Tough Love and Intergenerational Altruism*

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Abstract

This paper develops and studies a tough love model of intergenerational altruism. We model tough love by modifying the standard altruism model (Barro, Becker 1974) in two ways. First, the child's discount factor is endogenously determined, so that low consumption at young age leads to a higher discount factor later in her life. Second, the parent evaluates the child's life time utility with a constant high discount factor. One of the main findings of the paper is that the comparative static result from a change in an exogenous variable in the tough love model can be obtained by the standard altruism model with a certain parametric configuration. However, this observational equivalence is broken when the child is liquidity constrained and when the child's discount factor changes exogenously. In contrast to the

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predictions of the standard altruism model that transfers from parents are independent of exogenous changes in the child's discount factor, our tough love model predicts that transfers will fall.

1 Introduction

This paper develops and studies a *tough love* model of intergenerational altruism, in which the parent is purely altruistic to the child, but exhibits tough love: he cares about long run welfare of the child and hence may allow child to suffer in the short run. The main purpose of the paper is to find circumstances in which the tough love altruism model is not observationally equivalent to the Barro-Becker standard altruism model.

The infinite horizon dynamic macro models are based on the Barro-Becker standard altruism framework, and how different generations are connected is an important economic issue that can have nontrivial policy implications. Barro (1974) explored the effectiveness of the government redistribution and found that as long as there is some form of intergenerational transfer connecting current generations and future generations there will be no net wealth effect of a marginal change in government debt. This *Ricardian equivalence* result is based on the *standard altruism model*, which is discussed in the general context of interdependent preferences in Becker (1974). In the standard altruism model, the current generation derives utility form its own consumption and the utility level attainable by his descendant.

The empirical evidence for the standard altruism model is mixed at best. For example, Cox (1987) ran a horse race between altruism and exchange motive by studying the relationship between transfers received and income of the recipient. Using President's Commission on Pension Policy (PCPP) data he found support for exchange motive being the key factor behind inter vivos transfers. Another testable implication of the standard altruism model is the *redistributive neutrality* also known as the *transfer derivative restriction*: a dollar decrease in parent's income coupled with a dollar increase in child's income will lead to a dollar decrease in transfer from parent to the child. Altonji, Hayashi and Kotlikoff 1997) used PSID data and found in fact that transfers only decrease by 13 cent and hence strongly rejected the transfer derivative restriction implied by the standard altruism model. On other hand, Laitner and Thomas(1996) used TIAA-CREFF retirees data and focused on bequest as the channel for parental altruism. They found that for the subsample of respondents characterized by willingness to leave a bequest, the projected amount of the bequest is largest for the households with lowest assessments of their children's likely earnings in future. They view this as strong evidence in favor of the standard altruism model. Horioka (2002) analyze a variety of evidence for Japan and for the United States on bequest practices and motives. His results suggest that selfish life-cycle model is the dominant model of household behavior in both countries but that is is far more applicable in Japan.

Our introspection also indicates that the standard altruism model fails to capture some aspects of parents' love for their children. Imagine that a son makes friends with lazy people, becomes lazy by their influence, and quits his job. In reality, a loving parent may decide to let the son suffer by decreasing the transfer just because the parent hopes for the son's best in the long-run.

The standard altruism model does not predict this type of parents' behavior.

In this paper we modify the standard altruism model to account for, what we call, *tough love* exhibited by parents. We model parental tough love by combining the two ideas that have been studied in the literature in various contexts.First, the child's discount factor is endogenously determined, so that low consumption at young age leads to a higher discount factor later in his life. This is based on the endogenous discount factor models of Uzawa (1968) and Becker and Mulligan (1997). Second, the parent evaluates the child's life time utility function with a constant discount factor that is higher than the child's. Since the parent is the social planner in our simple model, this feature is related to the recent models (see Caplin and Leahy 2004; Sleet and Yeltekin 2005, 2007; Phelan 2006, and Farhi and Werning 2007) in which the discount factor of the social planner is higher than the agents'.

An argument for plausibility of the type of endogenous discounting is found in Becker and Mulligan (1997). They model an individual whose discounting factor depends on how remoteness or vividness of their imagination of future pleasures.[1]

It is necessary to be careful in evaluating the empirical evidence for endogenous discounting because of two problems. First, we have the endogeneity problem in that patient people with high discounting factors tend to accumulate financial and human wealth. Thus we may find rich people have higher discounting factors than poor people even when the discount factor of an individual is decreasing in wealth as in Uzawa's (1968) model. Sec-

[1]Becker and Mulligan's model involves investment to increase vividness of the imagination. We adopt Uzawa's (1968) model and do not study this investment aspect of endogenous discounting in this paper.

ond, endogenous discounting and wealth-varying intertemporal elasticities of substitution (IES) (see, e.g., Atkeson and Ogaki 1996) can have similar implications in growing economies, and may be hard to distinguish from each other.

The endogeneity problem is addressed in Ikeda, Ohtake, and Tsutsui (2005). If they do not control the endogeneity problem in analyzing experiment data, the discount factor appears to be an increasing function of income/wealth. After they control the endogeneity problem, they find evidence in favor of the view that the discount factor is decreasing in wealth.[2]

Another way to control the endogeneity problem is to give different levels of consumption to the subjects before an experiment to see which subjects are more patient. Implementing this idea is very difficult in experiments with human subjects. Rats were used to implement this in experiments. The results were in favor of the view that the discount factor is decreasing in wealth as reported in Kagel, Battalio, Green (1995, Chapter 7, Section 3).

Using the Panel Study of Income Dynamics (PSID), Lawrance (1991) employed the Euler equation approach to estimate the endogenous discount factor model. In principle, her instrumental variable method should take care of the endogeneity problem. Lawrance found evidence in favor of the discounting factor that is increasing in wealth. However, Ogaki and Atkeson (1997) point out that Lawrance did not allow the IES to vary with wealth. Ogaki and Atkeson allow both the IES and the discount factor to vary with the wealth for panel data of households in Indian villages. They find evidence in favor of the view that the discounting factor is constant and the IES is

[2]They control the endogeneity problem by analyzing how the discount factor changes with the size of a prize obtained in another experiment.

increasing in wealth. It is possible that the discounting factor is decreasing in wealth for richer households, and Lawrance found the opposite result because she did not allow the IES to change.

Turning to the plausibility of the parent using a higher discount factor than the child, an extreme case is a parent with a new born baby. When the baby is born, it is very impatient and it cries for food all the time but the parent does not give in to this persistent demand of the baby. This is likely because the parent evaluates the baby's utility over its life time with a higher discount factor than the baby's very low discount factor.

We think that it is likely that many parents continue to evaluate their children's life time utility when they are no longer babies. Parents may continue to do this until children learn to be as patient as the parents.

In our model, these two features (endogenous discount factor of the child and the parent's evaluation with a high discount factor) lead the parent to show tough love behavior in which the parent takes into account of the influence of the amount of transfer of income to the child on the discount factor of the child.

One question is that if there is empirical evidence that parents' behavior influence their children's discount factors and other economic preferences and attitudes. While the economic literature on the effect of parenting style on child's economic attitudes and behavior is sparse, there is substantial work that addresses this issue in Psychology literature. Diana Baumrind(1966) identifies three modes of parental control. The first mode is *Permissive*: Parent act as a resource to the child and does not actively involve himself in shaping the current as well as future behavior of the child. The second mode is *Authoritarian*: Parent uses a set standard of conduct, theologically or religiously motivated and try to shape and control child's behavior with overt use of power. The third mode is *Authoritative* Parent actively involves himself in shaping up child's behavior and attitudes and uses reasoning and discipline to ensure well rounded long run development of the child. He affirms child's current behavior, separating right from wrong, and also set standards for child's future behavior.

There is substantial evidence in Psychology literature in favor of the influence of parents in the development of children's willingness to delay reward. Mischel (1961) studied children in West Indian islands of Grenada and Trinidad. He found that the children of Grenada showed greater preference for a higher reward later than a smaller immediate reward when compared with the children of Trinidad. He also found that this difference is driven mainly by the critical role fathers played in handing down the cultural values of thrift to the children of Grenada and those of immediate gratification to the children of Trinidad.

Bandura and Mischel (1965) conducted an experiment on 250 school going children to explore the effects of an adult's discounting preferences of these children. They found that children who previously were more keen to get the immediate rewards now displayed increased willingness to wait for a more valued reward at a later date following their exposure to an adult exhibiting patience.

Carlson and Grossbart(1988) used survey data on the mothers of school going children(Kindergarten through sixth grade) and divided them into groups based on the parenting style staring from neglecting through rigid controlling. They found evidence for authoritative parents granting less consumption autonomy to the child, greater communication with the child about consumption related issues, higher consumer socialization goals and greater monitoring of children's consumption vis-a-vis both permissive and authoritarian parents.

More recently Webley and Nyhus (2006) used DNB household survey data and found evidence for the hypothesis that parental orientations have an effect on the economic behavior of the children as well as economic behavior in adulthood. In their analysis they observed high degree of association between child's savings and parental savings, household income and economic socialization of the parents.

In terms of the terminology used in the psychology, the parent in the standard altruism model acts like an permissive parent while the parent in the tough love altruism model acts like an authoritative parent.

We compare the tough love model and the standard altruism model by comparative statics in a three period economy without uncertainty. These two models are often observationally equivalent in the sense that the comparative statics result from a change in exogenous variables in the tough love model can be obtained by a standard altruism model with a certain parameter configuration.

We consider changes in the child's discount factor, in the parent's income and the child's income. If there are perfect markets to allow the child to borrow and lend freely, these models are observationally equivalent to these changes. However, if the child faces borrowing constraints, the two models are no longer observationally equivalent for changes in the child's discount factor. When there an exogenous decrease in child's discount factor (an increase in impatience of the child), the optimal transfer from the parent to the child does not change in the standard altruism model, while it falls in the tough love model with reasonable parameter configurations.

The remainder of the paper is organized as follows. Section 2 explains the structure and main findings of the tough love model with only consumption good[3] and contrast the implications of the model with those of the standard altruism model, section 3 proposes two alternative models of altruism aimed toward testing the robustness of the tough love model and section 4 concludes.

2 A consumption good economy

Consider a 3 period model economy with two agents- parent and child. For simplicity we consider the case of a single parent and a single child.The salient features of the model are:

i. Parent is an altruist while the child is a non-altruist and derives utility only from his own consumption.

ii. The life of the parent and the child overlap in period 1.

iii. Transfers are made only in period 1.

iv. Income of both the parent and the child is given exogenously.

v. Child is liquidity constrained in period 1.

[3]Introducing leisure as the second good along with the assumption of perfectly observable child's effort level does not change the main results of the paper.The results for the model with leisure as the second good are available upon request.

vi. There is no uncertainty in the economy.

2.1 Standard Altruism

In this model, both parent and child use the same constant discount factor while evaluating future utility. We call this model the *standard altruism* model. The parent solves the following maximization problem,

$$\begin{aligned}\max_{T} \Bigg[&\eta v(y_p - T) + (1-\eta)\Big[u(C_1^*) + \beta_2 u(C_2^*) \\ &+\beta_2\beta_3 u(R^2(y_1 + T + \frac{y_2}{R} - C_1^* - \frac{C_2^*}{R}))\Big]\Bigg]\end{aligned} \tag{1}$$

where,

$$\begin{aligned}\{C_1^*, C_2^*\} \equiv \arg\max_{C_1, C_2} &\Big[u(C_1) + \beta_2 u(C_2) \\ +\beta_2\beta_3 u(R^2(y_1 + T + \frac{y_2}{R} - C_1 - \frac{C_2}{R}))&\Big]\end{aligned} \tag{2}$$

subject to

$$C_1 = y_1 + T \tag{3}$$

Notations:

$u(C)$: the concave utility function of the child.

$v(C)$: the concave utility function of the parent.

η : weight attached by the parent to his own utility ; $0 < \eta < 1$.

$\beta_{t,p}$: discount factor used by the parent to evaluate child's future utility.... $t = 2, 3$.

$\beta_{t,k}$: discount factor used by the child in period t[4] ... $t = 2, 3$.

y_p : parent's exogenous period 1 income.

y_i : child's exogenous period t wage... $t = 1, 2$.

C_i : child's consumption in period t ... $t = 1, 2, 3$.

T : transfer made in period 1 by the parent.

R : gross real rate of interest.

We can substitute out the borrowing constraint faced by the child in the first period of his life(equation (3)) and rewrite the parent's optimization problem as follows:

$$\max_T \left[\eta v(y_p - T) + (1 - \eta) \Big[u(y_1 + T) + \beta_2 u(C_2^*) + \beta_2 \beta_3 u(R(y_2 - C_2^*)) \Big] \right] \tag{4}$$

where,

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_3 u(R(y_2 - C_2)) \Big] \tag{5}$$

[4]In this model we have $\beta_{t,p} = \beta_{t,k} = \beta_t$

Let us focus on the child's optimization program.From the first order condition for the child's problem described in (5), we get :

$$u_{C_2}(C_2) - \beta_3 R u_{C_2}(R(y_2 - C_2)) = 0 \tag{6}$$

where,

$$u_x(x) \equiv \frac{\partial u(x)}{\partial x}$$

Assuming that the utility function satisfy conditions for the *implicit function theorem*[5], we can in principle solve (6) for C_2 as a function of the model parameters and the state variables.Formally, we get,

$$C_2^* = C_2(y_2, \beta_3, R) \tag{7}$$

So the optimal period 2 consumption for the child is independent of the period 1 transfers of the parent and hence can be dropped from the parent's optimization program. Formally, we can rewrite the parent's problem described by (4), (5), as :

$$\max_T \left[\eta v(y_p - T) + (1 - \eta) u(y_1 + T) \right] \tag{8}$$

The first order condition for the above problem is given by,

[5] u(.) is continuously differentiable with non zero *Jacobian*

$$-\eta v_T(y_P - T) + (1-\eta)u_T(y_1 + T) = 0 \tag{9}$$

Again, using the *implicit function theorem*, we get,

$$T^* = T(y_P, y_1, \eta) \tag{10}$$

2.2 Comparative Statics

There are two kind of experiments we are interested in :

i. **Exogenous Shift in the Child's Time Preference**

   Here we consider an increase in the child's impatience as captured by a fall in the discount factor β_3. From (10) optimum period 1 transfers by the parent is in fact independent of the discount factor implying that an exogenous shift in the child's time preference will have no effect on the period 1 transfers made by parents.

ii. **Exogenous Wealth Redistribution**[6]

   Here we consider a dollar increase in the parent's income, y_p, followed by a dollar decrease in the child's period 1 income, y_1. To address this question, lets go back to (9) and rewrite it as,

$$\frac{v_T(y_p - T)}{u_T(y_1 + T)} = \frac{1-\eta}{\eta} \tag{11}$$

[6] The purpose behind this experiment is to illustrate the *observational equivalence* between the standard altruism model and the tough love altruism model

Hence, for a given η the ratio of marginal utilities of consumption for the parent and the child is constant. Given a standard, continuously differentiable, concave utility function (11) implies that a dollar increase in y_p and a dollar decrease in y_1 lowers the numerator and increases the denominator and hence T must increase by exactly one dollar to keep the ratio of marginal utilities constant.This is the *redistributive neutrality* property of the standard altruism model–a dollar increase in the parent's income, y_p, followed by a dollar decrease in the child's period 1 income, y_1 increases the period 1 transfers T by exactly one dollar.

2.3 Tough Love Altruism

In this section we describe our *tough love altruism* model that in essence introduces tough love motive of the parent via asymmetric discounting preference between generations. The main difference between this model and the standard altruism model is that in this model the parent uses a constant and high discount factor to evaluate child's life time utility while the child himself uses a discount factor which is endogenously determined as a decreasing function of his period 1 consumption.

$$\beta_{t,k}(C_1) \quad ; \quad \frac{\partial \beta_{t,k}}{\partial C_1} < 0$$

With the borrowing constraint faced by the child in period 1, the discount factor is given by $\beta_{t,k}(y1 + T)$.

The underlying motivation for this type of endogeneity of the child's discount

factor is the belief that the effect of parent's transfers on child's consumption habits is strongest in the period 1. This in turn is motivated by the evidence from child psychology literature.Maital (pg 54-81) provides a good analysis of the evolution of time preferences for an economic agent.His main finding is that learning to wait begins in the childhood and to a large extent is learned from parents and friends (see Maital,1991).
Now, the parent optimizes by solving the following optimization problem,

$$\begin{aligned}\max_{T} \Bigg[& \eta v(y_p - T) + (1-\eta)\Big[u(y_1 + T) + \beta_{2,p} u(C_2^*) \\ & + \beta_{2,p}\beta_{3,p} u(R(y_2 - C_2^*))\Big]\Bigg]\end{aligned} \tag{12}$$

where,

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_{3,k}(y1 + T)u(R(y_2 - C_2))\Big] \tag{13}$$

From the first order condition for the child's problem described in (13), we get :

$$u_{C_2}(C_2) - \beta_{3,k}(y1 + T)Ru_{C_2}(R(y_2 - C_2)) = 0 \tag{14}$$

where,

$$u_x(x) \equiv \frac{\partial u(x)}{\partial x}$$

Using the *implicit function theorem* we can in principle solve (14) for C_2 as a function of the model parameters and the state variables.Formally, we get,

$$C_2^* = C_2(y_2, \beta_{3,k}(y1 + T), R) \tag{15}$$

Unlike the standard altruism model, now the optimal period 2 consumption for the child is not independent of the period 1 transfers of the parent and hence cannot be dropped from the parent's optimization program. Due to the complexity of the problem we solve the problem described in (12), (13) numerically.

2.4 Comparative Statics

In this section we compare our tough Love altruism model with the standard altruism model.An important result of our paper is that these two models are often observationally equivalent. We define *observational equivalence* as follows-the comparative static result from a change in an exogenous variable in the Tough love model can be obtained by a standard altruism model for a given specification of the preference parameters.This in principle should allow for preference parameters that can change across time periods. However, for tractability, we assume time invariant preference parameters while carrying out our simulations.For the purpose of simulation we impose the following parametrization,

$$U(x) = \frac{x^{1-\sigma}}{1-\sigma} \tag{16}$$

The discount factor is given by,

$$\beta(y1+T) = \frac{1}{1+a(y1+T)} \quad where \quad a > 0 \tag{17}$$

Hence as we increase the parameter a then for any given level of period 1 income of the child and transfers the discount factor is lower implying more impatient behavior on the part of the child.

To bring out the contrast between the two models, we again consider two experiments :

i. **Exogenous Shift in the Child's Time Preference**

The evidence for the Tough Love motive is derived by comparing how parent's alter their period 1 transfers as child becomes impatient the hypothesis being a parent who has tough love motive will try to correct child's impatience by reducing his transfers as child becomes more impatient. We make child more impatient by increasing the preference parameter a monotonically.

The results for the assumed set of model parameters' values are summarized in Table 1 The main finding of the simulation exercise is that there is a monotonic decline in period 1 transfers by parents to the child with rise in child's impatience as captured by rising value of the parameter a.As we observe from Table 1, period 1 transfers fall monotonically from 0.9990 to 0.4969 as we increase the parameter a from 0.01 to 0.5. This is in sharp contrast with the comparative statics results for the standard altruism model (section 2.2) where the optimal period 1 transfers are independent of child's discounting preference.

Table 1. Tough Love Altruism Model

Global Parameters					
$\eta = 0.5;\ \sigma = 1.5;\ R = 1.2;$ $\beta_p = 1;\ y_1 = y_2 = 3;\ y_p = 5$					
Optimum	$a = 0.01$	$a = 0.07$	$a = 0.15$	$a = 0.3$	$a = 0.5$
T^*	0.9990	0.9628	0.8795	0.7066	0.4969
C_1^*	3.9990	3.9628	3.8795	3.7066	3.4969
C_2^*	1.5651	1.6673	1.7719	1.9088	2.0276
C_3^*	1.7218	1.5992	1.4738	1.3094	1.1669
$\beta(C_1^*)$	0.9615	0.7828	0.6321	0.4735	0.3638

These results hold for a wide range of parameters including $\sigma < 1$.

ii. **Exogenous Wealth Redistribution**

In this experiment we get observationally equivalence between the tough love model and the standard altruism model. Even with tough love altruism, a dollar increase in the income of the parent and a dollar decrease in the child's income would increase parental transfers to the child by exactly one dollar. Formally,

$$\frac{\partial T^*}{\partial y_p} - \frac{\partial T^*}{\partial y_1} = 1 \tag{18}$$
$$\Leftrightarrow Redistributive \quad Neutrality$$

$\| For\ proof\ see\ Appendix\ A \|$

To summarize, the tough love model is often *observationally equivalent* to the standard altruism model. However, this observational equivalence is broken when child is liquidity constraint and experiences an exogenous change in his discount factor: *with tough love, an exogenous decrease in the child's discount factor is followed by a decline in opti-*

mal transfers.In the standard altruism model parental transfers are not affected by child's rising impatience.

3 How important is Tough Love ?

In this section we show that in order to break the observational equivalence between the standard altruism model and the tough love model, both the elements that constitute tough love in our model, higher parental discount factor and endogenous discount factor for the child, are required. For the purpose, we carry out the above mentioned comparative statics for two models of altruism, namely, the paternalistic altruism model and the endogenous altruism model and show that the paternalistic altruism model is observationally equivalent to the standard altruism model and the endogenous altruism altruism model generates counterintuitive predictions.

3.1 Paternalistic Altruism Model

In this model both parent and child use a constant discount factor to evaluate future utility. However, unlike the standard altruism model, parent and child use different discount factors in this model with the discount factor used by the parent higher than the child's discount factor, i.e.,

$\beta_{t,p} > \beta_{t,k}$

where $\beta_{t,p}$ is the discount factor used by the parent to evaluate child's future utility and $\beta_{t,k}$ is the discount factor used by the child in period t.

The parent solves the following maximization problem,

$$\max_{T} \Big[\eta v(y_p - T) + (1-\eta)\Big[u(y_1 + T) + \beta_{2,p} u(C_2^*) \\ + \beta_{2,p}\beta_{3,p} u(R(y_2 - C_2^*))\Big]\Big] \tag{19}$$

where,

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_{3,k} u(R(y_2 - C_2))\Big] \tag{20}$$

From the first order condition for the child's problem, we get :

$$u_{C_2}(C_2) - \beta_{3,k} R u_{C_2}(R(y_2 - C_2)) = 0 \tag{21}$$

Using the *implicit function* theorem we get,

$$C_2^* = C_2(y_2, \beta_{3,k}, R) \tag{22}$$

Hence the optimal period 2 consumption for the child is independent of the period 1 transfers of the parent and so it can be dropped from the parent's optimization program.

Formally, we can rewrite the parent's problem described by (19), (20) as :

$$Max_T \; \left[\eta v(y_p - T) + (1-\eta)u(y_1 + T)\right] \tag{23}$$

The first order condition for the above problem is given by,

$$-\eta v_T(y_P - T) + (1-\eta)u_T(y_1 + T) = 0 \tag{24}$$

Again, using the *implicit function theorem*, we get,

$$T^* = T(y_P, y_1, \eta) \tag{25}$$

3.2 Comparative Statics

There are two kinds of experiment we are interested in :

i. **Exogenous Shift in the Child's Time Preference**

Here we consider an increase in the child's impatience as captured by a fall in the discount factor $\beta_{3,k}$. From (25) optimum period 1 transfers by the parent is in fact independent of the discount factor implying that an exogenous shift in the child's time preference will have no effect on the period 1 transfers made by parents.

ii. **Exogenous Wealth Redistribution**

Here we consider a dollar increase in the parent's income, y_p, followed by a dollar decrease in the child's period 1 income, y_1. To address this question, lets go back to (24) and rewrite it as,

$$\frac{v_T(y_p - T)}{u_T(y_1 + T)} = \frac{1-\eta}{\eta} \tag{26}$$

Hence, for a given η the ratio of marginal utilities of consumption for the parent and the child is constant. Given a standard, continuously differentiable, concave utility function (26) implies that a dollar increase in y_p and a dollar decrease in y_1 lowers the numerator and increases the denominator and hence T must increase by exactly one dollar to keep the ratio of marginal utilities constant.This is the *redistributive neutrality* property of the standard altruism model–a dollar increase in the parent's income, y_p, followed by a dollar decrease in the child's period 1 income, y_1 increases the period 1 transfers T by exactly one dollar.

To summarize, we find that if we only introduce higher parental discount factor then the comparative static results of the paternalistic model for both exogenous shift in the discount factor of the child and exogenous wealth redistributions are observationally equivalent to those obtained under the standard altruism model.

3.3 Endogenous Altruism Model

In this model as assumed in the Tough Love model, the discount factor used by the child is endogenously determined as a decreasing function of his period

1 consumption.

$$\beta_{t,k}(c_1) \quad ; \quad \frac{\partial \beta_{t,k}}{\partial C_1} < 0$$

With the borrowing constraint faces by child in period 1, the discount factor is given by $\beta_{t,k}(y1 + T)$.

However, unlike the Tough Love model now the parent also uses the above discount factor for evaluating the child's future utility, i.e.,

$\beta_{t,p}(x) = \beta_{t,k}(x)$.

Now, the parent optimizes by solving the following optimization problem,

$$\max_{T} \Big[\eta v(y_p - T) + (1-\eta)\Big[u(y_1 + T) + \beta_{2,p}(y1+T)u(C_2^*) \\ + \beta_{2,p}(y1+T)\beta_{3,p}(y1+T)u(R(y_2 - C_2^*))\Big]\Big] \tag{27}$$

where,

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_{3,k}(y1+T)u(R(y_2 - C_2))\Big] \tag{28}$$

From the first order condition for the child's problem we get :

$$u_{C_2}(C_2) - \beta_{3,k}(y1+T)Ru_{C_2}(R(y_2 - C_2)) = 0 \tag{29}$$

Using the *implicit function theorem* we can solve (29) for C_2 as a function of the model parameters and the state variables.Formally, we get,

$$C_2^* = C_2(y_2, \beta_{3,k}(y1 + T), R) \tag{30}$$

So the optimal period 2 consumption for the child is not independent of the period 1 transfers of the parent and hence cannot be dropped from the parent's optimization program. Due to the complexity of the problem we solve the problem described in (27), (28) numerically.

3.4 Comparative Statics

In this section we compare the endogenous altruism model with the standard altruism model.An important result of our paper is that these two models are often observationally equivalent. We use the same parameterization that was used for the tough love model.

To bring out the contrast between the two models, we again consider two experiments :

i. **Exogenous Shift in the Child's Time Preference**

   The question we ask in this experiment is how parent's alter their period 1 transfers as child becomes impatient the hypothesis being a parent who has tough love motive will try to correct child's impatience by reducing his transfers as child becomes more impatient. We make child more impatient by increasing the preference parameter a monotonically.

   The results for the assumed set of model parameters' values are sum-

Table 2. Endogenous Altruism Model

Global Parameters					
$\eta = 0.5$; $\sigma = 1.5$; $R = 1.2$; $y_1 = y_2 = 3$; $y_p = 5$					
Optimum	$a = 0.01$	$a = 0.07$	$a = 0.15$	$a = 0.3$	$a = 0.5$
T^*	1.4343	2.3043	2.3704	2.2049	1.9910
C_1^*	4.4343	5.3043	5.3704	5.2049	4.9910
C_2^*	1.5672	1.7022	1.8353	1.9965	2.1298
C_3^*	1.7193	1.5573	1.3977	1.2043	1.0442
$\beta(C_1^*)$	0.9575	0.7292	0.5538	0.3904	0.2861

marized in Table 2. The main finding of the simulation exercise is that there is a non monotonic response of transfers to an exogenous increase in child's impatience as captured by rising value of the parameter a.As we observe from Table 2, period 1 transfers first increase from 1.4343 to 2.3704 as we increase the parameter a from 0.01 to 0.15. However after that the transfer falls to 2.2049 when a is 0.3 and further to 1.9910 when a is 0.5. This *reversal* is in sharp contrast with the comparative statics results for the standard altruism model described in section 2.2 where optimal period 1 transfers are independent of child's discounting preference. It is also in contrast with the monotonically declining transfers observed in the tough love model.

ii. **Exogenous Wealth Redistribution** This result is observationally equivalent to the response we got with Standard altruism model and the tough love model. So even with endogenous altruism we get the

Ricardian equivalence result :

$$\frac{\partial T^*}{\partial y_p} - \frac{\partial T^*}{\partial y_1} = \frac{\eta v''(C_p) + A2}{A1} = \frac{A1}{A1} = 1$$
$$\Leftrightarrow Redistributive \quad Neutrality \tag{31}$$

$\| For\ proof\ see\ Appendix\ B \|$

4 Are parents loving in the Tough Love Altruism Model ?

In this section we present some simulation results with the objective of illustrating that the parent in the tough love is indeed loving. For the purpose we compare the utility of the child from last two period of the child's life and show that a child with low discount factor is better off in retrospect. For the purpose we solve our tough love model for two values of the impatience parameter a - $a_0 < a_1$. As usual higher value of a reflects more impatient behavior on the part of the child. We then compare the child's life time utility using the following procedure :

Step 0 : We solve the tough love model numerically first for a_0 and obtain corresponding $C^*_{0,1}$, $C^*_{0,2}$ and $C^*_{0,3}$ respectively where $C^*_{0,i}$ denotes child's consumption level in period i when impatience parameter is a_0.

Step 1 : We then solve the tough love model numerically first for a_1 and obtain corresponding $C^*_{1,1}$, $C^*_{1,2}$ and $C^*_{1,3}$ respectively where $C^*_{1,i}$ denotes child's optimum consumption level in period i when impatience parameter is a_1.

Step 2 : We evaluate the child's truncated life time utility (ignoring period 1 consumption) for consumption stream of Step 0 above but using $\beta(C^*_{1,1})$, the endogenous discount factor obtained from step 1.Suppose we denote it by V_0.Then,

$$V_0(\beta(C^*_{1,1})) = \frac{{C^*_{0,2}}^{1-\sigma}}{1-\sigma} + \beta(C^*_{1,1})\frac{{C^*_{0,3}}^{1-\sigma}}{1-\sigma}$$

Next we evaluate the child's truncated life time utility (ignoring period 1 consumption) for consumption stream of Step 1 above using $\beta(C^*_{1,1})$, the endogenous discount factor obtained from step 1. Suppose we denote it by V_1. Then,

$$V_1(\beta(C^*_{1,1})) = \frac{{C^*_{1,2}}^{1-\sigma}}{1-\sigma} + \beta(C^*_{1,1})\frac{{C^*_{1,3}}^{1-\sigma}}{1-\sigma}$$

The results of this exercise are provided in Table 3 where we keep $a_0 = 0.01$ fixed and compute $V_0(.)$ and $V_1(.)$ for several monotonically increasing values of a_1. We find that in many cases $V_1(\beta(C^*_{1,1})) > V_0(\beta(C^*_{1,1})$ as increase the value of the impatience parameter a_1. We interpret these results as implying that parent's in the tough love model were acting in accordance with the long run welfare of the child.

Table 3. Child's Utility Comparison

a_1 (1)	$\beta(C*_{1,1})$ (2)	$V_0(\beta(C^*_{1,1}))$ (3)	$V_1(\beta(C^*_{1,1}))$ (4)
0.1856	0.5840	-2.4640	-2.4888
0.2733	0.4947	-2.3121	-2.3527
0.3611	0.4321	-2.2022	-2.2573
0.4489	0.3857	-2.1185	-2.1866
0.5367	0.3499	-2.0523	-2.1320
0.6244	0.3213	-1.9984	-2.0884
0.7122	0.2980	-1.9535	-2.0528
0.8000	0.2785	-1.9154	-2.0231

For each simulation $a_0 = 0.01$, $\eta = 0.5$, $\sigma = 1.5$ & $R = 1.2$.

5 Conclusion

In the simple setting of 3 period consumption economy with a single parent and a single child, perfect information, liquidity constrained child and exogenous income , we propose a model of intergenerational transfers wherein the tough love motive for parents is the driving force behind family exchange.As the benchmark case, we consider the optimizing behavior across two generations in the standard Becker-type Standard altruism model. For the comparison of the tough love model with the benchmark Standard altruism model we conduct two experiments, namely, an Exogenous shift in the child's time preference to the child making him more impatient and an exogenous wealth redistribution wherein child's first period income/wage is reduced by a dollar coupled by a dollar increase in parent's income.

We have considered the case of a consumption good economy.The simulation results for the tough love model, for a reasonable range of parameter values, shows that as the child become more impatient, parent react by cutting down the transfers in an attempt to inculcate a more patient consumption behavior.This is consistent with our intuition of tough love parenting. On the other hand with standard altruism, parent seem to be oblivious to the child's increasingly impatient behavior by not changing transfers monotonically with rising impatience of the child.

However, in terms of wealth redistribution we get observational equivalence with both the benchmark Standard altruism model and the tough love model predicting a dollar increase in transfers in response to a dollar decrease in child's income and a dollar increase in parent's income.

We have also compared our tough love model with two special cases, the paternalistic altruism model and the endogenous altruism model. We found that the paternalistic model is observationally equivalent to the standard altruism in all respects and the endogenous altruism model though not observationally equivalent generates ambiguous response of transfers to an exogenous shift in the child's discount factor.

As a next step, we would like to find empirical evidence for tough love altruism using household level data like National Longitudinal Survey and PSID. Another useful extension for future research is to incorporate tough love altruism model in the standard neoclassical growth model and derive the implications for the steady capital accumulation and aggregate savings for the economy.

Appendix A

Consider the parent's maximization program in the tough love model:

$$\max_{T} \left[\eta v(y_p - T) + (1-\eta)\Big[u(y_1+T) + \beta_{2,p} u(C_2^*) + \beta_{2,p}\beta_{3,p} u(R(y_2 - C_2^*))\Big]\right] \tag{A-1}$$

where,

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_{3,k}(y1+T) u(R(y_2 - C_2))\Big] \tag{A-2}$$

Now from the first order condition of the child's maximization problem,

$$u_{C_2}(C_2) - \beta_{3,k}(y1+T) R u_{C_2}(R(y_2 - C_2)) = 0 \tag{A-3}$$

Then using the implicit function theorem we get,

$$C_2^* = C2(y_2, y1+T, R) \tag{A-4}$$

In this wealth redistribution experiment we only change y_p and y_1. Hence we can treat R and y_2 as constants. Also note that from child's first period borrowing constraint,

$$C_1 = y_1 + T \tag{A-5}$$

Using these facts, we can rewrite child's optimal period 2 consumption as,

$$C_2^* = C2(C_1) \tag{A-6}$$

Substituting child's optimal second period consumption in the parent's problem we get,

$$\begin{aligned} Max_T \ \ \eta v(y_p - T) + (1-\eta)\Big[& u(y_1 + T) + \beta_{2,p} u(C2(C_1)) \\ & +\beta_{2,p}\beta_{3,p} u(R(y_2 - C2(C_1)))\Big] \end{aligned} \tag{A-7}$$

Define $C_p = y_p - T$ for notational simplicity. Then the first order condition for the parent's problem is,

$$\begin{aligned} -\eta v'(C_p) + (1-\eta)\Big[& u'(C_1) + \beta_{2,p} u'(C2(C_1)) C2'(C_1) - \beta_{2,p} \\ & \beta_{3,p} R u'(R(y_2 - C2(C_1))) C2'(C_1)\Big] = 0 \end{aligned} \tag{A-8}$$

Now we totally differentiate equation (A-8) assuming R, y_2, $\beta_{2,p}$ and $\beta_{3,p}$ are constants. We get,

$$\begin{aligned}
&-\eta v''(C_p)dy_p + \eta v''(C_p)dT + (1-\eta)\Big[u''(C_1)dy_1 + u''(C_1)dT + \beta_{2,p}u''(C2(C_1))C2'(C_1)^2 dy_1 \\
&+\beta_{2,p}u''(C2(C_1))C2'(C_1)^2 dT + \beta_{2,p}u'(C2(C_1))C2''(C_1)dy_1 + \beta_{2,p}u'(C2(C_1))C2''(C_1)dT \\
&+\beta_{2,p}\beta_{3,p}R^2u''(R(y_2 - C2(C_1)))C2'(C_1)^2 dy_1 + \beta_{2,p}\beta_{3,p}R^2u''(R(y_2 - C2(C_1)))C2'(C_1)^2 dT \\
&+\beta_{2,p}\beta_{3,p}Ru'(R(y_2 - C2(C_1)))C2''(C_1)dy_1 + \beta_{2,p}\beta_{3,p}Ru'(R(y_2 - C2(C_1)))C2''(C_1)dT\Big] = 0
\end{aligned} \tag{A-9}$$

From equation (A-9) it is straightforward to show that,

$$\frac{\partial T^*}{\partial y_p} = \frac{\eta v''(C_p)}{A1} \tag{A-10}$$

$$\frac{\partial T^*}{\partial y_1} = -\frac{A2}{A1} \tag{A-11}$$

where

$$\begin{aligned}
A1 \equiv \eta v''(C_p) + (1-\eta)\Big[&u''(C_1) + \beta_{2,p}u''(C2(C_1))C2'(C_1)^2 + \beta_{2,p}u'(C2(C_1))C2''(C_1) \\
&+\beta_{2,p}\beta_{3,p}R^2u''(R(y_2 - C2(C_1)))C2'(C_1)^2 + \beta_{2,p}\beta_{3,p}Ru'(R(y_2 - C2(C_1)))C2''(C_1)\Big]
\end{aligned} \tag{A-12}$$

and

$$A2 \equiv (1-\eta)\Big[u''(C_1) + \beta_{2,p}u''(C2(C_1))C2'(C_1)^2 + \beta_{2,p}u'(C2(C_1))C2''(C_1) +$$

$$\beta_{2,p}\beta_{3,p}R^2u''(R(y_2 - C2(C_1)))C2'(C_1)^2 + \beta_{2,p}\beta_{3,p}Ru'(R(y_2 - C2(C_1)))C2''(C_1)\Big] \tag{A-13}$$

Hence,

$$\frac{\partial T^*}{\partial y_p} - \frac{\partial T^*}{\partial y_1} = \frac{\eta v''(C_p) + A2}{A1} = \frac{A1}{A1} = 1 \tag{A-14}$$
$$\Leftrightarrow Redistributive \quad Neutrality$$

Appendix B

Consider the parent's maximization program in the endogenous altruism model:

$$\max_{T} \left[\eta v(y_p - T) + (1-\eta)\Big[u(y_1 + T) + \beta_{2,p}(y1 + T)u(C_2^*) + \beta_{2,p}(y1+T)\beta_{3,p}(y1+T)u(R(y_2 - C_2^*))\Big]\right] \tag{B-1}$$

where,

$$\{C_2^*\} \equiv \arg\max_{C_2} \Big[u(C_2) + \beta_{3,k}(y1 + T)u(R(y_2 - C_2))\Big] \tag{B-2}$$

From the first order condition for the child's problem we get :

$$u_{C_2}(C_2) - \beta_{3,k}(y1+T)Ru_{C_2}(R(y_2 - C_2)) = 0 \tag{B-3}$$

Using the *implicit function theorem* we get,

$$C_2^* = C_2(y_2, \beta_{3,k}(y1+T), R) \tag{B-4}$$

In this wealth redistribution experiment we only change y_p and y_1. Hence we can treat R and y_2 as constants. Also note that from child's first period borrowing constraint,

$$C_1 = y_1 + T \tag{B-5}$$

Using these facts, we can rewrite child's optimal period 2 consumption as,

$$C_2^* = C2(C_1) \tag{B-6}$$

Substituting child's optimal second period consumption in the parent's problem we get,

$$\max_T \Bigg[\eta v(C_p) + (1-\eta)\Big[u(C_1) + \beta_{2,p}(C_1)u(C_2(C_1)) + \beta_{2,p}(C_1)\beta_{3,p}(C_1)u(R(y_2 - C_2(C_1)))\Big]\Bigg] \tag{B-7}$$

where,

$$\begin{aligned} C_p &= y_p - T \\ C_1 &= y_1 + T \end{aligned} \tag{B-8}$$

F.O.C for the parent's problem,

$$\begin{aligned} \Bigg[-\eta v'(C_p) + (1-\eta)\Big[u'(C_1) + \beta'_{2,p}(C_1)u(C_2(C_1)) + \beta_{2,p}(C_1)u'(C_2(C_1))C_2'(C_1) \\ \beta'_{2,p}(C_1)\beta_{3,p}(C_1)u(R(y_2 - C_2(C_1))) + \beta_{2,p}(C_1)\beta_{3,p}(C_1)'u(R(y_2 - C_2(C_1))) \\ -R\beta_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C_2'(C_1)\Big]\Bigg] = 0 \end{aligned} \tag{B-9}$$

Now we totally differentiate equation (B-9) assuming R and y_2 are constants. We get,

$$\Big[-\eta v''(C_p)dy_p + \eta v''(C_p)dT + (1-\eta)\Big[u''(C_1) + \beta''_{2,p}(C_1)u(C_2(C_1)) + 2\beta'_{2,p}(C_1)u'(C_2(C_1))C'_2(C_1)$$

$$+\beta_{2,p}(C_1)u''(C_2(C_1))C'_2(C_1)^2 + \beta_{2,p}(C_1)u'(C_2(C_1))C''_2(C_1) + \beta''_{2,p}(C_1)\beta_{3,p}(C_1)u(R(y_2 - C_2(C_1)))$$

$$+2\beta'_{2,p}(C_1)\beta'_{3,p}(C_1)u(R(y_2 - C_2(C_1))) - R\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)$$

$$+\beta_{2,p}(C_1)\beta''_{3,p}(C_1)u(R(y_2 - C_2(C_1))) - R\beta'_{2,p}(C_1)\beta'_{3,p}(C_1)u(R(y_2 - C_2(C_1)))C'_2(C_1)$$

$$+R^2\beta'_{2,p}(C_1)\beta'_{3,p}(C_1)u(R(y_2 - C_2(C_1)))C'_2(C_1)^2 - R\beta_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C''_2(C_1)$$

$$-R\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)$$

$$-R\beta_{2,p}(C_1)\beta'_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)\Big](dy_1 + dT)\Big] = 0 \tag{B-10}$$

From equation (B-10) it is straightforward to show that,

$$\frac{\partial T^*}{\partial y_p} = \frac{\eta v''(C_p)}{A1} \tag{B-11}$$

$$\frac{\partial T^*}{\partial y_1} = -\frac{A2}{A1} \tag{B-12}$$

where,

$$A1 \equiv \eta v''(C_p) + (1-\eta)\Bigg[u''(C_1) + \beta''_{2,p}(C_1)u(C2(C_1)) + 2\beta_{2,p}(C_1)'u'(C2(C_1))C2'(C_1)$$
$$+\beta_{2,p}(C_1)u''(C2(C_1))C2'(C_1)^2 + \beta_{2,p}(C_1)u'(C2(C_1))C2''(C_1)$$
$$+\beta''_{2,p}(C_1)\beta_{3,p}(C_1)u(R(y_2 - C_2(C_1))) + 2\beta'_{2,p}(C_1)\beta'_{3,p}(C_1)u(R(y_2 - C_2(C_1)))$$
$$-R\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1) + \beta_{2,p}(C_1)\beta''_{3,p}(C_1)u(R(y_2 - C_2(C_1)))$$
$$-R\beta_{2,p}(C_1)\beta'_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1) + R^2\beta_{2,p}(C_1)\beta_{3,p}(C_1)u''(R(y_2 - C_2(C_1)))C'_2(C_1)^2$$
$$-R\beta_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C''_2(C_1) - R\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)$$
$$-R\beta_{2,p}(C_1)\beta'_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)\Bigg] \tag{B-13}$$

$$A2 \equiv (1-\eta)\Bigg[u''(C_1) + \beta''_{2,p}(C_1)u(C2(C_1)) + 2\beta_{2,p}(C_1)'u'(C2(C_1))C2'(C_1)$$
$$+\beta_{2,p}(C_1)u''(C2(C_1))C2'(C_1)^2 + \beta_{2,p}(C_1)u'(C2(C_1))C2''(C_1)$$
$$+\beta''_{2,p}(C_1)\beta_{3,p}(C_1)u(R(y_2 - C_2(C_1))) + 2\beta'_{2,p}(C_1)\beta'_{3,p}(C_1)u(R(y_2 - C_2(C_1)))$$
$$-R\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1) + \beta_{2,p}(C_1)\beta''_{3,p}(C_1)u(R(y_2 - C_2(C_1)))$$
$$-R\beta_{2,p}(C_1)\beta'_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1) + R^2\beta_{2,p}(C_1)\beta_{3,p}(C_1)u''(R(y_2 - C_2(C_1)))C'_2(C_1)^2$$
$$-R\beta_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C''_2(C_1) - R\beta'_{2,p}(C_1)\beta_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)$$
$$-R\beta_{2,p}(C_1)\beta'_{3,p}(C_1)u'(R(y_2 - C_2(C_1)))C'_2(C_1)\Bigg] \tag{B-14}$$

Hence,

$$\frac{\partial T^*}{\partial y_p} - \frac{\partial T^*}{\partial y_1} = \frac{\eta v''(C_p) + A2}{A1} = \frac{A1}{A1} = 1 \quad \Leftrightarrow Redistributive \quad Neutrality \tag{B-15}$$

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