left\{\sum_{i=1}^{n-1}(\phi_{n+1-i} - \phi_{n-i})\theta_{II}(\cup_{j=1}^{i}\phi_{n+1-j})\right\} + \phi_1$ degenerated into ϕ_I and so does $\left\{\sum_{i=1}^{n-1}(\phi_i - \phi_{i+1})\theta_I(\cup_{j=1}^{i}\phi_j)\right\} + \phi_n$ into ϕ_{II}. This leads to the corollary. [Q.E.D.]

In the absence of Knightian uncertainty, ε is equal to zero, so that there exists no measurable range of parameters that satisfy the above inequalities. However, to the extent that ε > 0, some measurable range of parameters satisfy the above inequalities. Given the parameters, the range is wider as ε is larger.

Unless the condition (13) holds, the central bank changes its interest rate based on a feedback rule. The rule is, however, time-variant in the sense that the feedback parameters vary depending on realized exogenous shocks and parameters. Define $z_{A,B} \equiv \frac{\lambda(x_t^0 - x^*) + (\pi_t^0 - \pi^*)}{\alpha(\lambda + k) + (\lambda\eta + \mu)\phi^*}$, $\sigma_{x,I}^2 \equiv \nu(1\text{-}\varepsilon)(\alpha + \eta\phi_A)^2 + \{1\text{-}\nu(1\text{-}\varepsilon)\}(\alpha + \eta\phi_B)^2$, $\sigma_{\pi,I}^2 \equiv \nu(1\text{-}\varepsilon)(\mu\phi_A + \alpha k)^2 + \{1\text{-}\nu(1\text{-}\varepsilon)\}(\mu\phi_B + \alpha k)^2$, $\sigma_{x,II}^2 \equiv \{1\text{-}(1\text{-}\nu)(1\text{-}\varepsilon)\}(\alpha + \eta\phi_A)^2 + (1\text{-}\nu)(1\text{-}\varepsilon)(\alpha + \eta\phi_B)^2$, and $\sigma_{\pi,II}^2 \equiv \{1\text{-}(1\text{-}\nu)(1\text{-}\varepsilon)\}(\mu\phi_A + \alpha k)^2 + (1\text{-}\nu)(1\text{-}\varepsilon)(\mu\phi_B + \alpha k)^2$. We then obtain the following proposition.

Proposition 2: When $\lambda(x^0_t - x^*) + (\pi^0_t - \pi^*) > 0$, it is optimal for the central bank to set $\Delta i_t = z_{A,B}$ if and only if

(15) $$\frac{\lambda(x_t^0 - x^*)(\alpha + \eta\phi_I) + (\pi_t^0 - \pi^*)(\alpha k + \mu\phi_I)}{\omega + \lambda\sigma_{x,I}{}^2 + \sigma_{\pi,I}{}^2} < z_{A,B}$$

$$< \frac{\lambda(x_t^0 - x^*)(\alpha + \eta\phi_{II}) + (\pi_t^0 - \pi^*)(\alpha k + \mu\phi_{II})}{\omega + \lambda\sigma_{x,II}{}^2 + \sigma_{\pi,II}{}^2}.$$

It is also optimal for the central bank to set

(16) $$\Delta i_t = \frac{\lambda(x_t^0 - x^*)(\alpha + \eta\phi_I) + (\pi_t^0 - \pi^*)(\alpha k + \mu\phi_I)}{\omega + \lambda\sigma_{x,I}{}^2 + \sigma_{\pi,I}{}^2}$$

if and only if $0 < \Delta i_t < z_{A,B}$, and

(17) $$\Delta i_t = \frac{\lambda(x_t^0 - x^*)(\alpha + \eta\phi_{II}) + (\pi_t^0 - \pi^*)(\alpha k + \mu\phi_{II})}{\omega + \lambda\sigma_{x,II}{}^2 + \sigma_{\pi,II}{}^2}$$

if and only if $\Delta i_t < 0$ or $z_{A,B} < \Delta i_t$.

Similarly, when $\lambda(x^0_t - x^*) + (\pi^0_t - \pi^*) < 0$, it is optimal for the central bank to set $\Delta i_t = z_{A,B}$ if and only if

(18) $$\frac{\lambda(x_t^0 - x^*)(\alpha + \eta\phi_{II}) + (\pi_t^0 - \pi^*)(\alpha k + \mu\phi_{II})}{\omega + \lambda\sigma_{x,II}{}^2 + \sigma_{\pi,II}{}^2} < z_{A,B}$$

$$< \frac{\lambda(x_t^0 - x^*)(\alpha + \eta\phi_I) + (\pi_t^0 - \pi^*)(\alpha k + \mu\phi_I)}{\omega + \lambda\sigma_{x,I}{}^2 + \sigma_{\pi,I}{}^2}.$$

It is also optimal for the central bank to set

(19) $$\Delta i_t = \frac{\lambda(x_t^0 - x^*)(\alpha + \eta\phi_I) + (\pi_t^0 - \pi^*)(\alpha k + \mu\phi_I)}{\omega + \lambda\sigma_{x,I}{}^2 + \sigma_{\pi,I}{}^2}$$

if and only if $z_{A,B} < \Delta i_t < 0$, and

(20) $$\Delta i_t = \frac{\lambda(x_t^0 - x^*)(\alpha + \eta\phi_{II}) + (\pi_t^0 - \pi^*)(\alpha k + \mu\phi_{II})}{\omega + \lambda\sigma_{x,II}{}^2 + \sigma_{\pi,II}{}^2}$$

if and only if $\Delta i_t < z_{A,B}$ or $0 < \Delta i_t$.

Proof: See Appendix 2.

The monetary policy rules (16), (17), (19), and (20) as well as the rules $\Delta i_t = z_{A,B}$ are similar to a Taylor rule in the sense that the nominal interest rate is adjusted in response to "output gap" and "inflation". Except that $\Delta i_t = z_{A,B}$, they also reflect a version of Brainard's effect where the policymaker's optimal rule becomes less aggressive when the variances increase. However, the elasticity of Δi_t to output gap and inflation rate not only depends on the degree of uncertainty aversion (that is, ε) but also differs for different ranges of Δi_t. Consequently, the nominal interest rate shows different responses to "output gap" and "inflation" depending on whether Δi_t is positive or not and whether Δi_t is greater than $z_{A,B}$ or not.

It is also noteworthy that none of the Taylor type rules is optimal when the condition (14) holds. This implies that the policymaker, who has uncertainty aversion, sometimes keeps the interest rate unchanged and sometimes implements discontinuous jumps of the interest rate. This type of stop-go policy is different from standard interest-rate smoothing or monetary policy inertia that was regarded as a conventional wisdom in macroeconomics. The macroeconomic consequences for x_t and π_t are also different because x_t and π_t take different values depending on whether i_t was changed or not.

6. Predictability of Federal Fund Rates

Until the last section, we have demonstrated that the policymaker who has uncertainty aversion may maintain an unchanged policy stance for some significant periods and may make discontinuous changes of the target rate following time-variant Taylor rule. The purpose of this section is to examine how well this robust optimal monetary policy can predict actual central bank's policy changes in the United States. Specifically, we explore how our stop-and-go Taylor rule can track monthly targeted federal fund (FF) rates from January 2001 to December 2007.[6]

Our model has six constant parameters (α, β, k, δ, λ, and ω), two policy targets (x^* and π^*), and one random parameter ϕ_t. The discount factor β is set equal to 0.999, appropriate for interpreting the time interval as one month. We use the interest rate elasticity of the aggregate demand of $\alpha = 0.055$ and the slope of Phillips curve of $k = 0.02$, which imply $\alpha = 0.22$ and $k = 0.08$ by quarterly data.[7] We set a weight on output

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6 We used target FF rates to predict the policymaker's decision. Since effective FF rates are highly correlated with effective FF rates, the essential results will remain the same even if we use effective FF rates in the following analysis.

7 Since α is the inverse of the degree of relative risk aversion in New Keynesian models, $\alpha = 0.22$ indicates that the degree of relative risk aversion is about 4.5 which is consistent with

fluctuation of λ = 0.01 and a weight on interest change of ω = 0 in the loss function. We also set the elasticity of the private agents' expectations of $\delta = 0.25$. For the policy targets, we set x^* to be 2% and π^* to be 2.7%, which implies that the policymaker allows moderate inflation.

For the random parameter ϕ_t, we investigate the case of two states. As in the last section, we consider the convex probability capacity such that $\theta(\phi_A) = \nu(1-\varepsilon)$ and $\theta(\phi_B) = 1 - \theta(\phi_A)$ when $L_{A,t} > L_{B,t}$ and that $\theta(\phi_B) = (1-\nu)(1-\varepsilon)$ and $\theta(\phi_A) = 1 - \theta(\phi_B)$ when $L_{A,t} < L_{B,t}$, where ε denotes the degree of ε-contamination in the Choquet expectation. In the benchmark case, we set ϕ_A = 0.25, ϕ_B = -0.25, ν = 0.5 and ε = 0.01. It is noteworthy that we set the parameter ε to be very small. This implies that the policymaker's uncertainty aversion is very small in the benchmark case. Comparing the case where $\varepsilon = 0$, we will see how the very small uncertainty aversion improves the predictability of our model.

Given the parameter values and the policy targets, Corollary and Proposition 2 in the last section lead to the time-variant Taylor rule where Δi_t responds to "output gap", $x^0_t - x^*$, and "inflation gap", $\pi^0_t - \pi^*$. For π^0_t, we use monthly data of annual growth rate of consumer price index excluding food and energy. For x^0_t, we use monthly data of annual growth rate of Industrial Production Index (total industry excluding construction). Both of the data series are from OECD Main Economic Indicator. The use of an industry production index for output gap is not necessarily standard in literature. But since our main focus is to predict monthly changes of federal fund rate, the industry production index is one of the limited proxies for GDP. Allowing transmission lag of the monetary policy, we use the values of one month ahead for "output gap" and "inflation gap" in the following experiments.

Figure 2 depicts both predicted and actual targeted FF rates from January 2001 to December 2007. The prediction is a dynamic simulation in the sense that i_t is forecasted based on predicted i_{t-1}. We set the initial realized value of i_t by its realized value in January 2001. We also use realized values of x^0_t and π^0_t in the following periods. Although the experiment is based on a simple model and noisy data, our stop-and-go Taylor rule could track monthly FF rates remarkably well. Since our model does not incorporate sub-prime shocks, the predicted FF rates could not follow sharp decline of FF rates in the second half of 2007. However, they almost tracked sharp decline of FF rates in 2001 and rise of FF rates from late 2004 to early 2006.

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literature. The choice of k is also consistent with literature. For example, Roberts (1995) shows that a value for k is 0.075 by quarterly data. Jensen (2002) uses a baseline value of $k = 0.1$, while Walsh (2003) uses 0.05 for quarterly data.

The most noteworthy result is that our model could track infrequent policy changes in 2002 and 2003 well. In 2002 and 2003, FOMC decided not to change its target rates in fourteen out of sixteen meetings. Consequently, actual FF rates remained unchanged for significant periods in 2002 and 2003. The predicted FF rates capture this feature well in the figure, although they failed to track stable FF rates in early 2002 and rise of FF rates in early 2004.

The superiority of our model can be seen more clearly when comparing our benchmark model with the model where the policymaker has no uncertainty aversion, that is, $\varepsilon = 0$. Based on equation (11), Figure 3 depicts predicted FF rates from January 2001 to December 2007. The dynamic predictions were made not only for $\omega = 0$ but also for $\omega = 0.004$. Except for ε and ω, the parameters used for the prediction are the same as those in Figure 2. Even when $\varepsilon = 0$, the Taylor rule tracked long-run movements of FF rates. However, we see that the predicted series frequently showed significant downward deviations from the actual series in the short-run especially when $\omega = 0$. When $\omega = 0$, the deviations became serious in 2002 and 2003 when actual FF rates remained unchanged for significant periods. In particular, the predicted series fell below zero for significant periods. Since we set $\omega = 0$ for Figure 2, this suggests that removing uncertainty aversion in the model worsens the predictability dramatically.

Setting $\omega = 0.004$ improves the model predictability. This indicates that the model without uncertainty aversion could track actual FF rates if we allow some benefits from interest rate smoothing in the loss function. However, even when $\omega = 0.004$, the predicted series showed temporary increases in early 2003, which are followed by substantial decline from late 2003 to 2004. These short-run fluctuations never happened in actual FF rates. The predicted series also showed smaller increases in 2005 and 2006.

The superiority of our model can be also confirmed even when comparing with a standard Taylor rule whose coefficients are estimated by ordinary least squares. The standard Taylor rule we used is $i_t = \text{constant} + \rho i_{t-1} + \varphi_x (x^0_{t+1} - x^*) + \varphi_\pi (\pi^0_{t+1} - \pi^*)$. We estimated the coefficients by ordinary least squares for the sample period from January 2001 to December 2007. Using the estimated coefficients, a dynamic prediction is made based on an initial value of i_t and realized values of x^0_t and π^0_t in the following periods.[8] Figure 4 depicts both predicted and actual FF rates from January 2001 to December 2007. The standard Taylor rule with estimated coefficients tracked actual FF rates well for the first one and half years. However, it could not track

[8] From the estimation, we obtained the Taylor rule such that $i_t = 0.086 + 0.961\, i_{t-1} + 5.818\, (x^0_{t+1} - x^*) + 8.442\, (\pi^0_{t+1} - \pi^*)$.

unchanged policy decision from 2003 to the first half of 2004 in the dynamic simulation. It also under-predicted FF rates in 2006 and 2007.

7. Prediction under Different Degrees of Uncertainty

In the last section, we showed that our stop-and-go Taylor rule can track monthly FF rates very well. The most noteworthy result is that uncertainty aversion of the policymaker is useful in tracking infrequent policy changes in actual FF rates. In this section, we check how different degree of uncertainty will change the time series property of predicted FF rates. In our model, ϕ_t is the only random structural parameter that changes over time. In our benchmark case, we set ϕ_A = 0.25 and ϕ_B = -0.25 for the random parameter. We first examine how the predicted series will change when we use alternative combinations of (ϕ_A, ϕ_B) = (0.175, -0.175), (0.3, 0.3), and (0.5, -0.5). In all of the combinations, the average value of ϕ_t is set to be zero. The experiment thus explores how a mean-preserving spread will affect the predicted FF rates.

Figure 5 depicts the predicted FF rates for these alternative sets of ϕ_A and ϕ_B from January 2001 to December 2007. Like previous predictions, the dynamic predictions are based on an initial value of i_t and realized values of x^0_t and π^0_t in the following periods. However, unlike previous predictions, the initial value of i_t is chosen so that the predicted value is equal to the actual value in August 2001. It is easy to see that the interest rates change most frequently when (ϕ_A, ϕ_B) = (0.175, -0.175) and least frequently when (ϕ_A, ϕ_B) = (0.5, -0.5). This implies that a mean-preserving spread of ϕ_t will make the interest rates change less frequent.

In contrast with ϕ_t, the degree of ε-contamination changes the policymaker's aversion to uncertainty. Therefore, given the distribution of ϕ_t, changes of ε will capture another type of uncertainty changes. For the degree of ε-contamination, we set ε = 0.01 in our benchmark case. We examine how the predicted series will change when we use alternative values of ε = 0.001, 0.05, and 0.1. Figure 6 depicts the interest rates for these alternative values of ε from January 2001 to December 2007. Like Figure 5, the initial value of i_t is chosen so that the predicted value is equal to the actual value in August 2001. It is easy to see that the interest rate changes are more infrequent when ε = 0.05 and much more infrequent when ε =0.1. An increase of uncertainty aversion will make the interest rates change less frequent. They are, however, more frequent when ε = 0.001 for which the policymaker's uncertainty aversion is negligible.

The above results suggest that both a mean-preserving spread of ϕ_t and an increase of uncertainty aversion will make the interest rates change less frequent. However, in predicting actual FF rates, they have different implications. To see this, we examine how the predicted series will change when we use alternative combinations of (ϕ_A, ϕ_B, ε). Specifically, we examine how the predicted series will change when we use alternative combinations of (ϕ_A, ϕ_B, ε) = (0.175, -0.175, 0.025) and (0.4, -0.4, 0.005). Since (ϕ_A, ϕ_B, ε) = (0.25, -0.25, 0.01) in the benchmark case, the first is a combination that has smaller mean-preserving spread of ϕ_t but larger uncertainty aversion, while the second is a combination that has larger mean-preserving spread of ϕ_t but smaller uncertainty aversion. Figure 7 depicts the interest rates for these two alternative combinations from January 2001 to December 2007. The initial value of i_t is chosen so that the predicted value is equal to the actual value in August 2001. The combination of (0.175, -0.175, 0.025) shows that the predicted series remain constant from April 2002 to September 2004 but change more dramatically in the other periods. This implies that an increase of uncertainty aversion makes policy change more infrequent but smaller mean-preserving spread of ϕ_t makes policy changes more dramatic. In contrast, the combination of (0.4, -0.4, 0.005) shows that the predicted series are very flat not only in 2002 and 2003 but also in the other periods. This implies that larger mean-preserving spread of ϕ_t cannot predict unchanged policy that happens only for some specific periods. Larger mean-preserving spread of ϕ_t is helpful only in making the predicted series smooth throughout the period.

8. Predictability of ECB's Targeted Interest Rate

In section 6, we have demonstrated that our stop-and-go Taylor rule can track monthly policy changes in the United States very well. The purpose of this section is to explore whether the stop-and-go Taylor rule can also track policy changes by European Central bank (ECB). Specifically, we investigate how well our robust optimal monetary policy can predict monthly interest rates on the main refinancing operations (MRO) set by ECB from January 2001 to December 2007.

As in previous sections, we investigate the case of two states where Corollary and Proposition 2 in section 5 lead to the time-variant Taylor rule. Unlike section 6, we set π^* = 2.5% which implies that ECB has tighter inflation target than Federal Reserve Board (FRB) in the loss function. However, to make the following results comparable to those in previous sections, we use the exactly same parameters as those in section 6.[9]

[9] Specifically, we set α = 0.055, k = 0.02, and δ = 0.25 for structural parameters, x^* = 2%, λ = 0.01 and ω = 0 for the loss function, and ϕ_A = 0.25, ϕ_B = -0.25, ν = 0.5, and ε = 0.01 for the convex

For π^0_t, we use monthly data of annual growth rate of the Harmonised Index of Consumer Prices (all-items excluding energy and food). For x^0_t, we use monthly data of annual growth rate of Industrial Production Index (total industry excluding construction). Both of the data series are those of all Euro area and are downloaded from Euro Stat. The prediction is a dynamic simulation where MRO rates are forecasted based on an initial value of i_t in January 2001 and realized values of x^0_t and π^0_t from January 2001 to December 2007. Like previous sections, Δi_t is assumed to respond to one month ahead of $x^0_t - x^*$ and $\pi^0_t - \pi^*$.

Figure 8 depicts both predicted and actual rates of MRO from January 2001 to December 2007. Although based on a simple model and noisy data, our stop-and-go Taylor rule could track monthly rates of MRO remarkably well. The prediction underestimated the rates of MRO in 2002 and overestimated in the second half of 2007. However, the predicted rates are very similar to actual rates in the other periods. In particular, they almost tracked the unchanged rates of MRO from 2003 to 2005 and the continuous rise of MRO rates in 2006. From 2003 to 2005, the Governing Council of ECB decided not to change its target rates in thirty-three out of thirty-six meetings. Consequently, actual rates of MRO remained unchanged for significant periods from 2003 to 2005. The predicted rates of MRO capture this feature well in the figure. It is noteworthy that we used the exactly same parameters as those in section 6 except for π^*. This implies that both FRB' and ECB' policy decisions can be described by the same stop-and-go Taylor rule with the same feedback parameters.

The superiority of our model can be seen more clearly when comparing the benchmark model with the model where the policymaker has no uncertainty aversion, that is, $\varepsilon = 0$. Figure 9 depicts both predicted and actual rates of MRO from January 2001 to December 2007 in the case where $\varepsilon = 0$. Like section 6, the dynamic predictions were made not only for $\omega = 0$ but also for $\omega = 0.004$. Except for ε and ω, the parameters are the same as those in Figure 8. Even when $\varepsilon = 0$, the Taylor rule which is determined by equation (11) tracked long-run movements of MRO rates. However, we see that the predicted series frequently showed significant downward deviations from the actual series. In particular, the predicted series fell below zero from June 2005 to February 2006. This suggests that removing uncertainty aversion in the model worsens the predictability dramatically. Setting $\omega = 0.004$, that is, allowing some benefits from interest rate smoothing in the loss function, improves the model predictability. In particular, the predicted series tracked unchanged policy from March 2002 to November 2002. However, even when $\omega = 0.004$, they overestimated MRO

probability capacity.

rates from 2003 to 2005. They showed temporary increases in mid 2004, which are followed by substantial decline in the first half of 2005. These short-run fluctuations never happened in actual FF rates.

The superiority of our model can be also confirmed even when comparing with a standard Taylor rule whose coefficients are estimated by ordinary least squares. Like section 6, we estimated the coefficients of the standard Taylor rule by ordinary least squares for the sample period from January 2001 to December 2007.[10] Using the estimated coefficients, a dynamic prediction is made based on an initial value of i_t and realized values of x^0_t and π^0_t in the following periods. Figure 10 depicts both predicted and actual FF rates from January 2001 to December 2007. The standard Taylor rule with estimated coefficients tracked medium-run and long-run movements of MRO rates well. However, the predicted series moved smoother than actual series. They overestimated MRO rates from 2003 to 2005 and underestimated after 2006 in the dynamic simulation. In addition, they failed to detect unchanged policy decision from 2003 to 2005.

9. Concluding Remarks

In this paper, we explored why the central bank's policy instrument remains so unchanged under uncertainty. Although infrequent policy changes have been widely observed in many central banks, they have not been taken into account in previous macro models. This is true even in previous studies that investigated optimal monetary policy under model uncertainty or robust optimal policy rules. A large number of studies agreed that there is clearly much uncertainty over policy multipliers. However, most previous studies concluded that, under certain conditions, multiplier uncertainty may make optimal policy more conservative but does not lead to a policy of "doing nothing". A key departure of our paper from these studies is the introduction of a stop-go monetary policy under Knightian uncertainty or a robust control framework. This increases an incentive for the central bank to keep the policy instrument unchanged even when exogenous shocks change output gap and inflation rate. The calibrated optimal stop-go policy rules could track actual target rates of FRB and ECB quite well.

Needless to say, our stop-go monetary policy is not the only explanation for why the central banks' policy changes are so infrequent. Infrequent decision-making meetings would be one reason why the policy target changes so infrequently. Infrequent

[10] From the estimation, we obtained the Taylor rule such that $i_t = 0.049 + 0.975\ i_{t-1} + 2.926\ (x^0_{t+1} - x^*) + 3.331\ (\pi^0_{t+1} - \pi^*)$.

observations of macroeconomic data could be another reason. However, as we discussed briefly in the introduction, policy changes are less frequent than what these institutional constraints predict. This paper filled the gaps that the institutional constraints cannot explain. What we have not discussed in the paper but what seems important is an integer constraint where a unit of the target rate change is usually 25 basis points for most central banks. This could be another source for infrequent policy changes.

Appendix 1. Extension of Proposition 1.

In this Appendix, we extend Proposition 1 to the case where the policymaker faces uncertainty even without any policy change. Specifically, we consider the case where the policymaker cannot see exact values of x^0_t and π^0_t which are define by (5) and (6). We assume that the stochastic variables x^0_t and π^0_t are independent of the stochastic parameter ϕ_t for the policymaker.

Suppose that the random variables (x^0_t, π^0_t) takes k alternative values, (x^0_1, π^0_1), (x^0_2, π^0_2), ..., (x^0_k, π^0_k). Define $L_{j,t}(\iota) \equiv \lambda\{x^0_\iota - (\alpha + \eta\phi_j)\Delta i_t - x^*\}^2 + \{\pi^0_\iota - (\mu\phi_j + \alpha k)\Delta i_t - \pi^*\}^2 + \omega\Delta i_t^2$. It then holds that $L_{l,t}(\iota) \leq L_{m,t}(\iota)$ as $\Delta i_t \to 0$ if and only if $\{\lambda\eta(x^0_\iota - x^*) + \mu(\pi^0_\iota - \pi^*)\}(\phi_l - \phi_m)\Delta i_t \geq 0$. Since $\phi_1 \leq \phi_2 \leq \ldots \leq \phi_n$, this derives that $0 \leq L_{1,t}(\iota) \leq L_{2,t}(\iota) \leq L_{3,t}(\iota) \leq \ldots \leq L_{n,t}(\iota)$ as $\{\lambda\eta(x^0_\iota - x^*) + \mu(\pi^0_\iota - \pi^*)\}\Delta i_t \to -0$ and that $L_{1,t}(\iota) \geq L_{2,t}(\iota) \geq L_{3,t}(\iota) \geq \ldots \geq L_{n,t}(\iota) \geq 0$ as $\{\lambda\eta(x^0_\iota - x^*) + \mu(\pi^0_\iota - \pi^*)\}\Delta i_t \to +0$.

For $\iota = 1, 2, \ldots, k$, define the probability capacity π_ι with which $(x^0_t, \pi^0_t) = (x^0_\iota, \pi^0_\iota)$. Suppose that $\lambda\eta(x^0_\iota - x^*) + \mu(\pi^0_\iota - \pi^*)$ is greater than zero for $\iota = 1, 2, \ldots, k_0$ and is less than zero for $\iota = k_0+1, k_0+2, \ldots, k$. Then, we can show that as $\Delta i_t \to -0$,

$$E^Q L_t = \sum_{\iota=1}^{k_0} \pi_\iota \left[\left\{\sum_{i=1}^{n-1} (L_{i,t}(\iota) - L_{i+1,t}(\iota))\theta_I(\cup_{j=1}^{i}\phi_j)\right\} + L_{n,t}(\iota)\right]$$

$$+ \sum_{\iota=k_0+1}^{k} \pi_\iota \left[\left\{\sum_{i=1}^{n-1} (L_{n+1-i,t}(\iota) - L_{n-i,t}(\iota))\theta_{II}(\cup_{j=1}^{i}\phi_{n+1-j})\right\} + L_{1,t}(\iota)\right]$$

and that as $\Delta i_t \to +0$,

$$E^Q L_t = \sum_{\iota=1}^{k_0} \pi_\iota \left[\left\{\sum_{i=1}^{n-1} (L_{n+1-i,t}(\iota) - L_{n-i,t}(\iota))\theta_{II}(\cup_{j=1}^{i}\phi_{n+1-j})\right\} + L_{1,t}(\iota)\right]$$

$$+ \sum_{\iota=k_0+1}^{k} \pi_\iota \left[\left\{\sum_{i=1}^{n-1} (L_{i,t}(\iota) - L_{i+1,t}(\iota))\theta_I(\cup_{j=1}^{i}\phi_j)\right\} + L_{n,t}(\iota)\right].$$

Since $\left.\left(\frac{\partial L_{l,t}(\iota)}{\partial \Delta i_t} - \frac{\partial L_{m,t}(\iota)}{\partial \Delta i_t}\right)\right|_{\Delta i_t \to 0} = -2\{\lambda\eta(x^0_\iota - x^*) + \mu(\pi^0_\iota - \pi^*)\}(\phi_l - \phi_m)$, this leads to

$$\left.\frac{\partial E^Q L_t}{\partial \Delta i_t}\right|_{\Delta i_t = -0} = -2\Phi(+)\left[\left\{\sum_{i=1}^{n-1} (\phi_i - \phi_{i+1})\theta_I(\cup_{j=1}^{i}\phi_j)\right\} + \phi_n\right]$$

$$-2\Phi(-)\left[\left\{\sum_{i=1}^{n-1} (\phi_{n+1-i} - \phi_{n-i})\theta_{II}(\cup_{j=1}^{i}\phi_{n+1-j})\right\} + \phi_1\right]$$

$$-2\alpha\sum_{\iota=1}^{k} \pi_\iota \{\lambda(x^0_\iota - x^*) + k(\pi^0_\iota - \pi^*)\},$$

$$\left.\frac{\partial E^Q L_t}{\partial \Delta i_t}\right|_{\Delta i_t=+0} = -2\Phi(+)\left[\left\{\sum_{i=1}^{n-1}(\phi_{n+1-i}-\phi_{n-i})\theta_{II}(\cup_{j=1}^{i}\phi_{n+1-j})\right\}+\phi_1\right]$$

$$-2\Phi(-)\left[\left\{\sum_{i=1}^{n-1}(\phi_i-\phi_{i+1})\theta_I(\cup_{j=1}^{i}\phi_j)\right\}+\phi_n\right]$$

$$-2\alpha\sum_{\iota=1}^{k}\pi_\iota\,\{\lambda\,(x^0_\iota - x^*) + k(\pi^0_\iota - \pi^*)\}.$$

where $\Phi(+) \equiv \sum_{\iota=1}^{k_0}\pi_\iota\,\{\lambda\eta(x^0_\iota - x^*) + \mu\,(\pi^0_\iota - \pi^*)\}$ and $\Phi(-) \equiv \sum_{\iota=k_0+1}^{k}\pi_\iota\,\{\lambda\eta(x^0_\iota - x^*) + \mu\,(\pi^0_\iota - \pi^*)\}$. These equations imply that $\partial\, E^Q\, L_t\,/\Delta\partial i_t \geq 0$ as $\Delta i_t \to +0$ and $\partial\, E^Q\, L_t\,/\Delta\partial i_t \leq 0$ as $\Delta i_t \to -0$ if and only if

$$\Phi(+)\left[\left\{\sum_{i=1}^{n-1}(\phi_i-\phi_{i+1})\theta_I(\cup_{j=1}^{i}\phi_j)\right\}+\phi_n\right]+\Phi(-)\left[\left\{\sum_{i=1}^{n-1}(\phi_{n+1-i}-\phi_{n-i})\theta_{II}(\cup_{j=1}^{i}\phi_{n+1-j})\right\}+\phi_1\right]$$

$$\geq\ -\alpha\sum_{\iota=1}^{k}\pi_\iota\,\{\lambda\,(x^0_\iota - x^*) + k(\pi^0_\iota - \pi^*)\} \geq$$

$$\Phi(+)\left[\left\{\sum_{i=1}^{n-1}(\phi_{n+1-i}-\phi_{n-i})\theta_{II}(\cup_{j=1}^{i}\phi_{n+1-j})\right\}+\phi_1\right]+\Phi(-)\left[\left\{\sum_{i=1}^{n-1}(\phi_i-\phi_{i+1})\theta_I(\cup_{j=1}^{i}\phi_j)\right\}+\phi_n\right].$$

The above inequalities are the extended version of (12) to the case where x^0_t and π^0_t are uncertain for the policymaker. The central bank decides not to change the nominal interest rate if and only if the above inequalities hold. The result suggests that to derive infrequent policy changes, the assumption on information structure about x^0_t and π^0_t is not crucial in our analysis. What is crucial is that the change of the nominal interest rate induces additional independent uncertainty for the policymaker. This is the source for the central bank to decide no policy change under some circumstances.

Appendix 2. Derivation of Proposition 2

In this Appendix, we prove Proposition 2 in section 5. Suppose that $\lambda(x^0_t - x^*) + (\pi^0_t - \pi^*) > 0$. Then, $z_{A,B} \equiv \dfrac{\lambda(x^0_t - x^*) + (\pi^0_t - \pi^*)}{\alpha(\lambda+k)+(\lambda\eta+\mu)(\phi_l+\phi_m)/2} > 0$. It holds that $L_{A,t} < L_{B,t}$ if and only if $0 < \Delta i_t < z_{A,B}$ and that $L_{A,t} > L_{B,t}$ if and only if $\Delta i_t < 0$ or $\Delta i_t > z_{A,B}$. Equation (14) therefore leads to

(B1) $\frac{\partial E^Q L_t}{\partial \Delta i_t} = -2\nu(1\text{-}\varepsilon)\,[\lambda(\alpha+\eta\phi_A)\{x^0_t - (\alpha+\eta\phi_A)\Delta i_t - x^*\}$

$+ (\mu\phi_A+\alpha k)\{\pi^0_t - (\mu\phi_A+\alpha k)\Delta i_t - \pi^*\}\,]$

$-2\{1\text{-}\nu(1\text{-}\varepsilon)\}\,[\lambda(\alpha+\eta\phi_B)\{x^0_t - (\alpha+\eta\phi_B)\Delta i_t - x^*\}$

$+ (\mu\phi_B+\alpha k)\{\pi^0_t - (\mu\phi_B+\alpha k)\Delta i_t - \pi^*\}\,] + 2\omega\Delta i_t,$

when $0 < \Delta i_t < z_{A,B}$,

$= -2\{1\text{-}(1\text{-}\nu)(1\text{-}\varepsilon)\}[\lambda(\alpha+\eta\phi_A)\{x^0_t - (\alpha+\eta\phi_A)\Delta i_t - x^*\}$

$+ (\mu\phi_A+\alpha k)\{\pi^0_t - (\mu\phi_A+\alpha k)\Delta i_t - \pi^*\}^2\,]$

$-2(1\text{-}\nu)(1\text{-}\varepsilon)\,[\lambda(\alpha+\eta\phi_B)\{x^0_t - (\alpha+\eta\phi_B)\Delta i_t - x^*\}$

$+ (\mu\phi_B+\alpha k)\{\pi^0_t - (\mu\phi_B+\alpha k)\Delta i_t - \pi^*\}^2\,] + 2\omega\Delta i_t,$

when $\Delta i_t < 0$ or $\Delta i_t > z_{A,B}$.

This implies that $\frac{\partial E^Q L_t}{\partial \Delta i_t} > 0$ as Δi_t approaches to $z_{A,B}$ from below and that $\frac{\partial E^Q L_t}{\partial \Delta i_t} < 0$ as Δi_t approaches to $z_{A,B}$ from above if and only if

(B2) $$\frac{\lambda(x_t^0 - x^*)(\alpha+\eta\phi_I) + (\pi_t^0 - \pi^*)(\alpha k + \mu\phi_I)}{\omega + \lambda\sigma_{x,I}{}^2 + \sigma_{\pi,I}{}^2} < z_{A,B}$$

$$< \frac{\lambda(x_t^0 - x^*)(\alpha+\eta\phi_{II}) + (\pi_t^0 - \pi^*)(\alpha k + \mu\phi_{II})}{\omega + \lambda\sigma_{x,II}{}^2 + \sigma_{\pi,II}{}^2}.$$

This indicates that it is optimal for the central bank to set $\Delta i_t = z_{A,B}$ if and only if (B2) holds.

In addition, recall that $\phi_I \equiv \nu(1\text{-}\varepsilon)\phi_A+\{1\text{-}\nu(1\text{-}\varepsilon)\}\phi_B$, $\phi_{II} \equiv \{1\text{-}(1\text{-}\nu)(1\text{-}\varepsilon)\}\phi_A + (1\text{-}\nu)(1\text{-}\varepsilon)\phi_B$, $\sigma_{x,I}{}^2 \equiv \nu(1\text{-}\varepsilon)(\alpha+\eta\phi_A)^2 + \{1\text{-}\nu(1\text{-}\varepsilon)\}(\alpha+\eta\phi_B)^2$, $\sigma_{\pi,I}{}^2 \equiv \nu(1\text{-}\varepsilon)(\mu\phi_A+\alpha k)^2 + \{1\text{-}\nu(1\text{-}\varepsilon)\}(\mu\phi_B+\alpha k)^2$, $\sigma_{x,II}{}^2 \equiv \{1\text{-}(1\text{-}\nu)(1\text{-}\varepsilon)\}(\alpha+\eta\phi_A)^2 + (1\text{-}\nu)(1\text{-}\varepsilon)(\alpha+\eta\phi_B)^2$, and $\sigma_{\pi,II}{}^2 \equiv \{1\text{-}(1\text{-}\nu)(1\text{-}\varepsilon)\}(\mu\phi_A+\alpha k)^2 + (1\text{-}\nu)(1\text{-}\varepsilon)(\mu\phi_B+\alpha k)^2$. It thus holds that when $0 < \Delta i_t < z_{A,B}$, $\frac{\partial E^Q L_t}{\partial \Delta i_t} = 0$ if and only if

(B3) $$\Delta i_t = \frac{\lambda(x_t^0 - x^*)(\alpha + \eta\phi_I) + (\pi_t^0 - \pi^*)(\alpha k + \mu\phi_I)}{\omega + \lambda\sigma_{x,I}{}^2 + \sigma_{\pi,I}{}^2}.$$

and that when $\Delta i_t < 0$ or $\Delta i_t > z_{A,B}$, $\frac{\partial E_t^Q L_t}{\partial \Delta i_t} = 0$ if and only if

(B4) $$\Delta i_t = \frac{\lambda(x_t^0 - x^*)(\alpha + \eta\phi_{II}) + (\pi_t^0 - \pi^*)(\alpha k + \mu\phi_{II})}{\omega + \lambda\sigma_{x,II}{}^2 + \sigma_{\pi,II}{}^2}.$$

This proves the first part of Proposition 2. Since the second part of Proposition 2 can be proved similarly, we can obtain Proposition 2.

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Table 1. The Number of FOMC Release Dates and No Policy Change Announcement

	1991	1992	1993	1994	1995	1996	1997	1998	1999
Number of FOMC meetings	18	12	8	9	8	7	8	9	8
Number of meetings without policy change	9	9	8	3	4	6	7	6	5

	2000	2001	2002	2003	2004	2005	2006	2007
Number of FOMC meetings	8	11	8	8	8	8	8	11
Number of meetings without policy change	5	0	7	7	3	0	4	4

Table 2. The Number of Meetings and No Policy Change Announcement

(1) Bank of Japan (Monetary Policy Meetings)

	1999	2000	2001	2002	2003	2004	2005	2006	2007
Number of Monetary Policy Meetings	19	18	17	16	16	16	15	14	14
Number of meetings without policy change	18	17	12	13	12	15	15	12	13

(2) European Central Bank (The Governing Council)

	1999	2000	2001	2002	2003	2004	2005	2006	2007
Number of Government Council meetings	24	24	24	12	12	12	12	12	12
Number of meetings without policy change	19	17	20	11	10	12	11	7	10

(3) Bank of England (The Monetary Policy Committee)

	1999	2000	2001	2002	2003	2004	2005	2006	2007
Number of MPC meetings	12	12	13	12	12	12	12	12	12
Number of meetings without policy change	6	10	6	12	9	8	11	10	8

(4) Reserve Bank of Australia (Reserve Bank Board)

	1999	2000	2001	2002	2003	2004	2005	2006	2007
Number of RBB meetings	11	11	11	11	11	11	11	11	11
Number of meetings without policy change	10	7	5	9	9	11	10	8	8

(5) Reserve Bank of New Zealand

	1999	2000	2001	2002	2003	2004	2005	2006	2007
Number of OCR review meetings	6	8	9	8	8	8	8	8	8
Number of meetings without policy change	5	4	4	4	5	2	5	8	4

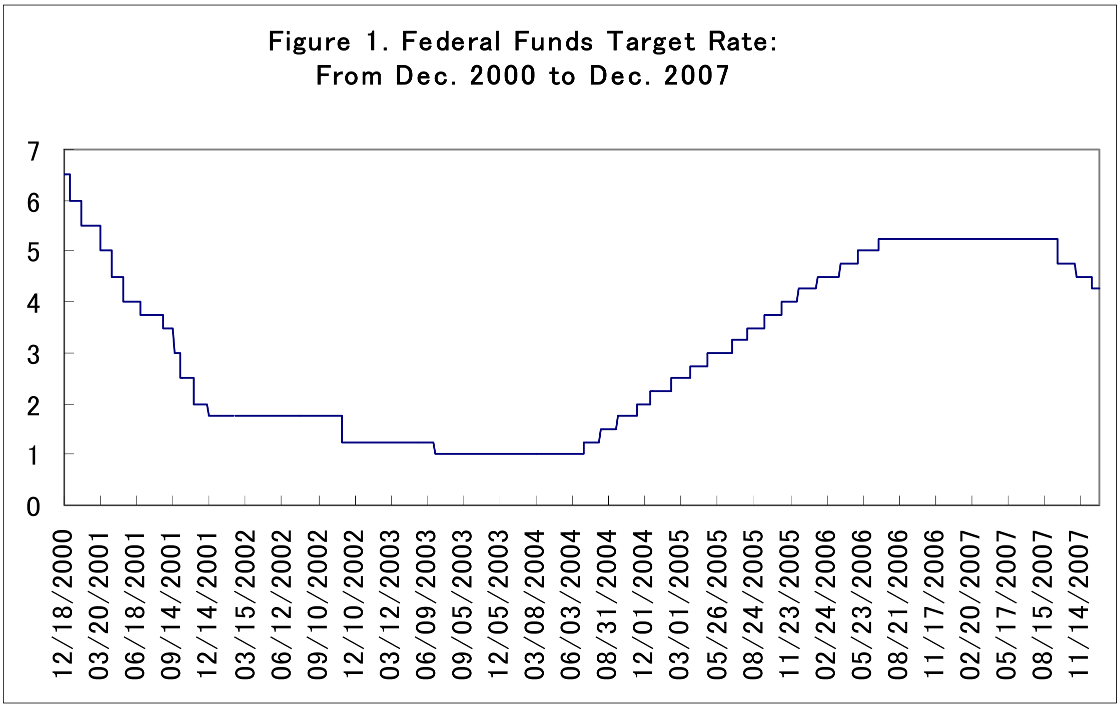

Figure 2.　Predicted and Actual FF Rates: Benchmark Case

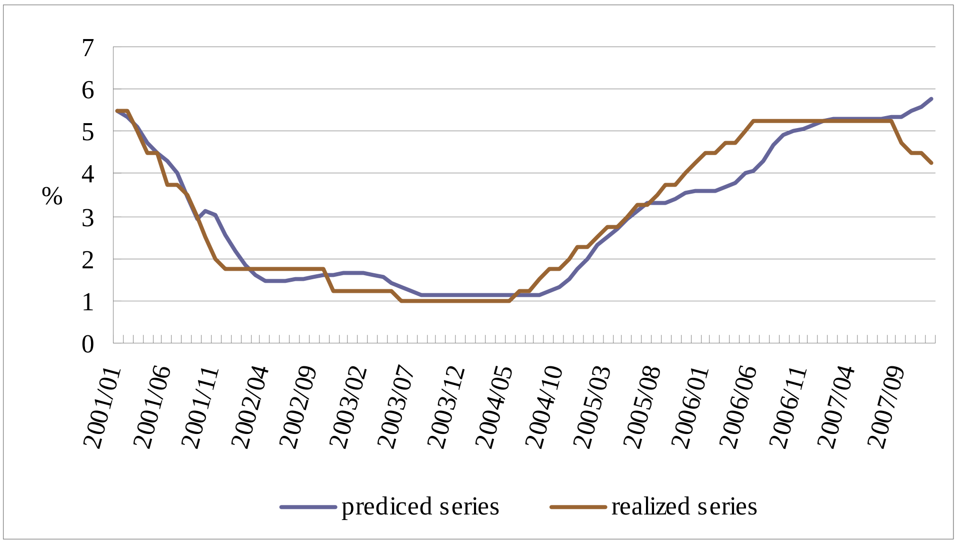

Figure 3. Predicted and Actual FF Rates: Cases of ε = 0

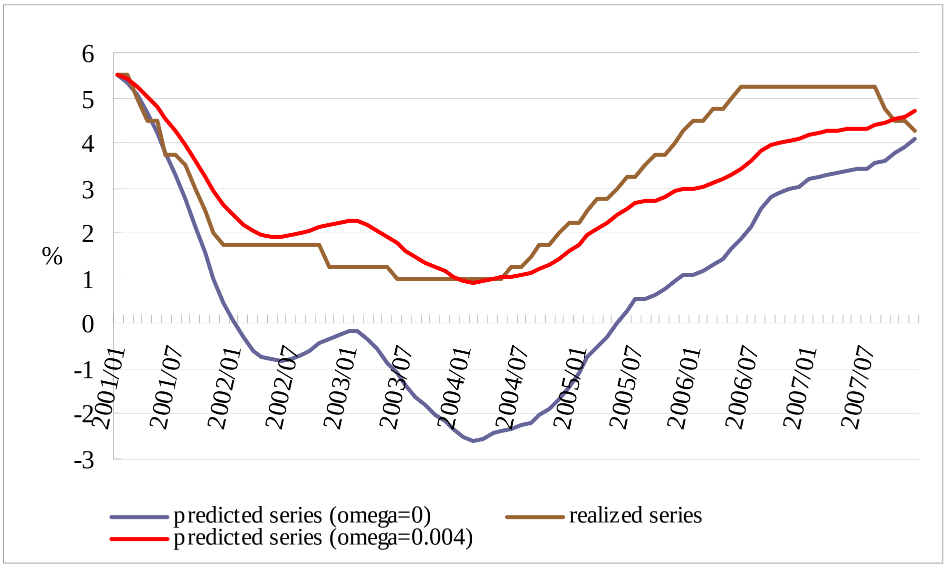

Figure 4. Predicted and Actual FF Rates: Case of the estimated coefficients

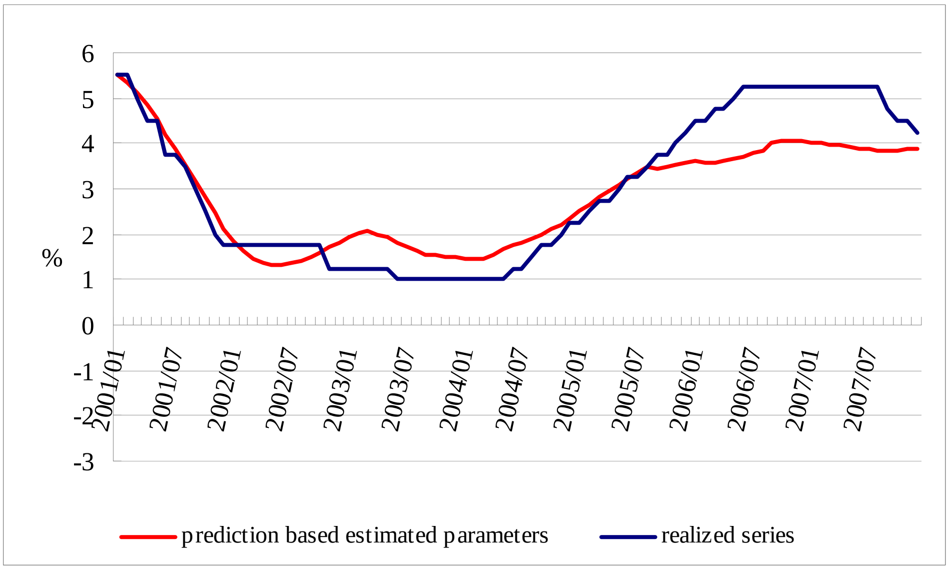

Figure 5. Predicted and Actual FF Rates for Alternative Combinations of ϕ_t

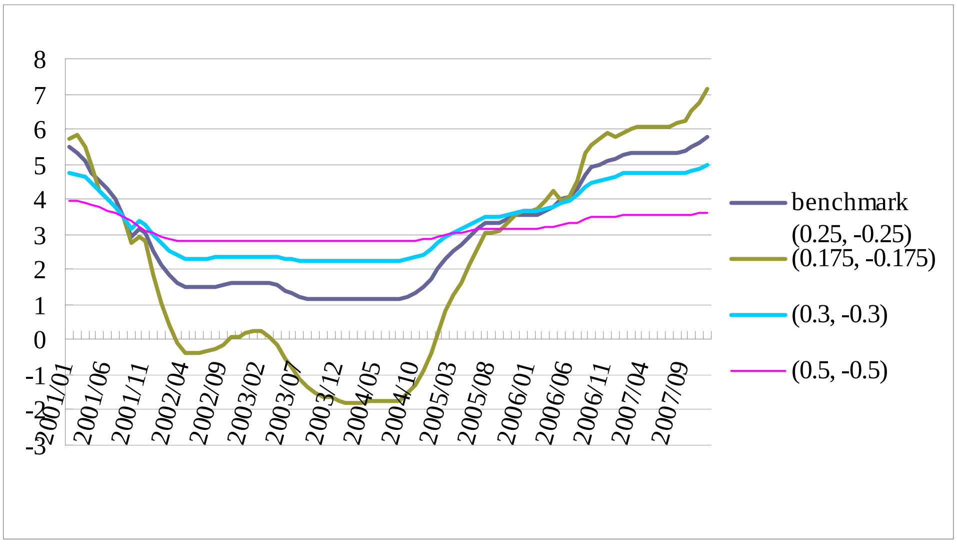

Figure 6. Predicted and Actual FF Rates for Alternative Values of ε

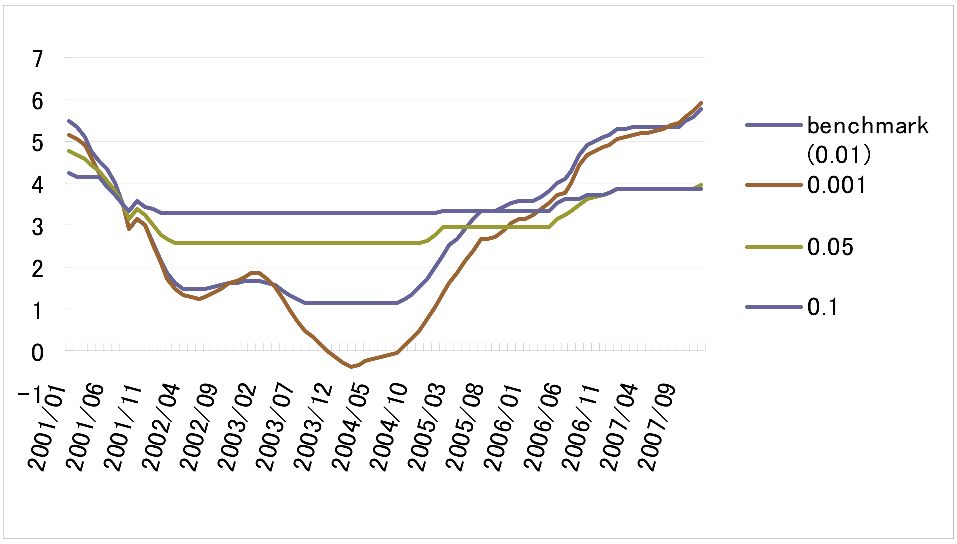

Figure 7. Predicted and Actual FF Rates for Alternative Combinations of ϕ_t and ε

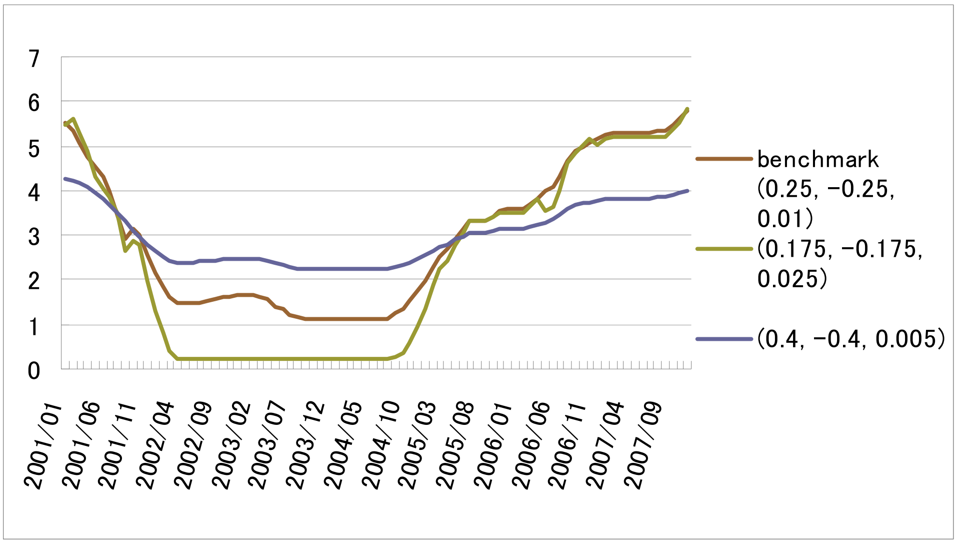

Figure 8. Predicted and Actual Rates of MRO: Benchmark Case

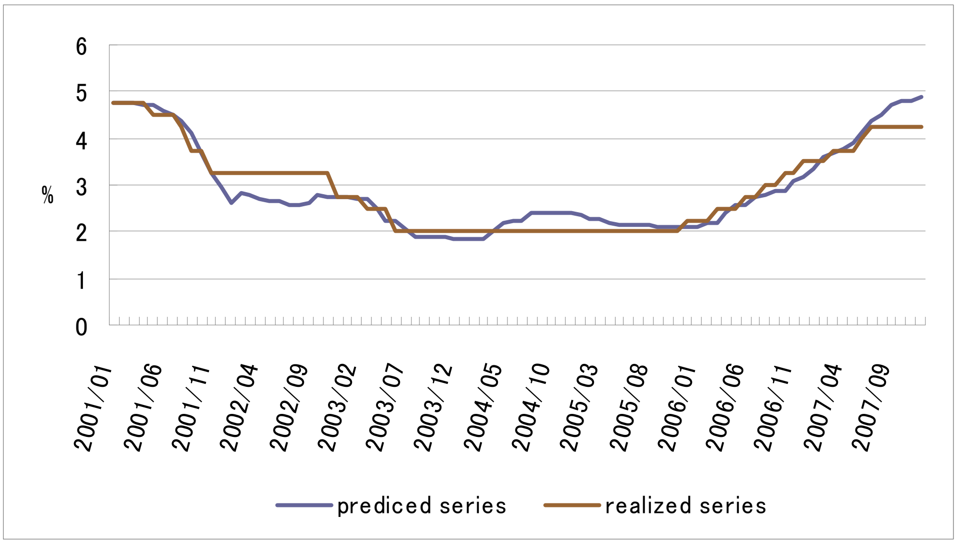

Figure 9. Predicted and Actual Rates of MRO: Cases of ε = 0

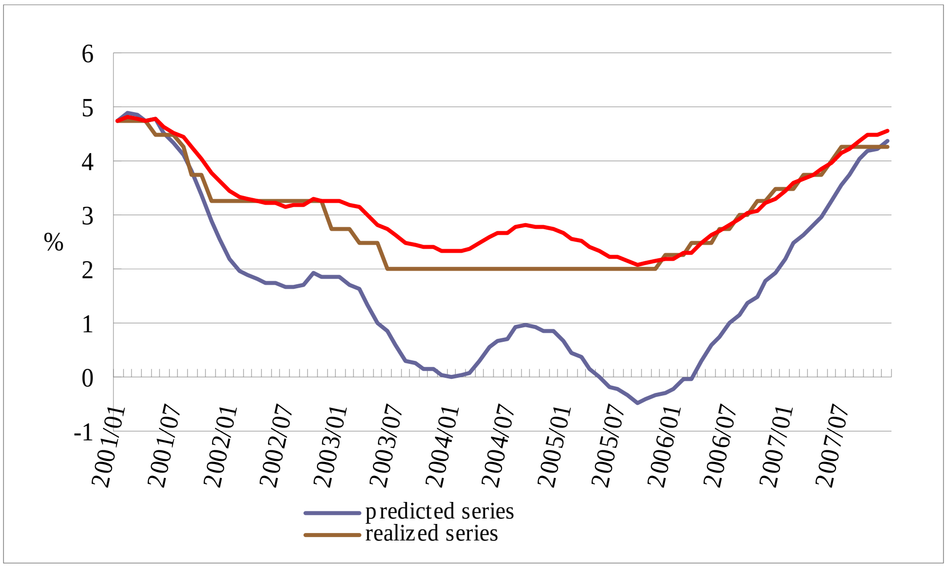

Figure 10. Predicted and Actual Rates of MRO: Case of the estimated coefficients