Does Ambiguity Matter? Estimating Asset Pricing Models with a Multiple-Priors Recursive Utility [1]

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Abstract

This paper considers continuous time asset pricing models with stochastic differential utility incorporating decision makers' concern with ambiguity on true probability measure. In order to identify and estimate key parameters in the models, we use a novel econometric methodology developed recently by Park (2008) for the statistical inference on continuous time conditional mean models. The methodology only imposes the condition that the pricing error is a continuous martingale to achieve identification, and obtain consistent estimates, of parameters. Under a representative agent setting, we empirically evaluate alternative preference specifications including a multiple-prior recursive utility. Our empirical findings are summarized as follows: Relative risk aversion is estimated around 2-5 with ambiguity aversion and 6-14 without ambiguity aversion. Related, the estimated ambiguity aversion is both economically and statistically significant and including the ambiguity aversion clearly lowers relative risk aversion. The elasticity of intertemporal substitution (EIS) is higher than 1, around 2-5 with ambiguity aversion, and quite high without ambiguity aversion. The identification of EIS appears to be fairly weak, as observed by many previous authors, though other aspects of our empirical results seem quite robust.

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1 Introduction

Since the seminal papers by Hansen and Singleton (1982) and Mehra and Prescott (1985), a large body of work has sought after more relevant forms of the preferences of economic agents to explain asset market behaviors. The main reason for this direction of the study is because time-separable expected utility functions equipped with a constant relative risk aversion (CRRA) impose a potentially restrictive relationship between the risk aversion and intertemporal substitution. Specifically, under power utility models, the elasticity of intertemporal substitution (EIS) is given by the reciprocal of the coefficient of relative risk aversion, which may result in various complications, such as equity premium, volatility and interest rate puzzles. Epstein and Zin (1989, 1991) investigated an important generalization of the standard power utility model by considering a class of recursive utility functions.[5] They provide a theoretical framework in which the agent can have distinct attitudes toward intertemporal substitution and risk. This flexibility may offer a possible solution for various asset price anomalies because a high (low) risk aversion does not necessarily imply a low (high) elasticity of intertemporal substitution.[6]

Even though the recursive utility model allows the distinction between risk aversion and willingness to substitute intertemporally, the preference toward Knightian uncertainty or ambiguity is difficult to model within the original recursive utility framework due to the assumption of single prior held by investors. However, the Ellsberg paradox suggests that decision makers prefer an unambiguous situation, other things being equal. In response to this, Gilboa and Schmeidler (1989) built a multiple-priors model to incorporate ambiguity aversion in an atemporal setting.[7]

Epstein and Wang (1994) develop a dynamic version of Gilboa and Schmeidler in a discrete-time framework. Chen and Epstein (2002) focused on the formulation of utility in continuous time that allows a distinction between risk aversion and ambiguity aversion, as well as the distinction from EIS. In order to achieve the additional dimension of flexibility, they extended the continuous time version of the recursive utility (stochastic differential utility) investigated by Duffie and Epstein (1992), such that the model includes a set of priors rather than a single prior.

According to Chen and Epstein (2002), the economic agents will have multiple prior beliefs on the state of the nature, and they form a set of expectations based on their beliefs. Due to the fact that fundamental shock processes are Brownian motion, the degree of ambiguity is described by an additional term distorting the conditional mean component

[5] The basic structure of recursive utility is due to Koopmans (1960) and Lucas and Stokey (1984), which decompose a utility function into current consumption and future utility in a non-linear fashion. In this context, Epstein and Zin (1989) can be regarded as a stochastic extension of the recursive utility framework.

[6] In addition, this preference has a preference ordering for temporal resolution of uncertainty. Recently, Bansal and Yaron (2004) and Hansen, Heaton, and Li (2008) exploit this aspect to explain equity premium puzzle together with a time-varying, conditional mean component. Kim, Lee, Park, and Yeo (2008) develop a stochastic volatility model with two asymptotic regimes and transition regimes and show that this type of preference can explain aversion to uncertainties in regimes.

[7] Simply put, they assume that economic agents have a class of probability distributions, say $\mathcal{P}$ on some events in a measurable space $(\Omega, \mathcal{F})$. Then the agents will make decisions following a max-min rule. For instance, if the agent decides consumption c to maximize utility $u(c)$, she solves $\max_c \min_{Q \in \mathcal{P}} E^Q[u(c)]$

of the implied asset return processes and the decision maker chooses a probability measure using the maxmin principle following Gilboa and Schmeidler (1989).[8]

Despite the appealing features of the multiple-priors recursive utility model, there has been little econometric work on estimating the model compared to other utility specifications. The purpose of this paper is to identify the important preference parameters, such as the ambiguity aversion as well as the elasticity of intertemporal substitution and risk aversion coefficients, and compare the extent to which each model explains financial market data.

The multiple-priors recursive utility model has a multi-factor beta representation of asset returns; (i) covariance between returns and consumption growth, (ii) covariance between returns and aggregate wealth return, and (iii) covariance between returns and ambiguity.[9] However, this structure makes identification of the model difficult because the aggregate wealth and the volatility of returns are unobservable latent variables, and there is a lack of econometric methodology for estimating continuous time models.

With regard to the unobservable aggregate wealth, several approaches have been suggested. The baseline approach would be to use a market return as a proxy for the aggregate wealth return (e.g., Epstein and Zin (1991), Bakshi and Naka (1997) and Normandin and St-Amour (1998)). However, the aggregate wealth portfolio is a broader measure than the market portfolio, because the former includes human capital, natural resources, and housing wealth etc. as well as the financial wealth. Therefore, the market return only covers a subset of the aggregate wealth returns. Another approach is to use a specific structure for the unobservable wealth by incorporating the dynamics of consumption growth and utility continuation value (e.g., Chen et al. (2008)) Given the imposed structure, the aggregate wealth is implicitly given by consumption and utility continuation value. Therefore, this approach enables them to replace the unobservable wealth return with the specific structure imposed on the consumption and the future utilities. Chen et al. (2008) exploits the Euler equation to estimate future continuation utility in a non-parametric way.

Although this method is attractive, it is difficult to use in our continuous-time framework handling mixed frequencies of data. Instead, we consider a different approach to overcome the difficulties from the unobservable aggregate wealth. The aggregate wealth return is a return on the claim which gives a stream of future consumption. In this sense, the consumption of each period is financed by the aggregate wealth return, and therefore, we can think of the aggregate wealth as the sum of financial wealth and human capital, which are the two largest sources of the income in an economy. That is, the unobservability of aggregate wealth falls mostly on the human wealth. Following Campbell (1993), we assume that the proportion of the financial wealth to the human wealth is stationary, and moreover, the labor income is homogeneous of degree one with respect to the human wealth. In this case, the unobservable wealth can be substituted by a linear combination of market return

[8] There exists a related line of work on robust decision making. Hansen and Sargent (2001) and their co-authors emphasize 'model uncertainty' and the concern on the misspecification, which is similar to ambiguity aversion à la Gilboa and Schmeidler.

[9] Note that this representation, especially in closed form is available only in continuous time due to Girsanov transformation which allows different subjective probability measures to be expressed via tilting the drift component in an equilibrium asset pricing equation.

and labor income growth. This simple structure makes the asset pricing formula tractable so that we can directly compare the results of alternative models.

For econometric evaluation of the model, we consider a martingale regression developed by Park (2008). The martingale regression estimator is a minimum distance estimator based on discrepancy between empirical distribution and normal distribution. The spirit of the estimator is similar to the GMM estimator for the nonlinear Euler equation models (e.g. Hansen and Singleton (1982)). If the parameter is the true parameter, the conditional expectation of pricing error will be zero and otherwise it is non-zero for any time interval. This implies that the pricing error as a process will be a continuous martingale, and moreover, it can be transformed into a Brownian motion by a proper time change. Especially, if the time change is defined by a generalized inverse of the quadratic variation of the pricing error process, then the error process read after time change will be given as a Brownian motion. Then we define our martingale estimator as a minimizer of a Cramer-von Mises distance between an empirical distribution of normalized error increments and the standard normal distribution.

There are several attractive features of our estimator. First, as mentioned earlier, the estimation does not assume any parametric model for the volatilities. Therefore it is robust to possible misspecification of the volatility process. For example, many empirical works on the financial data suggest that stock returns possess time-varying or stochastic volatilities while the exact nature of the volatilities is difficult to find in general. In this case, possible misspecification of the volatilities may occur, but the martingale estimator will be robust to the misspecification error. Second, the martingale estimator does not use the orthogonality condition to identify the true parameters. It utilizes the well-known time change theorem of Dambis (1965), Dubins and Schwarz (1965) that the martingale is essentially a Brownian motion, but only different with respect to its quadratic variations. Therefore, the martingale estimator is robust to any kind of endogeneity problem.

Last but not least, this method allows applied econometricians to directly tackle asset pricing models written in continuous time. Many asset pricing models are developed in continuous time partly because of its mathematical elegance. However, we believe that it is also because continuous time models better describe the observation that financial markets clear at a very high frequency. Choosing a model to estimate in a relevant frequency can greatly reduce the possibility of data missaggregation bias and decision bias. Alas, macro variables such as consumption growth are sampled at a lower frequency. To deal with this issue, we use a non-parametric method to compute volatilities of macro variables. According to our robustness checks with parametric estimations, our results are very robust. Thus, our estimation strategy can be understood as a semi-parametric approach to deal with mixed frequencies of data. Although it is still far from resolving those fundamental issues, this paper attempts to initiate a baby step toward this goal.

Using daily data on asset returns and monthly and quarterly macroeconomic data from 1960 to 2006, we estimated several specifications of recursive utility framework. According to our results, relative risk aversion is estimated around 2-5 with ambiguity aversion and 6-14 without ambiguity aversion. In addition, the estimates of ambiguity aversion is both economically and statistically significant. Ambiguity can be a source of uncertainty which may require premium to bear. Therefore, our results suggest that risk aversion parameter can have an upward bias sans an adjustment for ambiguity aversion. Another important preference parameter is the elasticity of intertemporal substitution (EIS). Recently, estimating the EIS has drawn much attention and existing studies report a wide range of values including even negative numbers. According to our estimations, the EIS is higher than 1; specifically 2-5 with ambiguity aversion, and quite high without ambiguity aversion. We find that the objective function of our minimum distance estimator measured by the Cramer-von Mises statistic is very flat around the values of the reciprocal of the EIS between 0 and 1.5. Based on extensive robustness checks, we argue that the weak identification issue of the EIS parameter results from the combination of smooth variations of consumption growth and parametric restrictions imposed in preferences.

The remainder of the paper begins with describing our theoretical model in Section 2. For comparison, we also consider other baseline models, which can be considered as special cases of our model. Section 3 accounts for the econometric methodologies in detail. Section 4 shows and discusses our main results. Then we conclude in Section 5.

2 A Recursive Utility Model with Ambiguity Aversion

Consider a probability space $(\Omega, \mathcal{F}, P)$ which describes the uncertain nature of the economy. Define a standard one dimensional Brownian motion (W_t) on $(\Omega, \mathcal{F}, P)$, and the Brownian filtration $(\mathcal{F}_t)_{0\leq t\leq T}$, where $\mathcal{F}_t$ is the σ-field generated by $(W_s)_{s\leq t}$. The time horizon $T \in (0, \infty]$ is finite. Suppose that the representative decision maker does not know the true probability measure and has to choose a subjective probability measure from the set of all priors $\mathcal{P}$, which are uniformly absolutely continuous with respect to the true P in $\mathcal{P}$.[10]. Duffie and Epstein (1992) show that for a fixed consumption process C and a probability measure $Q \in \mathcal{P}$, there exists a utility process V_t^Q uniquely solving

$$V_t^Q = \mathbb{E}^Q \left[\int_t^T f(C_s, V_s^Q) ds \middle| \mathcal{F}_t \right], \quad 0 \leq t \leq T, \tag{1}$$

where $\mathbb{E}^Q[\cdot|\mathcal{F}_t]$ is the conditional expectation operator and $f(C, V)$ is called a normalized aggregator function linking current consumption and the future value. From the Martingale representation theorem, we can express (1) in a differential form of

$$dV_t^Q = -f(C_t, V_t^Q)dt + \sigma_t^v dW_t^Q, \tag{2}$$

where $V_T^Q = 0$, W_t^Q is the standard Brownian motion under Q-measure, and σ_t^v is endogenously determined.

From now on, we use the following functional form

$$f(C, V) = \frac{C^{1-\beta} - \phi(\alpha V)^{\frac{1-\beta}{\alpha}}}{(1-\beta)(\alpha V)^{\frac{1-\beta}{\alpha}}} \tag{3}$$

[10] $\mathcal{P}$ is uniformly absolutely continuous with respect to P if for every $\varepsilon > 0$ there exists $\delta > 0$ such that $E \in \mathcal{F}$ and $P(E) < \delta$ imply $Q(E) < \varepsilon$, for all $Q \in P$.

for some $\phi \geq 0$, $\beta \neq 1$, $\alpha \leq 1$. This can be regarded as the continuous-time version of Kreps-Porteus utility function in which α and β measure the degree of relative risk aversion (RRA) and the elasticity of intertemporal substitution (EIS) respectively. Specifically, the RRA is measured by $(1-\alpha)$, and the EIS is $1/\beta$. In addition, following Epstein and Zin (1989), relative sizes of these two measures are related to the investor's attitude toward the speed of resolving uncertainty: If the RRA $(1-\alpha)$ is larger (smaller) than the reciprocal of the EIS (β), the investor prefers an early (a late) resolution of uncertainty. The additional feature of this model compared to the conventional recursive utility model is that the consumer chooses a probability measure from available priors, hence justifies the name, 'multiple-priors utility'. Under this extra layer of uncertainty which leads to the Ellsberg paradox, Gilboa and Schmeidler (1989) suggested the following minimax type of value function

$$V_t = \min_{Q \in \mathcal{P}} V_t^Q, \quad 0 \leq t \leq T. \tag{4}$$

The multiple-priors recursive utility is given by the lower envelope of the utility process V_t^Q which is determined by the conditional expectation of future consumption and utility values. Chen and Epstein (2002) showed that there exists a unique solution to (4) satisfying the dynamic consistency. As clearly seen from (2), the Girsanov transformation lies at the heart of constructing a set of priors $\mathcal{P}$ on $(\Omega, \mathcal{F}_T)$. Specifically, they define a density generator λ_t for which the process z_t^λ is a P-martingale, where

$$dz_t^\lambda = -z_t^\lambda \lambda_t dW_t, \quad z_0^\lambda = 1,$$

equivalently,

$$z_t^\lambda \equiv \exp\left(-\frac{1}{2}\int_0^t \lambda_s^2 ds - \int_0^t \lambda_s dW_s\right), \quad 0 \leq t \leq T.$$

Then, z_t^λ is set as the Radon-Nikodym derivative $dQ/dP|_{\mathcal{F}_t}$ and $\mathcal{P}$ is defined as the set of Q-measures produced by the density generator. Since all the prior beliefs are absolutely continuous with P, we can expect from the Girsanov's theorem that any subjective utility V^Q given an equivalent measure $Q \in \mathcal{P}$ will modify the drift function of the utility continuation process by $\lambda_t \sigma_t^v$. This is because (W_t) is the Brownian motion under P measure, but not under Q. That is, by shifting λ_t, we can generate a continuum of subjective utility functions differing in terms of probability distribution within the class of absolutely continuous multiple-priors. Chen and Epstein (2002) showed that the differential form of (4) is

$$dV_t = -\left\{f(C_t, V_t) + \max_{\lambda_t \in \mathcal{L}} \lambda_t \sigma_t^v\right\} dt + \sigma_t^v dW_t. \tag{5}$$

To further analyze the additional term in (5), assume that λ_t is bounded by some constant $\kappa > 0$. That is, the subjective beliefs have some boundary defined by κ. We can interpret this multiple-priors as the subjective beliefs for which the worst case scenario of the economic agents is confined by the case defined by κ. Hereafter, we examine the multiple-priors model with a boundary restriction for λ_t with $\kappa > 0$[11].

[11] Chen and Epstein (2002) call this specification "κ-ignorance" case.

Under the standard environment of the economy, first order conditions for optimal consumption choice can be expressed in terms of the supergradient of utility at the optimal consumption C.[12] Especially, C is optimal if

$$\Lambda_t = \exp\left\{\int_0^t f_v(C_s, V_s)ds\right\} f_c(C_t, V_t) z_t^{\lambda^*}, \quad \text{for all } t, \tag{6}$$

where Λ_t is the state-price process (or intertemporal marginal rate of substitution process, IMRS) and λ^* is the maximizer of the ambiguity compensation $\lambda_t \sigma_t^v$ for any given λ_t such that $|\lambda_t| \leq \kappa$ for all $0 \leq t \leq T$. Then the IMRS in our case is given as

$$\Lambda_t = \exp\left\{\int_0^t \left(-\phi + \frac{\phi}{1-\beta}\frac{(\phi - 1 - \beta)(C_s^{(1-\beta)} - (\alpha V_s)^{(1-\beta)/\alpha}}{(\alpha V_s)^{(1-\beta)/\alpha}}\right) ds\right\} \phi C_t^{-\beta} (\alpha V_t)^{\beta/\alpha} z_t^{\kappa}.$$

Using Ito's lemma and no arbitrage principle, we can show that

$$\begin{aligned} \frac{dp_t^i}{p_t^i} - r_t^f dt &= \mathbb{E}\left(\frac{dp_t^i}{p_t^i}\frac{d\Lambda_t}{\Lambda_t}\middle| \mathcal{F}_t\right) + \sigma_t^i dW_t \\ &= \left\{\frac{\beta\alpha}{1-\beta}\rho_c \sigma_t^c \sigma_t^i + \left(1 - \frac{\alpha}{1-\beta}\right)\rho_a \sigma_t^a \sigma_t^i + \kappa \sigma_t^i\right\} dt + \sigma_t^i dW_t, \end{aligned} \tag{7}$$

where σ_t^a is the volatility of aggregate wealth A_t for which the return is given by

$$dr_t^a = \mu_t^a dt + \sigma_t^a dW_t, \quad dA_t = A_t(dr_t^a) - C_t dt,$$

and σ_t^c is the volatility of consumption growth.

Equation (7) is a three-factor CAPM of the cross-sectional asset pricing model; the risk premium of any tradable asset with return dp^i/p^i is determined by the covariance between returns and consumption growth, covariance between returns and aggregate wealth, and covariance between returns and density generator. Notice that the standard CRRA utility specification, such as power utility, only has the first factor, while the single-prior recursive utility models (e.g. Epstein and Zin (1989, 1991) and Duffie and Epstein (1992)) have the first two factors.

In order to include unobservable wealth, we assume the wealth process A has two components - financial wealth M and human wealth H,

$$A_t = M_t + H_t. \tag{8}$$

From Ito's lemma we have

$$\sigma_t^a = \pi_t \sigma_t^m + (1 - \pi_t)\sigma_t^h, \tag{9}$$

where $\pi_t = M_t/A_t$ is the proportion of financial wealth to the total wealth at time t, and σ_t^a and σ_t^h are the diffusion coefficient of dM/M and dH/H respectively. In particular, we specify the human capital process by

$$dH_t = H_t(dr_t^h) - Y_t dt, \qquad dr_t^h = \mu_t^h dt + \sigma_t^h dW_t \tag{10}$$

[12]A supergradient for V at C is a process Λ_t with $\mathbb{E}\left\{\int_0^T \Lambda_t \cdot (C_t' - C_t)dt\right\} \geq V(C') - V(C)$ for all admissible C'. For more details, see Duffie and Skiadas (1994) and Chen and Epstein (2002).

where Y_t is real labor income at time t. Note that the labor income Y_t is financed from the return on the human capital H_t at time t. For simplicity, we assume that $\pi_t = \pi$ for all t. This is true, for instance, under steady state of the economy, in which the proportion of aggregate wealth to the financial wealth is constant over time. Moreover, we assume that the labor income is homogeneous of degree one with respect to human capital, especially, $Y_t = \psi H_t$ for some constant ψ. Furthermore, we use a proxy for market index to estimate the preference parameters. Thus, the asset pricing equation of the multiple-priors recursive utility models is expressed as

$$\begin{aligned}\frac{dp_t}{p_t} - r_t^f dt = {} & \frac{\beta\alpha}{1-\beta}\rho_c\sigma_t^c\sigma_t^m dt + \left(1 - \frac{\alpha}{1-\beta}\right)(\sigma_t^m)^2\pi dt \\ & + \left(1 - \frac{\alpha}{1-\beta}\right)\rho_y\sigma_t^m\sigma_t^y(1-\pi)dt + \kappa\sigma_t^m dt + \sigma_t^m dW_t,\end{aligned} \tag{11}$$

where p is the price of the market index, σ_t^y is the instantaneous conditional volatility of labor income growth, and ρ_c, ρ_y are the correlation coefficients of consumption growth and labor income with the market return, respectively. From now on, we turn our attention to estimating the key preference parameters (α, β, κ) in (11).

Note that (11) nests many popular asset pricing models as special cases. Thus by imposing a priori restrictions to (11), we can estimate different models to compare the common set of parameters. First three models estimated are the power (CRRA) utility case (Model I), recursive utility with financial wealth only (Model II), and a multiple-priors recursive utility with financial wealth only (Model III). Specifically, we can express Model I, II, III as

$$\begin{aligned}\frac{dp_t}{p_t} - r_t^f dt &= \beta\rho_c\sigma_t^c\sigma_t^m dt + \sigma_t^m dW_t, \\ \frac{dp_t}{p_t} - r_t^f dt &= \frac{\beta\alpha}{1-\beta}\rho_c\sigma_t^c\sigma_t^m dt + \left(1 - \frac{\alpha}{1-\beta}\right)(\sigma_t^m)^2 dt + \sigma_t^m dW_t, \\ \frac{dp_t}{p_t} - r_t^f dt &= \frac{\beta\alpha}{1-\beta}\rho_c\sigma_t^c\sigma_t^m dt + \left(1 - \frac{\alpha}{1-\beta}\right)(\sigma_t^m)^2 dt + \kappa\sigma_t^m dt + \sigma_t^m dW_t.\end{aligned}$$

As emphasized by many authors such as Epstein and Zin (1991), Bansal and Yaron (2004), Chen et. al. (2008), and Kim et. al. (2008), financial wealth may be insufficient to proxy the aggregate wealth of the representative investor. To address this issue, we include another source of risk premium resulting from labor income risk and this is the setup of (11). This is not a completely innocuous assumption because the fraction of human wealth to total wealth is assumed to be constant. However, it turns out that this restriction has little effect on empirical results according to our robustness checks.

One important observation from our empirical setting is that time-varying volatilities of macroeconomic variables and asset returns play key roles in both the conditional mean (drift) part and the error (diffusion) terms. Given the ample evidence that those volatilities are highly persistent, this makes identification of the models statistically challenging because of heteroskedasticity, endogeneity, and measurement problems. In addition, the equilibrium relationship (11) that continuously holds need to be properly treated for correct empirical evaluations with discretely sampled data points. In the below, we tackle those issues.

3 Econometric Methodology

3.1 Martingale Estimation

Here we explain how to specify and estimate our model (11). Tentatively, we assume that the volatility processes $(\sigma_t^m), (\sigma_t^c)$ and (σ_t^y) are observed. In the next subsection, we will explain in detail how we may extract theses processes. Moreover, we will set the correlation coefficients ρ_c and ρ_y of consumption and labor income with market returns, as well as the fraction π of financial wealth, to be known. These parameters will be calibrated using the values obtained or often assumed in the empirical literature. In what follows, we assume that $(\sigma_t^m), (\sigma_t^c)$ and (σ_t^y) are non-constant and time-varying, and that ρ_c and ρ_y are non-zero. These assumptions are necessary for the identification of our model.

Now we let $\theta = (\alpha, \beta, \kappa)$ be the parameter in our model with the true value $\theta_0 = (\alpha_0, \beta_0, \kappa_0)$, and define $(\Lambda_t(\theta))$ to be the state-price deflator (or IMRS) that is given by

$$\Lambda_t(\theta) = \frac{\beta\alpha}{1-\beta}\rho_c\sigma_t^c\sigma_t^m + \left(1 - \frac{\alpha}{1-\beta}\right)\{\pi\sigma_t^m + (1-\pi)\rho_y\sigma_t^y + \kappa\}\,\sigma_t^m. \tag{12}$$

Subsequently, we define the pricing error process $(Z_t(\theta))$ from our model as

$$dZ_t(\theta) = \frac{dp_t}{p_t} - r_t^f dt - \Lambda_t(\theta)dt,$$

and write

$$Z_t(\theta) = A_t(\theta) + M_t, \tag{13}$$

where $dA_t = -\{\Lambda_t(\theta) - \Lambda_t(\theta_0)\}dt$ and $dM_t = \sigma_t^m dW_t$.

It is clear that the pricing error process $(Z_t(\theta))$ is a semimartingale with the bounded variation component $(A_t(\theta))$ and the martingale component (M_t). Note in particular that (M_t) is a continuous martingale with respect to the filtration $(\mathcal{F}_t)$, to which the Brownian motion (W_t) is adapted. Furthermore, the bounded variation component $(A_t(\theta))$ vanishes if and only if $\theta = \theta_0$ under the trivial identification conditions introduced above.[13] Therefore, we may conclude that the pricing error process $(Z_t(\theta))$ becomes a continuous martingale if and only if $\theta = \theta_0$.[14]

Recently, Park (2008) developed a general methodology to estimate and test the continuous-time conditional mean model that is identified by this type of martingale condition for the error process. Below we explain how we can implement his methodology to estimate the unknown parameter θ in our model. The methodology relies on the celebrated theorem by Dambis, Dubins and Schwarz, which will be referred to the DDS theorem throughout the paper. To introduce the DDS theorem, we denote by $([M]_t)$ the quadratic variation of (M_t),

[13] As can be clearly seen, we may identify up to four unknown parameters in our model. Therefore, for instance, we may regard π as unknown and estimate it as an additional unknown parameter. However, the estimate for π is unstable and unreliable.

[14] We temporarily assume that there is no jump in the pricing error process to focus on the main idea of the methodology. Indeed, it can be applied to the processes with jumps with some simple modifications, which we will explain later in this subsection.

which is given by

$$[M]_t = \operatorname*{plim}_{|t_k|_t \to 0} \sum_k (M_{t_k} - M_{t_{k-1}})^2,$$

where $|t_k|$ is the mesh of partition (t_k) of the interval $[0,t]$. We assume that $[M]_t \to \infty$ a.s. as $t \to \infty$. Moreover, we introduce the time change (T_t), which is defined as

$$T_t = \inf\{s \geq 0 | [M]_s > t\}.$$

The DDS theorem says that if (M_t) is a continuous martingale, then there exists a standard Brownian motion B such that $M_t = B_{[Z]_t}$, or equivalently,

$$M_{T_t} = B_t.$$

The Brownian motion B is called the DDS Brownian motion of M. See, e.g., Revuz and Yor (2005) for the proof and more discussions about the DDS theorem. In most applications, $([M]_t)$ is strictly increasing, in which case T is just the time inverse of $([M]_t)$. Roughly, the DDS theorem implies that if we read a continuous martingale using a clock that is running at a speed inversely proportional to its quadratic variation, it reduces to a Brownian motion.

If we apply the time change to the original pricing error process $(Z_t(\theta))$, then we may deduce from (13) that

$$Z_{T_t}(\theta) = A_{T_t}(\theta) + M_{T_t} = A_{T_t}(\theta) + B_t.$$

Therefore, we may now claim that $(Z_{T_t}(\theta))$ becomes the standard Brownian motion if and only if $\theta = \theta_0$, due to the DDS theorem. Obviously, the bounded variation component $(A_{T_t}(\theta))$, even after time change, vanishes when and only when $\theta = \theta_0$. The martingale method by Park (2008) uses this fact and defines the value of θ, which makes the time-changed pricing error process best approximate the standard Brownian motion, to be the martingale estimator of the unknown parameter θ_0. It is important to note that we may obtain the time change (T_t) without any knowledge on the true parameter value θ_0, since the bounded variation component contributes nothing to the quadratic variation of a semimartingale. Therefore, for instance, the quadratic variation $([M]_t)$ of the martingale component, which is required to get the time change (T_t), is identical to the quadratic variation of (P_t), say, $dP_t = dp_t/p_t - r_t^f dt$, i.e., $d[M]_t = d[P]_t = d[p]_t/p_t^2$.

To implement the methodology, we set $\Delta > 0$ to be fixed,[15] and consider the normalized increments of the pricing error process that are given by

$$z_i(\theta) = \frac{1}{\sqrt{\Delta}} \left\{ Z_{T_{i\Delta}}(\theta) - Z_{T_{(i-1)\Delta}}(\theta) \right\}$$

for $i = 1, \ldots, N$. The discrete samples $(z_i(\theta))$ of size N obtained for each $\theta \in \Theta$ are then used to estimate the unknown parameter θ_0. Recall that the samples are obtained from the pricing error processes as their increments over the random intervals $[T_{(i-1)\Delta}, T_{i\Delta}]$ for

[15]The choice of Δ is more of an empirical matter, which we will discuss in detail later in our empirical section.

$i = 1, \ldots N$. It is quite clear that $(z_i(\theta))$ are i.i.d. normals for $\theta = \theta_0$, regardless of the choice of Δ. For all other values of $\theta \in \Theta$, this is not true at least for some value of Δ.

We let $z_i^d(\theta) = (z_i(\theta), \ldots, z_{i-d+1}(\theta))$ be the d-dimensional random vector consisting of d-adjacent samples starting from $i = 1, \ldots, N - d + 1$, so that $(z_i^d(\theta))$ is the d-dimensional standard multivariate random vector, i.e., the multivariate normal random vector with mean zero and identity covariance matrix, under $\theta = \theta_0$. Moreover, we denote by $\Phi_N(\cdot, \theta)$ the empirical distribution of $(z_i^d(\theta))$ for each $\theta \in \Theta$, and define the criterion function Q_N by

$$Q_N(\theta) = \int_{-\infty}^{\infty} \{\Phi_N(x, \theta) - \Phi(x)\}^2 \, d\Phi(x),$$

where Φ is the distribution function of the d-dimensional multivariate standard normal random vector. The martingale estimator $\hat{\theta}_N$ of θ_0 is then defined as the minimizer of the criterion function Q_N, i.e.,

$$\hat{\theta}_N = \operatorname*{argmin}_{\theta \in \Theta} Q_N(\theta).$$

The martingale estimator is therefore a minimum-distance estimator with the Cramer-von Mises (CvM) distance between the empirical distribution of the sample under the unknown parameter values and the distribution under the true parameter values. Park (2008) shows that this type of minimum distance estimator is consistent, and asymptotically normal, under mild regularity conditions. The asymptotic variance of the estimator can be obtained by the usual subsampling method.

To introduce the main idea of the methodology more effectively, we assume thus far that the pricing error process $(Z_t(\theta))$ is observed continuously in time for all $\theta \in \Theta$. This, of course, is not true in our analysis, as is the case for virtually all other potential applications. The methodology can be easily implemented and all the theoretical results continue to hold for discretely sampled observations, as long as the sampling intervals are sufficiently small relative to the time horizon of the samples. This was shown in Park (2008). For our empirical analysis, we use daily observations over approximately fifty years. The necessary modifications required to deal with discretely observed samples are largely trivial and obvious. To obtain the time change, for instance, we use the realized variance of (P_t), $dP_t = dp_t/p_t - r_t^f dt$, given by

$$[P]_t^\delta = \sum_{i\delta \leq t} (P_{i\delta} - P_{(i-1)\delta})^2,$$

instead of its quadratic variation $([P]_t)$, if (P_t) is observed at intervals of length $\delta > 0$ over time horizon $[0, T]$ with $T = n\delta$, where n is the size of discrete samples.

Finally, we may readily allow for the existence of jump components in our model (11). Indeed, we may easily deal with the presence of discrete jumps in our methodology, simply by discarding the observations of (P_t), $dP_t = dp_t/p_t - r_t^f dt$, over the random time interval $[T_{(i-1)\Delta}, T_{i\Delta}]$ that is believed to have jumps. All other procedures in our methodology are valid for the remaining observations. In our empirical studies, we use the Hausman-type test of Barndorff-Nielsen and Shephard (2006) for the detection of jumps for each of the random intervals $[T_{(i-1)\Delta}, T_{i\Delta}]$, $i = 1, \ldots, N$. Although it is well-known that the jumps are frequently observed for many intra-day samples, it appears that jumps are rare for the samples of daily or lower frequency observations. We detected some evidence of jumps in our daily observations, but their number is relatively small.

3.2 Measuring Volatilities of Macroeconomic Variables

Now we explain how to extract the volatility processes (σ_t^c) and (σ_t^y). It is much more difficult than to extract the volatility process (σ_t^m), since the observations on their underlying processes are available at relative low frequencies like many other macroeconomic variables. As we explained in the previous subsection, (σ_t^m) can be readily measured and estimated by the realized variance of market returns at high frequencies.[16] However, the identification and estimation of volatility for the processes that are not observed at high frequencies are not straightforward. In the paper, we let the underlying process (X_t) follow an Ito-diffusion

$$\frac{dX_t}{X_t} = \mu_t dt + \sigma_t dB_t,$$

where (B_t) is the standard Brownian motion, and consider the problem of estimating (σ_t), $\sigma_t = \sigma_t^c$ or σ_t^y, under some realistic assumptions, using discrete samples (X_{t_j}) of (X_t). It is assumed in our setup here that the sampling intervals $t_j - t_{j-1}$, $j = 1, \ldots, m$, are not sufficiently small.

Over the interval $[t_{j-1}, t_j]$, we have

$$\int_{t_{j-1}}^{t_j} \frac{dX_t}{X_t} = \int_{t_{j-1}}^{t_j} \mu_t dt + \int_{t_{j-1}}^{t_j} \sigma_t dB_t. \tag{14}$$

For many macroeconomic variables, the values of the level X_t is relatively much larger than its increment $X_t - X_{t_{j-1}}$ in any of the intervals $[t_{j-1}, t_j]$ of frequency such as monthly and quarterly. Therefore, it seems reasonable to approximate $\int_{t_{j-1}}^{t_j} dX_t/X_t$ by $(X_{t_j} - X_{t_{j-1}})/X_{t_{j-1}}$, i.e., the growth rate of (X_t) over the interval $[t_{j-1}, t_j]$, for $j = 1, \ldots, m$.[17] Moreover, if we assume the drift term (μ_t) is continuous, then there exists $s_j \in [t_{j-1}, t_j]$ such that $\mu_{s_j}(t_j - t_{j-1}) = \int_{t_{j-1}}^{t_j} \mu_t dt$ for all $j = 1, \ldots, m$, by the mean value theorem. If, furthermore, (μ_t) varies smoothly over time, then we may approximate (μ_{s_j}) by (μ_{t_j}). This appears to be realistic in our case, so we assume that (μ_t) is an exogenous function of time for which these approximations are valid. Given the assumption, the drift term (μ_t) can be consistently estimated by the standard nonparametric method applied to (14). We adopted the local linear estimation, using the least squares cross-validation method to obtain the optimal bandwidth parameter. The reader is referred to Li and Racine (2007, p.83) for more details.

[16] See, e.g., Barndorff-Nielsen and Shephard (2002) for more discussions on the estimation of volatility processes using high-frequency data.

[17] Note that the approximation error is given by $\int_{t_{j-1}}^{t_j} (X_t - X_{t_{j-1}})/(X_t X_{t_{j-1}}) dX_t$ and $(X_t - X_{t_{j-1}})/(X_t X_{t_{j-1}}) \approx 0$ for many macroeconomic variables including those we consider here.

We exploit two different approaches to extract the volatility process (σ_t^2). First, we consider

$$\left(\int_{t_{j-1}}^{t_j} \frac{dX_t}{X_t} - \int_{t_{j-1}}^{t_j} \mu_t dt\right)^2 = \int_{t_{j-1}}^{t_j} \sigma_t^2 dt + \left\{\left(\int_{t_{j-1}}^{t_j} \sigma_t dB_t\right)^2 - \int_{t_{j-1}}^{t_j} \sigma_t^2 dt\right\}, \tag{15}$$

the left-hand side of which we may approximate well using discrete observations (X_{t_j}) of (X_t) as explained above. Note that

$$\mathbb{E}\left\{\left(\int_{t_{j-1}}^{t_j} \sigma_t dB_t\right)^2 - \int_{t_{j-1}}^{t_j} \sigma_t^2 dt \,\middle|\, \mathcal{F}_{t_{j-1}}\right\} = 0$$

for $j = 1, \ldots, m$.

As for the drift term (μ_t), we may regard the diffusion term (σ_t) as an exogenous function of time varying smoothly over intervals $[t_{j-1}, t_j]$ for all $j = 1, \ldots, m$. In this case, we may approximate in (15)

$$\int_{t_{j-1}}^{t_j} \sigma_t^2 dt = \sigma_{s_j}^2 (t_j - t_{j-1}) \approx \sigma_{t_j}^2 (t_j - t_{j-1}),$$

where $s_j \in [t_{j-1}, t_j]$, $j = 1, \ldots, m$, and the volatility process (σ_t) can be estimated by the standard nonparametric method such as the local linear estimation. We use this approach to extract the volatility processes (σ_t^c) and (σ_t^y), again with the optimal choice of bandwidth based on the least squares cross-validation. The volatility processes extracted by this method appears to be overly smooth, though they correctly represent the overall fluctuations of the underlying processes.

Second, we suppose that the volatility process is stochastic with an additional source of randomness. For this approach, we let the volatility process (σ_t) be random, but remain to be constant over each of the intervals $[t_{j-1}, t_j]$, $j = 1, \ldots, m$. More specifically, we set

$$\int_{t_{j-1}}^{t_j} \sigma_t dB_t = \sigma_j (B_{t_j} - B_{t_{j-1}}) \tag{16}$$

and (σ_j^2) to be driven by the logistic transformation of a latent autoregressive factor (w_j), i.e.,

$$\sigma_j^2 = a + \frac{b}{1 + \exp\{-c(w_j - d)\}}$$

with $w_j = \rho w_{j-1} + \varepsilon_j$, where (ε_j) is assumed to be an i.i.d. sequence of standard normals. Note that (16) is the standard Gaussian volatility model in discrete time. We let (ε_j) be correlated with the Brownian motion (B_t) to allow for the leverage effect. The model parameters $a > 0, b > 0, c > 0$ and d determine the actual volatility function. In particular, a and $a + b$ represent the two asymptotic values of volatility, and c and d respectively the speed and location of transition.

The volatility model introduced above was developed and investigated recently by Kim, Lee and Park (2008). The model can be regarded as an extension of the usual discrete-time stochastic volatility model, which relies on the autoregressive modeling for the logarithmic transformation of volatility. The former is indeed much more flexible than the latter, and has implications that are much more realistic. The latent factor (w_j) and unknown parameters a, b, c and d can be estimated by the density-based Kalman filter, or by the Bayesian method using Gibbs sampling method. The reader is referred to Kim, Lee and Park (2008) for more details about the computation procedure and comparison with other existing discrete-time stochastic volatility models.

4 Empirical Results

4.1 Data

We use daily returns on the S&P500 index including dividends and monthly real per capita consumption of nondurables plus services as a measure of the market return and aggregate consumption. The data covers January 1, 1960 to December 29, 2006. The real consumption of nondurables plus services is obtained by the Bureau of Economic Analysis. The consumption level is adjusted by mid-month population from the Bureau of Economic Analysis in order to get per capita observation. We identify the parameters based on the martingale regression of market return. Hence, the time change and new observation in volatility time will be based on the market returns. This will provide a simpler form of the three-factor CAPM result because the original form in Equation (11) includes the market return in its IMRS. Especially, the covariance between the market return and the MRS will include the variance of the market return (or $\int(\sigma_t^{m2}dt)$ and the covariance between the market return and the consumption growth (or $\int \rho_c \sigma_t^c \sigma_t^m dt$ if their correlation is constant). We use the definition of labor income in Lettau and Ludvigson (2001), which is, wages and salaries plus transfer payments plus other labor income minus personal contributions for social insurance minus taxes. We use the quarterly labor income in real per capita, which is provided by Martin Lettau's webpage.

4.2 Implementation

Our martingale estimation framework involves the time change which enables us to observe the asset return in the volatility time, not in the usual calendar time. Before calculating the time change, one needs to preset a constant volatility length Δ which determines the degree on how often the data should be observed in terms of the volatility time. Since the total quadratic variation is finite for most of asset returns observed in finite time horizon, it is easy to deduce that higher volatility length would imply lower number of samples and vice versa. Common sense will choose the smallest Δ to obtain the largest number of samples. This is because usual estimators are more efficient as the number of samples gets larger. Adopting this idea, we find the volatility length Δ which is the smallest among all the admissible values of Δ. Note that the admissible range of Δ is determined by a number of factors that are difficult to evaluate in practice. In general, extremely small values of Δ can harm the effectiveness of the time change. For instance, if Δ is too small, then from the definition of the time change, $[\tau_{i-1}, \tau_i]$ often becomes the same as the observation interval of the data, and therefore the time changed data will have similar property as the original data. On the other hand, if Δ is too large, the number of samples will be too small, and this might affect the finite sample property of the estimator.

In this paper, we use the Cramer-von Mises (or CvM) distance for the standardized returns $dP_t = dp_t/p_t - r_t^f dt$ over the random interval $[T_{(i-1)\Delta}, T_{i\Delta}]$ divided by $\sqrt{\Delta}$ in order to determine the smallest possible value of Δ. The CvM distance can show how far the empirical distribution is departed from $N(0,1)$. If it is closer to 0, then the time change based on Δ works effectively, while if it is far from 0, then the Δ is supposedly too small. Figure 2 plots the CvM distance for each k with $\Delta = [P]_T k/K$. Note that k is the number of days to be considered to calculate the average quadratic variation and K is the total number of days in the dataset. The CvM measure drastically decreases as k increases from 0 (or daily frequency without time change) to 20 and then it stabilizes around the level of 0.1 - 0.3. This implies that the time change will work effectively if k is greater or equal to 20. Therefore, we use $k = 20$ as the volatility length which is used to generate a set of normal samples for the pricing errors.

4.3 Estimates

4.3.1 Baseline Case: Financial Wealth Only

Table 1 presents estimation results of three configurations for the recursive utility models; power utility (Model I), stochastic differential utility (Model II), and the stochastic differential utility with ambiguity aversion (Model III). In all three settings, it is assumed that financial wealth proxies the total wealth, i.e. $\pi = 1$ is imposed. As mentioned earlier, financial wealth is only a subset of the aggregate wealth and we may miss important interactions between human wealth and asset returns. However, to better understand the importance of excluding human wealth, we believe that it is important to compare the results across different specifications. In this light, we set this as our baseline case. The results from Model II are comparable to Epstein and Zin (1991), Baskshi and Naka (1997), and Normandin and St-Amour (1998). To the best of our knowledge, Model III which follows Chen and Epstein (2002) has not been empirically studied. Model I is used to verify if the equity premium puzzle arises in our setting and data set.

In estimating the models, we assume that the correlation between consumption growth and the market return, ρ_c is 0.2. We obtained this value by computing the sample correlation between the two variables and it is consistent with the existing studies.

Model I results state that there does exist the famous 'equity premium puzzle' showing roughly 258 as the relative risk aversion (RRA). Increasing the correlation coefficient to a counter-factual value of 1 still generates a high estimate of RRA around 52, confirming that the main reason for the puzzle is the smooth consumption growth. In this case, the elasticity of intertemporal substitution (EIS) is given by a reciprocal of the risk aversion, and it is estimated to be close to 0. This, in fact, is also consistent with the existing studies estimating the EIS such as Hall (1988) or Yogo (2004). In most cases, they use a linearized

Euler equation with consumption growth and asset returns especially, Treasury bills. In case of non-linear Euler equation, there are numerous studies such as Hansen and Singleton (1983). Interesting enough, both lines of literature often have conflicting results in terms of the EIS estimate with interest rate data. It is often ascribed to weak instrument problem, which reveals difficulty of identifying preference parameters. We now examine how this result is affected by alternative specifications of preferences.

With the stochastic differential utility, the estimates of the two parameters α and β in Model II are -3.6 and 0 respectively. This means that the estimated risk aversion $(1-\alpha)$ is dramatically decreasing to 4.6, while the estimated EIS $(1/\beta)$ is reaching a very high number. Although high values of EIS have no problem in explaining the behaviors of stock returns or risk-free rates, high standard errors of β estimates hint that it requires further investigation. In addition, β being zero implies that asset returns have no link to consumption growth, which is somewhat puzzling.

First of all, notice that the estimated EIS is measured by the reciprocal of β. Thus, a small perturbation of β coefficient can lead to a large swing of the EIS. Say $\beta = 0.1$ implies the EIS of 10, while $\beta = 0.8$ means 1.25. That is, if β is in the range of say $[0.1, 4]$, then the resulting EIS can be in the $[0.25, 10]$, which is much widely spread. Of course, this wide range of EIS esimates is not new in the related literature of estimating this preference parameter in both linearized and non-linear Euler equations. Several authors reported the EIS estimates between some negative numbers and large positive numbers. Our setup helps understand the difficulty in identifying the EIS parameter. Although Model II is an extension of Model I by adding the conditional return variance, the two models have very different implications for linking asset returns to conditional covariances of consumption growth and returns. In case of Model I, small volatility of consumption growth without an additional explanatory variable implies a large coefficient (i.e. a small EIS) to match the market risk premium. Meanwhile, Model II has an additional variable with non-linear parametric restrictions which can potentially yield a wide range of the EIS values even with small changes in the original coefficient.

Furthermore, between two explanatory variables, consumption growth and return from the aggregate wealth, the latter is likely to be more fluctuating, unless human wealth or labor income wealth significantly negatively correlated with financial wealth such that the resultant aggregate wealth is much less volatile. Assuming that both variables have similar degrees of correlations with asset returns, it is likely that there exists some statistical tension for estimating two parameters. Given that the EIS is related more closely to shifting consumptions across periods without uncertainty, its identification can be difficult. That is, all these problems lead to a weak identification problem of the EIS parameter.

To further analyze this issue, we draw both the surfaces and contours of the CvM measures in alternative forms of preferences. Figure 3 shows that estimating the risk aversion appears to be easy, while the elasticity of intertemporal substitution is not. The left panel in Figure 4 corroborates our conjecture. One clear pattern is that the reciprocal of the EIS is small and close to one.

Next we incorporate the ambiguity aversion to Model II. The result suggests that the RRA drops from 4.6 to 1.4, the EIS is estimated around 2, and the ambiguity aversion parameter (κ) is estimated around 0.36. κ measures how much the representative household distorts her beliefs to a worst case scenario given the ignorance of the true conditional probability distribution. Recall that the conventional notion of market price of risk measures the degree to which an investor will adjust her probability to be risk neutral. Thus, κ quantifies a constant adjustment of probability in order to be neutral against a Knightian sense of uncertainty.

Although it is true that a more sophisticated model of ambiguity aversion such as time-varying ambiguity aversion would further clarify the nature of this new source of premium, our empirical results state that modeling uncertainty differentiated from the usual sense of risk is an important, first-order business to understand the behaviors of asset returns. Given that, the lower RRA estimates in Model III is understandable because ambiguity aversion captured by conditional volatility in our setup is likely to alleviate the burden of the return variance in accounting for the average return behaviors.

Related but not expected, it appears that ambiguity aversion helps identify the EIS as well. Admittedly, it is still a noisy measurement. But the estimated EIS is little lower than 2, which is consistent with the recent empirical literature focusing on equity returns (Bansal, Kiku, and Yaron (2007), Kim et. al. (2008)). We conjecture that the inclusion of ambiguity aversion provides the other two explanatory variables, consumption growth and the rate of return from wealth, with fair chances of explaining asset returns by correctly specifying the existence of ambiguity aversion such that the contribution of the aggregate wealth return is evaluated with an upward bias.

It should be also noted that both Models II and III results show that agents prefer an early resolution of uncertainty whether or not there exists a static sense of uncertainty aversion. This makes economic agents unhappy about fluctuations in future utilities, often called the long-run risk channel. For more details on the mechanisms, see Bansal and Yaron (2004), Hansen, Heaton and Li (2008), and Kim et. al. (2008).

4.3.2 Human Wealth

Now we state our main results from our continuous-time recursive utility model with human wealth, (11). This involves fixing two more parameters π and ρ_y, the fraction of financial wealth, and the correlation between labor income growth and the market return. For the former, we tried two values (1/3 and 2/3). Our robustness checks reveal that different values give similar results to either of our chosen values.[18] Regarding the value of ρ_y, there is little consensus about it. Several empirical studies report that this correlation is positive, while other studies based on structural models such as Lustig and Van Nieuwerburgh (2006), and Chen et. al. (2008) report a strong negative correlation such as -0.7. According to our computation it was 0.03. Thus, we tried different values such as $0.03 and - 0.03$, and the results are reported in Table 2.

Major difference of the Table 2 in comparison with Table 1 is that the risk aversion coefficient increased. In case of Model II counterparts, the RRA increases from 4.6 up to 14. With ambiguity aversion, the RRA increases from 1.4 up to 5.2. The EIS estimates

[18] We also tried estimating this parameter directly and the estimated values are around 0.2 0.3 in some cases. But due to the weak identification problem, its identification is heavily affected by alternative model settings.

increase as well. With ambiguity aversion and $\rho_y = -0.03$, the estimated EIS is 7.24 but with high standard error or 21. When the fraction of financial wealth is 2/3, the estimated EIS is around 2 with standard errors of 2. Interestingly, when we impose a strong negative number as used in the papers mentioned above, we have somewhat lower EIS around 1.2, which is consistent with the literature. However, in all of the settings we have tried, the point estimates of the EIS is higher than one, meaning that economic agents will change their consumptions rather elastically when real interest rate changes.

Lastly, the estimates of ambiguity aversion rarely vary across settings and the estimates of κ is around 0.36.

In a summary, the recursive utility models with both financial and human wealth give most reasonable results when ambiguity aversion is included and the estimates of ambiguity aversion do not depend on alternative setting. Although the estimates of the risk aversion increase, those are still in an acceptable range of values. The weak identification problem of the EIS is also a prevalent feature across different model specifications.

5 Conclusion

Our paper began with a title asking if there is an important role played by decision makers' concern with ambiguity on true probability measure. Our answer to this question is positive from both economic and econometric perspectives. In terms of economic theory, the inclusion of ambiguity aversion can overcome the Ellsberg paradox. In addition, one can view that a multiple-priors utility as an extension of the rational expectation in that investors may be of insufficient knowledge about the true probability density. When ambiguity aversion is assumed, economic agents are basically endowed with a set of beliefs on the true probability distribution and choose the one that is the least ambiguous. Our estimation results strongly suggest that this is indeed the case. Even with various specifications, the preference parameter indicating the ambiguity aversion is both economically and statistically significant. Another interesting finding is that the models with ambiguity aversion have lower relative risk aversion. With regard to the elasticity of substitution, there exists a weak identification problem due to its non-linear parametric restrictions and the weak signal from consumption growth. That said, the models with ambiguity aversion still produce quite reasonable estimates of the intertemporal substitution. Therefore, ambiguity aversion not only matters in terms of explaining the behaviors of asset returns, but also helping identify key preference parameters.

In addition to the empirical findings, another contribution of our paper is that we provide a novel econometric approach estimating and testing for continuous-time asset pricing models including both financial and macroeconomic variables. In the empirical analysis of such models, it has long been a tradition that we ignore the availability of high-frequency observations on financial variables, mostly for the lack of ideas about how to use them constructively. Virtually all empirical studies of such models have been done only using lower-frequencies, at which all involved macroeconomic variables are also available. Our paper makes it clear that this is an important loss of information. In our analysis, we use the available high-frequency observations directly to identify our model, and also nonparametrically correct for time-varying stochastic volatility in the price equation errors. It is widely known that many asset returns show strong evidence for the presence of time-varying stochastic volatility. Unless properly and carefully taken care of, the time-varying stochastic volatility may well have a fatal effect on our estimation results. We believe that our method can be used in many other interesting applications, to unravel the complicated interactions between financial markets and macroeconomy.

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Table 1: Estimation Results for Baseline Models

	Model I		Model II		Model III	
β	258.161	(72.734)	0.000	(0.418)	0.507	(0.395)
α	-	-	-3.572	(1.386)	-0.445	(0.736)
κ	-	-	-	-	0.359	(0.134)
RA	258.161	(72.734)	4.572	(1.386)	1.445	(0.736)
EIS	0.004	(0.001)	∞	-	1.973	(1.540)
CvM	0.035		0.034		0.031	

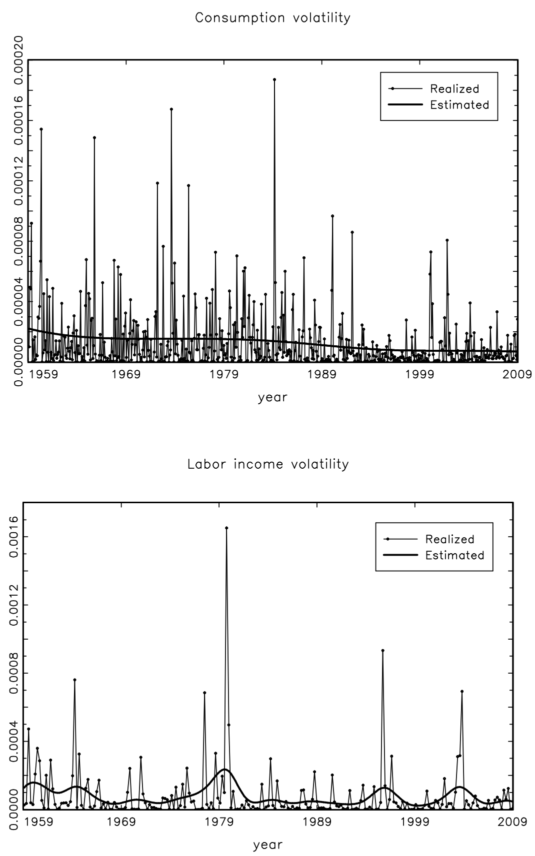
Note: The table reports the estimation results for the asset pricing models in which the aggregate wealth consists of only financial wealth. All results are for the sample 1/2/1960-12/29/2006. The first column is Model I with standard additive CRRA utility, the second column is Model II with recursive utility, and the third column is Model III with multiple-priors recursive utility. The correlation between the market return and the consumption growth (ρ_c) is set to be 0.2. The standard errors in parenthesis are obtained by the subsampling method.

Table 2: Implications of Human Wealth

	Without Ambiguity				With Ambiguity			
	Calibrated Values							
π	0.333	0.333	0.667	0.667	0.333	0.333	0.667	0.667
ρ_y	0.030	-0.030	0.030	-0.030	0.030	-0.030	0.030	-0.030
	Estimated Values							
β	0.000	0.000	0.000	0.000	0.100	0.138	0.245	0.435
	(0.428)	(0.425)	(0.436)	(0.437)	(0.412)	(0.412)	(0.420)	(0.422)
α	-12.629	-12.831	-5.842	-5.866	-4.234	-4.069	-1.397	-1.048
	(4.136)	(4.193)	(2.077)	(2.085)	(2.183)	(2.016)	(1.227)	(1.137)
κ	-	-	-	-	0.358	0.361	0.359	0.360
	-	-	-	-	(0.136)	(0.141)	(0.136)	(0.138)
RA	13.629	13.831	6.842	6.866	5.234	5.069	2.397	2.048
	(4.136)	(4.193)	(2.077)	(2.085)	(2.183)	(2.016)	(1.227)	(1.137)
EIS	∞	∞	∞	∞	10.003	7.237	4.075	2.301
	-	-	-	-	(41.206)	(21.601)	(6.977)	(2.233)
CvM	0.034	0.034	0.034	0.034	0.031	0.031	0.031	0.031

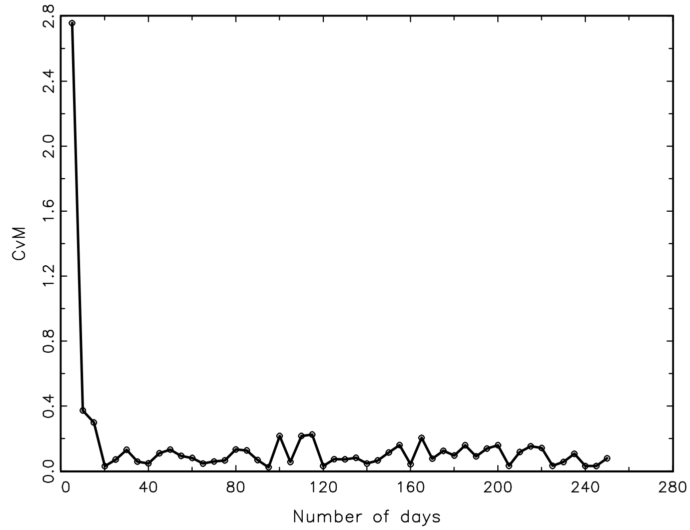
Note: The table reports the estimation results for the asset pricing models in which the aggregate wealth consists of financial wealth and human wealth. All results are for the sample 1/2/1960-12/29/2006. In each panel, each column represents the point estimates and their standard errors for the recursive utility model given the proportion of financial wealth to the aggregate wealth (π) and the correlation between the return on human wealth and financial wealth (ρ_y). The correlation between the market return and the consumption growth (ρ_c) is set to be 0.2. The standard errors in parenthesis are obtained by the subsampling method.

Figure 1: Extracted Volatilities of Macroeconomic Variables



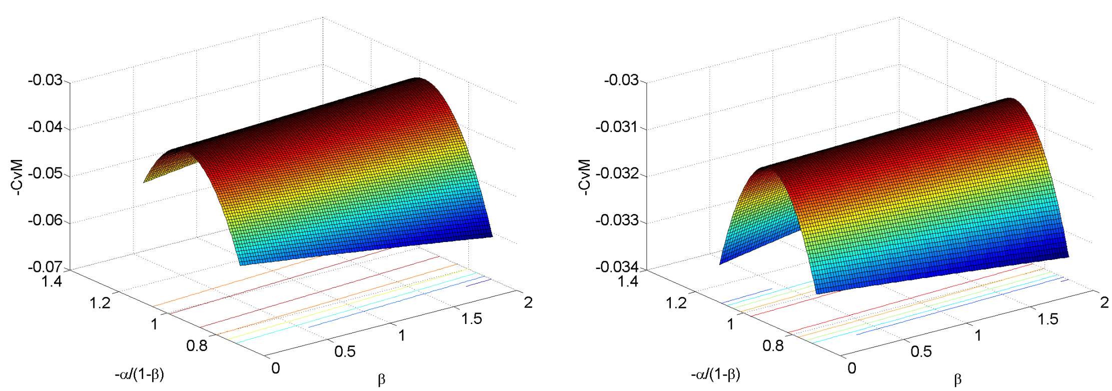
Note: The consumption and labor income growth are spanned from 1960 to 2006. The smoothing parameter of the local linear kernel estimation is based on the least squares cross-validation (see Li and Racine (2007, p. 83)).

Figure 2: Signature Plot of CvM Measure for S&P 500 Index Returns



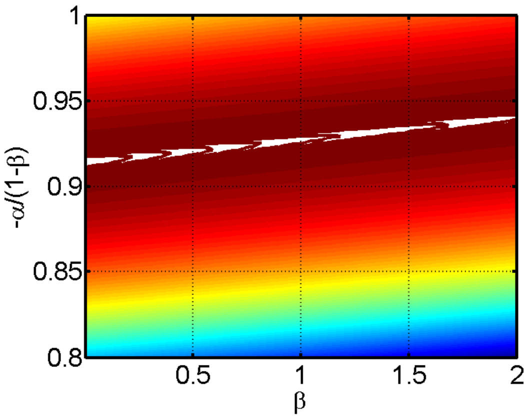
Note: The x-axis represents the number of days k included to calculate the volatility length Δ, i.e., $\Delta = [P]_T^\delta k/K$, where $[P]_t^\delta$ is the realized variance of (P_t), $dP_t = dp_t/p_t - r_t^f dt$, computed using daily observations over the time horizon $[0, T]$, and K is the total number of days. The y-axis represents the CvM distance for the standardized excess returns after time change.

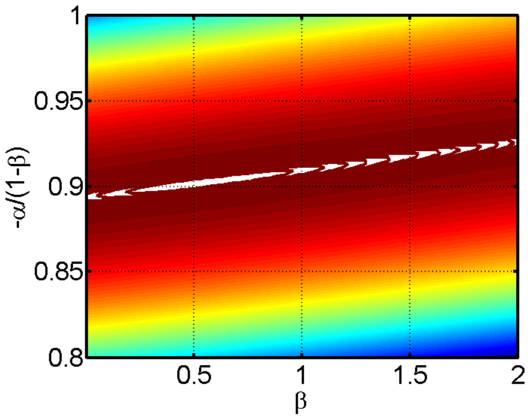
Figure 3: Surface Plots of CvM Measure for Model II and Model III



Note: The CvM measure is calculated for given parameter values of $(-\alpha/(1-\beta), \beta)$. In case of Model III, the surface plot is obtained with $\kappa = 0.359$.

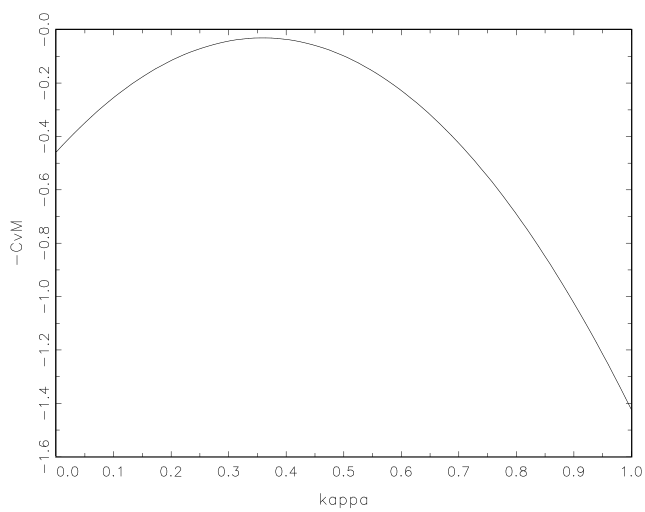
Figure 4: Contour Plots of CvM Measure for Model II and Model III





Note: The CvM measure is calculated for given parameter values of $(-\alpha/(1-\beta), \beta)$. In case of Model III, the contour plot is obtained with $\kappa = 0.359$.

Figure 5: Value Plot of CvM Measure for Model III



Note: The value plot of κ is obtained with $(\alpha, \beta) = (-0.445, 0.507)$.