The quantitative effects of monetary aggregate targeting in a zero interest rate environment: results from Japan

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Abstract

What are the effects of monetary aggregate targeting in a zero nominal interest rate environment? This paper develops a computable overlapping generations model to answer this question. In our model there are two sources of real balance effects: finite lifespans and borrowing constraints. Steady-state results reveal an asymmetry in the welfare costs of alternative monetary policies. A monetary aggregate targeting policy that is too tight has large and negative effects on welfare. A loose monetary aggregate targeting policy has much smaller effects on welfare. A dynamic analysis using data from Japan finds that the "quantitative easing"policy reduced deflationary pressure. Although the effects of this policy on GNP growth are small, there are important distributional effects. The biggest beneficiaries of this policy are the young who experience an easing in borrowing constraints and the old who benefit from lower taxes and higher interest rates. Individuals in other age groups experience consumption losses.

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1 Introduction

In April 1999 the Japanese call rate dropped to 0.01 percent. It remained at or about this level with the exception of a short period in 2000 until March 2006. With the nominal interest rate at effectively zero, the question arose as to what further actions the Bank of Japan could take to stimulate the Japanese economy of if not that at least quicken the end of deflation. In March 19, 2001 the Bank of Japan initiated a quantitative easing policy that targeted the level of excess reserves.[1] This policy was pursued until the call rate was raised to 0.25 percent in March 2006.

The objective of this paper is to use an economic model to assess the effects of quantitative easing on the price level and economic activity. When the nominal interest rate is zero money and short-term government bonds are perfect substitutes. In the infinite horizon models that are commonly used for analyzing the effects of monetary policy, a change in the timing of total government liabilities (including money) and lump-sum taxes has no real effects on economic activity. Krugman (1998) refers to this phenomenon as a liquidity trap. Ireland (2005) shows that allowing for positive population growth in a Blanchard (1985) model with infinite lived overlapping generations breaks Ricardian equivalence and induces a real balance effect. Auerbach and Obstfeld (2005) find that large open-market purchases of bonds can counteract deflationary price tendencies and lower the real value of government debt if households expect that the nominal interest rate will eventually rise above zero. Lower government debt reduces the need to tax and this raises household welfare.

We consider the effects of monetary aggregate targeting in a computable overlapping generations (OG) model. We choose this model because it produces real balance effects and its realizations can readily be compared with Japanese macroeconomic outcomes. The model period is a year, households are active for 80 years and they can save by accumulating capital, money or bonds. Savings patterns vary by age and there are active loan markets.

There are two factors that produce real balance effects in our model. First, households are finite lived and the timing of government borrowing can affect their present value tax liabilities and induce wealth effects. Second, we impose borrowing constraints that rule out uncollateralized lending. Lowering taxes increases disposable income and increases current consumption of households who are experiencing binding borrowing constraints.

A steady-state analysis indicates that a zero nominal interest rate is a good monetary policy that maximizes average welfare. However, the risks of small mistakes are asymmetric. As the growth rate of money is lowered from its optimal value welfare declines sharply. Welfare also declines if money growth is too high. However, the welfare losses associated with too rapid money growth are low. The reason for this asymmetry is due to the fact that once the nominal interest rate reaches zero money is a direct competitor with capital. In this situation lowering the growth rate of money further increases the real return

[1]Formally, the Bank of Japan set a target on "current account balances" that greatly exceeded required reserves.

on holding money and this directly crowds out the capital stock. On the other hand, when the growth rate of money is too high money is dominated in rate of return and households keep their holdings of money low. Households' efforts to economize on their holdings of money limits the crowding out effect of money on capital. This results provides a rationale for why a central bank might want to increase monetary supply when the nominal interest rate is zero.

Our dynamic analysis finds that Japan's quantitative easing has three main effects. First, it affects the inflation rate and thus the number of periods that the nominal interest rate is zero. In our model households have perfect foresight and the anticipation of quantitative easing lowers deflationary pressure in the 1990s. When we simulate a counter-factual experiment with no quantitative easing we find that there is more deflationary pressure in the 1990s and that the nominal interest rate is zero for longer. This effect is quantitatively large. In our counter-factual the number of years when the nominal interest rate is zero increases from 7 to 12.

Second, quantitative easing relaxes borrowing constraints for young households. We allow for age-dependent labor productivity. In this situation young households face binding borrowing constraints. They would like to borrow against their future high human capital but future human capital can not be collateralized and they face binding borrowing constraints. Quantitative easing increases the overall stock of government debt and lowers current taxes. This acts to relax the borrowing constraints of young people.

Third, quantitative easing increases the real interest rate. This increases consumption of retirees and also acts to ease borrowing constraints for the young. The old also are able to escape higher future taxes due to mortality risk and this transfers the burden on to younger individuals. In our model, a higher interest rate is also associated with a lower wage rate and this lowers consumption for many working households.

When we compare quantitative easing with a counter-factual we find that both the benefits and costs of this policy are concentrated among the old. Consumption of retirees rises by as much as 2.3 percent for some retirees between 2001 and 2005. Workers who have the highest labor productivity experience loses of as high as 1.3 percent due to lower wages and a transfer of tax burdens from older individuals to them. for younger workers the benefits of relaxed borrowing constraints are largely off set by lower wages and the consumption gains are small.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 explains how we parametrize the model. Section 4 contains our results and we conclude in Section 5.

2 The Model

We consider an economy that involves in discrete time. The structure of the real side of the economy is similar to economy considered by Braun, Ikeda and Joines (2007). Their model reproduces some of the principal macroeconomic facts of the Japanese economy between 1961 and 2002.

2.1 Demographics

Agents are born and become active at age 21. The growth rate of 21 year old individuals, n_1 is assumed to be constant in each period. Agents are subject to mortality risk in each period. If we let $N_{j,t}$ be the number of households of age j in period t, the dynamics of population are governed by a first-order Markov process:

$$\mathbf{N}_{t+1} = \begin{bmatrix} (1+n_1) & 0 & 0 & \dots & 0 \\ \psi_1 & 0 & 0 & \dots & 0 \\ 0 & \psi_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \psi_{J-1} & 0 \end{bmatrix} \mathbf{N}_t \equiv \mathbf{\Gamma N}_t, \tag{1}$$

where $\mathbf{N}_t$ is a $J \times 1$ vector that describes the population of each cohort in period t, ψ_j is the conditional probability that a household of age j survives to the next period and ψ_J is implicitly assumed to be zero. The aggregate population in period t, denoted by N_t, is given by

$$N_t = \sum_{j=1}^{J} N_{j,t}. \tag{2}$$

The population growth rate is then given by $n = N_{t+1}/N_t$. The unconditional probability of surviving from birth in period $t-j+1$ to age $j > 1$ in period t is:

$$\xi_j = \psi_{j-1}\xi_{j-1} \tag{3}$$

where $\xi_{1,t} = 1$ for all t.

2.2 Problem for a household born into cohort s

Households are born with zero assets and retire at age 65. The maximum life-span of an individual is $J = 100$ years. Money is introduced by assuming that households receive utility from two goods and assuming that one of the goods, which we refer to as the cash good, c_{1t}, is subject to a cash in advance constraint as in Lucas and Stokey (1987). The credit good, c_{2t}, may be purchased with cash or on credit. Households also value leisure, l_{jt}. Given these definitions expected present value utility of a household belonging to cohort j is:

$$\sum_{j=s}^{J+s} \beta^{j-s} \xi_j u(c^s_{1jt}, c^s_{2jt}, l^s_{jt}). \tag{4}$$

The specific functional form of preferences we will consider are:

$$u(c^s_{1jt}, c^s_{2jt}, l^s_{jt}) = \gamma \ln(c^s_{1jt}) + (1-\gamma)\ln(c^s_{2jt}) + \alpha \ln(l^s_{jt}). \tag{5}$$

This choice of preferences is consistent with balanced growth.[2] A household of age s in period t, who works h_t^s hours receives nominal earnings of $P_t w_t \varepsilon_j h_{jt}^s$. In this expression P_t is the price level, w_t is the wage rate, and ε_j is an age specific efficiency. A household can save by accumulating cash, M_{t+1}^s, bonds B_{t+1}^s, or capital k_{t+1}^s.

At the start of each period households visit a financial market where claims from the previous period are settled. Households also receive a lump-sum transfer from the government T_{jt}^s, and adjust their holdings of money and bonds. Total holdings of assets are restricted by the following borrowing constraint:

$$k_{j,t+1}^s + B_{j,t+1}^s + M_{j,t+1}^s \geq 0. \tag{6}$$

This borrowing constraint rules out uncollateralized borrowing.

After the financial market closes households separate into a worker and shopper. The shopper's purchases of the cash good and investment goods in any period are subject to the following cash in advance constraint:

$$\frac{B_{j,t+1}^s}{1+R_t} + P_t[k_{j,t+1}^s - k_{j-1,t}^s] + P_t c_{1jt}^s \leq \\ M_{j-1,t}^s + T_{jt}^s + B_{j-1,t}^s + P_t(1-\tau)(r_t - \delta)k_{j-1,t} \tag{7}$$

where δ is the depreciation rate on capital and τ is a tax on capital income.[3]The household's overall budget constraint is given by:

$$M_{j-1,t}^s + T_{jt}^s + B_{j-1,t}^s + P_t w_t \varepsilon_j h_{jt}^s + P_t(1-\tau)(r_t - \delta)k_{j-1,t}^s \geq \\ P_t(c_{1jt}^s + c_{2jt}^s) + \frac{B_{j,t+1}^s}{1+R_t} + M_{j,t+1}^s + P_t[k_{j,t+1}^s - k_{j-1,t}^s]. \tag{8}$$

Given these definitions the problem for a household born into cohort j is to choose the sequence $\{c_{1t}^s, c_{2t}^s, h_t^s, M_{t+1}^s, B_{t+1}^s, k_{t+1}^s\}_{t=s}^J$ that maximizes (4) subject to (7),(8), and (6). Some important household first order necessary conditions are:

$$\xi_j \gamma / c_{1jt}^s = P_t(\mu_t + \lambda_t) \tag{9}$$

$$\xi_j (1-\gamma)/c_{2jt}^s = P_t \lambda_t \tag{10}$$

$$\xi_j \frac{\alpha}{1 - h_{jt}^s} = \lambda_t P_t w_t \varepsilon_j \tag{11}$$

$$\beta(\mu_{t+1} + \lambda_{t+1})/P_{t+1} + \phi_t = (\mu_t + \lambda_t)/\{P_t(1+R_t)\} \tag{12}$$

$$\beta(\lambda_{t+1} + \mu_{t+1})[1 + (1-\tau)(r_{t+1} - \delta)] + \phi_t = (\lambda_t + \mu_t) \tag{13}$$

$$\beta\left(\mu_{t+1} + \lambda_{t+1}\right)/P_{t+1} + \phi_t = \lambda_t / P_t \tag{14}$$

[2]More generally preferences of the form: $\ln([\gamma(c_{1jt}^s)^\sigma + (1-\gamma)(c_{1jt}^s)^\sigma]^{1/\sigma}) + \alpha \ln(1 - n_t)$ are also consistent with balanced growth.

[3]It is more common to assume that investment is not subject to the cash in advance constraint. However, the expressions for the first order conditions are a bit more convenient using this formulation and since we are considering low inflation environments the distinction between this formulation and one that treats capital as a credit good should be small.

plus the CIA constraint, household budget constraint and the borrowing constraint.

The above expressions can be rearranged to yield the following restrictions on market clearing:

$$\frac{\alpha}{\gamma}\frac{c^s_{1jt}}{1-h^s_{jt}} = w_t\varepsilon_j/(1+R_t) \tag{15}$$

$$\frac{\alpha}{1-\gamma}\frac{c^s_{2jt}}{1-h^s_{jt}} = w_t\varepsilon_j \tag{16}$$

$$\beta[1+(1-\tau_{t+1})(r_{t+1}-\delta)]\xi_{j+1}\gamma/c^s_{1j,t+1} = \xi_j\gamma/c^s_{1jt} - \phi_t \tag{17}$$

$$(1+R_t)/(1+\pi_{t+1}) = 1+(1-\tau)(r_{t+1}-\delta) \tag{18}$$

$$\phi_t(M^s_{j,t+1}+k^s_{j,t+1}+B^s_{j,t+1}) = 0, \quad \phi_t \geq 0 \tag{19}$$

$$\mu_t\left\{\begin{array}{c} M^s_{j-1,t}+T^s_{jt}+B^s_{j-1,t}-\frac{B^s_{j,t+1}}{1+R_t}+(1-\tau)(r_t-\delta)k_{j-1,t}+ \\ P_t[k^s_{j-1,t}-k^s_{j,t+1}]-P_tc^s_{1jt} \end{array}\right\} = 0, \quad \mu_t \geq 0 \tag{20}$$

$$\begin{aligned} M^s_{j-1,t}+T^s_{jt}+B^s_{j-1,t}+P_tw_t\varepsilon_jh^s_{jt}+P_t(1-\tau)(r_t-\delta)k^s_{j-1,t} = \\ P_t(c^s_{1t}+c^s_{2t})+\frac{B^s_{j,t+1}}{1+R_t}+M^s_{j,t+1}+P_t[k^s_{j,t+1}-k^s_{j-1,t}]. \end{aligned} \tag{21}$$

2.3 The Firm's Problem

Firms produce consumption goods with a constant return to scale production technology. In each period t, firms choose labor, H_t, and capital, K_t, to maximize

$$A_tK_t^{\theta}H_t^{1-\theta} - w_tH_t - r_tK_t, \tag{22}$$

where w_t is the real wage, r_t is the real rental rate on capital, A_t evolves according to

$$A_{t+1} = g_tA_t, g_t > 0.$$

2.4 The Government and aggregate feasibility constraints

The government issues bonds, money and raises revenue through a tax on asset income. Government revenue is used to finance government purchases and lump-sum transfers:

$$P_tG_t + \sum_{j=1}^{J} N_{jt}T_{jt} = \frac{B_{t+1}}{1+R_t} - B_t + M_{t+1} - M_t + P_t\tau(r_t-\delta)K_t \tag{23}$$

The government expands (nominal) money supply at the rate σ_t by making lump-sum transfers to all households alive in a given period according to:

$$M_{t+1} = (1+\sigma_t)M_t.$$

We don't formally model a social security system. Instead we will assume that accidental bequests are lump-sum transferred back to surviving members of the same cohort.

The aggregate resource constraint for this economy is:

$$A_t K_t^{\theta} H_t^{1-\theta} = \sum_{j=1}^{J} N_{jt}(c_{1jt}^s + c_{2jt}^s) + K_{t+1} - (1-\delta)K_t + G_t \tag{24}$$

2.5 Competitive Equilibrium

Definition *Competitive Equilibrium*
Given an initial population wealth distribution, $\{M_{0j}, k_{0j}, B_{0j}\}_{j=1}^{J}$, a sequence of technologies, $\{A_t\}_{t=0}^{\infty}$, and government policies, $\{\tau, M_{t+1}, B_{t+1}, G_t, \{T_{jt}^s\}_{j=1}^{J}\}_{t=0}^{\infty}$, a competitive equilibrium is a price system $\{r_t, P_t, R_t, w_t\}_{t=0}^{\infty}$ and a sequence of allocations $\{c_{jt}^s, h_{jt}^s, k_{j,t+1}^s, M_{j,t+1}^s\}_{t=0}^{\infty}$ that solves the household problem, the firms problem and satisfies the following market clearing/feasibility conditions:

$$K_{t+1} = \sum_{j=1}^{J} N_{j,t} k_{j,t+1}^s \tag{25}$$

$$H_t = \sum_{j=1}^{J} N_{jt} h_{jt}^s \tag{26}$$

$$M_{t+1} = \sum_{j=1}^{J} N_{jt} M_{j,t+1}^s \tag{27}$$

$$A_t K_t^{\theta} H_t^{1-\theta} = \sum_{j=1}^{J} N_{jt}(c_{1jt}^s + c_{2jt}^s) + K_{t+1} - (1-\delta_t)K_t + G_t. \tag{28}$$

When solving the model we will specify an initial population wealth distribution and a terminal steady-state and then solve for the transitional dynamics. We thus define a steady-state equilibrium next.

Definition *Balanced Growth Equilibrium*
Suppose that technology grows at the constant rate: $g_t = g$, and that money supply grows at a constant rate: $\sigma_t = \sigma$, and the output shares of government purchases, and government debt are constant Then a balanced growth equilibrium is a competitive equilibrium in which the real wage rate grows at the rate of output, the real interest and nominal interest rates are constant and the output shares of capital and consumption are constant.

2.6 Computation of the equilibrium.

Before we compute the equilibrium we transform the economy. This is done using the transformations:

$$\begin{aligned}\hat{K}_t = \frac{K_t}{N_t A_t^{1/(1-\theta)}}, \hat{C}_t = \frac{C_t}{N_t A_t^{1/(1-\theta)}}, \hat{B}_t = \frac{B_t}{P_{t-1} N_t A_t^{1/(1-\theta)}}, \\ \hat{M}_t = \frac{M_t}{P_{t-1} N_t A_t^{1/(1-\theta)}}, \hat{T}_t = T_t/P_t, \hat{H}_t = H_t/N_t, \hat{w}_t = \frac{w_t}{A_t^{1/(1-\theta)}}.\end{aligned} \tag{29}$$

We first describe computation of the steady-state equilibrium. We are interested in considering situations where the nominal interest rate is positive and also in situations where it is zero. In the later situation the cash in advance constraint (7) ceases to bind and the steady-state conditions are different. When $R > 0$we start by guessing the aggregate values of hours, capital real balances and lump-sum transfers $(\hat{H}_0, \hat{K}_0, \hat{M}_0, \hat{T}_0)$. Given these objects we can derive the wage and rental rates $\widetilde{w}_0, r_0$ and solve the household's problem. (Note that the inflation rate can be derived from real balances using the following equation:

$$(1+\pi) = \frac{(1+\sigma)}{(1+n)(1+g_{TFP})} \tag{30}$$

where $1 + g_{TFP} = A_t^{1/(1-\theta)}/A_{t-1}^{1/(1-\theta)}$. When $R > 0$, the solution to the household's problem uniquely determines individual demand for real balances: $\hat{M}_0^{d,s}$ for each cohort $s = \{1, ...J\}$ and labor supply for each cohort $\hat{H}^s$. However, the household's problem only determines the sum of saving in the form of capital and bonds. We denote this sum as $\hat{S}_0^s$.

Given solutions to each cohort's optimization problem we then sum over households to derive aggregate assets supplied by households: $\hat{S}_0^{'}$, aggregate labor supply: $\hat{H}_0^{'}$ and aggregate demand for real balances: $\hat{M}_0^{'}$ Given these objects we can solve for the capital stock using the fact that the stock of government bonds is exogenous and: $\hat{S}' - \hat{B} = \hat{K}'$. Then using the initial guesses of the wage rate and rental rate we can update transfers using the steady-state version of the government budget constraint:

$$\begin{aligned}\left\{\frac{(1+g_{TFP})(1+n)}{1+R} - \frac{1}{1+\pi}\right\}\hat{B} + \left\{(1+g_{TFP})(1+n) - \frac{1}{1+\pi}\right\}\hat{M}' \\ +\tau(r-\delta)\hat{K}' = \hat{G} + \hat{T}'.\end{aligned} \tag{31}$$

Finally, we update our guess of capital, labor, real balances and government transfers by taking a weighted average of the initial guess plus the new values derived from household optimization:

$$\hat{K}_1 = \lambda\hat{K}_0^{'} + (1-\lambda)\hat{K}_0 \tag{32}$$

$$\hat{H}_1 = \lambda\hat{H}_0^{'} + (1-\lambda)\hat{H}_0 \tag{33}$$

$$\hat{T}_1 = \hat{T}_0^{\prime} \tag{34}$$

$$\hat{M}_1 = \lambda\hat{M}_0^{\prime} + (1-\lambda)\hat{M}_0 \tag{35}$$

When $R = 0$, the household problem only pins down household supply of *total* assets which now consists of the sum of real balances, capital and bonds: $\hat{S}_0' = \hat{K}_0' + \hat{B} + \hat{M}_0'$. In this case we derive real balances and the capital in the following way. First, we use the fact that:

$$(1 + \pi_0) = \frac{(1 + \sigma)}{(1 + n)(1 + g_{TFP})} \tag{36}$$

to pin down the inflation rate. Then we use

$$(1 + \pi_0) = (1 + r_0)^{-1} \tag{37}$$

to pin down the real interest rate. Given the real interest rate we derive a new guess of the capital stock, $\hat{K}_0'$, from aggregate labor supply plus the marginal product pricing relationship:

$$r_0 = (1 - \tau) \left\{ \theta \left(\hat{K}_0' / \hat{H}_0' \right)^{\theta - 1} - \delta \right\} \tag{38}$$

Then we derive real balances from the saving identity: $\hat{S}_0' - \hat{K}_0' - \hat{B} = \hat{M}_0'$. The updating of the guess proceeds in the same way as before.

When solving for the dynamic transition we proceed in an analogous way. The main distinction is that we now guess and update sequences of the form: $(\hat{H}_{i,t}, \hat{K}_{i,t}, \hat{M}_{i,t}, \hat{T}_{i,t})$ where i denotes the i^{th} iterate and t indexes time.

3 Model Parameterization

The calibration of the model is based on the calibration in Braun, Joines and Ikeda (2007). The preference discount rate β is calibrated to reproduce the average capital output ratio between 1984 and 2000. This results in a value of 0.97. The leisure weight in preferences, α is set to reproduce the value of labor input in the Japanese economy between 1984 and 2000. This yields $\alpha = 2.6$. The capital share parameter is set to 0.362 which is the average value of capital's share of GNP between 1984 and 2000. The depreciation rate calibrated in the same way is 0.085. The average tax rate on asset income over the same period is 0.46. The labor tax rate is currently zero. We assume a constant population growth rate of 1 percent per year. The ratio of government purchases in output is set to 0.14, its average value between 1984 and 2000 and the government debt ratio is set to 0.22. Below we will report results for a range of different average growth rates of money. We set the share weight on cash goods, $\gamma = 0.07$. This choice reproduces the ratio of real balances of monetary base to output between 1984 and 1994.

4 Steady-state Analysis

Considerable intuition into the workings of our model can be found by conducting a comparative steady-state analysis. The steady-state also exhibits an asymmetry that provides a new rationale for expanding money supply when the nominal interest rate is zero. We now turn to discuss these findings.

Table 1 reports the steady-state properties of our model for alternative settings of the growth rate of money. These results allow for age specific variation in the efficiency of work effort and assume that the population growth rate is 1 percent, the growth rate of TFP is 1.9 percent, the share of government purchases in output is 14.4 percent and the government debt ratio is 21.7 percent. These correspond to the average value of these variables in Japanese data over the 1984 to 2000 sample period. Table 1 has several noteworthy features. First, observe that there are a range of monetary policies that implement a zero nominal interest rate in our economy. Interestingly, the welfare maximizing choice occurs when the nominal interest rate reaches zero and is associated with a growth rate of money that declines at a rate of 1.96 percent per year. If we define the Friedman Rule as a policy that sets the nominal interest rate to zero as in e.g. Chari, Christiano and Kehoe (1991), then the Friedman Rule maximizes average welfare in our economy. Bhattacharya, Haslag and Russell (2005) consider the optimality of the Friedman rule in 2 period overlapping generations models and find that it is not optimal in their setting. One distinction between our setting and theirs is that we allow agents to borrow against their first period labor earnings.

Notice, that there is an asymmetry in the welfare costs of alternative growth rates of money. The welfare loss associated with large growth rates of money (e.g. 7 percent) is modest. However, lowering the growth rate of money below -1.96 percent has much larger effects on welfare. For instance, the welfare associated with negative money growth of -2.2 percent is about the same as welfare associated with a positive money growth of 7 percent per annum!

This asymmetry in welfare losses reflects the fact that monetary policy affects economic activity in a different way when the nominal interest rate is zero. When the average growth rate of money is higher than the optimal level, monetary policy acts as a tax on labor supply and capital. Households act to limit their holdings of cash and this limits the incidence of this tax. This can readily be seen in Figure 1. Higher growth rates of money are associated with lower consumption of cash goods. However, cash goods only constitute 14.8 percent of total consumption under the Friedman rule. The effect on the capital output ratio and thus the real interest rate is also modest when the growth rate of money exceeds -1.96 percent.

To understand why the welfare losses associated with money growth rates that are lower than the Friedman rule it is important to recall that under the Friedman rule money and capital earn the same real return. When the growth rate of money is instead lowered below its welfare maximizing level, the inflation rate falls and this directly increases the real return on money. This makes money a more attractive asset to households. Holdings of real balances rise and holdings of private capital fall in order to insure that the capital stock continues to earn the same return as money. It is worth pointing out that a lower growth rate of money is also associated with higher lump-sum taxes. This is because the government is offering the same real rate of return on money as it does on its other liabilities. Figure 1 indicates that these effects produce a sharp decline in welfare.[4]

This asymmetry has implications for the conduct of monetary policy. Even if we maintain the hypothesis that the model is correctly specified there is uncertainty about the values of the exogenous variables and model parameters. To illustrate how parameter uncertainty can affect policy decisions we conducted another steady-state analysis assuming that the growth rate of TFP is 4 percent rather than 2 percent. In this scenario welfare is maximized when the growth rate of money is -0.61 percent. However, setting the growth rate of money to -1.96 percent, which is the *optimal* monetary policy in Table 1, induces very large welfare losses. These results suggest that when a monetary authority finds itself in a zero interest rate environment there is a very good reason for pursuing an expansionary monetary policy.

One limitation of this steady-state analysis is that it is difficult to produce an empirically plausible calibrated specification of the model in which a zero nominal interest rate is associated with deflation and a positive growth rate of money as occurred in Japan between e.g. 2001 and 2005. We will see that these outcomes can be accounted for when one conducts a dynamic path analysis.

5 A Dynamic Analysis of "Quantitative Easing"

We now use the model to investigate the effects of monetary aggregate targeting in Japan during its recent episode with zero nominal interest rates. The collapse of Japan's bubble economy was associated with a steady decline in the uncollateralized call rate on overnight loans from 7.4 percent in 1990 to 0.06 percent in 1999. The nominal interest rate remained at effectively zero until 2006. Once the nominal interest rate reached zero policy makers discussed a variety of options for using monetary policy to stimulate the economy. The outcome of this discussion was the "Quantitative Easing" policy that was adopted on March 19, 2001. This policy which targeted the level of bank deposits at the Bank of Japan was effectively an excess reserve targeting policy. The Bank of Japan announced an end to the quantitative easing policy in March 9, 2006. But, it kept the call rate at zero until July 14, 2006 at which point the call rate was increased to 0.25 percent.

We investigate the quantitative effects of this policy using dynamic perfect foresight simulations. Chen, Imrohoroglu and Imrohoroglu (2007) and Braun, Ikeda and Joines (2007) have previously found that computable general equilibrium models that allow for variation in TFP and demographics can account for some of the principal movements in real economic activity in Japan from 1960 through 2002. Here we abstract from demographic variation and model only variation in TFP and government debt. Our government debt series is taken from Braun, Joines and Ikeda (2007). They construct a government debt series

[4] The asymmetry we are documenting here is even larger if investment is treated as a credit good. If investment is a credit good the welfare cost of 7 percent inflation is smaller. However, the welfare cost of too much deflation remains essentially unchanged.

following the methodology of Broda and Weinstein (2005). The initial period of our simulation is taken to be 1984. The initial wealth distribution is taken from the terminal steady-state but is rescaled to reproduce the capital stock in Japanese data in 1984.

We are interested in reproducing variations in the nominal interest rate and monetary base during the period 1986-2006. There are two issues that arise in doing this. First, when the nominal interest rate is zero the composition of government liabilities is indeterminate. Open market operations that exchange money for bonds have no real effects when the nominal interest rate is zero. Monetary policies that alter the total amount of outstanding government debt do have real effects. However, it is hard to tell what fraction of quantitative easing should be interpreted as having altered the amount of outstanding government debt. We dealt with these issues in the following way. First, we treated the sequence of government debt and the nominal interest rate as exogenous and solved for the equilibrium. The resulting sequence of real balances and lump-sum transfers does a good job of reproducing the path of the ratio of M0 to GNP in the period up to 1997. In the period after that though the model understated this ratio. Next we used the fact that when the nominal interest rate is zero the composition of government liabilities is indeterminate to adjust the composition of government liabilities so that we reproduce the actual trajectory of M0 to GNP during the period 1997-2006.

The resulting trajectories for M0 to GNP for the model and data are reported in Figure 1. We also report other descriptive statistics from the model and the data in the same figure. The general fit of the model is reasonably good. Our model successfully reproduces the increase in the capital-output ratio and the general pattern of decline in output relative to trend that Japan has experienced since 1990. The model somewhat understates the overall decline in output. This is due to the fact that we have abstracted from the demographic changes analyzed in Braun, Joines and Ikeda (2007) and changes in the number of holidays as in e.g. Hayashi and Prescott (2002). Consequently the model fails to produce a large enough decline in labor input. Modeling these factors brings labor input down more during this period. The model also systematically understates the inflation rate. Broda and Weinstein (2007) argue that problems in price measurement induce an upward bias of about 2 percentage points in the Japanese inflation rate. If we subtract 2 percent from the actual data, the model reproduces the overall level of the inflation rate and also some of its principal movements between 1986 and 2005.

Next we turn to assess the effects of quantitative easing. Table 2 assesses the price implications of quantitative easing. This table contains results from three scenarios: the first scenario assumes that the growth rate of money is 3 percent per year between 2000 and 2006, the second scenario is our baseline with quantitative easing and the third scenario is a counter-factual that keeps the ratio of real balances in output high for an additional seven year. The first scenario is a counter-factual that illustrates what might have happened if there had been no quantitative easing while the third scenario illustrates what might have happened of quantitative easing were pursued for a longer period of time.

This is accomplished by adjusting the growth rate of money to keep the ratio of real balances high until 2013. Between 2006 and 2013 the ratio of M0 to GNP averages 21.6 percent in this simulation as compared to 6.6 percent in the baseline simulation.

The results reported in Table 2 show that alternative monetary aggregate targeting rules have important effects on the trajectory of the inflation rate and the trajectory of the nominal interest rate. During the 1990s the 3 percent M0 monetary growth simulation exhibits inflation rates that average about 2 percent lower than the baseline. If the period of quantitative easing is extended the inflation rate that is about 1.4 percent higher than the baseline during the 1990s. Interestingly, these alternative policies have only minor implications for the inflation rate between 2001 and 2005. After 2006 the difference between the baseline and 3 percent M0 growth simulations is small. However, the third simulation exhibits considerably lower inflation rates after 2006.

These differences in the inflation rate across the three simulations have interesting implications for the trajectory of the nominal interest rate. The nominal interest rate falls to zero earliest in the 3 percent M0 growth simulation. It is zero from 1994 to 2006 or 6 years longer than in the baseline model which exhibits a zero nominal interest rate between 2000 and 2006. In the longer quantitative easing simulation the nominal interest rate falls to zero 2000 but remains zero until 2020.

Looking across these results we see that quantitative easing produces a higher overall average inflation rate than the 3 percent money growth policy.The inflation rates are respectively -2.7 percent and -3.4 percent. Longer quantitative easing produces an even higher average inflation rate of -2.4 percent. However, the nominal interest rate is zero for the shortest number of years under the quantitative easing monetary policy.

Table 2 also reports the real interest rate and wage implications of the three simulations. There are two noteworthy results. First, quantitative easing produces higher real interest rates in all periods than the 3 percent M0 growth counter-factual and extending the period of quantitative easing increases the real interest rate even more. Second, in our model a higher real interest rate is associated with a lower wage. These two properties of the model will be helpful when we discuss the welfare implications of quantitative easing below.

Table 3 reports simulation results for quantity variables. The implications of quantitative easing for the real economy are small. Output growth and consumption growth are nearly identical prior to 2000 under the Baseline and 3 percent M0 growth simulations. There are somewhat larger differences under the longer quantitative easing simulation. Output and consumption growth, for instance, are both lower during the 1990s. These same variables exhibit slightly more growth between 2001 and 2005 in this simulation as compared to the baseline simulation. However, the general conclusion that emerges is that the implications of quantitative easing for real macro-economic activity are small.

In the steady-state analysis above we saw that a tight monetary policy could crowd out private capital in a zero interest rate environment. How important is crowding out in a dynamic environment. Table 4 reports the ratio of real balances to output and the capital output ratio for the three simulations. Before discussing these results it should be pointed that in the dynamic analysis we are considering transitory changes in monetary policy. The ratio of real balances to output and the debt output ratios are the same in both the initial and terminal steady-states.

The dynamic effects of quantitative easing on real balances are different from the steady-state effects. We saw in the steady-state analysis above that a higher growth rate of money was associated with lower real balances. Here we see that a temporary increase in the growth rate of money acts to increase real balances between 2001 and 2005. The mechanism that is operating here is different. Here quantitative easing is temporarily increasing government indebtedness and lowering current taxation. Some individuals are benefiting from temporarily low taxes. Higher government debt crowds out private capital. This is why the baseline simulation exhibits lower capital output ratios than the 3 percent M0 simulation in all but the final sub-sample. This effect is more pronounced when quantitative easing is extended until 2013.

Next we turn to consider the micro-economic effects of quantitative easing. Quantitative easing acts to relax borrowing constraints in the period between 2000 and 2006 In our economy agents have age specific efficiencies. Young agents face binding borrowing constraints. They would like to shift consumption forward from future periods when their income is higher but are unable to collateralize their future high human capital. Under quantitative easing the number of periods that agents are borrowing constrained falls from 17 in 2000 to 12 in 2003. For purposes of comparison, in the 3 percent M0 growth simulation young individuals are borrowing constrained for the first 17 to 19 years of life during the 2000 to 2006 period. Table 5 reports some summary statistics. Notice that the average number of cohorts that households are constrained is 16.2 in the baseline simulation and 17.6 in the simulation with longer quantitative easing as compared to 16.6 in the 3 percent M0 growth simulation. There are two factors that work to relax borrowing constraints. First, the real interest rate is higher under quantitative easing. This induces households to save more and consume less. Secondly, taxes are lower between 2001 and 2005. In Table 5 lump-sum taxes are 7.8 percent of consumption between 2001 and 2005 in the 3 percent M0 growth scenario. Under the baseline simulation they fall to 6 percent of consumption. Issuing more currency reduces lump-sum taxation. Lower taxes increase disposable income and households can consume more. These two benefits of quantitative easing to the young, though are balanced against lower wages (see Table 2) and also the fact that the benefits of lower taxes are transient. As can be seen in Table 5 taxes rise quickly after 2006 in the baseline specification.

Consumption for households aged 21-35 rises by as much as 1.4 percent in 2000 and 2002 with the largest consumption gains concentrated among the youngest individuals. If we average over the 2000-2005 period individuals who are aged 21-25 in 2000 experience higher average consumption than the 3 percent M0 growth simulation. However, the increase is small and less than 0.2 percent per year.

Quantitative easing benefits older individuals most. For retirees higher a real interest rate increases the value of their saving and consumption increases. Moreover, older retirees enjoy the benefits of lower taxes and pass away before taxes rise. The magnitude of these increases are substantial. All individuals aged 75 and older as of 2000 see their consumption rise. Individuals who are aged 81 to 96 all experience consumption gains that exceed 2 percent per year.

Most other individuals experience lower average consumption between 2000 and 2005. For these individuals borrowing constraints are not binding and a temporary lower tax rate has no benefits since they must pay higher taxes in future years. In fact, for these individuals their future tax liabilities loom large since some of the burden has been transferred from the oldest households on to them. Moreover, for these individuals the benefit of a higher interest rate is more than offset by a lower wage rate. Individuals who are at or near their peak labor productivity experience the largest consumption declines. These declines amount to 1.3 percent per year for individuals aged 59 through 64 in 2000.

6 Conclusion

We have developed a model that is consistent with the facts from Japan. Our model reproduces some of principal movements in the real economy between 1986 and 2006. Our model also reproduces the evolution of nominal variables and in particular nominal interest rates of zero against a background of very rapid growth in the monetary base. Our results suggest that lower growth of monetary base would have been associated with more deflation in the 1990s and a longer period of zero nominal interest rates. Lengthening the period of quantitative easing lowers inflation after 2006 and also lengthens the period when nominal interest rates are zero.

We have found that quantitative easing was not effective policy for stabilizing output between 2001 and 2005. Output under quantitative easing was nearly the same as in a counter-factual with no quantitative easing. Quantitative easing did have important distributional effects. Young individuals experience some mild benefits due to relaxed borrowing constraints.

The most important effects of quantitative easing are for older individuals. Workers near the peak of their life-time earnings efficiencies experienced consumption losses due to lower wages and a higher tax burden. Older retirees experience substantial consumption gains since they on average don't survive long enough to face higher taxes.

In future work we plan to relax our current assumption that the government budget constraint is met by altering lump-sum taxes and instead make the more realistic assumption that a distortionary tax is adjusted instead. This will likely introduce stronger non-neutralities. Our model generates borrowing and lending in equilibrium. It is consequently a good framework for modeling financial intermediation and central bank lending. In future work we plan to pursue these extensions.

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Figure 1
Japanese Data and Model with Age Dependent Labor Efficiency.

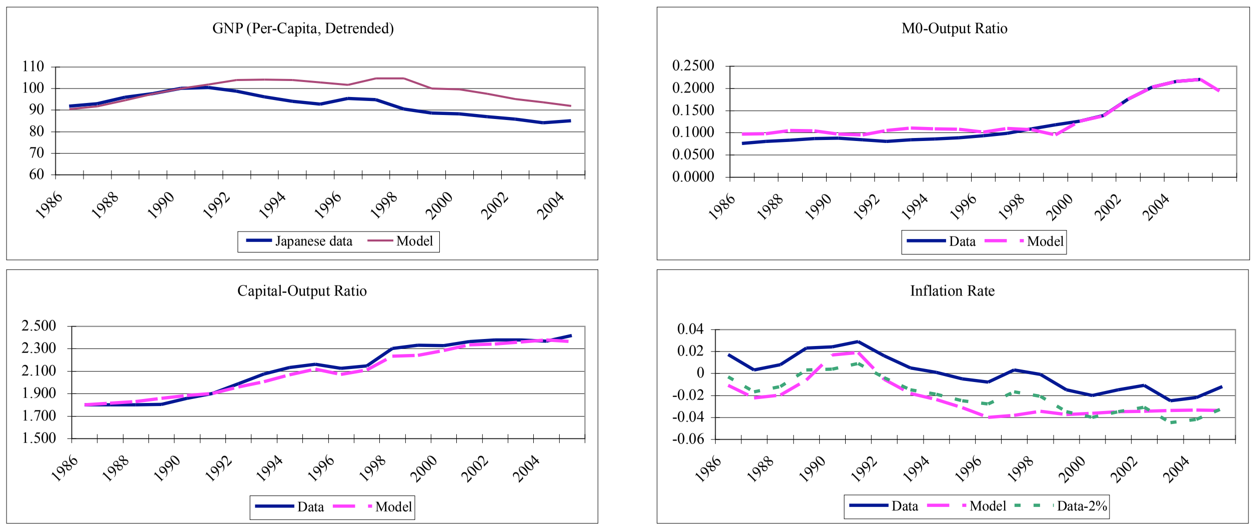

Table 1
Model Steadystates for Alternative Growth Rates of Money
Specification with Age Specific Efficiency Units

Growth rate of money (Percentage)	Cash Good Consumption*	Credit Good Consumption*	Money Output ratio	Capital Output Ratio	Output*	Real Interest Rate (Percentage)	Inflation Rate (Percentage)	Nominal Interest rate (Percentage)	Welfare
7.00	91.3	100.0	0.09	2.00	98.0	5.2	4.0	9.4	-64.20
3.00	95.0	100.0	0.09	2.02	98.8	5.1	0.1	5.2	-64.09
0.00	97.8	100.0	0.10	2.04	99.5	5.1	-2.9	2.1	-64.00
-1.96	100.0	100.0	0.10	2.05	100.0	5.0	-4.8	0.0	-63.95
-2.10	99.7	99.7	0.22	2.02	99.1	5.1	-4.9	0.0	-64.12
-2.20	99.4	99.5	0.32	2.00	98.6	5.3	-5.0	0.0	-64.27
-2.30	99.1	99.3	0.42	1.97	98.0	5.4	-5.1	0.0	-64.42
-2.40	99.1	99.2	0.53	1.95	97.5	5.5	-5.2	0.0	-64.59

* Cash good consumption, credit good consumption and output are expressed as a percentage of the respective variable under the Friedman rule.

Table 2

Prices

Period	Inflation			Nominal Interest rate			Real Interest rate			Wage rate		
	3% M0 Growth	Baseline	Longer Quantitative Easing	3% M0 Growth	Baseline	Longer Quantitative Easing	3% M0 Growth	Baseline	Longer Quantitative Easing	3% M0 Growth	Baseline	Longer Quantitative Easing
1991-1995	-3.69	-0.24	0.88	0.60	3.73	5.07	5.12	5.21	5.19	0.94	1.00	1.00
1996-2000	-3.99	-3.64	-1.89	0.00	0.28	1.79	4.15	4.32	4.34	1.00	1.05	1.05
2001-2005	-3.36	-3.46	-3.54	0.00	0.00	0.00	3.48	3.58	3.68	1.04	1.10	1.10
2006-2010	-3.18	-3.13	-3.50	0.32	0.45	0.00	3.49	3.56	3.63	1.04	1.11	1.10
2011-2015	-2.96	-2.91	-3.88	0.86	0.93	0.00	3.83	3.86	4.04	1.02	1.08	1.07

Table 3
Quantities

Year	Consumption*			Saving			labor input*			Output*		
	3% M0 Growth	Baseline	Longer Quantitative Easing	3% M0 Growth	Baseline	Longer Quantitative Easing	3% M0 Growth	Baseline	Longer Quantitative Easing	3% M0 Growth	Baseline	Longer Quantitative Easing
1991-1995	2.63	2.27	2.34	11.50	10.89	10.77	1.01	1.00	1.00	2.69	2.59	2.47
1996-2000	1.92	1.97	1.76	10.54	10.67	10.37	1.00	1.00	0.99	1.28	1.37	1.28
2001-2005	0.99	0.93	0.97	7.25	7.44	7.34	0.97	0.97	0.97	−0.15	−0.05	−0.01
2006-2010	0.78	0.88	0.81	4.43	4.25	4.69	0.97	0.97	0.96	0.89	0.93	0.79
2011-2015	1.73	1.75	1.78	3.65	3.65	3.24	0.97	0.97	0.97	1.58	1.60	1.84

* Consumption and Output are percentage growth rates. Labor input is expressed relative to the baseline simulation value of labor input between 1991-1995

Table 4

Real Balances, Capital and Total Debt as a Fraction of GNP

Period	Ratio of real balances to output			Ratio of capital to output		
	3% M0 growth	Baseline	Longer Quantitative Easing	3% M0 growth	Baseline	Longer Quantitative Easing
1991-1995	0.13	0.11	0.10	1.98	1.96	1.97
1996-2000	0.14	0.11	0.10	2.19	2.15	2.15
2001-2005	0.16	0.19	0.17	2.37	2.34	2.31
2006-2010	0.09	0.09	0.21	2.37	2.35	2.33
2011-2015	0.06	0.06	0.17	2.27	2.27	2.22

Table 5

Taxes, interest rate and borrowing constraints

	Lump-sum taxes*			Real interest rate			Number of borrowing constrained cohorts		
	3% M0 Growth	Baseline	Longer Quantitative Easing	3% M0 Growth	Baseline	Longer Quantitative Easing	3% M0 Growth	Baseline	Longer Quantitative Easing
1991-1995	7.0	6.9	7.0	5.12	5.21	5.19	17.8	18.4	18.6
1996-2000	2.8	1.3	1.1	4.15	4.32	4.34	14.4	14.4	14.8
2001-2005	7.8	6.0	6.2	3.48	3.58	3.68	17.6	16.2	16.6
2006-2010	17.9	19.6	13.8	3.49	3.56	3.63	18.0	18.6	15.4
2011-2015	17.7	17.7	21.8	3.83	3.86	4.04	22.0	22.0	17.2

* Lump-sum taxes are expressed as a percentage of average consumption.